Event-based H_∞ Consensus Control of Multi-agent Systems with Relative Output Feedback: the Finite-Horizon Case

Qinyuan Liu, Zidong Wang, Xiao He and D. H. Zhou

***Abstract*—In this paper, the H_∞ consensus control problem is investigated over a finite horizon for general discrete time-varying multi-agent systems subject to energy-bounded external disturbances. A decentralized estimation-based output feedback control protocol is put forward via the relative output measurements. A novel event-based mechanism is proposed for each intelligent agent to utilize the available information in order to decide when to broadcast messages and update control input. The aim of the problem addressed is to co-design the time-varying controller and estimator parameters such that the controlled multi-agent systems achieve consensus with a disturbance attenuation level γ over a finite horizon $[0, T]$. A constrained recursive Riccati difference equation approach is developed to derive the sufficient conditions under which the H_∞ consensus performance is guaranteed in the framework of event-based scheme. Furthermore, the desired controller and estimator parameters can be iteratively computed by resorting to the Moore-Penrose pseudo inverse. Finally, the effectiveness of the developed event-based H_∞ consensus control strategy is demonstrated in the numerical simulation.**

***Index Terms*—Multi-agent systems; event-based mechanism; H_∞ consensus; Riccati difference equation; output feedback.**

I. Introduction

During the past decade, the coordination problems of multi-agent systems have been a research focus attracting an increasing interest due primarily to their practical application insights in a variety of realms such as satellite formation control [3], collective behavior of flocking [22], attitude alignment among spacecraft [14], distributed estimation [7] and automated highway systems [2]. A critical issue in coordinated control problems is to design a distributed control protocol for communication behaviors and controller actuation based on the shared information (including graph topologies, real-time states of adjacent nodes and common control algorithms) in order to ensure that all the agents reach an agreement or collectively perform certain actions. Many important results have recently been reported on the cooperative control problems for multi-agent systems, see e.g. [4], [8], [10], [11], [16]–[19], [21], [24], [29] and the references therein.

Consider the practical situations where the states of the multi-agent systems are subject to real-time changes/variations/fluctuations such as time-varying temperature and mutative working conditions. In this circumstance, the evolution of the dynamics of local agents is inevitably dependent on the time. For such time-varying systems, in response to the changes in the environment, the intelligent agents should adopt the *time-varying* cooperative control strategies so as to better reflect the reality. A literature search has shown that several methodologies have recently been developed for time-varying systems and a great number of results have been available for the general control and filtering problems. These methodologies include, but are not limited to, the recursive linear matrix inequality (RLMI) technique [20] and the backward recursive Riccati difference equation (RDE) approach [25]. Unfortunately, up to now, the corresponding results on the coordination problems for time-varying multi-agent systems have been really scattered, and the first motivation of this paper is to shorten such a gap.

Owing to the recent advances in digital technologies, embedded microprocessors, which are responsible for communication between adjacent agents, have been becoming indispensable components of multi-agent systems. In an *ideal* world, the communication bandwidth is assumed to be unlimited and the traditionally periodically triggered communication won't bring any concern in terms of the network load. Such an assumption, however, is not true in some applications subject to certain resource constraints such as limited network bandwidth. For example, the frequent signal transmissions might give rise to network-related adverse phenomena such as communication delays and packet losses. In this case, a novel sporadic scheduling (called the event-based approach) seems to be more preferable, where the pre-described executions are triggered if and only if some 'interesting' events occur. With appropriately developed triggering events, it is predictable that both the reduction of bandwidth occupation and the desired properties of close-loop system (e.g. stability and convergence) can be guaranteed [1].

Up to now, many event-based schemes have been available in the literature for continuous- or discrete-time systems based on the input-to-state stable (ISS) theory, see for instance [15] and the references therein. Such sporadic event-based schemes have been applied in [27] for distributed network control systems (NCSs) with packet loss and transmission delays. Event-based control problems have also been addressed for multi-agent systems, see e.g. [5], [6], [9], [23], [28]. Specifically, in [5], both centralized and decentralized event-based control strategies have been proposed for a group of single-integrator multi-agents in order to reach an agreement according to a fixed undirected network topology. The event-based tracking control problem has been investigated in [9] for leader-follower multi-agent systems with and without communication delays, where the convergence analysis has been provided. Several linear matrix inequality (LMI) conditions have been reported in [28] for event-based control problem of discrete-time heterogeneous multi-agent systems. Additionally, in [6], the distributed event-based methods have been combined with an iterative algorithm to render the implementation more practical. A novel event-based strategy, which is independent of the real-time state of neighbors, has been examined in [23] for both single and double-integrator agents such that the continuous monitoring is no longer required. As for general agent dynamics, in [32], the authors have intensively investigated the consensus of multi-agent systems when the individual full state is available for its neighbors. Furthermore, in [30] and [31], the event-based consensus problems have been thoroughly studied for general linear or nonlinear system dynamics by assuming that each agent is passive.

It should be pointed out, despite the recent surge of research attention on the event-based schemes for multi-agent systems, several challenges still remain. First, most available results have been concerned with single or double-integrator *time-invariant* models *without* any external disturbances. Unfortunately, in real-world applications, the behaviors of local agents are usually complicated especially when they suffer from various stochastic disturbances and communication-induced noises. It is of vital importance to suppress the influence from the external noise disturbances for general multi-agent systems through analyzing and synthesizing the cooperative control schemes, for which the H_∞ disturbance rejection attenuation would be a suitable performance index. So far, some preliminary results [12], [13] have been reported on the H_∞ consensus control problem with the assumption that real-time information of adjacent nodes' *full states* is available in order for the agents to share their local information at every sampling instant. Such an assumption is somewhat restrictive

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in practice since the state of agents may not be easily available in some occasions. Also, the real-time full state feedback would increase the communication frequency, which is undesirable. To resolve the problem, a natural idea is to develop an appropriate event-based control scheme by using *accessible relative measurement outputs* instead of the full states, and this motivates us to investigate the H_∞ consensus control problem.

Summarizing the above discussions, in this paper, we aim to deal with the H_∞ *consensus* problems for a class of linear *time-varying* multi-agent systems with external disturbances by using *relative output feedback*. An event-based scheme is proposed to distributively calculate event times based on a predetermined data transmission condition in order to reduce the communication burden among agents. Moreover, we would like to derive sufficient conditions under which the consensus error is bounded in an H_∞ sense over a finite horizon. The corresponding time-varying estimation-based output feedback controller is designed via optimizing an H_2 performance index. The main contributions can be highlighted as follows: *1) the discrete-time intelligent agents with general dynamics are under consideration which cover the frequently investigated integrator models as special cases; 2) the transient behaviors are studied in order to reflect the time-varying nature of the addressed multi-agent systems; 3) a novel event-based control protocol is first proposed for the time-varying multi-agent systems so as to achieve the prespecified H_∞ constraints over a finite horizon $[0,T]$; and 4) different from the existing literature, relative measurements between adjacent agents are utilized for the event-based feedback control.*

Notation. Except where otherwise stated, the notations used throughout the paper are standard. $\mathcal{L}_2([0,T];\mathbb{R}^n)$ is the space of square-summable n-dimensional vector functions over $[0,T]$. $\mathbf{1}$ represents the $N\times 1$ column vector of with all the elements equal to 1 and $\mathbf{0}_n$ denotes the $n\times n$ zero matrix. $\mathrm{diag}\{.\}$ stands for a block-diagonal matrix and $\mathrm{col}_N\{x_i\}$ represents $[x_1^T,\cdots,x_N^T]^T$. M^T denotes the transpose of a matrix M, and $M^\dagger\in\mathbb{R}^{n\times m}$ describes the Moore-Penrose pseudo inverse of $M\in\mathbb{R}^{m\times n}$. $\|x\|$ stands for the Euclidean norm of a vector x and $\|f\|_\Omega^2$ represents $f^T\Omega f$.

II. Problem Formulation and Preliminaries

In this section, we introduce some preliminaries related to distributed control of multi-agent systems and then describe the problem setup.

A. Graph topology

The communication topology of the system is described by a fixed undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ of order N with the set of nodes $\mathcal{V}=\{1,2,\cdots,N\}$, the set of edges $\mathcal{E}=\mathcal{V}\times\mathcal{V}$, and the weighted adjacency matrix $\mathcal{A}=[a_{ij}]$. The weighted adjacency matrix of the graph is a matrix with nonnegative elements a_{ij} satisfying the property $a_{ij}>0\Longleftrightarrow(i,j)\in\mathcal{E}$, which means if there is an edge between nodes i and j, then they are called adjacent. Note that for an undirected graph $\mathcal{G}$, $\mathcal{A}$ is a symmetric $N\times N$ matrix given by $a_{ij}=1$ (if nodes i and j are adjacent) and $a_{ij}=0$ (otherwise). The graph $\mathcal{G}$ is assumed to be connected where there is a path between any of two nodes. The neighbors of node i is denoted by $\mathcal{N}_i\triangleq\{j|(i,j)\in\mathcal{E}\}$. The degree D is a diagonal matrix with elements d_i defined as the number of its adjacent vertices. The Laplacian of undirected graph $\mathcal{G}$ is a symmetric positive semidefinite matrix $H=D-A$.

B. Problem Formulation

Consider a multi-agent system with N identical agents, labelled by $1,2,\cdots,N$, respectively. The dynamics of agent i is governed by the following discrete time-varying systems:

$$\begin{cases}x_i(k+1)=A_kx_i(k)+B_ku_i(k)+D_kw_i(k),\\ y_i(k)=C_kx_i(k)+E_kv_i(k),\\ z_i(k)=M_kx_i(k)\end{cases}\tag{1}$$

where $x_i(k)\in\mathbb{R}^{n_x}$ is the system state, $w_i(k)\in\mathbb{R}^{n_w}$ and $v_i(k)\in\mathbb{R}^{n_v}$ are the external disturbances belonging to $\mathcal{L}_2[0,T]$, $y_i(k)$ is the measurement output, $z_i(k)$ is the controlled output, and $u_i(k)$ is a sequence of control inputs. Note that the system (1) under consideration is quite general which includes the single- and second-order integrators as special cases.

Owing to the time-varying manner of the system and the influence from external disturbances, it is difficult for the multi-agents to achieve asymptotic and accurate consensus. Therefore, in this paper, the finite-horizon H_∞ consensus problem is taken into account to alleviate the interferences from the time-varying parameters and the external disturbances on the desired agreement among the agents.

Definition 1: Let a disturbance attenuation level $\gamma>0$ and a positive definite matrix $W=W^T>0$ be given. The multi-agent system (1) with a prefixed connected topology is said to satisfy the H_∞ consensus performance constraint over the finite horizon $[0,T]$ if the following inequality holds:

$$\begin{aligned}&\sum_{i=1}^N\|\bar z_i(k)\|_{[0,T]}^2\\ &<\gamma^2\sum_{i=1}^N\Big\{\|w_i(k)\|_{[0,T]}^2+\|v_i(k)\|_{[0,T]}^2+\bar x_i^T(0)W\bar x_i(0)\Big\}\end{aligned}\tag{2}$$

where $\bar z_i(k)=z_i(k)-\frac{1}{N}\sum_{j=1}^N z_i(k)$, $\|\bar z_i(k)\|_{[0,T]}^2=\sum_{k=0}^T\|\bar z_i(k)\|^2$ and $\bar x_i(0)=x_i(0)-\frac{1}{N}\sum_{j=1}^N x_i(0)$.

Remark 1: The asymptotic (steady-state) H_∞ consensus problem has been dealt with in [12] for time-invariant system over an infinite horizon. To capture the behaviors of time-varying systems addressed in this paper, it makes more sense to study the finite-horizon (transient) H_∞ consensus control problem with hope to attenuate the effects from external disturbances over a specific time period.

C. Cooperative Estimators Design

Consider the situation where each agent has access to the measurements relative to its adjacent agents rather than the local measurements, which means the measurements $y_i(k)$ $(i=1,2,\cdots,N)$ cannot be obtained directly. In the following, a distributed estimator-type consensus protocol is proposed based on relative output measurements, which utilizes the state estimation information (for the controller design) and the output measurement information (for the estimator design).

The relative measurement of adjacent agents with respect to agent i is defined by

$$\zeta_i(k)=\sum_{j\in\mathcal{N}_i}a_{ij}(y_j(k)-y_i(k)),\quad \zeta_i(k)\in\mathbb{R}^{n_y}\tag{3}$$

and the relative full state is defined by

$$\xi_i^*(k)=\sum_{j\in\mathcal{N}_i}a_{ij}(x_j(k)-x_i(k)),\quad \xi_i^*(k)\in\mathbb{R}^{n_x}.\tag{4}$$

Note that the relative full state is unavailable but could be estimated. As such, an estimator-type consensus protocol is proposed as

$$\begin{cases}\xi_i(k+1)=A_k\xi_i(k)+B_k\sum_{j\in\mathcal{N}_i}a_{ij}(u_j(k)-u_i(k))\\ \qquad\qquad\quad+L_k(\zeta_i(k)-C_k\xi_i(k))\\ u_i(k)=K_k\xi_i(k)\end{cases}\tag{5}$$

where $\xi_i(k) \in \mathbb{R}^{n_x}$ is an estimate for the variable $\xi_i^*(k)$ and L_k (K_k) are the estimator (controller) parameters to be determined with appropriate dimensions. The control term $\sum_{j\in\mathcal{N}_i}(u_j(k)-u_i(k))$ in (5) requires the information exchanges between each agent and its neighbors, which renders the protocol (5) distributed and convenient for implementation in practical applications.

D. Event-based Mechanism

Thanks to its capabilities to reduce the information exchange frequency, the event-based mechanism has proven to be suitable for distributed control of multi-agent systems subject to limited network resources.

For the purpose of introducing the event-based scheduling, we first denote the triggering instant sequence of agent i by $s_0^i = 0 < s_1^i < s_2^i < \cdots$ and then define $\xi_i^t(k) = \xi_i(s_m^i)$ for $k \in [s_m^i, s_{m+1}^i)$ with the superscript "t" indicating triggering. To this end, the event-based estimator-type consensus protocol (5) can be rewritten as follows with a little abuse of the notation $\xi_i(k)$:

$$\begin{cases}\xi_i(k+1) = A_k\xi_i(k) + B_k\sum_{j\in\mathcal{N}_i} a_{ij}(u_j^t(k) - u_i^t(k)) \\ \qquad\qquad + L_k(\zeta_i(k) - C_k\xi_i(k)) \\ u_i^t(k) = K_k\xi_i^t(k)\end{cases} \tag{6}$$

and the dynamics of the close-loop system can be rewritten as

$$\begin{cases}x_i(k+1) = A_kx_i(k) + B_ku_i^t(k) + D_kw_i(k), \\ y_i(k) = C_kx_i(k) + E_kv_i(k), \\ z_i(k) = M_kx_i(k).\end{cases} \tag{7}$$

Moreover, let σ be a given positive scalar and the distributed triggering function $f_i(k, \xi_i(k), \xi_i^t(k), \sigma)$ be given by

$$f_i(k) = (\xi_i(k) - \xi_i^t(k))^T(\xi_i(k) - \xi_i^t(k)) - \sigma\xi_i^T(k)\xi_i(k) \tag{8}$$

which takes values in $\mathbb{R}$ for each agent. The control execution is triggered as long as the inequality $f_i(.) > 0$ is satisfied, i.e.

$$(\xi_i(k) - \xi_i^t(k))^T(\xi_i(k) - \xi_i^t(k)) > \sigma\xi_i^T(k)\xi_i(k). \tag{9}$$

Therefore, the next triggering instant is determined iteratively by

$$s_{m+1}^i = \min\{k \in \mathbb{N} | k > s_m^i, f_i(k, \xi_i(k), \xi_i^t(k), \sigma) > 0\} \tag{10}$$

Remark 2: From the event-based protocol (6), it can be seen that the estimator constructed for each agent updates the signals $\sum_{j\in\mathcal{N}_i}(u_j^t(k) - u_i^t(k))$ at event triggering instants for both the adjacent agents and itself, while the control input signals are corrected only when the agent triggers an event. On the other hand, once (9) is satisfied, a new event is triggered to correct difference of the estimation $\xi_i(k)$ (local knowledge) and the estimation at event times $\xi_i^t(k)$ (shared knowledge between adjacent nodes). According to the triggering rules (10), at triggering instants, we have $\xi_i^t(k) = \xi_i(k)$ which indicates that $f_i(k, \xi_i(k), \xi_i^t(k), \sigma) \le 0$ would never be violated during the system process. Moreover, the scalar σ regulates the triggering frequency. Obviously, more events would be triggered if such a scalar decreases. Particularly, when $\sigma = 0$, the event-based control approach reduces to the classical clock-driven control one.

To facilitate the subsequent formulation, the corresponding estimation error and the control input error (between actual and ideal input signals) are defined, respectively, by

$$e_i^s(k) = \xi_i(k) - \xi_i^*(k), \quad e_i^t(k) = \xi_i^t(k) - \xi_i^*(k).$$

For notational presentation convenience, here we denote

$$\begin{aligned}x(k) &= \text{col}_N\{x_i(k)\},\ \xi^*(k) = \text{col}_N\{\xi_i^*(k)\},\ \xi(k) = \text{col}_N\{\xi_i(k)\} \\ v(k) &= \text{col}_N\{v_i(k)\},\ w(k) = \text{col}_N\{w_i(k)\},\ e^s(k) = \text{col}_N\{e_i^s(k)\} \\ e^t(k) &= \text{col}_N\{e_i^t(k)\},\ u^t(k) = \text{col}_N\{u_i^t(k)\},\ u(k) = \text{col}_N\{u_i(k)\} \\ z(k) &= \text{col}_N\{z_i(t)\},\ \bar{z}(k) = \text{col}_N\{\bar{z}_i(t)\}\end{aligned}$$

Combining the consensus protocol (6) with the multi-agent systems (7), we have the following compact form:

$$\begin{aligned}x(k+1) &= (I_N \otimes A_k)x(k) + (I_N \otimes B_kK_k)e^t(k) \\ &\quad + (I_N \otimes B_kK_k)\xi^*(k) + (I_N \otimes D_k)w(k)\end{aligned} \tag{11}$$

It can be verified that $\xi^*(k) = -(H \otimes I_{n_x})x(k)$. Letting $s(k) \triangleq e^t(k) - e^s(k) = \xi^t(k) - \xi(k)$ represent the control error introduced by the event-based schedule, (11) becomes

$$\begin{aligned}x(k+1) &= (I_N \otimes A_k - H \otimes B_kK_k)x(k) + (I_N \otimes B_kK_k)e^s(k) \\ &\quad + (I_N \otimes B_kK_k)s(k) + (I_N \otimes D_k)w(k)\end{aligned}$$

Next, it follows from the relationship $e^s(k) = \xi(k) - \xi^*(k)$ that

$$\begin{aligned}e^s(k+1) &= (I_N \otimes (A_k - L_kC_k))\, e^s(k) \\ &\quad - (H \otimes L_kE_k)v(k) + (H \otimes D_k)w(k)\end{aligned} \tag{12}$$

Similar to $\bar{z}_i(k)$, we let $\bar{x}_i(k) = x_i(k) - \frac{1}{N}\sum_{i=1}^N x_i(k)$ and then get $\bar{x}(k) = (H_m \otimes I_{n_x})x(k)$, where $\bar{x}(k) = \text{col}_N\{\bar{x}_i(k)\}$ and $H_m = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T$. It can be derived that

$$\begin{aligned}\bar{x}(k+1) &= (H_m \otimes A_k - H_mH \otimes B_kK_k)x(k) + (H_m \\ &\quad \otimes B_kK_k)e^s(k) + (H_m \otimes B_kK_k)s(k) + (H_m \otimes D_k)w(k)\end{aligned} \tag{13}$$

Utilizing the properties of matrix H_m, we have $H_mH_m = H_m$ as well as $H_mH = HH_m = H$. Apparently, (13) can be converted into the form

$$\begin{aligned}\bar{x}(k+1) &= (H_m \otimes A_k - H \otimes B_kK_k)\bar{x}(k) + (H_m \otimes B_kK_k)e^s(k) \\ &\quad + (H_m \otimes B_kK_k)s(k) + (H_m \otimes D_k)w(k)\end{aligned}$$

By defining the variables $X(k) \triangleq [\ \bar{x}^T(k)\ \ (e^s(k))^T]^T$ and $\omega(k) \triangleq [\ w^T(k)\ \ v^T(k)\ ]^T$, we obtain the augmented system as follows

$$\begin{cases}X(k+1) = \mathcal{A}_kX(k) + \mathcal{B}_ks(k) + \mathcal{D}_k\omega(k) \\ \bar{z}(k) = \mathcal{M}_kX(k) \\ \xi(k) = \mathcal{H}X(k)\end{cases} \tag{14}$$

where

$$\begin{aligned}\mathcal{A}_k &= \begin{bmatrix}H_m \otimes A_k - H \otimes B_kK_k & H_m \otimes B_kK_k \\ 0 & I_N \otimes (A_k - L_kC_k)\end{bmatrix}, \\ \mathcal{B}_k &= \begin{bmatrix}H_m \otimes B_kK_k \\ 0\end{bmatrix}, \quad \mathcal{D}_k = \begin{bmatrix}H_m \otimes D_k & 0 \\ H \otimes D_k & -H \otimes L_kE_k\end{bmatrix}, \\ \mathcal{M}_k &= \begin{bmatrix}I_N \otimes M_k & 0\end{bmatrix}, \quad \mathcal{H} = \begin{bmatrix}-H \otimes I_{n_x} & I_N \otimes I_{n_x}\end{bmatrix}.\end{aligned}$$

We are now in a position to state the problem addressed in this paper as follows. We aim to design appropriate controller and estimator parameters to ensure that the controlled system (14) achieves the following H_∞ consensus performance constraint over the finite horizon $[0, T]$:

$$\|\bar{z}(k)\|_{[0,T]}^2 < \gamma^2\left\{\|\omega(k)\|_{[0,T]}^2 + \bar{x}^T(0)(I_N \otimes W)\bar{x}(0)\right\}. \tag{15}$$

III. Main Results

To start with, we first deal with the performance analysis problem, that is, derive the sufficient conditions under which the H_∞ consensus performance requirement (15) is guaranteed in terms of the feasibility of a backward RDE.

Lemma 1: Consider the multi-agent systems (1) with the estimator-type consensus protocol (5) and the event-based mechanism (10). Given a disturbance attenuation level $\gamma > 0$, a positive scalar $\theta > 0$ and a positive definite matrix W. For any disturbance sequence $\{\omega(k)\}_{0\le k\le T}$, the augmented system (14) satisfies the H_∞ consensus performance index if there exist a set of matrices $\{K_k\}_{0\le k\le T}$, $\{L_k\}_{0\le k\le T}$ and a set of non-negative definite matrices

$\{P_k\}_{0\leq k\leq T+1}$ (with final condition $P_{T+1}=0$) to the following backward RDE:

$$P_k = \mathcal{A}_k^T R_{k+1}\mathcal{A}_k + \mathcal{A}_k^T R_{k+1}\mathcal{D}_k\Omega_{k+1}^{-1}\mathcal{D}_k^T R_{k+1}\mathcal{A}_k + \mathcal{M}_k^T\mathcal{M}_k + \sigma\theta^2\mathcal{H}^T\mathcal{H} \tag{16}$$

subject to

$$\begin{aligned}\Phi_{k+1} &\triangleq \theta^2 I - \mathcal{B}_k^T P_{k+1}\mathcal{B}_k > 0,\\ \Omega_{k+1} &\triangleq \gamma^2 I - \mathcal{D}_k^T R_{k+1}\mathcal{D}_k > 0,\\ P_0 &< \gamma^2(I_{2N}\otimes W)\end{aligned} \tag{17}$$

where

$$R_{k+1} \triangleq P_{k+1}(I-\theta^{-2}\mathcal{B}_k\mathcal{B}_k^T P_{k+1})^{-1}. \tag{18}$$

Proof: First, define a Lyapunov-like quadratic function $V_k(X(k)) = X^T(k)P_kX(k)$. For a set of non-negative definite matrices $\{P_k\}_{0\leq k\leq T}$, the difference of $V_k(X(k))$ along the trajectory of (14) is calculated as follows:

$$\begin{aligned}Y_k^{(1)} &\triangleq V_{k+1}(X(k+1)) - V_k(X(k))\\ &= \|\mathcal{A}_kX(k)+\mathcal{B}_ks(k)+\mathcal{D}_k\omega(k)\|^2_{P_{k+1}} - \|X(k)\|^2_{P_k}\end{aligned}$$

Furthermore, by introducing the zero term $\|\bar{z}(k)\|^2-\gamma^2\|\omega(k)\|^2-\|\bar{z}(k)\|^2+\gamma^2\|\omega(k)\|^2+\theta^2\|s(k)\|^2-\theta^2\|s(k)\|^2$, we obtain that

$$\begin{aligned}Y_k^{(1)} &= X^T(k)(\mathcal{A}_k^TP_{k+1}\mathcal{A}_k+\mathcal{M}_k^T\mathcal{M}_k-P_k)X(k) - \|s(k)\|^2_{\Phi_{k+1}}\\ &- \|\omega(k)\|^2_{\gamma^2I-\mathcal{D}_k^TP_{k+1}\mathcal{D}_k} + 2X^T(k)\mathcal{A}_k^TP_{k+1}\mathcal{D}_k\omega(k)\\ &+ 2s^T(k)\mathcal{B}_k^TP_{k+1}\mathcal{D}_k\omega(k) + 2X^T(k)\mathcal{A}_k^TP_{k+1}\mathcal{B}_ks(k)\\ &- \|\bar{z}(k)\|^2+\gamma^2\|\omega(k)\|^2+\theta^2\|s(k)\|^2\end{aligned}$$

By using the matrix inversion lemma and substituting Φ_{k+1}, one has

$$R_{k+1} = P_{k+1}+P_{k+1}\mathcal{B}_k\Phi_{k+1}^{-1}\mathcal{B}_k^TP_{k+1} = P_{k+1}(I-\theta^{-2}\mathcal{B}_k\mathcal{B}_k^TP_{k+1})^{-1}$$

Completing the square for $s(k)$, we have the following equation:

$$\begin{aligned}Y_k^{(1)} &= X^T(k)(\mathcal{A}_k^TR_{k+1}\mathcal{A}_k+\mathcal{M}_k^T\mathcal{M}_k-P_k)X(k)\\ &+ 2X^T(k)\mathcal{A}_k^TR_{k+1}\mathcal{D}_k\omega(k) - \|s(k)-s^*(k)\|^2_{\Phi_{k+1}}\\ &- \|\omega(k)\|^2_{\Omega_{k+1}} - \|\bar{z}(k)\|^2+\gamma^2\|\omega(k)\|^2+\theta^2\|s(k)\|^2\end{aligned}$$

where $s^*(k) \triangleq \Phi_{k+1}^{-1}\mathcal{B}_k^TP_{k+1}\mathcal{D}_k\omega(k) + \Phi_{k+1}^{-1}\mathcal{B}_k^TP_{k+1}\mathcal{A}_kX(k)$. Furthermore, considering the definition of Ω_{k+1} in (17) and letting $\omega^*(k)\triangleq\Omega_{k+1}^{-1}\mathcal{D}_k^T\ R_{k+1}\mathcal{A}_kX(k)$, we complete the square for $\omega(k)$ as follows:

$$\begin{aligned}Y_k^{(1)} &= X^T(k)(\mathcal{A}_k^TR_{k+1}\mathcal{A}_k+\mathcal{M}_k^T\mathcal{M}_k+\mathcal{A}_k^TR_{k+1}^T\mathcal{D}_k\Omega_{k+1}^{-1}\\ &\times\mathcal{D}_k^TR_{k+1}\mathcal{A}_k-P_k)X(k) - \|s(k)-s^*(k)\|^2_{\Phi_{k+1}} - \|\bar{z}(k)\|^2\\ &+ \gamma^2\|\omega(k)\|^2+\theta^2\|s(k)\|^2 - \|\omega(k)-\omega^*(k)\|^2_{\Omega_{k+1}}\end{aligned}$$

Without loss of generality, the initial estimation error can be chosen as zero and let us consider a performance index

$$\begin{aligned}&J_1(K_k,L_k,\omega(k),s(k))\\ &\triangleq \|\bar{z}(k)\|^2_{[0,T]} - \gamma^2\|\omega(k)\|^2_{[0,T]} - \gamma^2X^T(0)(I_{2N}\otimes W)X(0)\end{aligned}$$

According to the triggering inequality (9), one has

$$s^T(k)s(k)\leq\sigma\xi^T(k)\xi(k)=\sigma X^T(k)\mathcal{H}^T\mathcal{H}X(k).$$

Then, it follows from the conditions $\Phi_{k+1}>0$, $\Omega_{k+1}>0$, $P_0<\gamma^2(I_{2N}\otimes W)$ and the final condition $P_{T+1}=0$ that

$$\begin{aligned}&J_1(K_k,L_k,\omega(k),s(k))\\ \leq\ &\sum_{k=0}^{T}Y_k^{(1)} + \sum_{k=0}^{T}(\|\bar{z}(k)\|^2-\gamma^2\|\omega(k)\|^2)\\ \leq\ &\sum_{k=0}^{T}\{-\|(s(k)-s^*(k)\|^2_{\Phi_{k+1}} - \|\omega(k)-\omega^*(k)\|^2_{\Omega_{k+1}}\}<0\end{aligned}$$

where (16) and (17) have been used in deriving the last inequality (19). To this end, it can be concluded that the H_∞ consensus of multi-agent systems is achieved. ■

Since the feasibility of (16) subject to (17) is difficult to tackle directly, let us now propose an approach for computing the controller parameters K_k and estimator parameters L_k in each step under the worst situation, i.e. $\omega(k)=\omega^*(k)=\Omega_{k+1}^{-1}\mathcal{D}_k^TR_{k+1}\mathcal{A}_kX(k)$ and $s(k)=s^*(k)=\Delta_{k+1}X(k)$ with

$$\Delta_{k+1}\triangleq\Phi_{k+1}^{-1}\mathcal{B}_k^T(P_{k+1}\mathcal{D}_k\Omega_{k+1}^{-1}\mathcal{D}_k^TR_{k+1}+P_{k+1})\mathcal{A}_k.$$

In the sequel, we rewrite the augmented system (14) as follows

$$X(k+1)=(\bar{\mathcal{A}}_k+\mathcal{D}_k\Omega_{k+1}^{-1}\mathcal{D}_k^TR_{k+1}\mathcal{A}_k)X(k)+\bar{\mathcal{B}}_ku^t(k) \tag{19}$$

where $\bar{\mathcal{A}}_k\triangleq\text{diag}\{H_m\otimes A_k, I_N\otimes(A_k-L_kC_k)\}$ and $\bar{\mathcal{B}}_k\triangleq[\ H_m\otimes B_k^T\quad 0\ ]^T$.

Before the statement of Lemma 2, we introduce the following notations in order to simplify the presentation.

$$\begin{aligned}&\tilde{\bar{\mathcal{A}}}_k\triangleq\text{diag}\{H_m\otimes A_k, I_N\otimes A_k\}, \mathcal{K}_k\triangleq I_N\otimes K_k, \mathcal{L}_k\triangleq I_N\otimes L_k,\\ &\bar{\bar{\mathcal{A}}}_k\triangleq\text{diag}\{\mathbf{0}_N\otimes I_{n_x}, -I_N\otimes L_kC_k\},\ \mathbf{I}=[\ 0\quad I_{n_xN}\ ]^T\end{aligned}$$

Lemma 2: Consider the multi-agent systems (1) with the estimator-type consensus protocol (5) and the event-based mechanism (10). Let the disturbance attenuation level $\gamma>0$, positive scalars $\theta>0$, $\varepsilon_1>0$ and the positive definite matrix W be given. For the worst disturbance sequence $\{\omega^*(k)\}_{0\leq k\leq T}$ and control error sequence $\{s^*(k)\}_{0\leq k\leq T}$, the augmented system (14) satisfies the H_∞ consensus performance requirement if there exist solutions $(P_k,Q_k,\mathcal{K}_k,\mathcal{L}_k)$ to the following backward RDEs

$$\begin{cases}P_k=\mathcal{A}_k^TR_{k+1}\mathcal{A}_k+\mathcal{A}_k^TR_{k+1}^T\mathcal{B}_k\Omega_{k+1}^{-1}\mathcal{B}_k^TR_{k+1}\mathcal{A}_k\\ \qquad+\mathcal{M}_k^T\mathcal{M}_k+\sigma\theta^2\mathcal{H}^T\mathcal{H}\\ P_k\geq0,\quad P_{T+1}=0\end{cases} \tag{20}$$

and

$$\begin{cases}Q_k=(\bar{\mathcal{A}}_k+\mathcal{D}_k\Omega_{k+1}^{-1}\mathcal{D}_k^TR_{k+1}\mathcal{A}_k)^TQ_{k+1}(\bar{\mathcal{A}}_k+\mathcal{D}_k\Omega_{k+1}^{-1}\\ \quad\times\mathcal{D}_k^TR_{k+1}\mathcal{A}_k)+\mathcal{M}_k^T\mathcal{M}_k+\Delta_{k+1}^T\mathcal{K}_k^T\Pi_{k+1}\mathcal{K}_k\Delta_{k+1}\\ \quad-\tilde{\bar{\mathcal{A}}}_k^TQ_{k+1}\bar{\mathcal{B}}_k\Pi_{k+1}^{-1}\bar{\mathcal{B}}_k^TQ_{k+1}^T\tilde{\bar{\mathcal{A}}}_k+\Xi_{k+1}^{(1)}+\Xi_{k+1}^{(1)T}\\ \quad-\varepsilon_1^{-1}\mathbf{I}\Xi_{k+1}^{(2)T}\Xi_{k+1}^{(2)}\mathbf{I}^T\\ Q_{T+1}=0\end{cases} \tag{21}$$

subject to

$$\begin{cases}\Phi_{k+1}\triangleq\theta^2I-\mathcal{B}_k^TP_{k+1}\mathcal{B}_k>0\\ \Omega_{k+1}\triangleq\gamma^2I-\mathcal{D}_k^TR_{k+1}\mathcal{D}_k>0\\ \Pi_{k+1}\triangleq\bar{\mathcal{B}}_k^TQ_{k+1}\bar{\mathcal{B}}_k+I>0\\ P_0<\gamma^2(I_{2N}\otimes W)\end{cases} \tag{22}$$

$\Xi_{k+1}^{(1)}$ and $\Xi_{k+1}^{(2)}$ are defined as follows:

$$\begin{aligned}\Xi_{k+1}^{(1)}&\triangleq(\mathcal{D}_k\Omega_{k+1}^{-1}\mathcal{D}_k^TR_{k+1}\mathcal{A}_k)^TQ_{k+1}\bar{\mathcal{B}}_k\mathcal{K}_k\mathcal{H}+(\bar{\mathcal{A}}_k+\mathcal{D}_k\Omega_{k+1}^{-1}\\ &\times\mathcal{D}_k^TP_{k+1}\mathcal{A}_k)^TQ_{k+1}\bar{\mathcal{B}}_k\mathcal{K}_k\Delta_{k+1}+\mathcal{H}^T\mathcal{K}_k^T\Pi_{k+1}\mathcal{K}_k\Delta_{k+1}\\ &+\mathbf{I}(I_N\otimes L_kC_k)^T\Xi_{k+1}^{(2)}(H\otimes I_{n_x})[\ I_{n_xN}\quad 0\ ]\\ \Xi_{k+1}^{(2)}&\triangleq Q_{k+1}^{(21)}(H_m\otimes B_kK_k)\end{aligned}$$

where $Q_{k+1}^{(ij)}(i,j=1,2)$ is the block elements of the matrix Q_{k+1} with appropriate dimensions.

Proof: Define a cost functional as $J_2(K_k,L_k,\omega^*(k),s^*(k))\triangleq\|\bar{z}(k)\|^2_{[0,T]}+\|u^t(k)\|^2_{[0,T]}+\varepsilon_1\|\tilde{e}^s(k)\|^2_{[0,T]}$, where $\tilde{e}^s(k)\triangleq(I_N\otimes L_kC_k)e^s(k)$, and ε_1 is introduced for more flexibility in the estimator parameter design. Furthermore, introducing the function $Y_k^{(2)}\triangleq$

$X^T(k+1)Q_{k+1}X(k+1) - X^T(k)Q_kX(k)$, it follows from (19) that

$$\begin{aligned}&Y_k^{(2)}\\&= X^T(k)(\bar{\mathcal{A}}_k + \mathcal{D}_k\Omega_{k+1}^{-1}\mathcal{D}_k^T R_{k+1}\mathcal{A}_k)^T Q_{k+1}(\bar{\mathcal{A}}_k + \mathcal{D}_k\Omega_{k+1}^{-1}\mathcal{D}_k^T\\&\times R_{k+1}\mathcal{A}_k)X(k) - X^T(k)Q_kX(k) + (u^t(k))^T\bar{\mathcal{B}}_k^T Q_{k+1}\bar{\mathcal{B}}_k u^t(k)\\&+ 2X^T(k)(\bar{\mathcal{A}}_k + \mathcal{D}_k\Omega_{k+1}^{-1}\mathcal{D}_k^T R_{k+1}\mathcal{A}_k)^T Q_{k+1}\bar{\mathcal{B}}_k u^t(k)\end{aligned}$$

which leads to

$$\begin{aligned}&J_2(K_k, L_k, \omega^*(k), s^*(k))\\&= \sum_{k=0}^{T}(Y_k + \|\bar{z}(k)\|^2 + \|u^t(k)\|^2 + \varepsilon_1\|\tilde{e}^s(k)\|^2) + X^T(0)Q_0X(0)\end{aligned}$$

Noting the fact that $u^t(k) = u(k) + \mathcal{K}_k s(k)$, one has

$$\begin{aligned}&J_2(K_k, L_k, \omega^*(k), s^*(k))\\&= \sum_{k=0}^{T}\{X^T(k)[(\bar{\mathcal{A}}_k + \mathcal{D}_k\Omega_{k+1}^{-1}\mathcal{D}_k^T R_{k+1}\mathcal{A}_k)^T Q_{k+1}(\bar{\mathcal{A}}_k + \mathcal{D}_k\Omega_{k+1}^{-1}\\&\times \mathcal{D}_k^T R_{k+1}\mathcal{A}_k) + \mathcal{M}_k^T\mathcal{M}_k - Q_k]X(k) + \varepsilon_1\|\tilde{e}^s(k)\|^2 + \|u(k)\|_{\Pi_{k+1}}^2\\&+ 2u^T(k)\Pi_{k+1}\mathcal{K}_k\Delta_{k+1}X(k) + X^T(k)\Delta_{k+1}^T\mathcal{K}_k^T\Pi_{k+1}\mathcal{K}_k\Delta_{k+1}X(k)\\&+ 2X^T(k)(\bar{\bar{\mathcal{A}}}_k + \tilde{\bar{\mathcal{A}}}_k)^T Q_{k+1}\bar{\mathcal{B}}_k u(k) + 2X^T(k)(\mathcal{D}_k\Omega_{k+1}^{-1}\mathcal{D}_k^T R_{k+1}\\&\times \mathcal{A}_k)^T Q_{k+1}\bar{\mathcal{B}}_k u(k) + 2X^T(k)(\bar{\mathcal{A}}_k + \mathcal{D}_k\Omega_{k+1}^{-1}\mathcal{D}_k^T R_{k+1}\mathcal{A}_k)^T Q_{k+1}\\&\times \bar{\mathcal{B}}_k\mathcal{K}_k\Delta_{k+1}X(k)\} + X^T(0)Q_0X(0)\end{aligned}$$

Completing the square with respect to $\tilde{e}^s(k)$ and $u(k)$, it follows from (21) that

$$\begin{aligned}&J_2(K_k, L_k, \omega^*(k), s^*(k))\\&= \sum_{k=0}^{T}\{\varepsilon_1\|\tilde{e}^s(k) - \varepsilon_1^{-1}\Xi_{k+1}^{(2)}e^s(k)\|^2 + X^T(0)Q_0X(0)\\&+ \|u(k) + \Pi_{k+1}^{-1}\bar{\mathcal{B}}_k^T Q_{k+1}^T\tilde{\bar{\mathcal{A}}}_k X(k)\|_{\Pi_{k+1}}^2\end{aligned} \quad (23)$$

which ends the proof. ■

Remark 3: Under the constraint of the event rules, the error $s(k)$ may not always be taken as the worst case $s^*(k)$ during the dynamics evolution. In other words, some non-worst cases may happen during the process. Fortunately, it is clear from (19) that, with the solutions $(P_k, Q_k, \mathcal{K}_k, \mathcal{L}_k)$ to (20)-(21) subject to (22), the H_∞ consensus performance requirement (15) can be satisfied even in the non-worst cases.

In the following theorem, an explicit algorithm is given to compute the controller parameters K_k and estimator parameters L_k in each step of the time-varying consensus process.

Theorem 1: Consider the multi-agent systems (1) with the estimator-type consensus protocol (5) and the event-based mechanism (10). Let the disturbance attenuation level $\gamma > 0$, positive scalars $\theta > 0$, $\varepsilon_1 > 0$ and the positive definite matrix W be given. The augmented system (14) satisfies the H_∞ consensus performance requirement if there exist solutions (P_k, Q_k, K_k^*, L_k^*) to the backward RDEs (20) and (21) subject to (22) with the controller and estimator parameters given as follows:

$$\begin{aligned}K_k^* &= -[\delta_k^{(1)}, \delta_k^{(2)}, \cdots, \delta_k^{(N)}][h^{(1)}, h^{(2)}, \cdots, h^{(N)}]^\dagger\\L_k^* &= [\ \kappa_k^{(1)}, \kappa_k^{(2)}, \cdots, \kappa_k^{(N)}\ ][\ \nu_k^{(1)}, \nu_k^{(2)}, \cdots, \nu_k^{(N)}\ ]^\dagger\end{aligned} \quad (24)$$

where

$$\begin{cases}\mathscr{D}_k \triangleq \Pi_{k+1}^{-1}\bar{\mathcal{B}}_k^T Q_{k+1}\tilde{\bar{\mathcal{A}}}_k \triangleq [\delta_k^{(1)T}, \delta_k^{(2)T}, \cdots, \delta_k^{(N)T}]^T\\\mathcal{H} \triangleq [h^{(1)T}, h^{(2)T}, \cdots, h^{(N)T}]^T\\\mathscr{V}_k \triangleq (I_N \otimes C_k) \triangleq [\ \nu_k^{(1)T}, \nu_k^{(2)T}, \cdots, \nu_k^{(N)T}\ ]^T\\\mathscr{W}_k \triangleq \varepsilon_1^{-1}Q_{k+1}^{(21)}(H_m \otimes B_kK_k) \triangleq [\ \kappa_k^{(1)T}, \kappa_k^{(2)T}, \cdots, \kappa_k^{(N)T}\ ]^T\end{cases}$$

Proof: It is easy to verify that the best choice of the controller parameter $\mathcal{K}_k = I_N \otimes K_k$ and the estimator parameter $\mathcal{L}_k = I_N \otimes L_k$ that suppress the cost function (23) is determined in each iteration backward as follows:

$$\begin{aligned}\mathcal{K}_k^* &= \arg\min_{\mathcal{K}_k}\ \text{norm}\ (\mathcal{K}_k\mathcal{H} + \Pi_{k+1}^{-1}\bar{\mathcal{B}}_k^T Q_{k+1}\tilde{\bar{\mathcal{A}}}_k),\\\mathcal{L}_k^* &= \arg\min_{L_k}\ \text{norm}\ (\mathcal{L}_k(I_N \otimes C_k) - \varepsilon_1^{-1}Q_{k+1}^{(21)}(H_m \otimes B_kK_k))\end{aligned}$$

The controller parameter can be rearranged as $K_k^* = \arg\min_{K_k}$ norm $(K_k[h^{(1)}, h^{(2)}, \cdots, h^{(N)}] + [\delta_k^{(1)}, \delta_k^{(2)}, \cdots, \delta_k^{(N)}])$, and therefore $K_k^* = -[\delta_k^{(1)}, \delta_k^{(2)}, \cdots, \delta_k^{(2N)}]\ [h^{(1)}, h^{(2)}, \cdots, h^{(N)}]^\dagger$. Similarly, we have $L_k^* = \arg\min_{L_k}$ norm $(L_k[\ \nu_k^{(1)}, \nu_k^{(2)}, \cdots, \nu_k^{(N)}\ ] - [\ \kappa_k^{(1)},\ \kappa_k^{(2)}, \cdots, \kappa_k^{(N)}\ ])$. By using the Moore-Penrose pseudo inverse, we can easily determine the estimator parameter as $L_k^* = [\kappa_k^{(1)}, \kappa_k^{(2)}, \cdots, \kappa_k^{(N)}][\nu_k^{(1)}, \nu_k^{(2)}, \cdots, \nu_k^{(N)}]^\dagger$. With the designed K_k^* and L_k^*, it follows from (20) and (22) that the performance index $J_1 < 0$. Therefore, the multi-agent system achieves the H_∞ consensus performance constraint over the finite horizon $[0, T]$. The proof is now complete. ■

Finally, let us outline the algorithm as follows,

Algorithm 1:

Step 1.	Set $k = T$, then $P_{T+1} = Q_{T+1} = 0$ are available.
Step 2.	Calculate the matrices Φ_{k+1}, Ω_{k+1} and Π_{k+1} by (22), respectively.
Step 3.	If Φ_{k+1} and Ω_{k+1} are all positive definite, then we can obtain the controller gain K_k and estimator gain L_k by (24) and step to the next procedure, else jump to *Step 6*.
Step 4.	Solve the backward RDEs of (20) and (21) to get P_k and Q_k.
Step 5.	If $k \neq 0$, set $k = k - 1$ and go back to *Step 2*, else turn to the next step.
Step 6.	If $\Phi_{k+1} \leq 0$, or $\Omega_{k+1} \leq 0$, or $\Pi_{k+1} \leq 0$ or $P_0 \geq \gamma^2 W$, this algorithm is infeasible. Stop.

IV. Numerical Example

To illustrate the effectiveness of the obtained theoretical results, we apply the event-based consensus control to an example system by numerical simulation in this section.

Consider a network of four agents, whose topology is represented by the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with the set of the nodes $\mathcal{V} = \{1, 2, 3, 4\}$, set of agents $\mathcal{E} = \{(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2), (3, 4), (4, 3)\}$. The individual dynamics of agents are given by (7) with the following parameters

$$A_k = \begin{bmatrix}0.98 + 0.04\sin(0.12k) & 0.4\\0.15 & -0.75 + 0.2\cos(0.1k)\end{bmatrix}$$

$$\begin{aligned}&B_k = [\ 0.8 + 0.2\sin(0.4k)\quad 0.5\ ]^T \qquad E_k = 0.04\\&C_k = [\ 0.82\quad 0.62 + 0.35\cos(0.3k)\ ] \qquad M_k = [\ 0.7\ \ -0.64\ ]\\&D_k = [0.16 + 0.05\cos(0.3k)\quad 0.18\ ]^T\end{aligned}$$

The process and measurement disturbances belonging to $\mathcal{L}_2[0,\ T]$ are selected as random variables uniformly distributed in the region $[-0.05, 0.05]$ and $[-0.5, 0.5]$, respectively. In this simulation, we choose the thresholds $\sigma = 0.28$, the scalars $\varepsilon_1 = 0.01$ and $\theta = 1.8$. The H_∞ performance index γ, the positive definite matrix W and the time horizon T are taken as $5, \text{diag}_2\{2.8, 2.8\}, 80$, respectively. The initial positions of four agents are uniformly distributed between -5 and 5. According to Theorem 1, the H_∞ performance index for the multi-agent system can be guaranteed with the controller parameters K_k and estimation parameters L_k computed in each iteration. Simulation results are presented in Figs. 1-4. Figs. 1-2 depict the state trajectories of $x_i(k)$ $(i = 1, 2, 3, 4)$ whose j-th element is denoted by $x_i^{(j)}(k)$ $(j = 1, 2)$. The consensus error $\bar{z}(k)$ can be found in Fig. 3 from which we can see that the time-varying

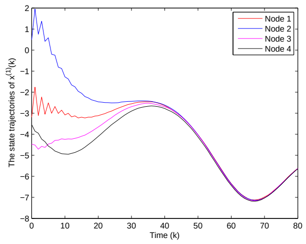

Fig. 1. The state trajectories $x_i^{(1)}(k)$

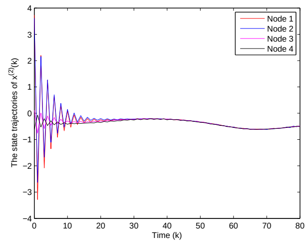

Fig. 2. The state trajectories $x_i^{(2)}(k)$

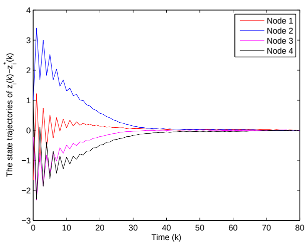

Fig. 3. The consensus error $\bar{z}_i(k)$

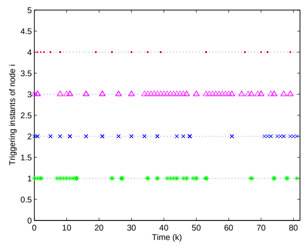

Fig. 4. The triggering instants

multi-agent system achieves H_∞ consensus over a finite horizon. In addition, Fig. 4 shows that the execution frequencies (for information broadcast and actuator adjustments) are dramatically decreased. As a result, the superiority of the proposed event-based mechanism is clearly shown.

V. Conclusions

This paper has addressed the consensus problem for discrete time-varying multi-agent systems with external disturbances. An event-based estimator-based output feedback protocol has been proposed to generate the control signals. Subsequently, by utilizing the H_∞ analysis techniques, a set of RDEs has been derived to determine whether the H_∞ performance constraint is met, and then the appropriate H_∞ controller as well as the estimator parameters have been designed under the worst situations. In the end, an illustrative example has been presented to demonstrate the effectiveness of the theoretical results proposed in this paper.

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