Event-triggered Distributed $\mathcal{H}_\infty$ State Estimation with Packet Dropouts through Sensor Networks

Derui Ding, Zidong Wang, Bo Shen and Hongli Dong

Abstract

This paper is concerned with the event-triggered distributed $\mathcal{H}_\infty$ state estimation problem for a class of discrete-time stochastic nonlinear systems with packet dropouts in a sensor network. An event-triggered communication mechanism is adopted over the sensor network with hope to reduce the communication burden and the energy consumption, where the measurements on each sensor are transmitted only when a certain triggering condition is violated. Furthermore, a novel distributed state estimator is designed where the available innovations are not only from the individual sensor but also from its neighboring ones according to the given topology. The purpose of the problem under consideration is to design a set of distributed state estimators such that the dynamics of estimation errors is exponentially mean-square stable and also the prespecified $\mathcal{H}_\infty$ disturbance rejection attenuation level is guaranteed. By utilizing the property of the Kronecker product and the stochastic analysis approaches, sufficient conditions are established under which the addressed state estimation problem is recast as a convex optimization one that can be easily solved via available software packages. Finally, a simulation example is utilized to illustrate the usefulness of the proposed design scheme of event-triggered distributed state estimators.

Keywords

Sensor networks, distributed state estimation, event-triggered protocol, $\mathcal{H}_\infty$ performance, packet dropouts

I. Introduction

A sensor network typically consists of a large number of geographically distributed sensor nodes which cooperatively monitor parameters or events of interest. Nowadays, sensor networks have a wide-scope domain of applications such as environment and habitat monitoring, health care applications, traffic control, distributed robotics, and industrial and manufacturing automation [1–3]. It is worth noting that, the network size, the communication constraints as well as the stringent energy limit inevitably result in great challenges to the applications of classical centralized estimation techniques that demand enormous storage space and centralized computation. As such, it is not surprising that the distributed filtering or estimation problem for sensor networks serves as one of the most fundamental collaborative information processing problems and has gained an ever-increasing research interest, see [4–12] and the references therein.

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D. Ding is with the Shanghai Key Lab of Modern Optical System, Department of Control Science and Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China.

Z. Wang is with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex UB8 3PH, U.K. He is also with the Communication Systems and Networks (CSN) Research Group, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia. (e-mail: Zidong.Wang@brunel.ac.uk)

B. Shen is with the School of Information Science and Technology, Donghua University, Shanghai 200051, China.

H. Dong is with the College of Electrical and Information Engineering, Northeast Petroleum University, Daqing 163318, China.

It is well recognized that the limited bandwidth of the communication channel inevitably leads to some network-induced phenomena whose occurrence is often of the random nature. It is worth pointing out that the desired system performances could be seriously degraded if such network-induced phenomena are ignored in the estimator design. Therefore, considerable attention has been focused on the state estimation issues for systems with various network-induced phenomena in the past few years [13–20]. Among others, the packet dropouts (also called missing measurements), which are known to be one of the most frequently occurred phenomena in the networked systems, have received particular research interests and a great number of estimator design algorithms have been proposed in the literature, with examples including Kalman filtering approaches and $\mathcal{H}_\infty$ estimation methods based on the linear matrix inequalities (LMI) or the recursive linear matrix inequalities (RLMI), see, e.g., [21–25]. On the other hand, the nonlinearity is ubiquitous in real-world systems. In a networked environment, the nonlinear disturbances might stem from the random fluctuation of the network load and the unreliability of communication links. In such a case, the disturbances themselves could experience random abrupt changes in their type or intensity [26–28]. Furthermore, for sensor networks of large size, the packet dropouts and stochastic nonlinearities become even severe due primarily to the inherent feature of such networks, for instance, the communication constraints, strong coupling, spatial deployment, and so forth. Therefore, it is desirable to examine how these two phenomena affect the estimation performance and this constitutes one of the motivations for the present research.

Very recently, the event-triggered communication mechanism has attracted some preliminary research attention for networked systems. The event-triggered strategy is deemed to be particularly necessary for the distributed real-time sensing and control due mainly to the need for reducing the communication burden and the energy consumption [29–31]. As far as the sensor networks are concerned, the sensor nodes are often battery operated with a limited energy resource. As such, in order to prolong the lifetime of such sensor networks, an effective approach in the implementation is to adopt a novel communication strategy (i.e. an event-triggered protocol) to avoid unnecessary data transmission. It is noted that, on event-triggered protocols, there have been a growing number of results covering a wide range of applications in various engineering systems such as networked control systems and multi-agent systems [29–33]. Unfortunately, in the presence of network-induced phenomena, available results in the literature have been scattered for state estimation problems through sensor networks. Such a situation results from the two challenging issues identified as follows. (1) A sensor network is often subject to various network-induced phenomena (e.g. packet dropouts and stochastic nonlinearities) even if the event-triggered communication protocol is exploited. So, the first difficulty is how to develop a reasonable model to describe event triggering communication mechanisms and network induced phenomena in a unified framework. (2) The key issue in designing distributed estimators for sensor networks is how to fuse the information available for the estimator both from itself (without event-triggered mechanism) and from its neighbors (with event-triggered mechanism). In other words, the second challenge is how to construct a suitable distributed estimator such that the information from different sources is adequately integrated.

Summarizing the above discussions, it can be concluded that there is a great need to examine how the packet dropouts and stochastic nonlinearities affect on the performance of event-triggered distributed estimators through sensor networks with the given topology. As such, the main purpose of this paper is to initiate a study on the distributed $\mathcal{H}_\infty$ state estimation problem with an event-triggered communication protocol. The main contributions of this paper can be highlighted as follows. *(1) Both the packet dropouts and the event-triggered communication protocol are considered within a unified framework in order to better reflect the* *reality. (2) A novel structure of distributed estimators is designed to adequately utilize the available innovations from not only itself (without event-triggered mechanism) but also its neighboring sensors (with event-triggered mechanism). (3) Intensive stochastic analysis is conducted, with help from the utilization of the Kronecker product, to enforce the $\mathcal{H}_\infty$ performance for the addressed distributed estimation problem in addition to the exponentially mean-square stability constraint.*

The rest of this paper is organized as follows. In Section II, a target plant described by a discrete-time stochastic nonlinear system is introduced, where the event-triggered communication mechanism and the phenomenon of packet dropouts are presented in the corresponding sensor networks. In Section III, by employing the Lyapunov stability theory combined with the Kronecker product, some sufficient conditions are established in the form of linear matrix inequalities, and then the gains of event-triggered distributed estimators are obtained by solving a convex optimization problem. In Section IV, an example is presented to demonstrate the effectiveness of the established design scheme of distributed estimators. Finally, conclusions are drawn in Section V.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n\times m$ real matrices. $l_2([0,\infty);\mathbb{R}^n)$ is the space of square-summable n-dimensional vector functions over $[0,\infty)$. I denotes the identity matrix of compatible dimension. The notation $X\geq Y$ (respectively, $X>Y$), where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). M^T represents the transpose of M. $\mathbb{E}\{x\}$ stands for the expectation of stochastic variable x. $\|x\|$ describes the Euclidean norm of a vector x. The shorthand $\mathrm{diag}\{M_1,M_2,\cdots,M_n\}$ denotes a block diagonal matrix with diagonal blocks being the matrices $M_1,\ M_2,\ \cdots\ M_n$. The symbol $\otimes$ denotes the Kronecker product. In symmetric block matrices, the symbol $*$ is used as an ellipsis for terms induced by symmetry.

II. Problem Formulation and Preliminaries

In this paper, it is assumed that the sensor network has n sensor nodes which are distributed in space according to a fixed network topology represented by a undirected graph $\mathscr{G}=(\mathscr{V},\mathscr{E},\mathscr{H})$ of order n with the set of nodes $\mathscr{V}=\{1,2,\cdots,n\}$, the set of edges $\mathscr{E}\in\mathscr{V}\times\mathscr{V}$, and the weighted adjacency matrix $\mathscr{H}=[h_{ij}]$ with nonnegative adjacency element h_{ij}. An edge of $\mathscr{G}$ is denoted by the ordered pair (i,j). The adjacency elements associated with the edges of the graph are positive, i.e., $h_{ij}>0\Longleftrightarrow(i,j)\in\mathscr{E}$, which means that sensor i can obtain information from sensor j. The set of neighbors of node $i\in\mathscr{V}$ is denoted by $\mathcal{N}_i=\{j\in\mathscr{V}:(i,j)\in\mathscr{E}\}$.

In this paper, a target plant is the system whose states are to be estimated through the distributed sensors. Let the target plant be described by the following discrete-time stochastic nonlinear system:

$$\begin{cases} x_{k+1}=Ax_k+A_dx_{k-\tau}+f(x_k,\vartheta_k)+Bw_k \\ \quad z_k=Lx_k \end{cases} \tag{1}$$

with n sensors modeled by

$$y_{i,k}=C_ix_k+D_iv_k,\quad i=1,2,\cdots,n \tag{2}$$

where $x_k\in\mathbb{R}^{n_x}$ is the state of the target plant that cannot be observed directly, $y_{i,k}\in\mathbb{R}^{n_y}$ is the measurement output from sensor i, $z_k\in\mathbb{R}^{n_z}$ is the output to be estimated, and $w_k,v_k\in l_2([0,\infty);\mathbb{R})$ are external disturbances. τ is a known positive scalar, and A, A_d, B, L, C_i and D_i are known constant matrices with compatible dimensions.

The function $f(x_k,\vartheta_k)$ with $f(0,\vartheta_k)=0$ is a stochastic nonlinear function having the following first moment for all x_k:

$$\mathbb{E}\left\{f(x_k,\vartheta_k)|x_k\right\}=0 \tag{3}$$

and the covariance given by

$$\mathbb{E}\left\{f(x_k,\vartheta_k)f^T(x_j,\vartheta_j)|x_k\right\}=0,\quad k\neq j, \tag{4}$$

and

$$\mathbb{E}\left\{f(x_k,\vartheta_k)f^T(x_k,\vartheta_k)|x_k\right\}=\sum_{i=1}^{s}\Pi_i x_k^T\Gamma_i x_k, \tag{5}$$

where s is a known nonnegative integer, Π_i and Γ_i $(i=1,2,\cdots,s)$ are known matrices with appropriate dimensions.

For the purpose of presentation clarity, on sensor node i, denote the estimation of x_k and the innovation sequence, respectively, as $\hat{x}_{i,k}$ and

$$r_{i,k}=y_{i,k}-C_i\hat{x}_{i,k}.$$

It should be pointed out that a distributed state estimation is capable of fusing the information available for the estimator on node i from both sensor i itself and its neighbors. A further objective of this paper is to take the event-triggered communication mechanism into consideration in order to reduce the communication burden. For this purpose, we define event generator functions $\psi_i(\cdot,\cdot):\mathbb{R}^{n_y}\times\mathbb{R}\rightarrow\mathbb{R}$ $(i=1,2,\cdots,n)$ as follows:

$$\psi_i(e_{i,k},\delta_i)=e_{i,k}^Te_{i,k}-\delta_i r_{i,k}^T r_{i,k}. \tag{6}$$

Here, $e_{i,k}=r_{i,k}^t-r_{i,k}$ where $r_{i,k}^t$ is the broadcast innovation at the latest event instant and δ_i is a given positive scalar. The executions are triggered as long as the condition $\psi_i(e_{i,k},\delta_i)>0$ is satisfied. Therefore, the sequence of event triggered instants $0\leq s_0^i<s_1^i<\cdots<s_l^i<\cdots$ is determined iteratively by

$$s_{l+1}^i=\inf\{k\in\mathbb{N}|k>s_l^i,\ \psi_i(e_{i,k},\delta_i)>0\}. \tag{7}$$

As is well known, due to the limited network bandwidth, the broadcast innovation could be subject to packet dropouts. To cater for the phenomenon of packet dropouts, the received information for neighbors of node i can be described as

$$\tilde{r}_{i,k}^t=\alpha_{i,k}r_{i,k}^t \tag{8}$$

where the stochastic variables $\alpha_{i,k}$ $(i=1,2,\cdots,n)$ are employed to govern the stochastic occurring packet dropouts. These variables are assumed to be mutually independent Bernoulli-distributed white sequences taking values on 0 or 1 with the following probabilities

$$\text{Prob}\{\alpha_{i,k}=0\}=1-\bar{\alpha},\quad \text{Prob}\{\alpha_{i,k}=1\}=\bar{\alpha}.$$

In this paper, the distributed state estimators are of the following structure:

$$\begin{cases}\hat{x}_{i,k+1}=A\hat{x}_{i,k}+A_d\hat{x}_{i,k-\tau}+K_{i,1}r_{i,k}+K_{i,2}\displaystyle\sum_{j\in\mathcal{N}_i}h_{ij}\tilde{r}_{j,k}^t\\ \hat{z}_{i,k}=L\hat{x}_{i,k}\end{cases} \tag{9}$$

where $\hat{z}_{i,k}\in\mathbb{R}^{n_z}$ is the estimated output on sensor node i. Here, $K_{i,1}$ and $K_{i,2}$ are the estimator gain matrices on node i to be determined.

Remark 1: For distributed state estimation problems, the information available on each node is not only from itself but also from its neighbors according to the given topology. From the engineering viewpoint, the event-triggered communication protocol is adopted to determine at what time the information needs to be broadcasted. Hence, for a given node, the amount of the data received from any neighboring sensors should be less in that of the data from the node itself due to the application of the event-triggered protocol. This explains why we divide the innovation into two parts in (9), i.e., $r_{i,k}$ concerning the data from the node itself and $\sum_{j\in\mathcal{N}_i} h_{ij}\tilde{r}^t_{j,k}$ concerning the data from the neiboring nodes. Therefore, the proposed estimator model (9) can be utilized to effectively cope with the complicated coupling issues between any sensor and its neighboring sensors and also adequately fuse these two kinds of information (i.e. $r_{i,k}$ and $\tilde{r}^t_{j,k}$) to improve the estimation performance.

Remark 2: For described state estimation issues, an event-triggered communication mechanism (7) is adopted with hope to reduce the communication burden and the energy consumption, where the innovation on each sensor is broadcasted to its neighbors only when the certain triggering condition in (8) is violated. In light of such a condition, it is not difficult to see that a smaller threshold δ_i gives rise to a heavier communication load, and therefore an adequate trade-off can be achieved between the threshold and the acceptable network load.

For notational simplicity, we define

$$
\begin{aligned}
&\xi_k = \mathbf{1}_n \otimes x_k - \hat{x}_k, \quad \hat{x}_k = [\hat{x}^T_{1,k} \quad \hat{x}^T_{2,k} \quad \cdots \quad \hat{x}^T_{n,k}]^T,\\
&\tilde{z}_{i,k} = z_k - \hat{z}_{i,k}, \quad \tilde{z}_k = [\tilde{z}^T_{1,k} \quad \tilde{z}^T_{2,k} \quad \cdots \quad \tilde{z}^T_{n,k}]^T,\\
&e_k = [e^T_{1,k} \quad e^T_{2,k} \quad \cdots \quad e^T_{n,k}]^T, \quad \tilde{f}(x_k,\vartheta_k) = \mathbf{1}_n \otimes f(x_k,\vartheta_k),\\
&\mathcal{A} = \mathrm{diag}_n\{A\}, \quad \mathcal{A}_d = \mathrm{diag}_n\{A_d\},\\
&\mathcal{C} = \mathrm{diag}\{C_1, C_2 \cdots, C_n\}, \quad \mathcal{K}_1 = \mathrm{diag}\{K_{1,1}, K_{2,1} \cdots, K_{n,1}\},\\
&\mathcal{D} = [\ D^T_1,\ D^T_2,\ \cdots,\ D^T_n\ ]^T, \quad \mathcal{K}_2 = \mathrm{diag}\{K_{1,2}, K_{2,2} \cdots, K_{n,2}\},\\
&\Xi = \mathrm{diag}_n\{\bar{\alpha}\}, \quad \Xi_k = \mathrm{diag}\{\alpha_{1,k} - \bar{\alpha},\ \alpha_{2,k} - \bar{\alpha},\ \cdots,\ \alpha_{n,k} - \bar{\alpha}\}.
\end{aligned}
$$

Using the defined notations, the dynamics of the estimation errors can be obtained as follows:

$$
\begin{aligned}
\xi_{k+1} = {} & \big(\mathcal{A} - \mathcal{K}_1\mathcal{C} - \mathcal{K}_2(\mathcal{H}\Xi \otimes I)\mathcal{C}\big)\xi_k + \mathcal{A}_d\xi_{k-\tau} - \mathcal{K}_2(\mathcal{H}\Xi \otimes I)e_k - \big(\mathcal{K}_1\mathcal{D} + \mathcal{K}_2(\mathcal{H}\Xi \otimes I)\mathcal{D}\big)v_k\\
& + (\mathbf{1} \otimes B)w_k + \tilde{f}(x_k,\vartheta_k) - \mathcal{K}_2(\mathcal{H}\Xi_k \otimes I)\mathcal{C}(\mathbf{1}_n \otimes x_k) - \mathcal{K}_2(\mathcal{H}\Xi_k \otimes I)e_k - \mathcal{K}_2(\mathcal{H}\Xi_k \otimes I)\mathcal{D}v_k
\end{aligned}
\tag{10}
$$

Setting $\eta_k = [x^T_k \quad \xi^T_k]^T$ and $\tilde{w}_k = [w^T_k \quad v^T_k]^T$, an augmented system can be derived from (1) and (10) as follows:

$$
\begin{cases}
\eta_{k+1} = \bar{\mathcal{A}}\eta_k + \tilde{\mathcal{A}}_k\eta_k + \mathscr{F}(x_k,\vartheta_k) + \bar{\mathcal{A}}_d\eta_{k-\tau} + \bar{\mathcal{B}}e_k + \tilde{\mathcal{B}}_k e_k + \bar{\mathcal{D}}\tilde{w}_k + \tilde{\mathcal{D}}_k\tilde{w}_k\\
\tilde{z}_k = \bar{\mathcal{L}}\eta_k
\end{cases}
\tag{11}
$$

where

$$
\begin{aligned}
&\mathscr{F}(x_k,\vartheta_k) = \mathbf{1}_{n+1} \otimes f(x_k,\vartheta_k), \quad \bar{\mathcal{L}} = [\ 0 \quad I \otimes L\ ], \quad \bar{\mathcal{A}}_d = \mathrm{diag}_{n+1}\{A_d\},\\
&\bar{\mathcal{A}} = \begin{bmatrix} A & 0\\ 0 & \mathcal{A} - \mathcal{K}_1\mathcal{C} - \mathcal{K}_2(\mathcal{H}\Xi \otimes I)\mathcal{C} \end{bmatrix}, \quad \bar{\mathcal{B}} = \begin{bmatrix} 0\\ -\mathcal{K}_2(\mathcal{H}\Xi \otimes I) \end{bmatrix},\\
&\tilde{\mathcal{A}}_k = \begin{bmatrix} 0 & 0\\ -\mathcal{K}_2(\mathcal{H}\Xi_k \otimes I)\mathcal{C}(\mathbf{1}_n \otimes I) & 0 \end{bmatrix}, \quad \tilde{\mathcal{B}}_k = \begin{bmatrix} 0\\ -\mathcal{K}_2(\mathcal{H}\Xi_k \otimes I) \end{bmatrix},\\
&\bar{\mathcal{D}} = \begin{bmatrix} B & 0\\ \mathbf{1}_n \otimes B & -\mathcal{K}_1\mathcal{D} - \mathcal{K}_2(\mathcal{H}\Xi \otimes I)\mathcal{D} \end{bmatrix}, \quad \tilde{\mathcal{D}}_k = \begin{bmatrix} 0 & 0\\ 0 & -\mathcal{K}_2(\mathcal{H}\Xi_k \otimes I)\mathcal{D} \end{bmatrix}.
\end{aligned}
$$

Before proceeding further, we introduce the following definition and assumption.

Definition 1: The augmented system (11) with $v_k = 0$ is said to be exponentially mean-square stable if there exist constants $\varepsilon > 0$ and $0 < \hbar < 1$ such that

$$\mathbb{E}\{||\eta_k||^2\} \leq \varepsilon \hbar^k \max_{i \in [-\tau,\ 0]} \mathbb{E}\{||\tilde{w}_i||^2\},\ k \in \mathbb{N}.$$

Assumption 1: The matrices Π_i and Γ_i $(i = 1, 2, \cdots, s)$ in (5) have the following decomposition

$$\Pi_i = \bar{\pi}_i \bar{\pi}_i^T = \begin{bmatrix} \pi_{1i} \\ \pi_{2i} \end{bmatrix} \begin{bmatrix} \pi_{1i} \\ \pi_{2i} \end{bmatrix}^T, \quad \Gamma_i = \bar{\theta}_i \bar{\theta}_i^T$$

where $\bar{\pi}_i$, π_{1i}, π_{2i} and $\bar{\theta}_i$ are known vectors with appropriate dimensions.

The purpose of this paper is to design a set of state estimators of form (9) for the discrete-time stochastic nonlinear system (1) through sensor networks. More specifically, we are interested in looking for the parameters $K_{i,1}$ and $K_{i,2}$ $(i = 1, 2, \cdots, n)$ such that the following requirements are met simultaneously:

$R1$) The augmented system (11) with $\tilde{w}_k = 0$ is exponentially mean-square stable;

$R2$) Under the zero-initial condition, for a given disturbance attenuation level $\gamma > 0$ and all nonzero $\tilde{w}_k$, the estimation error $\tilde{z}_k$ satisfies

$$\frac{1}{n} \sum_{k=0}^{\infty} \mathbb{E}\{||\tilde{z}_k||^2\} \leq \gamma^2 \sum_{k=0}^{\infty} ||\tilde{w}_k||^2. \tag{12}$$

III. Main Results

In this section, by resorting to the stochastic analysis techniques, we shall provide the analysis result of the $\mathcal{H}_\infty$ performance for the augmented system (11), and then proceed with the subsequent design stage of event-triggered estimators.

Theorem 1: Let the estimator parameters $K_{i,1}$ and $K_{i,2}$ $(i = 1, 2, \cdots, n)$ as well as a prescribed disturbance attenuation level $\gamma > 0$ be given. The dynamics of the estimation errors (11) is exponentially mean-square stable and also satisfies the prespecified $\mathcal{H}_\infty$ performance constraint (12) if there exist two positive definite matrices P, Q and a positive scalar λ satisfying

$$\mathcal{R} = \begin{bmatrix} \mathcal{R}_{11} & \mathcal{R}_{12} & \mathcal{R}_{13} & \mathcal{R}_{14} \\ * & \mathcal{R}_{22} & \mathcal{R}_{23} & \mathcal{R}_{24} \\ * & * & \mathcal{R}_{33} & \mathcal{R}_{34} \\ * & * & * & \mathcal{R}_{44} \end{bmatrix} < 0 \tag{13}$$

where

$$\Theta = \mathrm{diag}\{\delta_1,\ \delta_2,\ \cdots,\ \delta_n\}, \quad \mathcal{H}_i := \mathrm{diag}\{h_{i,1},\ h_{i,2},\ \cdots,\ h_{i,n}\},$$
$$\Psi = \bar{\alpha}(1-\bar{\alpha})\sum_{i=1}^{n}(\mathcal{H}_i^T\mathcal{H}_i)\otimes(K_{i,2}^TPK_{i,2}),$$
$$\mathbf{I} = [\, I\ \ 0\ \ 0\ \ \cdots\ \ 0\,], \quad \tilde{\mathbf{I}} = [\, 0\ \ I\ \ I\ \ \cdots\ \ I\,], \quad \tilde{\mathcal{D}} = [\, 0\ \ \mathcal{D}\,],$$
$$\begin{aligned}
\mathcal{R}_{11} &= \bar{\mathcal{A}}^T(I\otimes P)\bar{\mathcal{A}} + \Upsilon_1 + \sum_{i=1}^{s}(n+1)\mathrm{tr}\big[P\Pi_i\big]\mathbf{I}^T\Gamma_i\mathbf{I} \\
&\quad + \lambda(\mathbf{1}_n\otimes\tilde{\mathbf{I}})^T\mathcal{C}^T(\Theta\otimes I)\mathcal{C}(\mathbf{1}_n\otimes\tilde{\mathbf{I}}) - (I\otimes P) + (I\otimes Q) + \frac{1}{n}\bar{\mathcal{L}}^T\bar{\mathcal{L}}, \\
\mathcal{R}_{12} &= \bar{\mathcal{A}}^T(I\otimes P)\bar{\mathcal{A}}_d, \quad \mathcal{R}_{13} = \bar{\mathcal{A}}^T(I\otimes P)\bar{\mathcal{B}}, \\
\mathcal{R}_{14} &= \bar{\mathcal{A}}^T(I\otimes P)\bar{\mathcal{D}} + \lambda(\mathbf{1}_n\otimes\mathbf{I})^T\mathcal{C}^T(\Theta\otimes I)\tilde{\mathcal{D}}, \\
\mathcal{R}_{22} &= \bar{\mathcal{A}}_d^T(I\otimes P)\bar{\mathcal{A}}_d - (I\otimes Q), \quad \mathcal{R}_{23} = \bar{\mathcal{A}}_d^T(I\otimes P)\bar{\mathcal{B}}, \quad \mathcal{R}_{24} = \bar{\mathcal{A}}_d^T(I\otimes P)\bar{\mathcal{D}}, \\
\mathcal{R}_{33} &= \bar{\mathcal{B}}^T(I\otimes P)\bar{\mathcal{B}} + \Psi - \lambda I, \quad \mathcal{R}_{34} = \bar{\mathcal{B}}^T(I\otimes P)\bar{\mathcal{D}}, \\
\mathcal{R}_{44} &= \bar{\mathcal{D}}^T(I\otimes P)\bar{\mathcal{D}} + \Upsilon_2 + \lambda\tilde{\mathcal{D}}^T(\Theta\otimes I)\tilde{\mathcal{D}} - \gamma^2 I, \\
\Upsilon_1 &= \begin{bmatrix} (\mathbf{1}_n\otimes I)^T\mathcal{C}^T\Psi\mathcal{C}(\mathbf{1}_n\otimes I) & 0 \\ 0 & 0 \end{bmatrix}, \quad \Upsilon_2 = \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{D}^T\Psi\mathcal{D} \end{bmatrix}.
\end{aligned}$$

Proof: First, noting the stochastic matrix Ξ_k, one has

$$\begin{aligned}
&\mathbb{E}\big\{(\mathcal{H}\Xi_k\otimes I)^T\mathcal{K}_2^T(I\otimes P)\mathcal{K}_2(\mathcal{H}\Xi_k\otimes I)\big\} \\
&= \bar{\alpha}(1-\bar{\alpha})\sum_{i=1}^{n}(\mathcal{H}_i\otimes I)^T(I\otimes K_{i,2})^T(I\otimes P)(I\otimes K_{i,2})(\mathcal{H}_i\otimes I) \\
&= \bar{\alpha}(1-\bar{\alpha})\sum_{i=1}^{n}(\mathcal{H}_i^T\mathcal{H}_i)\otimes(K_{i,2}^TPK_{i,2}).
\end{aligned} \tag{14}$$

Then, by employing the property of matrix trace, it follows from (4) and (5) that

$$\begin{aligned}
&\mathbb{E}\big\{\mathscr{F}^T(x_k,\vartheta_k)(I\otimes P)\mathscr{F}(x_k,\vartheta_k)\big\} \\
&= \mathbb{E}\big\{(\mathbf{1}_{n+1}\otimes f(x_k,\vartheta_k))^T(I\otimes P)(\mathbf{1}_{n+1}\otimes f(x_k,\vartheta_k))\big\} \\
&= \mathbb{E}\big\{(\mathbf{1}_{n+1}^T\mathbf{1}_{n+1})\otimes(f^T(x_k,\vartheta_k)Pf(x_k,\vartheta_k))\big\} \\
&= (\mathbf{1}_{n+1}^T\mathbf{1}_{n+1})\otimes\mathbb{E}\big\{f^T(x_k,\vartheta_k)Pf(x_k,\vartheta_k)\big\} \\
&= (\mathbf{1}_{n+1}^T\mathbf{1}_{n+1})\otimes\mathbb{E}\left\{\mathrm{tr}\big[Pf(x_k,\vartheta_k)f^T(x_k,\vartheta_k)\big]\right\} \\
&= (\mathbf{1}_{n+1}^T\mathbf{1}_{n+1})\otimes\mathbb{E}\left\{x_k^T\sum_{i=1}^{s}\mathrm{tr}\big[P\Pi_i\big]\Gamma_i x_k\right\} \\
&= \mathbb{E}\left\{x_k^T\sum_{i=1}^{s}(n+1)\mathrm{tr}\big[P\Pi_i\big]\Gamma_i x_k\right\}.
\end{aligned} \tag{15}$$

In what follows, choose the Lyapunov function for system (11):

$$V_k = \eta_k^T(I\otimes P)\eta_k + \sum_{i=k-\tau}^{k-1}\eta_i^T(I\otimes Q)\eta_i. \tag{16}$$

Calculating the difference of V_k along the trajectory of system (11) with $\tilde{w}_k = 0$ and taking the mathematical expectation, one has

$$\begin{aligned}
\mathbb{E}\{\Delta V_k\} &:= \mathbb{E}\Big\{V_{k+1} - V_k\Big\} \\
&= \mathbb{E}\Big\{\Big(\bar{\mathcal{A}}\eta_k + \tilde{\mathcal{A}}_k\eta_k + \mathscr{F}(x_k,\vartheta_k) + \bar{\mathcal{A}}_d\eta_{k-\tau} + \bar{\mathcal{B}}e_k + \tilde{\mathcal{B}}_k e_k\Big)^T \\
&\quad \times (I\otimes P)\Big(\bar{\mathcal{A}}\eta_k + \tilde{\mathcal{A}}_k\eta_k + \mathscr{F}(x_k,\vartheta_k) + \bar{\mathcal{A}}_d\eta_{k-\tau} + \bar{\mathcal{B}}e_k + \tilde{\mathcal{B}}_k e_k\Big) \\
&\quad - \eta_k^T(I\otimes P)\eta_k + \sum_{i=k-\tau+1}^{k}\eta_i^T(I\otimes Q)\eta_i - \sum_{i=k-\tau}^{k-1}\eta_i^T(I\otimes Q)\eta_i\Big\} \\
&= \mathbb{E}\Big\{\eta_k^T\bar{\mathcal{A}}^T(I\otimes P)\bar{\mathcal{A}}\eta_k + 2\eta_k^T\bar{\mathcal{A}}^T(I\otimes P)\bar{\mathcal{A}}_d\eta_{k-\tau} + 2\eta_k^T\bar{\mathcal{A}}^T(I\otimes P)\bar{\mathcal{B}}e_k \\
&\quad + \eta_k^T\Upsilon_1\eta_k + \eta_k^T\sum_{i=1}^{s}(n+1)\mathrm{tr}\big[P\Pi_i\big]\mathbf{I}^T\Gamma_i\mathbf{I}\eta_k + \eta_{k-\tau}^T\bar{\mathcal{A}}_d^T(I\otimes P)\bar{\mathcal{A}}_d\eta_{k-\tau} \\
&\quad + 2\eta_{k-\tau}^T\bar{\mathcal{A}}_d^T(I\otimes P)\bar{\mathcal{B}}e_k + e_k^T\bar{\mathcal{B}}^T(I\otimes P)\bar{\mathcal{B}}e_k + e_k^T\Psi e_k - \eta_k^T(I\otimes P)\eta_k \\
&\quad + \eta_k^T(I\otimes Q)\eta_k - \eta_{k-\tau}^T(I\otimes Q)\eta_{k-\tau}\Big\}.
\end{aligned} \tag{17}$$

Furthermore, it follows from the event-triggering condition (7) that

$$\lambda e_k^T e_k - \lambda\eta_k^T(\mathbf{1}_n\otimes\tilde{\mathbf{I}})^T\mathcal{C}^T(\Theta\otimes I)\mathcal{C}(\mathbf{1}_n\otimes\tilde{\mathbf{I}})\eta_k \le 0. \tag{18}$$

Taking the above inequality into account, we have

$$\begin{aligned}
\mathbb{E}\{\Delta V_k\} &\le \mathbb{E}\Big\{\eta_k^T\bar{\mathcal{A}}^T(I\otimes P)\bar{\mathcal{A}}\eta_k + 2\eta_k^T\bar{\mathcal{A}}^T(I\otimes P)\bar{\mathcal{A}}_d\eta_{k-\tau} + 2\eta_k^T\bar{\mathcal{A}}^T(I\otimes P)\bar{\mathcal{B}}e_k \\
&\quad + \eta_k^T\Upsilon_1\eta_k + \eta_k^T\sum_{i=1}^{s}(n+1)\mathrm{tr}\big[P\Pi_i\big]\mathbf{I}^T\Gamma_i\mathbf{I}\eta_k + \eta_{k-\tau}^T\bar{\mathcal{A}}_d^T(I\otimes P)\bar{\mathcal{A}}_d\eta_{k-\tau} \\
&\quad + 2\eta_{k-\tau}^T\bar{\mathcal{A}}_d^T(I\otimes P)\bar{\mathcal{B}}e_k + e_k^T\bar{\mathcal{B}}^T(I\otimes P)\bar{\mathcal{B}}e_k + e_k^T\Psi e_k - \lambda e_k^T e_k \\
&\quad + \lambda\eta_k^T(\mathbf{1}_n\otimes\tilde{\mathbf{I}})^T\mathcal{C}^T(\Theta\otimes I)\mathcal{C}(\mathbf{1}_n\otimes\tilde{\mathbf{I}})\eta_k - \eta_k^T(I\otimes P)\eta_k \\
&\quad + \eta_k^T(I\otimes Q)\eta_k - \eta_{k-\tau}^T(I\otimes Q)\eta_{k-\tau}\Big\}
\end{aligned} \tag{19}$$

which results in

$$\mathbb{E}\{\Delta V_k\} \le \mathbb{E}\Big\{\bar{\eta}_k^T\tilde{\mathcal{R}}\bar{\eta}_k\Big\} \tag{20}$$

where $\bar{\eta}_k = [\ \eta_k^T \ \ \eta_{k-\tau}^T \ \ e_k^T\ ]^T$ and

$$\tilde{\mathcal{R}} = \begin{bmatrix} \mathcal{R}_{11} - \frac{1}{n}\bar{\mathcal{L}}^T\bar{\mathcal{L}} & \mathcal{R}_{12} & \mathcal{R}_{13} \\ * & \mathcal{R}_{22} & \mathcal{R}_{23} \\ * & * & \mathcal{R}_{33} \end{bmatrix}.$$

By considering (13), one has $\tilde{\mathcal{R}} < 0$ and, subsequently

$$\mathbb{E}\{\|\eta_k\|^2\} \le -\lambda_{\min}(-\tilde{\mathcal{R}})\|\bar{\eta}_k\|^2. \tag{21}$$

Finally, along the similar line of the proof of Theorem 1 in [34], once can prove that the augmented system (11) is exponentially mean-square stable.

To establish the $\mathcal{H}_\infty$ performance, we introduce the following:

$$
\begin{aligned}
&\mathbb{E}\Big\{\Delta V_k+\frac{1}{n}\|\tilde{z}_k\|^2-\gamma^2\|\tilde{w}_k\|^2\Big\}\\
\leq\ &\mathbb{E}\Big\{\bar{\eta}_k^T\Omega\bar{\eta}_k+2\eta_k^T\bar{\mathcal{A}}^T(I\otimes P)\bar{\mathcal{D}}\tilde{w}_k+2\eta_{k-\tau}^T\bar{\mathcal{A}}_d^T(I\otimes P)\bar{\mathcal{D}}\tilde{w}_k\\
&+2e_k^T\bar{\mathcal{B}}^T(I\otimes P)\bar{\mathcal{D}}\tilde{w}_k+\tilde{w}_k^T\bar{\mathcal{D}}^T(I\otimes P)\bar{\mathcal{D}}\tilde{w}_k+\tilde{w}_k^T\Upsilon_2\tilde{w}_k+\frac{1}{n}\eta_k^T\bar{\mathcal{L}}^T\bar{\mathcal{L}}\eta_k-\gamma^2\tilde{w}_k^T\tilde{w}_k\\
&+2\lambda\eta_k^T(\mathbf{1}_n\otimes\tilde{\mathbf{I}})^T\mathcal{C}^T(\Theta\otimes I)\mathcal{D}v_k+\lambda v_k^T\mathcal{D}^T(\Theta\otimes I)\mathcal{D}v_k\Big\}
\end{aligned}
\tag{22}
$$

which leads to

$$
\mathbb{E}\Big\{\Delta V_k+\frac{1}{n}\|\tilde{z}_k\|^2-\gamma^2\|\tilde{w}_k\|^2\Big\}\leq\mathbb{E}\big\{\tilde{\eta}_k^T\mathcal{R}\tilde{\eta}_k\big\}<0. \tag{23}
$$

It follows from the zero initial conditions and (23) that

$$
\frac{1}{n}\sum_{k=0}^{\infty}\mathbb{E}\{\|\tilde{z}_k\|^2\}\leq\gamma^2\sum_{k=0}^{\infty}\|\tilde{w}_k\|^2
$$

and therefore the proof is now complete. ■

Having established the analysis results, we are now ready to handle the distributed estimator design problem with an event-trigged communication mechanism. For this purpose, we firstly need to deal with the trace operation in Theorem 1, and then establish a sufficient condition for the existence of the desired $\mathcal{H}_\infty$ estimator.

Theorem 2: Let the estimator parameters $K_{i,1}$ and $K_{i,2}$ $(i=1,2,\cdots,n)$ as well as a prescribed disturbance attenuation level $\gamma>0$ be given. The dynamics of estimation errors (11) is exponentially mean-square stable and also satisfies the prespecified $\mathcal{H}_\infty$ performance constraint (12) if there exist two positive definite matrices P and Q, and positive scalars λ and ϖ_i $(i=1,2,\cdots,s)$ satisfying

$$
\begin{bmatrix}-\varpi_i & \bar{\pi}^TP\\ * & -P\end{bmatrix}<0,\quad i=1,2,\cdots,s \tag{24}
$$

$$
\begin{bmatrix}\bar{\mathcal{R}} & \bar{\mathcal{R}}_5^T & \mathcal{W}^T\mathcal{Y}^T(I\otimes P)\\ * & -I\otimes P & 0\\ * & * & -I\otimes P\end{bmatrix}<0 \tag{25}
$$

where

$$
\begin{aligned}
&\bar{\mathcal{R}}=\begin{bmatrix}\bar{\mathcal{R}}_{11} & 0 & 0 & \bar{\mathcal{R}}_{14}\\ * & \bar{\mathcal{R}}_{22} & 0 & 0\\ * & * & \bar{\mathcal{R}}_{33} & 0\\ * & * & * & \bar{\mathcal{R}}_{44}\end{bmatrix},\\
&\bar{\mathcal{R}}_5=(I\otimes P)\begin{bmatrix}\bar{\mathcal{A}} & \bar{\mathcal{A}}_d & \bar{\mathcal{B}} & \bar{\mathcal{D}}\end{bmatrix},\quad \mathcal{Y}=\begin{bmatrix}\mathcal{Y}_1 & \mathcal{Y}_2 & \cdots & \mathcal{Y}_n\end{bmatrix},\\
&\mathcal{W}=\sqrt{\bar{\alpha}(1-\bar{\alpha})}\mathrm{diag}\Big\{\mathrm{diag}\big\{\mathcal{C}(\mathbf{1}_n\otimes I),0\big\},\ 0,\ I,\ \mathrm{diag}\big\{0,\mathcal{D}\big\}\Big\},\\
&\mathcal{Y}_i=\mathrm{diag}\Big\{\mathrm{diag}\{\mathcal{H}_i\otimes K_{i,2},0\},\ 0,\ \mathcal{H}_i\otimes K_{i,2},\ \mathrm{diag}\big\{0,\mathcal{H}_i\otimes K_{i,2}\big\}\Big\},\\
&\bar{\mathcal{R}}_{11}=\lambda(\mathbf{1}_n\otimes\tilde{\mathbf{I}})^T\mathcal{C}^T(\Theta\otimes I)\mathcal{C}(\mathbf{1}_n\otimes\tilde{\mathbf{I}})\\
&\qquad\quad+(n+1)\sum_{i=1}^{s}\varpi_i\mathbf{I}^T\Gamma_i\mathbf{I}-I\otimes P+I\otimes Q+\frac{1}{n}\bar{\mathcal{L}}^T\bar{\mathcal{L}},\\
&\bar{\mathcal{R}}_{14}=\lambda(\mathbf{1}_n\otimes\tilde{\mathbf{I}})^T\mathcal{C}^T(\Theta\otimes I)\tilde{\mathcal{D}},\quad \bar{\mathcal{R}}_{22}=-I\otimes Q,\\
&\bar{\mathcal{R}}_{33}=-\lambda I,\quad \bar{\mathcal{R}}_{44}=\lambda\tilde{\mathcal{D}}^T(\Theta\otimes I)\tilde{\mathcal{D}}-\gamma^2I.
\end{aligned}
$$

Proof: First, it is not difficult to see that (13) is equivalent to

$$\mathcal{S}+(n+1)\sum_{i=1}^{s}\mathrm{diag}\Big\{\mathrm{tr}\big[P\Pi_i\big]\mathbf{I}^T\Gamma_i\mathbf{I}-\varpi_i,\ 0,\ 0,\ 0\Big\}<0 \tag{26}$$

where $\mathcal{S}_{11}=\mathcal{R}_{11}-\sum_{i=1}^{s}(n+1)\mathrm{tr}\big[P\Pi_i\big]\mathbf{I}^T\Gamma_i\mathbf{I}+(n+1)\sum_{i=1}^{s}\varpi_i$ and

$$\mathcal{S}=\begin{bmatrix}\mathcal{S}_{11} & \mathcal{R}_{12} & \mathcal{R}_{13} & \mathcal{R}_{14}\\ * & \mathcal{R}_{22} & \mathcal{R}_{23} & \mathcal{R}_{24}\\ * & * & \mathcal{R}_{33} & \mathcal{R}_{34}\\ * & * & * & \mathcal{R}_{44}\end{bmatrix}.$$

On the other hand, in light of the Schur complement lemma, (24) is equivalent to

$$\bar{\pi}_i^T P\bar{\pi}_i<\varpi_i,\quad (\varpi_i=1,2,\cdots,s)$$

which, by using the property of matrix trace, can be rewritten as

$$\mathrm{tr}\big[P\Pi_i\big]<\varpi_i,\quad (\varpi_i=1,2,\cdots,s).$$

Therefore, if $\mathcal{S}<0$, one has that (13) is true. Furthermore, by using the Schur complement lemma again, it follows that (25) is equivalent to $\mathcal{S}<0$. Finally, according to Theorem 1, the design requirements R1) and R2) are simultaneously satisfied. The proof is complete. ■

Finally, by utilizing variable substitution, we have the following theorem whose proof is omitted for space saving.

Theorem 3: Let the disturbance attenuation level $\gamma>0$ be given. Assume that there exist two positive definite matrices P and Q, matrices $\tilde{K}_{i,1}$ and $\tilde{K}_{i,2}$ ($i=1,2,\cdots,s$), and positive scalars λ and ϖ_i ($i=1,2,\cdots,s$) satisfying the following linear matrix inequalities

$$\begin{bmatrix}-\varpi_i & \bar{\pi}^T P\\ * & -P\end{bmatrix}<0,\quad i=1,2,\cdots,s \tag{27}$$

$$\begin{bmatrix}\bar{\mathcal{R}} & \mathcal{Z}^T & \mathcal{W}^T\tilde{\mathcal{Y}}^T\\ * & -I\otimes P & 0\\ * & * & -I\otimes P\end{bmatrix}<0 \tag{28}$$

where

$$\mathcal{Z}=\begin{bmatrix}\mathcal{Z}_1 & \mathcal{Z}_2 & \mathcal{Z}_3 & \mathcal{Z}_4\end{bmatrix},\quad \tilde{\mathcal{Y}}=\begin{bmatrix}\tilde{\mathcal{Y}}_1 & \tilde{\mathcal{Y}}_2 & \cdots & \tilde{\mathcal{Y}}_n\end{bmatrix},$$

$$\tilde{\mathcal{Y}}_i=\mathrm{diag}\Big\{\mathrm{diag}\{\mathcal{H}_i\otimes\tilde{K}_{i,2},0\},\ 0,\ \mathcal{H}_i\otimes\tilde{K}_{i,2},\ \mathrm{diag}\{0,\mathcal{H}_i\otimes\tilde{K}_{i,2}\}\Big\},$$

$$\mathcal{Z}_1=\begin{bmatrix}PA & 0\\ 0 & (I\otimes P)\mathcal{A}-\tilde{\mathcal{K}}_1\mathcal{C}-\tilde{\mathcal{K}}_2(\mathcal{H}\Xi\otimes I)\mathcal{C}\end{bmatrix},\quad \mathcal{Z}_2=\mathrm{diag}_{n+1}\{PA_d\},$$

$$\mathcal{Z}_3=\begin{bmatrix}0\\ -\tilde{\mathcal{K}}_2(\mathcal{H}\Xi\otimes I)\end{bmatrix},\quad \mathcal{Z}_4=\begin{bmatrix}PB & 0\\ \mathbf{1}_n\otimes(PB) & -\tilde{\mathcal{K}}_1\mathcal{D}-\tilde{\mathcal{K}}_2(\mathcal{H}\Xi\otimes I)\mathcal{D}\end{bmatrix}.$$

In this case, with the estimator gain matrices given by $K_{i,1}=P^{-1}\tilde{K}_{i,1}$ and $K_{i,2}=P^{-1}\tilde{K}_{i,2}$ ($i=1,2,\cdots,s$), the dynamics of estimation errors (11) is exponentially mean-square stable while achieving the prespecified $\mathcal{H}_\infty$ performance constraint (12).

Remark 3: In this paper, a novel distributed estimator is first proposed in order to properly fuse two classes of information (i.e. the innovation for the node itself without the event-triggering mechanism and the innovation for neighboring nodes subject to the event-triggering mechanism). It can be seen that, in the main results in Theorems 1-3, the information about the given topology, the probability of packet dropouts and the threshold of event-triggering conditions on the estimation performance are all involved. For instance, the matrix $\Theta = \mathrm{diag}\{\delta_1, \delta_2, \cdots, \delta_n\}$ mainly lies in matrices $\mathcal{R}_{11}$ and $\mathcal{R}_{44}$ in Theorem 1. It can be found that, with increased the threshold δ_i, the inequality $\mathcal{R}_{11} < 0$ and $\mathcal{R}_{44} < 0$ are more difficultly satisfied which reduces the feasibility of the matrix inequality (13). The main technical contributions lie in that 1) a reasonable model is established to describe the event-triggered communication mechanism and the network-induced phenomena in an unified framework; and 2) the gains of proposed distributed estimators are obtained by solving a set of linear matrix inequalities reflecting both the threshold and the desired $\mathcal{H}_\infty$ performance.

Remark 4: Note that the estimator design scheme provided is in form of LMI techniques. As is well known, the algorithm based on the standard LMI system has a polynomial-time complexity. Specifically, the number $N(\varepsilon)$ of flops needed to compute an ε-accurate solution is bounded by $O(MN^3 \log(V/\varepsilon))$, where M is the total row size of the LMI system, N is the total number of scalar decision variables, V is a data-dependent scaling factor, and ε is relative accuracy set for algorithm [34]. To handle the computational complexity of the developed LMI-based algorithm, we recall that the sensor network size is n and the variable dimensions can be seen from $x_k, \hat{x}_{i,k} \in \mathbb{R}^{n_x}$, $y_{i,k} \in \mathbb{R}^{n_y}$, $z_k, \hat{z}_{i,k} \in \mathbb{R}^{n_z}$, and $w_k, v_k \in \mathbb{R}$. Furthermore, according to Theorem 3, one has both $M = 12(n+1)n_x + sn_x + s$ and $N = (n_x+1)n_x + 2sn_xn_y + 1$. Therefore, the computational complexity of the established result can be represented as $O(nn_x^7)$. In other words, such a computational complexity depends polynomially on the variable dimensions.

IV. A Simulation Example

In this section, a simulation example is presented to illustrate the effectiveness of the proposed design scheme of distributed $\mathcal{H}_\infty$ estimators for discrete-time stochastic nonlinear systems with both event-triggered communication protocol and packet dropouts through sensor networks.

The considered target plant and sensor dynamics are, respectively, modeled by (1) and (2) with the following parameters:

$$A = \begin{bmatrix} 0.72 & 0.40 \\ 0.25 & -0.56 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.15 \end{bmatrix}, \quad B = \begin{bmatrix} 0.20 \\ 0.25 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -0.30 & 0.10 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.27 & 0.12 \end{bmatrix}, \quad C_3 = \begin{bmatrix} -0.32 & 0.10 \end{bmatrix},$$
$$C_4 = \begin{bmatrix} -0.30 & 0.12 \end{bmatrix}, \quad C_5 = \begin{bmatrix} -0.29 & 0.09 \end{bmatrix}, \quad D_1 = 0.02,$$
$$D_2 = D_3 = 0.025, \quad D_4 = D_5 = 0.015, \quad L = \begin{bmatrix} 0.20 & 0.30 \end{bmatrix},$$

and the stochastic nonlinear function $f(x_k, \vartheta_k)$ is chosen as

$$f(x_k, \vartheta_k) = \left(0.1\mathrm{sign}(x_k^1)x_k^1\vartheta_k^1 + 0.13\mathrm{sign}(x_k^2)x_k^2\vartheta_k^2\right) \begin{bmatrix} 0.06 \\ 0.09 \end{bmatrix}$$

where x_k^i $(i = 1, 2)$ denotes the i-th element of the system state, and ϑ_k^1 and ϑ_k^2 are zero mean, uncorrelated Gaussian white noise sequences with unity covariance. It is not difficult to verify that the above stochastic

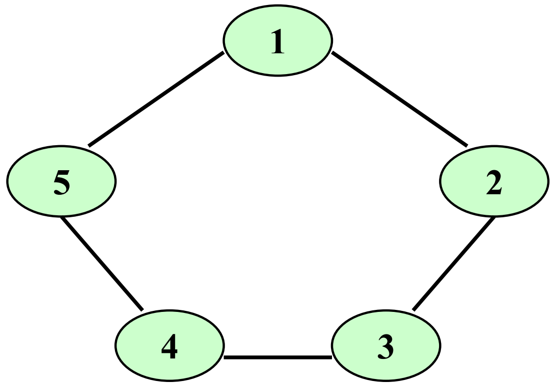


Fig. 1. Topological structure of the sensor network

nonlinear function satisfies

$$\mathbb{E}\left\{f(x_k,\vartheta_k)|x_k\right\}=0,\quad \mathbb{E}\left\{f(x_k,\vartheta_k)f^T(x_k,\vartheta_k)|x_k\right\}=\begin{bmatrix}0.06\\0.09\end{bmatrix}\begin{bmatrix}0.06\\0.09\end{bmatrix}^T x_k^T\begin{bmatrix}0.01 & 0\\0 & 0.0169\end{bmatrix}x_k.$$

The sensor network shown in Fig. 1 is represented by a graph $\mathscr{G}=(\mathscr{V},\mathscr{E},\mathscr{H})$ with the set of nodes $\mathscr{V}=\{1,2,3,4,5\}$, the set of edges $\mathscr{E}=\{(1,2),(1,5),(2,1),(2,3),(3,2),(3,4),(4,3),(4,5),(5,1),(5,4)\}$ and the following adjacency matrix

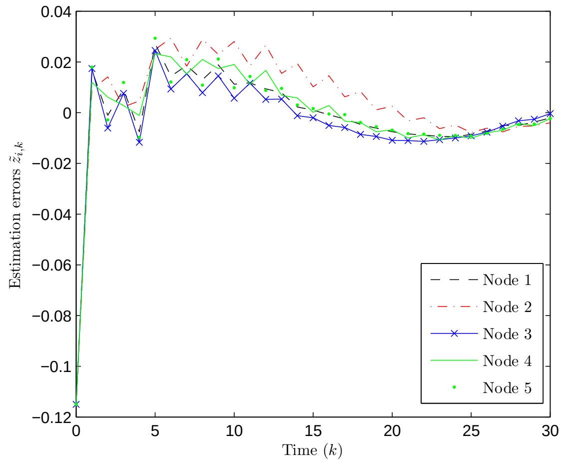
$$\mathscr{H}=\begin{bmatrix}0 & 0.20 & 0 & 0 & 0.12\\0.12 & 0 & 0.20 & 0 & 0\\0 & 0.12 & 0 & 0.20 & 0\\0 & 0 & 0.20 & 0 & 0.20\\0.20 & 0 & 0 & 0.12 & 0\end{bmatrix}.$$

The $\mathcal{H}_\infty$ performance level γ, the threshold δ_i $(i=1,2,\cdots,5)$, the time delay τ and the probability $\bar{\alpha}$ are taken as 0.98, 0.04, 3 and 0.95, respectively. Using the Matlab software (with the YALMIP 3.0), a set of solutions to linear matrix inequalities (27)-(28) in Theorem 3 is obtained as follows:

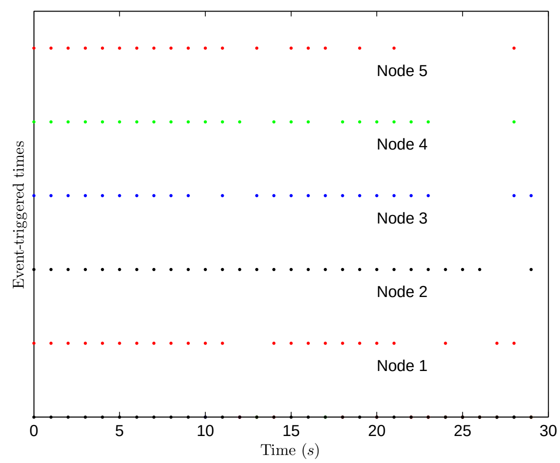
$$\begin{aligned}
&P=\begin{bmatrix}1.0075 & -0.0168\\-0.0168 & 1.0980\end{bmatrix},\quad Q=\begin{bmatrix}0.1681 & -0.0601\\-0.0601 & 0.3167\end{bmatrix},\\
&\tilde{K}_{1,1}=\begin{bmatrix}-1.6208\\-1.4858\end{bmatrix},\quad \tilde{K}_{1,2}=\begin{bmatrix}0.0030\\0.0004\end{bmatrix},\quad \tilde{K}_{2,1}=\begin{bmatrix}-1.5140\\-1.7262\end{bmatrix},\quad \tilde{K}_{2,2}=\begin{bmatrix}-0.0024\\0.0041\end{bmatrix},\\
&\tilde{K}_{3,1}=\begin{bmatrix}-1.5697\\-1.3806\end{bmatrix},\quad \tilde{K}_{3,2}=\begin{bmatrix}0.0039\\-0.0002\end{bmatrix},\quad \tilde{K}_{4,1}=\begin{bmatrix}-1.4643\\-1.5319\end{bmatrix},\quad \tilde{K}_{4,2}=\begin{bmatrix}-0.0008\\0.0016\end{bmatrix},\\
&\tilde{K}_{5,1}=\begin{bmatrix}-1.7357 & -1.5194\end{bmatrix}^T,\quad \tilde{K}_{5,2}=\begin{bmatrix}0.0040 & -0.0003\end{bmatrix}^T,\quad \lambda=1.1497,\quad \varpi=0.4374.
\end{aligned}$$

Furthermore, the desired estimator parameters are

$$\begin{aligned}
&K_{1,1}=\begin{bmatrix}-1.6317\\-1.3781\end{bmatrix},\quad K_{1,2}=\begin{bmatrix}0.0030\\0.0004\end{bmatrix},\quad K_{2,1}=\begin{bmatrix}-1.5293\\-1.5954\end{bmatrix},\quad K_{2,2}=\begin{bmatrix}-0.0024\\0.0037\end{bmatrix},\\
&K_{3,1}=\begin{bmatrix}-1.5794\\-1.2815\end{bmatrix},\quad K_{3,2}=\begin{bmatrix}0.0039\\-0.0001\end{bmatrix},\quad K_{4,1}=\begin{bmatrix}-1.4770\\-1.4177\end{bmatrix},\quad K_{4,2}=\begin{bmatrix}-0.0008\\0.0014\end{bmatrix},\\
&K_{5,1}=\begin{bmatrix}-1.7463 & -1.4104\end{bmatrix}'\quad K_{5,2}=\begin{bmatrix}0.0039 & -0.0002\end{bmatrix}^T.
\end{aligned}$$

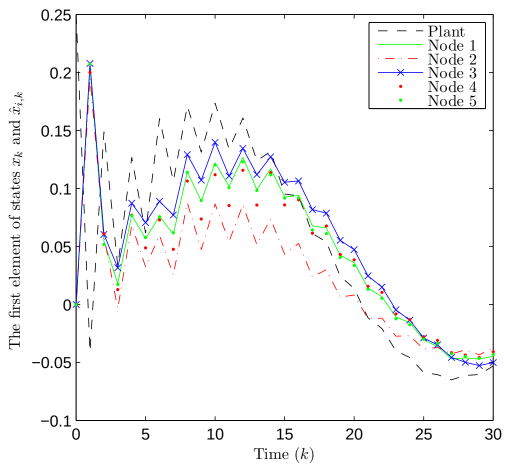


(a) The estimation errors.

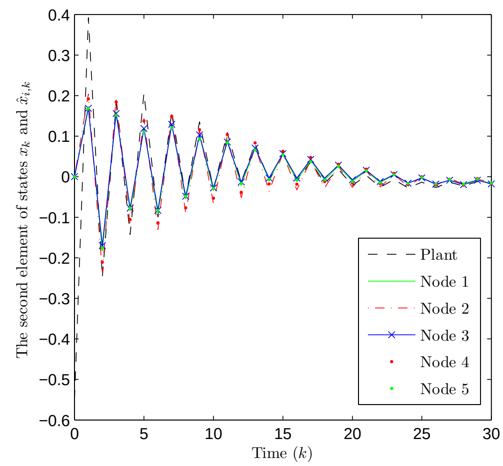


(b) The event-triggered times.

Fig. 2. The estimation errors and event-triggered times.



(a) The first element of dynamic trajectories.



(b) The second element of dynamic trajectories.

Fig. 3. The dynamic trajectories.

In the simulation, the exogenous disturbance inputs are selected as

$$w_k = \frac{0.25\sin(0.2k)}{0.1k+1}, \quad v_k = 0.25\cos(0.2k)\exp(-0.1k).$$

The initial conditions are set as $x_{-3} = [-0.10 \ \ 0.15]^T$, $x_{-2} = [0.20 \ \ -0.27]^T$, $x_{-1} = [0.125 \ \ -0.17]^T$, $x_0 = [0.25 \ \ -0.55]^T$ and $\hat{x}_{i,k} = [0 \ \ 0]^T$ ($k = -3, -2, -1, 0$, $i = 1, 2, \cdots, 5$). Simulation results are shown in Figs. 2-3, where Fig. 2 depicts the estimation errors $\tilde{z}_{i,k}$ as well as the event-triggered times, and Fig. 3 plots the trajectories for the states and the estimates. The simulation results show that estimators have a satisfactory estimation performance which confirms that the distributed estimation scheme presented in this paper is indeed effective.

V. Conclusions

In this paper, we have dealt with the event-triggered distributed $\mathcal{H}_\infty$ state estimation problem for a class of discrete-time stochastic nonlinear systems through sensor networks. To reduce the network burden and the energy consumption, we have considered the event-triggered communication mechanism, where the innovation on each sensor has been transmitted only when a certain triggering condition has been violated. By employing the Lyapunov stability theorem, some sufficient conditions have been established to ensure that the dynamics of the estimation error satisfies the desired $\mathcal{H}_\infty$ performance constraint. Finally, an illustrative example has been provided to confirm the usefulness of the developed state estimation approach. Further research topics include the extension of the main results to the distributed filtering for more general stochastic nonlinear systems with different triggering rules.

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