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On the analysis of lead-time disturbances in production and inventory control models

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Abstract

Changes in the lead-time can lead to supply chain inefficiencies and risks. In this paper, we investigate the effects of lead-time disturbances on the system's output responses of a production and inventory control model.
In the adaption process of the control system for lead-time disturbance analysis, the resulting model becomes nonlinear. Hence nonlinear control theory in combination with simulation is used to analyse the impact of lead-time changes on the transient and steady state responses of order rate, inventory and work in process. Assuming constant customer demand, small perturbation theory is applied to linearise the model and to find the transfer functions relating the system's outputs to the lead-time input.
We find that the order rate, inventory and work in process transfer functions are input-dependent. In order words, the output responses depend on the input type, amplitude and direction of changes in the lead-time. When lead-time increases, the system has a relatively slow transient response and, as expected, work in process inventory levels increase and order rates are higher. However, step decreases in the lead-time can cause significant underdamped dynamics in the system.
This work demonstrates that, although lead-time reduction is associated with service level improvement, increased flexibility and cost reductions, its implementation has to be carefully planned since a quick time compression may lead to undesirable oscillations in the supply chain system. In contrast, increased lead-times, associated say with a disturbance, yield slow recovery requiring adjustment of control parameters to increase resilience.

Keywords: lead-time disturbances, inventory and production control models, nonlinear systems

1. Introduction

The main goal of supply chain managers is to match customer demand with supply effectively, in order to minimise stockout rate as well as to reduce operating costs. Production and inventory control plays an important role in balancing supply chain-wide operational costs and customer service level. However, this goal is made more challenging given the uncertainties originated from the demand side, the supply side, manufacturing processes and control systems (Mason-Jones and Towill, 1998; Christopher and Peck, 2004).

Disturbances and uncertainties in production and supply lead-times are reported to be the main sources of supply chain risk (Colicchia et al., 2010). Notwithstanding the relevance of lead-time disturbances, previous research using control theory has focused on understanding the impact of demand uncertainty and on improving demand forecasting methods. Lead-time fluctuations can lead to performance degradation and increased production costs, just as demand uncertainties can (Dolgui et al., 2013).

Spiegler et al. (2012) proposed a methodology to assess supply chain resilience to disturbances caused by changes in demand. They used a well-known production and inventory control model: the Automatic Pipeline Inventory and Order Based Production Control System (APIOBPCS) (John et al, 1994). Using the same model, in this paper we investigate the impact of lead-time disturbances on the system's responses. Lead-time disturbances can be triggered by variations in internal manufacturing process time or external supplier lead-time. We initially assume constant demand/consumption to ease the modelling process but we changes in lead-time coupled with changes in demand are suggested for future research.

2. Literature Review

Without strong inventory management strategies, companies will fail to compete on a global scale (Schwartz and Rivera, 2010). In order to address production and inventory control problems, many researchers (e.g. Towill, 1982; John et al., 1994; Disney and Towill, 2005; Dejonckheere, et al., 2003; Aggelogiannaki and Sarimveis, 2008) have applied classical control engineering techniques such as block diagram development, transfer function formulation and Liénard–Chipart or Routh–Hurwitz stability analysis for discrete- or continuous-time systems, respectively. More recently, nonlinear control theory has been used as an alternative to simulation to investigate the effects of nonlinearities in production and inventory control systems (Jeong et al., 2000; Wang and Disney, 2012; Wang et al., 2015; Spiegler et al., 2015a, Spiegler et al., 2015b).

These works have generated knowledge to help supply chain designers to determine replenishment rules and policies that optimise order variances and stockouts across the supply chain system. However, focus has been given on the analysis of disturbances caused by the demand side. Although Dolgui et al. (2013), in their literature review paper on supply planning and inventory control under lead-time uncertainty, have pointed out a small number of articles using analytical modelling techniques to investigate the impact of stochastic lead-time on supply chain performance, we have not found any work applying control theory to examine the impact of even deterministic disturbances, such as step responses, triggered by sudden changes in lead-time. Step responses provide great insights into the characteristics of system dynamics, including peak response and settling time. Moreover, broader and deeper picture of the system dynamics can be obtained from determining the frequency-domain descriptors (Tangirala, 2015).

The literature regarding the application of control theory in supply chain management recognises the importance of estimating lead-time with accuracy since a mismatch between actual and estimated lead-times may lead to an inventory drift (Towill et al., 1997; Disney and Towill, 2005; Aggelogiannaki and Sarimveis, 2008; Schwartz and Rivera, 2010; Garcia et al., 2012). These works have made efforts to develop alternative production and inventory control models or to propose controllers with adaptive capabilities for predicting lead-time, generating new decision replenishment policies and tracking lead-time changes online.

In this work, we investigate the impact lead-time disturbances have on system response by assuming constant demand. We use and adapt the APIOBPCS model to predict the transient and steady state responses of order rate, inventory and work in process.

3. Model Formulation

3.1 The original APIOBPCS model

We chose to analyse the APIOBPCS model (John et al., 1994) since it is very similar to the classical discrete order-up-to (OUT) policy inventory control system (Disney and Towill, 2005). This model belongs to a family of decision support systems, the IOBPCS family (Towill, 1982), which takes into account a demand forecasting method, production and distribution lead-times, an inventory feedback loop, a work in process feedback loop and target inventory levels.

The production and inventory control model in Figure 1a is characterised by three control parameters T_a, T_i and T_w and a physical parameter, the actual lead-time T_p. The expected lead-time $\bar{T}_p$ is assumed to be equal to the actual lead-time in this model. In the demand policy, the value of the current demand or consumption *CONS* is exponentially smoothed. The parameter T_a represents the time to average demand/consumption, creating *AVCON*. The exponential smoothing constant is given by $\alpha = \frac{1}{1+T_a/\Delta t}$, where Δt is the sample time interval.

The inventory and work in process policies are characterised by feedback loops. The inventory control is concerned with the rate ($1/T_i$) at which a deficit in inventory is recovered. This policy is responsible for reducing the discrepancy between desired inventory *DINV* and actual inventory *AINV*. The pipeline policy considers the actual work in process *WIP* and the time T_w it takes to recover to target (desired) levels *DWIP*. While the *DINV* is a constant value, the *DWIP* is function of $\bar{T}_p$ and *AVCON*. Finally the order (rate) *ORATE* placed onto the supplier will take into account *AVCON*, the fraction of errors in inventory (*DINV-AINV*)/ T_i and work in process (*DWIP-WIP*)/ T_w. The delay between *ORATE* and completion rate *COMRATE* is represented by a first order delay of T_p time units.

3.2 Adapting APIOBPCS for lead-time input response analysis

The first step taken to adapt the APIOBPCS for the analysis of lead-time disturbances was to make *CONS* constant (Figure 1b). Given a constant *CONS*, the demand-forecasting element is no longer needed to predict *AVCON*. In this way the block containing the smoothing constant T_a was removed.

The second step was to detach T_p from the block that represents the first order delay and to predict $\bar{T}_p$. In the detachment process of T_p a new negative feedback loop from *COMRATE* to *ORATE* has been established. Moreover, a division equation between *WIP* and T_p was introduced. To estimate $\bar{T}_p$ we introduced a new parameter T_L that represents the time to average lead-time using its past values. Exponential smoothing was chosen to ease calculation. More sophisticated methods for lead-time estimation can be found in Towill et al. (1997); Aggelogiannaki and Sarimveis (2008); Schwartz and Rivera (2010); Garcia et al. (2012).

Figure 1c represents the resulting model for lead-time disturbances analysis. This model is nonlinear due to the presence of nonlinear algebraic differential equations, which in the block diagram are represented by the symbol Ⓝ. To further consider this model analytically, linearisation techniques based on Taylor series approximation are applied.

3.3 Linearisation through small perturbation theory

The system in Figure 1c can be described by the nonlinear differential equations (1-3), where $\dot{wip}(t)$, $\dot{ainv}(t)$ and $\dot{\bar{T}}_p(t)$ are the state variables $\dot{x} = f(x,u)$ of the system and $T_p(t)$ is the system input *u*.

$$f_1(x,u) = \dot{wip}(t) = orate(t) - \frac{wip(t)}{T_p(t)} \quad (1)$$

$$f_2(x,u) = \dot{ainv}(t) = \frac{wip(t)}{T_p(t)} - CONS \quad (2)$$

$$f_3(x,u) = \dot{\bar{T}}_p(t) = \frac{T_p(t) - \bar{T}_p(t)}{T_L} \quad (3)$$

The outputs $y = g(x,u)$ we are interested in analysing are the *comrate(t), orate(t), wip(t) and ainv(t)*.

$$g_1(x,u) = comrate(t) = \frac{wip(t)}{T_p(t)} \quad (4)$$

$$g_2(x,u) = orate(t) = CONS + \frac{1}{T_i}\Big(DINV - ainv(t)\Big) + \frac{1}{T_w}\Big(CONS.\bar{T}_p(t) - wip(t)\Big) \tag{5}$$

$$g_3(x,u) = wip(t) \tag{6}$$

$$g_4(x,u) = ainv(t) \tag{7}$$

The mathematical model given by Equations 1-7 is nonlinear due to the presence of nonlinear algebraic differential equations given by the division between the state variable *wip(t)* and the input $T_p(t)$. We can linearise the overall model about a nominal operating state space x^* and for a given input u^* by using small perturbation theory with Taylor series expansion. The assumption of this linearisation approach is that departures from a steady state point are small enough to produce transfer function coefficients. Hence, by assuming small amplitude of the excitation signal, we can replace the nonlinear differential equations by a set of linearised differential equation with coefficients dependent upon the steady state operating point.

The first-order Taylor Series approximation of the nonlinear state derivatives leads to the following linearised function:

$$\Delta\dot{x} = A\Delta x + B\Delta u \tag{8}$$

$$\Delta y = C\Delta x + D\Delta u \tag{9}$$

where the state perturbations $\Delta x = x - x^*$ and $\Delta y = y - y^*$, and the input perturbation $\Delta u = u - u^*$ are sufficiently small. Coefficients A, B, C, D can be found through the following partial derivatives:

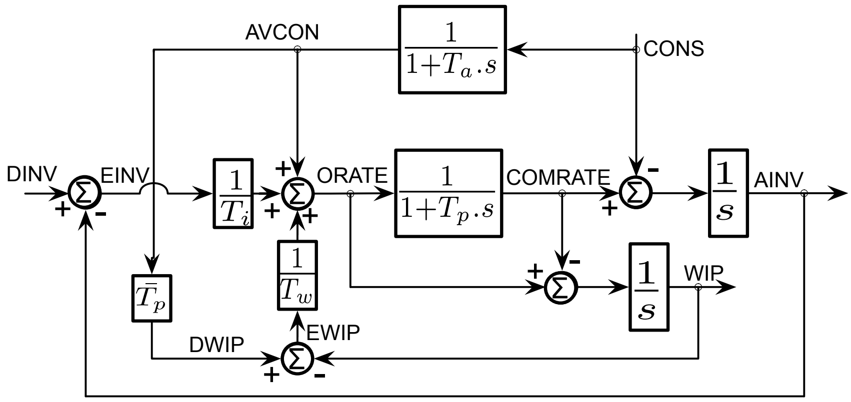
$$\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) = \left(\begin{array}{ccc|c} \frac{\partial f_1(x^*,u^*)}{\partial wip} & \cdots & \frac{\partial f_1(x^*,u^*)}{\partial \dot{T}_p} & \frac{\partial f_1(x^*,u^*)}{\partial T_p} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial f_3(x^*,u^*)}{\partial wip} & \cdots & \frac{\partial f_3(x^*,u^*)}{\partial \dot{T}_p} & \frac{\partial f_3(x^*,u^*)}{\partial T_p} \\ \hline \frac{\partial g_1(x^*,u^*)}{\partial wip} & \cdots & \frac{\partial g_1(x^*,u^*)}{\partial \dot{T}_p} & \frac{\partial g_1(x^*,u^*)}{\partial T_p} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\partial g_4(x^*,u^*)}{\partial wip} & \cdots & \frac{\partial g_4(x^*,u^*)}{\partial \dot{T}_p} & \frac{\partial g_4(x^*,u^*)}{\partial T_p} \end{array}\right) \tag{10}$$

The equilibrium or resting points (x^*,u^*) can be found by considering a step change in lead-time. In this way, the input can be defined as a function of an initial value (T_{pi}) and a step change (*STEP*). Hence, the final lead-time value $T_p^* = T_{pi} + STEP$ will be the steady state operating point for the input. To find the equilibrium point for the state variables x^* we solve the system of equations where all state derivatives are equal to zero. In this way the equation's equilibrium point for a generic input T_p^* is:

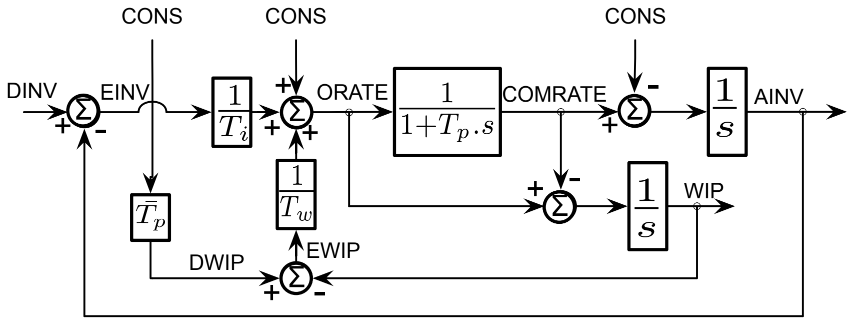
$$wip^* = T_p^*.CONS \tag{11}$$

$$ainv^* = DINV \tag{12}$$

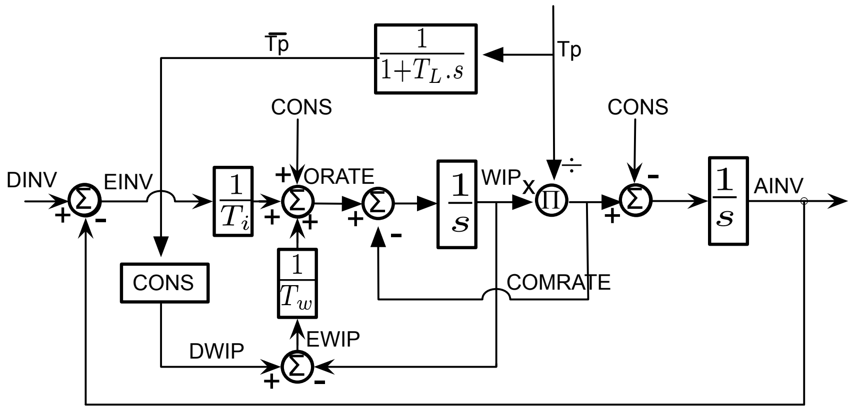
$$\bar{T}_p^* = T_p^* \tag{13}$$



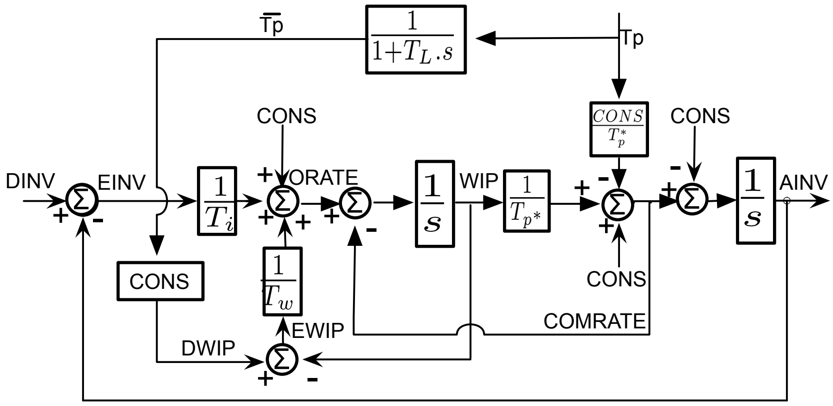
a) Original APIOBPCS



b) Making consumption constant



c) Inserting lead-time as input and determining expected lead-time



d) Linearised model

Figure 1. Adapting and linearising APIOBPCS for lead-time disturbance analysis

Finding the partial derivatives as given by Equation 10 and replacing them with the steady state points of Equations 11-13 will result in Equation 14, which can then be converted back to a block diagram representation as in Figure 1d).

$$\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) = \left(\begin{array}{ccc|c} -\frac{T_p^*+T_w}{T_p^*.T_w} & -\frac{1}{T_i} & \frac{CONS}{T_w} & \frac{CONS}{T_p^*} \\ \frac{1}{T_p^*} & 0 & 0 & -\frac{CONS}{T_p^*} \\ 0 & 0 & -\frac{1}{T_L} & \frac{1}{T_L} \\ \hline \frac{1}{T_p^*} & 0 & 0 & -\frac{CONS}{T_p^*} \\ -\frac{1}{T_w} & -\frac{1}{T_i} & \frac{CONS}{T_w} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) \tag{14}$$

Note that the state and outputs responses are input-dependent since they depend on the lead-time equilibrium state. When comparing Figures 1c) and 1d), we can see that the product functions (Ⓟ) are replaced by summing comparators (Ⓢ) after linearisation.

4. Analysis

Because the adapted APIOBPCS model for lead-time disturbance analysis is nonlinear, we can investigate the model by two attributes a) the T_p^*- dependent transfer functions of the linearised system around the equilibrium state and b) the input-output static characteristic function that specifies how a constant value T_p^* for the system input relates to the corresponding equilibrium value $y^* = g(x^*,u^*)$. This can be found by applying the final value theorem. In addition, it is possible to discuss the stability properties around the equilibrium point of the nonlinear system.

4.1 T_p^*-dependent transfer functions

Equations 15-18 display the system transfer functions for *ORATE*, *AINV, WIP* and *COMRATE*. Note that although *CONS* is assumed constant, its value is considered important in the calculation of the transfer functions. All transfer functions will equal zero at *CONS=0*. This result is reasonable since without demand there will be no changes in the inventory levels and order rates will be zero.

$$\frac{ORATE}{T_p} = CONS + \frac{CONS.s\big(T_w+s(T_i.T_p^*-T_i.T_L+T_L.T_w)\big)}{(1+s.T_L)(T_w+s(T_i.T_p^*+T_i.T_w)+s^2.T_i.T_w.T_p^*)} \tag{15}$$

$$\frac{AINV}{T_p} = -\frac{CONS.s(T_i.(T_L+T_w)+s.T_i.T_L.T_w)}{(1+s.T_L)(T_w+s(T_i.T_p^*+T_i.T_w)+s^2.T_i.T_w.T_p^*)} \tag{16}$$

$$\frac{WIP}{T_p} = \frac{CONS.(T_w+s(T_i.T_p^*+T_i.T_w+T_L.T_w)+s^2T_i.T_w.T_L)}{(1+s.T_L)(T_w+s(T_i.T_p^*+T_i.T_w)+s^2.T_i.T_w.T_p^*)} \tag{17}$$

$$\frac{COMRATE}{T_p} = CONS - \frac{CONS.s^2(T_i(T_L+T_w)+sT_i.T_L.T_w)}{(1+s.T_L)(T_w+s(T_i.T_p^*+T_i.T_w)+s^2.T_i.T_w.T_p^*)} \tag{18}$$

Another important point is that for *ORATE* and *COMRATE* transfer functions, the constant term *CONS* is being added to the transient term. For dynamic analysis only the transient term (second part of Equations 15 and 18) are needed but the constant term is necessary for determining the input- output static characteristic functions.

4.2 The input-output static characteristic function

The input-output static characteristic function is defined to be the equilibrium point from the system as a function of the value of a constant input T_p to the system. When the reference input signal is a constant (step input), the output signal (position) is a constant in steady-state. Hence the Final Value Theorem of the Laplace transforms will be used to determine the steady-state values:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s.Y(s) \tag{19}$$

By applying Equation 19 to Equations 15-18, we can determine the steady state values for the responses given a step input on T_p. The steady state values for *AINV* and *WIP* were already calculated in Equations 11 and 12 when determining the points where the state derivatives of the system equal zero. We find that the steady state values for ORATE and *COMRATE* are equal to the *CONS* regardless the value of $T_{p.}$ Hence, the only steady state response which is input-dependent is *WIP* since its final value is equal to T_p^*. *CONS.*

The input-dependent transfer functions can be used to determine the transient behaviour of the system around a pre-specified equilibrium. When combined with the input-output static characteristic functions, it is possible to assess other properties about the nonlinear system, such as whether the system has asymptotically stable equilibrium points.

4.3 Root locus analysis

The denominator of the transient part of Equations 15-18 gives the characteristic equation of the system:

$$(1 + s.T_L)(T_w + s(T_i.T_p^* + T_i.T_w) + s^2.T_i.T_w.T_p^*) \tag{20}$$

We note that Equation 20 is very similar to the original APIOBPCS characteristic equation, where T_a is analogous to T_L in the first part of the equation. In the second part of Equation 19, the only difference is that while in the APIOBPCS the lead-time T_p is constant, in our model the characteristic equation changes depending on the final value of the input, T_p^*. In this way the system's natural frequency (ω_n) and damping ratio (ζ) also depend on T_p^* and can be determined as:

$$\omega_n = \sqrt{\frac{1}{T_i.T_p^*}} \tag{21}$$

$$\zeta = \frac{T_i(T_p^* + T_w)}{2T_w}\sqrt{\frac{1}{T_i.T_p^*}} \tag{22}$$

Equations 21 and 22 demonstrate that when there is a step increase in lead-time (T_p^* increases), the system will have a relatively slow transient response given by increasing values of ζ potentially making the system overdamped. On the other hand, when T_p^* decreases (caused by a step decrease in lead-time) underdamped dynamics in the system may occur. Hence, it is recommended to set the parameters T_i and T_w as function of T_p, so that they are adapted as lead-time changes.

To verify the system's stability we will use the following theorems of Liapunov on the stability of linearised systems (Popov, 1961):

1. If all the roots of the characteristic equation of a linearised system have negative real parts, both the nonlinear and the linearised will be asymptotically stable.
2. If the characteristic equation of a linearised system has at least one root with positive real part, both the nonlinear and linearised systems will be unstable.
3. In there is at least one purely imaginary root the behaviour of the nonlinear system cannot be always be determined by its linearised equations.

Taking that in consideration, we calculated the roots of the characteristic equation which are:

$$R_1 = -\frac{1}{T_L} \tag{23}$$

$$R_{2,3} = \frac{-T_i T_w - T_i T_p^* \pm \sqrt{{T_i}^2({T_w}^2 + {T_p^*}^2) + 2T_i T_w T_p^*(T_i - 2T_w)}}{2T_i T_w T_p^*} \tag{24}$$

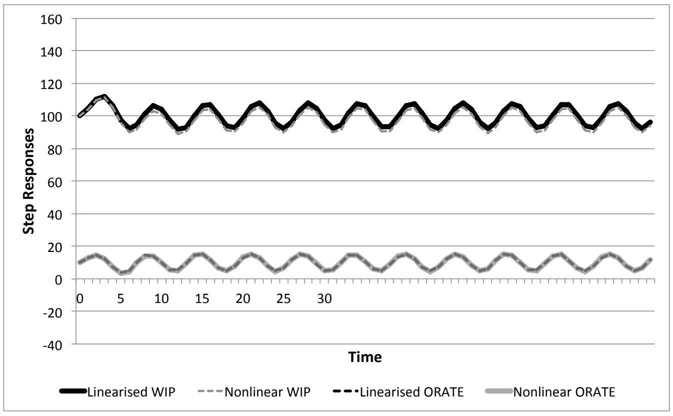
The stability analysis becomes very complex since the values of the roots change with the input. According to Equation 23, positive values of T_L are needed for system stability. On the other hand, Equation 24 demonstrates that as T_p^* increases the unstable region also increases. The value Tw=- T_p^* should be avoided since purely imaginary roots will be produced. Hence, it is advisable to adjust the values of the control parameters according to changes in the lead-time.

4.4 Simulation results

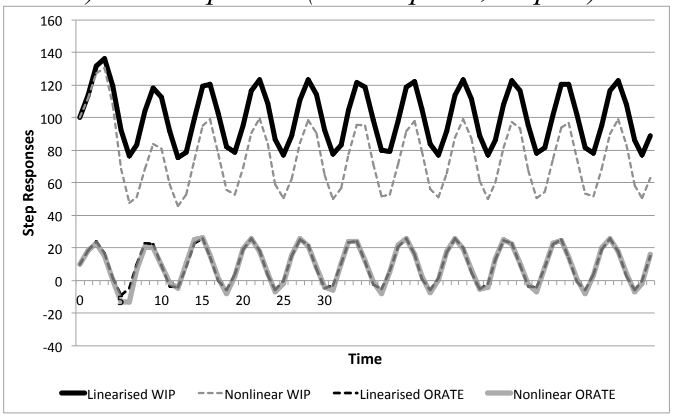
In this section, we will discuss the accuracy of the linearised model in comparison with the nonlinear model and illustrate the asymmetrical behaviour of the system responses since they are input-dependent.

When comparing the step responses of the linearised and the nonlinear models via simulation we found that both models yield exactly the same output responses for any chosen parameters lying within the stable region. Although the linearised model is only valid for step input response analysis, we decided to compare how both models behave with sinusoidal inputs by assuming T_p^* to be equal to the average lead-time. Figure 2 illustrates the sinusoidal responses for *ORATE* and *WIP* when *CONS*=10 units. When the amplitude of lead-time is low (Figure 2a) the linearised model is more accurate than when lead-time amplitude is high (Figure 2b). Interestingly, the *WIP* and *AINV* responses in the nonlinear model never recover target average values and this finding needs further investigation.

Figure 3 illustrates the asymmetrical behaviour of the nonlinear system. Both Figures 3a and 3b illustrate the output responses of *WIP, AINV, ORATE* and *COMRATE* for a step of 3 units change in lead-time, but the former picture displays a step increase while the latter, a step decrease. This result confirms the analysis made in the previous sections and it calls for attentions to the dangers of time compression without proper planning. When lead-time increases, the system has a relatively slow transient response. However, step decreases in the lead-time can cause significant underdamped dynamics in the system.

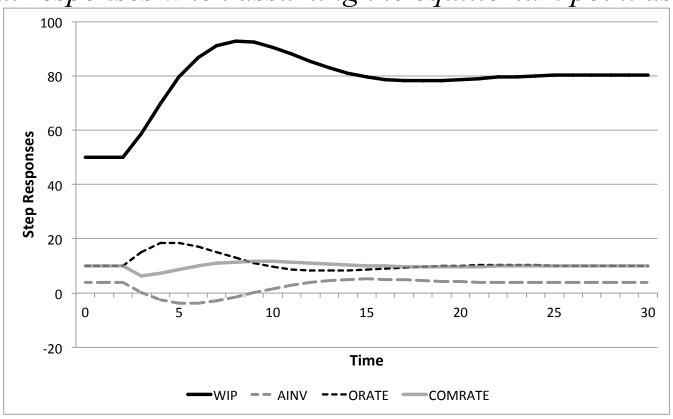


a) Low amplitude (mean Tp=10, amp=2)

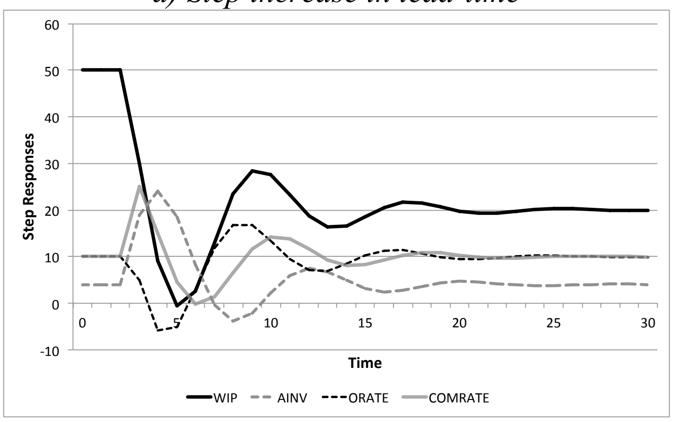


b) High amplitude (mean Tp=10, amp=5)

Figure 2. Sinusoidal responses when assuming the equilibrium point as average lead-time



a) Step increase in lead-time



b) Step decrease in lead-time

Figure 3. Effects of increasing/decreasing lead-time without re-adjusting control parameters

5. Conclusion and future research

In this paper we investigated the effects of lead-time disturbances on the system's output responses of a production and inventory control model. We adapted the APIOBPCS model so that deterministic disturbances on lead-time could be generated assuming a constant demand. Since the adapted model is intrinsically nonlinear, we used a linearisation technique to analytically evaluate the impact of lead-time changes on the system's transient and steady state responses.

We found that the transient response of the order rate, inventory and work in process are input-dependent as well as the steady state value of work in process. Because the output responses depend on the input type, amplitude and direction of changes in the lead-time, the system's characteristics, such as natural frequency, damping ratio and stability, are not fixed making the system design more complex. In order to overcome this problem, we recommended that the control parameters (T_i and T_w) should be time-varying and dependent on T_p . John et al. (1994) suggested that the "best design variables" are when $T_w = 2T_p$ and $T_i = T_p$. Taking into consideration these recommendations, we present the model in Figure 4 where we introduce intentional nonlinearities so that the control parameters are adaptive in relation to the lead-time and to the gain parameters i and w. The latter represent the weight given in the calculation of the control parameter T_i and T_w, respectively. Hence, if we want to design the system with 'fast' responses, the values of i and w should be small and for 'slow' responses they should be large. According to John et al. (1994)'s nominal design i should be equal to 1 and w equal to 2.

The current analysis complements the previous work of Spiegler et al. (2012). In the next steps of research the more realistic setting of changes in demand coupled by changes in lead times is to be considered. Also for future research we intend to analyse stochastic lead-time disturbances and their impact on supply chain resilience. For this, the ripple effect of lead-time disturbance impacting upstream and downstream the supply chain should be investigated. Moreover, we want to investigate the efficacy of deliberately introducing nonlinearities in production and inventory control models for better system design, such as the case of the model in Figure 4.

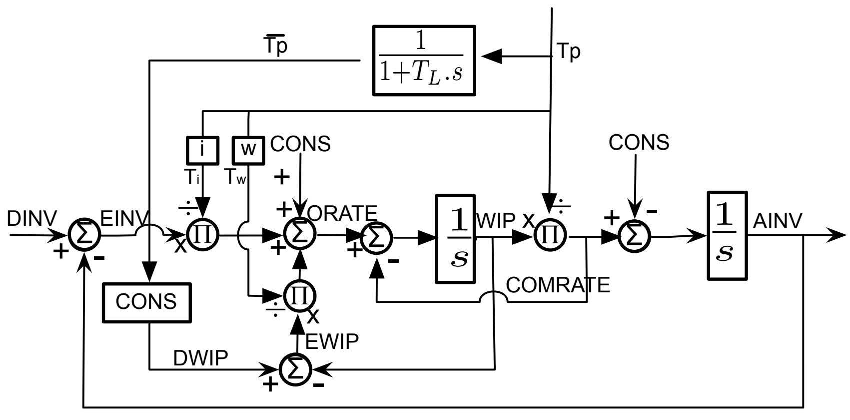


Figure 4. Proposed model for further investigation

This work demonstrated that the implementation of lead-time reduction has to be carefully planned and the system may need re-designing so that undesirable oscillations in the supply chain system are avoided.

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