

Development of a Low Cost 3x3 Coupler  
Mach-Zehnder Interferometric Optical Fibre Vibration  
Sensor

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## **Abstract**

In this paper, a low cost 3x3 coupler Mach-Zehnder interferometric optical fibre vibration sensor is proposed. Unlike traditional Mach-Zehnder interferometer with a strain free reference arm, two parallel optical fibres with different strain transfer sensitivity is adopted. With proper demodulation algorithm, the time rate of change of the strain can be computed from the light signal of the outputs of the 3x3 coupler. The proposed optical fibre vibration sensor is verified by experiment. The vibrating frequency spectrum of a 2-layer metal frame is measured by both the proposed optical fibre vibration sensor and MEMS-based accelerometer. It is found that the proposed vibration sensor is more sensitive unless it is located near zero strain region compared to MEMS-based accelerometer. Several proposed sensor can be easily multiplexed to minimise the overall cost.

# 1 Introduction

Vibrating frequency is a common parameter for global structural health monitoring. When there is structural degradation or damage after excessive loading or natural disaster, the stiffness of the structure is reduced and hence the natural frequencies. Nowadays, the frequency spectrum is popularly obtained from MEMS-based accelerometers. The advantages of accelerometer are that the amplitude of the frequency spectrum is magnified by the frequency itself so that the higher resonant mode can be retrieved with higher signal-to-noise ratio. Also, the output voltage of the accelerometer is directly proportional to the acceleration after calibration. However, it is prone to electromagnetic interference. Based on the developed  $3\times 3$  coupler Mach-Zehnder interferometer [1], in this paper, a low cost  $3\times 3$  coupler Mach-Zehnder interferometric optical fibre vibration sensor is proposed. With the proposed technique, several vibration sensors can be multiplexed by single light source. By the Mach-Zehnder interferometry [2], the temperature induced strain is self-compensated and the ultra-high sensitivity can show the frequency spectrum to be more sensitive to higher mode. In this paper, the sensor design for practical application and the demodulation algorithm of the received signals is introduced first. The proposed sensor is verified experimentally and the frequency spectrum from the proposed sensor is compared to the traditional MEMS based accelerometers.

## 2 Sensing Principles

### 2.1 Sensor design

The schematic diagram of the optical vibration sensor is shown in Figure 1. A Mach-Zehnder interferometer consists of a distributed feedback laser diode (DFB-LD), a  $2\times 2$  single-mode directional coupler, two SMF28 optical fibres, a  $3\times 3$  coupler and three photodetectors (PD). The wavelength of the DFB-LD is temperature dependent, so there is a thermal controlling circuit to maintain the working temperature at  $25^{\circ}\text{C}$  to keep the wavelength of the DFB-LD at 1550 nm. The bandwidth of the DFB-LD is 0.1 nm. The maximum optical path difference (OPD) between the two arms of the interferometer is

17 mm to maintain the coherence. During the sensor fabrication, the length of each arm of the interferometer is measured carefully to ensure the coherence requirement is fulfilled. It should be noted that a lower OPD is desirable because it can result in higher resistance of the interferometer to environmental noise. The 2x2 single-mode directional coupler splits the light into the two separated optical paths to the sensing plate. The polymeric coating of one of the fibres is removed so it is more strain sensitive than the other under the same strain field [3]. The frequency of the strain field induced to the optical fibre is the same as the strain field of the substrate but the magnitude of it is different with a fixed ratio. The different strain in the fibre along the gauge length induces phase shift of the optical signal. The optical signal of two optical fibres in the sensing plate interfere in the 3x3 coupler. The interfered signals are measured by 3 photodetectors separately as  $y_1(t)$ ,  $y_2(t)$  and  $y_3(t)$ , respectively. When strain is applied to the sensing arm, the phase shift is changed and it is given by Equation 1 [4],

$$\phi = \frac{2n_0\pi}{\lambda} \int_L \left[ 1 + \varepsilon_{xx} + \frac{1}{4}n_0^2 (2P_{12}\varepsilon_{xx} + (P_{11} + P_{12})(\varepsilon_{yy} + \varepsilon_{zz})) \right] ds \quad (1)$$

where  $\phi(t)$  is the total optical phase shift,  $n_0$  is the refractive index of the core of the unstrained optical fibre,  $L$  is the gauge length of the interferometer,  $P_{11}$  and  $P_{12}$  are the Pockels strain-optic constants of the fibre,  $\varepsilon_{xx}$  is the longitudinal strain of the fibre and  $\varepsilon_{yy}$  and  $\varepsilon_{zz}$  are the transverse strains of the fibre. The gauge length of the sensor is the length of the stripped fibre. The advantage of the configuration is that the strain field of both fibres in the sensing plate is identical except the gauge length and hence the environmental noise is minimised.

## 2.2 Demodulation Algorithm

The three photodetectors measure the light intensities of the interfered signals from the 3 arms of the 3x3 coupler in time domain that are denoted as  $y_1(t)$ ,  $y_2(t)$  and  $y_3(t)$ , respectively. The phase shift is demodulated from the three signals. For an ideal 3x3 coupler, the light is evenly split into each output with a phase difference of  $2\pi/3$ . Math-

ematically, the intensities measured by the three photodetectors can be expressed in the form of Equation 2

$$y_n(t) = C_1 + C_2 \cos \left[ \phi(t) - (n - 1) \frac{2\pi}{3} \right] \quad (2)$$

where  $n = 1, 2, 3$ ,  $C_1$  and  $C_2$  are the background intensity and fringe contrast, respectively.  $\phi(t)$  is the optical phase difference between the two optical fibres in the sensing plate. The phase shift can be expressed in the form of Equation 3.

$$\phi(t) = C_3 \cos(\omega t) \quad (3)$$

where  $C_3$  and  $\omega$  are the amplitude of the optical phase difference and the angular frequency of the external excitation, respectively. The block diagram describing the technique of symmetric demodulation using the three outputs from a  $3 \times 3$  coupler [5] is shown in Figure 2. As the phases of the three outputs of the interferometer are complementally symmetric, the cosine terms are cancelled by taking an average of the three outputs with the trigonometric identity in Equation 4,

$$\cos x + \cos \left( x - \frac{2\pi}{3} \right) + \cos \left( x + \frac{2\pi}{3} \right) = 0 \quad (4)$$

By subtracting each output by the average of the three outputs, the constant term  $C_1$  in Equation 4 is eliminated. There are only the alternating terms of the output signal named a, b and c in Figure 2. By taking the first derivative of each of them with respect to time, d, e and f can be computed, respectively. Each of them contains a common factor  $\dot{\phi}(t)$ , which is the time derivative of  $\phi(t)$ . Each signal a, b and c is then multiplied by the difference of the derivatives of the other two signals. The sum of the three products is named  $N$  and with the manipulation of algebra and trigonometric identities, the output  $N$  is equal to it is given by Equation 5.

$$N = -\frac{3\sqrt{3}}{2} C_2^2 \dot{\phi}(t) \quad (5)$$

By the trigonometric identity in Equation 6, the cosine square terms are eliminated.

$$\cos^2 x + \cos^2 \left( x - \frac{2\pi}{3} \right) + \cos^2 \left( x + \frac{2\pi}{3} \right) = \frac{3}{2} \quad (6)$$

Then, the sum of the square of all signals  $D$  are computed. The ratio between  $N$  and  $D$  is independent to the background intensity  $C_1$  and fringe contrast  $C_2$  in the outputs of the interferometer, and hence  $\dot{\phi}(t)$  can be computed. Finally, the phase change  $\phi(t)$  is obtained by an integration of  $\dot{\phi}(t)$  with respect to time. The demodulation algorithm is manipulated in Matlab.

## 3 Experiments

### 3.1 Experimental Setup

The proposed optical fibre vibration sensor is verified by a free vibration test of a metal frame. The optical fibre vibration sensor is attached on the side of a two-layer metal frame as shown in Figure 3. Two MEMS-based accelerometers are installed at the top and middle layers of the frame (Figure 3). The vertical members of the frame were made of aluminum and the horizontal members were made of polymethyl methacrylate (PMMA). The width, depth and height of the metal frame are 312 mm, 108 mm and 991 mm, respectively. The thicknesses of the vertical and horizontal members are 2 mm and 12 mm, respectively. The frame is fixed on the ground by a 20 kg mass. In order to trigger different vibration modes, there are two different initial conditions applied on the frame for free vibration. The first initial condition of free vibration is to apply a force  $F_1$  on the top layer of the frame as shown in Figure 3. The second initial condition of the free vibration is to apply two forces  $F_2$  and  $F_3$  simultaneously at the midpoints of each floor as shown in Figure 3. For the first case, the acceleration along x-direction is monitored by the two accelerometers installed on the top and middle layers. For the second case, the accelerations along x- and y-directions of the top layer only are monitored. All the outputs of the proposed vibration sensor and accelerometers are recorded at 400 kHz sampling

rate. To improve the signal-to-noise ratio of the interferometer, moving average technique (averaged by 100 consecutive points) is applied on the three signals from photodetectors. The sampling duration is 8 seconds. The magnitude of  $F_1$ ,  $F_2$  and  $F_3$  is not measured but the initial displacements of the frame by the force are small enough to avoid nonlinear vibration and uplift of the metal frame.

### 3.2 Experimental Result and Discussion

The light intensity signal of the 3 photodetectors as well as the accelerometer in time domain is recorded. The time variation of the phase shift  $\phi(t)$  of the proposed vibration sensor is computed by the aforementioned demodulation algorithm. The time domain signal of phase shift as well as acceleration is transformed to frequency domain by standard fast Fourier transform to compute the frequency spectrum. The experimental results are shown in Figures 4 and 5. The amplitude of the frequency spectrum is normalised by the maximum amplitude in the spectrum. For the first case of the experiment, only  $F_1$  is applied on the frame as the initial condition. All the optical fibre vibration sensor and the two accelerometers at the top and middle layer show a sharp peak at 2.2 Hz in the frequency spectrum. It is the fundamental frequency of the frame in x-direction. There is a small peak shown from the frequency spectrum of the two accelerometers near 6.5 Hz while the frequency spectrum of the optical fibre vibration sensor shows a sharp peak at that frequency. It is because of the ultra-high sensitivity of the interferometric based vibration sensor. There are several sharp peaks in the frequency spectrum of the optical fibre vibration sensor at higher frequencies. Since the initial displacement by the force  $F_1$  may not be perfectly along x-direction, there may be torsional modes and they induce strain along the gauge length of the sensing plate.

For the second case, two forces act on the frame in opposite direction at different floor as the initial condition. The accelerations in both x- and y-directions on the top of the frame are measured by a dual-axis accelerometer. The frame vibrates freely and the fundamental frequency measured by both the optical fibre vibration sensor and the accelerometer in x- and y-directions is 6.3 Hz. The frequency spectrum of the accelerom-

eter shows a peak at 2.3 Hz with small amplitude while the optical fibre vibration sensor shows a sharp peak at 2.3 Hz which is close to the fundamental frequency of the first case. The frequency spectrum of the acceleration in y-direction shows three more peaks than the x-direction. They are 7.6 Hz, 12.4 Hz and 18.6 Hz. The optical fibre vibration sensor shows sharp peaks with much higher amplitude than the accelerometer in y-direction at 12.4 Hz and 18.6 Hz. However, the optical fibre vibration sensor does not show any peak at 7.6 Hz. It may be due to the skew-symmetric strain of the mode shape at this frequency. However, the optical fibre vibration sensor shows a peak at 0.4 Hz while no peak is shown of the acceleration in both directions. That means the proposed optical fibre vibration sensor is sensitive to some vibration modes that cannot be detected by a 2D accelerometer.

### **3.3 Extension of the proposed sensor**

The proposed optical fibre vibration sensor can be extended to multiplex several sensors by single light source. The highest cost of the whole instrumentation is the DFB-LD. The  $2 \times 2$  coupler can be replaced by an  $1 \times 16$  coupler. Two of the 16 output signal form two independent optical path as shown in Figure 1 and they are connected with a  $3 \times 3$  couplers and 3 photodetectors. For example, with an  $1 \times 16$  coupler from a DFB-LD, 8 pairs of sensors are divided to 8  $3 \times 3$  couplers. The 8 independent optical fibre vibration sensors are measured by 24 photodetectors but they can be logged by single DAQ device. Hence, the averaged cost of multiplexed sensors can be significantly reduced.

## **4 Conclusions**

In this paper, a new, low cost Mach-Zehnder interferometric fiber optic vibration sensor utilising differential strain transfer is proposed. The sensing principle is based on the strain variation of the two arms of the interferometer. The phase shift of the two arms of the interferometer is obtained by a demodulation technique from the sensor outputs. Experimental results indicate that the sensor can measure the resonant frequencies of a

free-vibrating structure under different initial conditions. Also, the frequency spectrum from the sensor output combines the frequency in x- and y-directions. With the proposed sensing principle, several optical fibre vibration sensors can be multiplexed easily.

## 5 Acknowledgments

The work described in this paper was fully supported by Prof Christopher K.Y. Leung of Department of Civil and Environmental Engineering of the Hong Kong University of Science and Technology in Hong Kong.

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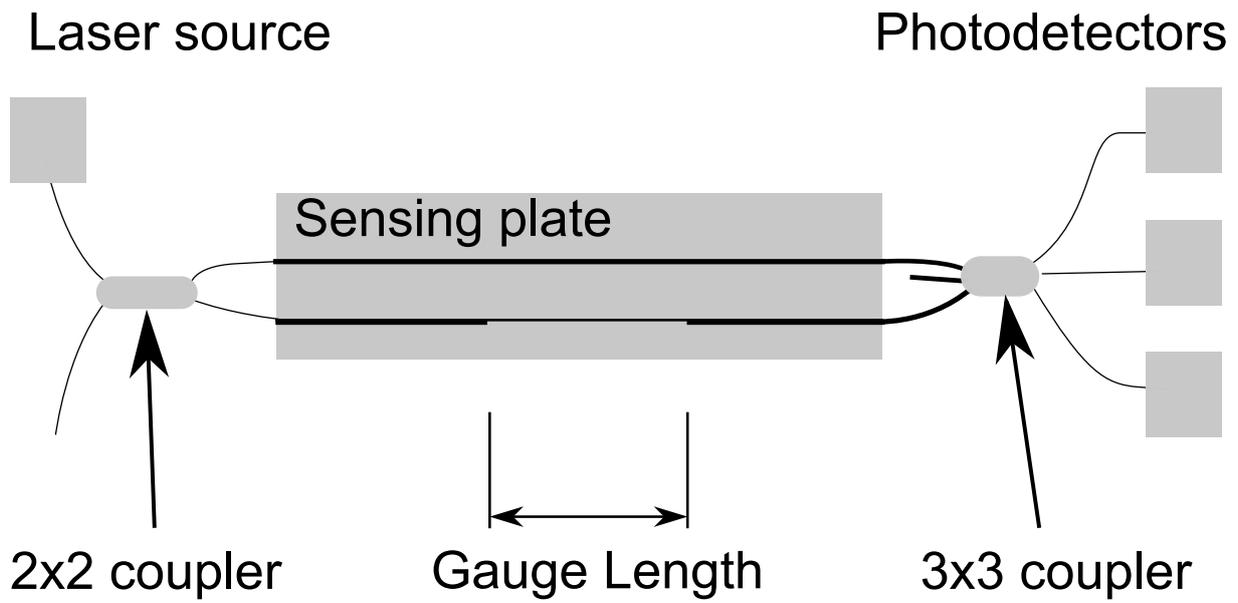


Figure 1: Schematic diagram of vibration sensor.

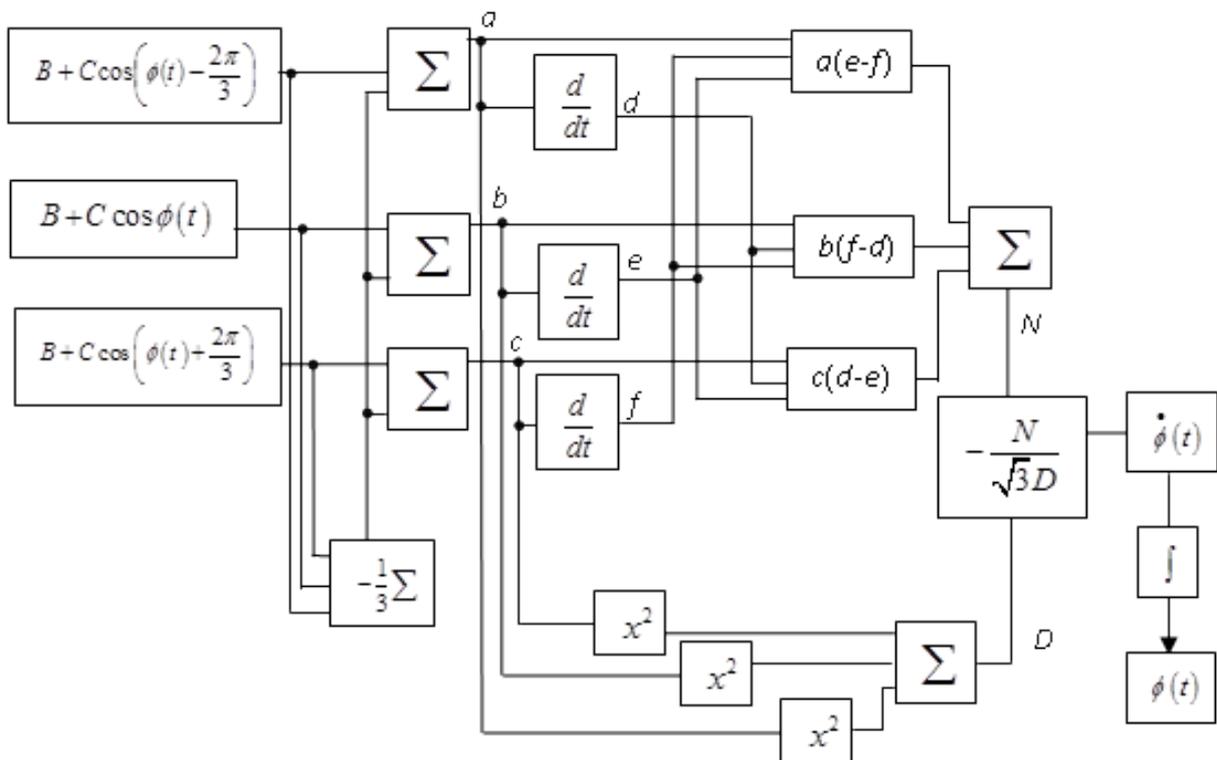


Figure 2: Schematic diagram of demodulation algorithm to obtain the frequency of the strain field from the signals of the photodetectors.

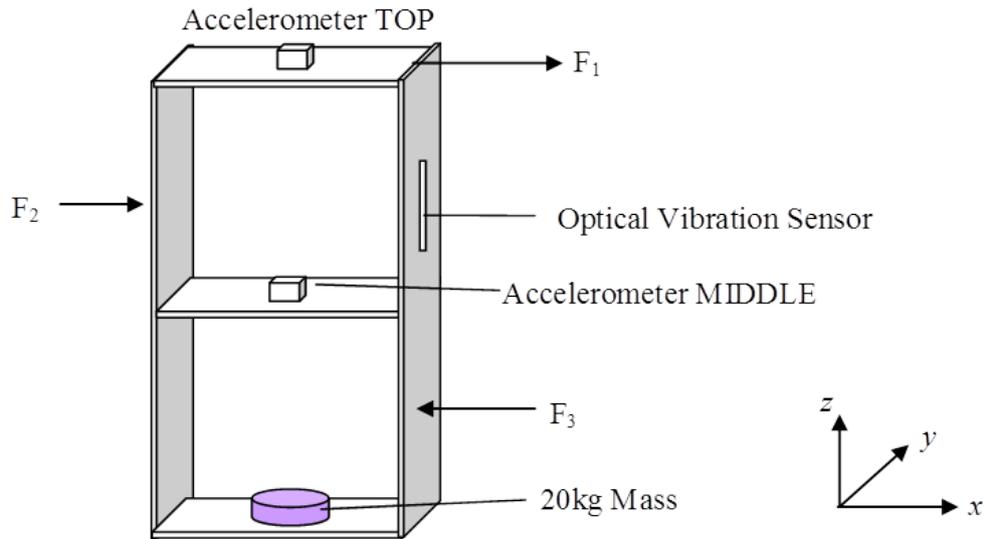


Figure 3: Schematic diagram of experimental setup.

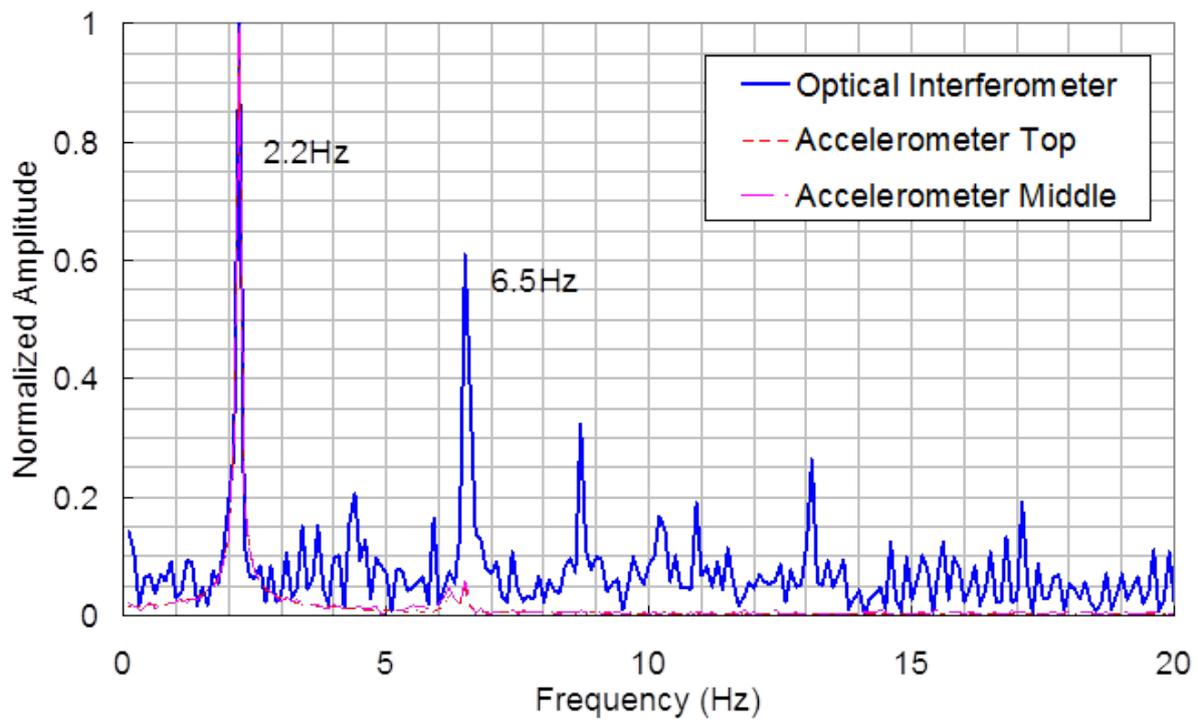


Figure 4: Comparison of the frequency spectrum of the free vibrating two-layer frame by  $F_1$ .

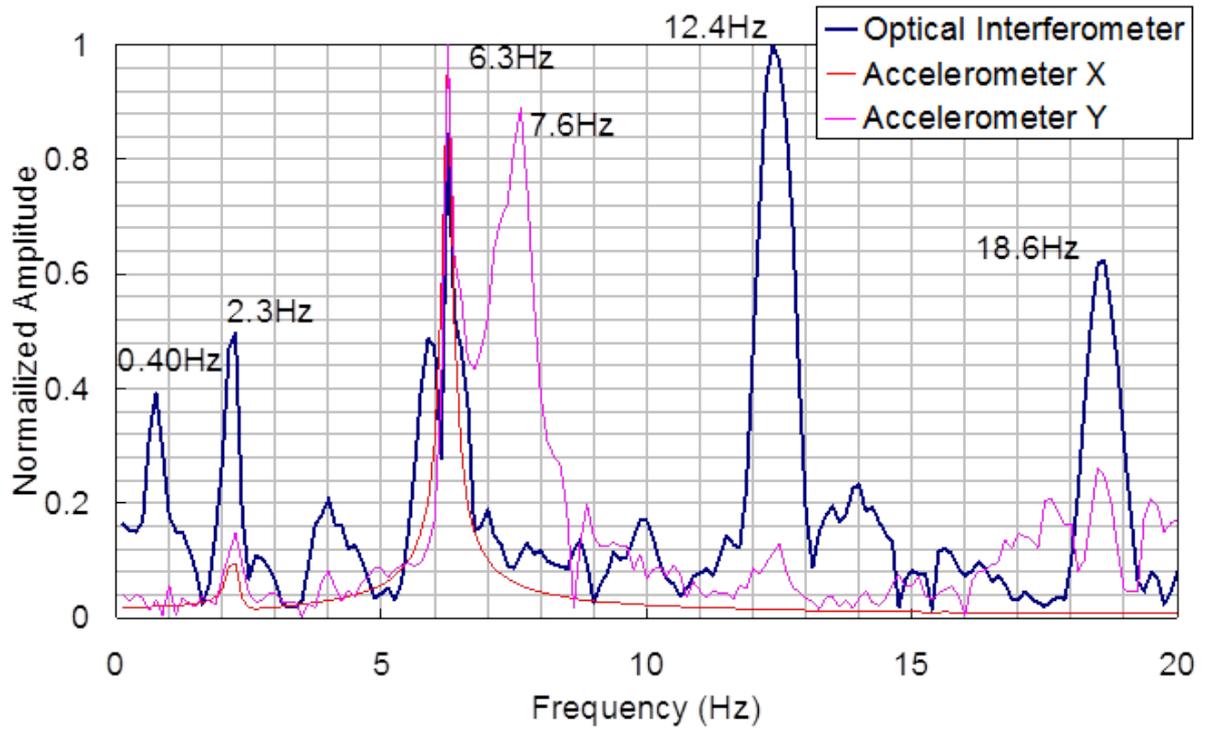


Figure 5: Comparison of the frequency spectrum of the free vibrating two-layer frame by  $F_2$  and  $F_3$ .