Optimal Design of Single-Tuned Passive Filters Using Response Surface Methodology

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Abstract — This paper presents an approach based on Response Surface Methodology (RSM) to find the optimal parameters of the single-tuned passive filters for harmonic mitigation. The main advantages of RSM can be underlined as easy implementation and effective computation. Using RSM, the single-tuned harmonic filter is designed to minimize voltage total harmonic distortion (THDV) and current total harmonic distortion (THDI). Power factor (PF) is also incorporated in the design procedure as a constraint. To show the validity of the proposed approach, RSM and Classical Direct Search (Grid Search) methods are evaluated for a typical industrial power system.

Keywords — Harmonic mitigation, power factor improvement, passive harmonic filters, response surface methodology.

NOMENCLATURE

Ψₙ, Ψₑ: hᵗʰ harmonic voltage and current phasors,
Ψᵥ, Ψᵢ: hᵗʰ harmonic voltage and current rms values,
ϕᵦ: Phase angle difference between hᵗʰ harmonic voltage and current,
Yᵤ: The corresponding response,
Xᵤ: Coded values of the fᵦ input parameters,
βᵢ: Constant of regression equation,
βᵢ: Regression coefficients of linear terms,
βᵢ: Regression coefficients of square terms,
βᵢ: Regression coefficients of interactions,
eᵦ: The residual experimental error of the nth observation.

INTRODUCTION

Harmonic distortion of voltage and current waveforms significant concerns today’s power systems due to the large proliferation of the nonlinear loads, or harmonic producing loads [1]. The basic capacitors may not provide the desired power factor value under the harmonically distorted voltage and current conditions [2], [3]. Thus, regarding harmonic mitigation and reactive power compensation, passive and active filters have been presented in the literature [4]-[6]. Active filters have superior performance on harmonic mitigation and power factor correction when compared to the passive filters [4]. However, they suffer from high costs [4] and require advanced control algorithms [7]-[9]. Accordingly, passive filters are still extensively used in the industry [10].

Since power factor, current harmonic distortion, voltage harmonic distortion, filter loss and filter investment cost can be contradictory to each other, the design of passive filters is not a straightforward problem. Accordingly, in the literature, minimization of current harmonic distortion (THDI) and/or voltage total harmonic distortion (THDV) [11]-[16], power factor (PF) maximization [17]-[21] and minimization of the filter investment cost (FC) and/or filter loss (FL) objectives [22]-[24] were taken into account for optimal passive filter design. In [25], the optimal passive filter design problem has an objective as minimization of FC and THDV. Reference [26] aimed to achieve minimization of FC, THDV and THDI. [27] and [28] employed passive filters for minimization of the objective function including FC, FL, THDI and THDV. In [29], four objectives as maximization of PF and minimization of FC, THDV and THDI are collectively considered to find optimal passive filter design. In addition to the above mentioned approaches, [30] and [31] employed passive filters for minimization of the harmonic loss factor or maximization of transformer’s loading capability under harmonically contaminated load current conditions.

In these studies, the heuristic methods such as the differential evolution (DE) [13], [14], [22], [26], genetic algorithms [18], [19], [25] and particle swarm optimization method [15], [16], [27], [28] were extensively utilized to solve the optimal passive filter design problem. The most important advantage of the heuristic methods is that they provide a reasonable solution (near globally optimal) in a short time or less iterations [32]. However, their results are sensitive to a large number of specific parameters, which are set by...
designers. Therefore, the specific parameters should be well determined for the success of the methods.

RSM is a statistical technique, which is employed to obtain the functional relationships between the inputs and outputs of a system [33], [34]. In the terminology of experimental design, the inputs and outputs are called as the factors and the responses, respectively. By using the functional relationships based on RSM, the optimal filter design problem can be solved with less computational effort [12]. In addition to that, due to the fact that there is no need for any specific parameter setting in the optimization process with RSM, its implementation is quite straightforward when it is compared to the above mentioned heuristic methods.

This paper presents an application of RSM for the solution of the multi-objective optimization problem of shunt single-tuned passive filters. The purpose is to minimize THDV and THDI while holding PF at its desired value. The major attribute of the proposed approach is that it can easily be implemented for computational efficient solution of the problem.

ANALYSIS OF THE SYSTEM UNDER STUDY

A typical industrial power system is considered to demonstrate the proposed multi-objective optimization approach based on RSM. The single-line diagram of the considered system, which consists of a transformer, the consumer with the linear and nonlinear loads and LC filter connected to load bus, is shown in Figure 1. By taking into account its single-phase equivalent circuit given in Figure 2, the voltage, current and powers can practically be calculated. In the single-phase equivalent circuit, the linear and nonlinear loads are modelled as the parallel connection of an impedance \(( R_h + j h X_h )\) and constant current sources as per harmonics \(( I_{h_h} )\) [35], [36]. For each harmonic, utility side is represented as Thevenin equivalent voltage source \(( V_{th} )\) and Thevenin equivalent impedance \(( Z_{th} )\), which is seen from the load bus.

Fig. 1: A typical industrial power system.

![Diagram of a typical industrial power system](image)

As a result, the following current and voltage equations can be written for the single-phase equivalent circuit of the system using the superposition principle, as follows:

\[
I = \frac{V_{th}}{Z_{th} + Z_{ph}} + \frac{Z_{th} I_{ph}}{Z_{th} + Z_{ph}}
\]

\[
V_s = V_{th} - \frac{Z_{th} I_{ph}}{Z_{th} + Z_{ph}}
\]

where \(Z_{th}\) is the equivalent of the load side’s \( h^{th}\) harmonic impedance \(( R_h + j h X_h )\) and \( h^{th}\) harmonic impedance of single-tuned LC filter (\( Z_{th} = j \left( h X_L - \frac{X_{th}}{h} \right)\)).

\[
Z_{ph} = \frac{Z_{th}}{(Z_{th} + R_h + j h X_h)}
\]

Note that the subscript \(( _{...} )\) denotes phasor values of the respective voltage, current and impedances.

Considering the voltage and current harmonics found from (1) and (2), voltage and current total harmonic distortions (THDV and THDI) can be calculated as follows:

\[
THDV = \sum_{h=1}^{22} \frac{V_h^2}{V_i^2}
\]

\[
THDI = \sum_{h=1}^{22} \frac{I_h^2}{I_i^2}
\]

Using the voltage and current quantities, one can also express active and apparent powers consumed by the load:

\[
P = \sum_i V_i I_i \cos \phi_i
\]

\[
P = V_{th} I_{ph} \cos \phi_s + \sum_i V_i I_i \cos \phi_s = P + P_{th}
\]

\[
S = \sqrt{\sum_i V_i^2} \sqrt{\sum_i I_i^2}
\]

\[
S = V_{th} \sqrt{1 + \left( \sum_i V_i^2 / V_{th}^2 \right) \left( \sum_i I_i^2 / I_{ph}^2 \right)} = S_i \sqrt{1 + THDV^2} \sqrt{1 + THDI^2}
\]

Thus, in terms of these powers, displacement power factor (DPF) and power factor (PF) can be found:

\[
DPF = \frac{P}{S_i}
\]

\[
PF = \frac{P}{S}
\]
\[ PF = \frac{P}{S} = \frac{P + P_f}{S \sqrt{1 + THDV^2 + THDI^2}} \]  

By means of the above mentioned voltage and current relations, power quality indices and power quantities, the multi-objective optimization problem of single-tuned passive filter will be formulated and solved for the studied system in the next sections.

**FORMULATION OF OPTIMAL FILTER DESIGN PROBLEM**

The design of a harmonic filter, which minimizes voltage and current total harmonic distortions, should be useful to prevent malfunctions of the harmonic sensitive system equipments. Accordingly, this paper aims to provide optimal passive harmonic filter for the minimization of THDI and THDV in the studied system. THDI and THDV indices are also considered as two constraints in the optimization problem due to the fact that in IEEE Std. 519-2014 [37] both indices are taken under limitation for several levels of the supply voltage and short circuit power of the system. On the other hand, in various countries, utilities charge consumers an extra fee if their PF is less than 90% [38]. Thus, PF becomes a third constraint of the optimization problem.

Hence, considering the presented objectives and constraints, design of the harmonic filter can be formulated as a multi-objective optimization problem as follows:

Find: \( X_{LF} \) and \( X_{CF} \) values of the passive filter to minimize,

\[ f(X_{LF}, X_{CF}) = w_1 THDV + w_2 THDI \]

Subjected to:

\[ 0 \leq THDI \leq \text{Maximum allowable THDI} \ (\text{IEEE Std. 519}), \]

\[ 0 \leq THDV \leq \text{Maximum allowable THDV} \ (\text{IEEE Std. 519}), \]

\[ 90\% \leq PF \leq 100\%. \]

where \( w_1 \) and \( w_2 \) denotes the weighting factors of the objectives, which are assumed equal (\( o_{w1}, o_{w2}=0.5 \)).

**IV. NUMERICAL EXAMPLE**

As mentioned before, proposed design approach based on RSM has been demonstrated for the system given in Figure 1. For the demonstration, the fundamental frequency supply bus voltage and short circuit power of the system are predetermined as 4.16 kV (line-to-line) and 175 MVA. In the exemplary system without filter, power quality indices and power quantities, measured at the load bus, are presented in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>THDV</th>
<th>THDI</th>
<th>DPF</th>
<th>PF</th>
<th>( P_f )</th>
<th>( S_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.98%</td>
<td>10.06%</td>
<td>72.82%</td>
<td>72.27%</td>
<td>5058kW</td>
<td>6947kVA</td>
</tr>
</tbody>
</table>

According to the modelling issues presented in section II, for the single-phase equivalent circuit of the system, the impedance parameters and the harmonic spectrums of voltage and current sources are given in Table II. For the equivalent circuit, fundamental frequency line-to-neutral source voltage \( (V_s) \) and fundamental frequency line current \( (I_s) \) are calculated as 2400 V and 995.92 A.

**TABLE II**

<table>
<thead>
<tr>
<th>Impedance Parameters (Ω)</th>
<th>Voltage Source Harmonics (V)</th>
<th>Current Source Harmonics (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_L = 1.7 )</td>
<td>( V_{s1} = 48.0 )</td>
<td>( L_{s1} = 99.56 \circ135^\circ )</td>
</tr>
<tr>
<td>( X_L = 1.6 )</td>
<td>( V_{s2} = 48.0 )</td>
<td>( L_{s2} = 44.79 \circ45^\circ )</td>
</tr>
<tr>
<td>( R_p = 0.01 )</td>
<td>( V_{s11} = 24.0 )</td>
<td>( L_{s11} = 19.91 \circ135^\circ )</td>
</tr>
<tr>
<td>( X_p = 0.1 )</td>
<td>( V_{s12} = 24.0 )</td>
<td>( L_{s12} = 9.96 \circ135^\circ )</td>
</tr>
</tbody>
</table>

IEEE Std. 519 recommended limits for THDV and THDI levels of the exemplary system are 5% and 8%. It is seen from Table I that the THDI limit, according to the standard, is not satisfied. In addition, the system has very low PF percentage, 72.27%. Thus, a passive filter should be employed for both power factor correction and harmonic mitigation in the exemplary case.

**A. Application of RSM to Solve Formulated Optimization Problem**

For the solution of the optimal filter design problem, RSM can be performed to establish the mathematical relationships between the responses (PF, THDI, THDV) and the factors \( (X_{LF} \) and \( X_{CF} \). To obtain these mathematical relationships, initial experiments should be done for actual and corresponding coded values of \( X_{LF} \) and \( X_{CF} \), which are given in Table III. For the initial experiments, the intervals of \( X_{LF} \) and \( X_{CF} \) values can be set by considering filter’s tuning harmonic order \( h = \sqrt{X_{CF}/X_{LF}} \) around the dominant harmonic order \( (h=5) \) of the system and the DPF interval (95%-100% lagging). Consequently, calculated THDV, THDI and PF values are presented in Table IV. Here it should be noted that the design type is selected as the central composite face centered RSM design. The selected design type requires standard 8 experiments for cube and axial points and 1 experiment (custom) for the center point (0,0), with totally 9 experiments.

Equation (10) shows the general second-order polynomial response surface mathematical model (full quadratic model) for the experimental design [33, [34] [39]:

\[ Y_a = \beta_0 + \sum_{i=1}^{n} \beta_i X_{a_i} + \sum_{i=1}^{n} \beta_i X_{a_i}^2 + \sum_{i<j}^{n} \beta_{ij} X_{a_i} X_{a_j} + e_a \]  

**TABLE III**

<table>
<thead>
<tr>
<th>Level</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_{LF}(Ω) )</td>
<td>0.1000</td>
<td>0.2000</td>
<td>0.3000</td>
</tr>
<tr>
<td>( X_{CF}(Ω) )</td>
<td>2.0000</td>
<td>3.0000</td>
<td>4.0000</td>
</tr>
</tbody>
</table>
MINTAB 16 statistical package [39] is used to establish the mathematical models for minimizing THDV and THDI while holding PF at its desired percentage (90%). In the optimization process, it is considered that the THDV and THDI percentages must meet the limits specified in IEEE Std. 519 (5% and 8%, respectively). According to the results of the initial experiments (Table IV), mathematical models based on RSM for correlating responses such as the THDV, THDI and PF have been established with 95% confidence, and are represented in the following regression equations (11)-(13) with $R^2$ value (coefficient of determination) of 66.72%, 90.35% and 98.54%, respectively.

\[
THDV = 1.9922 + 0.0804X_{LF} + 0.6924X_{CF} + 0.6779X_{LF}^2 + 0.7913X_{CF}^2 - 1.3041X_{LF}X_{CF}
\]  

(11)

\[
THDI = 4.0530 - 10.4940X_{LF} + 8.3000X_{CF} + 10.2790X_{LF}^2 + 3.5860X_{CF}^2 - 10.9760X_{LF}X_{CF}
\]  

(12)

\[
PF = 98.6210 + 0.7154X_{LF} + 9.5663X_{CF} - 2.2594X_{LF}^2 - 11.2994X_{CF}^2 + 4.47832X_{LF}X_{CF}
\]  

(13)

Using (11)-(13), the contours of responses for THDV, THDI and PF are plotted in Figure 3 (a), (b) and (c). When the contours of three responses are superimposed on each other, it is seen that the solution of the formulated optimization problem can be found in the region, which is limited by the coded $X_{LF}$ values between -0.5 and 0.5 and the coded $X_{CF}$ values between -0.6 and 0. Thus, the parameter set of $X_{LF}$ and $X_{CF}$, which were given in Table III, should be updated. Thus, a new parameter set is determined as shown in Table V. Accordingly, the second experiment results, which are obtained with respect to the new parameter set, are presented in Table VI.

Using the new (second) experimental results presented in Table VI, for the correlating responses THDV, THDI and PF the models have been established with 95% confidence, and are expressed as in (14)-(16) with $R^2$ values of 99.99%, 98.60%, and 100.00%, respectively.

\[
THDV = 2.6562 + 0.5253X_{LF} - 0.0907X_{CF} - 0.1191X_{LF}^2 - 0.0059X_{CF}^2 + 0.0474X_{LF}X_{CF}
\]  

(14)

\[
THDI = 5.9548 - 0.9168X_{LF} + 0.6208X_{CF} + 0.9962X_{LF}^2 - 0.0091X_{CF}^2 - 0.4932X_{LF}X_{CF}
\]  

(15)

\[
PF = 94.3923 - 0.7447X_{LF} + 4.5175X_{CF} - 0.0926X_{LF}^2 - 1.2167X_{CF}^2 + 0.4383X_{LF}X_{CF}
\]  

(16)

With respect to (14)-(16) the contours of the responses for THDV, THDI and PF are plotted in Figure 4 (a), (b) and (c). By searching these contours, minimum values of the THDV and THDI are found as 2.6519% and 5.087 % for the $X_{LF}$ and $X_{CF}$ coded values such as -0.1092 and -0.8182, respectively. For the optimum case, it is also observed that PF attains 90.0010 %. Normalizing the coded values, actual values of the optimal $X_{LF}$ and $X_{CF}$ parameters are calculated as 0.1945 Ω and 2.4545 Ω. In order to show the validity of the proposed approach, it will be evaluated with respect to Classical Direct Search Method (CDSM) [40].
Fig. 3: Contour plots of (a) THDV, (b) THDI, (c) PF responses for initial parameter set.

### TABLE V

<table>
<thead>
<tr>
<th>Levels</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{LF} (\Omega)$</td>
<td>0.1500</td>
<td>0.2000</td>
<td>0.2500</td>
</tr>
<tr>
<td>$X_{CF} (\Omega)$</td>
<td>2.4000</td>
<td>2.7000</td>
<td>3.0000</td>
</tr>
</tbody>
</table>

### TABLE VI

<table>
<thead>
<tr>
<th>$X_{LF}$</th>
<th>$X_{CF}$</th>
<th>THDV</th>
<th>THDI</th>
<th>PF</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>2.1476</td>
<td>6.6685</td>
<td>89.7701</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>3.1041</td>
<td>5.7660</td>
<td>87.3746</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td>1.8634</td>
<td>9.1105</td>
<td>97.9138</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3.0097</td>
<td>6.2351</td>
<td>97.2716</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>2.0122</td>
<td>7.8065</td>
<td>95.0160</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>3.0616</td>
<td>6.0834</td>
<td>93.5853</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>2.7330</td>
<td>5.5326</td>
<td>88.6445</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2.5672</td>
<td>6.3465</td>
<td>97.7087</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>2.6566</td>
<td>5.9672</td>
<td>94.3904</td>
</tr>
</tbody>
</table>

B. Solution of the Formulated Optimization Problem with respect to Classical Direct Search Method

CDSM is one of the oldest and the most reliable optimization techniques, which searches all possible choices for finding the optimal solution. When CDSM is employed to design a single-tuned passive filter, by considering the formulated optimization problem, the optimal $X_{LF}$ and $X_{CF}$ values are found as 0.1880 $\Omega$ and 2.4480 $\Omega$, for three digits precise. For the optimal $X_{LF}$ and $X_{CF}$ parameters, THDV, THDI and PF are calculated as 2.5941%, 5.7060% and 90.0203%, respectively. Thus, it can be concluded that the results of the proposed approach and CDSM are very close to each other, and optimal $X_{LF}$ and $X_{CF}$ values of the passive harmonic filter can successfully be found using the proposed approach.

VI. CONCLUSION

In the literature, the optimization problem of single-tuned passive filters is generally solved by the heuristic methods. These methods provide a reasonable solution (near globally optimal) in a short time or less iterations when they are compared to the conventional optimization methods. However, the success of the heuristic methods depends on the estimation of their specific parameters, which is not a straightforward process.

Consequently, in this paper, a new approach based on Response Surface Methodology (RSM) is implemented to solve the multi-objective optimization problem of the single-tuned passive filters. The objective of the proposed approach is to minimize total harmonic distortions of voltage and current (THDV and THDI) while maintaining the power factor (PF) at its desired value.

The main advantage of the RSM is that it provides the mathematical expressions of PF, THDI and THDV in terms of the filter parameters ($X_{CF}$ and $X_{LF}$). Thus, by using these expressions, it can be possible to find the optimal combination of $X_{CF}$ and $X_{LF}$. For a typical industrial power plant, the numerical results show that the proposed approach based on RSM can be used for simple, fast and accurate design of harmonic passive filters.
Fig. 4: Contour plots of (a) THDV, (b) THDI, (c) PF responses for new (second) parameter set.

VII. REFERENCES


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