A New Genetic Algorithm Approach to Smooth Path Planning for Mobile Robots

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Abstract

In this paper, the smooth path planning problem is considered for a mobile robot based on the genetic algorithm and the Bezier curve. The workspace of a mobile robot is described by a new grid-based representation ($2^n \times 2^n$ grids) that facilitates the operations of the adopted genetic algorithm. The chromosome of the genetic algorithm is composed of a sequence of binary numbered grids (i.e., control points of the Bezier curve). Ordinary genetic operators including crossover and mutation are used to search the optimum chromosome where the optimization criterion is the length of a piecewise collision-free Bezier curve path determined by the control points. A numerical experiment is given to demonstrate the effectiveness of the proposed smooth path planning approach for a mobile robot.

Index Terms

Smooth path planning; mobile robot; genetic algorithm; Bezier curve.

I. Introduction

With the rapid development of modern industry, mobile robots have been widely used in a wide range of applications such as manufacturing, assembly, logistics and transportation [14], [25]. The path planning is one of the most important topics in mobile robotics whose objective is to find a feasible and optimal path from a start position to a target position. A path is said to be "feasible" and "optimal" if the mobile robot moving along it could avoid collisions with obstacles and also satisfy certain optimal criteria. In other words, the path planning can be considered as an optimization problem on certain indices (e.g. shortest distance) under certain constraints (e.g. collision-free route). As shown in [20], the path planning problem of a mobile robot is a NP-hard optimization one that can only be solved by heuristic algorithms such as evolutionary computation techniques. Among various algorithms capable of handling NP-hard problems, the genetic algorithm (GA) has proven to be the simple yet effective one that has been frequently used in industry especially in mobile robotics [20].

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Recently, a variety of GA-based approaches have been developed for the mobile robot path planning problems [24]. A problem-specific GA for the path planning of a mobile robot has been proposed in [13] that incorporates the domain knowledge into its specialized operators. A new mutation operator has been presented in [31] for the GA and applied to the path planning problem of mobile robots in dynamic environments. In addition to the ordinary crossover, another new mutation operator has been developed in [21] as a subset of mutation in order to manipulate an individual. A vibrational GA has been put forward in [23] to reduce the possibility of premature convergence and therefore help the candidate solution to reach the global optimum. A parallel elite GA has been proposed in [30] along with a migration operator to maintain better population diversity, avoid premature convergence and keep parallelism in comparison with the conventional GA.

In almost all aforementioned literature, the genetic-algorithm-based path planning approaches have been concerned with the issue of planning a feasible path with certain simple optimal criterion (e.g. the minimum length of the path). Actually, a collision-free shortest path is often not sufficient for the planed movement of a mobile robot. For example, it is quite common that a traditional path planning algorithm gives rise to a path that contains some polygonal lines or even sharp turns. When moving along the polygonal lines, a mobile robot would have to switch between different modes (e.g. stop, rotate and restart) frequently, and such a switching process is both time- and energy-consuming. Such undesirable switches are even impermissible when the smoothness of the movement is a requirement for some service tasks [35]. Therefore, in addition to the distances, some other optimization criteria should be included such as the path smoothness, energy evaluation, time consumption and robot speed, see [19], [32], [34] for more details. Note that, apart from the length of the path, the path smoothness has been considered to be another important criterion because the smoothness is a closely related to other optimization criteria [2].

In recent years, the Bezier curve has been increasingly applied in the smooth path planning problems [1], [22], [29]. For example, a Bezier-curve-based approach has been proposed in [15] for the path planning of a mobile robot in a multi-agent robot soccer system which is compatible with the velocity and acceleration limits. A collision-free curvature-bounded smooth path planning technique has been presented in [11] whose idea is to divide the nodes on the piecewise linear path into control point subsequences so as to generate a collision-free composite Bezier curve under the curvature constraint. A new cooperative collision-avoidance method has been developed in [27] for multiple and nonholonomic robots based on the Bernstein-Bezier curves, and a model-predictive trajectory tracking algorithm has been used to drive the robots on the obtained reference paths. In order to form a smooth path based on the path points, the Bezier curve and other parameter curves are usually produced by the Voronoi diagram, the Dijkstra algorithm, the $A*$ algorithm and the $D*$ algorithm, etc.

Up to now, some scattered results have been available in literature on the smooth path planning problem of mobile robots or multi-agent systems by combining the heuristic intelligent optimization algorithm (e.g. GA) with the path smoothing approach (e.g. Bezier curve). For example, in [33], the path planning algorithm has been developed for obstacle avoidance problems of mobile robots by adopting the Bezier curve based on the GA, but the general mathematical description of the optimal path planning problems as well as the representation issue of the workspace have not been thoroughly discussed and this makes it inconvenient to implement the GA in practice. In [26], the Bezier curve-based flyable trajectories have been generated for multi-UAV systems with parallel genetic algorithm where the Bezier curve has been used for smoothing the obtained path and, as such, it is difficult to guarantee the optimality of the eventually planned path. *The purpose of this paper is to improve the existing results by making the following three distinctive contributions: 1) a rigorous mathematical formulation of the path planning optimization problem is formulated; 2) a general grid-based representation ($2^n \times 2^n$ grids) is proposed to describe the workspace of the mobile robots in order to facilitate the implementation of the GA where n is chosen according to the trade-off between the accuracy and the computational burden; and 3) the control points of the Bezier curve are directly linked to the optimization criteria so that the generated paths are guaranteed to be optimal without any need for smoothing afterwards.*

In this paper, a new approach is proposed to solve the smooth path planning problem of a mobile robot based on the genetic algorithm and the Bezier curve. A new grid-based representation of the workspace is presented in this paper which simplifies the population initialization, chromosome encoding and the genetic operators in existing literature (e.g. [31]). The workspace of a mobile robot is divided into several orderly numbered square grids and the center of a grid is defined as a candidate control point of a Bezier curve. A sequence of binary number represented control points, namely, the genes, represent a chromosome of genetic algorithm, and ordinary genetic operators are used to search the optimal chromosome. The optimization criterion of the genetic algorithm is the length of all piecewise collision-free Bezier curves determined by the control points. The numerical experiment results verify the effectiveness of the proposed smooth path planning approach.

The remainder of this paper is organized as follows. In Section II, the Bezier curve is introduced and some preliminaries are briefly outlined. In Section III, the key stages of the smooth path planning approach are presented based on the genetic algorithm. In Section IV, the effectiveness of the obtained results is illustrated by a simulation example and the expected performance is evaluated as well. Concluding remarks and future work are given in Section V.

II. Preliminary of Bezier curve

The Bezier curve is contained within the convex hull of a sequence of control points. In this case, the Bezier curve is different from the traditional interpolation-based curves such as polynomials and cubic splines. The control points, which define the Bezier curve, are not on the curve except the start and end points. The high-order derivative continuity of the Bezier curve can guarantee the smooth variation of the curve between the start point and the end point.

Given a set of control point vectors $\mathbf{P}_0, \mathbf{P}_1, \cdots, \mathbf{P}_n$, the corresponding Bezier curve is defined as

$$\mathbf{P}(t) = \sum_{i=0}^{n} B_i^n(t)\mathbf{P}_i,\ 0 \le t \le 1, \tag{1}$$

where t is the normalized time variable, $B_i^n(t)$ is a Bernstein polynomial and $\mathbf{P}_i = (x_i, y_i)^T$ stands for the coordinate vector of the ith control point with x_i and y_i being the components corresponding to the X and Y coordinate, respectively. The Bernstein polynomial is the base function in the expression of Bezier curve, which is defined as

$$B_i^n(t) = \begin{pmatrix} n \\ i \end{pmatrix} t^i(1-t)^{n-i},\ i = 0, 1, \ldots, n. \tag{2}$$

The derivatives of a Bezier curve can also be determined by the control points. The first derivative of a Bezier curve is expressed as

$$\dot{\mathbf{P}}(t) = \frac{d\mathbf{P}(t)}{dt} = n\sum_{i=0}^{n-1} B_i^{n-1}(t)(\mathbf{P}_{i+1} - \mathbf{P}_i) \tag{3}$$

and the higher order derivative of a Bezier curve can be obtained by repeatedly using (3). For example, the second derivative of a Bezier curve is expressed as

$$\ddot{\mathbf{P}}(t) = n(n-1)\sum_{i=0}^{n-2} B_i^{n-2}(t)(\dot{\mathbf{P}}_{i+2} - 2\dot{\mathbf{P}}_{i+1} + \dot{\mathbf{P}}_i). \tag{4}$$

Accordingly, in the two-dimensional plane, the curvature of a Bezier curve with respect to t can be represented as

$$\kappa(t) = \frac{1}{R(t)} = \frac{\dot{\mathbf{P}}_x(t)\ddot{\mathbf{P}}_y(t) - \dot{\mathbf{P}}_y(t)\ddot{\mathbf{P}}_x(t)}{(\dot{\mathbf{P}}_x^2(t) + \dot{\mathbf{P}}_y^2(t))^{3/2}}, \tag{5}$$

where $R(t)$ is the radius of curvature, $\dot{\mathbf{P}}_x(t)$, $\dot{\mathbf{P}}_y(t)$, $\ddot{\mathbf{P}}_x(t)$ and $\ddot{\mathbf{P}}_y(t)$ are X and Y coordinate components of the first and second derivatives of the Bezier curve $\mathbf{P}(t)$, respectively.

In this paper, piecewise Bezier curves are connected to form a complete path in the smooth path planning of a mobile robot, where second and lower order continuities are considered for the smoothness of the path. The zero-order continuity (i.e., continuous position) is held by coincident end and start points of the connected Bezier curves. The first-order continuity is ensured by equivalent tangent vectors at the connection of two curves, and the second-order continuity is ensured by equivalent curvatures. For simplicity, the curvature at the connection of two connected Bezier curves is usually set to zero, i.e., a few adjacent points of the connected Bezier curves are on the same line. Therefore, it is easy to satisfy the continuity requirements in most cases.

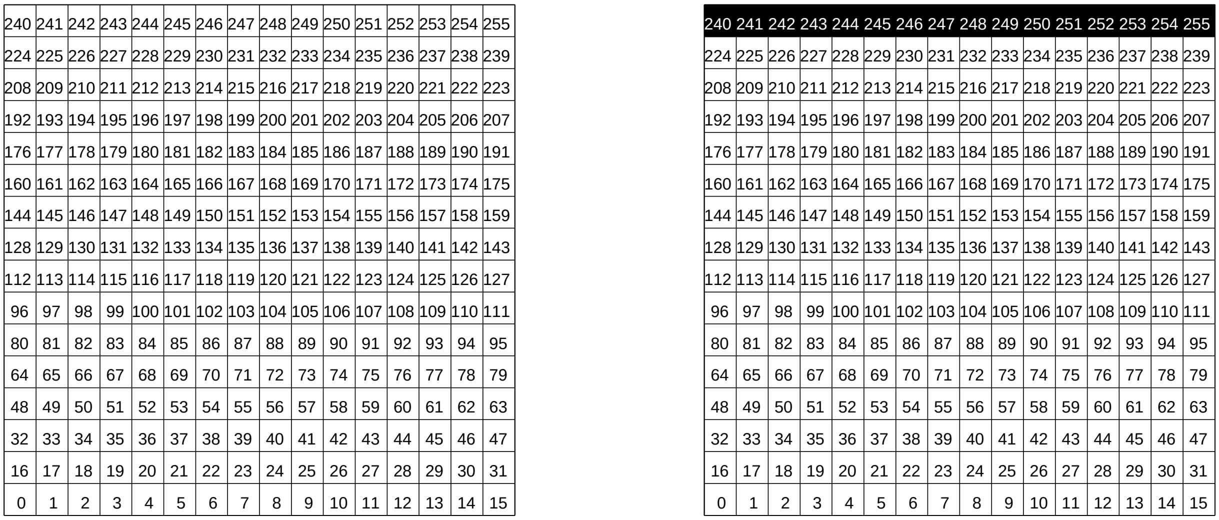
III. Smooth path planning based on genetic algorithm

In this paper, the genetic algorithm is combined with the Bezier curve for the smooth path planning of a mobile robot. The genetic algorithm is applied to search the optimal control points which are used to define the smooth Bezier curve. The feasible and shortest Bezier curve path is the optimum solution for the smooth path planning problem of a mobile robot. Several key stages of the approach are presented in this section as follows.

A. Problem description

In this paper, the working environment of a mobile robot is supposed to be a two-dimensional workspace. The proposed genetic algorithm based smooth path planning is used to find a feasible and optimal Bezier curve path for a mobile robot, which include three conditions, i.e., firstly the path should be collision-free, secondly the smooth movement of a mobile robot should be considered by satisfying the second-order continuity of the path, and thirdly the path should be the shortest distance from a start point to a target point under the above two constraints. The formal expressions of above conditions are as follows:

$$\begin{aligned} \min \quad & \|\mathbf{P}(t)\|,\ 0 \le t \le 1, \\ \text{s.t.} \quad & \mathbf{P}(t) \in C^2, \\ & \mathbf{P}(t) \in \mathbf{P}_{free}, \end{aligned} \tag{6}$$



(a) (b)

Fig. 1. (a) The workspace with 16×16 grids; (b) The workspace with 15×16 grids

where t is normalized time variable, $\|\mathbf{P}(t)\|$ represents the length of the Bezier curve path, C^2 is a set of second-order differentiable function and $\mathbf{P}_{free}$ indicates a set of collision-free path. Since the Bezier curve is defined by its control points, the above optimization problem is to find a sequence of control points which determine the Bezier curve path under constraint conditions.

B. Representation of workspace

The workspace is an environment where the mobile robot and obstacles both exist. The grid-based model is usually utilized to represent the workspace in the path planning of a mobile robot, since it is easy to calculate distances and represent obstacles. The boundary of obstacles is formed by their actual boundary plus minimum safety distance considering the size of a mobile robot, so that a mobile robot could be treated as a point in the workspace [13]. The whole workspace is represented by orderly numbered grids, and the size of the grids determines how many numbers there is. For each grid, it is defined to be either empty (i.e., the white square grid in the workspace) or occupied (i.e., the black square grid in the workspace), which depends on whether the boundary of obstacles is in the grid.

The whole workspace is divided into $M \times N$ grids in this paper, where both M and N are positive integer power of number 2. M could be equal to N or not in practice, because a workspace with $M \times N$ grids could be treated as a workspace with $M \times M$ grids and some of the grids are occupied. Fig. 1(a) and Fig. 1(b) show a workspace with $M \times M$ grids ($M = 16$, $N = 16$) and $M \times N$ grids ($M = 15$, $N = 16$) respectively. This representation approach simplifies the chromosome encoding and population initialization of genetic algorithm. For example, if the workspace is divided into 10×10 grids, then the numbers of the grids are from 0 to 99. Binary encoding is used for chromosome in this paper, and this means a 7 bit binary number is necessary to represent one grid. However, some of the numbers (e.g., from 100 to 127) exceed all of the grid numbers in the workspace, so that additional process has to be added to check whether the randomly generated chromosomes (defined in next section) are reasonable.

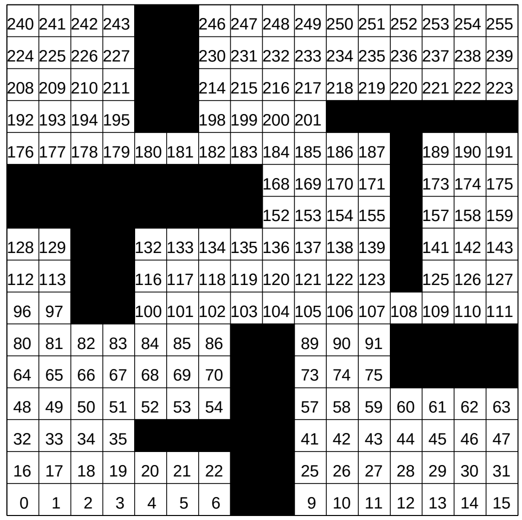


Fig. 2. The workspace of a mobile robot in this paper

C. Chromosome encoding and population initialization

A sequence of grid numbers is used to represent the chromosomes of the genetic algorithm in this paper and chromosomes are encoded by binary numbers for easier bit operations of genetic operators. Take the 160×160 units workspace with 16×16 grids (i.e., each grid is a 10×10 units square) in Fig. 2 for example hereinafter, each grid is assigned a number between 0 and 255, which could be represented by a 8 bit binary number. All of the binary numbers are connected orderly to form a chromosome. This kind of chromosome encoding is suitable for both $M \times M$ and $M \times N$ grids. Moreover, genetic operators will not bring unreasonable chromosome by using this approach because both crossover and mutation operators will keep a grid number in the chromosome between 0 and 255. This is another advantage of the workspace representation approach mentioned above.

All of the control points are defined at the center of the grids in the workspace. The transformation from grid numbers to coordinate values is expressed as

$$\begin{aligned} \mathbf{P}_x(t) &= (\text{Number}\%16) \times 10 + 5, \\ \mathbf{P}_y(t) &= \lfloor \text{Number}/16 \rfloor \times 10 + 5, \end{aligned} \tag{7}$$

where $\%$ indicates the complementation, $\lfloor \ \rfloor$ denotes the rounding down, $\mathbf{P}_x(t)$ and $\mathbf{P}_y(t)$ are X and Y coordinate components of the center of the grid respectively. On the other hand, the transformation from coordinate components of any point on the path to the number of a grid that contains the point is expressed as

$$\text{Number} = \lfloor \mathbf{P}_x(t)/10 \rfloor + \lfloor \mathbf{P}_y(t)/10 \rfloor \times 16. \tag{8}$$

Considering the representation of the workspace and chromosome encoding approaches, it is very easy to implement the population initialization of the generic algorithm. For example, if a chromosome has n grid numbers, then the initialized population is a set of 8×n bit binary numbers.

D. Fitness function and selection method

The purpose of the genetic algorithm based smooth path planning in this paper is to find an optimal path under constraint conditions (6). The fitness function is defined as

$$f = \begin{cases} \frac{1}{\sum_{i=1}^{n} \|\mathbf{P}_i(t)\|}, & \text{for feasible paths} \\ \frac{1}{\sum_{i=1}^{n} \|\mathbf{P}_i(t)\| + \text{penalty}}, & \text{for infeasible paths} \end{cases} \tag{9}$$

where $\mathbf{P}_i(t)$ is the ith segment of the piecewise Bezier curves with n segments and *penalty* is added when the Bezier curve passes through an occupied grid. A shorter path will has a larger fitness value and the optimal path is a shortest feasible Bezier curve path.

The proportional selection strategy is used in the selection method of the genetic algorithm, i.e., the probability that the selected chromosome is proportional to the fitness value. Suppose the fitness value of the ith chromosome is f_i and the population size is S_p, the selection probability of the ith chromosome can be expressed as

$$p_i = \frac{f_i}{\sum_{i=1}^{S_p} f_i}, \tag{10}$$

where p_i is the selection probability and the roulette wheel method is used for selection operation afterwards.

E. Genetic operators

Crossover and mutation are two heart genetic operators in the genetic algorithm. The crossover operator is to combine the features of two parent chromosomes to produce two offspring chromosomes. The crossover probability is randomly generated to determine whether the crossover operator is implemented on two parent chromosomes. The single point crossover operator is used in this paper, i.e., the genes of two chromosomes after a randomly generated crossover point are swapped.

The mutation operator is implemented after the crossover operator on randomly selected chromosomes in the population. The binary complement operation is implemented on a randomly generated mutation point of the chromosomes. Above genetic operators will not produce genes (i.e., grid numbers) out of the workspace by using the proposed approaches in Sections III-B and III-C.

Remark 1: The grid-based representation of the workspace is common in the literature of mobile robot path planning. However, a general grid-based representation is proposed in this paper. The whole workspace is divided into $2^n \times 2^n$ grids in spite of whether the workspace is a square (the workspace could be treated as a square with some occupied grids if it is not a square). This approach facilitates the implementation of the GA in the processes of initialization, crossover and mutation.

Remark 2: In this paper, the control points of the Bezier curve are directly linked to the optimization criteria, which is different from most Bezier-curve-based path planning approaches where Bezier curve is used to smooth the path produced by some path planning methods [26]. Although the path planning adopting the Bezier curve based on the GA has been developed in [33], the details of the implementation have not been thoroughly discussed.

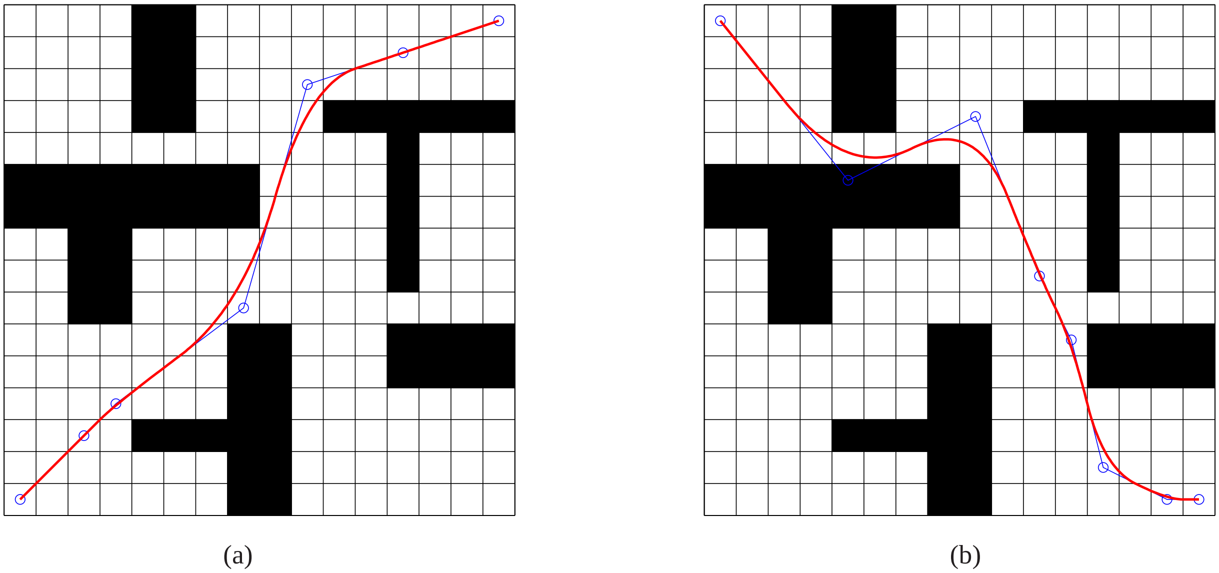


Fig. 3. (a) Smooth path planning from grid 0 to grid 255; (b) Smooth path planning from grid 15 to grid 240

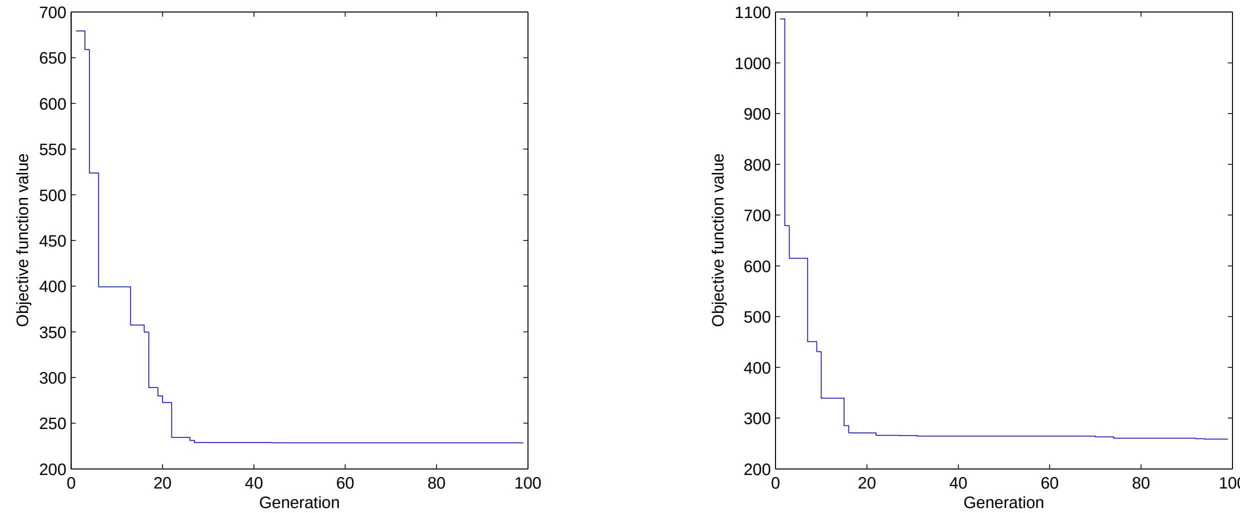
IV. Numerical experiments and performance evaluation

In this section, the smooth path planning based on the genetic algorithm will be applied to the workspace in Fig. 2 to demonstrate the effectiveness of the proposed approach. The parameters of the genetic algorithm are as follows: the population size is taken as 200, the maximum generation is taken as 100, the crossover probability is taken as 0.5, the mutation probability is taken as 0.1 and the *penalty* is taken as 10 for each infeasible point of the Bezier curve path.

Fig. 3(a) and Fig. 3(b) show the numerical experiment results with different start positions and target positions in the workspace, where blue circles indicate the control points of the Bezier curve path, blue solid lines compose the convex hull and red solid lines is the optimum smooth path. Eight control points (i.e., eight grid numbers for each chromosome) are used for the Bezier curve path in this paper. In spite of the difficulty in the two cases, the proposed approach can accomplish the smooth path planning task successfully. Define the objective function as the reciprocal of the fitness function f in (9). The objective function value of the optimum chromosome in each generation is depicted in Fig. 4, which shows the fast convergence of the genetic algorithm in this problem.

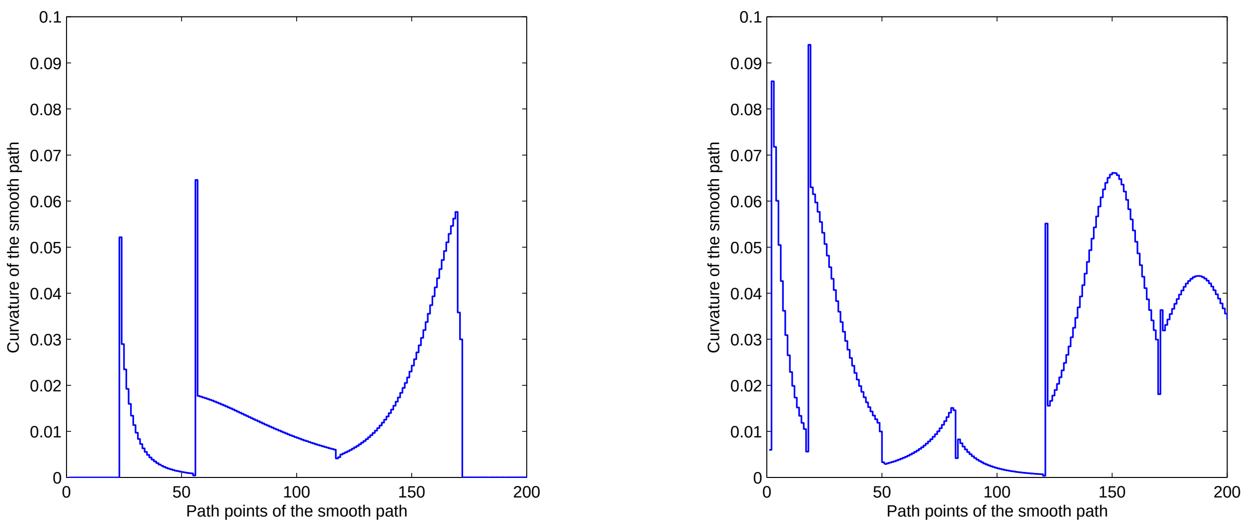
Fig. 5 shows the curvature of each point on the Bezier curve path, and it is obvious that the maximal curvature is lower than 0.1 according to the coordinates of the workspace. The low curvature values reflect the smoothness of the obtained path. Table I presents some comparisons on smooth path planning and non-smooth path planning both based on the genetic algorithm. The experiment results with different start positions and target positions indicate that, by using the proposed smooth path planning approach, we can derive an optimal path with the similar length as in the non-smooth path planning. Moreover, there is little difference on the minimal convergence generation between smooth and non-smooth path planning.

Remark 3: It is very important to choose the control point vectors in practical applications. Generally speaking, more control point vectors are required if the workspace is more complicated (e.g., with more obstacles). The GA could be implemented with different numbers of control points in practice. It is worth mentioning that the redundant control point, which is coincide with the other, will be rejected in the



(a) (b)

Fig. 4. (a) Optimum objective function value in the case of Fig. 3(a); (b) Optimum objective function value in the case of Fig. 3(b)



(a) (b)

Fig. 5. (a) Path curvature in the case of Fig. 3(a); (b) Path curvature in the case of Fig. 3(b)

optimum control point sequence at the end of the procedure (e.g., eight control points are used for the GA and seven control points are remained in Fig. 3(a)).

Remark 4: Compared with other smooth path planning methods [1], [22], [29], one of the advantages of the proposed approach is the general grid-based representation of the workspace, which facilitates the implementation of the GA. The other but more meaningful one is that this approach is a “real” smooth path planning approach combining a heuristic intelligent optimization algorithm (e.g., the GA in this paper) with a path smoothing approach (e.g., Bezier curve in this paper). In this paper, the control points of the Bezier curve are directly linked to the optimization criteria of the GA, so that the generated paths are guaranteed to be optimal instead of smoothing the paths after some path planning processes.

Remark 5: In (9), the *penalty* is added when the Bezier curve passes through an occupied grid. In some special cases, a path will has larger fitness value, even it’s control points lie in the occupied grids and

TABLE I
COMPARISON OF GENETIC ALGORITHM BASED SMOOTH PATH PLANNING AND NON-SMOOTH PATH PLANNING

	Minimal generation non smooth path	Objective value non smooth path	Minimal generation smooth path	Objective value smooth path
Grid 0 to 15	26	225.4982	49	221.0383
Grid 15 to 255	88	211.2393	67	218.5059
Grid 240 to 255	76	178.3131	47	195.3404
Grid 0 to 240	69	244.9847	73	247.8467

the *penalty* is added in the fitness function. For example, it can be found that optimum control points can be in the *occupied* grids as shown in Fig. 3(b). Moreover, how to select the *penalty* to improve the performance of the algorithm is still an open problem for further discussion.

V. CONCLUSIONS AND FUTURE WORK

This paper has proposed a new smooth path planning for a mobile robot by resorting to the genetic algorithm and the Bezier curve. A new grid-based representation of the workspace has been presented, which makes it convenient to perform operations in the genetic algorithm. The genetic algorithm has been used to search the optimum control points that determine the Bezier curve based smooth path. The effectiveness of the proposed approach has been verified by a numerical experiment, and some performances of the obtained method have also been analyzed. There still remain many interesting topics, for example, 1) how to solve the specific smooth path planning problem by using the genetic algorithm; 2) how to promote the computational efficiency in the more grids case; 3) how to select the number of control points and value of the "*penalty*"; and 4) how to apply the developed algorithms to more complicated situations (e.g. mobile navigations in networked environments [3]–[10], [12], [16]–[18], [28], [36]. These issues deserve further research.

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