A Tale of Five Bridges; the use of GNSS for Monitoring the Deflections of Bridges.

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Abstract

The first Bridge Monitoring surveying was carried out in 1996 by the authors, through attaching Ashtech ZXII GPS receivers onto the Humber Bridge' parapet, and gathering and further analysing the resulting 1Hz RTK GPS data. Various surveys have subsequently been conducted on the Humber Bridge, the Millennium Bridge, the Forth Road Bridge, the Severn Suspension Bridge and the Avonmouth Viaduct. These were all carried out using survey grade carrier phase/pseudorange GPS and later GNSS receivers. These receivers were primarily dual frequency receivers, but the work has also investigated the use of single frequency receivers, gathering data at 1Hz, 10Hz, 20Hz and even 100Hz. Various aspects of the research conducted are reported here, as well as the historical approach. Conclusions are shown in the paper, as well as lessons learnt during the development of this work. The results are compared to various models that exist of the bridges' movements, and compare well. The results also illustrate that calculating the frequencies of the movements, as well as looking at the magnitudes of the movements, is an important aspect of this work. It is also shown that in instances where the magnitudes of the movements of the bridge under investigation are small, it is still possible to derive very accurate frequencies of the movements, in comparison to the existing models.

1. Introduction

The use of GPS and GNSS for monitoring the deflections of bridges has been an ongoing thread of work by the authors for almost 20 years. The work started on the Humber Bridge in the UK, followed by the London Millennium Bridge, an 80m long motorway viaduct near Bristol in the UK, the Forth Road Bridge and more recently the Severn Suspension Bridge. Other pieces of research have been conducted by the authors on other bridges in Australia, South Korea, the UK as well as China, but so far the five bridges named above are the ones from which test results have been used to advance the concepts and ideas.

The data gathered has been mainly used twofold. Firstly, the movements of the structures, both on the deck and on the support towers have been analysed. Secondly the frequencies of the movements of the various locations on the bridges have been derived from the 3D GPS coordinates with associated time. All this work has been carried out in a coordinate system relative to the bridge i.e. transforming the GPS coordinates into bridge coordinates. This is simply carried out by measuring the bearing of the bridge, usually through the various GPS antennas located on the side rail, and then transforming the coordinate system into one that makes more sense in terms of the movements; usually referred to as the bridges' lateral, longitudinal and vertical directions.

During the historical field tests carried out, GPS and GNSS data have been gathered at an increasing data rate, from the original 1Hz to 10Hz, and even 100Hz in some recent cases. This in itself has been an important step change in allowing higher natural frequencies of the structures to be derived from the GPS data.

The general concept behind this research is that the GNSS data can be used to derive the movements of the bridge, and in fact the simultaneous movement of a variety of locations on the bridge. These data can then be used to compare to existing models of how the bridges should behave, such as Finite Element Models (FEM). If specific loading is recorded upon the bridge, such as wind loading, traffic loading or temperature loading, then these can also be compared with the expected movements from the models. The long term theory is that the ongoing GNSS derived movements and frequencies can be used to validate the models. As the bridges become older, their deflection characteristics may change. In addition, some bridges have much more loading applied to them than when designed. The Forth Road Bridge, for example, was opened in 1964. During its first year of operation, some 4 million vehicles passed over it, and at the time the largest Heavy Goods Vehicles (HGV) on UK highways weighed 24 tonnes. In 2002, 23 million vehicles passed over the Bridge, and now the maximum sized HGVs in the UK are 44 tonnes.

Over the past 20 years or so, an increasing number of long span suspension bridges have been built. Interestingly, today 30 of the top 100 longest spans in the World are in China, 23 of which were opened after the year 2000. The oldest was opened in 1984; the Dazi Bridge, a one-lane suspension bridge with a main span length of 500m, spanning the river Kyi in Dagzê, Tibet. Many of these have GPS receivers located upon them, gathering data to allow the analysis of their performance to be ascertained.

The following paper gives an overview of the work carried out by the authors to date, illustrating some of the main findings during each of the pieces of work over the past 18 years on the Humber Bridge, Millennium Bridge, Forth Road Bridge, Avonmouth M5 Viaduct and more recently the Severn Suspension Bridge.

2. Case Study 1; The Humber Bridge

2.1 Field Test Number 1; Initial RTK Demonstration Results.

The Humber Bridge was the first test bed used by the authors for measuring the deflections of such a bridge using kinematic GPS. The Bridge was opened in 1981, and had the longest suspended span in the World until the Akashi Kaikyō Bridge in Japan was opened in 1998. The Akashi Kaikyō Bridge still holds this record by over 300m above its nearest competitor. The Humber Bridge was number one in this claim when the first set of surveys were conducted by the authors upon it, today it lies number 7, with 3 Chinese bridges in the top 6. The Humber Bridge lies in an almost north-south direction. The first tests were conducted on the 7 March 1996, whereby Ashtech ZXII dual frequency code-carrier phase GPS receivers were used. During the initial tests, one GPS receiver was placed on top of the Bridge's control building (Figure 1), and a rover GPS receiver attached to the parapet at the side of the public footpath at the middle of the 1,410m long mid-span (Figure 2). During these field tests, RTK GPS was incorporated at a rate of 1Hz using Ashtech's real time version of their PNAV software. The initial study showed that the use of kinematic GPS was indeed suitable for such deflection monitoring of large suspension bridges, such as the Humber Bridge [Ashkenazi *et al.*, 1996; Brown *et al.*, 1999; Roberts *et al.*, 1999a]. Movements of the order of 300mm were observed in the vertical direction, Figure 3, due to normal traffic loading.

Figure 1, The Ashtech ZXII reference GPS antenna located on top of the Humber Bridge office.

Figure 2, Gethin Roberts on the Humber Bridge on the 7 March 1996 [Ashkenazi *et al.*, 1998].

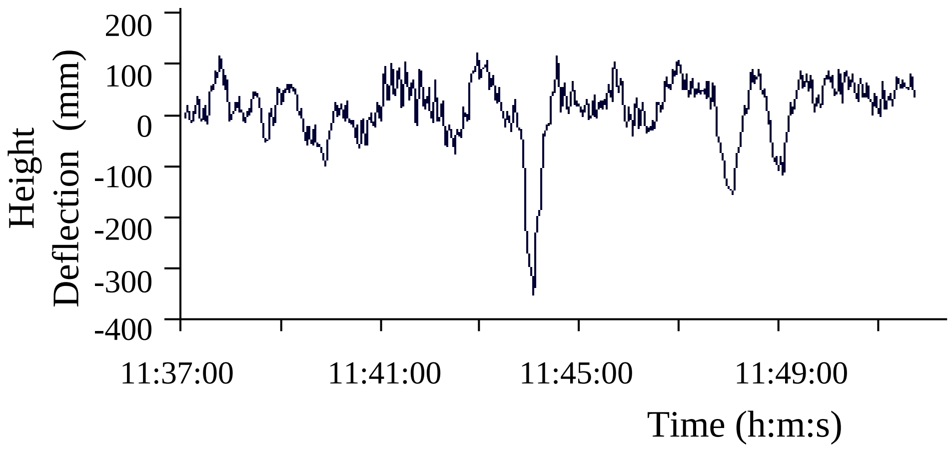


Figure 3, Humber Bridge RTK Results in the Height Direction, 7 March 1996 [Ashkenazi *et al.*, 1998].

It was also deduced that the pole used to support the GPS ground plane antenna was not ideal, as this in itself would have its own vibration characteristics as the Bridge moved, and all subsequent field work used a low rising clamp to attach the antenna directly onto the bridges.

2.2 Field Test Number 2; Validation of Finite Element Numerical Models

Subsequent field tests on the Bridge included one whereby 5 fully laden HGVs, with a measured known total mass of 160.19 tonnes, travelled over the Bridge in various formations at 45km/h on the 16 February 1998 [Roberts *et al.*, 1999a, Brown *et al.*, 1999]. The tests were carried out in almost windless conditions in the early hours of the morning when other traffic loading was extremely light. Four Ashtech ZXII and one Ashtech GG24 single frequency GPS/GLONASS receivers were located on the bridge at key locations, on both sides of the deck, gathering raw data at 5Hz to allow post processing. Again the reference receivers were located on top of the control building for the Humber Bridge. An FEM of the Humber Bridge had been developed at Brunel University [Karuna *et al.*, 1998a, Brown *et al.*, 1999], the tests carried out using GNSS on the Bridge allowed the theoretical model to be compared to the GPS derived results.

The trials consisted of the HGVs being driven over the bridge in three tight formations, whereby three HGVs were in the outer carriageway, next to two in the inner carriageway. The first consisted of all five HGVs travelling southbound together at approximately 45 km/h. The second run consisted of all five HGVs travelling together northbound at approximately 45 km/h. Two HGVs were then taken over the bridge southbound to begin the final run that consisted of two pairs of HGVs travelling from each end of the bridge to meet at the middle of the midspan. Here they remained stationary for approximately 5 minutes before departing. Although the southbound carriageway was closed to other traffic, there was no way of stopping northbound traffic. This, however, was very light and intermittent due to the time of the trial; therefore the majority of the movement seen is due to the loading resulting from the five HGVs.

Figure 4 illustrates the movements measured at the various GPS locations whilst the 4 HGVs passed over the Bridge. The fundamental frequency of the Bridge in the vertical and lateral directions, calculated using Fast Fourier Transformation (FFT) analysis using Matlab were 0.116Hz and 0.052Hz respectively. These compared well to previous measurements [Brownjohn *et al.*, 1986; Brownjohn *et al.,* 1994] and FEM calculated frequencies [Karuna *et al*., 1997a; Karuna *et al*., 1997b; Karuna *et al*., 1998a; Karuna *et al*., 1998b; Brown *et al.,* 1999].

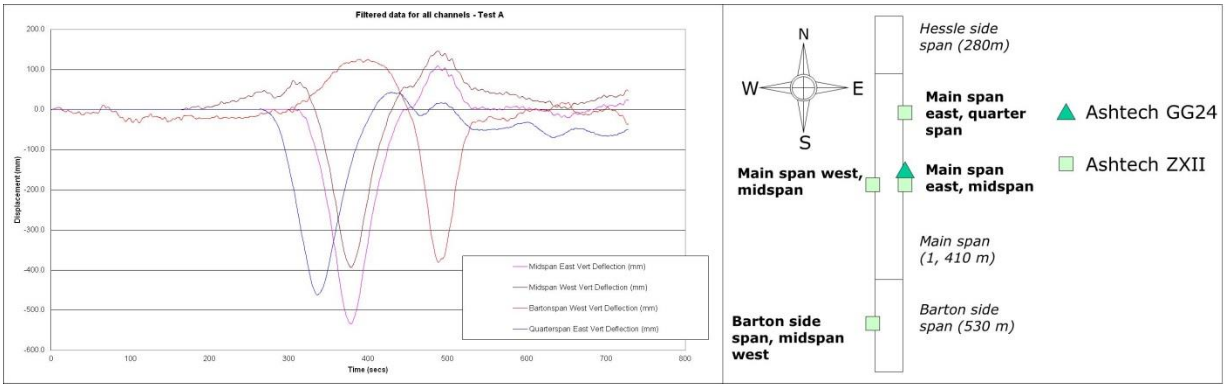


Figure 4, Height deflections at the various GPS stations during the passage of the four HGVs over the Humber Bridge.

Figures 5 and 6 illustrate two of the 5 height plots for the GNSS receivers located on the Bridge. These are at the middle of the mid-span, which would experience the largest deflections. It is evident from these where the two initial manoeuvres described exist i.e. at 1:33 and 1:52. At 2:00 there is a small dip due to the two HGVs travelling to the south side of the bridge, in preparation for the final manoeuvre starting at 2:07. These results also illustrated the twisting or torsional movement of the Bridge deck due to off centre loading. This phenomenon has been witnessed again and again in subsequent work, and the frequency of the movement measured as well as its magnitude.

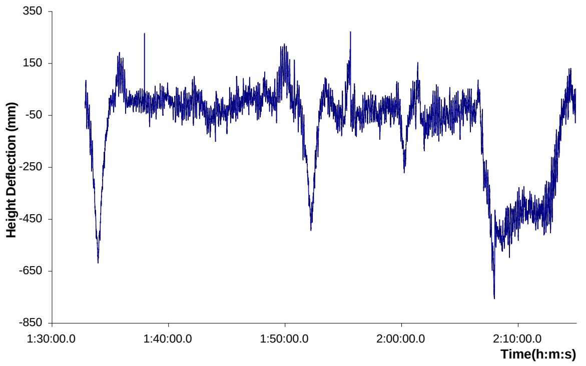


Figure 5, Height deflection of main span east mid span on the Humber Bridge [Roberts *et al.,* 1999b].

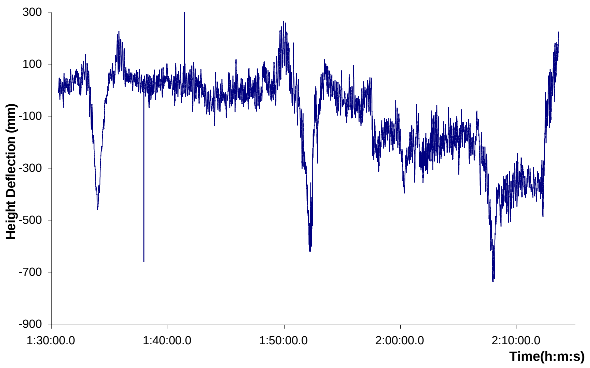


Figure 6, Height deflection of main span west mid span on the Humber Bridge [Roberts *et al.,* 1999b].

2.3 Field Test Number 3; 14 GPS Receivers.

Further tests were carried out on the Humber Bridge from the 1-4 March 2004, whereby 7 GPS receivers and 4 triple axis accelerometers were placed upon the Bridge's main span, with another on each of the two side spans. These were Leica SR530 and SR510 receivers using choke ring antennas. Data were gathered at a rate of 10Hz. A weather station was placed next to one of the antennas, Figure 8, and used to relate the wind speed and direction with the movements [Roberts *et al.*, 2008]. During these tests, a further GPS receiver was placed at shore level, giving a height difference of 155m between this and the two GPS receivers on top of the support tower connected to a single antenna through a splitter [Mohd Suldi, 2006]. This data was used to research into the residual tropospheric error. Figure 7 illustrates the locations of all the various GPS receivers, as well as the 4 accelerometers, whilst Figure 8 illustrates the shore level GPS receiver with the Bridge in the background (left), and a GPS receiver with weather station located on the Bridge (right).

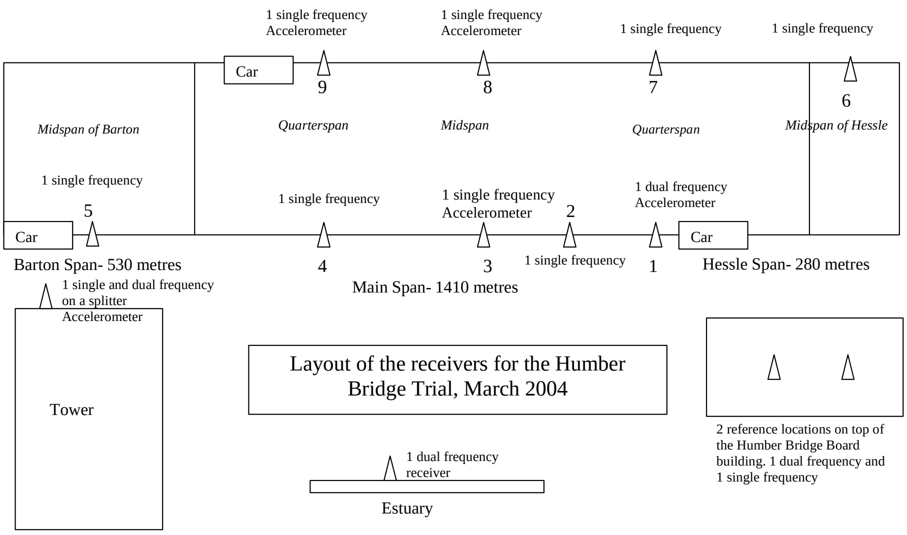


Figure 7, The Locations of the various instruments during the field trials on the 1-4 March 2004 on the Humber Bridge [Roberts *et al.,* 2005a].

The datasets created are very large, with, in this example, 14 GPS receivers gathering data at 10Hz over a period of 3 working days, in addition to the weather station and 4 accelerometer data. It was deduced at this stage that due to the increasing size of the data, the processing was not the issue, but being swamped with results was.

Figure 8, A Leica SR530 GPS receiver and antenna located at water level (left), and a Leica SR530 and lightweight choke ring antenna on the Humber Bridge adjacent to a weather station (right) [Roberts *et al.,* 2005a].

Various results were analysed and produced from this large dataset [Cosser *et al.*, 2004; Cosser, 2005]. Various researchers used the output of the data, investigating for example the residual tropospheric effects due to the 155m difference in height between the shore and tower top antennas [Mohd Suldi, 2006], the use of single frequency GPS receivers for such work [Cosser *et al.*, 2004; Cosser, 2005; Roberts *et al.*, 2005a], as well as general analysis of movements and frequency deduction using various approaches [Roberts *et al*., 2013]. Such approaches included FFT and Power Spectral Density (PSD) analysis using Matlab, and the development of Adaptive Filtering techniques [Roberts *et al.*, 2002] in order to compare common signals in various pieces of data. This included consecutive days' data from the same GPS receiver at the same location, in order to assess the multipath. The common signal from the filter would be the multipath, and the uncommon the movement. Band-pass filtering approaches were also developed [Roberts *et al.,* 2001; Cosser, 2005] in order to focus the frequency analysis within a specific range, hence ignoring the GPS noise region.

Figure 9 illustrates the height movements of two GPS antennas at locations 1 and 7 over a period of 18 minutes and 20 seconds. These are at the same location along the Bridge's length, but on opposite sides of the deck. Here it is evident that the Bridge during this period deflects down by up to 270mm. In particular, it can be seen that there is a torsional movement in this data between the two locations from 22mm to 119mm.

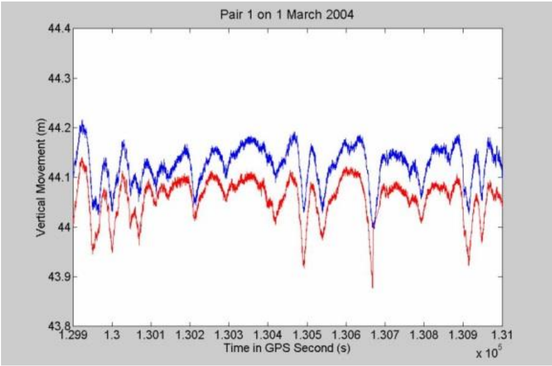


Figure 9, Two GPS antenna results from locations on opposite sides of the Humber Bridge, showing the effect of twisting on the Bridge deck [Roberts *et al.,* 2005b].

Figure 10 illustrates the analysis of the fundamental frequency in the vertical direction derived from one of the GPS data sets using Matlab's FFT analysis tool. Here it can be seen that the frequency is 0.117 Hz, which compares to the previously derived frequency from GPS of 0.116Hz, as well as the Brunel model of 0.116Hz. Further tests by the authors on the Forth Bridge illustrated that the frequencies do change slightly, mainly due to external factors; such as traffic loading.

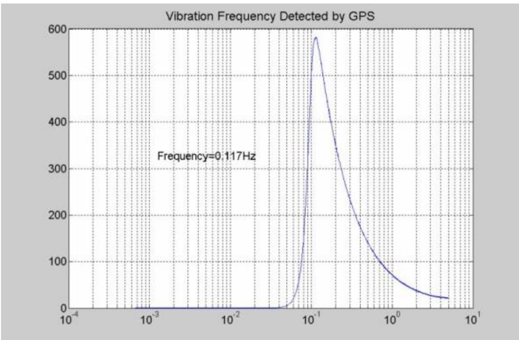


Figure 10, The Natural Frequency of the vertical direction of the Humber Bridge obtained from the GPS data using FFT analysis in Matlab [Roberts *et al.,* 2005b].

Figure 11 illustrates the vertical component of the GPS results at one of the GPS locations for 3 different days over 12 ½ hours for each of 2 and 4 March. The graphs have been overlapped with each other relative to the time of day, starting at 9am. The data from the 1 March starts later in the day than the other two, as the receivers and equipment were set up during the morning which took some time. The deflections due to the traffic loading can be seen here, but also the overall dip in the Bridge's level over the period of time. On the 2 March, the temperature at 09:15 was 4°C, and changed down to 1.8°C at 09:40 and finally up to 8°C by 15:00 [Cosser, 2005]. On the 4 March, the temperature started at 14.5°C at 09:00, changed down to 9°C by 10:05 and then up to 15°C by 12:00, then again dropping to 11°C from 12:45 to 14:10, rising again to 13°C at 14:50, then gradually dropping from 15:15 to 10°C at 16:30. It can be seen from Figure 11 that the data corresponding to the 4 March started off at a lower level than the other two days, and continued downwards at a slower rate [Cosser, 2005]. The Bridge deck lies 110mm lower at 09:00 on the 4th March than on the 2 March. There is evidently a relation between the temperature and the level of the Bridge. Similar results have been seen on other bridges, but there will be a time delay between the air temperature and the temperature of the bridge's steel work. A lesson was learnt here in that ideally the temperature of the steel would be required for future work if possible, in addition to that of the air.

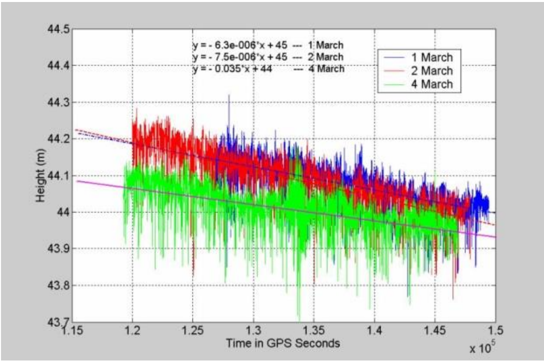


Figure 11, Three consecutive days' of data from the same location on the Humber Bridge, illustrating the effect of temperature on the overall level of the Bridge deck [Roberts *et al.,* 2005b].

3. Case Study 2; The London Millennium Bridge

The London Millennium Bridge is a pedestrian bridge, connecting St Paul's Cathedral on the north side of the River Thames with the Tate Modern Gallery on the south side, in London. It has span lengths of 81m, 144m and 108m, north to south. It was opened on the 10 June 2000, and a charity walk was organised, whereby 100,000 people crossed over to celebrate the opening. It is estimated that there was a maximum of 2,000 people on the deck at any one time. This resulted in greater than expected lateral movements, to the extent that some people stopped walking and grabbed onto the handrail in fear. The Bridge was subsequently closed on the 12 June 2000 in order to retrofit hydraulic dampers in order to minimise such future movements. The Bridge re-opened in 2002. It is thought that due to the fact that the natural frequency of the Bridge is very similar to that of someone walking, a forced oscillation was caused, and due to the fact that so many people were walking close together they were likely to be walking in step with each other [Dallard *et al.*, 2001]. The result was that the bridge moved with its resonant frequency, a similar concept to the ill-fated Takoma Narrows Bridge that suffered such a spectacular collapse in November 1940.

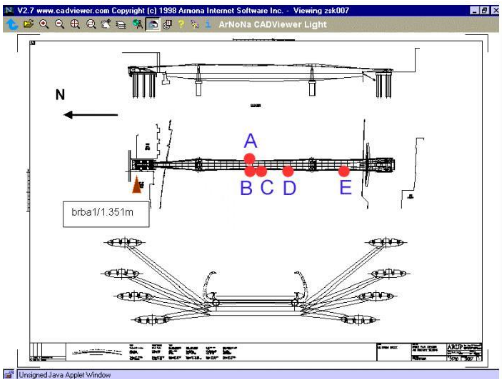
After its closure, and before the refurbishment was underway, the authors were given permission to carry out a survey upon the Bridge using GPS. The survey was conducted 22-24 November 2000, the authors placed 3 Leica SR530 dual frequency code/carrier GPS receivers, with AT504 choke ring antennas, as well as a Kistler triple axis accelerometer clamped directly onto the Bridge's handrail. A Leica SR530 GPS receiver, with AT504 choke ring antenna, was established on the top of an adjacent building, within a few tens of metres of the Bridge. Figure 12 illustrates the reference station being setup adjacent to the Bridge, and Figure 13 illustrates one of the GPS receivers and antenna on the Bridge, with the Kistler accelerometer housed directly below the antenna.

Figure 12, The Leica GR530 GPS receiver and Choke Ring antenna reference station being set-up adjacent to the Millennium Bridge [Roberts *et al.,* 2004].

Figure 13, A Leica choke ring antenna attached to the Millennium Bridge’s handrail, with a Kistler triple axis accelerometer attached underneath [Roberts *et al.,* 2004].

Due to the university only having 4 suitable GPS receiver at the time, it was decided to occupy one of the Bridge points for the whole of the time, but the other two receivers would rotate around the remaining 4 points for periods of time, gathering data at a rate of 10Hz in order to be able to post process using both the Leica SkiPro and the University of Nottingham's KinPOS softwares. Figure 14 illustrates the locations of the survey points on the Bridge, as well as the occupation periods. These were easily re-occupied every time the antennas were moved, as they consisted of clamps which remained in situ throughout the survey, and the antennas were moved from one to another, sitting on the 5/8” thread located on the top. However, at a data rate of 10Hz, and an 11 hour occupation period, this results in 396,000 epochs of data for each of the GPS receivers. The accelerometer recording at 200Hz over this period results in 7.92 million epochs of data points. Again, this illustrates the vast amount of data that is gathered through such surveying, which can then be very time consuming to analyse completely.

The movements experienced on the Bridge during the survey period were close to the noise level of the GPS. Only the authors were present on the Bridge at this time, as it had been shut to the general public since June. In addition to this, small wind loading was experienced. However, the authors made every effort to excite the Bridge by walking in step on the Bridge.



Date	Session No	Start Time	End Time	Antenna Location
22/11/00	1	14.35.42	16.25.52	A
	1	14.36.03	16.24.58	B
	1	14.35.52	16.25.26	C
	1	15.08.12	16.08.14	B (accelerometer)
23/11/00	2	09.51.23	12.13.26	B
	2	09.38.25	12.13.37	C
	2	09.38.58	12.13.22	D
	3	12.37.24	14.50.04	B
	3	12.39.24	14.50.21	C
	3	12.37.57	14.50.13	A
	3	12.42.08	13.42.10	B (accelerometer)
	3	13.46.26	14.46.28	B (accelerometer)
	4	15.14.24	16.47.12	B
	4	15.16.23	16.47.17	C
	4	15.15.24	16.47.28	A
24/11/00	5	08.56.09	12.06.13	B
	5	09.05.15	12.06.36	D
	5	09.15.28	12.07.20	E
	5	09.04.35	10.04.38	B (accelerometer)

Figure 14, The locations and times of locations of the 3 GPS receivers at the 5 monitoring points on the Millennium Bridge [Roberts *et al.,* 2004].

Figure 15 illustrates the movements in the lateral direction at the midpoint (location B), as well as the derived frequencies obtained through the use of FFT analysis in Matlab. The modelled frequencies of the Bridge were known to be 0.5Hz and 0.95Hz for the lateral movements of the midspan, and 0.77Hz for the south side span. It is evident from Figures 15 and 16 that there are spikes in the frequency results that relate well with the modelled results. However, there is a lot of noise around this, so it may not be possible to pin point these frequencies without any prior knowledge.

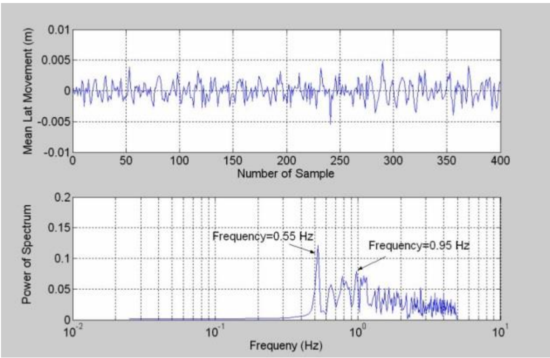


Figure 15, Lateral dynamics on the Millennium Bridge's midspan [Roberts *et al.,* 2005b].

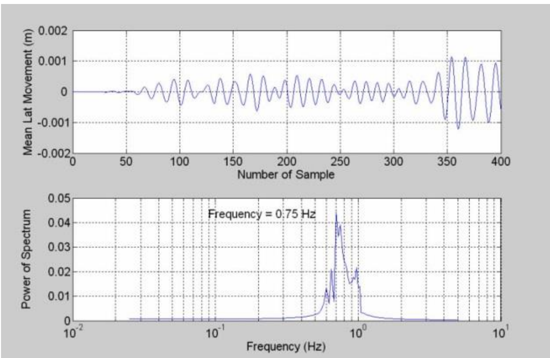


Figure 16, Lateral dynamics on the Millennium Bridge's south span [Roberts *et al.,* 2005b].

Figure 17 illustrates the midspan results from the two opposite locations i.e. A and B in Figure 14. The GPS position results from the GPS data processing has had a 20 second moving average filter applied. At first glance, the results look good, as the apparent movements at both sides seem to relate in all three axes. However, on closer inspection is can be seen that the longitudinal movements of the Bridge are larger than those in the lateral direction. This is not as expected, as the lateral movements should be larger than the longitudinal. The Millennium Bridge is orientated in a north to south direction, as are the Humber and Forth bridges. However, the magnitude of the deflection on the Millennium Bridge is far smaller than the other two, and close to the noise level of the GPS carrier phase derived position. Therefore, the resulting noise in the GPS solution, due to the resolution of the carrier phase signal, due to any additional multipath, as well as satellite geometry induced noise, resulted in a solution whose noise was in the same ball park as the magnitude of the movement. This resulted in "apparent" movements. Further analysis of this data and general satellite geometry showed that due to the GPS satellite geometry, there is always a "hole" where one would never see a GPS satellite in the sky at latitudes such as the UK. Figure 18 illustrates the skyplot for London on the 24 November 2000. Subsequently, the data was simulated, and a simulated pseudolite introduced in order to assess the effectiveness of a psuedolite in such an environment [Meng *et al.*, 2004]. The results showed that pseudolite technology could indeed improve the results due to such gaps in the sky. This led to more research into this area [Meng *et al*., 2004; Barnes *et al*., 2004], and a change of direction to using Locatalites when the use of pseudolites and transmitting on GPS frequencies became outlawed in many countries. However, the results from this showed that even with small movements, it was still possible to pick up the frequencies of the deflections, however, when the movements are relatively small, other error sources become dominant.

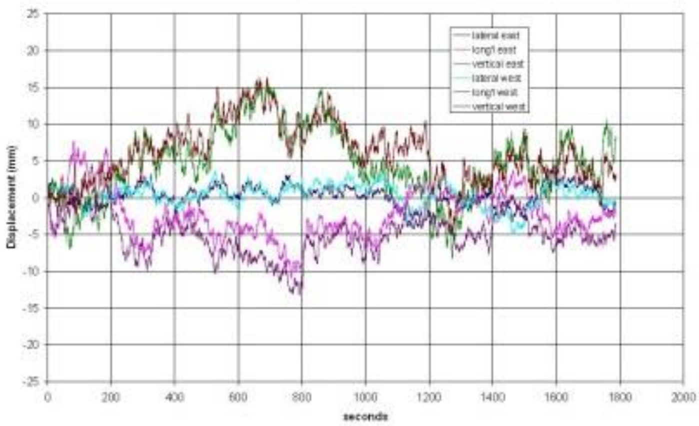


Figure 17, The two Millennium Bridge's midspan GPS receivers' deflections having had a 20s moving average filter applied [Roberts *et al.,* 2004].

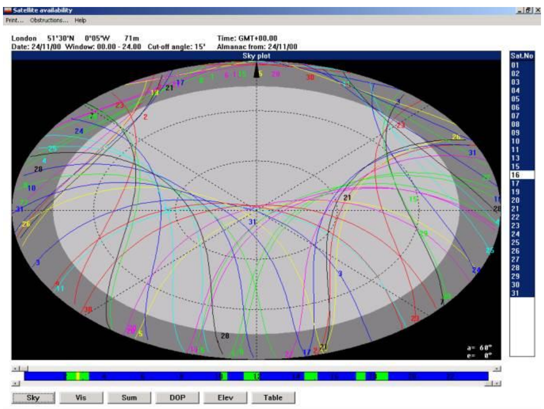


Figure 18, A 24 hour GPS satellite skyplot over London on the 24 November 2000 [Roberts *et al.,* 2004].

During this period further analytical techniques – such as wavelet analysis – were implemented, and the use of Adaptive Filtering was used to try and improve the quality of the movement signal in the results. This was achieved through comparing two successive days' of data ("Desired" and "Ref" in Figure 19), and removing the common part (the multipath), leaving behind a cleaner signal ("Out" in Figure 19). The wavelet analysis had shown the multipath to be relatively low frequency. Figure 19, a sub-set of the results, illustrates the height results for two consecutive days on the middle of the Bridge. At first glance, the graph showing the desired results could be thought to be movement, but some of this is due to multipath. By using the adaptive filtering it is possible to split the two input signals into a common element (multipath) and output result, which includes the receiver noise and the movement.

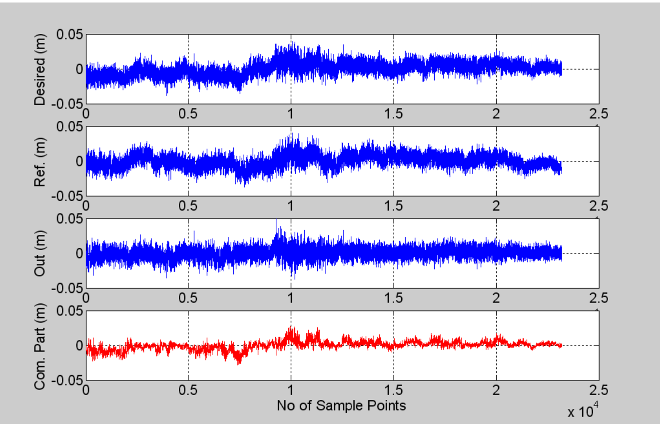


Figure 19, Multipath analysis of the GPS data through the use of Adaptive Filtering.

4. Case Study 3; The Forth Road Bridge

The Forth Road Bridge is currently the 27th longest single suspended span in the World, at a length of 1,006metres. It was the 6th longest suspended span when it was opened in 1964. This Bridge is very heavily used, and the traffic at peak times was queuing up over the Bridge during the field surveys carried out. Figure 20 illustrates one of the monitoring locations, location D, as well as the two reference stations. Leica SR530, SR510 and GX1230 GPS receivers were used, as well as a Novatel OEM4 GPS receiver linked directly to an Applanix POS-RS inertial measurement unit [Hide *et al*., 2005; Roberts *et al*., 2012]. Previous work by the authors illustrated that there was an improvement in the quality of the carrier phase data between successive generations of receivers ie the Leica SR530 to the Leica GX1230 [Roberts *et al*., 2012]. A zero baseline test was conducted simultaneously using two pairs of each of these receivers, the results showing that the standard deviation or the zero baseline OTF processed data were 0.9mm, 1.3mm and 2.1mm for the SR530 in East, North and Vertical directions respectively, and 0.4mm, 0.7mm and 1.2mm for the GX1230 in East, North and Vertical directions respectively [Roberts *et al*., 2012]. Figure 21 illustrates the weather station placed adjacent to location F; continuously gathering wind speed and direction, temperature and relative humidity data, as well as the Leica AT502 GPS antenna on top of one of the support towers at location A2.

Figure 20, The Leica AT504 choke ring antenna placed at location D (left) and the two Leica AT503 choke ring antennas placed at the two reference sites at the Forth Road Bridge (right) [Roberts *et al*., 2012].

Figure 21, The Omni Instrument weather station placed at location F (left) and the Leica AT502 GPS antenna located at location A2 (right) on the Forth Road Bridge [Roberts *et al*., 2012].

Figure 22 illustrates the locations of all the GPS antennas. The layout was planned so that the magnitudes and frequencies of the movements could be observed at the middle of the mid-span at two locations on opposite sides of the Bridge deck (locations D and F), a quarter of the way along the mid-span at two locations (locations C and E), and also one eighth of the way along the mid-span (location B). In addition to this, two GPS antennas were placed on the tops of one pair of support towers (locations A1 and A2). The layout was planned in this way in order to investigate the relationship between the frequencies and movements of the deck and towers, as well as to investigate how some of the frequencies were in existence at some locations and not others.

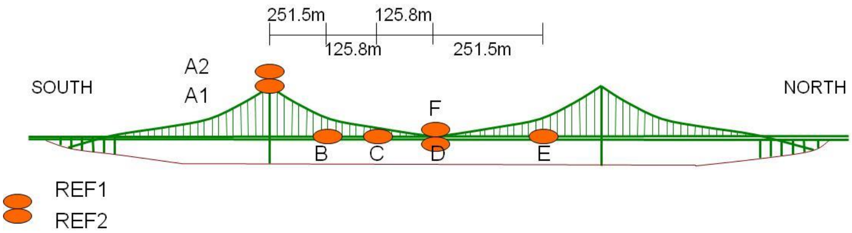


Figure 22, The location of the GPS antennas upon the Forth Road Bridge [Roberts *et al*., 2012].

In total, 46 hours of data were gathered on the 8-10 February 2005. Data were typically gathered with download periods every 5 or 6 hours in order to prevent long data spans from being lost if there were any faults with the equipment. Two reference GPS receivers were used, mainly as a backup plan in case one failed. Fortunately, there were no problems with the equipment, and all the data were gathered. The Forth Estuary Transport Authority (FETA) supplied mains electricity at all the GPS receiver locations, which made the survey easier to perform and only backup battery power was required in case of power cuts. Again, there were none. The Bridge itself, compared to the Humber Bridge, is very heavily loaded. The construction of a second Forth Road Crossing, mainly due to the high usage, is underway. The traffic can indeed be seen to be queuing over the Bridge in Figure 20 (right). One of the data processing and analysis methodologies used was to investigate whether the natural frequencies of the Bridge changed due to the traffic loading. Data were analysed between 02:00 – 03:30 (light traffic loading) and between 07:30 – 09:00 (rush hour). The results show that the frequencies of all the locations become slower from 0.1040Hz or 0.1041Hz to 0.1026Hz or 0.1027Hz in the vertical directions at locations B, C, D and F, and from 0.2699Hz to 0.2684Hz in the torsional movement between locations D and F [Roberts *et al.,* 2012].

During the early hours of the 10 February 2005 between 01:00 to 01:45, two HGVs with known loading were chartered by FETA to cross over the Bridge carrying out specific manoeuvres. The HGVs weighed 40 tonnes each, and the Bridge was closed to other traffic during the specific moves. The manoeuvres were as follows, where the HGVs travelled at 30km/h:

(a) one HGV ran from north to south.
(b) one HGV ran from south to mid-span on the west side, stopped then the other HGV moved north to south.
(c) one HGV moved from north to south and stopped at mid-span, while the other moved south to north.
(d) one HGV moved from south to north, and then both moved side by side north to south.

Figure 23 illustrates the height component for the 5 GPS receivers on the Bridge deck over the period of the manoeuvers above. The figure illustrates the periods where the above manoeuvres occur. It is evident from Figure 23 that he Bridge deflects during the loading. It is also evident that there are other movements on the Bridge outside of the manoeuvres. This is due to the ambient traffic having to wait for the manoeuvres to be conducted, and the Bridge opening up to the traffic in between. In fact, after manoeuvre b, the traffic loading, and subsequent maximum deflection of the Bridge, is more than that of the 2 HGVs.

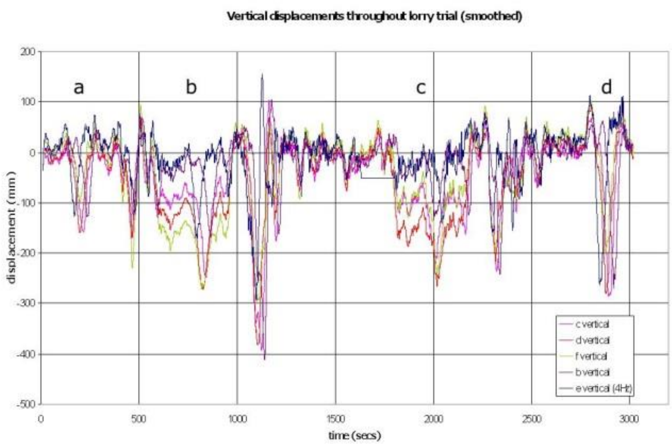


Figure 23, Vertical displacements during the Two HGV trials on the Forth Road Bridge [Roberts *et al*., 2012].

Figure 24 illustrates in more detail manoeuvre d, whereby the two HGVs travel from the north to the south side by side. At the start of the graph, there is no mass on the Bridge, as the Bridge was closed to ambient traffic loading. For the first 2,770 seconds or so, the Bridge moves with its natural frequency, and the sinusoidal nature of this is evident. The two HGVs then move from north to south, firstly crossing onto the north span, with a length of 408m. The mass pushes down onto this span, causing the deck to pull down on the suspension cable, and hence pull the mid-span deck up. This lift is of the order of 53.1mm, 71.8mm, 96.7mm, 103.1mm and 90.6mm for locations b, c, d, e and f respectively from their mean height before the movement. Figure 24 illustrates the instances when the HGVs cross each of the GPS antennas, as well as the towers. The Bridge deck dips down by 317.2mm, 328.1mm, 329.7mm, 331.3mm and 253.1mm at locations b, c, d, e and f respectively from the mean heights just before the initial manoeuvre to the peak of the dip. The FEM prediction for the movement at the mid-span under such loading was 280mm.

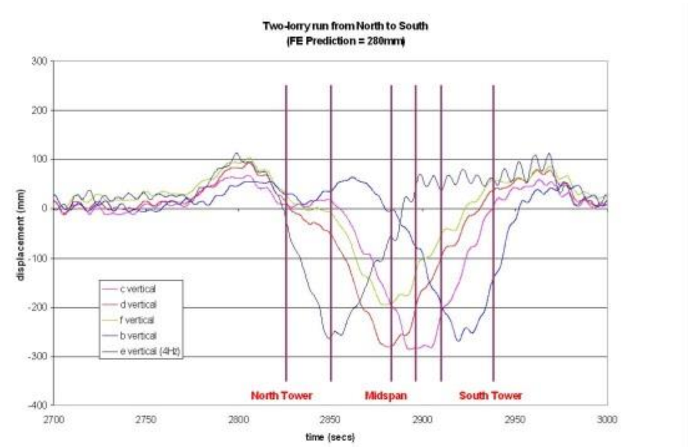


Figure 24, Vertical displacement during the Two HGV trials with the two HGVs travelling side by side from north to south on the Forth Road Bridge [Roberts *et al*., 2012].

Figure 25 illustrates the FEM output for the shape of the movement at the mid-span under such loading compared to the raw carrier phase OTF derived position and the height of the GPS antenna. A moving average filter of 100 0.1 second epochs, or 10 seconds was applied, through the GPS data in order to smooth out the sinusoidal frequency characteristics within the data, in order to result in a graph more comparable to the overall dip. Here, again, it is seen that there is good correlation between the two sets of results.

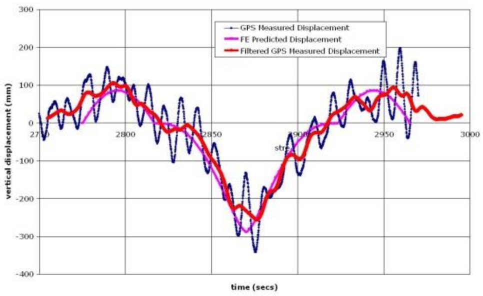


Figure 25, Comparison between the GPS vertical displacements at location D with the FEM on the Forth Road Bridge [Roberts *et al.*, 2012].

The correlation between the GPS derived movements and the wind speeds during the surveys are illustrated in Figure 26. It can be seen that there is a relation between these. For example, it can be seen at around 23:00 on the 9th February the wind speeds reach values of up to 80 kph. In relation to this, the mean lateral movements of the Bridge reach their maximum of almost up to 1 metre in the East direction, from the maximum in the west direction. The wind speeds during this night were large, to the extent that high sided vehicles had been stopped from crossing, and small vehicles were about to be stopped, but then the wind speed died down slightly.

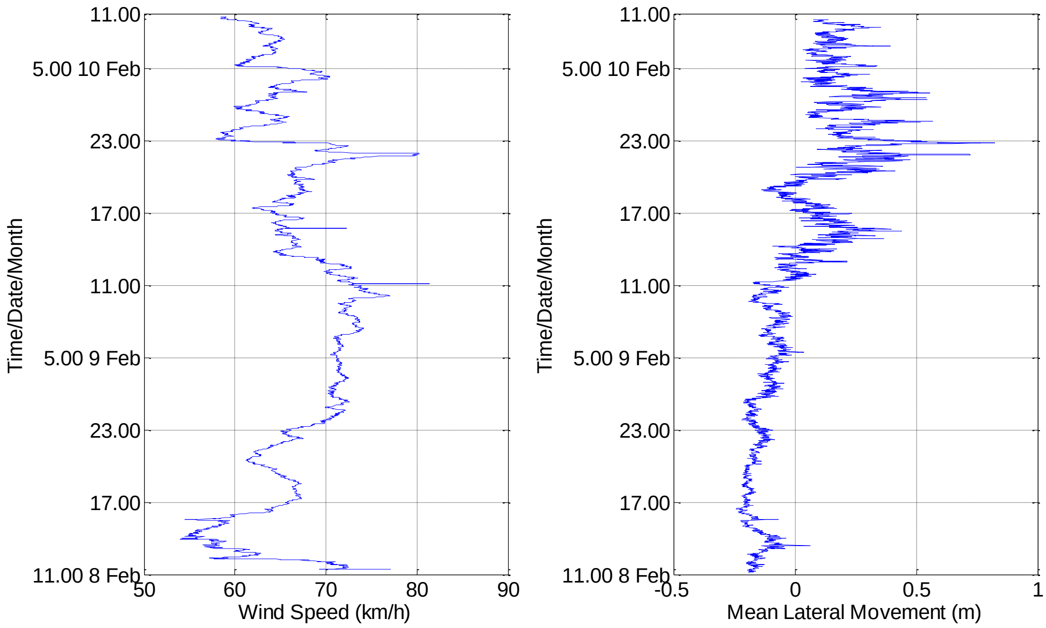


Figure 26, Comparison between the GPS derived lateral movements on the Bridge and the wind speed at location F [Roberts *et al.*, 2012].

5. Case Study 4; The Avonmouth M5 Viaduct

The Avonmouth M5 viaduct is a 173.7m long span over the River Avon in the UK. It carries a motorway, carrying six running lanes, as well as hard shoulders and a single public footpath and cycle path on the east side. This is an example of a survey whereby small movements were expected. The survey was conducted on the 29th and 30th November 2007 between 14:40 to 17:00 and 07:30 to 15:00 respectively, when five GPS receivers were placed upon the Bridge's parapets. These were a mixture of Leica SR530 and Leica 1200 receivers, but all using choke ring antennas. Figure 27 illustrates the viaduct, and the locations of the 5 GPS receivers located upon it for the survey. Locations M and D are at the middle of the span, located on each side of the footpath on the parapets. Locations A and B at 40 metre distances from M, and C is 50 metres beyond the end of the support pier, thus allowing any bending movements of the road over the pier to be measured. Figure 28 illustrates the Leica lightweight choke ring antennas located on the Bridge's parapets, as well as a close up of location C. The reference receivers were placed adjacent to the Bridge, some 2 kilometres to the North West of the Bridge. The main reference station was a Leica SR530 GPS receiver located at ground level, and a second static receiver was placed on top of an adjacent derelict storage building, which was approximately the same height as the viaduct, Figure 29. Again with this survey, the antennas were attached as close as possible to the Bridge's parapets, clamping the antenna directly onto this.

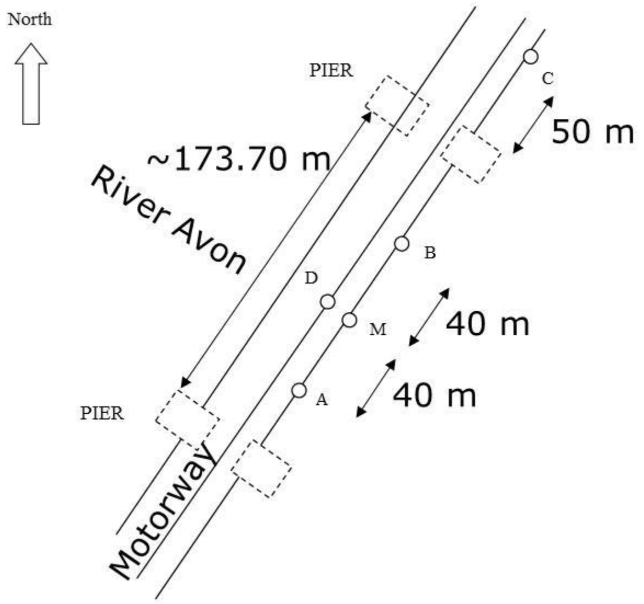


Figure 27, An illustration of the Avonmouth M5 Motorway Viaduct and the location of the GPS antennas.

Figure 28, Four of the five survey points on the Bridge (left), and a close-up of location C (right), all using Leica lightweight At503 choke ring antennas on the Avonmouth Viaduct.

Figure 29, The main reference station (left), and the second reference station located on top of a derelict grain store (right) used for the Avonmouth survey.

Even though small movements were expected, movements of the order of 20mm were evident in the lateral direction, 30mm in the longitudinal direction and 80mm in the vertical direction. Figure 30 illustrates the lateral, longitudinal and vertical movements on the 30th November at location M. Both the vertical and in particular the longitudinal movements drift away from the mean over the 7.5 hour survey. This is thought to be due to the heating effect of the Bridge, as has been seen previously in the vertical direction, but the longitudinal drift movement may be due to expansion in one direction.

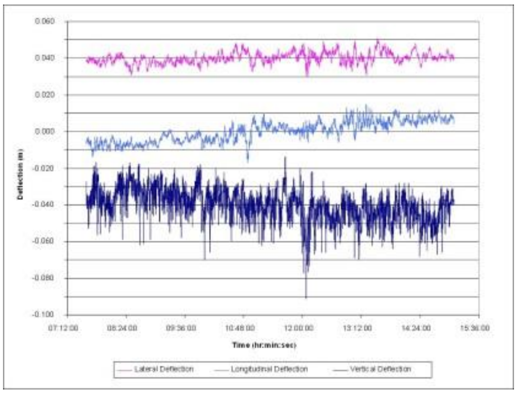


Figure 30, Lateral, longitudinal and vertical movements of location M on the 30 November 2007, the Avonmouth viaduct.

One of the reasons for having the second reference station on top of the granary building, which is approximately the same level as the viaduct, was to look at any residual tropospheric effects. Figure 31 illustrates the vertical movements at all the survey points on the viaduct, as well as the height results from the granary location over approximately a 3.5 hour period. The granary results help to illustrate the noise in the GPS results in the vertical direction, and illustrates that the movements at the other locations.

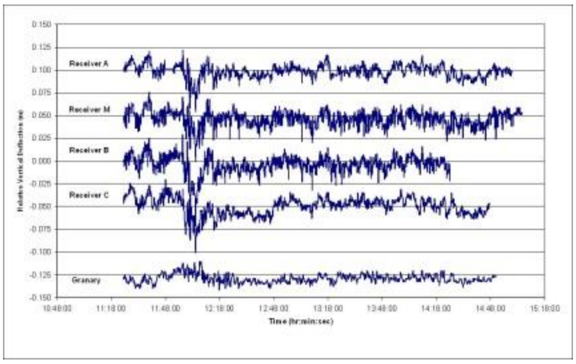


Figure 31, The vertical movements at the granary location as well as the viaduct locations A, M, B and C over a 3.5 hour period on the Avonmouth viaduct [Ogundipe *et al.*, 2014].

There seems to be movement at around 12:00 at all the locations, but the results from the granary site also seem noisy. The noise at the granary site is not as large as the apparent movement at the viaduct locations, but they do coincide. Figure 32 illustrates that at that time, there is a very big spike in the DOP values, which could contribute to some of this apparent movement.

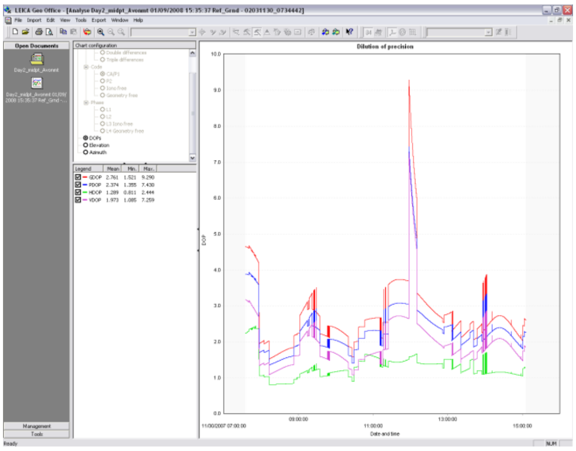


Figure 32, Dilution of Precision results for the 30 November 2007 at the Avonmouth viaduct.

Figure 33 illustrates the frequency response of the viaduct through using Amplitude Spectrum analysis of the GPS results in the vertical direction at location M. Here, again, it can be seen that there is a clear response, showing a frequency of 0.061Hz, 0.5257Hz and 1.139Hz on the 29th November. Similar results of 0.068Hz, 0.521Hz and 1.106Hz were seen on the 30th November.

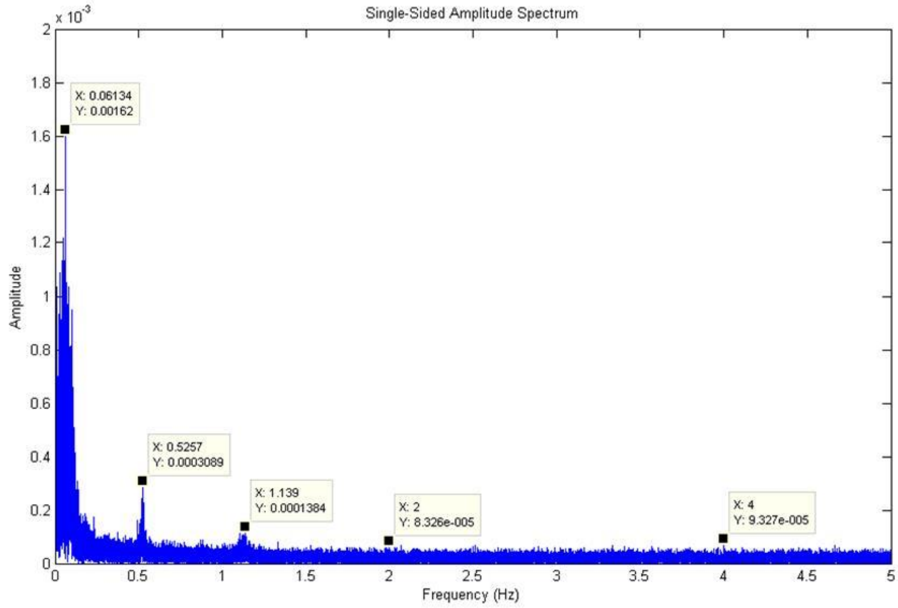


Figure 33, The Amplitude Spectrum analysis of location M on the 29th November 2007, the Avonmouth viaduct.

6. Case Study 5; The Severn Suspension Bridge

The Severn Bridge has the 29th longest suspended span in the World at 988m, and was opened in 1966. The Bridge carries the M48 motorway as a dual carriageway over the Severn Estuary, connecting Bristol area to South Wales. On the 10-12 and 18th March 2010, the authors carried out some very extensive surveying of the Bridge using GPS and GNSS receivers, located at key locations. Figure 34 illustrates the locations of the various antennas upon the Bridge. Four were places on top of each of the two pairs of support towers (locations T1, T2, T3, T4), a further four were placed on the northern side suspension cable (A, B, C, D), and the ninth (location E) was placed on the south side support cable adjacent to location C on the north side cable.

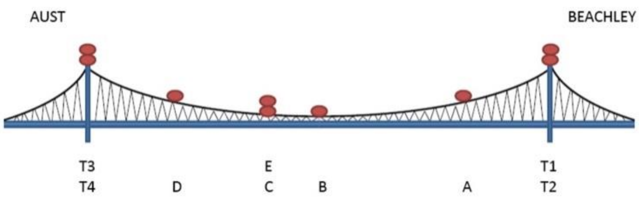


Figure 34, The locations of the 9 GPS/GNSS receivers upon the Severn Bridge.

Two reference stations were used at different locations, Figure 35. The main reference station was located on top of the Bridge's toll office, adjacent to the Bridge. This allowed for increased security for the equipment, and also allowed mains power supply to be available. The two reference GPS receivers were Leica 1200 dual frequency, gathering GPS and GLONASS carrier phase and pseudorange data. Leica AT504 choke ring antennas were used, with the secondary station having a radome cover. The coordinates of the main reference station was coordinated relative to the UK's Ordnance Survey's active network stations, and the secondary station relative to the main station. The secondary reference station was established in order to be used as a backup in case data was lost from the main station, and also to allow relative OTF positioning to be carried out, in order to help assess the precision at specific times of day, to then be able to compare to the moving locations on the Bridge.

Figure 35, The main reference station (left) and secondary reference station (right) used during the Severn Suspension Bridge survey.

The GNSS antennas attached to the Bridge were located on the very tops of the pairs of support towers, Figure 36 (left), as well as directly to the suspension cables, Figure 36 (right). It is thought that this was the first time so many GNSS antennas have been attached directly to a suspension cable in this manner. The Tsing Ma Bridge in Hong Kong has two of its 14 GPS receivers located on the suspension cables in the middle of the mid-span, but all the others are located on the Bridge deck [Wong *et al.*, 2001]. The antennas were carefully attached by staff from the Severn River Crossing plc (SRC) at all the locations, due to the risk involved. The 50m antenna cables were trailed down to easier and safer access locations, either within the enclosed crossway between each pair of towers, or at Bridge deck level, so that access was possible from the public footpath at the side of the Bridge. In fact, mains electricity was available within the crossways, which made longer term data gathering possible, with battery backup. The receivers located at the Bridge deck level had to be battery powered, and the batteries needed to be changed every 4 hours or so. The data was also downloaded at this interval in order to ensure it was captured safely. The 9 GNSS receivers upon the Bridge consisted of four Leica SR530 dual frequency GPS receivers at the four tower locations, all connected to Leica AT504 choke ring antennas. The GNSS receivers on the support cables were all Leica 1200 dual frequency GPS/GLONASS receivers, apart from location A on the 18th March, which was a Leica SR510 single frequency GPS receiver, due to equipment availability.

Figure 36, A Leica choke ring AT504 antenna placed at location T2 (right), and a Leica AT503 lightweight choke ring antenna placed at location C (right) on the Severn Suspension Bridge.

Over the survey period, large amounts of data were gathered. The SR530 GPS receivers were gathering data at 10Hz, whilst the Leica 1200 receivers gathered data at 20Hz. In all, it is estimated that just fewer than 7.8 million 3-dimensional coordinates were generated for every 24 hours of survey at the 9 Bridge survey locations at a rate of 10Hz. Figure 37 illustrates a typical lateral, longitudinal and vertical plot at location A over a 40 minute period. It is evident that deflections of 300mm or more are seen in the vertical direction. There is also correlation seen between the other two Bridge axes with the vertical deflections.

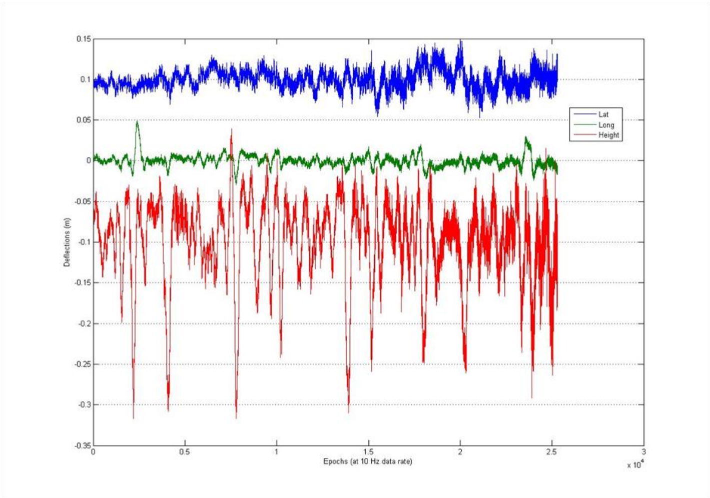


Figure 37, The Lateral, Longitudinal and Vertical displacements at location B over a 40 minute period on the Seven Suspension Bridge.

If this data is looked at in more detail, Figure 38, it can be seen that there is a clear sinusoidal pattern in the vertical displacement.

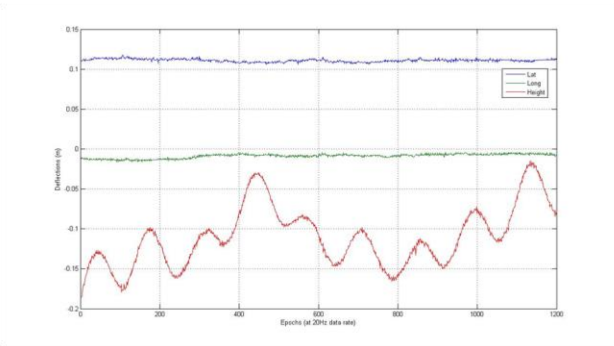


Figure 38, The Lateral, Longitudinal and Vertical displacements at location B over a 1 minute period on the Seven Suspension Bridge.

Figure 39 illustrates the vertical movements of the four Bridge locations over a period of 10 minutes. Clear displacements are seen, with a maximum at around the 4500 epoch mark. Clear sinusoidal characteristics in the data are also seen, in particular when the Bridge has little loading, such as around the 9000 epoch mark. It is also seen in Figure 39 that the vertical movements at the various locations are slightly offset from each other, as was seen in the earlier Figure 24.

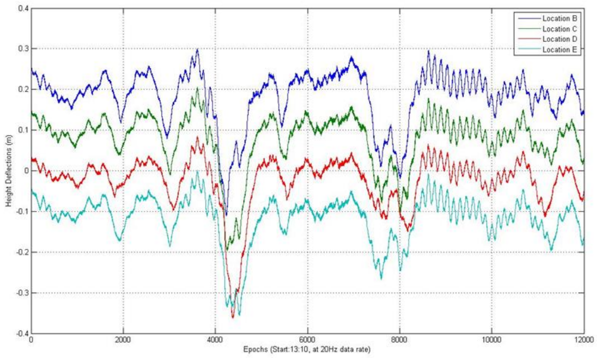


Figure 39, The vertical displacements at the 4 Bridge deck locations over a 10 minute period on the Seven Suspension Bridge.

In addition to the movements of the Bridge deck, those of the towers are also evident. Figure 40 shows the longitudinal movements of the four tower top GPS antennas. Here again there is evidence of correlation between the pairs of locations with each other, as well as the opposite pairs. The relationship between the 4 locations can be seen at around the 700 epoch mark, and again at the 5,700 epoch mark. There are also instances where there is movement at one pair of towers, but not the other such as at the 1,600 epoch mark and again at the 5,000 epoch mark.

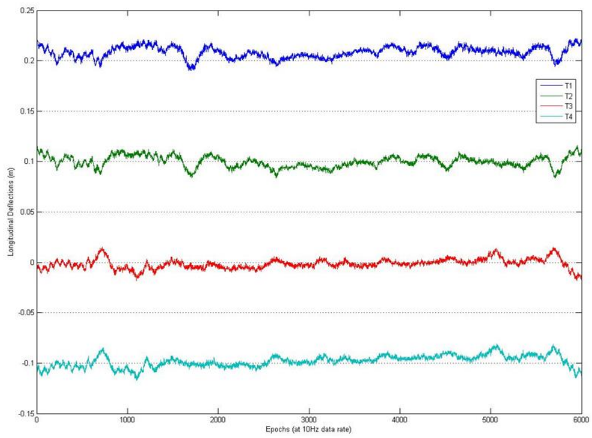


Figure 40, The longitudinal displacements of the four tower locations over a 10 minute period on the Seven Suspension Bridge.

The GNSS data can be used to derive the frequency of the movements as well as the magnitudes of the movements. By using the time associated with each 3-dimensional coordinate upon the Bridge locations, a Fast Fourier Transformation (FFT) analysis within Matlab can be used to extract these frequencies. Figure 41 illustrates the frequencies obtained in the vertical direction at location B. Here it can be seen that there is a strong signal at 0.146Hz and a weaker signal at 0.2265Hz. By carrying out such analysis at all the locations, Table 1 is derived. Here it can be seen that there is a fundamental frequency that is present throughout the Bridge of around 0.146Hz. This is true for the vertical directions on the Bridge’s suspension cable, as well as in the longitudinal direction at the tower tops.

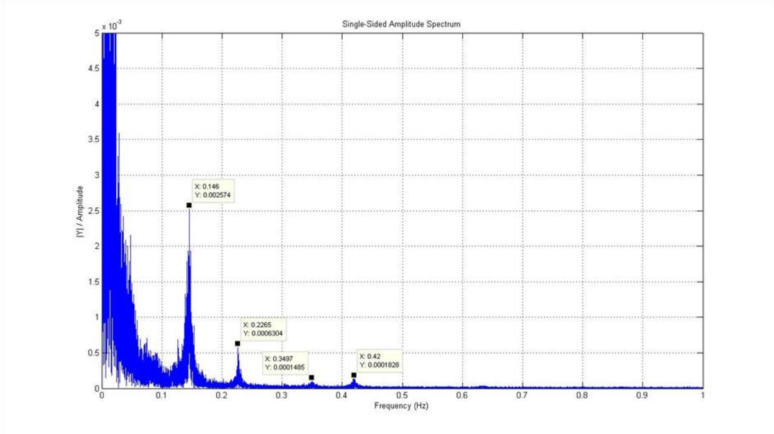


Figure 41, FFT analysis in the vertical direction at location B on the Seven Suspension Bridge.

	vertical					longitudinal			
Position	A	B	C	D	E	T1	T2	T3	T4
Freq	0.0595								
Hz	0.1453	0.146	0.1448	0.1457	0.1457	0.1457	0.1455	0.1461	0.1457
	0.1862		0.1847	0.1847					
		0.2265	0.2264		0.2264				

Table 1, All the GPS derived frequencies at the various survey locations on the Seven Suspension Bridge.

Normally, for a suspension bridge the cable frequencies will dominate the range of frequencies identified from the analyses. However, where data is simultaneously available from both the towers and the cable/deck – as in the case study of the Severn Bridge - the frequency range given from the Towers may reveal results pertinent to the towers alone that are not present in the cables. This is the subject of an ongoing separate study.

7. Summary of the Research

The progression of the work described in Sections 2 to 6 above has been significant, and has demonstrated increasing reliability and sensitivity as research has progressed. The authors now feel in a strong position to state that the use of GNSS to measure deformations, deflections and frequencies on such structures is a well-established technique.

The clear advantage of GNSS is the capability to measure 3-dimensional deformations at millimetre scale, and at precise times, at rates up to 100Hz. This means that frequencies and deformations can be measured simultaneously, at numerous synchronized locations. The second advantage is that these measurements may be obtained at any location for which line-of-sight to satellites can be obtained. In some of the cases described in Sections 2 to 6 above, alternative methods would have led to very complex logistics at the very least, and possible loss of accuracy at worst. Using GNSS in this way can also lead to an accurate 3D coordinate that can be compared between subsequent surveys in order to measure the deformations over time, as well as the short term movements.

It is not suggested here that GNSS monitoring is appropriate for all forms of structural monitoring. Accelerometers can be more appropriate for higher frequencies where deformations are small [Meng *et al*., 2007]. Laser scanners and more recently Ground Based Synthetic Aperture Radar (GBSAR) have been used for deformation monitoring, in particular of landslides [Farina *et al*., 2012]. Deflection monitoring of structures and bridges using GBSAR have also been carried out [Gentile, 2009; Mayer *et al*., 2010]. The use of laser scanners for deflection monitoring of bridges is not seen as being very feasible, for two reasons. Firstly, the range is limited to a couple of hundred metres, depending on the laser type used, but more so; the laser scanner measures millions of discrete points over a period of time, which could be over many minutes. This means that each laser-scanned measurement would be taken at a slightly different time, and hence the bridge would have moved over this interval. The use of laser scanners is more for long term deformation measurements, but any short term deflections would make this end result noisy. Further to this, the distance from the shore-line to the bridge would have to lie within the range of the instrument's capabilities. The use of GBSAR, however, has many opportunities for measuring both deformations as well as deflections. Such devices have a claimed precisions of 0.2mm and can measure at rates of up to 200Hz [Gentile, 2009]. However, in order to obtain a 3D coordinate, two synchronized scanners are required, taking orthogonal measurements to each other, and also the GBSAR technique suffers the same problem as laser scanner, in that the measurements are to many random locations on the structure. The correlation of these measurements from one measurement to the next is not straight forward. Measurements to corners or discrete points on a structure, or even targets, are easier to correlate from one instant in time to another, but measurements to plain open surfaces are not as easily correlated from one instance in time to another. However, GBSAR looks like a very useful tool for deflection monitoring of structures. Nevertheless, GNSS adds a very powerful tool to the armoury, and enables a significant range of deformations, deflections and frequencies to be determined using the same equipment, at synchronized multiple locations, and also at ranges of many kilometres from the shore-line.

8. Conclusions

This paper brings together the work carried out, so far, in this subject area by the authors. It illustrates how advancement in GPS and GNSS technology and availability has improved the results. It also shows that the frequency response of structures is sometimes easier to pick up than looking for the movements, in particular when surveying bridges that have small movements, which are close to the noise of the GPS.

The results have shown that it is possible to relate the movements derived by GPS at various locations to each other, as well as to established models of the bridges.

The use of such GNSS derived movements and frequency responses could well be used as part of a Structural Health Monitoring System, whereby the actual results can be compared to predictions such as those obtained from Finite Element Models. If a bridge is deteriorating, it could be that the characteristics of the movements may change over time. Such data and results could also help to show when a bridge requires refurbishment, or even help to develop models in the first place, as well as help future bridge designs.

Post processing of the data allows better analysis of the results, using various reference stations and processing parameters. Real time is not necessarily required unless the data is used to control the flow of traffic related to the movements.

The use of GPS has to be carefully handled as there are still changes in Dilution of Precision values at various times of days, as well as due to blockages to the view of the open sky in particular directions due to buildings and other infrastructure. This means that it is still required to have a knowledgeable eye to look at and interpret the results, in case any spurious data results in apparent movements. The introduction and adoption of a multi GNSS approach could reduce the instances where spurious results are given. Currently at the University of Nottingham's campus in China, typically 30+ GNSS satellites are visible, including GPS, GLONASS, BeiDou, QZSS, and Galileo. By 2020, it is expected to be able to track even more than this number from the planned 110 or more GNSS satellites available globally.

It could be possible to survey such structures continuously; however this would result in an unmanageable amount of data, which would be difficult to fully interpret. It is very important to realise what is required from this data, and a decision has to be made as to whether data is required at every 10th, 20th or even 100th of a second every single moment of the bridge's life in an operational system. Indeed, if the approach is to investigate the changes in a bridge's characteristics over time, then a survey once per year for 3 or so days would be required. If the engineer is interested in looking at the change in characteristics due to specific external factors, then the surveys should be coordinated to coincide. For example, the engineer may be interested in looking at the effect of temperature changes, therefore, data in the height of summer and depths of winter could provide relevant results. This paper has illustrated that a temperature change of 12.7°C can result in a dip in the main span of 110mm on the Humber Bridge. At such a location, a temperature differential of 40°C or more could be possible over a year's period. In addition to which, high wind loading or high traffic loading could also be investigated. Therefore, the approach could be to occupy the bridge with GPS receivers periodically, or during specific times, or even all the time, but only to look at specific pieces of data rather than trying to process absolutely everything. However, it is very important to establish a benchmark for this data very early on in the bridge's life. Recording GNSS data at key locations to enable future surveys to be compared to. These locations should be marked or a permanent antenna mounting attached, so that re-occupation is possible. Even locating an antenna permanently in place should be considered. However, it is also important to think ahead, as such structures are usually planned to have a life of 100 years or more, therefore the use of a permanent antenna housing in terms of a 5/8" thread attachment on the bridge is more feasible.

The future for the application of GNSS-based technology to structural monitoring is good, based as it is on technology development and not on any theoretical restrictions. In the early days, the cost of equipment was relatively high, and this has reduced over the period. The need to use good antennas will remain. Much infrastructure, particularly in China and neighbouring countries, has GNSS technology fitted onto new major bridge infrastructure. Developments should focus on two main areas:

a) the use of GNSS to remotely monitor infrastructure, and

b) longer-term studies to demonstrate the stability and trends in the performance of major structures.

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