Event-Triggered Distributed State Estimation with Randomly Occurring Uncertainties and Nonlinearities over Sensor Networks: A Delay-Fractioning Approach

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Abstract

This paper is concerned with the problem of event-triggered distributed state estimation for a class of discrete nonlinear stochastic systems with time-varying delays, randomly occurring uncertainties and randomly occurring nonlinearities. Both the uncertainties and nonlinearities enter into the system in a random way characterized by random variables obeying the Bernoulli distribution. An event-triggered scheme is introduced to reduce the number of excessive executions of the signal transmissions. The aim of this paper is to design a distributed state estimator such that the estimation error dynamics is asymptotically mean-square stable. By constructing a Lyapunov-Krasovskii functional and employing the delay-fractioning approach, sufficient conditions are established to guarantee the desired performance requirements and then the explicit form of the distributed estimator gains is parameterized. An illustrative example is finally provided to demonstrate the effectiveness of the developed distributed state estimation scheme with the event-triggered communication mechanism.

Keywords

Sensor networks, distributed state estimation, randomly occurring uncertainties, randomly occurring nonlinearities, time-varying delay.

I. Introduction

A sensor network is composed of a group of sensor nodes equipped with the communication infrastructure monitoring and collecting information at diverse locations [26]. During the past decade, the sensor networks have received an increasing research interest due to their successful applications in a variety of domains such as industrial automation, traffic and environmental monitoring, medical device monitoring, and wireless networks etc [6, 24, 31]. In terms of theoretical research, considerable effort has been devoted to the synchronization,

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estimation and filtering problems over sensor networks [7, 32, 34]. To be specific, the design of the centralized time-varying Kalman filtering has been studied in [30] over a wireless sensor network with correlated fading channels, where the stochastic stability of the developed filtering algorithm has been discussed. A stochastic sampled-data approach has been proposed in [35] to design the distributed H_∞ filtering in sensor networks, and a sufficient condition has been given to ensure the exponential mean-square stability of the resulting filtering error dynamics as well as the H_∞ performance requirement. In [38–40], the distributed synchronization problems have been addressed for dynamical networks with nonlinear disturbances and stochastic coupling.

As is well known, time delays are inevitable in various practical engineering systems including networked control systems and process control systems [9, 10, 15, 41]. The occurrence of time delays causes lasting impact on the desirable system performance and may even yield the instability of the controlled systems [17, 19, 44]. So far, a number of approaches have been developed to deal with the analysis and synthesis problems for time-delay systems. It should be pointed out that, so far, much effort has been made to design the state estimators for time-delay systems over sensor networks [18, 20, 21]. To mention a few, in [18], the Gaussian mixture Kalman particle filtering algorithm has been developed to cope with the Gaussian or non-Gaussian nature of the random network delays. The distributed state estimation problems have been investigated in [20, 21] for discrete nonlinear systems with time-delays and nonlinear disturbances.

On the other hand, note that the measured signals collected by the sensors are commonly transmitted over a shared communication channel in the networked systems [33, 43]. Since a network is typically of limited communication capacity, it would be significant yet challenging to avoid unnecessary waste of the communication and computation resources. In this regard, the event-triggering mechanism has recently gained particular research focus because of its capability of decreasing the executions of the signal transmissions while maintaining the admissible performance [8, 16, 22, 29, 37]. To be specific, the event-triggered filtering and fault detection problems have been investigated in [16,22] for networked systems with communication delays. In [37], the optimal event-triggered approach to state estimation has been developed for linear discrete time systems where the multiple sensors provide the measurement updates according to the individual event-triggering conditions. It is worth mentioning that, however, little research attention has been paid on the distributed state estimation problem for time-delay systems with the event-triggered communication mechanism, and this constitutes one of the motivations of this paper.

Nonlinearities and uncertainties are arguably two of the most important kinds of complexities that have received considerable research attention in the past few decades [1, 4, 14, 23]. With the rapid development of the network technologies, it has now well recognized that both the nonlinearities and uncertainties may occur in a probabilistic way with certain types/intensity due primarily to the random changes of the network circumstances. As such, the randomly occurring uncertainties (ROUs) and the randomly occurring nonlinearities (RONs) should be properly taken care of when designing the control systems. In [45], sufficient conditions have been established to guarantee the asymptotic synchronization in the mean-square sense for discrete stochastic complex networks subject to RONs, multiple stochastic disturbances and mixed time delays. Very recently, in [11], a sliding mode control scheme has been developed for a class of discrete networked nonlinear systems with both ROUs and RONs. Up to now, despite its practical significance, the event-based distributed state estimation problem has not yet been tackled for discrete stochastic systems in the simultaneous presence of

ROUs and RONs, not to mention the case where the communication delays are also involved. It is, therefore, the purpose of this paper to shorten such a gap by initiating a study on the distributed state estimation problem with both ROUs and RONs by using the delay-fractioning approach [12, 28] with hope to reduce the possible conservatism.

Motivated by the above discussions, in this paper, we aim to discuss the distributed state estimation problem for a class of discrete nonlinear stochastic systems with ROUs, RONs and time-varying delay over sensor networks. By constructing a Lyapunov-Krasovskii functional based on the delay-fractioning approach, sufficient conditions are given such that the resulting estimation error systems are asymptotically stable in the mean-square sense. Moreover, the derived explicit form of the distributed estimator gains can be easily solved by using the semi-definite programme approach. Finally, an illustrative example is provided to show the usefulness of the distributed estimation scheme. The main contributions of this paper lie in the following two aspects: (i) the delay-fractioning approach is, for the first time, introduced to deal with the event-based distributed state estimation problem; and (ii) intensive stochastic analysis is conducted to handle the ROUs, RONs and time-varying delay in a unified framework under the event-triggered communication mechanism.

The rest of this paper is organized as follows. In Section II, the problem addressed is formulated and some preliminaries are briefly introduced. By constructing the Lyapunov-Krasovskii functional based on the delay-fractioning approach, sufficient conditions are presented in Section III to ensure the asymptotical mean-square stability of the estimation error systems. Subsequently, the explicit form of the distributed estimator gain are derived that can be easily solved by using the semi-definite programme method. In Section IV, a numerical example is provided to illustrate the feasibility and effectiveness of the proposed estimation method. Finally, conclusions are drawn in Section V.

Notations. The notations used throughout the paper are standard except where otherwise stated. The superscript "T" stands for matrix transposition. $\mathbb{R}^n$ ($\mathbb{R}^{n\times m}$) denote the n-dimensional Euclidean space and the set of all $n\times m$ matrices, respectively. The notation $P>0$ ($P\geq 0$) means that matrix P is real symmetric and positive definite (positive semi-definite). $(\Omega, \mathcal{F}, \mathrm{Prob})$ is a probability space, where Prob, the probability measure, has a total mass 1. $\mathbb{E}\{x\}$ stands for the expectation of x. $\otimes$ represents the Kronecker product. 1_N is the N-dimensional vector with all elements being 1. I and 0 denote the identity matrix and a zero matrix with appropriate dimensions, respectively. $\mathrm{diag}\{X_1, X_2, \cdots, X_n\}$ stands for a block-diagonal matrix with matrices $X_1, X_2\cdots, X_n$ on the diagonal. $\|\cdot\|$ denotes the Euclidean norm of a vector or its induced norm of a matrix. In symmetric block matrices or long matrix expressions, we use a star ($*$) to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. Problem Formulation and Preliminaries

Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{L})$ be a directed graph of order N with the set of nodes $\mathcal{V}=\{1,2,\ldots,N\}$, the set of edges $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$, and the weighted adjacency matrix $\mathcal{L}=[l_{ij}]$ with nonnegative adjacency element l_{ij}. Here, the matrix $\mathcal{L}$ characterizes the interconnection topology of the nodes and the ordered pair (i,j) denotes an edge of $\mathcal{G}$. The adjacency elements associated with the edges of the graph are positive, i.e., $l_{ij}>0 \Longleftrightarrow (i,j)\in\mathcal{E}$ which represents that there exists the information transmission from sensor j to sensor i. Also, assume that $l_{ii}=1$

for all $i \in \mathcal{V}$. The set of the neighbors of node $i \in \mathcal{V}$ plus the node itself is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} | (i,j) \in \mathcal{E}\}$.

In this paper, we consider the following class of discrete nonlinear stochastic systems:

$$x_{k+1} = (A + \alpha_k \Delta A)x_k + A_d x_{k-d_k} + \beta_k B f(x_k) + D x_k \omega_k \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $f(x_k)$ is the known nonlinear function, and ω_k is a one-dimensional, zero-mean Gaussian white noise sequence on a probability space $(\Omega, \mathcal{F}, \text{Prob})$ with $\mathbb{E}\{\omega_k^2\} = 1$. The positive integer d_k describes the discrete time-varying delay satisfying $d_m \le d_k \le d_M$, where d_m and d_M are known positive integers representing the lower and upper bounds of d_k, respectively. The lower bound of delay d_m can always be described by $d_m = \tau m$ with τ and m being positive integers. $x_s = \varphi_s$ $(s = -d_M, -d_M + 1, \ldots, 0)$ is the initial condition. A, A_d, B and D are known real matrices with appropriate dimensions.

The real-valued matrix ΔA stands for the norm-bounded parameter uncertainty:

$$\Delta A = HFM \tag{2}$$

where H and M are known real constant matrices, and $F \in \mathbb{R}^{n_1 \times n_2}$ is an unknown matrix satisfying $F^T F \le I$. The nonlinearity $f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ satisfies

$$\|f(x) - f(y)\| \le \|\Phi(x - y)\|, \quad \forall x, y \in \mathbb{R}^n \tag{3}$$

with Φ being a known matrix.

The random variables $\alpha_k \in \mathbb{R}$ and $\beta_k \in \mathbb{R}$ are Bernoulli distributed white noise sequences taking the values 0 or 1 by

$$\begin{aligned} \text{Prob}\{\alpha_k = 1\} &= \mathbb{E}\{\alpha_k\} = \bar{\alpha}, \ \ \text{Prob}\{\alpha_k = 0\} = 1 - \bar{\alpha} \\ \text{Prob}\{\beta_k = 1\} &= \mathbb{E}\{\beta_k\} = \bar{\beta}, \ \ \text{Prob}\{\beta_k = 0\} = 1 - \bar{\beta} \end{aligned} \tag{4}$$

with $\bar{\alpha} \in [0,1]$ and $\bar{\beta} \in [0,1]$ being known scalars. Here, we assume that α_k, β_k and ω_k are mutually independent.

Assume that the sensors are distributed in the space according to certain rules. The states of the target plant are to be estimated based on the measurement outputs collected by sensors. For the i-th sensor, the measurement output is

$$y_{k,i} = C_i x_k, \tag{5}$$

where $y_{k,i} \in \mathbb{R}^p$ is the output measured by sensor i from the target plant, C_i $(i = 1, 2, \ldots, N)$ are known real matrices with appropriate dimensions.

To reduce the data transmission frequency, the following event generator functions $\psi_i(\cdot,\cdot)$ $(i = 1, 2, \ldots, N)$ are introduced

$$\psi_i(\phi_{k,i}, \delta_i) = \phi_{k,i}^T \Omega_i \phi_{k,i} - \delta_i r_{k,i}^T \Omega_i r_{k,i} \tag{6}$$

where $r_{k,i} = y_{k,i} - C_i \hat{x}_{k,i}$ is the innovation sequence exchanged via the network with $\hat{x}_{k,i} \in \mathbb{R}^n$ being the state estimation of the target plant in the i-th sensor node, $\Omega_i > 0$, $\delta_i \in [0,1)$, and $\phi_{k,i} = r_{k',i} - r_{k,i}$ with $r_{k',i}$ being the broadcast innovation at the latest event time.

The execution is triggered if

$$\psi_i(\phi_{k,i}, \delta_i) > 0 \tag{7}$$

holds. Hence, the sequence of the event-triggered instants $0 = k_{0,i} \le k_{1,i} \le \cdots \le k_{k',i} \le \cdots$ is determined iteratively by

$$k_{k'+1,i} = \inf\{k \in \mathbb{N} | k > k_{k',i}, \ \ \psi_i(\phi_{k,i}, \delta_i) > 0\}, \tag{8}$$

where $k_{0,i} = 0$ is the initial event-triggered instant.

Remark 1: In the extreme case, when $\delta_i = 0$ $(i = 1, 2, \ldots, N)$, we can see that $\{k_{0,i}, k_{1,i}, k_{2,i}, \ldots\} = \{0, 1, 2, \ldots\}$. It means that all measurements are transmitted to the side of the state estimator at each sampling instant. Then, the addressed event-triggered state estimation problem reduces to the traditional one.

In this paper, we construct the following event-based distributed state estimator for sensor node i:

$$\hat{x}_{k+1,i} = A\hat{x}_{k,i} + \bar{\beta}Bf(\hat{x}_{k,i}) + \sum_{j\in\mathcal{N}_i} l_{ij}K_{ij}r_{k_{k'},j} \tag{9}$$

where $\hat{x}_{0,i} = 0$ and K_{ij} $(j \in \mathcal{N}_i)$ are the estimator gains on the sensor node i to be designed.

By letting $\tilde{x}_{k,i} = x_k - \hat{x}_{k,i}$, we have

$$\begin{aligned} \tilde{x}_{k+1,i} \ &= \ A\tilde{x}_{k,i} + \alpha_k\Delta Ax_k + A_dx_{k-d_k} + \bar{\beta}B\tilde{f}(\tilde{x}_{k,i}) \\ &\quad +(\beta_k - \bar{\beta})Bf(x_k) + Dx_k\omega_k - \sum_{j\in\mathcal{N}_i} l_{ij}K_{ij}(\phi_{k,j} + C_j\tilde{x}_{k,j}), \end{aligned} \tag{10}$$

where $\tilde{f}(\tilde{x}_{k,i}) = f(x_k) - f(\hat{x}_{k,i})$. For convenience of later developments, set

$$\begin{aligned} \tilde{x}_k &= \begin{bmatrix} \tilde{x}_{k,1}^T & \tilde{x}_{k,2}^T & \cdots & \tilde{x}_{k,N}^T \end{bmatrix}^T, \ \ \tilde{A} = I_N \otimes A, \ \ \Delta\tilde{A} = 1_N \otimes \Delta A, \\ \tilde{A}_d &= 1_N \otimes A_d, \ \ \tilde{B}_1 = I_N \otimes B, \ \ \tilde{B}_2 = 1_N \otimes B, \\ \tilde{C} &= \text{diag}\{C_1, C_2, \cdots, C_N\}, \ \ \tilde{D} = 1_N \otimes D, \\ \tilde{f}(\tilde{x}_k) &= \begin{bmatrix} \tilde{f}^T(\tilde{x}_{k,1}) & \tilde{f}^T(\tilde{x}_{k,2}) & \cdots & \tilde{f}^T(\tilde{x}_{k,N}) \end{bmatrix}^T, \\ \tilde{\phi}_k &= \begin{bmatrix} \phi_{k,1}^T & \phi_{k,2}^T & \cdots & \phi_{k,N}^T \end{bmatrix}^T, \end{aligned} \tag{11}$$

and

$$\tilde{K} = \begin{bmatrix} \tilde{K}_{ij} \end{bmatrix}_{N\times N} \ \text{ with } \ \tilde{K}_{ij} = \begin{cases} l_{ij}K_{ij}, & i = 1, 2, \ldots, N, \ \ j \in \mathcal{N}_i \\ 0, & i = 1, 2, \ldots, N, \ \ j \notin \mathcal{N}_i \end{cases} \tag{12}$$

Noting that $l_{ij} = 0$ if $j \notin \mathcal{N}_i$, it is easy to see that $\tilde{K}$ is a sparse matrix in $\mathcal{W}_{n\times p}$ with

$$\mathcal{W}_{n\times p} = \left\{\tilde{U} = [U_{ij}] \in \mathbb{R}^{Nn\times Np} | U_{ij} \in \mathbb{R}^{n\times p}, U_{ij} = 0 \ \text{ if } \ j \notin \mathcal{N}_i\right\}. \tag{13}$$

Then, (10) can be rewritten by

$$\begin{aligned} \tilde{x}_{k+1} \ &= \ (\tilde{A} - \tilde{K}\tilde{C})\tilde{x}_k + \alpha_k\Delta\tilde{A}x_k + \tilde{A}_dx_{k-d_k} + \bar{\beta}\tilde{B}_1\tilde{f}(\tilde{x}_k) \\ &\quad +(\beta_k - \bar{\beta})\tilde{B}_2f(x_k) + \tilde{D}x_k\omega_k - \tilde{K}\tilde{\phi}_k. \end{aligned} \tag{14}$$

It follows from (3) and the definition of $\tilde{f}(\tilde{x}_k)$ that

$$\|\tilde{f}(\tilde{x}_k)\| \leq \|\tilde{\Phi}\tilde{x}_k\| \tag{15}$$

with $\tilde{\Phi} = I_N \otimes \Phi$.

By denoting $\eta_k = \begin{bmatrix} x_k^T & \tilde{x}_k^T \end{bmatrix}^T$, we have

$$\begin{aligned} \eta_{k+1} &= (\mathcal{A} + \bar{\alpha}\Delta\mathcal{A})\eta_k + (\alpha_k - \bar{\alpha})\Delta\mathcal{A}\eta_k + \mathcal{A}_d\eta_{k-d_k} + \bar{\beta}\mathcal{B}_1 F(\eta_k) \\ &\quad +(\beta_k - \bar{\beta})\mathcal{B}_2 F(\eta_k) + \mathcal{D}\eta_k\omega_k - \bar{K}\tilde{\phi}_k, \end{aligned} \tag{16}$$

with

$$\begin{aligned} \mathcal{A} &= \begin{bmatrix} A & 0 \\ 0 & \tilde{A} - \tilde{K}\tilde{C} \end{bmatrix}, \ \Delta\mathcal{A} = \begin{bmatrix} \Delta A & 0 \\ \Delta\tilde{A} & 0 \end{bmatrix}, \ \mathcal{A}_d = \begin{bmatrix} A_d & 0 \\ \tilde{A}_d & 0 \end{bmatrix}, \\ \mathcal{B}_1 &= \begin{bmatrix} B & 0 \\ 0 & \tilde{B}_1 \end{bmatrix}, \ \mathcal{B}_2 = \begin{bmatrix} B & 0 \\ \tilde{B}_2 & 0 \end{bmatrix}, \ F(\eta_k) = \begin{bmatrix} f(x_k) \\ \tilde{f}(\tilde{x}_k) \end{bmatrix}, \\ \mathcal{D} &= \begin{bmatrix} D & 0 \\ \tilde{D} & 0 \end{bmatrix}, \ \bar{K} = \begin{bmatrix} 0 \\ \tilde{K} \end{bmatrix}. \end{aligned} \tag{17}$$

It is not difficult to test that

$$\|F(\eta_k)\| \leq \|\bar{\Phi}\eta_k\| \tag{18}$$

where $\bar{\Phi} = \mathrm{diag}\{\Phi, \tilde{\Phi}\}$ and $\tilde{\Phi}$ is defined in (15).

To proceed, we introduce the following lemmas that will be used for further developments.

Lemma 1: [5] Given constant matrices $\mathcal{S}_1$, $\mathcal{S}_2$ and $\mathcal{S}_3$ where $\mathcal{S}_1 = \mathcal{S}_1^T$ and $\mathcal{S}_2 = \mathcal{S}_2^T > 0$. Then $\mathcal{S}_1 + \mathcal{S}_3^T\mathcal{S}_2^{-1}\mathcal{S}_3 < 0$ if and only if

$$\begin{bmatrix} \mathcal{S}_1 & \mathcal{S}_3^T \\ * & -\mathcal{S}_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\mathcal{S}_2 & \mathcal{S}_3 \\ * & \mathcal{S}_1 \end{bmatrix} < 0.$$

Lemma 2: [42] Let $Q = Q^T$, N and H be real matrices of appropriate dimensions. Then, for any F satisfying $F^TF \leq I$, $Q + NFH + H^TF^TN^T < 0$ if and only if there exists a scalar $\varepsilon > 0$ such that $Q + \varepsilon^{-1}NN^T + \varepsilon H^TH < 0$, or equivalently,

$$\begin{bmatrix} Q & N & \varepsilon H^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0.$$

Lemma 3: [34] Let $P = \mathrm{diag}\{P_1, P_2, \ldots, P_N\}$ with $P_i \in \mathbb{R}^{n\times n}$ $(i = 1, 2, \ldots, N)$ being invertible matrices. If $X = PU$ for $U \in \mathbb{R}^{nN\times pN}$, then we have $U \in \mathcal{W}_{n\times p} \Longleftrightarrow X \in \mathcal{W}_{n\times p}$.

The purpose of this paper is to design the event-based distributed state estimator of form (9) on each sensor node i such that the resulting estimation error system (16) is mean-square asymptotically stable irrespective of the time-varying delays, ROUs and RONs. To be specific, by using the delay-fractioning approach, we aim to design the estimator gains K_{ij} so as to ensure the asymptotical stability of the estimation error system in the mean-square sense.

III. Main Results

In this section, by constructing a Lyapunov-Krasovskii functional and using the delay-fractioning approach, a sufficient condition is established to ensure the mean-square asymptotical stability of the estimation error system. Subsequently, the explicit form of the distributed state estimator gains is provided.

Theorem 1: For given $\bar{K}$ and a scalar $\gamma \in (0,1)$, assume that there exist matrices $\mathcal{P} = \mathrm{diag}\{\mathcal{P}_1, \mathcal{P}_2, \cdots, \mathcal{P}_{N+1}\}$ with $\mathcal{P}_i > 0$ $(i = 1, 2, \cdots, N+1)$, $\mathcal{Q} > 0$, $\mathcal{R} > 0$, $\mathcal{S} > 0$, $\mathcal{X}_1 \geq 0$, $\mathcal{X}_2 \geq 0$, matrices $\mathcal{Y}_i$ $(i = 1, 2, 3)$, and scalars $\lambda_1 > 0$, $\lambda_2 > 0$ satisfying

$$\Pi_1 + \Pi_2 + \Pi_2^T + \Pi_3 < 0, \tag{19}$$

$$\Lambda_1 = \begin{bmatrix} \mathcal{X}_1 & \mathcal{Y}_1 \\ * & \gamma\mathcal{P} \end{bmatrix} \geq 0, \quad \Lambda_2 = \begin{bmatrix} \mathcal{X}_2 & \mathcal{Y}_2 \\ * & \gamma\mathcal{P} \end{bmatrix} \geq 0, \quad \Lambda_3 = \begin{bmatrix} \mathcal{X}_2 & \mathcal{Y}_3 \\ * & \gamma\mathcal{P} \end{bmatrix} \geq 0, \tag{20}$$

where

$$\begin{aligned}
\Pi_1 &= \tau\mathcal{X}_1 + (d_M - d_m)\mathcal{X}_2, \\
\Pi_2 &= \begin{bmatrix} \mathcal{Y}_1 & \mathcal{Y}_2 & \mathcal{Y}_3 \end{bmatrix} \\
&\quad \times \begin{bmatrix} I_{(N+1)n\times(N+1)n} \quad -I_{(N+1)n\times(N+1)n} \quad 0_{(N+1)n\times[(N+1)(m+2)n+Np]} \\ \hline 0_{(N+1)n\times(N+1)nm} \quad I_{(N+1)n\times(N+1)n} \quad -I_{(N+1)n\times(N+1)n} \quad 0_{(N+1)n\times[2(N+1)n+Np]} \\ \hline 0_{(N+1)n\times(N+1)(m+1)n} \quad I_{(N+1)n\times(N+1)n} \quad -I_{(N+1)n\times(N+1)n} \quad 0_{(N+1)n\times[(N+1)n+Np]} \end{bmatrix}, \\
\Pi_3 &= (1+2\gamma\hbar)\Xi_1^T\mathcal{P}\Xi_1 + \tilde{\alpha}(1+2\gamma\hbar)\Xi_2^T\mathcal{P}\Xi_2 + \tilde{\beta}(1+2\gamma\hbar)\Xi_5^T\mathcal{B}_2^T\mathcal{P}\mathcal{B}_2\Xi_5 \\
&\quad + \Xi_3^T[(1+2\gamma\hbar)\mathcal{D}^T\mathcal{P}\mathcal{D} + (2\gamma\hbar - 1)\mathcal{P} + (d_M - d_m + 1)\mathcal{Q} + \mathcal{S} + \lambda_1\bar{\Phi}^T\bar{\Phi} + \lambda_2\mathcal{C}^T\Psi\Omega\mathcal{C}]\Xi_3 \\
&\quad - \Xi_4^T\mathcal{Q}\Xi_4 - \Xi_5^T\mathcal{S}\Xi_5 - \lambda_1\Xi_6^T\Xi_6 - \lambda_2\Xi_7^T\Omega\Xi_7 + \Xi_{\bar{\mathcal{R}}}^T\bar{\mathcal{R}}\Xi_{\bar{\mathcal{R}}}, \\
\Xi_1 &= \begin{bmatrix} \mathcal{A} + \bar{\alpha}\Delta\mathcal{A} & 0_{(N+1)n\times(N+1)mn} & \mathcal{A}_d & 0_{(N+1)n\times(N+1)n} & \bar{\beta}\mathcal{B}_1 & -\bar{K} \end{bmatrix}, \\
\Xi_2 &= \begin{bmatrix} \Delta\mathcal{A} & 0_{(N+1)n\times[(N+1)(m+3)n+Np]} \end{bmatrix}, \\
\Xi_3 &= \begin{bmatrix} I_{(N+1)n\times(N+1)n} & 0_{(N+1)n\times[(N+1)(m+3)n+Np]} \end{bmatrix}, \\
\Xi_4 &= \begin{bmatrix} 0_{(N+1)n\times(N+1)(m+1)n} & I_{(N+1)n\times(N+1)n} & 0_{(N+1)n\times[2(N+1)n+Np]} \end{bmatrix}, \\
\Xi_5 &= \begin{bmatrix} 0_{(N+1)n\times(N+1)(m+2)n} & I_{(N+1)n\times(N+1)n} & 0_{(N+1)n\times[(N+1)n+Np]} \end{bmatrix}, \\
\Xi_6 &= \begin{bmatrix} 0_{(N+1)n\times(N+1)(m+3)n} & I_{(N+1)n\times(N+1)n} & 0_{(N+1)n\times Np} \end{bmatrix}, \\
\Xi_7 &= \begin{bmatrix} 0_{Np\times(N+1)(m+4)n} & I_{Np\times Np} \end{bmatrix}, \\
\hbar &= d_M - \tau m + \tau, \quad \tilde{\alpha} = \bar{\alpha}(1-\bar{\alpha}), \quad \tilde{\beta} = \bar{\beta}(1-\bar{\beta}), \\
\Xi_{\bar{\mathcal{R}}} &= \begin{bmatrix} I_{(N+1)mn\times(N+1)mn} \quad 0_{(N+1)mn\times[4(N+1)n+Np]} \\ \hline 0_{(N+1)mn\times(N+1)n} \quad I_{(N+1)mn\times(N+1)mn} \quad 0_{(N+1)mn\times[3(N+1)n+Np]} \end{bmatrix}, \\
\bar{\mathcal{R}} &= \mathrm{diag}\{\mathcal{R}, -\mathcal{R}\}, \quad \Omega = \mathrm{diag}\{\Omega_1, \Omega_2, \cdots, \Omega_N\}, \\
\Psi &= \mathrm{diag}\{\delta_1 I, \delta_2 I, \cdots, \delta_N I\}, \quad \mathcal{C} = \begin{bmatrix} 0 & \tilde{C} \end{bmatrix}.
\end{aligned} \tag{21}$$

Then, the estimation error system (16) is mean-square asymptotically stable.

Proof: Based on the delay-fractioning idea, let us construct the following Lyapunov-Krasovskii functional for (16):

$$V(\eta_k) = \sum_{l=1}^{4} V_l(\eta_k) \tag{22}$$

where

$$\begin{aligned}
V_1(\eta_k) &= \eta_k^T \mathcal{P} \eta_k, \\
V_2(\eta_k) &= \sum_{l=k-d_k}^{k-1} \eta_l^T \mathcal{Q} \eta_l + \sum_{j=-d_M+1}^{-\tau m} \sum_{l=k+j}^{k-1} \eta_l^T \mathcal{Q} \eta_l, \\
V_3(\eta_k) &= \sum_{l=k-\tau}^{k-1} \Gamma_l^T \mathcal{R} \Gamma_l + \sum_{l=k-d_M}^{k-1} \eta_l^T \mathcal{S} \eta_l, \\
V_4(\eta_k) &= \sum_{j=-\tau+1}^{0} \sum_{l=k+j-1}^{k-1} \zeta_l^T \gamma \mathcal{P} \zeta_l + \sum_{j=-d_M+1}^{\tau m} \sum_{l=k+j-1}^{k-1} \zeta_l^T \gamma \mathcal{P} \zeta_l \\
\Gamma_l &= \begin{bmatrix} \eta_l^T & \eta_{l-\tau}^T & \cdots & \eta_{l-(m-1)\tau}^T \end{bmatrix}^T, \quad \zeta_l = \eta_{l+1} - \eta_l
\end{aligned}$$

with $\mathcal{P} > 0$, $\mathcal{Q} > 0$, $\mathcal{R} > 0$ and $\mathcal{S} > 0$ to be determined. Along the state trajectory of the system (16), one has

$$\begin{aligned}
\mathbb{E}\{\Delta V_1(\eta_k)\} &= \eta_k^T (\mathcal{A} + \bar{\alpha}\Delta\mathcal{A})^T \mathcal{P} (\mathcal{A} + \bar{\alpha}\Delta\mathcal{A}) \eta_k + 2\eta_k^T (\mathcal{A} + \bar{\alpha}\Delta\mathcal{A})^T \mathcal{P} \mathcal{A}_d \eta_{k-d_k} \\
&\quad + 2\bar{\beta}\eta_k^T (\mathcal{A} + \bar{\alpha}\Delta\mathcal{A})^T \mathcal{P} \mathcal{B}_1 F(\eta_k) - 2\eta_k^T (\mathcal{A} + \bar{\alpha}\Delta\mathcal{A})^T \mathcal{P} \bar{K} \tilde{\phi}_k \\
&\quad + \tilde{\alpha}\eta_k^T \Delta\mathcal{A}^T \mathcal{P} \Delta\mathcal{A} \eta_k + \eta_{k-d_k}^T \mathcal{A}_d^T \mathcal{P} \mathcal{A}_d \eta_{k-d_k} \\
&\quad + 2\bar{\beta}\eta_{k-d_k}^T \mathcal{A}_d^T \mathcal{P} \mathcal{B}_1 F(\eta_k) - 2\eta_{k-d_k}^T \mathcal{A}_d^T \mathcal{P} \bar{K} \tilde{\phi}_k \\
&\quad + \bar{\beta}^2 F^T(\eta_k) \mathcal{B}_1^T \mathcal{P} \mathcal{B}_1 F(\eta_k) - 2\bar{\beta} F^T(\eta_k) \mathcal{B}_1^T \mathcal{P} \bar{K} \tilde{\phi}_k \\
&\quad + \tilde{\beta} F^T(\eta_k) \mathcal{B}_2^T \mathcal{P} \mathcal{B}_2 F(\eta_k) + \eta_k^T \mathcal{D}^T \mathcal{P} \mathcal{D} \eta_k \\
&\quad + \tilde{\phi}_k^T \bar{K}^T \mathcal{P} \bar{K} \tilde{\phi}_k - \eta_k^T \mathcal{P} \eta_k,
\end{aligned} \tag{23}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are defined in (21). Similarly, through straightforward algebraic manipulations, we have

$$\begin{aligned}
\mathbb{E}\{\Delta V_2(\eta_k)\} &= \eta_k^T \mathcal{Q} \eta_k - \eta_{k-d_k}^T \mathcal{Q} \eta_{k-d_k} + \sum_{l=k+1-d_{k+1}}^{k-1} \eta_l^T \mathcal{Q} \eta_l - \sum_{l=k+1-d_k}^{k-1} \eta_l^T \mathcal{Q} \eta_l \\
&\quad + (d_M - \tau m) \eta_k^T \mathcal{Q} \eta_k - \sum_{l=k-d_M+1}^{k-\tau m} \eta_l^T \mathcal{Q} \eta_l \\
&\leq (d_M - \tau m + 1) \eta_k^T \mathcal{Q} \eta_k - \eta_{k-d_k}^T \mathcal{Q} \eta_{k-d_k},
\end{aligned} \tag{24}$$

$$\mathbb{E}\{\Delta V_3(\eta_k)\} = \Gamma_k^T \mathcal{R} \Gamma_k - \Gamma_{k-\tau}^T \mathcal{R} \Gamma_{k-\tau} + \eta_k^T \mathcal{S} \eta_k - \eta_{k-d_M}^T \mathcal{S} \eta_{k-d_M}, \tag{25}$$

$$\begin{aligned}
\mathbb{E}\{\Delta V_4(\eta_k)\} &= \sum_{j=-\tau+1}^{0}\Big(\sum_{l=k+j}^{k}\zeta_l^T\gamma\mathcal{P}\zeta_l-\sum_{l=k-1+j}^{k-1}\zeta_l^T\gamma\mathcal{P}\zeta_l\Big)\\
&\quad+\sum_{j=-d_M+1}^{\tau m}\Big(\sum_{l=k+j}^{k}\zeta_l^T\gamma\mathcal{P}\zeta_l-\sum_{l=k-1+j}^{k-1}\zeta_l^T\gamma\mathcal{P}\zeta_l\Big)\\
&= \hbar\zeta_k^T\gamma\mathcal{P}\zeta_k-\sum_{l=k-\tau}^{k-1}\zeta_l^T\gamma\mathcal{P}\zeta_l-\sum_{l=k-d_k}^{k-\tau m-1}\zeta_l^T\gamma\mathcal{P}\zeta_l-\sum_{l=k-d_M}^{k-d_k-1}\zeta_l^T\gamma\mathcal{P}\zeta_l\\
&\le 2\gamma\hbar\eta_{k+1}^T\mathcal{P}\eta_{k+1}+2\gamma\hbar\eta_k^T\mathcal{P}\eta_k-\sum_{l=k-\tau}^{k-1}\zeta_l^T\gamma\mathcal{P}\zeta_l\\
&\quad-\sum_{l=k-d_k}^{k-\tau m-1}\zeta_l^T\gamma\mathcal{P}\zeta_l-\sum_{l=k-d_M}^{k-d_k-1}\zeta_l^T\gamma\mathcal{P}\zeta_l, \qquad (26)
\end{aligned}$$

where $\hbar$ is defined in (21).

Subsequently, for any appropriately dimensioned matrices $\mathcal{X}_1 \ge 0$ and $\mathcal{X}_2 \ge 0$, we have

$$0 = \tau\xi_k^T\mathcal{X}_1\xi_k-\sum_{l=k-\tau}^{k-1}\xi_k^T\mathcal{X}_1\xi_k, \qquad (27)$$

$$0 = (d_M-d_m)\xi_k^T\mathcal{X}_2\xi_k-\sum_{l=k-d_k}^{k-d_m-1}\xi_k^T\mathcal{X}_2\xi_k-\sum_{l=k-d_M}^{k-d_k-1}\xi_k^T\mathcal{X}_2\xi_k, \qquad (28)$$

where $\xi_k = \begin{bmatrix}\Gamma_k^T & \eta_{k-d_m}^T & \eta_{k-d_k}^T & \eta_{k-d_M}^T & F^T(\eta_k) & \tilde{\phi}_k^T\end{bmatrix}^T$. Noting $\zeta_l = \eta_{l+1}-\eta_l$, for any matrices $\mathcal{Y}_i$ ($i = 1,2,3$), the following equations are true:

$$0 = 2\xi_k^T\mathcal{Y}_1\Big[\eta_k-\eta_{k-\tau}-\sum_{l=k-\tau}^{k-1}\zeta_l\Big], \qquad (29)$$

$$0 = 2\xi_k^T\mathcal{Y}_2\Big[\eta_{k-d_m}-\eta_{k-d_k}-\sum_{l=k-d_k}^{k-d_m-1}\zeta_l\Big], \qquad (30)$$

$$0 = 2\xi_k^T\mathcal{Y}_3\Big[\eta_{k-d_k}-\eta_{k-d_M}-\sum_{l=k-d_M}^{k-d_k-1}\zeta_l\Big]. \qquad (31)$$

In addition, according to the event-triggering condition (7), we obtain

$$\tilde{\phi}_k^T\Omega\tilde{\phi}_k-\eta_k^T\mathcal{C}^T\Psi\Omega\mathcal{C}\eta_k \le 0, \qquad (32)$$

where Ω, $\mathcal{C}$ and Ψ are defined in (21). Then, together with (18), (20), (23)-(32), for two scalars $\lambda_1 > 0$ and $\lambda_2 > 0$, we have

$$\begin{aligned}
\mathbb{E}\{\Delta V(\eta_k)\} \leq\; & \eta_k^T(\mathcal{A}+\bar{\alpha}\Delta\mathcal{A})^T\mathcal{P}(\mathcal{A}+\bar{\alpha}\Delta\mathcal{A})\eta_k + 2\eta_k^T(\mathcal{A}+\bar{\alpha}\Delta\mathcal{A})^T\mathcal{P}\mathcal{A}_d\eta_{k-d_k} \\
& +2\bar{\beta}\eta_k^T(\mathcal{A}+\bar{\alpha}\Delta\mathcal{A})^T\mathcal{P}\mathcal{B}_1F(\eta_k) - 2\eta_k^T(\mathcal{A}+\bar{\alpha}\Delta\mathcal{A})^T\mathcal{P}\bar{K}\tilde{\phi}_k \\
& +\tilde{\alpha}\eta_k^T\Delta\mathcal{A}^T\mathcal{P}\Delta\mathcal{A}\eta_k + \eta_{k-d_k}^T\mathcal{A}_d^T\mathcal{P}\mathcal{A}_d\eta_{k-d_k} \\
& +2\bar{\beta}\eta_{k-d_k}^T\mathcal{A}_d^T\mathcal{P}\mathcal{B}_1F(\eta_k) - 2\eta_{k-d_k}^T\mathcal{A}_d^T\mathcal{P}\bar{K}\tilde{\phi}_k - \eta_k^T\mathcal{P}\eta_k \\
& +\bar{\beta}^2F^T(\eta_k)\mathcal{B}_1^T\mathcal{P}\mathcal{B}_1F(\eta_k) - 2\bar{\beta}F^T(\eta_k)\mathcal{B}_1^T\mathcal{P}\bar{K}\tilde{\phi}_k \\
& +\tilde{\beta}F^T(\eta_k)\mathcal{B}_2^T\mathcal{P}\mathcal{B}_2F(\eta_k) + \eta_k^T\mathcal{D}^T\mathcal{P}\mathcal{D}\eta_k + \tilde{\phi}_k^T\bar{K}^T\mathcal{P}\bar{K}\tilde{\phi}_k \\
& +(d_M-\tau m+1)\eta_k^T\mathcal{Q}\eta_k - \eta_{k-d_k}^T\mathcal{Q}\eta_{k-d_k} + \Gamma_k^T\mathcal{R}\Gamma_k \\
& -\Gamma_{k-\tau}^T\mathcal{R}\Gamma_{k-\tau} + \eta_k^T\mathcal{S}\eta_k - \eta_{k-d_M}^T\mathcal{S}\eta_{k-d_M} \\
& +2\gamma\hbar\eta_{k+1}^T\mathcal{P}\eta_{k+1} + 2\gamma\hbar\eta_k^T\mathcal{P}\eta_k + \xi_k^T[\tau\mathcal{X}_1 + (d_M-d_m)\mathcal{X}_2]\xi_k \\
& +2\xi_k^T\mathcal{Y}_1(\eta_k-\eta_{k-\tau}) + 2\xi_k^T\mathcal{Y}_2(\eta_{k-d_m}-\eta_{k-d_k}) + 2\xi_k^T\mathcal{Y}_3(\eta_{k-d_k}-\eta_{k-d_M}) \\
& -\sum_{l=k-\tau}^{k-1}\varsigma_{k,l}^T\Lambda_1\varsigma_{k,l} - \sum_{l=k-d_k}^{k-\tau m-1}\varsigma_{k,l}^T\Lambda_2\varsigma_{k,l} - \sum_{l=k-d_M}^{k-d_k-1}\varsigma_{k,l}^T\Lambda_3\varsigma_{k,l} \\
& +\lambda_1\eta_k^T\bar{\Phi}^T\bar{\Phi}\eta_k - \lambda_1F^T(\eta_k)F(\eta_k) + \lambda_2\eta_k^T\mathcal{C}^T\Psi\Omega\mathcal{C}\eta_k - \lambda_2\tilde{\phi}_k^T\Omega\tilde{\phi}_k \\
\leq\; & \xi_k^T(\Pi_1+\Pi_2+\Pi_2^T+\Pi_3)\xi_k - \sum_{l=k-\tau}^{k-1}\varsigma_{k,l}^T\Lambda_1\varsigma_{k,l} \\
& -\sum_{l=k-d_k}^{k-\tau m-1}\varsigma_{k,l}^T\Lambda_2\varsigma_{k,l} - \sum_{l=k-d_M}^{k-d_k-1}\varsigma_{k,l}^T\Lambda_3\varsigma_{k,l}
\end{aligned} \tag{33}$$

where $\varsigma_{k,l} = \begin{bmatrix} \xi_k^T & \zeta_l^T \end{bmatrix}^T$, Π_1, Π_2, Π_3, Λ_1, Λ_2 and Λ_3 are defined in (21). According to (19)-(20), we can conclude that the estimation error system (16) is mean-square asymptotically stable. ■

Now, we are in a position to deal with the uncertainties in (19) and then derive the explicit form of the distributed estimator gains K_{ij}.

Theorem 2: For a given scalar $\gamma \in (0,1)$, assume that there exist matrices $\mathcal{P} = \text{diag}\{\mathcal{P}_1, \mathcal{P}_2, \cdots, \mathcal{P}_{N+1}\}$ with $\mathcal{P}_i > 0$ $(i = 1,2,\cdots,N+1)$, $\mathcal{Q} > 0$, $\mathcal{R} > 0$, $\mathcal{S} > 0$, $\mathcal{X}_1 \geq 0$, $\mathcal{X}_2 \geq 0$, matrices $\mathcal{Y}_i$ $(i = 1,2,3)$, matrix $\mathcal{K} \in \mathcal{W}_{n\times p}$, and scalars $\lambda_1 > 0$, $\lambda_2 > 0$, $\varepsilon > 0$ satisfying (20) and

$$\begin{bmatrix}
\Pi_1+\Pi_2+\Pi_2^T+\bar{\Pi}_3 & \sqrt{1+2\gamma\hbar}(\bar{\Xi}_1^T\mathcal{P}+\hat{\Xi}_1^T) & 0 & 0 & \varepsilon\bar{\mathcal{M}}^T \\
* & -\mathcal{P} & 0 & \bar{\alpha}\sqrt{1+2\gamma\hbar}\mathcal{P}\bar{H} & 0 \\
* & * & -\mathcal{P} & \sqrt{\tilde{\alpha}(1+2\gamma\hbar)}\mathcal{P}\bar{H} & 0 \\
* & * & * & -\varepsilon I & 0 \\
* & * & * & * & -\varepsilon I
\end{bmatrix} < 0, \tag{34}$$

with

$$\begin{aligned}
\bar{\Xi}_1 &= \left[\begin{array}{cccccc} \mathcal{A}_0 & 0_{(N+1)n\times(N+1)mn} & \mathcal{A}_d & 0_{(N+1)n\times(N+1)n} & \bar{\beta}\mathcal{B}_1 & 0_{(N+1)n\times Np} \end{array}\right],\\
\hat{\Xi}_1 &= \left[\begin{array}{ccc} -\bar{\mathcal{K}}\mathcal{C} & 0_{(N+1)n\times(N+1)(m+3)n} & -\bar{\mathcal{K}} \end{array}\right],\ \ \bar{\mathcal{K}} = \left[\begin{array}{cc} 0 & \mathcal{K}^T \end{array}\right]^T,\\
\mathcal{A}_0 &= \mathrm{diag}\{A, \tilde{A}\},\ \ \bar{H} = 1_{N+1}\otimes H,\\
\bar{\Pi}_3 &= \tilde{\beta}(1+2\gamma\hbar)\Xi_5^T\mathcal{B}_2^T\mathcal{P}\mathcal{B}_2\Xi_5 + \Xi_3^T[(1+2\gamma\hbar)\mathcal{D}^T\mathcal{P}\mathcal{D} + (2\gamma\hbar-1)\mathcal{P}\\
&\quad +(d_M-d_m+1)\mathcal{Q} + \mathcal{S} + \lambda_1\bar{\Phi}^T\bar{\Phi} + \lambda_2\mathcal{C}^T\Psi\Omega\mathcal{C}]\Xi_3 - \Xi_4^T\mathcal{Q}\Xi_4\\
&\quad -\Xi_5^T\mathcal{S}\Xi_5 - \lambda_1\Xi_6^T\Xi_6 - \lambda_2\Xi_7^T\Omega\Xi_7 + \Xi_{\bar{\mathcal{R}}}^T\bar{\mathcal{R}}\Xi_{\bar{\mathcal{R}}},\\
\bar{\mathcal{M}} &= \left[\begin{array}{cc} M & 0_{n_2\times[(N+1)nm+4Nn+3n+Np]} \end{array}\right],
\end{aligned} \tag{35}$$

then the estimation error system (16) is mean-square asymptotically stable. If the above linear matrix inequalities are feasible, the matrix $\tilde{K}$ can be given by

$$\tilde{K} = (V^T\mathcal{P}V)^{-1}\mathcal{K} \tag{36}$$

with $V = \left[\begin{array}{cc} 0_{Nn\times n} & I_{Nn\times Nn} \end{array}\right]^T$. Accordingly, the distributed state estimator gains K_{ij} $(i = 1, 2, \ldots, n, j \in \mathcal{N}_i)$ can be obtained by (12).

Proof: By using Lemma 1, $\Pi_1 + \Pi_2 + \Pi_2^T + \Pi_3 < 0$ is equivalent to

$$\left[\begin{array}{ccc} \Pi_1 + \Pi_2 + \Pi_2^T + \bar{\Pi}_3 & \sqrt{1+2\gamma\hbar}\,\Xi_1^T\mathcal{P} & \sqrt{\tilde{\alpha}(1+2\gamma\hbar)}\,\Xi_2^T\mathcal{P}\\ * & -\mathcal{P} & 0\\ * & * & -\mathcal{P} \end{array}\right] < 0 \tag{37}$$

where $\bar{\Pi}_3$ is defined in (35). Then, we can rewrite (37) into the following form:

$$\Theta + \mathcal{H}F\mathcal{M} + \mathcal{M}^TF^T\mathcal{H}^T < 0, \tag{38}$$

where

$$\begin{aligned}
\Theta &= \left[\begin{array}{ccc} \Pi_1 + \Pi_2 + \Pi_2^T + \bar{\Pi}_3 & \sqrt{1+2\gamma\hbar}(\bar{\Xi}_1^T + \tilde{\Xi}_1^T)\mathcal{P} & 0\\ * & -\mathcal{P} & 0\\ * & * & -\mathcal{P} \end{array}\right],\\
\tilde{\Xi}_1 &= \left[\begin{array}{ccc} -\bar{K}\mathcal{C} & 0_{(N+1)n\times(N+1)(m+3)n} & -\bar{K} \end{array}\right],\\
\mathcal{H} &= \left[\begin{array}{ccc} 0_{n_1\times[(N+1)(m+4)+p]n} & \bar{\alpha}\sqrt{1+2\gamma\hbar}\,\bar{H}^T\mathcal{P} & \sqrt{\tilde{\alpha}(1+2\gamma\hbar)}\,\bar{H}^T\mathcal{P} \end{array}\right]^T,\\
\mathcal{M} &= \left[\begin{array}{cc} M & 0_{n_2\times[(N+1)nm+6Nn+5n+Np]} \end{array}\right].
\end{aligned}$$

By denoting $\mathcal{P}\bar{K} = \bar{\mathcal{K}}$ and using Lemma 2, it is not difficult to verify that (38) holds if (34) is true. Moreover, according to Lemma 3, it is easy to show that $\tilde{K} \in \mathcal{W}_{n\times p}$. Hence, the proof of this theorem is complete. ■

Remark 2: Up to now, the distributed state estimation problem is studied for a class of discrete nonlinear stochastic systems with time-varying delay, ROUs and RONs. Based on the event-triggered mechanism, a new distributed state estimator is constructed where the available neighbor information of the sensor nodes is employed. It is shown that the explicit form of the estimator gains can be easily solved by using the semi-definite programme approach.

IV. An Illustrative Example

In this section, we provide a numerical example to demonstrate the feasibility and effectiveness of the distributed state estimation scheme.

The sensor network is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{L})$ with the set of nodes $\mathcal{V} = \{1, 2, 3, 4\}$, the set of edges $\mathcal{E} = \{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3), (4,3), (4,4)\}$, and the following adjacency matrix

$$\mathcal{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

The system parameters are given as follows:

$$\begin{aligned}
A &= \begin{bmatrix} -0.24 & -0.32 \\ 0.4 & -0.08 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.8 & 0.8 \\ -1.2 & -0.4 \end{bmatrix}, \\
B &= \begin{bmatrix} -0.6 & -0.02 \\ 0.02 & -0.7 \end{bmatrix}, \quad D = \begin{bmatrix} -0.4 & 0 \\ -0.3 & -0.2 \end{bmatrix}, \\
H &= \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}^T, \quad M = \begin{bmatrix} 0.05 & 0.1 \end{bmatrix}, \quad F = \sin(0.5k) \\
C_1 &= \begin{bmatrix} 0.8 & 1.6 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 5.6 & 2.4 \end{bmatrix}, \\
C_3 &= \begin{bmatrix} 0.8 & 0 \end{bmatrix}, \quad C_4 = \begin{bmatrix} 2.4 & 3.2 \end{bmatrix}.
\end{aligned}$$

Assume that the time-varying delay d_k satisfies $2 \leq d_k \leq 5$, $\Phi = \mathrm{diag}\{0.2, 0.3\}$, $\bar{\alpha} = 0.9$, $\bar{\beta} = 0.85$, $\Omega_i = 1$ and $\delta_i = 0.5$ $(i = 1, 2, 3, 4)$. Set $m = 1$ and $\gamma = 0.01$. By solving (20) and (34) in Theorem 2, we have

$$\begin{aligned}
K_{11} &= \begin{bmatrix} 0.0097 & -0.0984 \end{bmatrix}^T, \quad K_{21} = \begin{bmatrix} -0.0701 & 0.0079 \end{bmatrix}^T, \\
K_{22} &= \begin{bmatrix} -0.1277 & 0.0652 \end{bmatrix}^T, \quad K_{31} = \begin{bmatrix} -0.0064 & 0.0518 \end{bmatrix}^T, \\
K_{32} &= \begin{bmatrix} 0.0177 & -0.3597 \end{bmatrix}^T, \quad K_{33} = \begin{bmatrix} 0.0151 & 0.0401 \end{bmatrix}^T, \\
K_{43} &= \begin{bmatrix} -0.0378 & -0.0236 \end{bmatrix}^T, \quad K_{44} = \begin{bmatrix} 0.0178 & 0.0086 \end{bmatrix}^T,
\end{aligned}$$

which confirm the feasibility of the proposed state estimation approach.

V. Conclusions

In this paper, the event-based distributed state estimation problem has been investigated for a class of nonlinear stochastic systems with time-varying delay, ROUs and RONs. The phenomena of the ROUs and RONs have been depicted by introducing two random variables obeying the Bernoulli distribution. By considering the event-triggered mechanism and the characteristic of the sensor works, a new distributed state estimator has been constructed. Based on the delay-fractioning idea, a sufficient condition has been proposed to guarantee the mean-square asymptotical stability of the estimation error system and the explicit form of the estimator gains has also been given. Finally, a numerical example has been provided to demonstrate the usefulness of the developed estimation scheme. Further research topics include the extension of the proposed results to more complex systems with event-triggering communication mechanism such as nonlinear polynomial systems [2, 3], time-varying systems [13, 43], mechanical systems [25] and multi-machine power systems [27].

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