Essays in Ricardian Trade Theory

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Abstract

We build a general Ricardian model of international trade, which extends Eaton and Kortum (2002), in order to analyze the sources of the gains from trade, the effects of trade openness on productivity, and the role of nominal exchange rates.

For general distributions of industry efficiencies, welfare gains can always be decomposed into a selection and a reallocation effect. The former is the change in average efficiency due to the selection of industries that survive international competition. The latter is the rise in the weight of exporting industries in domestic production, due to the reallocation of workers away from non-exporting industries. This decomposition, which is hard to calculate in the general case, simplifies dramatically with Fréchet-distributed efficiencies, providing easy-to-quantify model-based measures of these two effects. For an average of 46 countries in 2000 and 2005, the selection effect turns out to be somewhat more important than the reallocation effect.

By analyzing the relationship between trade openness and total factor productivity (TFP), we propose a novel methodology to measure the latter. The logic of our approach is to use a structural model and measure TFP not from its "primitive" (the aggregate production function), but from its observed implications. We estimate TFP levels of the manufacturing sector of 19 OECD countries, relative to the United States, in 1985-2002, as the average productivity – a proxy for aggregate TFP – that best fits data on trade, production and wages. Our measures turn out to be easy to compute and are no longer mere residuals.

To examine the role exchange rates in a model of real consumption and production decisions with no money, we follow an insight of Keynes (1931) and replicate a currency depreciation with an increase in import barriers and a symmetric decline in export barriers. By mimicking changes in exchange rates with changes in the model parameters, we can demonstrate a series of classical results and conjectures, in a very general framework with many countries, tradeable goods and non-tradeable goods. We show not only that a depreciation has no real effects with flexible wages, but, with sticky wages, we are able to prove that an undervalued currency causes involuntary unemployment abroad, while at home it determines inefficiently high employment in the export sector, raising real GDP but lowering welfare. If the currency is overvalued, we also show that there exists an appropriate depreciation that restores competitive prices, with welfare-enhancing effects, proving Friedman’s conjecture (1953).
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Declaration

Chapter 1 was written with Stefano Bolatto (University of Bologna). It is forthcoming in the Review of International Economics.

Chapter 2 was written with Andrea Finicelli (Bank of Italy) and Patrizio Pagano (Bank of Italy). It is under revision for the Journal of International Economics.

Chapter 3 was written with Virginia Di Nino (Bank of Italy) and Barry Eichen-green (University of California, Berkeley). It is a strongly revised version of two papers that we previously circulated as: "Real Exchange Rates, Trade, and Growth: Italy 1861-2011" and "Real Effects of Nominal Exchange Rates: A View from Trade Theory".
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Introduction

The importance of cross-country differences in industry efficiencies for international trade flows has been recognized in the economic literature since at least the work of David Ricardo (1817).\(^1\) During the XIX century, Ricardo’s famous example — which explained how England and Portugal could both benefit from international trade by exploiting their comparative advantage in making clothes and wine — was gradually turned into a formal model. Solving for equilibrium quantities and relative prices, however, was quite tedious even in the case of a world economy with only few countries and goods, making it difficult to derive general comparative statics results (Eaton and Kortum, 2012). Moreover, the enduring lack of a treatable general-equilibrium model prevented to use Ricardian trade theory for empirical applications as well as for answering meaningful theoretical questions.

In recent years, instead, the theory of comparative advantage has experienced a revival, favored by two major breakthroughs.

First, Dornbusch, Fischer and Samuelson (1977) showed that, by considering a continuum of tradeable goods, the model simplified neatly with respect to the discrete many-commodity case. In fact, one could represent industry efficiencies (of a country relative to another one) with a mathematical function and then use the tools of calculus to derive equilibrium quantities and relative prices. This model simplified the task of deriving the full competitive equilibrium, but could handle only the case of a world economy with just two countries.

Second, after further 25 years, it was finally laid out the full-fledged many-country many-good model. This happened when Eaton and Kortum (2002) focused on specific functions, namely cumulative distribution functions, to represent industry efficiencies in different countries. In particular, they assumed that the efficiencies of the various industries in each country could be described by a Fréchet cumulative distribution

\(^1\) Chipman (1965) provides a famous survey and a discussion of the contribution of Ricardo (1817) to the so-called "classical" theory of international trade, comparing it to the previous studies of Torrens (1808 and 1815). See also Seligman (1903) for an early comparison of the contributions of Ricardo and Torrens.
function. By adding an hypothesis about the strength of the cross-country correlations of industry efficiencies, their model exploited the language of probability to obtain equilibrium quantities and relative prices.

In this model, the full general equilibrium is the solution of a system of equations, with parameters that depend on consumer preferences, labor endowments, trade barriers and the probability distributions representing industry efficiencies. Due to the presence of non-linearities, the system does not have a closed-form solution. Nevertheless, results by Alvarez and Lucas (2007) grant that a solution of the system exists and that it is unique. In addition, the parameters can all be estimated or calibrated, including those of the probability distributions representing technologies. The unique equilibrium quantities and relative prices can then easily be obtained by resorting to standard numerical methods to solve the system.

Thus, the lack of a closed form solution does not prevent to quantify the model and perform counterfactual simulations. In the last few years, in fact, several exercises have been performed and various empirical questions have been explored, including the quantification of the welfare effects of changes in trade barriers, the impact of an improvement in domestic and foreign technologies, the importance of capital endowment and technology for shaping industry specialization, and the size of the change in factor costs needed to balance current accounts across all countries.

In this thesis, we take an alternative route and explore the possibility of using the Ricardian model in order to tackle a set of theoretical questions. Specifically, we consider variants and extensions of the Eaton-Kortum model, in order to analyze different, albeit strictly related, issues, concerning the sources of the welfare gains from

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2 The Fréchet distribution is a heavy-tailed distribution that, together with the Pareto and Lognormal distributions, is commonly used to model efficiencies at the industry or the firm level (see Eaton and Kortum, 2002).

3 The most convenient assumption about the strength of the cross-country correlations of industry efficiencies is the hypothesis of independence, which is the one adopted in the basic model. This restrictive assumption, however, can be easily relaxed in favor of positive or negative correlation (see Eaton and Kortum, 2002, and Finicelli, Pagano and Sbracia, 2013).

trade, the effects of trade openness on total factor productivity, and the role of nominal exchange rates.

The first chapter, written with Stefano Bolatto (University of Bologna), relaxes the assumption of Fréchet-distributed efficiencies, extending the Eaton-Kortum model to general technologies represented by generic distributions of industry efficiencies. For this very general Ricardian framework, we show that the welfare gains from trade can always be decomposed into a selection effect and a reallocation effect. The former is the effect on average efficiency of the mechanism of selection of industries that, thanks to sufficiently low marginal costs of production relative to foreign industries, make only some industries survive to international competition. The latter effect, instead, is related to the rise in the weight of exporting industries in domestic production, which is due to the reallocation of workers away from the less-efficient non-exporting industries to the industries that start servicing the foreign market.

Although the model provides very precise theoretical definitions for both effects, their analytical expression is, in general, too cumbersome to be used for empirical purposes. In particular, with $N$ countries, one should compute the distributions of the efficiencies for the industries that export in each of the $N - 1$ foreign countries, the distributions of the efficiencies for the industries that export in all the possible $N (N - 1) / 2$ couples of countries, etc. In most applications, this calculation would require computing several billions of distributions of efficiencies. For example, in the 46-country application that we consider in the chapter, one would have to compute more than 35,000 billions of different distributions.

By contrast, this decomposition simplifies dramatically if we impose that industry efficiencies are Fréchet distributed. This assumption makes our general Ricardian model return to the original Eaton-Kortum model. Under this assumption, we derive exact model-based measures of these two effects, which can be easily quantified using data on trade flows and domestic production.

A quantification for a sample of 46 advanced and developing economies in the years 2000 and 2005 shows that the selection effect is, on average, somewhat more important than the reallocation effect (accounting for about 60% of the gains from trade). In particular, the former effect is dominant for large countries: only in the United States and Japan, among the advanced economies, and in Brazil, Russia, India,
and China, among the developing countries, does the share of gains pertaining to the selection effect exceeds 80%. However, for small open economies such as Denmark, Ireland, the Netherlands, Singapore, Thailand, and Vietnam, it is the reallocation effect that dominates, as it is responsible for over 70% of the gains.

The second chapter, written with Andrea Finicelli (Bank of Italy) and Patrizio Pagano (Bank of Italy), focuses on the effects of trade openness on total factor productivity (TFP), which is closely related to that of the welfare gains from trade. The relationship between welfare and TFP stems from the fact that, at a global level, the growth in world-wide aggregate TFP induced by international trade is the basic source of the welfare gains for all countries. In other words, countries benefit from the fact that, after opening to trade, specialization makes the world to produce more of each good. This additional production comes from the selection and the reallocation effect discussed in Chapter 1. In particular, the selection effect is such that only a set of domestic industries survives the competition from foreign industries and, therefore, the average of the efficiencies across domestic industries, on which Chapter 2 focuses, changes due to international trade.

We first prove formally that the average productivity across "active" domestic industry is a good proxy for the aggregate TFP of both the closed and the open economy.\footnote{With the term average productivity we refer to the first moment of the efficiency distribution of the active industries. In the closed economy, this is simply the first moment of the Fréchet distribution, which describes the efficiencies of all the industries. In the open economy, average productivity of the industries that survive international competition is the same as the average productivity of the closed economy, augmented by a measure of trade openness.} We then take the former as our measure of productivity, which we dub trade-revealed TFP, and introduce a novel methodology to measure the relative TFP of the tradeable-goods sector of various countries. This new approach is based on the theoretical relationship between trade openness and TFP in the Eaton-Kortum model. The logic of our methodology is to use a structural model and measure TFP not from its "primitive" (which is the aggregate production function in the standard development-accounting approach), but from its observed implications. Specifically, our trade-revealed TFP is the average productivity that best fits data on trade, production and factor costs.
The main advantage of our methodology is that TFP is no longer a mere residual. Moreover, our measures turn out to be easier to compute than in the standard development-accounting approach.

Using annual data from 1985 to 2002, we thus estimate TFP levels, relative to the United States, of 19 OECD countries. Results show a close resemblance between the trade-revealed TFP and the TFP derived from the standard approach. In addition, our measures do not yield common "anomalies," such as the higher TFP of Italy relative to the United States.

The third chapter, written with Virginia Di Nino (Bank of Italy) and Barry Eichengreen (University of California, Berkeley), builds on the previous two, which focused on welfare and productivity, and studies these variables (together with GDP and employment) in a more general setting, in which the Eaton-Kortum model is extended to incorporate also the non-tradeable-goods sector. The main purpose of Chapter 3 is to analyze the domestic and international effects on welfare, productivity, GDP and employment of "misaligned currencies," i.e. of currencies that are either undervalued (say "excessively competitive") or overvalued ("scarcely competitive") with respect to their long-run equilibrium level.

The main challenge that we have to face in order to examine this question is how to introduce a nominal variable like the exchange rate into a model of real consumption and production decisions, in which there is no money. We do so by building on the insight of Keynes (1931) that the combination of an import tariff and an export subsidy — which are two parameters in our model — is isomorphic to an exchange rate depreciation. By mimicking changes in exchange rates with changes in the model parameters, we can demonstrate a series of classical results and conjectures in a very general framework with a multiplicity of countries, tradeable goods and non-tradeable goods.

This chapter obtains three main theoretical results:

- First, we show that a depreciation has no real effects with flexible wages. The decline in marginal costs due to the depreciation is, in fact, completely offset by a proportional rise in relative wages, as it is to be expected in a frictionless economy.
Second, by assuming sticky wages, we are able to prove that an undervalued currency causes involuntary unemployment abroad, while at home it determines inefficiently high employment in the high-productivity export sector. This employment misallocation raises real GDP but lowers welfare, as real wages are too low.

Finally, we show that if the currency is overvalued, then there exists an appropriate depreciation that restores the relative prices of the long-run competitive equilibrium across countries, with welfare-enhancing effects — a result that provides the first formal proof of Friedman's conjecture (1953).6

Thus, our results show formally, in a general equilibrium framework, the domestic and international effects of a nominal depreciation of a currency (and, by the same token, of a nominal appreciation).

In particular, if wages are sticky and the depreciation makes the currency under-valued (i.e. relative wages become lower than their long-run value), then workers shift from the non-tradeable-goods to the tradeable-goods sector. The relative size of latter, however, becomes inefficiently large and, although the domestic economy preserves full employment and real GDP rises, welfare declines. Moreover, undervaluation causes involuntary unemployment abroad, because foreign workers are displaced by the "excessive competitiveness" of the domestic economy and only some of them, but not all, find a job in the non-tradeable-goods sectors.

On the other hand, if the depreciation takes place at a time in which the currency is overvalued (i.e. relative wages are higher than in the long-run equilibrium), then it facilitates the return of the economy to its competitive equilibrium, with a small inflationary impact and welfare-enhancing effects. The increase in consumer prices is "small" because, following the depreciation, domestic wages do not rise and, importantly, would not rise even if they were perfectly flexible. If domestic wages (relative to foreign wages) were higher than what they should have been in the long-run competitive equilibrium, in fact, an appropriate currency depreciation can bring them to

6Friedman (1953, p. 173) conjectured that, in case of misalignments in nominal wages across countries, one could quickly restore the long-run competitive equilibrium just by allowing the exchange rate to properly adjust, rather than by changing the entire internal wage-price structure. In other words, exchange rates could solve the enormous coordination problem of wage and price setters.
their equilibrium level. Thus, depreciation can substitute for the adjustment of relative wages, confirming Friedman’s (1953) intuition.

The fourth chapter summarizes the most important findings, draws the main conclusions and, together with a discussion of some limits of the various models presented in the thesis, offers some related suggestions for future research.
CHAPTER 1

Deconstructing the Gains from Trade:
Selection of Industries vs. Reallocation of Workers
1 Introduction

In a very influential paper, Arkolakis, Costinot, and Rodríguez-Clare (2012) have shown that the welfare gains from trade implied by a very large class of models depend on only two sufficient statistics: (i) the share of expenditure on domestic goods (which is often called "domestic trade share"); and (ii) the elasticity of imports with respect to variable trade costs ("trade elasticity"). This result is remarkable because it applies to frameworks as different as the simple Armington model, in which goods are differentiated by country of origin; the Ricardian model with heterogeneous industries and Fréchet-distributed efficiencies of Eaton and Kortum (2002); the monopolistic competition model of Krugman (1980); as well as variants of the monopolistic competition model of Melitz (2003), with heterogeneous firms and Pareto-distributed efficiencies (such as those developed by Chaney, 2008, and Eaton, Kortum, and Kramarz, 2011). Given their importance for empirical studies, these models are now commonly referred to as "quantitative trade models."

Following this result, the literature appears to be taking two main directions. One analyzes how the measurement of the gains from trade changes when some assumptions of quantitative trade models are relaxed (see Arkolakis, Costinot, Donaldson, and Rodríguez-Clare, 2015, and Melitz and Redding, 2014 and 2015). The other focuses on the empirical implications of the result. In particular, it is now clear that the various models have different implications for the estimated value of the trade elasticity, so that even though the analytical formulation of the gains from trade is the same, the resulting quantification still differs across models (Simonovska and Waugh, 2014a).

In this chapter we explore a different route, by focusing on the sources of the welfare gains of the open economy with respect to the autarky economy as well as on their quantification. In particular, we study whether quantitative trade models allow us to measure not only the overall welfare gains, but also the contribution of the different sources — a key issue in both the theoretical and the empirical literature in international trade. Answering this question, however, is in general very difficult, because different quantitative models entail different predictions on the sources of the welfare gains. For example, the gains from consuming a greater variety of goods are key in Armington and monopolistic competition models, but are absent in Ricardian
models. Given these sharp differences, we analyze this question for one specific family of models and investigate whether belonging to the class of quantitative trade models facilitates the measurement of the contribution of the different sources.

The family on which we focus is the Ricardian model with many countries and goods, CES preferences, and general distributions of industry efficiencies. Thus, with respect to Arkolakis, Costinot, and Rodríguez-Clare (2012), although we restrict the attention to only one family of models, we extend the scope of the analysis by providing general results for Ricardian models in which industry efficiencies follow a generic distribution, and not necessarily a Fréchet.

For this general family of models, we show that the welfare gains of the open economy with respect to the autarky economy can always be decomposed into two distinct sources: a selection and a reallocation effect. The former is the effect on average efficiency of the selection of domestic industries that, thanks to their sufficiently low marginal costs of production relative to foreign industries, survive international competition. Such average efficiency is computed by considering, for the sole industries that survive international competition, the same relative weights in domestic production as the autarky economy. The latter effect, instead, is related to the rise in the weight in domestic production of the exporting industries, which is due to the reallocation of workers away from the less-efficient non-exporting industries to the industries that start servicing the foreign market.

While the model provides very precise theoretical definitions for both effects, their analytical expression is, in general, too cumbersome to be used for empirical purposes. In most applications, in fact, it would require computing several billions of distributions of efficiencies. By contrast, this decomposition simplifies dramatically if we impose that industry efficiencies are Fréchet distributed — the assumption that makes our Ricardian model belong to the class of quantitative trade models. Under this assumption, we can derive exact model-based measures of these two effects, which can be quantified using only data on trade flows and domestic production.

The Fréchet assumption entails this simplification for the following reasons. First, it allows us to easily quantify the overall gains from trade, as in Arkolakis, Costinot, and Rodríguez-Clare (2012). Second, it implies that the selection effect is a measurable share of the overall gains from trade, making it possible to easily obtain the contribu-
tion to welfare of this effect. Third, as a consequence, the reallocation effect (whose quantification is, in the general case, extremely difficult) can be calculated simply as the complement of the selection effect. Therefore, a key insight of our analysis is that quantitative trade models may be useful not only to assess the overall welfare gains, but also to properly measure their sources.

Using the Fréchet assumption, we also demonstrate that, when the gains from trade are small and there are still few exporters in the domestic economy, the largest share of the welfare gains is due to the selection effect. As the export sector grows and the gains from trade increase, the importance of the reallocation effect also rises. Because the contribution of the reallocation effect grows with the size of the overall gains from trade, it follows that the factors affecting the former are exactly the same factors affecting the latter. In particular, both the welfare gains and the contribution of the reallocation effect are higher for small, open and very productive economies, located near to markets that are large, rich, and less productive and, therefore, easier to penetrate. Another interesting feature of our result is that the specific value of the trade elasticity, which is key to determine the overall welfare gains, does not affect the shares of the gains pertaining to the selection and the reallocation effect, making their measurement even more straightforward and robust than that of the welfare gains.

A quantification for a sample of 46 advanced and developing economies in the years 2000 and 2005 shows that the selection effect is, on average, somewhat more important than the reallocation effect (accounting for about 60% of the gains from trade). In particular, the selection effect is dominant for large countries: only in the United States and Japan, among the advanced economies, and in Brazil, Russia, India, and China, among the developing countries, does the share of gains pertaining to the selection effect exceeds 80 percent. However, for small open economies such as Denmark, Ireland, the Netherlands, Singapore, Thailand, and Vietnam, it is the reallocation effect that is dominant, as it is responsible for over 70 percent of the gains.

These findings have important policy implications. Suppose that the export sector is less similar to other sectors of the economy in terms of, for example, skills that are required to workers, as documented by the empirical literature.\(^1\) This feature of the

\(^1\)Bernard, Jensen, Redding and Schott (2007) show, in fact, that exporting firms are more skill intensive than their domestic competitors.
export sector could make the resource reallocation from other industries slower or more difficult. In this case, our theoretical and empirical results suggest that, in the initial stages of trade liberalization (i.e. when trade barriers are still high), these frictions do not prevent to reap the benefits from trade, because most of the gains obtain from the selection effect, that is from the closure of less efficient industries and the reallocation of workers across all the surviving industries, which are mostly non-exporters. Similarly, large countries can expect to enjoy welfare gains almost in full, even in the hypothesis of a cumbersome reallocation to the export sector, thanks to the considerable size of their non-exporting industries. On the other hand, reallocation of workers to the export sector is crucial in small open economies. Therefore, to fully benefit from trade, these countries must be ready to favor the resource reallocation to this sector, in particular by enhancing education and training for unskilled workers.

Our chapter is related to several strands of the literature. Many recent empirical and theoretical studies have focused on one specific source of the welfare gains, that is aggregate productivity. An early example is Pavcnik (2002), who estimates productivity improvements in Chile using firm-level data. This study confirms the importance of the mechanisms described in this chapter, as it finds that the exit of plants and the reshuffling of resources from less efficient to more efficient producers are the main sources of the productivity gains. Many other papers, instead, have focused on model-based measures of the "productivity gains from trade," computed as increases in the average efficiency. To better grasp the link between these papers and our own, it is worth recalling that, in the Ricardian model, the growth in world-wide aggregate productivity induced by international trade is the basic source of the welfare gains for all countries. In other words, countries benefit from the fact that, by specializing in the production of the goods for which they have a comparative advantage, the world production of the optimal consumption bundle increases. Thus, our chapter sheds light on how each individual country, through the mechanisms of selection and reallocation induced by trade liberalization, contributes to the improvement of the world-wide aggregate productivity and reaps the benefits of international trade for its own welfare.

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Another related strand of the literature is the wave of papers focusing on empirical estimates of the gains from trade, such as Feenstra (1994 and 2010), Broda and Weinstein (2006), Goldberg, Khandelwal, Pavcnik, and Topalova (2009), and many others. These papers use different econometric techniques to quantify either the contribution of specific sources of gains (usually those from consuming new varieties) or the size of the overall welfare gains. Our approach, instead, grounded on the derivation of model-based measures of the welfare gains, follows more closely the one of Eaton and Kortum (2002), Alvarez and Lucas (2007), Arkolakis, Demidova, Klenow, and Rodríguez-Clare (2008), Chor (2010), Arkolakis, Costinot, and Rodríguez-Clare (2012), and Ravikumar and Waugh (2015). Unlike those papers, however, we are also able to quantify the contribution of the different sources of gains.3

Our chapter complements Finicelli, Pagano and Sbracia (2013), who focus on the average efficiency of domestic industries (instead of welfare), which is affected only by the selection effect. In an open economy, welfare differs from the average efficiency of domestic industries, because it depends not only on the efficiencies of domestic industries (which determine the price of domestically-produced goods), but also on the efficiencies of foreign industries (which determine import prices). Thus, welfare and the average efficiency of domestic industries are distinct concepts. In this chapter we show that the balanced-trade condition allows us to derive the welfare contribution of imports by using exports; this makes it possible to compute such contribution starting from the efficiency distribution of domestic industries. By using this technique, we can decompose the welfare gains into the selection and the reallocation effect discussed above. As we show, the selection effect turns out to be related to the average price of domestically-produced goods and the reallocation effect to the average price of imported goods.4

3A close relative of our study is also the paper by Demidova and Rodríguez-Clare (2009), who decompose the welfare gains from trade of a small open economy under monopolistic competition into four terms: productivity, terms of trade, number of varieties, and curvature (i.e. the degree of heterogeneity across varieties). Here, instead, we consider a general equilibrium model with perfect competition and, most importantly, we derive a quantifiable expression of the two sources that, in our Ricardian framework, provide the welfare gains.

4It is worth noting that Finicelli, Pagano and Sbracia (2013, pg. 100) also mention a "market-share reallocation effect" but, in that paper, that is the effect of reallocation on labor productivity and not on welfare. Unlike their counterparts on welfare, the selection and the reallocation effect on labor
The rest of the chapter is organized as follows. Section 2 describes the model, which extends Eaton and Kortum (2002) to general distributions of industry efficiencies. Section 3 shows that the welfare gains induced by international trade can be decomposed into two distinct effects, related to the selection of industries and the reallocation of workers. Section 4 introduces the assumption of Fréchet-distributed industry efficiencies, shows that the analytical expressions of the two effects simplify, and quantifies them for a sample of countries and years. Section 5 draws the main conclusions.

2 The model

We consider a continuum of tradable goods, indexed by \( j \in [0, +\infty) \), that can potentially be produced in any of the \( N \) countries of the world economy. Each good \( j \) can be produced in country \( i \) with an efficiency \( z_i(j) \) that, in turn, is defined as the amount of output that can be produced with one unit of input — where both output and input are measured in units of constant quality. Any country has a fixed labor endowment \( L_i \). Inputs include labor as well as a bundle of intermediates goods, which comprises the full set of tradable goods \( j \). Technology is described by a Cobb-Douglas production function with constant returns to scale, in which labor has a constant share \( \beta \leq 1 \) for all industries and countries; namely:

\[
q_i(j) = z_i(j) L_i^\beta(j) L_i^{1-\beta}(j),
\]

where \( q_i(j) \) is the quantity of output \( j \) in country \( i \), \( L_i(j) \) is the number of workers, and \( I_i(j) \) is the quantity of the bundle of intermediate goods.

Consumer preferences are the same across countries. The representative consumer in country \( i \) purchases individual goods in amounts \( c_i(j) \) in order to maximize a CES utility function:

\[
U_i = \left( \int [c_i(j)]^{\frac{\sigma+1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}},
\]

\( productivity \) are analytically indistinguishable and hard to quantify, even in the two-country case. On the contrary, the selection and the reallocation effect on welfare are analytically distinct and easily measurable.

\( ^5 \)We can ignore physical capital in the production function because the model is static and, then, intermediate inputs play a very similar role.
where $\sigma > 0$ is the elasticity of substitution. While the model allows us to deal with both inelastic ($\sigma \leq 1$) and elastic demand ($\sigma > 1$), we will focus on the latter case, because the goods that we consider are all tradable and, in this setting, the typical calibration is $\sigma > 1$.\(^6\)

Consumers maximize their utility function subject to a standard budget constraint. Because we assume that trade is balanced in the open economy, income available for consumption is $Y_i = w_iL_i$, where $w_i$ is the (nominal) wage.

International trade is constrained by barriers, which are modeled using the standard assumption of iceberg costs; i.e., delivering one unit of a good from country $i$ to country $n$ requires shipping $d_{ni}$ units, with $d_{ni} > 1$ for $i \neq n$ and $d_{ii} = 1$ for any $i$. By arbitrage, trade barriers obey the triangle inequality, so that $d_{ni} \leq d_{nk} \cdot d_{ki}$ for any $n$, $i$ and $k$.

Perfect competition implies that the price of one unit of good $j$ produced by country $n$ and delivered to country $i$ is:

$$p_{in}(j) = \frac{c_n d_{in}}{z_n(j)},$$

where $c_n = w_n^\beta p_n^{1-\beta}$ is the cost of one unit of input in the source country $n$, with $p_n$ being the unit price of the optimal bundle of intermediate goods, which is the same as the unit price of the optimal bundle of final goods (see equation (3) below). In other words, we assume (as Eaton and Kortum, 2002) that producers combine intermediate goods using the same CES aggregator that consumers use to combine final goods.

Consumers purchase each good from the country that can supply it at the lowest price; therefore, the price of good $j$ in country $i$ is:

$$p_i(j) = \min_n \left( \frac{c_n d_{in}}{z_n(j)} \right).$$

We assume that, in each country $i$, industry efficiencies $z_i(j)$ are the realizations of a random variable $Z_i$, with a country-specific cumulative distribution function (c.d.f.) $F_i$. Because the $z_i(j)$ represent industry efficiencies and there is a continuum of goods, it is natural to assume that $Z_i$ is non-negative and absolutely continuous.

\(^6\)For an extension of the model that encompasses both tradable and non-tradable goods, see Di Nino, Eichengreen, and Sbracia (2013).
for each country $i$. These are the only conditions that we impose, in this and in the following section, on the $Z_i$’s (in Section 4, instead, we assume that the $Z_i$ are Fréchet distributed). As the expert reader may have noticed, we do not impose the standard restriction that the $Z_i$ are mutually independent across countries, but we allow for dependent (correlated) variables.

The continuum-of-goods assumption and the conventional application of the law of large numbers imply that the share of goods for which country $i$’s efficiency is below any real number $z$ is the probability $\Pr (Z_i < z) = F_i (z)$. It is worth noting that, in the autarky economy, all goods are made at home and, then, $Z_i$ is the efficiency distribution of the closed economy.

Given the cost of inputs, the distribution of industry efficiencies translates into a distribution of good prices. More formally, let us denote with $P_i$ the random variable that describes the distribution of good prices in country $i$; this random variable is defined as:

$$P_i = \min_n \left( \frac{c_n d_{in}}{Z_n} \right) = \left[ \max_n \left( \frac{Z_n}{c_n d_{in}} \right) \right]^{-1}. \quad (2)$$

The price index in country $i$, $p_i$, computed using the correct CES aggregator, is simply the moment of order $1 - \sigma$ of the random variable $P_i$, at the $1/(1 - \sigma)$ power; that is:

$$p_i = \left[ E (P_i^{1-\sigma}) \right]^{1/(1-\sigma)}. \quad (3)$$

After a simple manipulation of equations (2) and (3), we obtain:

$$p_i = c_i \cdot \left[ E \left( M_i^{\sigma-1} \right) \right]^{1/(1-\sigma)};$$

$$\text{where } M_i = \max_n \left( \frac{c_i Z_n}{c_n d_{in}} \right), \quad (4)$$

that leads to the real wage, which measures welfare:\footnote{Recall that, in the competitive equilibrium of both the open and the closed economy, welfare is $U_i = w_i L_i / p_i$, where $L_i$ is exogenous.}

$$\frac{w_i}{p_i} = \left[ E \left( M_i^{\sigma-1} \right) \right]^{1/\beta(\sigma-1)}. \quad (5)$$

The welfare gains from trade can be obtained by comparing the real wage of the open and the closed economy, where the latter can be obtained from the former, letting
$d_{i,n} \rightarrow +\infty$ for $i \neq n$ (using equations (4) and (5)). In this case, we have $M_i \rightarrow Z_i$ and the real wage is $\left[ E \left( Z_i^{\sigma-1} \right) \right]^{1/\beta(\sigma-1)}$. Hence, the gain from trade for country $i$ is:

$$g_i = \left[ \frac{E \left( M_i^{\sigma-1} \right)}{E \left( Z_i^{\sigma-1} \right)} \right]^{1/\beta(\sigma-1)}. \quad (6)$$

Equation (6) shows that welfare gains arise from the transformation, that occurs in the open economy, of the "source of the production efficiencies" (efficiencies that, in turn, determine good prices) from $Z_i$ to $M_i$. Note, in particular, that the latter random variable is a maximum between a set of random variables that includes also $Z_i$. Because the maximum of a set of random variables first-order stochastically dominates any variable included in the set, then $M_i \succeq Z_i$, so that $g_i \geq 1$. In other words, the real wage is higher in the open economy. Thus, the result that trade is welfare improving is here proven using the language of probability, rather than the tools of general equilibrium.

3 Welfare decomposition

Let us now focus on how labor units are reallocated after opening to trade. To foster intuition, we start by considering the case of two countries, say $i$ and $n$, before generalizing the result to $N$ countries.

3.1 A 2-country example

The first-order conditions (FOCs) of the consumer’s problem imply that the demand for good $j$ in country $i$ is:

$$c_i (j) = \left[ \frac{p_i (j)}{p_i} \right]^{-\sigma} \cdot U_i, \quad (7)$$

where $U_i = w_i L_i / p_i$ is the level of utility achieved by country $i$.

---

8We remind the reader that the random variable $X$ first-order stochastically dominates the random variable $Y$, and we write $X \succeq Y$, if and only if $F_X (z) \leq F_Y (z)$ for any $z \in \mathbb{R}$, where $F_X$ and $F_Y$ are the c.d.f. of, respectively, $X$ and $Y$. If this condition holds, then $E \left( X^k \right) \geq E \left( Y^k \right)$, for any $k > 0$.

9The finding that $g_i \geq 1$ for any $i$, proven using basic probability theory, generalizes a result of Finicelli, Pagano, and Sbracia (2013), extending it to a framework in which there are also intermediate goods.
The FOCs of the producer’s problem, on the other hand, imply that the quantities of labor and intermediate goods used to produce good \( j \) in country \( i \) are chosen according to the following proportions:

\[
I_i(j) = \frac{1 - \beta w_i}{\beta} \cdot \frac{L_i(j)}{p_i}.
\]  

By aggregating across industries both sides of equation (8), we find that the overall amount of intermediate goods used in country \( i \) is \( I_i = \frac{1 - \beta}{\beta} \cdot \frac{(w_i/p_i) \cdot L_i}{} \).

The assumption that intermediate goods are combined using the same CES aggregator used to combine final goods implies that, for any country \( i \), the demand for \( j \) as intermediate good, \( m_i(j) \), is proportional to the demand as consumption good, \( c_i(j) \); that is: \( m_i(j) / U_i = m_i(j) / I_i \). Because \( I_i / U_i = (1 - \beta) / \beta \), it follows that, in country \( i \), the demand for good \( j \) as an intermediate input is \( m_i(j) = (1 - \beta) \cdot c_i(j) / \beta \). Hence, in any country \( i \), the overall demand for good \( j \) is \( c_i(j) / \beta \).

In the two-country model that we are examining, each good can either be produced abroad and imported at home; or be produced at home and sold only in the domestic market; or be produced at home and sold both in the domestic and the foreign market. Therefore, the resource constraint for country \( i \) requires that:

\[
q_i(j) = \begin{cases} 
0 & \text{if } j \in O_{i,z} \\
\frac{1}{\beta} c_i(j) & \text{if } j \in O_{i,d} \\
\frac{1}{\beta} [c_i(j) + c_n(j) d_{ni}] & \text{if } j \in O_{i,e} 
\end{cases},
\]  

for any \( j \), where \( O_{i,z} \) denotes the set of "zombie" industries of country \( i \), i.e. those industries that shut down right after trade liberalization;\(^{10}\) \( O_{i,d} \) is the set of industries that sell their goods only on the domestic market; and \( O_{i,e} \) is the set of industries that sell both at home and in country \( n \).\(^{11}\) By construction, the sets \( O_{i,z}, O_{i,d}, \) and \( O_{i,e} \) form a partition of the set of tradable goods; hence, the intersection between any subset of them is empty and their union spans the whole set of tradable goods. The

\(^{10}\)We borrow the terminology "zombie industries" from Caballero, Hoshi, and Kashyap (2008), who use it to refer to industries that are kept alive only by misdirected or subsidized bank lending. In the context of our model, instead, these industries would be kept alive by trade protectionist policies.

\(^{11}\)In the two-country model, these sets are defined as follows: \( O_{i,z} = \{ j : \frac{z_i(j)}{c_i} > \frac{z_n(j) d_{ni}}{c_n} \} \), \( O_{i,d} = \{ j : \frac{z_i(j)}{c_i} \leq \frac{z_i(j)}{c_i} < \frac{z_n(j) d_{ni}}{c_n} \} \), and \( O_{i,e} = \{ j : \frac{z_i(j)}{c_i} \geq \frac{z_n(j) d_{ni}}{c_n} \} \).
set $O_{i,o} \equiv O_{i,d} \cup O_{i,e}^o$, on the other hand, includes the sole industries that survive international competition.\textsuperscript{12}

By plugging equations (1) and (7) into equation (9) (using also equation (8)), and solving the resource constraint for the number of workers in industry $j$, we obtain:

$$L_i(j) = \begin{cases} 
0 & \text{if } j \in O_{i,z} \\
{z}_i^{\sigma-1}(j) \cdot \left( \frac{w_i}{p_i} \right)^{\beta(1-\sigma)} L_i & \text{if } j \in O_{i,d} \\
{z}_i^{\sigma-1}(j) \cdot \left( \frac{w_i}{p_i} \right)^{\beta(1-\sigma)} L_i \cdot (1 + k_{ni}) & \text{if } j \in O_{i,e}^o
\end{cases} \tag{10}$$

where:

$$k_{ni} = \frac{w_n L_n}{w_i L_i} \left( \frac{p_i d_{ni}}{p_n} \right)^{1-\sigma} \tag{11}$$

The term $k_{ni}$ measures the rise in the weight of the exporting relative to non-exporting industries. It is related to the demand that comes from country $n$, since it depends positively on the size of this country in terms of relative GDP, and negatively on the iceberg cost between countries $i$ and $n$, and their relative price levels.

In the autarky economy, $O_{i,z} = O_{i,e}^o = \emptyset$ and the resource constraint returns, for any good $j$, $L_i(j) = z_i^{\sigma-1}(j) \cdot (w_i/p_i)^{\beta(1-\sigma)} L_i$. Let us consider, then, how labor is re-allocated after trade liberalization. With respect to the autarky economy, in the open economy the number of workers in the zombie industries goes to zero. The number of workers in the industries that produce goods that are sold only domestically declines (provided that $\sigma > 1$), because these industries face a tougher competition, due to the fact that imported goods are cheaper than those that were made at home under the autarky regime.\textsuperscript{13} The number of workers in the exporting industries rises, absorbing all the workers "in excess" from the other domestic industries. More specifically, these industries sell less in the domestic market (as international competition brings in cheaper imported goods), so they would need less workers to serve this market, but foreign demand allows them not only to keep their workers, but also to hire new ones.

\textsuperscript{12}The term $c_n(j) d_{ni}/\beta$ in equation (9) represents the foreign demand that benefits only the exporting industries. In particular, the representative consumer of country $n$ demands the quantity $c_n(j)/\beta$, but iceberg costs imply that $d_{ni}$ units must be shipped from country $i$ to deliver one unit of good to country $n$. Thus, the overall quantity produced to serve the latter market is $c_n(j) d_{ni}/\beta$.

\textsuperscript{13}If $\sigma < 1$ ($\sigma = 1$), industries producing goods that are sold only at home would employ more (the same number of) workers.
in order to produce more goods to be sold abroad.\textsuperscript{14}

Notice that, in any industry, the number of workers is proportional to the efficiency of this industry, at the $\sigma - 1$ power (i.e. to $z_i^{\sigma - 1}(j)$). By aggregating across industries both sides of equation (10), we obtain:

$$L_i = \left(\frac{w_i}{p_i}\right)^{\beta(1-\sigma)} L_i \cdot \left\{ \int_{j \in O_{i,d}} z_i^{\sigma - 1}(j) \, dj + (1 + k_{ni}) \int_{j \in O_{i,e}^n} z_i^{\sigma - 1}(j) \, dj \right\}$$

$$= \left(\frac{w_i}{p_i}\right)^{\beta(1-\sigma)} \int_{j \in O_{i,o}} z_i^{\sigma - 1}(j) \, dj + k_{ni} \int_{j \in O_{i,e}^n} z_i^{\sigma - 1}(j) \, dj,$$

from which we can derive the following decomposition of the real wage (which is proven in Appendix A for the general $N$-country case):\textsuperscript{15}

$$w_i = \left[ \lambda_{i,o} \cdot E \left( Z_{i,o}^{\sigma - 1} \right) + \lambda_{i,e} \cdot k_{ni} \cdot E \left( Z_{i,e,n}^{\sigma - 1} \right) \right]^{1/\beta(1-\sigma)}, \quad (12)$$

where $\lambda_{i,o}$ is the probability that an industry of country $i$ survives international competition; $\lambda_{i,e}$ is the probability that it is also an exporter (with $\lambda_{i,e} \leq \lambda_{i,o}$);\textsuperscript{16} $Z_{i,o}$ is the random variable that describes the efficiencies of the surviving industries; and $Z_{i,e,n}$ describes the efficiencies of the industries that export in country $n$.

Equation (12) shows — together with equation (10), from which it is derived — the two sources of the welfare gains of this model. The first one comes from impact of the selection of industries due to international competition, that transforms the average efficiency of the economy from $E\left( Z_i^{\sigma - 1} \right)$ into $E\left( Z_{i,o}^{\sigma - 1} \right)$. The second one comes from the reallocation of workers to the exporting industries, which provides a contribution to welfare that is separate and additional to the previous one (measured by the second term inside the square brackets on the right-hand side of (12)).\textsuperscript{17} This contribution

\textsuperscript{14}For $j \in O_{i,e}^n$, the two terms of equation (10) represent exactly these factors: the number of workers in the exporting industry that serve the domestic market (which declines after trade liberalization) and the number of workers hired to start servicing the foreign market.

\textsuperscript{15}Recall that $E \left( Z_i | Z_i \in A \right) = [\Pr (Z_i \in A)]^{-1} \cdot \int_{j \in A} z_i (j) \, dj$

\textsuperscript{16}The triangle inequality implies that if an industry is an exporter, then it must necessarily sell its goods also in its domestic market.

\textsuperscript{17}The efficiencies of the exporting industries are included also in $Z_{i,o}$ (that describes the efficiency of all the surviving industries, including the exporters). Therefore, the contribution of the reallocation effect is distinct from the one that comes from the selection effect.
depends on the strength of foreign demand (as measured by $k_{ni}$) and is key to the result that trade is welfare improving. In fact, although the real wage always rises after trade openness, the average efficiency does not necessarily rise.\textsuperscript{18} Hence, economies in which average efficiency is lower under trade openness, still benefit from trade thanks to this additional reallocation effect. Under broad conditions about the distribution of industry efficiencies, however, also the selection effect provides a positive contribution to welfare and, in the next section, we discuss and quantify both effects for one specific model that fulfills those conditions.\textsuperscript{19}

It is interesting to see how the main variables in equation (12) vary as the world economy converges to zero gravity, that is as the barriers, $d_m$ and $d_{ni}$, tend to 1. For the sake of simplicity, let us focus on the case of two identical countries, with, in particular, $d_m = d_{ni} = d \geq 1$ (and $L_i = L_n$) and independent distributions of efficiencies. As $d$ declines, the probability of surviving decreases (more domestic producers are displaced by foreign exporters) while the probability of exporting increases (selling abroad becomes easier), until $\lambda_{i,o} = \lambda_{i,e} = 1/2$ for $d = 1$.\textsuperscript{20} Analogously, $Z_{i,e,n}$ and $Z_{i,o}$ also tend to converge: as $d$ diminishes, more and more industries become exporters, including those with lower efficiencies, so that the moments of $Z_{i,e,n}$ decrease; on the other hand, more and more industries shut down, so that those that survive have higher efficiency and the moments of $Z_{i,o}$ increase. Eventually, it becomes $Z_{i,e,n} = Z_{i,o}$ for $d = 1$.\textsuperscript{21}

To foster the intuition on the nature of the sources of welfare identified in equation

\begin{itemize}
  \item \textsuperscript{18}In other words, the result that $M_i \geq Z_i$ implies that $E(M_i^{\sigma-1}) \geq E(Z_i^{\sigma-1})$ (i.e. welfare rises after trade openness), even though $E(Z_i^{\sigma-1})$ can be either larger of smaller that $E(Z_{i,o}^{\sigma-1})$ (average efficiency does not necessarily rise).
  \item \textsuperscript{19}Finicelli, Pagano, and Sbracia (2013) examine the theoretical conditions under which average efficiency across industries rises after opening to trade. In particular, they show that it always rises under very broad assumptions about the country distributions of industry efficiencies; namely: (i) if the distributions of efficiencies are independent across countries; (ii) for many types of distributions, if their correlations are sufficiently low; (iii) regardless of cross-country correlations, if industry efficiencies belong to families of distributions that are widely used in the literature, such as the Fréchet, Pareto and Lognormal.
  \item \textsuperscript{20}With identical countries, $k_{ni} = d^{1-\sigma}$; thus, as $d$ diminishes, $k_{ni}$ rises (if $\sigma > 1$, as in the standard calibrations), increasing the weight of the reallocation effect, until it becomes $k_{ni} = 1$ for $d = 1$.
  \item \textsuperscript{21}In the general case, even if countries are not identical and the distribution of their industries are not independent, it still holds that $\lambda_{i,o} = \lambda_{i,e}$ and $Z_{i,e,n} = Z_{i,o}$ for $d = 1$.
\end{itemize}
Appendix B shows that welfare depends on the average price of domestically-produced goods and the average price of imported goods. The selection effect turns out to be related to the former average and the reallocation effect to the latter. The average price of imported goods, in particular, depends on the efficiency distribution of foreign exporters. However, by using the resource constraint, which is equivalent to the balanced-trade condition, in equation (12) we are able to use the efficiency distribution of domestic instead of foreign exporters (i.e., we use exports instead of imports) and then obtain a term that can be easily quantified.

Before turning to the quantification, however, let us show how the welfare decomposition (12) generalizes to the case of many countries ($N \geq 2$).

### 3.2 The $N$-country case

For the general multi-country framework, in Appendix A we prove that the real wage in each country $i$ has still two components, the selection effect ($SE_i$) and the reallocation effect ($RE_i$):

$$\frac{w_i}{p_i} = (SE_i + RE_i)^{1/\beta(\sigma-1)}.$$  \hspace{1cm} (13)

The first term inside the brackets of the right hand side of (13) has the same expression as the corresponding term of the two-country case:

$$SE_i = \lambda_{i,o} \cdot E \left(Z_{i,o}^{\sigma-1}\right).$$  \hspace{1cm} (14)

The second term is now more cumbersome:

$$RE_i = \sum_{n \neq i} \lambda_{i,e;n} \cdot k_{ni} \cdot E \left(Z_{i,e;n}^{\sigma-1}\right) +$$
$$+ \sum_{n \neq i, k \neq i, n \neq h} \lambda_{i,e;n,h} \cdot (k_{ni} + k_{hi}) \cdot E \left(Z_{i,e;n,h}^{\sigma-1}\right) +$$
$$+ ... + \lambda_{i,e;1,...,N} \cdot (k_{1i} + ... + k_{Ni}) \cdot E \left(Z_{i,e;1,...,N}^{\sigma-1}\right),$$  \hspace{1cm} (15)

where $\lambda_{i,e;n,h,...,k}$ is the probability that an industry of country $i$ exports in (and only) countries $n$, $h$, ..., and $k$; while $Z_{i,e;n,h,...,k}$ is the distribution of the efficiencies of these industries.

As shown by equations (12) and (15), in both the cases $N = 2$ and $N > 2$ the magnitude of the reallocation effect is governed by $k_{ni}$ (equation (11)). In particular,
$k_{ni}$ and the size of the reallocation effect are larger if country $i$ is relatively more productive ($p_i/p_n$ is low), and if the destination market $n$ is rich ($w_n/w_i$ high), large ($L_n$ is high relative to $L_i$) and not too far away ($d_{ni}$ low). Thus, geography, which is key in the Ricardian model as shown by Eaton and Kortum (2002), exerts its effects mostly through the reallocation of workers to the export sector.

Given that a big chunk of the related literature focuses on monopolistic competition models à la Melitz (2003), it is worth clarifying how the welfare sources are different in these frameworks with respect to the Ricardian model, with welfare in the latter being described by (13). On the production side, the adjustment that takes place after trade liberalization is very similar in the two frameworks. In both models, in fact, domestic production concentrates on only a subset of the goods that were made under autarky: these are the goods that are made more efficiently with monopolistic competition, and those in which the country has a comparative advantage in Ricardo (in (13), this is represented by the fact that (14) is an average across a set of goods which includes only the industries that survive international competition). In addition, in both perfect and monopolistic competition models, domestic production becomes tilted towards exporters, who benefit from foreign demand (and, in Ricardo, this is represented by the term (15)).

On the consumption side, in both Ricardian and monopolistic competition models, households consume less of those tradeable goods whose production remains domestic. In the Ricardian model, however, households purchase more of the remaining tradeable goods (because imports are cheaper), so that overall consumption increases (thus the gain from trade is positive), even though they do not gain access to more varieties. In the monopolistic competition model, households start consuming a greater variety of goods. In light of Arkolakis, Costinot, and Rodríguez-Clare (2012), if the trade elasticity implied by the two models were the same, then the gains from consuming a larger quantity of imported goods in the Ricardian model would be the same as the gains from consuming more imported varieties in frameworks à la Melitz (2003). To put it differently, if trade elasticities were identical, "Ricardo’s intensive margin" would be equal to "Melitz’s extensive margin".\footnote{We recall, however, the important caveat, established by Simonovska and Waugh (2014a), that different trade models have different implications about the value of the trade elasticity. These authors,}
In principle, quantifying the expressions of (14) and (15) is not an impossible task, although it may be rather daunting. Given the joint distribution of \((Z_1, ..., Z_N)\), in fact, one can always derive the distribution of any of the \(Z_{i;e,n,h,\ldots,k}\), which are just univariate conditional distributions (see Appendix A). However, in empirical applications their number might be extremely large, making their computation a very challenging task. With \(N\) countries, one has to compute the distributions of the efficiencies for the industries that export in each of the \(N-1\) foreign countries, those for the industries that export in all the possible \(N(N-1)/2\) couples of countries, etc.. For instance, in the 46-country application that we consider in the next section, one should have to compute a total of more than 35,000 billions of different distributions (that is \(2^{N-1}-1\)). In the next section, instead, we show that, by introducing an assumption that transform our general Ricardian model into one of the quantitative trade models of Arkolakis, Costinot, and Rodríguez-Clare (2012), the quantification of the two effects simplifies dramatically.

4 Fréchet-distributed efficiencies

We now assume that, in any country \(i\), industry efficiencies are Fréchet distributed, with parameters \(T_i\) and \(\theta\);\(^{23}\) hence, the probability that an industry of country \(i\) has an efficiency lower that a positive real number \(z\) is \(F_i(z) = \exp\{-T_i z^{-\theta}\}\). For the sake of simplicity, we also assume that these distributions are mutually independent across

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\(^{23}\)Kortum (1997) and Eaton and Kortum (2009) show that the Fréchet distribution emerges from a dynamic model of innovation in which, at each point in time: (i) the number of ideas that arrive about how to produce a good follows a Poisson distribution; (ii) the efficiency conveyed by each idea is a random variable with a Pareto distribution; (iii) firms produce goods using always the best idea that has arrived to them.
countries. The moment of order $k$ of $Z_i$ is:

$$
E \left( Z_i^k \right) = T_i^{k/\theta} \cdot \Gamma \left( \frac{\theta - k}{\theta} \right),
$$

which exists if and only if $\theta > k$, where $\Gamma$ is Euler’s Gamma function. Because welfare is related to the moment of order $\sigma - 1$ of $Z_i$, we assume $\theta > \sigma - 1$. The parameter $T_i$, usually defined as the "state of technology" of country $i$, captures country $i$’s absolute advantage: an increase in $T_i$ relative to $T_n$ implies an increase in the share of goods that country $i$ produces more efficiently than country $n$. The shape parameter $\theta$, common to all countries, is inversely related to the dispersion of $Z_i$. It is related to the concept of comparative advantage because, in the Ricardian model, gains from trade depend on the heterogeneity in efficiencies. In this model, a decrease in $\theta$ (i.e. higher heterogeneity), coupled with mutual independence, generates larger gains from trade for all countries.

An important property of the model with Fréchet-distributed efficiencies is that the price distribution in country $i$ for the goods imported from country $n$ is the same for any $n$ (and equal to $P_i$). Thus, for example, source countries with a higher state of technology or lower iceberg costs exploit these advantages by selling a wider range of goods to that country but, in the equilibrium, the price distributions of the goods that the various foreign sources supply to the destination market $i$ are identical (see Eaton and Kortum, 2002, and Arkolakis, Costinot, and Rodríguez-Clare, 2012). A related

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24 The key assumption is that industry efficiencies are Fréchet distributed, while independence can easily be relaxed. In particular, Eaton and Kortum (2002) propose a multivariate Fréchet distribution for industry efficiencies that allows for correlation across countries, and Finicelli, Pagano and Sbracia (2013) use it to compute the "productivity gains from trade" for different degrees of correlation.
key property is that, in the open economy: \( M_i = Z_{i,o} \). Hence, equation (5) becomes:

\[
\frac{w_i}{p_i} = \left[ E(\gamma_{i-1}) \right]^{1/(\sigma-1)}.
\]  

(17)

We now show how the analytical decomposition of welfare simplifies and how its sources can be quantified under the Fréchet assumption. Combining equation (17) with (13) and using equation (14), it turns out that:

\[
RE_i = (1 - \lambda_{i,o}) \cdot E(\gamma_{i-1}) ,
\]

while it is still \( SE_i = \lambda_{i,o} \cdot E(\gamma_{i-1}) \).

The welfare gain induced by trade openness (equation (6)) becomes:

\[
g_i = E(\gamma_{i-1})^{1/(\sigma-1)} ,
\]

that, in turn, can be decomposed as:

\[
g_i = \left[ \frac{E(\gamma_{i-1})}{E(\gamma_i)} \right]^{1/(\sigma-1)},
\]

In other words, given the overall gain from trade \( g_i \), a share \( \lambda_{i,o} \) of the gain is due to the selection effect, while its complement, \( 1 - \lambda_{i,o} \), is due to the reallocation effect.

26If the random variables \( X \sim \text{Fréchet} (\xi, \theta) \) and \( Y \sim \text{Fréchet} (\lambda, \theta) \) are independent, then \( \max (X, Y) \sim X | X \geq Y \sim \text{Fréchet} (\xi + \lambda, \theta) \). Thus, in particular, \( E[\max (X, Y)] = E(X | X \geq Y) \).

This property is important to quantify the overall welfare gains and the welfare decomposition, because it enables to focus on the change of the distribution of industry efficiencies induced by trade openness (from \( Z_i \) to \( M_i = Z_{i,o} \)), which is in turn summarized by the change of the scale parameter of the Fréchet distribution. It is worth noticing that \( E[\max (X, Y)] = E(X | X \geq Y) \) always holds if the random variables \( X \) and \( Y \) are i.i.d.. Unlike the Fréchet case, instead, for Pareto- and Lognormally-distributed variables, the hypothesis that \( X \) and \( Y \) are identical (and not just independent) is essential.

In the Fréchet case, instead, not only the identity assumption, but also the independence assumption can be relaxed. (We thank a referee for stimulating this discussion; a proof for these results is available from the authors upon request.)

26In interpreting the shares of the welfare gains due to the selection and the reallocation effect, we can safely ignore the complication due to the exponent \( 1/\beta (\sigma - 1) \). In fact, a monotone transformation of the utility function, such as the one that can be obtained by taking \( U_i \) at the \( \beta (\sigma - 1) \) power, would yield the same equilibrium quantities and relative prices. In this transformed model, then, welfare would be the same as in the original model, but at the \( \beta (\sigma - 1) \) power, making the exponent of the gains from trade equal to 1 (while leaving the base unchanged).
We can now turn to the measurement. The properties of the Fréchet distribution imply that $Z_{i;o}$ is still a Fréchet, with parameters $\Lambda_i$ and $\theta$, where:

$$\Lambda_i = T_i + \sum_{i\neq k} T_k \left( \frac{c_k d_{ik}}{c_i} \right)^{-\theta}.$$ 

It follows that:

$$\frac{E \left( Z_{i;o}^{\sigma-1} \right)}{E \left( Z_i^{\sigma-1} \right)} = \left( \frac{\Lambda_i}{T_i} \right)^{(\sigma-1)/\theta}.$$ 

To quantify $g_i$, we borrow from Finicelli, Pagano and Sbracia (2013, Proposition 5) the result that:

$$\frac{\Lambda_i}{T_i} = \frac{IMP_i}{PRO_i - EXP_i},$$

in which $IMP_i$ is the value of country $i$’s aggregate imports, $PRO_i$ is the value of its production, and $EXP_i$ is the value of aggregate exports. Thus:

$$g_i = (\Omega_i)^{1/\beta \theta}.$$ 

This is the same result established by Arkolakis, Costinot, and Rodríguez-Clare (2012) for the larger class of quantitative trade models. In fact, $\Omega_i^{-1}$, which is equal to one minus the import penetration ratio, is the so-called "trade domestic share" (i.e. the share of expenditure on domestic goods), while in this Ricardian model the trade elasticity is $\beta \theta$.

The quantification of the selection and the reallocation effect can be completed once that we derive $\lambda_{i,o}$, which is the probability that an industry of country $i$ survives international competition. Using the properties of the Fréchet distribution, it is easy to find that:

$$\lambda_{i,o} = \frac{T_i (c_i)^{-\theta}}{\sum_k T_k (c_k d_{ik})^{-\theta}} = \frac{1}{\Omega_i}.$$ 

27 The result follows immediately from the property described in footnote 22 and the fact that if $X \sim \text{Fréchet}(\xi, \theta)$ and $a > 0$, then $aX \sim \text{Fréchet}(a\xi, \theta)$.

28 Note that $\Lambda_i > T_i$. In other words, if industry efficiencies are Fréchet distributed, then the average efficiency of the surviving industries is always higher than that of the whole set of domestic industries (i.e. of the set that includes also the industries that shut down after trade liberalization). This feature of the "quantitative Ricardian trade model" is both consistent with the available empirical evidence and it is shared by a large class of Ricardian models (see footnote 20).
Note that, because welfare gains are increasing in $\Omega_i$, it follows that, when the gains are larger, the selection effect is less important and the reallocation effect is more important. This result can be readily explained. When the gains from trade are small, the selection effect matters mostly because there are few exporters in the domestic economy and, then, the possibilities of reallocating workers in these industries are fewer. On the other hand, as the export sector grows and the gains from trade increase, the importance of the reallocation effect also rises because exporting industries (which are on average more productive) absorb more workers.

What does the data show about the size of these two effects? Table 1 provides a quantification of the welfare gains from trade as well as the contribution of the selection and reallocation effect for a sample of 46 advanced and developing countries in two different years, 2000 and 2005. Given that the Ricardian theory laid out in this chapter best describes trade in manufactures, rather than in natural resources or primary goods, we follow the literature and consider data on the values of domestic production, exports and imports — which is all is needed to compute the gains from trade as well as the contribution of their sources — all referred to the manufacturing sector.\footnote{Data on the value of output (i.e. value added plus intermediate goods) of the manufacturing sector is often available only at five year intervals, especially for small countries (see also Levchenko and Zhang, 2013). In addition, emerging countries typically have data only for very short time horizons. Here we solve the trade-off between number of countries and number of years by including in the sample 46 countries (a larger number than the 19 OECD countries considered by Eaton and Kortum, 2002, albeit smaller than the 60 countries considered by Alvarez and Lucas) and by considering two different years (against the practice of considering only one single year). A remarkable exception is Levchenko and Zhang (2013), who set up a dataset encompassing 72 countries over 5 decades, using the UNIDO Industrial Statistics Database. Here we prefer to stick to the OECD STAN dataset, which is generally considered to have higher quality data.} In addition, given that the model assumes that trade is balanced, in the application we impose that exports are identical to imports (equal to their average).

The gains are computed using equation (20), taking the value of the main parameters from literature. In particular, we assume that the shape parameter is $\theta = 4$, as advocated by Simonovska and Waugh (2014b), and the share of intermediate goods in production is $\beta = 0.33$, a conventional measure of the share of value added in total output. The share of the gains from trade pertaining to the selection and reallocation effects, respectively equal to $\lambda_{i,o}$ and $1 - \lambda_{i,o}$, are computed using equation (21).
Table 1: Gains from trade and their sources (1)

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<tr>
<th>OECD countries</th>
<th>Welfare gain (%)</th>
<th>Selection effect (%)</th>
<th>Reallocation effect (%)</th>
<th>Welfare gain (%)</th>
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<td>min</td>
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Source: authors’ calculations on OECD STAN data.

(1) Real wage relative to the autarky economy (values of \((g_i - 1)\)% and contributions of the selection and the reallocation effect (in percentage).
For each year, Table 1 shows the percentage increase in welfare due to international trade and the shares (in percentage) due to the selection and the reallocation effect. Results show that the gains from trade are considerable (for the cross-country average welfare is almost 60 and 70 percent higher than in autarky in 2000 and 2005). As it is well known, the size of the gains is quite sensitive to the assumptions about the value of the shape parameter and the share of intermediate goods in production. For instance, by taking $\theta = 6.66$ instead of $\theta = 4$ (as Alvarez and Lucas, 2007), the gains would be about 60 percent of those reported in Table 1. By the same token, in the model without intermediate goods ($\beta = 1$), gains from trade would be about one third of those reported in the table.

Overall, the size of the selection effect is somewhat more important than the reallocation effect in our sample of countries (it is close to 60 percent in the year 2000 and around 55 per cent in 2005). It is worth noting that, unlike the gains from trade, the two shares remain unchanged irrespectively of the exact value of $\theta$ and $\beta$. Unsurprisingly, the reallocation effect is more important in small open economies, such as Denmark, Estonia, Ireland, the Netherlands, Slovenia, Singapore, Thailand, and Vietnam. For these countries, the share of the welfare gains pertaining to the reallocation effect is above 70 percent in at least one year. On the other hand, for large and relatively more closed countries, it is the selection effect that it is dominant. For instance, among the OECD economies, only the United States and Japan record a share of the welfare gains pertaining to the selection effect above 80 percent in at least one year. Among non-OECD economies, only the BRIC countries (Brazil, Russia, India, and China) show the same record as the United States and Japan.

5 Conclusion

This chapter provides a deconstruction of the sources of the welfare gains from trade in a Ricardian model. Under general distributions of industry efficiencies, welfare gains arise from two distinct sources. The former is an effect due to the selection of industries that survive international competition. The latter is related to the reallocation of workers away from the industries that shut down, as well as from those selling only in the domestic market, to the industries that start servicing the foreign market. If industry
efficiencies are Fréchet distributed, so that the model becomes one of the quantitative trade models of Arkolakis, Costinot and Rodríguez-Clare (2012), these two effects can be easily measured.

Our results also show that the share of the welfare gains due the reallocation effect is larger, the larger is the overall welfare gains. Thus, countries that can potentially gain more from trade — i.e. small open economies that are close to large, rich, and less efficient markets — would gain mostly from the reallocation effect. Therefore, to fully reap the benefits from international trade, they must be ready to favor the reallocation of resources towards exporting industries, for example supporting workers’ education and training.

The key insight from our analysis, however, is that quantitative trade models seem to be useful not only in order to assess the overall welfare gains, but also to properly measure their sources — an issue that deserves to be further explored in future studies tackling other models in this class. The route taken in this chapter of using quantitative trade models to measure not only the overall welfare gains from trade, but also the contribution of their sources, appears to be a promising area for theoretical and empirical research.
Appendix

A Welfare decomposition with many countries

In order to prove equation (13), let us start by generalizing the resource constraint (9) to a context with more than just two countries. As in the two-country case, we still have: 

\[ q_j (i) = 0 \], if \( j \in O_{i,z} \) and \( q_j (i) = c_i (j) / \beta \), if \( j \in O_{i,d} \). Now consider the set of industries of country \( i \) that export in (and only) the countries \( n, h, \ldots, k \), for any \( \{ n, h, \ldots, k \} \in \{ 1, \ldots, N \} \setminus \{ i \} \), and denote this set by \( O_{i,e}^{n,h,\ldots,k} \); the resource constraint for these industries becomes:

\[ q_j (i) = \frac{1}{\beta} [c_i (j) + c_n (j) d_{ni} + c_h (j) d_{hi} + \ldots + c_k (j) d_{ki}] . \]

Solving the resource constraint for the number of workers in industry \( j \), we obtain:

\[
L_i (j) = \begin{cases} 
0 & \text{if } j \in O_{i,z} \\
\frac{z_i^{\sigma-1} (j) \cdot \left( \frac{w_i}{p_i} \right)^{\beta (1-\sigma)} L_i}{\frac{1}{\beta} [c_i (j) + c_n (j) d_{ni} + c_h (j) d_{hi} + \ldots + c_k (j) d_{ki}]} & \text{if } j \in O_{i,d} \\
\frac{z_i^{\sigma-1} (j) \cdot \left( \frac{w_i}{p_i} \right)^{\beta (1-\sigma)} L_i \cdot (1 + k_{ni} + k_{hi} + \ldots + k_{ki})}{\frac{1}{\beta} [c_i (j) + c_n (j) d_{ni} + c_h (j) d_{hi} + \ldots + c_k (j) d_{ki}]} & \text{if } j \in O_{i,e}^{n,h,\ldots,k}
\end{cases}, \quad (22)
\]

where the terms \( k_{li} \) are defined as in equation (11), for any destination market \( l \).

Note that the sets \( O_{i,z}, O_{i,d}, O_{i,e}^{n,h,\ldots,k} \) (for any \( \{ n, h, \ldots, k \} \) as above) form a partition of the set of tradable goods. By aggregating across industries both sides of equation (22), we obtain the following:

\[
\left( \frac{w_i}{p_i} \right)^{\beta (\sigma-1)} = \lambda_{i,d} E \left( Z_{i,d}^{\sigma-1} \right) + \ldots + \lambda_{i,e,n,h,\ldots,k} (1 + k_{ni} + k_{hi} + \ldots + k_{ki}) \cdot E \left( Z_{i,e,n,h,\ldots,k}^{\sigma-1} \right) + \ldots
\]

(23)

where \( \lambda_{i,d} \) is the probability that an industry of country \( i \) survives international competition and serves only the domestic market (i.e. \( \lambda_{i,d} = \text{Pr} (Z_i \in O_{i,d}) \)); \( \lambda_{i,e,n,h,\ldots,k} \) is the probability that an industry of country \( i \) exports in (and only) countries \( n, h, \ldots, k \), and \( k, i.e. those for which \( z_i (j) / c_i < z_l (j) d_{li} / c_l \), for \( l = n, h, \ldots, k \); and excludes those that export in countries different from \( n, h, \ldots, k \), i.e. those for which \( z_i (j) / c_i > z_l (j) d_{li} / c_l \) for \( l \neq n, h, \ldots, k \).

---

The analytical definition of \( O_{i,e}^{n,h,\ldots,k} \) is as follows: this set includes all the industries that export in countries \( n, h, \ldots, k \), and \( k, i.e. those for which \( z_l (j) / c_l > z_l (j) d_{li} / c_l \), for \( l = n, h, \ldots, k \); and excludes those that export in countries different from \( n, h, \ldots, k \), i.e. those for which \( z_l (j) / c_l < z_l (j) d_{li} / c_l \) for \( l \neq n, h, \ldots, k \).
\[ k \text{ (i.e. } \lambda \text{, } \lambda_{i,e,n,h,...,k} = \Pr(Z_i \in O_{i,e,n,h,...,k})) \text{; } \]  
\[ Z_{i,e,n,h,...,k} \] is the distribution of the efficiencies of these industries (i.e. \( Z_i \mid Z_i \in O_{i,e,n,h,...,k} \)). Considering that:

\[ \lambda_{i,o} \cdot E(Z_{i,o}^\sigma-1) = \lambda_{i,d} \cdot E(Z_{i,d}^\sigma-1) + \ldots + \lambda_{i,e,n,h,...,k} \cdot E(Z_{i,e,n,h,...,k}) + \ldots, \]

we can conveniently rearrange the right-hand side of equation (23) into the sum of two terms, given by equations (14) and (15). By taking the \( 1/\beta (\sigma - 1) \) power of both sides, we finally obtain equation (13).

### B Welfare decomposition and average prices

Recall that, in the two-country example, the price of good \( j \) in country \( i \) is \( c_i/z_i(j) \) if the good is domestically produced; it is \( c_n d_{in}/z_n(j) \) if the good is imported (from country \( n \)). Then, we can write:

\[
    p_i = \left[ \int_j \left[ p_i(j) \right]^{1-\sigma} \, dz \right]^{1/(1-\sigma)}
\]

\[
    = c_i^{1-\sigma} \int_{j \in O_{i,o}} \left[ z_i(j) \right]^{1-\sigma} \, dz + (c_n d_{in})^{1-\sigma} \int_{j \in O_{i,s}} \left[ z_n(j) \right]^{1-\sigma} \, dz
\]

\[
    = c_i \cdot \left[ \lambda_{i,o} \cdot E(Z_{i,o}^\sigma-1) + (1 - \lambda_{i,o}) \cdot \left( \frac{c_n d_{in}}{c_i} \right)^{1-\sigma} \cdot E(Z_{n,e}^\sigma-1) \right]^{1/(1-\sigma)}.
\]

The first term in the square bracket, multiplied by \( c_i \), is the average price of domestically-produced goods, and depends only on the efficiency distribution of the domestic industries that survive international competition, \( Z_{i,o} \). The second term, multiplied by \( c_i \), is instead the average price of imported goods, and depends on the efficiencies of foreign exporters, \( Z_{n,e} \).

Using the fact that \( c_i = w_i^\beta p_i^1 - 1 \) for any country \( i \), we obtain:

\[
    \frac{w_i}{p_i} = \left[ \lambda_{i,o} \cdot E(Z_{i,o}^\sigma-1) + \lambda_{i,e} \left( \frac{c_n d_{in}}{c_i} \right)^{1-\sigma} \cdot E(Z_{n,e}^\sigma-1) \right]^{1/\beta(\sigma-1)}.
\]  

Equation (24) shows two main facts. First, it shows that country \( i \)'s welfare depends on the efficiency distribution of domestic industries as well as on the efficiency distribution of those foreign industries from which country \( i \) imports goods, i.e. the efficiencies of foreign exporters. Second, by comparing it with equation (12), it shows that the
selection effect (which coincides with first term in the square bracket in (24)) measures the welfare gains obtained from lower average domestic prices; on the other hand, the reallocation effect, which is the complement of the selection effect (just like the second term in the square bracket is the complement of the first term in (24)), is equivalent to the welfare gains due to lower import prices.

For what concerns the reallocation effect, in particular, notice that in Section 3, by using the resource constraint (which is equivalent to the balanced-trade condition), we have been able to use the distribution of domestic exporters in the decomposition (12), instead of the distribution of foreign exporters as in the alternative decomposition (24). In other words, the resource constraint makes it possible to shift the focus from imports to exports, i.e. from the efficiencies of foreign exporters to the efficiencies of domestic exporters.
CHAPTER 2

Trade-Revealed TFP
1 Introduction

Estimating the level of a country’s total factor productivity (TFP) is a very difficult task. The standard development-accounting (or "level-accounting") approach consists in choosing a functional form for the aggregate production function, measuring the quantities of output and inputs, and then obtaining TFP as a residual (King and Levine, 1994). In general, the derivation of quantity data is a very critical part of this approach. Recent literature, such as Burstein and Cravino (2015), has in fact shown that the common practice of obtaining quantity data by deflating value data may be flawed, because the price indexes calculated by statistical agencies that are adopted for this purpose differ from the ideal deflators that would be appropriate from a theoretical standpoint.¹

Another important problem of the standard methodology is related to the fact that TFP is measured as a residual. As such, TFP incorporates any statistical error in the data and ends up being a "measure of our ignorance" (Abramovitz, 1956).

The need to refine existing methodologies and complement them with new ones is warranted by the importance of TFP for understanding the distribution and growth of wealth across nations. In particular, studies based on development accounting show that differences in TFP account for a big chunk of cross-country differentials in per capita income (e.g. Hall and Jones, 1999). In addition, while recent research has conjectured that these results might be due to sectoral differences in TFP levels, this hypothesis can be tested only on very rough sectoral classifications.²

¹Similarly, the measurement of the quantity of physical capital is especially troubling, because even the basic value data have to be constructed. In addition, the perpetual inventory method commonly adopted for this purpose is very demanding in terms of data (for instance, it requires long time series on fixed investments and price deflators), entails heroic assumptions about the depreciation rate, mixes up types of capital with different efficiencies, and neglects key issues regarding the quality of capital. Difficulties then escalate if one is interested in calculating homogeneous measures of TFP across several countries or sectors. In fact, cross-country heterogeneities in the quality of capital and in the diffusion and accuracy of price deflators are especially large across industrial and developing countries. The lack of sectoral data on fixed investment, which affects even some industrial economies, is also stunning.

²For different views about this hypothesis see Caselli (2005), who considers data disaggregated into two sectors (agriculture and non-agriculture), and Herrendorf and Valentinyi (2012), who extend the
In this chapter, we use a structural model of international trade to measure TFP. The logic of our approach is to derive TFP not from its "primitive" (the aggregate production function), but from its observed implications. The model, in fact, maps international trade flows, production, and wages into the average productivity of the tradeable goods sector, allowing to obtain a proxy for the TFP of this sector.

The new road that we explore builds on the Ricardian model developed by Eaton and Kortum (2002) and some theoretical results obtained in a companion paper by Finicelli, Pagano, and Sbracia (2013), which we extend in this chapter. In the Eaton-Kortum model, industry productivities in the tradeable goods sector of each country are described by a Fréchet distribution, whose country-specific scale parameter is related to the average productivity of the closed economy. Eaton and Kortum (2002) have shown that this parameter can be estimated, relative to that of a benchmark country, using value data on bilateral trade flows, production and nominal wages. For the open economy, Finicelli, Pagano, and Sbracia (2013) have proven that the productivity distribution of the industries that survive international competition remains a Fréchet and that the mean of this distribution is equal to the average productivity under autarky, augmented by an easy-to-quantify measure of trade openness. Here, we first show formally that the average productivity across domestic industry provides a proxy for the TFP and, then, we take the former as our measure of productivity, which we dub trade-revealed TFP. The approximation error is nil in the extreme cases of autarky and zero gravity; in the intermediate case of positive but finite trade barriers, the approximation error, however, is low not only when the set of exporters is small (as the bias affects few exporting industries), but also when it is large. In the latter case, in fact, because foreign demand does not alter the composition of the demand for domestic goods (foreign and domestic consumers have the same preferences), then almost all industries are equally affected by the same bias. In other words, in this case the problem becomes the larger weight than in TFP that the simple average gives to the industries that do not export, which are few when trade barriers are small.

We then turn these theoretical results into estimates of the TFP of the tradeable goods sector of 19 OECD countries, relative to the United States, with annual data analysis to five sectors.
from 1985 to 2002.\(^3\)

With respect to the development-accounting approach, our estimates have three main advantages. First, they are no longer mere residuals, but are the productivities that best fit data on trade, production and wages. The reason is that the country-specific scale parameter related to average productivity affects, together with wages, all the bilateral exports of this country as well as the domestic production sold domestically. Therefore, in a model with \(N\) countries, each scale parameter can be estimated using \(N - 1\) observations (i.e. \(N(N - 1)\) observations to estimate \(N\) parameters). The estimation process that is involved, in turn, potentially allows to reduce the impact of the statistical error that affects, instead, the standard methodology.

Second, our estimates are obtained from value data about trade, production and wages and do not require hard-to-get quantity data. This feature eliminates the critical step related to the deflation of quantity data, and ensures a higher degree of homogeneity and comparability of data across several countries.

Third, the wide availability of sectoral data on trade flows, production and wages makes it possible to compute sectoral estimates of TFP. While in this chapter we only consider the aggregate tradeable goods sector and do not pursue any finer classification, Levchenko and Zhang (2011), based on our methodology, estimate average productivities for several countries over a 50-year period. Similarly, Shikher (2004) performs sectoral estimates of the parameter related to average productivity, from which one could easily retrieve sectoral TFPs following our methodology.

These advantages do not come at no cost. Our estimates refer to the restricted universe of the tradeable goods sector, which we identify as the manufacturing sector, rather than embracing the whole economy. They measure TFP in each country as the average productivity across domestic industries and, therefore, are an approximation of the TFP that would obtain by aggregating production across industries.\(^4\) They provide

---

\(^3\)We use the same data sources and sample of countries as Eaton and Kortum (2002). In addition, while they focus only on one year (1990), we consider a time horizon spanning 18 years. Unfortunately, one data source was discontinued in 2004, preventing us to analyze a longer time span.

\(^4\)Finicelli, Pagano, and Sbracia (2013) show that average productivity of domestic industries approximates the TFP of the economy’s aggregate production function. In the extreme cases of autarky and zero gravity, the approximation error is nil. In fact, the aggregate production function derived
relative levels of TFP across countries, but not their absolute values. Finally, while our measure is obtained from a model which neglects physical capital, the last is not necessary because in the model it is the cost of inputs that matters for bilateral trade shares, not their quantities. When we introduce capital, however, results are broadly unchanged.

A noteworthy result from our analysis is that the TFP rankings and relative values that we obtain appear more plausible than those delivered by the standard development-accounting approach. One key difference with respect to development-accounting studies, most notably Hall and Jones (1999), is that while in their samples Italy is usually found to have the highest TFP, a surprising result given the relative weakness of institutions and government policies ("social infrastructure") in this country, according to our analysis Italy ranks only 6th or 7th over the whole sample period, and the most productive country is invariably the United States.\(^5\) Interestingly, in our sample of countries the correlation between TFP and Hall and Jones’ social infrastructure index is higher if TFP is measured using our methodology than with their own TFP data.

We then provide a zoom shot of the manufacturing TFP of Italy relative to the United States, comparing the dynamics of our measure with one obtained from development accounting. We view this case study as especially intriguing because of the just mentioned "anomaly" of development-accounting results. The focus on this country pair also allows us to offer a more detailed and data-enhanced analysis. We find that our measure yields a sharp difference in levels with respect to development

\(^5\)For a brief discussion of this point, see Hall and Jones (1999) and Lagos (2006).
accounting, while preserving a very similar time pattern.

The focus on input costs (instead of quantities) to measure TFP makes our methodology reminiscent of the dual method for computing TFP growth rates developed by Hsieh (2002). However, we do not obtain our TFP as a residual, and we compute TFP (relative) levels instead of growth rates. Another closely related method for comparing TFP across countries is the "revealed-superiority" approach of Bar-Shira, Finkelshtain, and Simhon (2003), which in turn is inspired by Samuelson's principle of revealed preferences. With this paper, our methodology shares the idea of measuring TFP not from its "primitive" (the production function) but from its observed implications. Our approach distinguishes from Bar-Shira, Finkelshtain, and Simhon's in that they extract information about the TFP for the whole economy from observed aggregate profits, while we focus on the TFP of the tradeable goods sector and derive it from countries' shares in international trade. In addition, we quantify relative TFPs, while their methodology only delivers a ranking.

Traces of the idea of exploiting the effects of TFP on trade flows to retrieve a measure of the TFP itself appear, in different forms, also in other papers. Treffer (1995) obtains Hicks-neutral factor-augmenting productivities for several countries (relative to the United States) as the productivities that minimize the gap between observed trade data and the trade pattern implied by factor intensities according to the Hecksher-Ohlin-Vaneck theory. Waugh (2010) obtains a relationship between model parameters and TFP using a variant of the Eaton-Kortum model with traded intermediate goods and a non-traded final good; then, he quantifies the contribution of international trade to the TFP without estimating the latter. Fadinger and Fleiss (2011) develop a model with monopolistic competition and homogeneous firms — whereby we assume perfect competition and heterogeneous industries — but end up with an empirical framework to measure TFP that turns out to be similar to ours, in that it requires only data on trade flows, production and input costs.

Here is a roadmap of the chapter. Section 2 briefly summarizes the Eaton-Kortum model and the main results that provide the theoretical background for the empirical methodology, which is presented in Section 3. Section 4 computes and describes the trade-revealed TFPs, compares them with results from a sample of previous studies, and checks their robustness to the inclusion of physical capital. Section 5 analyzes
more closely the case of Italy versus the United States. Section 6 concludes, with some suggestions for future research.

2 Theoretical underpinnings

The Eaton-Kortum model considers a framework with many countries and a continuum of tradeable goods produced by industries operating under perfect competition. Making the quantity \( q_i(j) \) of the tradeable good \( j \) in country \( i \) requires the bundle of inputs \( I_i(j) \), in a constant-returns-to-scale technology; namely: \( q_i(j) = z_i(j)I_i(j) \), where \( z_i(j) \) is an efficiency parameter which varies across countries and industries.

The key hypothesis in Eaton-Kortum is that each \( z_i(j) \) is the realization of a country-specific random variable \( Z_i \), with \( Z_i \sim Fréchet (T_i, \theta) \), where \( T_i > 0, \theta > 1 \), and the \( Z_i \) are mutually independent across countries. The two parameters of the distribution are the theoretical counterparts of the Ricardian concepts of absolute and comparative advantage. The former — \( T_i \), the state of technology — captures country \( i \)'s absolute advantage: an increase in \( T_i \), relative to \( T_n \), implies a higher share of goods that country \( i \) produces more efficiently than country \( n \). The latter — \( \theta \), the precision of the distribution — which is assumed identical across countries, is inversely related to the dispersion of \( Z_i \) and its connection with the concept of comparative advantage stems from the fact that Ricardian gains from trade depend on cross-country heterogeneities in technologies.\(^6\) In this perspective, Eaton and Kortum (2002) demonstrate that a decrease in \( \theta \) (higher heterogeneity) generates larger gains from trade for all countries.

Another important assumption concerns trade barriers, which are modeled as iceberg costs: delivering one unit of any good from country \( i \) to country \( n \) requires producing \( d_{ni} \) units (with \( d_{ni} > 1 \) for \( i \neq n \) and \( d_{ii} = 1 \)). Trade barriers lift the price at which countries can sell their products in foreign markets above the one at which they sell the same goods at home.

If representative consumers in all countries have identical CES preferences across tradeable goods, it is possible to show two fundamental properties of the model. First,\(^6\) Denoting Euler’s gamma function by \( \Gamma \), the moment of order \( k \) of \( Z_i \) is given by \( T_i^{k/\theta}\Gamma [(\theta - k)/\theta] \) if \( \theta > k \). The connection between \( \theta \) and the dispersion of \( Z_i \) can be appreciated by considering that the standard deviation of \( \log Z_i \) is \( \pi/(\theta \sqrt{6}) \).
the market share of country $i$ in country $n$ — i.e. the ratio between the value of the imports of country $n$ from country $i$ ($X_{ni}$) and the value of the total expenditure (or total absorption) of country $n$ ($X_n$) — is given by

$$
\frac{X_{ni}}{X_n} = \frac{T_i \cdot (c_i d_{ni})^{-\theta}}{\Phi_n}, \text{ where: } \Phi_n = \sum_{k=1}^{N} T_k \cdot (c_k d_{nk})^{-\theta},
$$

(1)

with $c_i$ denoting the cost of the bundle of inputs. This share is increasing in the state of technology $T_i$ and decreasing in the input cost $c_i$ and the trade barrier $d_{ni}$. Its value depends also on the technologies, costs and trade barriers of any other country $k$: it increases with costs $c_k$ and distances $d_{nk}$, and decreases if technologies $T_k$ increase.

Second, the exact price index of the bundle of tradeable goods in country $n$ resulting from the CES aggregator and the prices $p_n(j)$ is

$$
p_n = \gamma \cdot \Phi_n^{-1/\theta}, \text{ where: } \gamma = \left[ \Gamma \left( \frac{\theta + 1 - \sigma}{\theta} \right) \right]^{1/(1-\sigma)},
$$

(2)

with $\Gamma$ denoting Euler’s Gamma function and $\theta > \sigma - 1$.

The solution of the model is given by a system of non-linear equations in relative wages, relative prices and trade flows. The main testable implication can be obtained by assuming that intermediate inputs comprise the full set of tradeable goods, so that the input cost is $c_i = w_i^{\beta} p_i^{1-\beta}$, where $w_i$ is the nominal wage in country $i$ and $p_i$ is given by equation (2). Rearranging equations (1) and (2) we obtain:

$$
\log \left( \frac{X_{ni}}{X_{nn}} \right) \left( \frac{X_{ii}/X_i}{X_{nn}/X_n} \right)^{1-\beta} = S_i - S_n - \theta \log (d_{ni}), \quad (3)
$$

where

$$
S_i \equiv \frac{1}{\beta} \log (T_i) - \theta \log (w_i), \quad (4)
$$

and $X_{ni}$ is the value of imports of country $n$ from country $i$, $X_n$ the value of the total expenditure (or total absorption) of country $n$, $X_{nn}$ the value of expenditure on domestically produced goods, $w_i$ the nominal wage in country $i$. The variable $S_i$, given by the state of technology adjusted for labor costs, is a measure of the competitiveness of country $i$. The left-hand side of equation (3) is a "normalized" share of the imports

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7There is also a non-tradeable goods sector in the economy, and a constant fraction of the aggregate final expenditure (equal to $1 - \alpha$, $0 < \alpha < 1$) is spent on these goods. For the whole solution of the model, see Eaton and Kortum (2002, pp. 1756-1758).
of country \( n \) from country \( i \). This equation shows that the ability of country \( i \) to sell its own products in country \( n \) is increasing in the relative competitiveness of country \( i \) vis-à-vis \( n \) and decreasing in the iceberg cost of exporting from \( i \) to \( n \).

Equation (3) can be used, as in Eaton and Kortum (2002), to obtain estimates of the relative states of technology in a cross section of countries (i.e. the ratios \( T_i/T_n \)). However, we are interested in estimating TFPs, which are related but far from identical to the states of technology. In fact, while the mean of \( Z_i \) is the average productivity in country \( i \) across all existing tradeable goods, with open markets there exist some industries in country \( i \) that cease to produce because they eventually succumb to foreign competition. The latter happens, precisely, to the industries that make their goods less efficiently than their foreign competitors, so that these goods are cheaper to import than to produce at home, despite the advantage provided by trade barriers. Therefore, \( E(Z_i) \) corresponds to the TFP of country \( i \) only under autarky, while if markets are open then the TFP must be calculated over the subset of tradeable goods that are actually made by country \( i \).

This issue is addressed from a theoretical standpoint in Finicelli, Pagano, and Sbracia (2013), who derive, within the Eaton-Kortum model, the productivity distribution of the industries that survive international competition, also a Fréchet. The mean of this distribution calculated for country \( i \), that is the TFP of the tradeable goods sector of this country, denoted with TFP, can be expressed as follows:

\[
\text{TFP}_i = E(Z_i) \cdot \Omega_i^{1/\theta} = T_i^{1/\theta} \cdot \Gamma \left( \frac{\theta - 1}{\theta} \right) \cdot \Omega_i^{1/\theta},
\]

where

\[
\Omega_i \equiv 1 + \frac{\text{IMP}_i}{\text{PRO}_i - \text{EXP}_i}.
\]

The factor \( \Omega_i^{1/\theta} \) is a measure of trade openness that captures the effect of international competition in selecting industries that have a competitive advantage.\(^9\) Equation (5)\(^8\)

---

\(^8\) The fact that quantity-data on physical capital are not needed in our methodology is by no means driven by the omission of this factor from the production function. As equations (3) and (4) show, in fact, although labor is included in the production function, its cost, and not its quantity, is relevant for bilateral trade shares. By the same token, if physical capital is added to the production function, equations (3) and (4) become functions of both wages and the cost of capital, but do not depend on the capital stock (see Section 4 and Appendix C for details).

\(^9\) Notice that the selection effect is always positive (TFP, > E(Z_i)). In other words, industries
forms the basis of our estimates of cross-country relative TFPs: once the relative states of technology are estimated, measuring relative TFPs requires only widely available data on trade and production.

3 Empirical methodology

In this section, we illustrate the methodology to estimate the TFP of the manufacturing sector and apply it to a sample of 19 OECD countries for each year between 1985 and 2002. The methodology follows three main steps. First, equation (3) is used to estimate the competitiveness indexes $S_i$. Second, the states of technology $T_i$ are derived from the estimated $S_i$, using equation (4). In applying these two steps, we provide an extension of the cross-section analysis performed by Eaton and Kortum (2002) with 1990 data, to a sample period spanning 18 years. In addition, we update the original methodology of Eaton and Kortum (2002) in that we convert nominal wages into U.S. dollars using PPP instead of market exchange rates, as suggested by Finicelli, Pagano, and Sbracia (2011). Once states of technology are obtained, it is immediate to compute our trade-revealed TFPs from equations (5) and (6), a step that we finalize in Section 4. We defer to Appendix A a detailed description of our dataset.

Let us consider equation (3). The left-hand side can be measured with production and trade data, and a calibration for $\beta$. For $\beta$, we follow Alvarez and Lucas (2007) who define it as the cross-country average of manufacturing value added over gross manufacturing production. By doing so, they consider labor and capital goods as part of a single production factor, which they label as "equipped labor". Over the period 1985-2002 this calibration delivers annual values of $\beta$ between 0.31 and 0.34.\(^{10}\) that survive international competition are on average more productive than those that are crowded out, implying that the TFP of the open economy is above the autarky level. Finicelli, Pagano, and Sbracia (2013) focus on this result and show that it holds under very general assumptions about the distribution of productivities. In particular, it holds irrespectively of the correlation among country technologies, for several classes of joint distributions, including the multivariate Fréchet, Pareto, normal, and lognormal. With independent technologies, the result always holds, irrespectively of the type of their joint distribution.

\(^{10}\)Eaton and Kortum (2002) use an alternative calibration, setting $\beta$ equal to the cross-country average of the labor share in gross manufacturing production. This calibration implies that labor is
On the right-hand side, trade barriers can be modeled using the proxies suggested by the gravity literature. Following Eaton and Kortum (2002), we proxy trade barriers between \( i \) and \( n \) with a set of standard dummy variables, namely:

\[
\log d_{ni} = d_k + b + l + e + m_n,
\]

where the dummy variables associated with each effect are suppressed for notational simplicity. In equation (7), \( d_k \) is the effect of the distance between \( i \) and \( n \) lying in the \( k \)-th interval \( (k = 1, \ldots, 6) \); \( b \) is the effect of \( i \) and \( n \) sharing a border; \( l \) is the effect of \( i \) and \( n \) sharing the language; \( e \) is the effect of both \( i \) and \( n \) belonging to the European Economic Community (EEC), from 1985 to 1992, or to the European Union (EU), from 1993 onwards; \( m_n \) \( (n = 1, \ldots, 19) \) is a destination effect.

Using (7), equation (3) becomes

\[
\log \left[ \left( \frac{X_{ni}}{X_{nm}} \right) \left( \frac{X_{ii}}{X_{mn}} \right)^{\frac{1-\beta}{\beta}} \right] = S_i - S'_n - \theta d_k - \theta b - \theta l - \theta e,
\]

where \( S'_n = S_n + \theta m_n \). The competitiveness of country \( i \) is estimated as the source country effect \( (S_i) \), while the destination dummies \( (S'_n) \) are the sum of country \( n \)'s competitiveness \( (S_n) \) and destination effect \( (\theta m_n) \). To avoid perfect multicollinearity, we impose the same restriction as Eaton and Kortum (2002) that \( \sum_n S_n = \sum_n S'_n = 0 \); therefore, the estimated coefficients of these dummy variables measure the differential competitiveness effect with respect to the average (equally-weighted) country.

We estimate equation (8) by ordinary least squares for each year (separately) in the period 1985-2002.\(^{12}\) With 19 countries, we have 342 informative observations for each regression (the equation is vacuous when \( n = i \)). Table 1 reports the results of the regressions for the first and last year of our sample, and for 1990 (the benchmark year in Eaton and Kortum, 2002). The coefficients of the distance dummies indicate, as expected, that geographic distance inhibits trade. However, the size of this effect

\(^{11}\)Intervals are specified in Table 1, with distance calculated in miles.

\(^{12}\)By running a regression for each year, we end up with yearly estimates of the main parameters and, then, of average productivities.
tends to decline over time, perhaps suggesting an increasing degree of integration not captured by other effects. In addition, the decline appears to be sharper for the biggest distances. The dumping effect of distance is mitigated by positive border and language effects. Belonging to the EEC/EU also tends to foster trade, although this effect is not statistically significant, which comes as no surprise given that most countries in the sample are European.

Estimates of the source dummies $S_i$ indicate that in 1985 Japan is the most competitive country, followed by the United States, while the ranking between these two countries inverts towards the end of the sample period. On the other hand, Greece and Belgium stand out as the least competitive countries throughout the whole period. Relative to the United States, competitiveness of most countries in the sample peaks towards the end of the 1980s, then declines until 2000, and recovers somewhat in 2001-02.

Estimates of $-\theta m_n$ provide a measure of how cheap it is to export manufacturing goods to country $n$, compared to the average.\(^\text{13}\) The values of $-\theta m_n$ reflect the presence of tariffs and non-tariff costs that have to be paid by foreigners to sell a good in the domestic market, such as local distribution costs, legal obligations, product standards. Over the entire sample period, the country ranking of $-\theta m_n$ is similar to that $S_i$; for instance, Japan is the cheapest destination, while Belgium stands out as the most expensive one.\(^\text{14}\)

From $S_i$, we can now extract the states of technology $T_i$ simply by inverting equation (4), i.e. $T_i = \exp (\beta S_i) \cdot w_i^{\beta \theta}$. This step requires data on nominal wages and

\(^{13}\)Waugh (2010) estimates equation (8) by including a source- instead of a destination-country dummy. In his sample, which includes both advanced and developing economies, this method returns a positive correlation between the $S_i$ and income per worker, consistently with some stylized facts documented in that paper. In our sample, which comprises only developed economies, Waugh’s method would return a negative correlation of the $S_i$ with income per worker (in the order of -0.2) against a positive correlation (0.3) from our method.

\(^{14}\)Eaton and Kortum (2002) estimate equation (8) by generalized least squares, using only 1990 data, obtaining similar results in terms of sign and significance of the coefficients and of country ranking. (See, in particular, their discussion concerning the apparently surprising result about the high degree of openness of Japan.) The small differences between our results and theirs are due only to the different calibration of $\beta$ and to the older update of the OECD data used in their paper, and not to the different estimation method.
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(1) Estimates of equation (8) using OLS; standard errors in brackets.
a calibration for $\theta$.

Following Eaton and Kortum (2002), nominal wages are adjusted for education to account for the different degrees of "worker quality" among the countries in our sample. We set $w_i = \text{comp}_i \cdot \exp (-g \cdot h_i)$, where $\text{comp}_i$ is the nominal compensation per worker, $g$ the return on education (which we set to 0.06 as Eaton and Kortum, 2002), $h_i$ the average years of schooling.\textsuperscript{15} Wages are converted into a common currency using PPP exchange rates, as suggested by Finicelli, Pagano, and Sbracia (2011).\textsuperscript{16} This approach is also consistent with the standard practice in development accounting, which is the yardstick for our trade-revealed TFPs.

The parameter $\theta$ is set equal to 6.67 as in Alvarez and Lucas (2007), who exploit the fact that the expression for market shares derived in Eaton and Kortum (2002) is identical to one obtained in a model à la Armington (1969), with $\theta$ replacing Armington’s $\sigma_a - 1$, where $\sigma_a$ is the Armington elasticity. Based on Anderson and van Wincoop (2004), Alvarez and Lucas pick their preferred calibration from a range of values between 4 and 10.\textsuperscript{17}

Table 2 shows the values of the resulting states of technology, at the $1/\theta$ power, relative to those of the United States in selected years. We report the values of $(T_i/T_{us})^{1/\theta}$, where the subscript $us$ stands for the United States, because this ratio is equal to $E(Z_i)/E(Z_{us})$ (see footnote 7 for the mean of the Fréchet), that is, as discussed in

\textsuperscript{15} Setting $g = 0.06$ is a conservative calibration according to Bils and Klenow (2000). See Appendix B for results with the somewhat larger (and non-linear) values of the return on education used by Hall and Jones (1999) and Caselli (2005).

\textsuperscript{16} Finicelli, Pagano, and Sbracia (2011) document that, by converting wages into a common currency using market exchange rates, as originally suggested by Eaton and Kortum (2002), the resulting estimates of relative technologies show implausible swings for several countries. Most importantly, the time-series of these estimates exhibit a correlation with nominal exchange rates vis-à-vis the US dollar that, for many countries, is not significantly different from $-1$ (a negative correlation means that a depreciation of a country’s currency vis-à-vis the US dollar is associated with a decrease in its relative state of technology).

\textsuperscript{17} Following a different approach, Eaton and Kortum (2002) estimate $\theta$ using other testable implications of the model and find values between 3 and 13 (their benchmark is 8.28). Notice that both Alvarez and Lucas (2007) and Eaton and Kortum (2002) consider cross-sectional data. In our empirical analysis spanning 18 years, we take $\theta$ time-invariant. Finicelli, Pagano, and Sbracia (2011) provide some evidence supporting this assumption.
Table 2: States of technology in selected years (1)

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(1) Values of $(T_i/T_{us})^{1/6.67}$.

Section 2, the TFP of the manufacturing sector of country $i$, relative to the United States, under an autarky regime.

Over the whole sample period, the United States stands out as the country with the highest state of technology, followed by the other major industrial countries (the second place is taken by France, Japan, or the United Kingdom, depending on the sample year). On average, the state of technology of the United States is about 15% above that of the rest of the sample. Portugal occupies invariably the bottom place of our sample, with a state of technology that is 35% lower than that of the United States. In the next section, we transform these estimates into values of relative TFPs.

4 Results

We are now equipped to calculate TFP levels relative to a benchmark country. Denoting with $\lambda_i$ the TFP of country $i$ relative to the United States, from equations (5) and (6) one obtains

$$\lambda_i = \left( \frac{T_i}{T_{us}} \frac{\Omega_i}{\Omega_{us}} \right)^{1/\theta}.$$  \hspace{1cm} (9)

By construction, then, the TFP in the United States is normalized to 1 in every year.
Table 3 shows that, over the whole sample period, the manufacturing TFP of the United States is the highest among the 19 OECD countries considered, followed by Belgium, the United Kingdom and France. Portugal, New Zealand, and Australia have the lowest average TFPs. Over time, the average relative TFP across all countries (excluding the United States) exhibits tiny fluctuations around 80%.\textsuperscript{18}

In Figure 1, we focus on the relative TFPs of Japan, the United Kingdom, and the four largest euro area countries. In the early 1990s, the TFPs of these countries are close to each other and become more dispersed thereafter. The divergent path of the TFPs of Italy and the United Kingdom, in particular, is noteworthy. In 1985 they are not dissimilar. Afterwards, Italy loses ground with respect to the other countries, while the United Kingdom’s relative TFP grows rapidly. In 2001-2002, Italy’s TFP is the lowest among the group of countries in the figure, also surpassed by Spain, while the United Kingdom ranks first, not too distant from the United States. Finicelli, Pagano, and Sbracia (2013) show that an important driver of the UK’s TFP has been the selection effect of international competition (according to their estimates, the contribution of trade openness to the UK’s TFP has grown from 5.6% in 1985 to 10.1% in 2005, the largest increase among the countries in Figure 1).

\textsuperscript{18}Our results are robust to alternative calibrations of the main parameters in the model, i.e. $\beta$, $\theta$, and $g$ (see Appendix B for details).
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The finding that the United States’ TFP is the highest throughout the two-decade period is worth stressing. According to a number of studies based on development accounting, in fact, in the mid-1980s to early-1990s it was Italy’s TFP that ranked first among the 19 countries in our sample. These findings appear rather odd given the well known relative weakness of Italy’s institutions. For example, Lagos (2006) is puzzled by the result that TFP is higher in Italy than in the United States, which is at odd with the observation that Italy has a more distorted labour market vis-à-vis the United States. Similarly, Hall and Jones (1999) underscore that hours per worker "are higher in the United States than in France and Italy, making their [high] productivity levels more surprising." Our methodology returns a more plausible assessment, whereby in our sample of high-income countries Italy ranks 6th or 7th, with a manufacturing TFP that is 13% to 17% lower than that of the United States.

Besides the specific result for Italy, which is analyzed with greater detail in the next section, our findings are broadly in line with those from a sample of other studies that use different methodologies. The rank correlation of our 1990 results with the

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See, for example, Hall and Jones (1999), Chari, Restuccia, and Urrutia (2005), or the development-accounting exercise performed by Fadinger and Fleiss (2011). In Klenow and Rodríguez-Clare (1997), the TFP of Italy is third, but it is still higher than that of the United States.
TFP ranking estimated by Bar-Shira, Finkelshtain, and Simhon (2003) is above 0.8. The (linear) correlation of our 1985 results with the 1983 "trade-revealed type" of TFP provided by Trefler (1995) is about 0.7. The broad picture delivered by our methodology is also not too different from that in Klenow and Rodriguez-Claire (1997) and Hall and Jones (1999): the correlation between their relative TFPs and ours are fairly high, equal to about 0.65 in both cases.

It is worth recalling the result documented by Hall and Jones (1999) who find that, in a sample of 127 countries, differences in social infrastructure drive differences in capital accumulation, productivity, and output per worker. The positive correlation between their measure of TFP and their index for social infrastructure remains also if one narrows the analysis to the 19 advanced economies of our sample (left panel of Figure 2). Yet, in that scatter plot some countries — notably Italy, but also France and Spain — display very large residuals from a simple OLS regression, featuring a much higher TFP than the predicted one. Interestingly, using our trade-revealed TFPs (right panel of Figure 2) delivers a stronger correlation and a better fit of the data ($R^2$

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20 Trefler (1995) obtains the Hicks-neutral factor-augmenting productivities of several countries (relative to the United States) that provide the smallest gap between observed trade data and the trade pattern implied by factor intensities. While the purpose of his study was not that of measuring TFP (but, rather, that of vindicating the predictions of the Hecksher-Ohlin-Vaneck theory), his results provide the first example of a trade-revealed measure of TFP.

21 The estimates in Klenow and Rodriguez-Claire refer to year 1985, those in Hall and Jones to 1988. The correlations are obviously calculated with respect to our estimates for the corresponding years.
climbs from 19 to 34 percent), while solving the TFP "anomalies" of Italy, France, and Spain, that present a much smaller residual in the new regression.

Are our measures robust to the inclusion of physical capital in the model? In this case, estimating the relative states of technology requires data on the rental cost of capital. Assume that the production function for good \( j \) in country \( i \) is given by

\[
q_i(j) = z_i(j) [L_i^\alpha(j) K_i^{1-\alpha}(j)]^\beta I_i^{1-\beta}(j) .
\]

Denoting with \( T^K_i \) the state of technology in country \( i \) when capital is included in the production function, in Appendix C we show that

\[
\frac{T^K_i}{T^K_j} = \frac{T_i}{T_j} \cdot \left( \frac{r_i}{w_i} \right)^{(1-\alpha)\beta \theta}
\]

where \( r_i \) denotes the rental cost of capital, and \( T_i \) is still the state of technology in the benchmark model without capital. Note, first, that if \( r_i/w_i = r_j/w_j \ \forall i, j \), then the models with and without capital bear identical results for the states of technology and the TFPs. For the case in which this restrictive assumption is not fulfilled, we verify the robustness of our findings to the inclusion of capital by computing the corresponding TFPs relative to the United States that, in analogy with (9), are given by

\[
\lambda_i^K = \left( \frac{T^K_i}{T^K_{us}} \frac{\Omega_i}{\Omega_{us}} \right)^{\frac{1}{2}}
\]

Maintaining \( \beta = 0.33 \) and \( \theta = 6.67 \), and calibrating the additional parameter \( \alpha = 0.33 \), we compute the \( \lambda_i^K \) using data for 1996 and different sources for \( r_i \).

In particular, we provide four different estimates (the results are shown in Figure 3, which plots \( \lambda_i^K \) against \( \lambda_i \)). As a first special case, we assume that \( r_i = r \ \forall i \), so that in (11) the cost of capital cancels out and the corresponding data are not necessary (north-west panel in the figure). Second, after removing the hypothesis of cross-country equality of the cost of capital, we compute \( \lambda_i^K \) using the estimates of \( r_i \) from Caselli and Feyrer (2007; north-east panel). Third, using the first-order condition from the cost minimization problem that \( r_i/w_i = (1-\alpha)/\alpha k_i \), where \( k_i \) is the capital-labor ratio, we express (11) as a function of \( k_i \), instead of \( r_i/w_i \), and compute the \( \lambda_i^K \) using the estimates of \( k_i \) from Caselli and Feyrer (2007; south-east panel). Finally, we change the estimates of both \( r_i \) and \( w_i \) and take them from Waugh (2010; south-west panel).

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22Waugh (2010) computes equilibrium wages from trade shares by imposing balanced trade (equation 3, page 7. in his paper), instead of using actual data. Wages, in turn, in combination with
Figure 3 shows that the results obtained with capital in the production function compare rather well with our benchmark estimates, with a high correlation between $\lambda_i^K$ and $\lambda_i$ in all four cases. We can thus conclude that our results are robust to the fact that the model by which they are drawn does not provide an explicit role for the stock of physical capital.\textsuperscript{23}

Figure 3: A comparison of TFPs with and without capital (1)

(1) Data refer to 1996 in all pictures. TFP is relative to the United States.

aggregate capital-labor ratios, are used to determine the rental costs of capital. We thank Mike Waugh for sharing his data with us.

\textsuperscript{23}We also estimate the relative TFPs using, alternatively, data on the capital-labor ratios in Kleenow and Rodriguez-Claire (1997) and in Hall and Jones (1999). Consistently with these papers, we use data, respectively, for 1985 and 1988. In both cases, the results with and without capital are comparable, with correlations between $\lambda_i^K$ against $\lambda_i$ equal to 0.65.
5 A case study: Italy vs. the United States

The methodology that we propose to estimate TFPs marks a neat departure from the standard approach. It is therefore interesting to enhance the comparison of our results with those from development accounting. We perform this exercise for the whole 1985-2002 period for Italy versus the United States, which is a particularly interesting case given the aforementioned "Italian anomaly" from development-accounting studies. This case also allows us to refine the measurement of labor inputs by adjusting wages for working hours, which are available for both countries at the sectoral level.\textsuperscript{24} The limited availability of the necessary data to implement the development-accounting methodology prevents us from extending the comparison to all the countries in the sample.\textsuperscript{25}

We start from the standard assumption in development accounting that output in country \( i \) (\( Y_i \)) is given by: \( Y_i = A_i K_i^\eta H_i^{1-\eta} \), where \( A_i \) is the TFP, \( K_i \) the stock of physical capital with share \( \eta \), and \( H_i \) the stock of human-capital augmented labor.

Assuming that each worker in country \( i \) has been trained with \( h_i \) years of schooling, human-capital augmented labor is given by \( H_i = L_i \cdot \exp (g \cdot h_i) \), where \( L_i \) is the total number of worked hours and \( g = 0.06 \) as in the previous section.

\textsuperscript{24}Recall that Hall and Jones (1999) were especially concerned by the high TFP of Italy because of the lower number of hours worked in this country vis-à-vis the US. Therefore, accounting for working hours also allows us to explicitly address their concern.

\textsuperscript{25}The measurement of physical capital is the step in which data limitations are stronger. For instance, from OECD STAN, the main source of comparable cross-country data on production at the sectoral level, the \textit{volume of net capital stock} — a common proxy for physical capital — is available for the whole sample period for the manufacturing sector of only four countries (Denmark, France, Italy, and Spain). The \textit{volume of gross capital stock} — a measure in which capital depreciation is neglected and different capital assets are not weighted — is available only for six additional countries (which do not include major countries such as the United States and Japan). Similar problems arise if one tries to calculate the stock of capital from manufacturing investments. OECD STAN provides the \textit{volume of fixed investment} in the manufacturing sector of 11 countries during our sample period (and, again, not for large countries such as Japan and the United Kingdom). The \textit{value of manufacturing investment} is available for almost all countries (15 out of 19) but, then, one faces the critical issue of finding an appropriate price deflator. Schreyer and Webb (2006) provide a useful survey of definitions and data availability of capital stock measures.
Figure 4: Manufacturing TFP of Italy relative to the US (including worked hours)

Setting $\eta = 1/3$ — which is broadly consistent with the national accounts of developed countries — and using data on output per worker, capital/output ratios, and schooling, one can calculate the level of manufacturing TFP from the production function:

$$A_i = \left( \frac{Y_i}{L_i} \right)^{1-\eta} \left( \frac{K_i}{Y_i} \right)^{-\eta} \left( \frac{H_i}{L_i} \right)^{-(1-\eta)}.$$  \hspace{1cm} (12)

Except for the years of schooling, which are not sector specific, all data refer to the manufacturing sector. In particular, we measure the capital stock with the perpetual inventory method as in Caselli (2005).26

Figure 4 shows the TFP of Italy relative to the United States obtained with this methodology, and compares it with the one that results from the trade-revealed approach. Note that the two series are measured on different axes and scales. The similar time pattern exhibited by the two TFPs, evident at first sight, is quite remarkable given that they are derived from unrelated methodologies and completely different data series (quantity data on production and inputs on the one hand, value data on trade flows, production and wages on the other). According to our development accounting calculations, at the beginning of the sample period Italy’s TFP is 21% higher than that of the United States; afterwards it falls by as much as 27 percentage points. When measured on the basis of our trade-revealed approach, instead, in 1985 Italy’s TFP lies

26Appendix A provides all the details on the methodology, as well as on data sources.
below that of the United States and records a much smaller cumulative loss, falling by 9 percentage points (to 0.89).\footnote{27}

Our TFP measures seem to provide a more reasonable picture of the productivity divide between Italy and the United States. In fact, on the one hand, our trade-revealed TFP is not blurred by the surprising result that in the mid-1980s to the early-1990s Italy’s TFP was higher than that of the United States. On the other hand, this improvement is obtained while preserving a very similar time pattern.

\section{Conclusion}

We have proposed a new methodology to measure the relative TFP of the tradeable goods sector across countries, based on the relationship between trade and TFP in the state-of-the-art model of Eaton and Kortum (2002). With respect to the standard development-accounting approach, our methodology has two main advantages. First, it is based on easy-to-get value data on trade, production, and wages. Second, our TFPs are no longer mere residuals, but are the productivities that best fit those data.

Applying this methodology to estimate the TFPs of the manufacturing sector of 19 OECD countries (with respect to the United States) from 1985 to 2002 provides promising results. Our findings, while broadly in line with those of many previous studies, including the standard development accounting approach, appear more reasonable in some respects. In particular, they fix the "anomaly" produced by the standard method that Italy’s TFP is the highest among a large pool of developed countries in the mid-1980s to the early-1990s. Similarly to other "alternative" methodologies existing in the literature (such as the "revealed superiority" approach of Bar-Shira, Finkelshtain, and Simhon, 2003, and the measures based on the Hecksher-Ohlin-Vaneck theory provided by Trefler, 1995), we obtain that the TFP of the United States ranked first throughout our two-decade sample period. Interestingly, the case study about the TFP of Italy versus the United States shows that our measure yields a difference in levels with respect to development accounting, while preserving a very similar time pattern.

\footnote{27By comparing the results of Figure 4 with those from Table 3, note that accounting for working hours raises the TFP of Italy versus the US by 11 percentage points in 1985, and then delivers a richer dynamics.}
These results are encouraging. The new road we have explored takes the observed implications of TFP on trade data as the starting point. Future research is needed, however, to enhance our methodology along at least two main dimensions. The Ricardian framework of Eaton and Kortum (2002) needs to be generalized into a truly dynamic model, in order to meaningfully include physical capital among the production factors. Second, the model requires a better treatment of the non-tradeable goods sector, in order to extend the methodology with the aim of estimating the TFP of the whole economy.
Appendix

A Data

Manufacturing production and trade data. The source for production, total imports, and total exports of manufacturing goods in local currency is OECD-STAN. Bilateral manufacturing imports from each of the other 18 countries (as a fraction of total manufacturing imports) are from the Statistics Canada’s World Trade Analyzer. The reconciliation between the ISIC and SITC codes follows Eurostat-RAMON (http://europa.eu.int/comm/eurostat/ramon/index.cfm).

Gravity data. Geographic distances and border dummies are from Jon Haveman’s International Trade Data (http://www.macalester.edu/research/economics/page/Haveman/Trade/Resources/TradeData.html). Countries are grouped by language as in Eaton and Kortum (2002): (i) English: Australia, Canada, New Zealand, United Kingdom, United States; (ii) French: Belgium and France; (iii) German: Austria and Germany.

Wages and schooling data. Annual compensation per worker in the manufacturing sector is from OECD-STAN. Values are converted into a common currency using the PPP exchange rates available from the OECD. Wages are then adjusted for education, as explained in Section 3. Years of schooling are obtained from de la Fuente and Doménech (2006). We deal with missing data by interpolation and extrapolation using the most recent update of the dataset first presented in Barro and Lee (2000).

Development-accounting methodology and data. Capital stock data are obtained from real investment using the perpetual inventory method, according to the following relationship:

$$K_t = I_t + (1 - \delta) K_{t-1}$$

where $I_t$ is real investment and $\delta$ the depreciation rate, which we set equal to 0.06 as in Caselli (2005). Real investment in PPP in the manufacturing sector is computed as RGDPL-POP-KI-IM, where RGDPL is real income per capita in PPP, POP is population, KI is the total investment share in total income, and IM is the investment share of the manufacturing sector in total investment. The variables RGDPL, POP, and KI are from the Penn World Tables 6.2; IM is computed from OECD STAN.
Following the standard practice, initial capital stock is computed as \( K_0 = I_0 / (\delta + \kappa) \), where \( I_0 \) is the oldest available value in the investment series (which start in 1970 for both Italy and the Unites States) and \( \kappa \) is the geometric growth rate of investments over the first ten years of data.

Real output in PPP in the manufacturing sector \( (Y_t) \) is computed as \( \text{RGDPL} \times \text{POP} \times \text{YM} \), where \( \text{YM} \) is the manufacturing value added share in total value added, from OECD STAN.

The number of employees in the manufacturing sector \( (L_t) \) comes from OECD STAN. The total amount of working hours per worker in the same sector, used in the case study, are from the Bank of Italy for Italy and the Bureau of Labor Statistics for the United States.

### B Sensitivity analysis

This section provides a brief analysis about the sensitivity of the estimates of the states of technology to alternative calibrations of the main parameters, i.e. \( \theta \), \( \beta \), and \( g \). Recall that states of technology represent an essential intermediate step for the quantification of countries’ relative TFP.\(^{28}\) In our empirical analysis we have chosen as benchmarks \( \theta = 6.67 \), annual values of \( \beta \) set equal to the ratio between manufacturing value added and production, and \( g = 0.06 \). As alternative values for \( \theta \), we set \( \theta = 4 \) and \( \theta = 10 \), which are the lower and upper bounds in the range that Alvarez and Lucas (2007) consider reasonable, and \( \theta = 8.3 \) (Eaton-Kortum’s preferred calibration). The alternative calibration for \( \beta \) is given by the ratio between labor compensation and production (see footnote 10), as in Eaton and Kortum (2002). Finally, for the return on education \( g \) we adopt a non-linear function as in Hall and Jones (1999) and Caselli (2005), setting \( g = 0.13 \) for \( h_i \leq 4 \), \( g = 0.10 \) for \( 4 < h_i \leq 8 \), and \( g = 0.07 \) for \( h_i > 8 \).

Combining the above set of parameter values results in 16 alternative estimates of the states of technology, including our benchmark. Since states of technology vary both across countries and over time, we analyze the sensitivity of the results by computing, in

\(^{28}\)Given the relationship between the two parameters, the sensitivity evidence provided for the relative states of technology can be safely applied to the relative TFPs.
Table 4: Correlation of alternative calibrations with benchmark estimates (1)

<table>
<thead>
<tr>
<th>Choice of $g$</th>
<th>lab comp / production</th>
<th>value added / production</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 4$</td>
<td>0.81 0.95</td>
<td>$\theta = 4$ 0.95 0.98</td>
</tr>
<tr>
<td>$\theta = 6.67$</td>
<td>0.93 0.98</td>
<td>$\theta = 6.67$ 1.00 1.00</td>
</tr>
<tr>
<td>$\theta = 8.3$</td>
<td>0.95 0.99</td>
<td>$\theta = 8.3$ 0.99 1.00</td>
</tr>
<tr>
<td>$\theta = 10$</td>
<td>0.96 0.99</td>
<td>$\theta = 10$ 0.98 0.99</td>
</tr>
</tbody>
</table>

(1) The number on the left (right) of each cell is obtained by computing, for each country (year), the time-series (cross-country) correlation between the $T_i$ resulting from an alternative calibration and the corresponding benchmark estimates and, then averaging across countries (years).

The correlations shown in the table reveal, at a glance, that results are robust to the alternative calibrations. Cross-country correlations (right-hand values in each panel) are in most cases very close to one, and never below 0.9. As far as time-series correlations are concerned, results are also quite comforting. We never get a value below 0.8, except in the case in which we change all the parameters and set $\theta$ equal to 4, and the non-linear specification for returns on education, which nonetheless results in an average time-series correlation...
of about 0.7, still within an acceptable range of values. A deeper analysis of time-series for individual countries reveals that the largest impact on our estimates comes from the influence of the non-linearity assumption on Greece, the only case in which we get a negative correlation. Once this country is excluded, there is a significant improvement, with the lowest correlation now close to 0.8.

C The model with physical capital

Suppose that physical capital in the amount $K_i(j)$ is employed in the production of good $j$ in country $i$, as in equation (10). Denote with $T^K_i$ the state of technology in country $i$. As in Eaton and Kortum (2002), one can use the expression for the cost of a bundle of inputs, given by $c^K_i = (w_i^{\alpha} l_i^{1-\alpha})^{\beta} p_i^{1-\beta}$, together with the equations for market shares and prices to obtain the relationship

$$\log \left[ \left( \frac{X_{ni}}{X_{nm}} \right) \left( \frac{X_{ii}/X_i}{X_{nm}/X_n} \right)^{1-\beta} \right] = S'_i - S'_n - \theta \log (d_{ni}) ,$$

where $S'_i \equiv \frac{1}{\beta} \log (T^K_i) - \theta [\alpha \log (w_i) + (1 - \alpha) \log (r_i)]$. By inverting with respect to $T^K_i$,

$$T^K_i = \left[ \exp (S_i) \cdot (w_i^{\alpha} l_i^{1-\alpha})^{\theta} \right]^{\beta} .$$

The ratio between the states of technology between any two countries ($i$ and $j$) is:

$$\frac{T^K_i}{T^K_j} = \left[ \exp (S_{i,j}) \right]^{\beta} \cdot \left( \frac{w_i}{w_j} \right)^{\alpha \theta} \cdot \left( \frac{r_i}{r_j} \right)^{(1-\alpha) \beta \theta} ,$$

where $\exp (S_{i,j}) = \exp (S_i) / \exp (S_j)$. Recalling that in the model without capital

$$\frac{T_i}{T_j} = \left[ \exp (S_{i,j}) \right]^{\theta} \cdot \left( \frac{w_i}{w_j} \right)^{\beta \theta} ,$$

then one obtains

$$\frac{T^K_i}{T^K_j} = \frac{T_i}{T_j} \cdot \left( \frac{r_i/w_i}{r_j/w_j} \right)^{(1-\alpha) \beta \theta} ,$$

which is equation (11) in the chapter.

In order to derive $T^K_i$ as a function of the capital-labor ratio $k_i = K_i/L_i$, consider the first-order condition that $r_i/w_i = (1 - \alpha) / \alpha k_i$ and, substituting it into the above
equation for $T^K_i/T^K_j$, obtain

$$\frac{T^K_i}{T^K_j} = \frac{T_i}{T_{us}} \cdot \left( \frac{1}{k_i/k_{us}} \right) ^{\beta \theta} ,$$

which is the equation used to estimate the relative TFPs with data on the capital-labor ratios.
CHAPTER 3

Exchange Rates in a General Equilibrium Model of Trade without Money
1 Introduction

A recurrent issue in the macroeconomic debate concerns the domestic and international effects of "excessively competitive currencies," i.e. of currencies that are depreciated with respect to their long-run equilibrium level. This phenomenon can take place when a country intervenes in the exchange rate market in order to maintain a persistently undervalued currency (a critique that in the past has been frequently made to China) or, under fixed exchange rates, when it keeps domestic wages at artificially low values (a policy for which Germany has often been blamed in the eve and aftermath of the euro area crisis of 2011-12).\textsuperscript{1} Despite the flurry of commentary that are repeatedly made about these measures, the economic literature still lacks a systematic analysis of such effects in the context of a general equilibrium model.

In this chapter, we tackle this issue using the Ricardian general-equilibrium model of international trade of Eaton and Kortum (2002), which we further extend to encompass both tradeable and non-tradeable goods. The main challenge that we have to face to examine this question is how to introduce a nominal variable like the exchange rate into a model of real consumption and production decisions, in which there is no money. We do so by building on the insight of Keynes (1931) that the combination of an import tariff and an export subsidy is isomorphic to an exchange rate depreciation.\textsuperscript{2} The advantage of this strategy is that it allows us to replicate changes in exchange rates with changes in the \textit{real parameters} of the model and to obtain, as a result, the response of the endogenous variables.

In particular, we model import tariffs and export subsidies by using a variant of the standard formulation of iceberg costs. This alternative formulation was introduced by Samuelson (1952), in the same study where he laid out the standard specification.\textsuperscript{3}

\textsuperscript{1}See, for example, Bergsten and Gagnon (2012), Krugman (2010 and 2013), Subramanian (2008), and Wolf (2010).

\textsuperscript{2}During the gold standard, Keynes (1931) conjectured the equivalence between an exchange rate devaluation and the combination of an increase in import barriers and a simultaneous decline in export barriers. Hence, he proposed a tariff-cum-subsidy policy as a tool to cut relative wages and raise employment, leaving the "sterling international obligations unchanged in terms of gold."

\textsuperscript{3}The standard specification of iceberg costs was introduced to model \textit{transport costs} (Samuelson, 1952, pp. 268-271). The alternative specification, instead, was formulated to model \textit{trade tariffs}...
The key difference with the latter is that, in the alternative formulation, changes in iceberg costs affect product prices but not, when demand is given, also quantities. Hence, this alternative formulation makes it possible to replicate precisely the effects of a currency depreciation with an increase in import barriers and a simultaneous and symmetric decline in export barriers. In fact, these changes capture the essence of what a nominal depreciation does: it makes exports cheaper and imports more expensive.

Under the Ricardian assumptions of perfect competition, constant returns to scale and fully flexible wages, "depreciation," modeled as above, has no effect on equilibrium quantities and relative product prices. The decline in marginal costs due to the depreciation is completely offset by a proportional rise in relative wages. Following the depreciation, the economy jumps to a new equilibrium with higher nominal wages and product prices, so that a nominal depreciation does not carry over a real depreciation. This result, which is expected in this type of model, is for the first time proved in this chapter into a general equilibrium framework with a multiplicity of countries, tradeable and non-tradeable goods.

We then turn to the analysis of the effects of a depreciation in the case of sticky wages and make two further assumptions. The first is the standard hypothesis of a low (less than 1) elasticity of substitution between tradeable and non-tradeable goods. The second is the assumption that productivity is higher in the tradeable-goods than the non-tradeable-goods sector, which is consistent, for developing countries, with the premise of Lewis (1954) about the productivity differential between the modern and the traditional sector.

If wages are sticky, during the transition from a depreciation to a new long-run equilibrium some real effects obtain. While the persistence of these effects depends on the strength of the frictions that prevent wages from rising, their impact depends on the initial "competitiveness" conditions of the economy. If the depreciation makes the

\[ \text{(Samuelson, 1952, pp. 273-276) and is suitable to replicate the effects of a "price shock," like the one that it is needed to mimic a change in the external value of the currency. Consider, for example, a decline in the barriers to exports, whose direct effect is, for both types of iceberg costs, to reduce export prices in the destination market. For given foreign demand, with standard iceberg costs exporters produce less, because smaller product quantities dissipate in transit. With the alternative specification, instead, exporters keep producing the same quantities, because iceberg costs affect only prices.} \]
currency undervalued, i.e. if relative wages become lower than their long-run value, then workers shift from the non-tradeable-goods to the tradeable-goods sector.\textsuperscript{4} The relative size of latter, however, becomes inefficiently large. Hence, although the domestic economy preserves full employment and real GDP rises, welfare declines. Moreover, undervaluation causes involuntary unemployment abroad. Intuitively, this happens because foreign workers are displaced by the "excessive competitiveness" of the domestic economy and, therefore, employment declines in the tradeable-goods sectors of foreign countries; some, but not all, displaced foreign workers find a job in the non-tradeable-goods sectors, whose relative prices increase, but unemployment is completely absorbed only in the long run, when foreign wages decline relative to domestic wages.

On the other hand, if the depreciation takes place at a time when the currency is overvalued (i.e. relative wages are higher than in the long-run equilibrium), then it facilitates the return of the economy to its competitive equilibrium, with a small inflationary impact and welfare-enhancing effects. The increase in consumer prices is "small" because, following the depreciation, domestic wages do not rise and, importantly, would not rise even if they were perfectly flexible.\textsuperscript{5} If domestic wages (relative to foreign wages) were higher than in the equilibrium, in fact, an appropriate currency depreciation brings them to their competitive equilibrium level. Thus, depreciation can substitute for the adjustment of relative wages, confirming Friedman’s (1953) "daylight saving time" intuition.\textsuperscript{6}

\textsuperscript{4}We assume that nominal wages are sticky in each and every country. Under fixed exchange rates, however, it is clear that if countries with undervalued currencies resist wage inflation, then downward nominal wage rigidities would be enough to determine real effects.

\textsuperscript{5}Some increase in consumer prices takes place only because imported goods become more expensive, but this effect is attenuated by import substitution. This finding is consistent with empirical studies such as Goldfajn and Valdés (1999) and Burstein, Eichenbaum and Rebelo (2005).

\textsuperscript{6}In making the case for flexible exchange rates, Friedman (1953, p. 173) explained that: "The argument for a flexible exchange rate is, strange to say, very nearly identical with the argument for daylight savings time. Isn’t it absurd to change the clock in summer when exactly the same result could be achieved by having each individual change his habits? All that is required is that everyone decide to come to his office an hour earlier, have lunch an hour earlier, etc. But obviously it is much simpler to change the clock that guides all than to have each individual separately change his pattern of reaction to the clock, even though all want to do so. The situation is exactly the same in the exchange market. It is far simpler to allow one price to change, namely, the price of foreign exchange, than to rely upon changes in the multitude of prices that together constitute the internal

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Other studies have focused on the equivalence between a depreciation and a tariff-cum-subsidy policy. In particular, Chipman (2006) demonstrated this equivalence in a small open economy with two tradeable goods, one non-tradeable good and flexible prices. Staiger and Sykes (2009) further analyzed the equivalence in a model with two countries and two tradeable goods, and in the cases of both sticky and flexible wages.

We are able to derive new results in a setting with many countries and many tradeable and non-tradeable goods by exploiting the generality of the Ricardian framework of Eaton and Kortum (2002), which we further extend. This extension is an important by-product of our analysis, because the modeling choices that we introduce (such as the representation of productivities of non-tradeable-goods industries with an appropriate probability distribution) allow us to obtain analytically simple solutions.

This chapter is also related to the recent debate on fiscal devaluations, which are generally proposed in the form of an increase in import tariffs and export subsidies, as in Keynes (1931), or an increase in VAT and a reduction in payroll taxes. In this context, our chapter is close to the spirit of Farhi, Gopinath and Itskhoki (2013), who study the effects of an increase in tariffs and subsidies (as well as that of a value-added tax increase and a payroll tax reduction). Their analysis uses a dynamic New Keynesian open economy model to explore the role of alternative asset market structures and pricing assumptions (producer and local currency pricing), and distinguishes between anticipated and unanticipated "devaluations," whereas we use a static international trade model to investigate different assumptions on wage flexibility and derive broad result in a general-equilibrium context.
The remainder of the chapter is organized as follows. Section 2 introduces the model with perfect competition and constant returns to scale. Section 3 presents the main results with both flexible and sticky wages. Section 4 draws the main conclusions.

2 The model

We consider an economy with the following features: a tradeable-goods and a non-tradeable-goods sector, each of them producing a continuum of goods; industries with heterogeneous efficiencies, described by Fréchet distributions; labor, the only production factor, is perfectly mobile across sectors within each country and immobile across countries; the market structure is perfect competition. We analyze first the closed economy and then the open economy. In the latter, we introduce asymmetric trade barriers, modeled as a variant of the standard formulation of iceberg costs. The resulting framework extends the model of Eaton and Kortum (2002) by adding the non-tradeable-goods sector. The reason for this modification is that the interplay between tradeable-goods and non-tradeable-goods industries plays a key role in our analysis.

2.1 Closed economy

Consumer’s problem is

$$\max_{c_i^N(j), c_i^T(j)} \left\{ \left[ (c_i^N)^{\frac{\sigma-1}{\sigma}} + (c_i^T)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right\}; \text{ subj. to: } \sum_{m=N,T} \int p_i^m(j) c_i^m(j) \, dj \leq w_i L_i \right\}$$

(1)

with: $$c_i^m = \left\{ \int [c_i^m(j)]^{\frac{\sigma-1}{\sigma}} \, dj \right\}^{\frac{\sigma}{\sigma-1}}, \forall m = N, T$$

where the superscripts $N$ and $T$ distinguish non-tradeable from tradeable goods and $i$ denotes the country; $c_i^N \ (c_i^T)$ is the consumption bundle of non-tradeable (tradeable) goods; $c_i^N (j) \ (c_i^T (j))$ is consumption of the non-tradeable (tradeable) good $j$, where goods $j$ are indexed in the interval $[0, +\infty)$; $p_i^N (j) \ (p_i^T (j))$ is the price of the non-tradeable (tradeable) good $j$; $w_i$ is the nominal wage; $L_i$ is the number of workers; and $\sigma, \eta > 0$ are elasticities.

While we could also consider only one single non-tradeable good, by assuming a continuum of non-tradeable goods we preserve some symmetry with the tradeable-goods sector that allows us to simplify the results.
The parameter $\sigma$ is the elasticity of substitution between two tradeable goods and between two non-tradeable goods; $\eta$ governs the elasticity of substitution between tradeable and non-tradeable goods. This framework is consistent with both elastic ($\sigma, \eta \geq 1$) and inelastic demand ($\sigma, \eta < 1$). In the following, however, we assume $\sigma > 1$, while for $\eta$ we explicitly consider both $\eta < 1$ – which is, empirically, the most relevant case – and $\eta \geq 1$.

Goods are produced with constant returns to scale: $q_i^m (j) = z_i^m (j) L_i^m (j)$, $m = N, T$, where $q_i^m (j)$ is the quantity of good $j$ of sector $m$ produced by country $i$, $z_i^m (j)$ is the efficiency (productivity) of industry $j$, and $L_i^m (j)$ is the number of workers employed in that industry. Perfect competition implies $p_i^m (j) = w_i / z_i^m (j)$, for any $i, m, j$.

Industry productivities in the non-tradeable-goods and the tradeable-goods sector are respectively described by $Z_i^N \sim Fréchet (N_i, \theta)$ and $Z_i^T \sim Fréchet (T_i, \theta)$, with $N_i, T_i > 0$ and $\theta > \sigma$, and where all distributions are assumed to be mutually independent. The parameters $N_i$ and $T_i$ are related to the first moments of, respectively, $Z_i^N$ and $Z_i^T$: an increase in $N_i$ ($T_i$) implies an increase in the share of non-tradeable (tradeable) goods that country $i$ produces more efficiently. The parameter $\theta$ is inversely related to the dispersion of $Z_i^N$ and $Z_i^T$.

The key equations of the autarky equilibrium (see Appendix A for details as well

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10 The assumption that the elasticity of substitution between non-tradeable goods is the same as the one between tradeable goods (equal to $\sigma$) can be relaxed, at the cost of a slightly more cumbersome algebra.

11 Using cross-sectional data from the International Comparison Program, Stockman and Tesar (1995) have estimated an elasticity of substitution between tradeable and non-tradeable goods equal to 0.44. Following their study, most open-economy models usually calibrate $\eta$ at around 0.5.

12 If $X \sim Fréchet (\xi, \theta)$, the moment of order $k$ of $X$ (which exists iff $\theta > k$) is $\xi^{k/\theta} \cdot \Gamma ([\theta - k] / \theta)$, where $\Gamma$ denotes Euler’s Gamma function. In an open economy, $T_i$ and $\theta$ are the theoretical counterparts of, respectively, absolute advantage (due to the close link of $T_i$ with the mean of $Z_i^T$) and comparative advantage (because $\theta$ is closely connected with the dispersion of $Z_i^T$ and the gains from trade). See Eaton and Kortum (2002) for some background, and Finicelli, Pagano and Sbracia (2013) for a model with productivity distributions that are correlated across countries.
as for analytical definitions) are:\(^{13}\)

\[
\frac{p_i^T}{p_i^N} = \left( \frac{N_i}{T_i} \right)^{1/\theta} \tag{2}
\]

\[
\frac{L_i^N}{L_i^T} = \left( \frac{N_i}{T_i} \right)^{(\eta-1)/\theta} \tag{3}
\]

\[
\frac{c_i^N}{c_i^T} = \left( \frac{N_i}{T_i} \right)^{\eta/\theta} \tag{4}
\]

\[
\frac{w_i}{p_i} = \gamma_w \left[ N_i^{(\eta-1)/\theta} + T_i^{(\eta-1)/\theta} \right]^{1/(\eta-1)} \tag{5}
\]

where \( \gamma_w \) is a constant.\(^{14}\) These equations show: the price of the bundle of the tradeable goods relative to that of the non-tradeable goods (equation (2)); the size of the non-tradeable-goods sector relative to the tradeable-goods sector (equation (3)), measured in terms of number of workers, with \( L_i^N + L_i^T = L_i \); the demand for the bundle of non-tradeable goods relative to that of the tradeable goods (equation (4)); the real wage (equation (5)), which is related to welfare that, in turn, is \( U_i = w_i L_i / p_i \).

By aggregating the production of non-tradeable and tradeable goods in the equilibrium, we can also obtain an expression for real GDP (see Appendix A), which is given by

\[
Q_i = A_i^N L_i^N + A_i^T L_i^T \tag{6}
\]

with:

\[
A_i^N / A_i^T = \left( \frac{N_i}{T_i} \right)^{1/\theta} \tag{7}
\]

where equation (7) shows the aggregate productivity of the non-tradeable-goods sector relative to that of the tradeable-goods sector.

### 2.2 Open economy

Representative consumers have identical preferences in all countries and solve the problem (1) described above. International trade is constrained by barriers: delivering one

\(^{13}\)Here we are mostly interested in the main macroeconomic aggregates, rather than in the single tradeable and non-tradeable goods, whose equilibrium quantities and relative prices are nevertheless determined. To economize on the notation, we report ratios not only for prices, but also for some quantities, deferring all the details to Appendix A.

\(^{14}\)It is \( \gamma_w = \left[ \Gamma \left( \frac{\theta - \sigma + 1}{\theta} \right) \right]^{1/\theta} \), which is the same as the constant \( \gamma \) in Eaton and Kortum (2002).
unit of a good from country $n$ to country $i$ costs the price of this good multiplied by $d_{in} \geq 1$ for $i \neq n$, while $d_{ii} = 1$ and the triangle inequality holds ($d_{in} \leq d_{ij} \cdot d_{jn}$). With standard iceberg costs, to satisfy country $n$’s demand for good $j$, country $i$ would produce $c_n (j) d_{ni}$ and would be paid the nominal value $w_i c_n (j) d_{ni}/z_i (j)$. With the iceberg costs used here, instead, country $i$ produces $c_n (j)$, is paid $w_i c_n (j)/z_i (j)$, but country $n$ spends $w_i c_n (j) d_{ni}/z_i (j)$. Thus, in the standard specification, it is the quantity $c_n (j) (d_{ni} - 1)$ that dissipates in transit, while here it is the nominal value $(d_{ni} - 1) c_n (j) w_i/z_i (j)$. In both specifications, some consumption goods are paid for, although they are not consumed.\footnote{The two formulations of iceberg costs imply different resource constraints. For example, if good $j$ is produced by country $i$ and it is sold both in the home market and in country $n$, then the resource constraint is $q_i (j) = c_i (j) + c_n (j) d_{ni}$ in the standard case; it is $q_i (j) = c_i (j) + c_n (j)$, in the alternative case considered here.}

**Prices.** As in the standard Ricardian model, production and trade are governed by comparative advantages and each good is bought from the producer who sells it at the lowest price. Hence, the price of the tradeable good $j$ in country $i$ is $p_i^T (j) = \min_n [w_n d_{in}/z_n^T (j)]$. That is: (i) $p_i^T (j) = w_i/z_i^T (j)$ if $j$ is domestically produced; (ii) $p_i^T (j) = w_n d_{in}/z_n^T (j)$, if $j$ is imported from country $n$. The price of the non-tradeable good $j$ is simply $p_i^N (j) = w_i/z_i^N (j)$.

Using the Fréchet assumption, it is easy to obtain the following price indices (analytic details are in Appendix B):

\[
p_i^N = \frac{w_i}{N_i^{1/\theta}} \tag{8}
\]

\[
p_i^T = \frac{w_i}{T_i + \sum_{n \neq i} T_n \left( \frac{w_i}{w_n d_{in}} \right)^{\theta} } \frac{1}{\theta} \tag{9}
\]

where $\gamma_w$, as before, represents a constant which is function of the parameters $\sigma$ and $\theta$. Therefore, the ratio $p_i^T / p_i^N$ is

\[
\frac{p_i^T}{p_i^N} = \left[ \frac{N_i}{T_i + \sum_{n \neq i} T_n \left( \frac{w_i}{w_n d_{in}} \right)^{\theta} } \right]^{1/\theta} \tag{10}
\]
Not surprisingly, the ratio $p^T_i/p^N_i$ is lower in the open economy than in autarky, because the latter has access to potentially cheaper foreign goods. Of course, $p^T_i/p^N_i$ is increasing in $N_i$ and $d_{in}$, and decreasing in $T_i$, $T_n$ and $w_i/w_n$.

**Sector sizes.** The number of workers in the tradeable-goods sector is

$$L^T_i = \frac{\left(\frac{p_i^T}{p_i^N}\right)^{1-\eta}}{\left(\frac{p_i^N}{p_i^T}\right)^{1-\eta} + \left(\frac{p_i^T}{p_i^N}\right)^{1-\eta} L_i}$$

while the number of workers in the non-tradeable-goods sector is $L^N_i = L_i - L^T_i$. Hence, the relative size of the non-tradeable-goods sector is

$$\frac{L^N_i}{L^T_i} = \left(\frac{p_i^T}{p_i^N}\right)^{-\eta-1} \left[ \frac{N_i}{T_i + \sum_{n \neq i} T_n \left(\frac{w_i}{w_n d_{in}}\right)^\theta} \right]^{(\eta-1)/\theta}$$

Recall that $p^T_i/p^N_i$ is lower in the open economy. Thus, equation (12) shows that the relative size of the tradeable-goods sector after opening to trade depends on the exact value of the elasticity $\eta$. If $\eta > 1$, then the share of workers in the tradeable-goods sector rises after trade liberalization, even though some domestic industries shut down. On the contrary, in the empirically-relevant case $\eta < 1$, the share of workers in the tradeable-goods sector declines, following the exit of some domestic industries.

**Demand.** Solving the consumer’s problem, we also obtain:

$$c^N_i = \frac{(p_i)^{\eta-1}}{(p_i^N)^\eta - w_i L_i} \text{ and } c^T_i = \frac{(p_i)^{\eta-1}}{(p_i^T)^\eta - w_i L_i}$$

where $p_i = \left[\left(\frac{p_i^N}{p_i^T}\right)^{1-\eta} + \left(\frac{p_i^T}{p_i^N}\right)^{1-\eta}\right]^{1/(1-\eta)}$.

The main difference with respect to the autarky case is that the price index $p^T_i$ of the open economy includes the prices of both domestically-produced and imported goods. Relative consumption then is:

$$\frac{c^N_i}{c^T_i} = \left[ \frac{N_i}{T_i + \sum_{n \neq i} T_n \left(\frac{w_i}{w_n d_{in}}\right)^\theta} \right]^{\eta/\theta};$$

clearly, thanks to the decline in $p^T_i/p^N_i$, country $i$ consumes a larger share of tradeable goods after opening to trade.
Expenditures on non-tradeable and tradeable goods are, respectively, $p_i^N c_i^N = w_i L_i^N$ and $p_i^T c_i^T = w_i L_i^T$.

Before turning to trade flows, it is worth to sum up the effects of trade liberalization on the tradeable-goods sector. First, the production of some tradeable goods ceases (and these goods are imported). Second, the goods (non-tradeables and tradeables) whose production continues to take place at home and that are sold only domestically face tougher competition (from foreign producers) and meet lower demand. Third, the tradeable goods whose production continues and that are sold both domestically and abroad meet a larger demand (less demand at home, but some additional demand from other countries). Fourth, the relative size of the tradeable-goods sector depends on the elasticity $\eta$: this size decreases (increases) if $\eta < 1$ ($\eta > 1$).

**Trade.** It is easy to compute the value of exports from country $i$ to country $n$, using the fact that the tradeable good $j$ made in country $i$ is exported in $n$ if and only if $w_i d_{ni} / z_i^T (j) = \min_k [w_k d_{nk} / z_k^T (j)]$. We only have to calculate the share of these goods:

$$\pi_{ni} = \frac{X_{ni}}{X_n} = \frac{T_i (w_i d_{ni})^{-\theta}}{\sum_k T_k (w_k d_{nk})^{-\theta}}$$

where $X_{ni}$ is the value of exports from country $i$ to country $n$, and $X_n = c_n^T p_n^T$ is the total expenditure of country $n$ on tradeable goods.

**Average productivity.** We can also compute the productivity distribution of the surviving industries. In autarky, where all tradeable goods are produced at home, this is described by $Z_i^T \sim Fréchet (T_i, \theta)$; in the open economy, instead, it is described by a new random variable, call it $Z_{i,o}^T$, such that $Z_{i,o}^T \sim Fréchet (\Lambda_i, \theta)$, where

$$\Lambda_i = T_i + \sum_{n \neq i} T_n \left( \frac{w_i}{w_n d_{in}} \right)^\theta$$

(see Appendix B). Thus, the average productivity of the tradeable-goods sector of the open economy, $E \left( Z_{i,o}^T \right)$, is larger than that of the closed economy (because $\Lambda_i > T_i$).

---

16Finicelli, Pagano and Sbracia (2013) show that the latter result holds under very general conditions about the distribution of industry productivities.
The "productivity gain" from trade is

\[
\frac{E(Z_{i,o}^T)}{E(Z_{i}^T)} = \left[ 1 + \sum_{n \neq i} \frac{T_{n}}{T_{i}} \left( \frac{w_{i}}{w_{n}d_{ni}} \right)^{\theta} \right]^{1/\theta}.
\]

This gain comes from a selection effect, as domestic industries that are less efficient than foreign industries are forced to exit the market. Of course, the productivity distribution in the non-tradeable-goods sector remains the same, because all non-tradeable goods are made domestically.

**Wages, welfare, and output.** The model is closed by determining relative wages. Income in country \( i \), which is \( w_{i}L_{i} \), must be equal to what the country obtains from selling non-tradeable goods in the domestic market and tradeable goods around the world, including at home; that is: \( w_{i}L_{i} = w_{i}L_{i}^{N} + \sum_{n} X_{ni}/d_{ni} \), which implies that trade is balanced.\(^{17}\) Hence, \( w_{i}L_{i}^{T} = \sum_{n} \pi_{ni}X_{n}/d_{ni} \) from which we obtain:\(^{18}\)

\[
w_{i}L_{i}^{T} = \sum_{n \neq i} \pi_{ni} \frac{w_{n}L_{n}^{T}}{d_{ni}}.
\]

Using equations (8)-(9), we can also compute the real wage as a measure of welfare:

\[
\frac{w_{i}}{p_{i}} = \gamma_{w} \left\{ N_{i}^{(\eta-1)/\theta} + \left[ T_{i} + \sum_{n \neq i} T_{n} \left( \frac{w_{i}}{w_{n}d_{ni}} \right)^{\theta} \right]^{(\eta-1)/\theta} \right\}^{1/(\eta-1)},
\]

which is always higher than in autarky, irrespectively of the exact value of relative wages or of the elasticity \( \eta \).

Real GDP is given by \( Q_{i} = A_{i,o}^{N}L_{i}^{N} + A_{i,o}^{T}L_{i}^{T} \), where \( A_{i,o}^{N} = A_{i}^{N} \) and \( A_{i,o}^{T} > A_{i}^{T} \) (see Appendix B for details). In other words, aggregate productivity of the non-tradeable-good sector of the open economy is same as that of the closed economy, while aggregate productivity of tradeable-good industries rises in the open economy.

\(^{17}\)Although the model assumes trade balance and ignores tariff revenues that trade barriers might generate, it is possible to extend it to incorporate imbalances (see Dekle, Eaton, and Kortum, 2007) and to take revenue effects into account (see Eaton and Kortum, 2002, and Staiger and Sykes, 2009).

\(^{18}\)Recall that \( X_{ni} \) is calculated on a c.i.f. basis (see Eaton and Kortum, 2002, page 1749) and includes the iceberg cost \( d_{ni} \) that, in our formulation, is dissipated in transit and does not contribute to the income of country \( i \).
Equilibrium. To sum up, the full general equilibrium is given by the solutions of equations (8) (9), (11), (14), and (17), which form a system of $M^2 + 4M$ non-linear equations, where $M$ is the number of countries. The $M^2 + 4M$ unknowns are: $p_i^N$, $p_i^T$, $L_i^T$, $w_i$ and $\pi_{ni}$. The parameters of the model — which are $\sigma$, $\eta$, $\theta$, $T_i$, $N_i$, $L_i$, $d_{in}$ and $d_{ni}$ — can all be estimated or calibrated. Because of non-linearities, there is no closed-form solution, but it is possible to simulate the model and analyze some counterfactuals.\textsuperscript{19} In addition, for parameter changes such as those concerning trade barriers, the model is simple enough to make it possible to derive analytic results, as we show below.

3 Changes in exchange rates

To mimic the effects of a depreciation of country $i$’s currency in a model that has no money, we consider an increase in the barriers to its imports from all the other countries and a simultaneous and symmetric decrease in the barriers to its exports to the other countries. The following proposition holds (see Appendix C):

**Proposition 1** Under constant returns to scale, perfect competition and flexible wages, consider an increase of country $i$’s import barriers from $d_{in}$ to $d'_{in} = \delta d_{in}$ and a decline of its export barriers from $d_{ni}$ to $d'_{ni} = d_{ni}/\delta$, with $\delta \geq 1$, for any $n \neq i$. Then, relative wages rise from $w_i/w_n$ to $w'_i/w'_n = \delta w_i/w_n$. The economy under $(d'_{in}, d'_{ni})$ features the same equilibrium quantities and relative product prices as the one under $(d_{in}, d_{ni})$.

The proposition shows that the depreciation, represented by the change in trade barriers, has no effect on equilibrium quantities and relative product prices: all that happens is just an increase in domestic wages relative to foreign wages that completely offsets the depreciation.

To provide some intuition, let us focus on the model with two countries only ($i$ and $n$). The change in trade barriers illustrated above can be interpreted as a depreciation in country $i$’s nominal exchange rate from $e_{in} = 1$ to $e_{in} = \delta \geq 1$, where the exchange rate is expressed in terms of units of country $i$’s currency for one unit of $\ldots$

\textsuperscript{19}Results by Alvarez and Lucas (2007), however, grant that a solution of the model exists and is unique.
foreign currency. For given wages $w_i$ and $w_n$, these changes in trade barriers replicate exactly what happens right after a depreciation: if good $j$ is imported, its price increases from $w_n d_n z_n (j)$ to $w_n d_n \delta/z_n (j)$; if good $j$ is exported, its price in country $n$ decreases from $w_i d_n z_i (j)$ to $w_i d_n /\delta z_i (j)$; if good $j$ is domestically produced and sold only at home, its price does not change (and remains equal to $w_i / z_i (j)$).

Let us now focus on wages. On impact (that is before wages change), the increase in import barriers makes purchases from country $n$ more expensive, favoring import substitution and boosting demand for domestic goods (both tradeables and non-tradeables). By the same token, the decline in export barriers makes country $i$’s goods cheaper abroad, raising foreign demand for domestic tradeable goods. Hence, after the depreciation all domestic industries (producing either tradeables or non-tradeables) are willing to hire more workers, thanks to the rise in both domestic and foreign demand.

With flexible wages and full employment, however, the rise in demand puts pressure on domestic wages. For the sake of simplicity, let us normalize wages in country $n$, setting $w_n = 1$. Under full employment, relative wages in country $i$ increase from $w_i$ to $\delta w_i$. This rise in (relative) wages offsets the effects of the depreciation and restores equilibrium quantities and relative product prices to their pre-depreciation levels. In other words, the result of the depreciation is just a change in all the nominal variables (nominal wages and prices), so that all the real variables (quantities and relative product prices) return to the previous equilibrium levels.

We now turn to the assumption that relative wages are no longer flexible. In particular, let us suppose that they are sticky and set to levels $w_i^* / w_n^*$ that, for country $i$, are higher than the equilibrium relative wages $w_i / w_n$ and, therefore, too high to deliver full employment in this country. In other words, while so far we have considered $L_i$ given and $w_i$ endogenous (so that the economy achieves its full employment level $L_i$), now we consider the polar case in which $w_i$ is given and $L_i$ is endogenous.$^{20}$ We

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$^{20}$Models of international trade typically assume that trade is balanced and that the number of workers is exogenous, while wages are endogenous. Dekle, Eaton and Kortum (2007) perform an exercise in which, instead, wages and the number of workers are exogenous, while the resulting trade imbalance is endogenous. Here we consider a complementary exercise in which, instead, trade is balanced and wages are exogenous, while the number of employed is endogenous. A similar exercise, but in a different context with Bertrand competition and constant returns to scale, is carried on by Bernard, Eaton, Jensen, and Kortum (2003), who examine the effects on total employment of an
can prove the following result (see Appendix D):

**Proposition 2** Consider an economy (economy A) in which country i has trade barriers \((d_{in}, d_{ni})\) and where relative wages are sticky and set to \(w_i^*/w_n^*\), such that \(w_i^*/w_n^* = \delta w_i/w_n\) for any \(n \neq i\), where \(w_i/w_n\) are the competitive-equilibrium relative wages and \(\delta \geq 1\). Employment in country i is \(L_i^* \leq L_i\) (\(L_i^* < L_i\) if \(\delta < 1\)). Now consider a rise of country i’s import barriers \(d_{in}\) to \(d_{in}^* = \delta d_{in}\) and a simultaneous decline of its export barriers \(d_{ni}\) to \(d_{ni}^* = d_{ni}/\delta\). The economy under \((d_{in}^*, d_{ni}^*)\) and \(w_i^*/w_n^*\) (economy B) features the same equilibrium quantities and relative product prices as the one under \((d_{in}, d_{ni})\) and \(w_i/w_n\) (economy C). In particular, in economy B country i has full employment and higher output than in economy A.

Proposition 2 shows that if domestic wages are too high (in the proposition this happens in country i in economy A), the country experiences involuntary unemployment \((L_i^* \leq L_i)\). Then, an appropriate depreciation of the exchange rate, which is here modeled with a change in trade barriers, can replace for the adjustment of relative wages (equilibrium in economy B is the same as in economy C). This result vindicates the assertions of both Keynes (1931) and Friedman (1953) (see footnotes 2 and 4).

In particular, the currency depreciation raises domestic and foreign demand for domestic tradeable goods. Thus, the number of workers in the tradeable-goods sector increases (because both \(\pi_{ni}\) and \(\pi_{ii}\) increase; equation (17)). On the other hand, the increase in import barriers raises the price of the bundle of tradeable goods \(p_i^{T}\) (because the newly domestically-produced tradeable goods and those that are still imported are more expensive) and, therefore, demand for non-tradeable goods increases (equation (13)). The consequent rise in employment boosts output and welfare. If the depreciation of country i’s currency is of the "appropriate" extent (which is \((\delta - 1)\%\)), full employment and competitive equilibrium quantities and relative product prices are restored despite wage stickiness. Therefore, should wages become fully flexible right after the depreciation, they would nonetheless remain flat because equilibrium conditions have been restored by through the exchange rate.

The relative size of the tradeable sector resulting from the depreciation depends on the exact value of the elasticity of substitution between tradeable and non-tradeable exogenous change in relative wages.
goods. If $\eta > 1$, then the absolute size of the tradeable-goods sector rises (equation (17)), but the relative size of this sector (with respect to the non-tradeable-goods sector) declines (equation (12)); that is, workers move from unemployment to employment in both sectors, and flow to the non-tradeable sector more than proportionally. In the empirically relevant case $\eta < 1$, instead, both the absolute and the relative size of the tradeable-goods sector rise.

Proposition 2 considers a country with "uncompetitive" relative wages and shows that an appropriate depreciation can restore the competitive equilibrium. One might also wonder what happens in a country that depreciates its currency starting from a competitive equilibrium, before wages adjust (Proposition 1 shows that no real variable changes after the full adjustment). The following proposition answers this question, assuming that relative wages are sticky while workers adjusts by moving across industries and sectors (see Appendix E):

**Proposition 3** Consider an economy with trade barriers $(d_{in}, d_{ni})$ and where relative wages are equal to their competitive equilibrium levels $w_i/w_n$. Consider also an increase of $d_{in}$ to $d'_{in} = \delta d_{in}$ and a decline of $d_{ni}$ to $d'_{ni} = d_{ni}/\delta$, with $\delta \geq 1$ for any $n \neq i$. If relative wages are sticky, then in country $i$ we observe that: (i) $L^T_i/L^N_i$ rises; (ii) average productivity in the tradeable sector declines, but real output $Q_i$ rises (provided that $T_i$ is sufficiently larger than $N_i$); (iii) the real wage declines.

Proposition 3 shows the transitory effects of the attempt of a country to bring down relative wages through a currency depreciation, while its economy features full employment. The depreciation raises the size of the tradeable-goods sector as domestic goods become cheaper abroad and imported goods become more expensive. The rise in $L^T_i/L^N_i$, in turn, increases output: each workers that goes from the non-tradeable-goods to the tradeable-goods sector contributes to raise total output, due to the higher productivity of the latter sector. The increase in output, however, occurs at the cost of reducing welfare, as wages become inefficiently low in this country. In particular, to provide intuition in a two-country example, let us normalize wages in country $n$, setting $w_n = 1$. Before the depreciation, relative wages were equal to their competitive level $w_i$. After the depreciation, relative wages should go up to $\delta w_i$ in order to restore equilibrium quantities and relative prices (Proposition 1). If they are sticky and remain
set at $w_i$, the real wage diminishes, because the price level in country $i$ increases due to the fact that imports have become expensive.

Taken together, Propositions 2 and 3 provide a nice illustration of the inefficiencies of hampering the normal functioning of the competitive economy and of the law of comparative advantage.\footnote{Imposing the Fréchet assumption returns simple and elegant analytical expressions for equilibrium quantities and relative prices, but it is by no means necessary for the main results of this chapter. Appendices A and B clarify that, for any distribution of industry productivities, equilibrium quantities and relative prices depend on the moment of order $\sigma$ and $\sigma - 1$ of such distribution. These moments, in turn, feature the variables $w_i/w_n d_i$ and $w_i d_{ni}/w_n$ in their argument — variables that do not change when $d_i$ and $d_{ni}$ change into $d'_i$ and $d'_{ni}$ (defined above) and $w_i/w_n$ changes into $\delta w_i/w_n$ (see also Bolatto and Sbracia, 2015, for a full-fledged version of the model with general distribution of productivities).}

The countries where relative wages are too high, experience unemployment and welfare losses; but also the countries where relative wages are too low, despite preserving full employment, incur in welfare losses (even though output rises), as the real wage is inefficiently low.\footnote{To fully exploit our multi-country setting, Appendices C and D prove some more general statements for Propositions 1 and 2 (see Propositions 4 and 5). In Proposition 4, it is assumed that trade barriers change from $d_i$ to $d'_i = \delta_i d_i/\delta_n$ and from $d_{ni}$ to $d'_{ni} = \delta_n d_{ni}/\delta_i$. Therefore, while in Proposition 1 it was just country $i$ that depreciated its currency (while all the others appreciated their exchange rate vis-à-vis country $i$’s currency), in Proposition 4 more than one country can depreciate its currency (while the others appreciate) with, again, no effect on equilibrium quantities and relative prices. Similarly, in Proposition 5 we consider relative wages set to levels $w'_i/w'_n$ such that $w'_i/w'_n = \delta_i w_i/\delta_n w_n$. Hence, while in Proposition 2 there was just one country with "high" domestic wages, in Proposition 5 this is possible in many countries and, again, there exists an appropriate change in exchange rates (i.e. in trade barriers) that restores equilibrium quantities and relative prices. Finally, Proposition 3 could be generalized as well, but deriving precise implications for the number of workers when one country’s currency is undervalued with respect to some currencies and overvalued with respect to others is much more cumbersome.}

4 Conclusion

Our study about the effects of nominal exchange rates in open economies has been grounded on a very general model of international trade. In particular, the finding that under perfect competition, constant returns to scale and flexible wages, exchange...
rates do not yield any effect on the equilibrium is obtained with a multiplicity of countries, tradeable goods and non-tradeable goods. By the same token, the result that if sticky wages are too high in a country, then there is an appropriate depreciation that restores the competitive equilibrium, demonstrates, in a very general analytical framework, the intuitions of Keynes (1931) and Friedman (1953).

In this chapter, we have not exploited one of the most important feature of our model, that is the possibility of quantifying it in order to run counterfactual simulations. This exercise — which we leave for future research — would allow us to provide model-based measures of the domestic and international effects of an misaligned currencies on employment, output and welfare.

By mimicking changes in the value of the currency with changes in trade barriers, we have been able to analyze these questions in a real framework, where there is no money. A limit of this analysis is that it neglects real effects that may come from changes in interest rates and, more in general, from the financial sector. Nonetheless, our approach yields very simple results and can be applied in many standard models of international trade and economic growth. Therefore, our modeling strategy may turn out to be useful also in other studies about the role of exchange rates in open economies.
Appendix

A The closed economy model

To simplify the computations, we proceed in three stages.

Stage 1. Let us rewrite the consumer’s problem as:

\[
\max_{c_T, c_N} \left[ \left( \frac{c_T}{p_T} \right)^{1-n} + \left( \frac{c_N}{p_N} \right)^{1-n} \right]^{\frac{n}{n-1}} \text{ subj. to: } p_i^N c_N^i + p_i^T c_T^i \leq w_i L_i
\]

where: \( p_i^m c_i^m = \int p_i^m (j) c_i^m (j) dj \), that is: \( p_i^m = (c_i^m)^{-1} \int p_i^m (j) c_i^m (j) dj \), \( \forall m = T, N \).

From the first order conditions and the budget constraint, we get the demand for the non-tradeable and the tradeable bundles of goods:

\[
c_i^N = \frac{\left( p_i \right)^{\eta-1}}{(p_i^N)^{\eta} - \left( p_i^T \right)^{\eta}} w_i L_i \text{ and } c_i^T = \frac{\left( p_i \right)^{\eta-1}}{(p_i^T)^{\eta} - \left( p_i^N \right)^{\eta}} w_i L_i
\]

where \( p_i = \left[ \left( p_i^N \right)^{1-\eta} + \left( p_i^T \right)^{1-\eta} \right]^{1/(1-\eta)} \).

Of course, expenditures for non-tradeable and tradeable goods are:

\[
p_i^N c_i^N = \frac{\left( p_i^N \right)^{1-\eta}}{(p_i^T)^{1-\eta} - \left( p_i^N \right)^{1-\eta}} w_i L_i
\]

\[
p_i^T c_i^T = \frac{\left( p_i^T \right)^{1-\eta}}{(p_i^T)^{1-\eta} - \left( p_i^N \right)^{1-\eta}} w_i L_i
\]

Stage 2. Now consider the two problems:

\[
\max_{c_i^N (j)} \left\{ \int \left[ c_i^N (j) \right]^{\frac{\sigma-1}{\sigma}} dj \right\}^{\frac{\sigma}{\sigma-1}} \text{ subj. to: } \int p_i^N (j) c_i^N (j) dj \leq p_i^N c_i^N
\]

\[
\max_{c_i^T (j)} \left\{ \int \left[ c_i^T (j) \right]^{\frac{\sigma-1}{\sigma}} dj \right\}^{\frac{\sigma}{\sigma-1}} \text{ subj. to: } \int p_i^T (j) c_i^T (j) dj \leq p_i^T c_i^T
\]

From the first order conditions and the budget constraint we obtain:

\[
c_i^m (j) = \frac{(p_i^m)^{\sigma-1}}{(p_i^m)^{\sigma} - \left( p_i^N \right)^{\sigma}} \left( p_i^T \right)^{1-\eta} w_i L_i \text{ , } \forall m = T, N ,
\]

where: \( p_i^m = \int [p_i^m (j)]^{1-\sigma} dj \) \( \frac{1}{1-\sigma} \), \( \forall m = T, N \).\(^{23}\)

\(^{23}\)It is easy to check that: \( \int [p_i^m (j)]^{1-\sigma} dj \) \( \frac{1}{1-\sigma} \) = \( (c_i^m)^{-1} \int p_i^m (j) c_i^m (j) dj \), \( \forall m = T, N \). In other words, the price index defined in the first step is the same as the one defined in the second step.
Stage 3. Given \( p_i^N (j) = w_i / z_i^N (j) \), we now solve for the resource constraint, \( c_i^N (j) = q_i^N (j) \), and obtain:

\[
L_i^N (j) = \frac{[z_i^N (j)]^{\sigma-1}}{\int [z_i^N (j)]^{\sigma-1} \, dj} L_i^N \quad \text{and} \quad L_i^T (j) = \frac{[z_i^T (j)]^{\sigma-1}}{\int [z_i^T (j)]^{\sigma-1} \, dj} L_i^T
\]

where \( L_i^N = \frac{1 - \eta}{(p_i^N)^{1-\eta}} L_i \) and \( L_i^T = \frac{1}{(p_i^T)^{1-\eta} + (p_i^N)^{1-\eta}} L_i \).

By aggregating across industries the quantities \( q^m_i (j) \), we can find:

\[
Q_i^m = \int q^m_i (j) \, dj = \int z^m_i (j) L^m_i (j) \, dj = \frac{\int [z_i^N (j)]^\sigma \, dj}{\int [z_i^N (j)]^{\sigma-1} \, dj} L_i^m, \quad \forall m = T, N.
\]

where we have used the fact that \( L_i^m (j) = [z_i^m (j)]^{\sigma-1} L_i^T / \int [Z_i^T (j)]^{\sigma-1} \, dj \).

Thus:

\[
Q_i^N = A_i^N L_i^N \quad \text{and} \quad Q_i^T = A_i^T L_i^T
\]

\[
A_i^m = \frac{\int [z_i^m (j)]^\sigma \, dj}{\int [z_i^m (j)]^{\sigma-1} \, dj}, \quad \forall m = T, N.
\]

Note also that aggregate production and real GDP are:

\[
Q_i = Q_i^N + Q_i^T = A_i^N L_i^N + A_i^T L_i^T
\]

\[
\frac{Q_i}{L_i} = \frac{A_i^N}{L_i^N + L_i^N} + \frac{A_i^T}{L_i^T + L_i^T}.
\]

Similarly, we can compute the real wage as:

\[
\frac{w_i}{p_i} = \left[ \left( \int [z_i^N (j)]^{\sigma-1} \, dj \right)^{1-\eta} \cdot \int [z_i^T (j)]^{\sigma-1} \, dj \right]^{1-\eta} \cdot \left( \int [z_i^T (j)]^{\sigma-1} \, dj \right)^{1/(\eta-1)}.
\]

Key equations. Summing up, the main equations of the autarky equilibrium

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24 Finicelli, Pagano and Sbracia (2015) consider different ways for aggregating production across industries. All the different aggregations provide expressions of the type: \( Q_i^m = A_i^m L_i^m \), where \( A_i^m \) is always proportional, at least to a first approximation, to the first moment of the distribution of \( Z_i^m \).
are:

\[
\frac{p_i^T}{p_i^N} = \left[ \frac{E \left[ (Z_i^N)^{\sigma-1} \right]}{E \left[ (Z_i^T)^{\sigma-1} \right]} \right]^{\frac{1}{\sigma-1}} \tag{19}
\]

\[
\frac{L_i^N}{L_i^T} = \left( \frac{p_i^T}{p_i^N} \right)^{\eta-1} \tag{20}
\]

\[
\frac{c_i^N}{c_i^T} = \left( \frac{p_i^T}{p_i^N} \right)^{\eta} \tag{21}
\]

\[
\frac{w_i}{p_i} = \left[ \left( E \left[ (Z_i^N)^{\sigma-1} \right] \right)^{\frac{1}{\sigma-1}} \frac{1}{1-\eta} + \left( E \left[ (Z_i^T)^{\sigma-1} \right] \right)^{\frac{1}{\sigma-1}} \frac{1}{1-\eta} \right]^{1/(\eta-1)} \tag{22}
\]

\[
Q_i = A_i^N L_i^N + A_i^T L_i^T \tag{23}
\]

\[
\frac{A_i^N}{A_i^T} = \frac{E \left[ (Z_i^N)^{\sigma} \right]}{E \left[ (Z_i^T)^{\sigma} \right]} \left( \frac{E \left[ (Z_i^T)^{\sigma} \right]}{E \left[ (Z_i^T)^{\sigma-1} \right]} \right)^{-1} \tag{24}
\]

By exploiting the assumption that efficiencies are Fréchet distributed (see footnote 12 for the moments of this distribution), equations (19)-(24) immediately turn into the equations (2)-(5) shown in Section 2.1.

**B The open economy model**

In this section we use three simple properties of the Fréchet distribution. Specifically, if \( X \sim \text{Fréchet}(\xi, \theta) \) and \( Y \sim \text{Fréchet}(\lambda, \theta) \), with \( X \perp Y \), then: (P1) \( aX \sim \text{Fréchet}(a\xi, \theta) \), for \( a > 0 \); (P2) \( \max(X, Y) \sim X|X \geq Y \sim \text{Fréchet}(\xi + \lambda, \theta) \); (P3) \( \Pr(X \geq Y) = \xi / (\xi + \lambda) \).

We start by computing prices. In the open economy consumers buy goods from the producers who sell them at the lower price, that is: \( p_i^T(j) = \min_n \left[ w_n d_{in} / z_i^T(j) \right] \). Let \( P_i^T \) be the random variable that describes the prices of the tradeable goods sold in country \( i \). Then:

\[
P_i^T = \min_n \left( \frac{w_n d_{in}}{Z_i^T} \right) = w_i \cdot M_i^{-1}, \text{ where: } M_i = \max_n \left( \frac{w_i}{w_n d_{in}} \frac{Z_i^T}{Z_n^T} \right).
\]

By the properties (P2) and (P3), it is

\[
M_i \sim \text{Fréchet}(A_i, \theta)
\]

with:

\[
A_i = T_i + \sum_{n \neq i} T_n \left( \frac{w_i}{w_n d_{in}} \right)^{\theta}
\]
Thus, \( p^T_i = w_i \cdot \{ E[(M_i)^{\sigma-1}] \}^{1/(\sigma-1)} = \gamma w_i \Lambda_i^{-1/\theta} \), while \( p^N_i = \gamma w_i N_i^{1/\theta} \) (just like in the closed economy). Therefore, equations (8), (9) and (10) immediately follow.

The labor sizes, \( L^T_i \) and \( L^N_i \), and the consumption bundles, \( c^T_i \) and \( c^N_i \), can be derived using the expressions of \( p^T_i \) and \( p^N_i \), so that equations (12) and (13) obtain.

The market share \( \pi_{ni} \) is simply equal to \( \Pr (Z_i/w_i d_{ni} \geq M_n/w_n) \). The properties \((P1)\) and \((P3)\) of the Fréchet distribution imply that \( \pi_{ni} = T_i (w_i d_{ni})^{-\theta} / \Lambda_n (w_n)^{-\theta} \), which is equation (14).

The productivity distribution for the industries that survive international competition is \( Z^T_{i,o} = Z_i Z_i \max \left( \frac{w_i}{w_n d_{ni}} Z^T_n \right) \). From \((P1)\) and \((P2)\) it follows that \( Z^T_{i,o} \sim M_i \), proving equations (15) and (16).

The definition of \( p_i \) and the expressions of its components \( p^N_i \) and \( p^T_i \) imply that the real wage is

\[
\frac{w_i}{p_i} = \left( \left( \{ E[(Z_i^N)^{\sigma-1}] \}^{\frac{1}{\sigma-1}} \right) + \left( \{ E[(M_i)^{\sigma-1}] \}^{\frac{1}{\sigma-1}} \right) \right)^{1/(1-\eta)}
\]

from which equation (18) obtains.

Computing real GDP is somewhat more cumbersome. For the sake of exposition, we do it for the two country model and we then generalize it to \( M \) countries. Let us consider the resource constraint. For tradeable goods, the condition that demand is equal to supply translates into \( q^T_i (j) = 0 \), if \( j \) is imported by country \( i \); \( q^T_i (j) = c_i (j) \), if \( j \) is sold only domestically; \( q^T_i (j) = c_i (j) + c_n (j) \), if \( j \) is exported to country \( n \).

Solving such constraint for the number of workers, we obtain: \( L^T_i (j) = 0 \), if \( j \) is imported, while for the industries that are active it is:

\[
L^T_i (j) = \frac{\left[ \frac{z^T_i (j)}{E \left[ \left( Z^T_{i,o} \right)^{\sigma-1} \right]} \right]^{\sigma-1} L^T_i}{L^T_i (1 + k_{ni})}
\]

where

\[
k_{ni} = \left( \frac{p^T_{i,d_{ni}}}{p^T_n} \right)^{-\sigma} \frac{w_n L^T_n / p^T_n}{w_i L^T_i / p^T_i}
\]

the dyadic term \( k_{ni} \) measures the strength of the demand that comes from country \( n \): it depends on the relative size of this country’s tradeable sector in terms
of real expenditure in tradeables, the iceberg cost between \( i \) and \( n \), and the relative price levels of tradeables. Aggregate production then is \( Q_i = A_{i,o}^N L_i^N + A_{i,o}^T L_i^T \), where aggregate productivity for non-tradeable goods is still given by \( A_{i,o}^N = \frac{E \left[ (Z_i^N)^\sigma \right]}{E \left[ (Z_i^N)^{\sigma-1} \right]} \), while the expression of \( A_{i,o}^T \) is:

\[
A_{i,o} = \frac{\lambda_{i,o} E \left[ (Z_{i,o}^T)^\sigma \right] + \lambda_{i,c,n} k_{mi} E \left[ (Z_{i,c,n}^T)^\sigma \right]}{\lambda_{i,o} E \left( Z_{i,o}^{\sigma-1} \right) + \lambda_{i,c,n} k_{mi} E \left( Z_{i,o}^{\sigma-1} \right)}
\]

where \( \lambda_{i,o} (\lambda_{i,c,n}) \) is the probability that an industry of country \( i \) survives international competition (exports into country \( n \)) and \( Z_{i,c,n}^T \) is the random variable that describes the productivities of the industry that export to country \( n \). Not surprisingly, one can find that \( Z_{i,c,n}^T \sim Fréchet \left( \Lambda_{i,n}, \theta \right) \) where

\[
\Lambda_{i,n} = T_i + T_n \left( \frac{w_i d_{ni}}{w_n} \right)^\theta.
\]

Let \( c_{i,n} = \Lambda_{i,n}/\Lambda_i \) (\( c_{i,n} > 1 \)); by the properties of the Fréchet distribution, we find:

\[
A_{i,o} = \gamma_q \frac{\lambda_{i,o} \Lambda_i^{\sigma/\theta} + \lambda_{i,c,n} k_{mi} \Lambda_{i,n}^{\sigma/\theta}}{\lambda_{i,o} \Lambda_i^{(\sigma-1)/\theta} + \lambda_{i,c,n} k_{mi} \Lambda_{i,n}^{(\sigma-1)/\theta}}
\]

\[
= \gamma_q \Lambda_i^{1/\theta}
\]

Then, \( A_{T,o}^T/A_{i}^T = (\Lambda_i/T_i)^{1/\theta} \geq 1 \).

The generalization to \( M \) countries is straightforward. With similar passages one can prove that \( Q_i = A_{i,o}^N L_i^N + A_{i,o}^T L_i^T \), with

\[
A_{i,o} = \frac{\lambda_{i,o} E \left[ (Z_{i,o}^T)^\sigma \right] + f_{0,i}}{\lambda_{i,o} E \left( Z_{i,o}^{\sigma-1} \right) + f_{1,i}}
\]

where

\[
f_{0,i} = \sum_{n \neq i} \lambda_{i,c,n} \cdot k_{mi} \cdot E \left[ (Z_{i,c,n}^T)^\sigma \right] + \sum_{n \neq i, h \neq i, n \neq h} \lambda_{i,c,n,h} \cdot (k_{mi} + k_{hi}) \cdot E \left[ (Z_{i,c,n,h}^T)^\sigma \right] + \cdots + \lambda_{i,c;1,\ldots,N} \cdot (k_{1i} + \cdots + k_{Ni}) \cdot E \left( Z_{i,c;1,\ldots,N}^\sigma \right),
\]

(25)

and

\[
f_{1,i} = \sum_{n \neq i} \lambda_{i,c,n} \cdot k_{mi} \cdot E \left[ (Z_{i,c,n}^T)^{\sigma-1} \right] + \sum_{n \neq i, h \neq i, n \neq h} \lambda_{i,c,n,h} \cdot (k_{mi} + k_{hi}) \cdot E \left[ (Z_{i,c,n,h}^T)^{\sigma-1} \right] + \cdots + \lambda_{i,c;1,\ldots,N} \cdot (k_{1i} + \cdots + k_{Ni}) \cdot E \left( Z_{i,c;1,\ldots,N}^{\sigma-1} \right),
\]

(26)
where \( \lambda_{i;c;n,h,...,k} \) is the probability that an industry of country \( i \) exports in (and only) countries \( n, h, ..., \) and \( k \), while \( Z_{i;c;n,h,...,k}^T \) is the distribution of the productivities of these industries. By the properties of the Fréchet distribution and, just like in the

2-country case, factoring in the common component \( \Lambda_0^{\sigma/\theta} \) in \( f_{0,i} \) and \( \Lambda_1^{(\sigma-1)/\theta} \) in \( f_{1,i} \), one still obtains \( A_{i,o} = \gamma_q \Lambda_1^{1/\theta} \).

### C Proof for Proposition 1

We prove the following proposition, which generalizes Proposition 1:

**Proposition 4** Under constant returns to scale, perfect competition and flexible wages, consider a change in trade barriers from \( d_{in} \) to \( d_{in}' = \delta_id_{in}/\delta_n \), with \( \delta_i, \delta_n \geq 1 \), \( \forall i, n \). Then, nominal wages change from \( w_i \) to \( w_i' = \delta_iw_i \) \( \forall i \). The economy under \((d_{in}',w_i')\) features the same equilibrium quantities and relative product prices as the one under \((d_{in},w_i)\).

**Proof.** We need to check that if \( p_i^N, p_i^T, L_i^T, \pi_{ni} \) and \( w_i \) form a solution for the system of equations (8), (9), (11), (14) and (17) for given trade barriers \( d_{in} \), then \( \delta_i p_i^N, \delta_i p_i^T, L_i^T, \pi_{ni} \) and \( w_i' \) are a solution for the same equations when the barriers \( d_{in} \) are replaced with \( d_{in}' \) (while the other parameters are given).

By replacing \( d_{in}' \) and \( w_i' \) into (8) and (9), the resulting prices of the bundles of non-tradeable and tradeable goods become indeed equal to, respectively, \( \delta_i p_i^N \) and \( \delta_i p_i^T \) for any country \( i \). The relative price of the two bundles remains set at \( p_i^N/p_i^T \) in each country \( i \) and, therefore, \( L_i^T \) still solves equation (11).

By the same token, it takes simple algebra to show that if \( \pi_{ni} \) solves equation (14), then it still solves it when \( d_{in} \) and \( w_i \) are replaced with, respectively, \( d_{in}' \) and \( w_i' \). Similarly, if equation (17) holds with \((d_{in},w_i)\), then it also holds with \((d_{in}',w_i')\). \( \blacksquare \)

Proposition 1 follows from Proposition 4 by taking \( \delta_i = \delta \) for some \( i \) and \( \delta_n = 1 \) for any \( n \neq i \).
D Proof for Proposition 2

The following proposition is a more general result than Proposition 2:

**Proposition 5** Consider an economy (economy A) with trade barriers \(d_{ni}\) and where relative wages are sticky and set to \(w_i/w_n\), such that \(w_i/w_n = \delta_i w_i/\delta_n w_n\), where \(w_i/w_n\) are the competitive-equilibrium relative wages and \(\delta_i, \delta_n \geq 1\) \(\forall i, n\). Now consider a change in trade barriers from \(d_{in}\) to \(d'_{in} = \delta_i d_{in}/\delta_n \forall i, n\). The economy under \(d'_{in}\) and \(w_i/w_n\) (economy B) features the same equilibrium quantities and relative product prices as the one under \(d_{in}\) and \(w_i/w_n\) (economy C).

**Proof.** This result follows directly from Proposition 4, which implies that if \(p_i^N, p_i^T, L_i^T, \pi_{ni}\) and \(w_i\) are a solution for equations (8), (9), (11), (14) and (17) for given trade barriers \(d_{in}\), then \(\delta_i p_i^N, \delta_i p_i^T, L_i^T, \pi_{ni}\) and \(\delta_i w_i\) are a solution for the same equations when the barriers \(d_{in}\) are replaced with \(d'_{in}\). In other words, economy C has the same equilibrium quantities and relative product prices as economy B. \(\blacksquare\)

Proposition 1 follows from Proposition 4 by taking \(\delta_i = \delta\) for some \(i\) and \(\delta_n = 1\) for any \(n \neq i\). To complete the proof, we need to show that, in economy A, \(L_i^* \leq L_i\). Without loss of generality, we can consider relative wages in economy A as given by \(w_i^* = \delta w_i\) and \(w_n^* = w_n\) \(\forall n \neq i\).

Recall that \(L_i^T\) solves equation (17) for given relative wages \(w_i/w_n\). Denote by \(L_i^{T,*}\) the number of workers in the tradeable-goods sector of country \(i\) in economy A, where wages are set to \(w_i^*/w_n^*\). Replacing \(w_i/w_n\) with \(w_i^*/w_n^*\) in equation (17) shows that, if the market shares of country \(i\) remained set at \((\pi_{ni}, \pi_{ii})\) and the number of workers in the tradeable-goods sector of country \(n\) remained equal to \(L_n^{T,*} = L_n^T\), then it would be \(L_i^{T,*} = L_i^T/\delta \leq L_i^T\). In other words, if the right-hand side of equation (17) remains constant, then, to maintain the same income as \(w_i L_i^T\), the number of workers \(L_i^{T,*}\) must be lower than \(L_i^T\) because wages are higher.

Equation (14), in addition, shows that in economy A the market shares of country \(i\) decline to \(\pi_{ni}^* \leq \pi_{ni}\) and \(\pi_{ii}^* \leq \pi_{ii}\), further reducing \(L_i^{T,*}\).

If all goods were tradeable, the proof that country \(i\) experiences unemployment would have been complete, because \(L_n^{T,*}\) could not be higher that its full employment
level. Then, economy A would experience an excess demand of workers in country n and unemployment in country i.

Similarly, the proof is immediate if tradeable and non-tradeable goods are substitute ($\eta > 1$). In this case, in fact, $L^T_{n,\ast}$ cannot be higher than $L^T_n$ because, in country n, the relative price of tradeables rises ($p^T_{n,\ast}/p^N_{n,\ast} > p^T_n/p^N_n$) and, therefore, tradeable-goods industries want to employ less workers.

The proof is more complex, instead, if tradeable and non-tradeable goods are complements. In this case, the two effects that tend to lower employment in country i (the rise in relative wages and the related loss in market shares) are contrasted by two other effects. The first is that the number of workers in the tradeable-goods sector of country n increases, because the relative price of tradeables rises in this country. Therefore, country n spends more on all tradeable goods, including those that are sold by country i. Second, non-tradeable-goods industries of country i are willing to hire some workers that are eventually fired from the tradeable-goods sector, because the relative price of non-tradeables rises in this country ($p^T_{i,\ast}/p^N_{i,\ast} < p^T_i/p^N_i$). We can show, however, that these two effects are not sufficient to offset those that tend to lower employment in country i. Intuitively, this happens because the changes in relative prices that raise demand for workers in the tradeable-goods industries of country n and in the non-tradeable-goods industries of country i are not large enough. We remind the reader that the model does not have a closed-form solution (which is the result of a non-linear system of transcendental equations). For the sake of simplicity, we focus on the 2-country case for the proof that follows, but the extension to M countries is straightforward.

Consider, first, the rise in the number of workers in the tradeable-goods sector of country n. By using the expression of relative prices $p^T_{n}/p^N_{n}$, it takes simple algebra to
show that:

\[
\frac{L^T \ast}{L^T_n} = 1 + \left( \frac{T_n}{N_n} \right)^{1-\eta} \left[ 1 + \frac{T_i}{T_n} \left( \frac{w_n}{w_i d_{ni}} \right)^\theta \right]^{1-\eta} \frac{1-\eta}{\sigma} \\
\leq \frac{T_n}{N_n} \left[ 1 + \frac{T_i}{T_n} \left( \frac{w_n}{w_i d_{ni}} \right)^\theta \right]^{1-\eta} \frac{1-\eta}{\sigma} \\
\leq \frac{\left[ \frac{T_i}{T_n} \left( \frac{w_n}{w_i d_{ni}} \right)^\theta \right]^{1-\eta} \frac{1-\eta}{\sigma}}{\left[ \frac{1}{\delta^\theta \frac{T_i}{T_n} \left( \frac{w_n}{w_i d_{ni}} \right)^\theta} \right]^{1-\eta}} = \delta^{1-\eta}.
\]

In other words, the number of workers in the tradeable-goods sector of country \( n \) rises less than proportionally to \( \delta^{1-\eta} \).

Now let us turn to the market shares:

\[
\frac{\pi_{ni}}{\pi_{ni}^*} = \frac{1 + \delta^\theta \frac{T_n}{T_i} \left( \frac{w_i d_{ni}}{w_n} \right)^\theta}{1 + \frac{T_n}{T_i} \left( \frac{w_i}{w_n} \right)^\theta} \\
1 - \pi_{ii}^* = \delta^\theta \frac{1 + \delta^\theta \frac{T_n}{T_i} \left( \frac{w_i}{w_n} d_{ii}^* \right)^\theta}{1 + \frac{T_i}{T_n} \left( \frac{w_i}{w_n} d_{ii}^* \right)^\theta}.
\]

Therefore:

\[
\frac{\pi_{ni}}{1-\pi_{ii}} = \delta^\theta \frac{1 + \frac{T_n}{T_i} \left( \frac{w_i}{w_n d_{ni}} \right)^\theta}{1 + \left( \frac{T_n}{T_i} \frac{w_i}{w_n} \right)^\theta} \\
1 - \pi_{ii} = \delta^\theta \frac{1 + \frac{T_n}{T_i} \left( \frac{w_i}{w_n} d_{ii}^* \right)^\theta}{1 + \left( \frac{T_i}{T_n} \frac{w_i}{w_n} \right)^\theta}.
\]

\[
\frac{\pi_{ni}}{1-\pi_{ii}} = \delta^\theta \frac{(d_{in} d_{ni})^\theta + \frac{T_n}{T_i} \left( \frac{w_i d_{ni}}{w_n} \right)^\theta}{1 + \frac{T_n}{T_i} \left( \frac{w_i}{w_n} d_{ni} \right)^\theta} \\
\geq \delta^\theta,
\]

where the last step follows from the fact that, because \( d_{in} d_{ni} \geq 1 \), then the two factors that multiply \( \delta^\theta \) are larger than 1. Thus, the decline in market shares is more than proportional to \( \delta^\theta \).

Taking into account also the fact that \( w_i^* = \delta w_i \), then the changes in market shares and in the number of workers in the tradeable-goods sector of country \( n \) reveal
that the number of workers in the tradeable-goods sector of country \( i \) declines at least by \( \delta^{\theta+\eta} \) (i.e. \( L_i^{T,\ast} \leq L_i^T / \delta^{\theta+\eta} \)).

To complete the proof, we show that the rise in the number of workers in the non-tradeable-goods sector of country \( i \) is not sufficient to restore full employment. Recall that:

\[
L_i^* = L_i^{T,\ast} + L_i^{N,\ast} = L_i^{T,\ast} \left[ 1 + \left( \frac{p_i^{N,\ast}}{p_i^{T,\ast}} \right)^{1-\eta} \right]
\]

and, therefore, we have unemployment \( (L_i^* \leq L_i) \) if:

\[
\frac{1 + \left( \frac{p_i^{N,\ast}}{p_i^{T,\ast}} \right)^{1-\eta}}{\left[ 1 + \left( \frac{p_i^{N,\ast}}{p_i^{T,\ast}} \right)^{1-\eta} \right]} \leq \delta^{\theta+\eta}.
\]

And, in fact:

\[
\frac{1 + \left( \frac{p_i^{N,\ast}}{p_i^{T,\ast}} \right)^{1-\eta}}{\left[ 1 + \left( \frac{p_i^{N,\ast}}{p_i^{T}} \right)^{1-\eta} \right]} \leq \frac{1 + \left( \frac{p_i^{N,\ast}}{p_i^{T}} \right)^{1-\eta}}{\left[ 1 + \left( \frac{p_i^{N}}{p_i^{T}} \right)^{1-\eta} \right]} = \frac{T_i + \delta^\theta T_n \left( \frac{w_i}{w_n d_{in}} \right) \theta^{\frac{1-\eta}{\theta}}}{T_i + T_n \left( \frac{w_i}{w_n d_{in}} \right) \theta^{\frac{1-\eta}{\theta}}} = \delta^{1-\eta}.
\]

Thus, total employment in country \( i \) declines for any positive elasticity (because, we recall, \( \theta > 1 \)).

**E  Proof for Proposition 3**

Result (i) follows immediately from the fact that, with sticky wages, the relative price of tradeables rises after the change in trade barriers.

The result that average productivity declines follows from the fact that \( \Lambda_i \) decreases (equation (15)). Each workers that goes from the tradeable-goods to the non-tradeable-goods sector produces more goods if the average productivity of the tradeable goods industries that become active is larger that the average productivity of the non-tradeable goods industries (while the precise condition is rather technical, it is enough that \( T_i \) is sufficiently larger than \( N_i \)).
Result \((iii)\) follows immediately from equation (18).
CHAPTER 4

Summary and Conclusions
The development of the Eaton-Kortum model has favored a resurgence of studies focusing on comparative advantage as the main determinant of international trade flows. Despite the lack of a closed form solution, this framework can be easily quantified and has already been intensively used in order to provide an empirical answer to different policy questions, by running appropriate counterfactual simulations.

The model is also very tractable and suitable to tackle meaningful theoretical questions. This is the route that we have explored in this thesis, in which we have considered variants and extensions of the Eaton-Kortum model, with the purpose of analyzing the sources of the gains from trade, the effects of trade openness on total factor productivity, and the role of nominal exchange rates.

The first chapter has shown that, in a generalization of the Eaton-Kortum model in which industry efficiencies can have any continuous distribution, the welfare gains from trade can be always decomposed into two effects. The former is a selection effect, which is the change in average efficiency due to the selection of industries that survive international competition. The latter is a reallocation effect, which is the rise in the weight of exporting industries in domestic production. Interestingly, the analytical expression of these two effects, which is too cumbersome to be used for empirical purposes in the general model, simplifies dramatically if we impose that industry efficiencies are Fréchet distributed. This assumption makes our general Ricardian framework go back to the original Eaton-Kortum model, which, in turn, belongs to the class of the so-called "quantitative trade models" analyzed by Arkolakis, Costinot, and Rodríguez-Clare (2012). The distinguishing feature of this class of models is that they entail a particularly simple quantification of the overall gains from trade.

A key insight from our analysis, then, is that quantitative trade models may be useful not only in order to assess the overall welfare gains, but also to properly measure their sources. This is an important issue that deserves to be further explored in future studies tackling other models in this class.

The second chapter has focused on the relationship between total factor productivity — as proxied by the average productivity calculated across the domestic industries that survive international competition — and trade openness in the original Eaton-Kortum model. Building on this relationship, we introduce a novel methodology
to measure the relative total factor productivity of the tradeable-goods sector of various countries. The logic of this new approach is to use a structural model and estimate the total factor productivity from its observed implications on trade flows, production and factor costs, rather than just as a residual of the aggregate production function. The results of this methodology, while broadly in line with those of many previous studies, including those stemming from the standard development-accounting approach, appear more reasonable in some respects. In particular, they fix the "anomaly" produced by the standard method that, from the mid-1980s to the early-1990s, the total factor productivity of the manufacturing sector of Italy — a country with strong labor-market and product-market rigidities — is the highest of our sample, which includes all the major developed countries. Our methodology, instead, returns estimates of the total factor productivity in which Italy ranks 6th or 7th in the entire sample period (1985-2002), broadly in line with the rank of Italy in terms of real GDP per capita. The United States turns out to be the economy with the highest total factor productivity throughout our two-decade sample period.

While these empirical results are quite encouraging, future research is still needed to enhance our methodology, which can provide only estimates of the total factor productivity of the tradeable-goods sector (proxied with the manufacturing sector in the empirical application). Incorporating the non-tradeable-goods sector in the model could then allow to derive estimates the total factor productivity of the whole economy, just like with the standard development-accounting approach.

The third chapter makes a first step in this direction, as it extends the Eaton-Kortum model to encompass the non-tradeable-goods sector. The purpose of the analysis, however, is different, as it focuses on the effects of nominal exchange rates. By mimicking changes in the value of the currency with changes in trade barriers, we have been able to analyze these effects in a very general real framework, in which there is no money. In particular, the result that under perfect competition, constant returns to scale and flexible wages, exchange rates do not yield any effect on the competitive equilibrium is obtained with a multiplicity of countries, tradeable goods and non-tradeable goods. By the same token, the result that if sticky wages are too high in a country, there is an appropriate depreciation that restores the competitive equilibrium, demonstrates, in a very general framework, the intuitions of Keynes (1931) and Friedman.
The analysis performed in the third chapter has not exploited one of the most important feature of our extension of the Eaton-Kortum model, that is the possibility of quantifying it in order to obtain model-based measures of the domestic and international effects of misaligned currencies on employment, output and welfare. This is an exercise that warrants further investigation, which we leave for future research.

The different models introduced in this thesis have three main limitations, which they share with the original set-up of Eaton and Kortum (2002): (i) they are all based on a (clearly restrictive) representative-agent assumption; (ii) they are all static; (iii) they are all real frameworks with no money and no financial sector. Overcoming these limitations would give rise to promising areas for both theoretical and empirical research.

In particular, models with heterogeneous agents would allow to study the effect of international trade on income inequality. Extensions of the model aimed at studying the factor distribution of income — i.e. either the distribution to capital and labor and/or the distribution to skilled workers and unskilled workers — have already been developed in the literature (see, for example, the recent papers by Parro, 2013, and Burstein and Vogel, 2015, and the references cited therein). What is still missing, instead, is a satisfactory treatment of the personal distribution of income. These issues could also have important implications for the quantification of the welfare gains from trade analyzed in Chapter 1, especially once they are associated with the introduction of frictions in the labor market.

The inclusion of capital accumulation in the production function, discussed (but not implemented) also by Alvarez and Lucas (2007), could affect the estimates of the total factor productivity performed in Chapter 2. It could potentially help to improve the methodology and return dynamic, instead of static, estimates of the total factor productivity. In addition, considering that international trade is more intensive in capital goods, the shift from the autarky to the open economy could have strong effects on capital accumulation and, possibly, imply larger gains from trade than those estimated by Chapter 1.

Finally, extending the model to incorporate also money and the financial sector
would allow to study the real effects of nominal exchange rates that may stem from the related changes in interest rates as well as from exchange rate expectations. Notice that, in the case of misaligned currencies, the model discussed in Chapter 3 is agnostic on whether exchange rates gradually return to their long-run equilibrium level or whether they overshoot or undershoot. Thus, incorporating money and the financial sector would allow to associate the theory of the real effects of nominal exchange rates outlined in Chapter 3, with a theory concerning their dynamics.

Overall, the theoretical and empirical results obtained in this thesis confirm that, after two hundred years, the Ricardian theory of comparative advantage is still alive and kicking. It is easy to predict that this field will continue to be a fruitful line of research still for many years to come.
References


