A Credit Scoring Model Based on Classifiers Consensus System Approach

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ABSTRACT

Managing customer credit is an important issue for each commercial bank; therefore, banks take great care when dealing with customer loans to avoid any improper decisions that can lead to loss of opportunity or financial losses. The manual estimation of customer creditworthiness has become both time- and resource-consuming. Moreover, a manual approach is subjective (dependable on the bank employee who gives this estimation), which is why devising and implementing programming models that provide loan estimations is the only way of eradicating the 'human factor' in this problem. This model should give recommendations to the bank in terms of whether or not a loan should be given, or otherwise can give a probability in relation to whether the loan will be returned. Nowadays, a number of models have been designed, but there is no ideal classifier amongst these models since each gives some percentage of incorrect outputs; this is a critical consideration when each percent of incorrect answer can mean millions of dollars of losses for large banks. However, the LR remains the industry standard tool for credit-scoring models development. For this purpose, an investigation is carried out on the combination of the most efficient classifiers in credit-scoring scope in an attempt to produce a classifier that exceeds each of its classifiers or components.

In this work, a fusion model referred to as 'the Classifiers Consensus Approach' is developed, which gives a lot better performance than each of single classifiers that constitute it. The difference of the consensus approach and the majority of other combiners lie in the fact that the consensus approach adopts the model of real expert group behaviour during the process of finding the consensus (aggregate) answer. The consensus model is compared not only with single classifiers, but also with traditional combiners and a quite complex combiner model known as the 'Dynamic Ensemble Selection' approach.

As a pre-processing technique, step data-filtering (select training entries which fits input data well and remove outliers and noisy data) and feature selection (remove useless and statistically insignificant features which values are low correlated with real quality of loan) are used. These techniques are valuable in significantly improving the consensus approach results. Results clearly show that the consensus approach is statistically better (with 95% confidence value, according to Friedman test) than any other single classifier or combiner

analysed; this means that for similar datasets, there is a 95% guarantee that the consensus approach will outperform all other classifiers. The consensus approach gives not only the best accuracy, but also better AUC value, Brier score and H-measure for almost all datasets investigated in this thesis. Moreover, it outperformed Logistic Regression. Thus, it has been proven that the use of the consensus approach for credit-scoring is justified and recommended in commercial banks.

Along with the consensus approach, the dynamic ensemble selection approach is analysed, the results of which show that, under some conditions, the dynamic ensemble selection approach can rival the consensus approach. The good sides of dynamic ensemble selection approach include its stability and high accuracy on various datasets.

The consensus approach, which is improved in this work, may be considered in banks that hold the same characteristics of the datasets used in this work, where utilisation could decrease the level of mistakenly rejected loans of solvent customers, and the level of mistakenly accepted loans that are never to be returned. Furthermore, the consensus approach is a notable step in the direction of building a universal classifier that can fit data with any structure. Another advantage of the consensus approach is its flexibility; therefore, even if the input data is changed due to various reasons, the consensus approach can be easily re-trained and used with the same performance.

DEDICATION

To my lovely parents and family

This thesis is dedicated to my ever-loving mother, Khadija, for her continuous love and support. She always believed in me and made it all possible. Her endless love and encouragement has absolutely helped me to achieve my dream. Without her standing by my side and her permanent support, this work would not have been possible.

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I also dedicate this work to my wonderful brothers and only sister, Mohammed, Ahmed, Omar, Ibrahim and Noor, for their incessant love, care and encouragement, also my lovely nephew Abdelhamid. I owe everything I have achieved or will achieve to my lovely family.

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DECLARATION

It is hereby declared that the thesis in focus is the author's own work and is submitted for the first time to the Post-Graduate Research Office. The study was originated, composed and reviewed by the mentioned author and supervisors in the Department of Electronic and Computer Engineering, College of Engineering, Design and Physical Sciences, Brunel University London UK. All the information derived from other works has been properly referenced and acknowledged.

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LIST OF ABBREVIATIONS

AUC	Area Under Curve
AVG	Average Rule
ConsA	Consensus Approach
D-ENS	Dynamic Ensemble Selection
DT	Decision Tree
GNG	Gabriel Neighbourhood Graphs
LDA	Linear Discriminant Analysis
LR	Logistic Regression
MajVot	Majority Voting Rule
MARS	Multiple Adaptive Regression Splines
MAX	Maximum Rule
MIN	Minimum Rule
NB	Naïve Bayes
NN	Neural Networks
PROD	Product Rule
RF	Random Forests
ROC	Receiver Operating Characteristics
SVM	Support Vector Machines
WAVG	Weighted Average Rule
WVOT	Weighted Voting Rule

CHAPTER 1

INTRODUCTION

1.1 Background

Credit granting to lenders is considered a key business activity that generates profits for banks, financial institutions and shareholders, as well as contributing to the community; however, it also can be a great source of risk. The recent financial crises resulted in huge losses globally and, hence, increased the attention directed by banks and financial institutions to credit risk models. That is, as a result of the crises, banks are now more conscientious when considering the need to adopt rigorous credit evaluation models in their systems when granting a loan to an individual client or a company.

The problem associated with credit-scoring is that of categorising potential borrowers into either good or bad. Models are developed to help banks to decide whether or not to grant a loan to a new borrower using their data characteristics (Kim & Sohn, 2004). The area of credit-scoring has become a widely researched topic by scholars and the financial industry (Kumar & Ravi, 2007; Lin *et al*., 2012) since the seminal work of Altman in 1968 (Altman, 1968). Subsequently, many models have been proposed and developed using statistical approaches, such as LR and linear discriminate analysis (LDA) (Desai *et al*., 1996; Baesens *et al*., 2003). Recently, the Basel Committee on Banking Supervision (Lessmann *et al*., 2015) requested that all banks and financial institutions implement rigorous and complex credit-scoring systems in order to help them improve their credit risk levels and capital allocation.

Despite developments in technology, LR remains the industry-standard baseline model used for building credit-scoring models (Lessmann *et al*., 2015); many studies have demonstrated that artificial intelligence (AI) techniques, such as NN, SVM, DT, RF and NB, which may act as substitutes for statistical approaches in building credit-scoring models (Atiya, 2000; Bellotti & Crook, 2009; Brown & Mues, 2012; Hsieh & Hung, 2010).

The utilisation of the different techniques in building credit-scoring models have varied with time, with researchers tending to use each technique individually, and then later to overcome shortcomings of applying these techniques individually, with researchers tending to customise

the design of credit-scoring models. Researchers leant towards complexity in their designs and trying new modelling approaches, such as hybrid and ensemble modelling, with both approaches showing better performance than the use of individual techniques. However, hybrid and ensemble approaches can be utilised independently or in combination. The basic idea behind hybrid and ensemble modelling, for the former, is to conduct a pre-processing step for the data that is fed to the classifiers whilst for the latter is to use and garner benefit of group of classifiers trained on the dame problem and use their opinions to reach a proper classification decision. However, modelling complexity can be associated with financial and computational cost; nonetheless, it is believed that complexity could lead to a better and universal classification models for credit-scoring, which in fact is the main aim of this thesis investigation.

Generally, there is no overall best classification technique used in building credit-scoring model; selecting a model that could discriminate between two groups, depends on the nature of the problem, data structure, variables used and the market and environment (developed by Hand& Henley, (1997)).

1.2 Research Motivations

In recent years, the research trend has been actively moving towards using single AI techniques in building ensemble models (Wang *et al*., 2011). According to Tsai (2014), the idea of ensemble classifiers is based on the combination of a pool of diversified classifiers, such that their combination achieves higher performance than single classifiers since each complements the other classifier's errors. However, in the literature on credit-scoring, most of the classifier combination techniques adopt the form of homogenous and heterogeneous classifier ensembles, where the former combines the classifiers of the same algorithm, whilst the latter combine classifiers of different algorithms (Lessmann *et al*., 2015; Tsai, 2014). As Nanni & Lumini (2009) point out, an ensemble of classifiers is a set of classifiers, where the decisions of each are combined using the same approach.

Recent studies have shown ensemble models perform better than single AI classifiers in credit-scoring (Lessmann *et al*., 2015; Nanni & Lumini, 2009). Most of the related work in ensemble studies in the domain of credit-scoring have focused on homogenous ensemble classifiers via simple combination rules and basic fusion methods, such as majority voting, weighted average, weighted voting, reliability-based methods, stacking and fuzzy rules

(Wang *et al*., 2012; Tsai, 2014; Yu *et al*., 2009; Tsai & Wu, 2008; West *et al*., 2005; Yu *et al*., 2008). A few researchers have employed heterogeneous ensemble classifiers in their studies, but still with the aforementioned combination rules (Lessmann *et al*., 2015; Wang *et al*., 2012; Hsieh & Hung, 2010; Tsai, 2014). In ensemble learning, all classifiers are trained independently to produce their decisions, to be combined via a heuristic algorithm to produce one final decision (Zang *et al*., 2014; Rokach, 2010).

1.3 Aim and Objectives

This core aim of this research is to explore a new combination method in the field of credit-scoring that can replace the existing combination methods by developing a new combination rule whereby the ensemble classifiers can work and collaborate as a group or a team in which their decisions are shared between classifiers. The classifier ConsA is where classifier ensembles work as a team to interact and cooperate to solve the same problem. Another aim is centred on addressing the question as to whether or not complexity in modelling credit-scoring problems is worth investigation by encompassing several stages to reach the core aim of this thesis. The stages involved in the proposed model include starting with simple models, followed by steady complexity carried out through the implementation of developments, investigations and comparisons on the models for the goal of achieving better results and validating it is effectiveness. However, the main objectives of this research are as follows:

- Implement five single classifiers: RF, DT, NB, NN and SVM. Moreover, implement a LR benchmark classifier for comparison with all the results achieved during this work.
- Investigate the influence of data-filtering and feature selection over training data on a single classifier performance.
- Implement D-ENS Selection model and investigate how the Accuracy of this combiner exceeds the Accuracy of single classifiers and classical combiners over the selected datasets.
- Improve the performance of existing models by combining them into one model using ConsA.
- An important step in ConsA that it cannot be used without information about conditional ranking for all pairs of single classifier, the intermediary task which remains in front of us is to estimate conditional rankings in a logically relevant way.
- Using Friedman and Bonferroni-Dunn statistical methods prove that ConsA is really better than any other classifier or combiner considered in this work.

1.4 Contributions to Knowledge

In this work, several algorithms have been improved and developed in an effort to increase the performance of classifiers and combiners to an even greater degree. The main contributions of this thesis are as follows:

- This thesis delivers a critical related literature on the different hybrid and ensemble techniques, taking into consideration different aspects and sides of their modelling approaches for the period of 2002–2015.
- Use improved filtering method, based on the weighted average of Gabriel Graph neighbour labels, rather than simple average. Use various threshold values to filter data instead of simple 0.5 threshold value for good and bad loan entries.
- Evaluate local Accuracy using weighted average, instead of simple Average. Weights are inversely proportional to the distance from the target point to the neighbours.
- Improve ConsA to be able to use it as credit-scoring model. To do this, conditional rankings for all classifiers were estimated using local Accuracy.
- Introduce new technique to evaluate ranking vector, which are based on mean squared error rather than iterations procedure.
- Introduce several parameters inside ConsA to be able to fine-tune it to obtain better performance.

1.5 Structure of the Thesis

The thesis is made up of nine individual chapters, which are structured as follows:

- **Chapter 2** presents the background and literature review in two-fold: the first fold provides a theoretical background on credit-scoring and related issues, in addition to the quantitative tools utilised in the developed models; the second fold focuses on the related work of credit-scoring models that are correlated to the proposed modelling approach of this thesis, followed by a critical review and analysis of the selected studies tracked by drawings and findings.

- **Chapter 3** explains the process of the methodological experimental design of this thesis, where the experimental procedure is described in stages and each stage

discusses several issues relating to the best modelling approach for selection in order to achieve a stable and reliable model.

- **Chapter 4** applies and develops the base classifiers used in this thesis (RF, DT, NB, NN and SVM). Their results are discussed, analysed and then compared with the benchmark model of this thesis (LR).

- **Chapter 5** seeks to improve the performance of the single base classifiers by producing hybrid classifiers through applying two data pre-processing techniques, namely data-filtering and feature selection. The results demonstrated are based on three experiments with data-filtering and feature selection and by combining both techniques. All results are discussed, analysed and compared with the benchmark model of this thesis (LR).

- **Chapter 6** delves into more depth by investigating the ensemble classifiers using seven traditional combinations rules. Each combination rule is analysed in terms of strength and weaknesses for each. Finally, the results are discussed, analysed and compared with the results of the single base classifiers' results and LR.

- **Chapter 7** presents the new hybrid ensemble proposed method based on the classifiers ConsA, along with another combination technique based on local Accuracy estimates for comparison purposes and to investigate to extent to which modelling complexity can enhance classification performance. This chapter discusses the theoretical aspects of the proposed model components supported with illustrative examples of their implementation.

- **Chapter 8** demonstrates the experimental results of /her and D-ENS Classifier approach. Results of ConsA are discussed, analysed and compared with all models developed (single classifiers, hybrid classifiers, ensemble classifiers with traditional combiners, D-ENS Classifiers approach and LR) and then followed by statistical significance test to validate its superiority over all models.

- **Chapter 9** highlights the conclusions, limitations and suggests future research directions of this thesis.

1.6 List of Publications

➢ Journals

- **Ala'raj, M**., Abbod, M. Classifiers consensus system approach for credit scoring. Knowledge-Based Systems (In Press).
- **Ala'raj, M**., Abbod, M. A new hybrid ensemble credit scoring model based on classifiers consensus system approach. Expert Systems with Applications (Submitted 27 Mar. 2015, Under 2nd review).
- **Ala'raj, M**., Abbod, M. On the use of credit-scoring models in emerging economies: The case of the Jordanian retail banking. Review of Development Finance (Submitted 27 Mar. 2015, Under review).

➢ Conferences

- Al-hnaity, B., Abbod, M., **Alar'raj, M.** (2015). Predicting FTSE 100 close price using hybrid model. SAI Intelligent Systems Conference (IntelliSys), 49-54.London, UK.
- **Ala'raj, M.**, Abbod, M. (2015). A systematic credit-scoring model based on heterogeneous classifier ensembles. Innovations in Intelligent SysTems and Applications (INISTA), 1-7. Madrid, Spain.
- **Ala'raj, M.**, Abbod, M., Al-Hnaity, B. (2015). Evaluation of Consumer Credit in Jordanian Banks: A Credit-scoring Approach.' 17th UKSIM-AMSS International Conference on Modelling and Simulation. Cambridge, UK.
- **Ala'raj, M.**, Abbod, M., Hunaiti, Z. (2014). Evaluating consumer loans using NN ensembles. International Conference on Machine-learning, Electrical and Mechanical Engineering. Dubai, UAE.

CHAPTER 2

BACKGROUND AND REVIEW OF THE LITERATURE

2.1 Introduction

In this section, a comprehensive literature review is conducted related to credit-scoring and its modelling approaches. At the beginning, a theoretical background on credit-scoring in terms of its definition, its importance and the systems used to assess credit and evaluation techniques to validate the developed credit-scoring models is demonstrated. Following this, the techniques used in developing credit-scoring models, from statistical to machine-learning methods and the modelling approaches used to develop these models, are described and discussed in detail. To date, a large number of studies have been undertaken to propose an efficient approach that can lead to better loan classification. In order to clarify the research aim and accordingly establish a theoretical framework for this study, only the related and most relevant literature utilising quantitative methods to develop credit-scoring systems on real world datasets is collected for analysis, discussion and comparison, especially for binary classification problem. Finally, a summary of the findings is demonstrated, along with identifying and addressing the interesting research trends in the field of classification and credit-scoring.

2.2 What is Credit-Scoring

For banks or any financial institution, credit lending activities are the principal of their business. However, good lending action leads to high profits, otherwise loss will take place. In order to minimise risk and choose where the money should be granted, a critical evaluation of loan applications should be carried out in order to reach to a reliable and effective decision. Hence, it is important for each lender, bank or financial institution to have methods that help them in identifying borrowers risk levels.

Credit-scoring has become an essential tool in banks' credit management process, with banks recognised as facing a lot of risks, especially those associated with the granting of loans to customers. Banks collect data, analyse it and then give a final decision on the loan, i.e. whether to accept or reject it. The important role of credit-scoring is to help analysts to reduce

the expected risks that might occur when a customer defaults. It gives signs and indicators about which customers are 'good' and which are 'bad'; this could prevent a wrong decision that causes financial losses.

Theoretically, many useful definitions of credit-scoring have been provided by many scholars in the field, which mention some are described. According to Thomas *et al*. (2002) they define credit-scoring as a *'set of decision models and their underlying techniques that aid lenders in the granting of consumer credit'*. Hand (1996) believes that credit-scoring '*is the term used to describe formal statistical methods used for classifying applicants for credit into "good" and "bad" risk classes*'. Another definition is based on *'assigning a single quantitative measure, or score, to a potential borrower representing an estimate of the borrower's future loan performance'* (Frame *et al*., 2001). Moreover, Anderson (2007) has a different view on how to define credit-scoring; he proposes the term be split into two components: the first one 'Credit', meaning 'buy now, pay later'; the second one 'Scoring', referring to the use of numerical formulae to rank different loan applications according to the available data and to their level of risk.

All the aforementioned definitions of credit-scoring lie in the use of quantitative methods or decision tools that are able to derive a score that can be used to help lenders to assess the risk level of each borrower and accordingly assign them to the appropriate risk class based on the available data. In order to quantify or measure the associated risk, lenders aimed at developing and building automated systems known as credit-scoring. Figure 2.1 illustrates the procedure of credit-scoring.

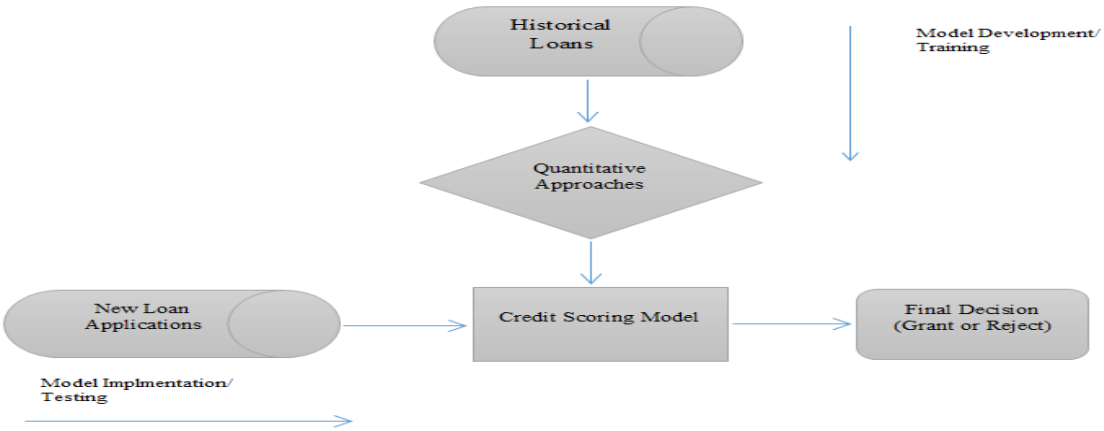

Figure 2.1 The procedure of credit-scoring

As can be observed from the above figure, the process of credit-scoring comprises two main phases, namely the model development and the model implementation. The first phase of the process starts with collecting samples of good and bad loan applications of past borrowers, where the selected sample is used develop and train a model that can capture the payment behaviour patterns between different borrowers. Formally, let $x^* = \{x_i, y_i\}$ be a pool of past loan application, where x_i is number of loan applications and y_i is the status or the target for each loan application which is either good or bad loan. Each loan application is characterised by a number of m attributes or variables $x_i = (x_{i1}, x_{i2}, \ldots, x_{im})$ which make up the loan application form. Consequently, a model is developed based on quantitative methods that construct a function $f(x)$ with the ability to map the attributes of each loan applicant to measure their probability of default. After the model has been developed and trained, the second phase is aimed at bringing the model in action by implementing and testing its ability to classify new loan applicants. The final measurement or score given to the applicant is based on a pre-determined threshold or cut-off score of (T_c) in which the lender will make a decision as to whether or not to grant the loan. The status of a loan applicant y is recognised either as good (whom can repay) or bad (whom cannot repay loan) usually labelled as (0) for good loan and (1) for bad loan. In the case of a new loan application, the developed model will generate a score specified by $f(x)$. If this score is below the cut-off score T then the loan is approved; otherwise, it is rejected. The cut-off score value is assigned by lender in a way that meets its financial business objectives and strategies, such as through fulfilling their default loans target rate. Equation 2.1 explains the decision process.

$$y = \begin{cases} 0, f(x) \leq T \\ 1, otherwise \end{cases} \tag{2.1}$$

From the discussion, it can noted that the entire process can be seen as a binary classification problem, where the problem is associated in categorising the loans of potential borrowers into either good or bad loan class using models and decision tools which will help to decide the optimal $f(x)$ to derive the accurate credit score. For this reason, the area of credit-scoring has become a topic researched by scholars and the financial industry (Kumar & Ravi, 2007; Lin *et al*., 2012) since the seminal work of Altman in 1968. In this context, the focus of this thesis is centred on investigating and developing decision models that are able to classify loan applicants in line with their appropriate class, taking into account all issues emerging throughout the model development process.

It is worth noting that credit-scoring contains two main types (Liu, 2001): the first is *application scoring*, where a score is used to give a decision on new credit application; the second type is *behavioural scoring*, where this type of score is used to deal with existing loan customers. Therefore, the main focus of the thesis is on the first type. The coming subsections will focus on the importance and the need of credit-scoring systems along with its evaluation techniques and approaches adopted by lenders.

2.2.1 The Importance of Credit-Scoring

As is obvious from the previous section, credit-scoring has many forms of definitions, and there is no doubt that it has become an important tool in banking system. With this noted, this section will show how credit-scoring has developed in importance. Banks face a wealth of risks, such as credit, market and operational, etc., and these risks are subject to economic, political and environmental factors or inappropriate policies and regulations. For this purpose, the role of effective management is important to banks and bankers. As credit risk considered the most effective risk on banks' performance, banks should be strict and sound in their credit granting policies in order to minimise risk and accordingly increase profit. The motivation comes here in developing reliable credit-scoring systems for evaluating and discrimination of risk classes.

The quality of credit in banks is a very important issue and, in order to control it efficiently, Basel Committee on Banking Supervision (2000) required banks and financial institutions to use solid credit-scoring systems to help them in estimating degrees of credit risk and different risk exposures, and to improve capital allocation and credit pricing. The Basel Committee, which consists of the Central Bank and other banks from different countries, have formulated a number of guidelines and standards for banks to implement. Credit-scoring is used by banks and financial institutions in order to predict default, make loan decisions and accordingly estimate the probability of default (PD) and exposure at default (EAD), as required by Basel II (BCBS, 2010). In relation to the quick growth of the credit industry, granting loans is one of the significant decisions that needs to be handled in a special way due to the huge demand on loans (Huang *et al*., 2007) by adopting credit-scoring systems credit analysts could reduce the cost of analysis, reduce bad loan risks, and speed-up the evaluation process, observing existing clients' accounts, improvements in cash flow and the collection process (Brill, 1998; West, 2000; Mester, 1997).

In reference to Mester (1997), and in line with the Federal Reserve 1996 option survey, credit-scoring models have been reported as used by approximately 97% of banks in evaluating loan applications, with approximately about 82% of banks using them as a guide to deciding which applicant is eligible for a credit card. According to Lee *et al*. (2002 and West (2000), it has been stated that having a consistent credit-scoring model can lead to great advantages to the bank in terms of cash flow improvement, appropriate credit collection, credit losses reduction, time consumption and the evaluation of the purchase behaviour of current customers. In developing a robust credit-scoring model, economically significant changes will be seen in credit portfolio performance (Blochlinger & Leippold, 2006).

In conclusion, credit-scoring derived its importance from being employed widely to solve or to be an indicator to serious problems. Robust credit-scoring systems could lead to the better estimation of different risks, improved credit management process, enhanced decision-making, and a greater degree of reliability, in addition to being an effective tool for indicating a serious problem that could result in huge financial losses in future, which might end up to business distress or failure.

2.2.2 Credit-Scoring Evaluation Techniques

Banks and financial institution do not grant a loan to anyone who asks for it; rather, an evaluation is conducted to measure the risk level of the applicants and then coming out with a decision to either grant the credit or not. In other words, if the characteristics of new applicants are similar to those of previous applicants (either defaulted or not) and based on their history performance, whether or not a loan can be granted is decided. Generally, the results or scores generated by the scoring systems can be obtained in two ways or approaches, namely judgmental approach and statistical approach (Liu, 2001; Thomas, 2000).

2.2.2.1 Judgmental Scoring Systems

In banking systems, before using numerical scoring systems, the traditional method of the decision of granting credit to customers was originally based on the credit officer judgmental and subjective decision (Hand & Henley, 1997; Thomas, 2000). In addition, these subjective decisions integrate guidelines and other credit rules established by bank policies (Chandler & Coffman, 1979). According to Liu (2001), in judgmental scoring systems, points/weight are assigned to the borrower in terms of his given characteristic; these then are weighted and turned to a score, which decides whether or not to grant a loan. The final decisions are made by credit officer based on his experience, his common sense and simple numerical support. Other helpful tools that can support the subjective decision of the credit officer are the well-known 5Cs, which are useful in determining borrower's creditworthiness, as stated by Thomas (2000), which are: (1) Character (the background and reputation of the borrower); (2) Capital (borrower's contribution to the investment); (3) Collateral (guarantees to back-up the loan in case of default); (4) Capacity (the financial ability of the borrower to pay the loan); and (5) Condition (the overall economy of the borrower).

There are many arguments concerning the efficiency and reliability of judgmental approaches. On the one hand, Capon (1982) states that the importance of judgmental evaluation have been criticised due to some shortcomings, such as the possibility of human error, high costs of training and inconsistency in the application of credit policies across credit officers; therefore, lenders are searching for more computerised ways of completing credit evaluation and decision. Limsombunchai *et al*. (2005) have given the assurance that the results of the judgmental approach are inefficient, inconsistent and lacking in uniformity.

In contrast, Anderson (2007) states that judgmental techniques are still used with lending decisions, based on little or unstructured data and experience. Moreover, Jensen (1992) states why lending institutions have resisted using credit-scoring systems that might be centred on the unwillingness to lose the credit officers with high experience, errors that might exist in the models, and the shortness of quantitative skills in credit management. Moreover, they believe that the credit-granting process does not achieve the level of using statistical systems. However, due to the fast growth of consumer credit industry and the significant amount of data, banks and financial institutions tend to have objective, fast, consistent and uniformed methods to replace or supplement the judgmental methods (Thomas, 2000; Hand & Henley,

1997; Anderson, 2007). These methods are built up using statistical techniques that have been developed year by year.

2.2.2.2 Credit-Scoring Systems

As noted earlier, the intense growth of consumer lending and technology in recent decades has caused banks and financial institutions to upgrade their credit strategies to a level that can cope with this growth, with changes made in the way credit is assessed, with greater engagement in more sophisticated methods that can replace and overcome the shortcomings of their approaches.

Capon (1982), in his paper, drew attention to the importance of using statistical credit-scoring; he showed that the awareness by lenders increased where the use statistical credit-scoring could be the best replacement of the judgmental methods when considering objectivity, consistency and reliability. The widespread use of quantitative and statistical methods did not arise until the development of the necessary computer technology in the early 1960s. This was augmented by economic pressures, which eased as a result of the development of objective credit decisions structure, known as credit-scoring (Thomas, 2000).

Thomas (2000) states that, these days, credit-scoring is based on statistical or operational methods, such as discriminant analysis, logistic regression, decision tress and neural networks. According to Jensen (1992), banks seek to reduce delinquency rates and achieve better control on their credit policies, which eventually leads them to experiment the credit-scoring*: 'They found that credit-scoring provided (1) lower processing costs, (2) more efficient credit control, (3) racially and ethnically non-discriminatory lending, (4) ease in adjusting credit standards, and (5) faster loan approval decisions'*. Along with this, banks experienced an increase in the number of customers without a corresponding increase in delinquency rates. According to Reichert *et al*. (1983), objective techniques can be a good tool for understanding credit evaluation process up to the decision-making stage. On the other hand, albeit the growth of consumer lending, it can be argued that the use of statistical systems in markets with limited number of customers can be costly. Anderson (2007) supported that judgmental evaluation should be used in certain conditions, such as granting a highly valued customer whom can bring high profit, where the statistical scoring system cannot capture customers' information. Mays (2004) argues that costs and benefits generated from using statistical scoring limited from the use of judgmental evaluation in credit

decisions. Credit-scoring techniques (statistical and machine-learning) are discussed in detail with their empirical studies in the coming sections.

2.3 Quantitative Credit-Scoring

The main purpose, when building a credit-scoring model, is to establish the best classification techniques that can discriminate between good and bad credit, and accordingly predict new loan applicants. Credit-scoring models are being applied widely in the area of finance, and in banking in particular. A wide range of classification techniques can be used to build credit-scoring varying from statistical (e.g., LDA, LR and NB) to machine-learning (e.g., NN, SVM) classification techniques. The use of statistical techniques was first introduced to solve a classification problem by Fisher (1936), and it was proposed in building credit-scoring in the late 1960s by the Fair Isaac Company (Thomas, 2000). Since then, statistical techniques have been adopted in developing credit-scoring methods until the appearance of the machine-learning or artificial intelligence (AI) techniques, which was stirred up by evolution of computer technologies. However, these techniques are believed to have better performance than statistical techniques (Desai *et al*., 1996; Huang *et al*., 2004). The main significant difference between statistical methods and machine-learning methods is that statistical techniques focus on analysing existing data and study the relationship between them by making assumptions in order to predict an outcome, whilst machine-learning techniques do not require any assumptions about data and focus by constructing systems that can acquire knowledge directly from the available data (Huang *et al*., 2004; Ratner, 2012).

Regardless of the variety differences in the methods and techniques used, the main purpose is to build a model that can predict borrower loan applications, and classify and measure borrowers' repayment behaviour accurately (Lee *et al*., 2002; Thomas, 2000). In this section, the mostly used state-of-the art statistical and machine-learning classification techniques that are relevant to this thesis and used to develop credit-scoring models are summarised.

2.3.1 LDA

LDA is a parametric statistical method first proposed by Fisher (1936) in order to classify and disseminate between two objects in a population. Moreover, in the field of credit-scoring, LDA was first recognised and proposed by Durand (1941) to measure the good credit payments. Moreover, LDA was one of the first methods used in developing credit-scoring models (Thomas, 2000; West, 2000).

Practically, in a credit-scoring classification problem, assume there is a dataset of n customers, where each customer in the dataset has certain m characteristics or variables $x = (x_1, x_2, \ldots, x_m)$ that are used to classify the customers to their appropriate class or group y (good/0 or bad/1 loans). Within the dataset customers are categorised in n_g which is the group of customers with good loan status and n_b which are the group of customers with bad loan status. The objective of LDA is to estimate the probability of a customer as either good or bad loan group $p(y|x)$ given a vector of its characteristics, features or variables x. LDA proposes separating the objects by linearly combining their independent variables in order classify the objects in its appropriate groups or classes (Lee *et al*., 2002). The linear combination of discriminate analysis can be expressed as:

$$Z = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_n X_n. \quad (2.2)$$

where Z represents the discriminate score, β_0 is the intercept term, and βi represent the coefficient or weights related to the variables x_i $(i=1, 2 \ldots n)$. The above equation constructs the discriminate model, which helps in predicting and classifying the customer's loan to the suitable group of credit. In LDA model, data are necessary to be independent, distributed normally, variance and the distribution of good and bad loans should be homogenous and equal (West, 2000; Lee *et al*., 2002). The values of coefficients $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ are adjusted based on the covariance of matrix and the mean feature vectors of the two group of loans. After attaining the values of the coefficient vector, the discriminate score can be calculated. Finally, when a new loan is received, it is classified by projecting it onto the maximally separating direction represented by the function $Z = \beta^T x + \beta_0$. Classification is achieved by comparing $\beta^T x$ to a threshold T_c. If $\beta^T x < T_c$. The loan then is a good loan, otherwise it's a bad loan. The threshold is chosen based on the prior probabilities of the loan customers in each loan group. The classification problem lies in finding coefficient values that can maximise the distance between the good and bad loans and minimise it within the good and bad loans. The goal is to select a projection that best separates the two loan classes.

LDA is considered a simple method in classifying linear data, but one weakness could occur when the data are non-linearly related, as LDA assumes the relationship between variables are linear, and for that reason, LDA might be considered as having a lack of Accuracy when dealing with non-linear data (Sustersic *et al*., 2009). Other limitations of LDA are improper grouping definitions, estimation of classification errors and improper prior probabilities (Eisenbeis, 1978; Rosenberg & Gleit, 2004). However, LDA has been widely adopted in building scoring models (Reichert *et al*., 1983; Boyes *et al*., 1989; Deasi *et al*., 1996; Lee *et al*, 2002).

2.3.2 LR

LR is another broadly used statistical technique and the most popular tool for classification problems. In general, LDA and regression models (e.g., linear regression) gives a continuous output that ranges from [-∞, +∞] by linear combining the independent variables. In credit-scoring, the classification is a binary or dichotomous problem in which the decision is characterised by 0 (grant the loan) or 1 (reject the loan) (Thomas, 2000). For this reason, LR was developed to address this issue by reducing the output to either 0 (good loan) or 1 (bad loan). LR studies the relationship between several independent variables and the probability of a loan being granted by fitting them to a logistic curve (Sweet & Martin, 1999). As shown in Figure. 2.2, the LR model outcome is either a good loan (0) or bad loan (1). In this context, the classification cannot be modelled using a linear relationship as it must be estimated at a continuous level. However, LR will construct an 's-shape' logistic curve, where the values are between 0 and 1; this curve expresses the relationship between the independent variables and the probability of a binary outcome of interest, using non-linear function of independent variables, as in Equation (2.3).

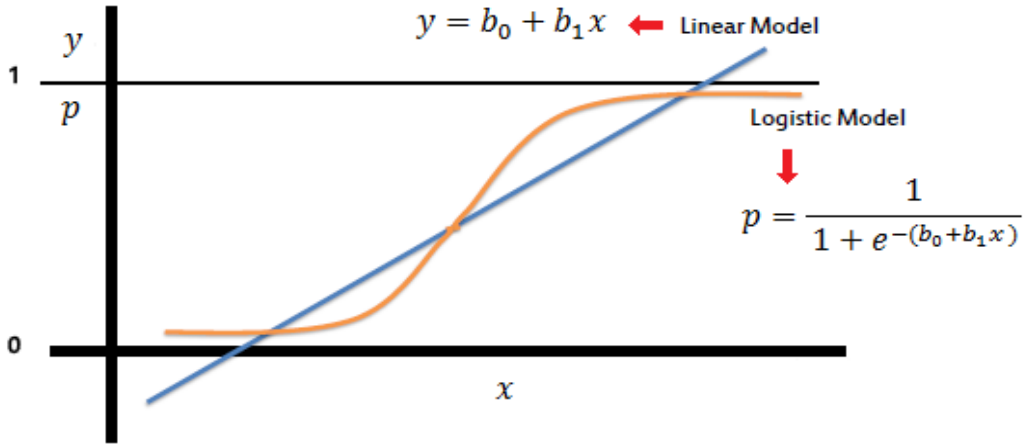

Figure 2.2 Logistic curve (Sayad, 2010)

$$p = 1/\,1+e^{-(\beta_o+\beta_1 X)} \tag{2.3}$$

where p is the probability of the interest outcome, β_0 is the intercept term and β_1 represent the coefficient related to the variables X. In the context of credit-scoring, if the probability of having a good loan is p, then the probability of bad loan is $(1-p)$. This is concept is known as *odds*, which is calculated as the ratio of probability of having good loan relative to the probability of not having a good loan. Odds can be expressed as follows:

$$Odds = p\,/\,1-p \tag{2.4}$$

Intuitively, it's more relevant to take the odds of event happening than take its probability. Hence, the curve equation can be described in terms of odds ratio as:

$$(p\,/\,1-p) = exp^{\beta_o+\beta_1 X} \tag{2.5}$$

According to Thomas (2000), a distraction in the above equation is that the right-hand side of the equation can take any value whilst the left-hand side takes only values between 0 and 1. In order to solve this distraction, the natural log of both side of the equation can be taken to have the following transformation in term of logit (log-odds):

$$Logit\,(y) = ln\,[p_g/(1-p_g)] = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots. + \beta_n X_n. \tag{2.6}$$

where $ln\,[p\,/(1-p)]$ is the dependent variable or the credit decision which is the log odds of having a loan as good or bad. The logit function transforms the non-linear 's-curve' to

approximately straight line and to change the range of the proportion from 0–1 to $-\infty$ to $+\infty$ (Bewick *et al*., 2005). The intercept and coefficients of the logit function are estimated using the maximum likelihood function, in which it is an iterative process that is responsible in finding the best values that raise log likelihood (Bewick *et al*., 2005; Kleinbaum & Klein, 2010). Once knowing the odds of the logit function, the probability can be known and hence the outcome between 0 and 1 can be achieved.

$$P = odds / 1+odds \tag{2.7}$$

Unlike LDA, LR does not require multivariate normal distribution of data. However, the LR is exposed to a full linear relationship between the independent variable when related to the logit of dependent variable (Lee & Chen, 2005). Moreover, if data are non-linear, the Accuracy of LR decreases (Ong *et al*., 2005). Furthermore, according to West (2000), Thomas (2000) and Akkoc (2012), LR and LDA are designed for problems where the relationship between variables is linear; therefore, this might lead to deficiency in models' predictive performance.

Despite the limitations, LR is an easy technique and is more suitable for credit-scoring than LDA (Press & Wilson, 1978) and is recognised as the industry standard for building credit-scoring models (Lessmann *et al*., 2015); this has been widely adopted in the literature (Wiginton, 1980; Steenackers & Goovaerts, 1989; Hand & Henley, 1996; Desai & Crook, 1996; Baesens *et al*., 2003; Deasi *et al*., 2009; Crook *et al*., 2007).

2.3.3 DT

DT is another commonly used approach for classification purposes in credit-scoring applications. It is also well-known as recursive partitioning method (Hand & Henley, 1997) and classification and regression trees' CART (Breiman *et al*,, 1984; Lee *et al*., 2006; Zekic-Susac *et al*., 2004). DT are non-parametric classification techniques used to analyse dependant variables as a function of independent variables (Arminger *et al*., 1997; Lee *et al*., 2006). As shown in Figure 2.3, DT use graphical tools; the node is shown in the box with lines to show the possible events and consequences, until reaching the best and optimal outcome. The idea behind the DT in credit-scoring is to provide a classification between two classes of credit 'good' and 'bad' loans. It begins with a root node that contain the two types of class, with the node then being split into two subsets with possible events based on the

chosen variable or attribute, and so are the nodes, until the decision algorithm loops on all the splits to find the optimal split and select the wining sub-tree which gives the best partitioning of mostly 'good' and 'bad credit based on its overall error rate and lowest misclassification cost (Biermann *et al*., 1984; Thomas, 2000).

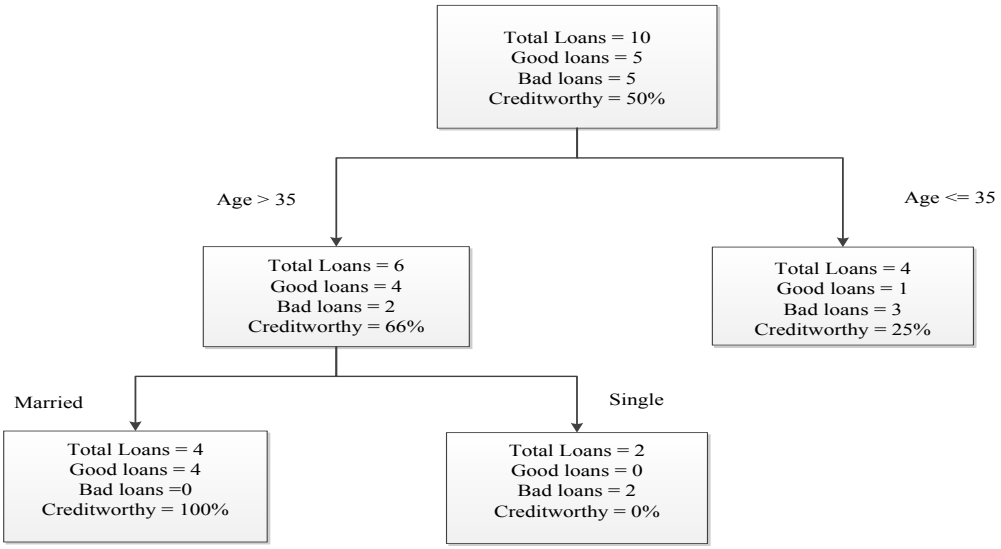

Figure 2.3 Example of DT for credit-scoring

A problem seen to arise in building DT is the goodness of split, which is what variable to choose to grow the tree that could discriminate between classes (Mingers, 1989). In general, the selection of the variable is based on the purity of the split, where several approaches are available for splitting (Berzal *et al*., 2002), where the common one used is the information entropy, which is based on the highest information gained from each attribute (Berzal *et al*., 2002; Osei-Bryson, 2006). After finishing building the DT, some sections of the tree with less predictive ability can be removed or pruned; this can reduce the complexity of the tree, hence achieving better Accuracy by reducing overfitting (Mansour, 1997). Some pros of DTs are: 1) Easy and interpretable; 2) Handles missing values; 3) They hold nominal and numeric attributes; and 4) They don't make assumptions about data distribution. Some cons include the fact that DTs are: 1) sensitive to irrelevant attributes and noise; 2) DTs take a great deal of effort in handling missing values (Rokach & Maimon, 2008); and 3) DT lacks robustness and performance optimality (Lariviere & Van den poel, 2005).

Makowski (1985) was the first to introduce DTs in credit-scoring, and then it was investigated in building credit-scoring models in various contexts, such as credit cards,

consumer loans, business failure and corporate loans (Srinivasan & Kim, 1987; McKee & Greenstein, 2000). On the other hand, different forms of DT models were used as a comparison with other techniques, such as LDA and LR: for example, in Lee *et al*. (2006), Henley & Hand (1996) and Yeh & Lien (2009). DT was superior to LDA and LR, whilst it was inferior in other works (West, 2000; Baesens *et al*., 2003; Zhou *et al*., 2009).

2.3.4 NB

NB classifiers are statistical classifiers that predict a particular class (good or bad) loan. Bayesian classification is based on Bayesian theory and is a valuable and helpful measure when the input feature space is high (Bishop, 2006). This is considered a very simple method for making classification rules that are more accurate than those made from other methods; therefore, very little attention and focus was assigned to the credit-scoring domain (Antonakis & Sfakianakis, 2009). NB is calculated using the posterior probability of a class by multiplying the prior probability of a class before seeing any data with the likelihood of the data given its class: for example, in the credit-scoring context, the assumption can be made that the training sample set $D = \{ x_1, x_2, ...,x_n\}$, where each x is made up of n characteristics or attributes $\{ x_{11}, x_{12}, ..., x_{1n}\}$ and assisted with a class label c either good or bad loan. The task of the NB classifier is centred on analysing these training set instances and determining a mapping function $f: (x_{11}, ...,x_{1n}\} \rightarrow (c)$, which can decide the label of an unknown example $x = (x_1, ..., x_n)$:

$$P (c_i| x_1, ..., x_n) = P (x_1, ..., x_n | c_i)* P (c_i)/ P(x_1, ..., x_n) \quad (2.8)$$

where $P (c| x_1, ..., x_n)$ is the posterior probability of class c after seeing data x. $P (x_1, ..., x_n | c_i)$ is the likelihood of the data x belonging class c_i. $P (c_i)$ is the prior probability of class c_i before seeing any data and finally $P(x_1, ..., x_n)$ is the probability of data on class c_i .Bayesian classifiers choose the class that has the greatest posterior probability $P(c_i |x_1 ... x_n)$ as the class label, according to minimum error probability criterion or maximum posterior probability criterion. That is, if $P (c_i | x) = max P(c_i | x)$, then assigning x to a particular class c_i can be determined. For the purpose of structure simplicity, the NB classifier (Bishop, 2006) strongly assumes that the variables of data are independent and not correlated; however, this is considered a weakness since dependences between variables can exist (Antonakis & Sfakianakis, 2009). Some advantages of NB classifiers include that they are easy to explain and understand; it is fast to train and classify; resistant to irrelevant attributes and outliers

(Kohavi, 1996; Oded & Rokash, 2005). Regardless, the advantages of NB include good performance n many classification tasks but poor performance in others (Ratanamahatana & Gunopulos, 2003). However, NB has been investigated in some credit-scoring studies and has not shown valuable performance compared to other methods (e.g., Baesens *et al*., 2003; Yeh & Lien, 2009; Antonakis & Sfakianakis, 2009).

2.3.5 MARS

It is a non-parametric and non-linear regression technique developed by Friedman (1991), which models the complex relationships between the independent input variables and the dependent target variable. Multivariate Adaptive Regression Splines (MARS) is built using piece-wise linear regression by modelling a sequence of linear regressions on different intervals or splines (Briand *et al*., 2004). Each spline should be specified by finding the suitable independent variable to fit.

Figure.2.4 demonstrates how MARS fit data onto each linear regression spline. As can be seen, each spline or interval is separated by what is referred to as a knot, which indicates the end of an interval and the beginning of another one. This also indicates changing in the behaviour of the dependent variable *y* or, in other words, knots are the place where the behaviour of linear regression function change.

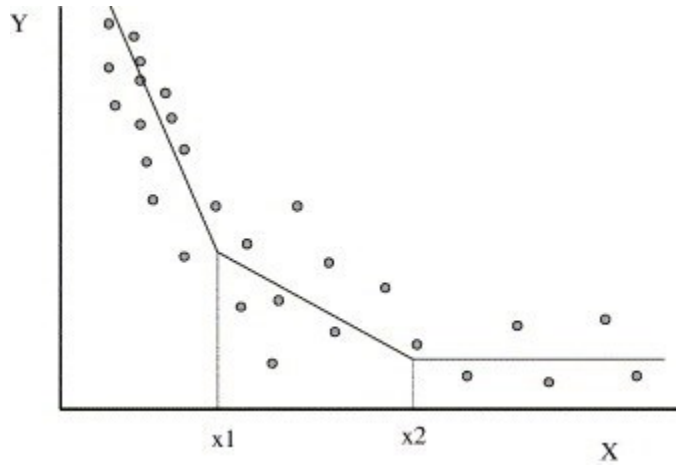

Figure 2.4 MARS example on linear regression intervals and knot locations (Briand *et al*., 2004)

In the figure, knots are labelled by x_1 and x_2, identifying three different linear relationships on each interval or spline. In MARS, identifying and determining the knots' locations is done by a searching mechanism. MARS built a model in the form of:

$$y = c_0 + \sum_{i=1}^{k} c_i * B_i(x) \quad (2.9)$$

where c_0 is a constant coefficient, $B_i(x)$ is the basis functions and c_i is a coefficient for the basis functions. MARS use what is referred to as the basis function, taking numerous forms of independent variables interactions. The common functions used are the hinge functions that are used to find variables, which are selected as knots; hence, the function takes the following form (Miguéis *et al*., 2013):

$$max\ (0, X - c) \text{ or,} \quad (2.10)$$

$$max\ (0, c - X) \quad (2.11)$$

where c is constant, threshold or knot location, X is the independent variable. The goal behind the basis functions is to transform the independent variable X in to a new variable (e.g., X*). Based on equations 2.10 and 2.11 X* will take the value of X if X is greater than c and it will take value of zero if the value of X is less than c (Briand *et al*., 2004). The knots' locations c is determined by forward/backward stepwise criteria, where many basis functions create many knots, allowing interactions between different independent variables. Later, these knots are pruned based on their contribution to the overall fit of the model (Lee *et al*., 2006; Miguéis *et al*., 2013). An important advantage of MARS is that it can identify the importance of each independent variable to the dependent variable when various possible independent variables are measured (Lee *et al*., 2006). MARS refits the model after removing all terms involving the variable to be assessed and accordingly calculates the reduction in model error, with all variables categorised according to their influence on the performance of the model; the optimal MARS model is based on the lowest generalised cross-validation (GCV) measure (Briand *et al*., 2004; Miguéis *et al*., 2013) (for more insight of the MARS modelling, refer to Friedman (1991) and Hastie *et al*. (2001)). Therefore, in the context of credit-scoring, this feature of MARS is exploited in building credit-scoring model, as it is mostly utilised to obtain a subset of the most significant variables based on their relative importance on the model and use it as an inputs for the ordinary model (Lee *et al*., 2005; Lee *et al*., 2006; Chen *et al*., 2009; Chuang & Lin, 2009). A strong advantage of MARS is that it is easy to understand the model and it does not require a long training process (Lee *et al*., 2005).

2.3.6 NN

NN are machine-learning systems based on the concept of Artificial Intelligence (AI) inspired by the design of the biological neuron (Haykin, 1999). NN models are modelled in such a way to be able to mimic the human brain functions in terms of capturing complex relationships between the inputs and outputs (Bhattacharyya and Maulik, 2013). In credit-scoring context, NNs has been identified as an effective tool used in building credit-scoring models as an alternative to the statistical techniques such as LDA and LR (Atiya, 2001; Angelini, 2008). One of the most common architectures for NNs is the multi-layer perceptron (MLP), which consists of one input layer, one or more hidden layers, and one output layer (Jensen, 1992). According to Angelini (2008), one of the key issues needing to be addressed in building NNs is their topology, structure and learning algorithm. The most commonly utilised topology of NNs model in credit-scoring is the three-layer feed-forward back propagation (FFBP). Figure 2.5 illustrates the architecture of common topology of NN based on interconnected three layers FFBP.

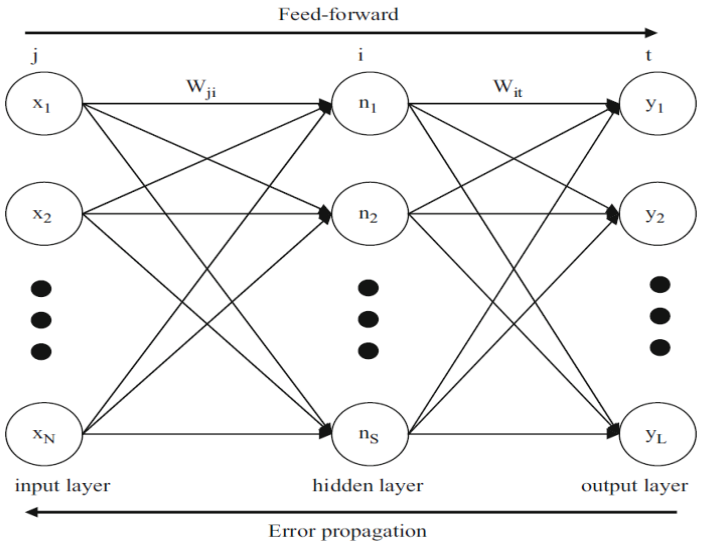

Figure 2.5 The topology of back propagation (FFBP) NN (Al-Hnaity *et al*., 2015)

Consider the input of credit-scoring training set $x = \{x_1, x_2, \ldots, x_n\}$, the NN model works in one direction, starting from feeding the data x to input layer (x includes the customer's attribute or characteristics). These inputs then are sent to a hidden layer through links or synapsis, associated with random initial weight for every input. The hidden layer will process what it has received from the input layer and accordingly will apply it to an activation function. The results are served as a weighted input to the output layer, which will further

process weighted inputs and apply activation function, which will lead to a final decision. The following steps the steps illustrate the training computations of the NN, all neurons in the hidden layers are calculated as follows:

$$n_i = f_H(\sum_{j=1}^{N} x_j.w_{ji}),\ i = 1,2,.....,S \quad (2.12)$$

where n_i is the output of the hidden layer, f_H is the activation function (threshold) applied on the value of each node in the hidden layer, activation function can take many forms the most common one is the sigmoid function. Hence, the output of the hidden layer is passed with other weights to the output layer, which is computed as follows:

$$y_t = f_t(\sum_{i=1}^{S} n_i.w_{it}),\ t = 1,2,.....,L \quad (2.13)$$

where y_t is the output of the final decision after applying the activation function (threshold) on the value of each node in the output layer. If the difference between final decision and the actual target is significant, the back propagation learning algorithm will revert and update the weights between hidden layer and output layer, and the weights between inputs and hidden layer, with computation completed again until the difference between, with the NN computed again until the final decision and the actual target is minimised. Building an NN model involves training procedure for the variables in order to differentiate between that to get a better decision and results. However, if the results are improper, the estimated values are changed by the NN model until they become proper and acceptable (Abdou & Pointon, 2011).

The NN tend to establish the relationship between a customer's probability of default and their given characteristics, which then are filtered, and with the most important prediction variables amongst them identified. The main advantages of NNs models is that it can handle incomplete, missing or noisy data and requires no prior assumptions relating to the distribution of the variables (Vellido *et al*., 1999). Moreover, it has the ability to recognise complex patterns between input and output variables and accordingly predict the outcome of new independent input data (Keramati & Youssefi, 2010). On the other hand, NNs are criticised and lack explanatory capability, such that it cannot give an explanation or justification as to how customers are chosen as either 'good' or 'bad', accepted or rejected. Another shortcoming in NN is the selection of parameters as there is no official method to select the appropriate parameters for the model, which might affect its prediction Accuracy

(Vellido *et al*., 1999; Lahsasna *et al*., 2010). Several studies have proven that NNs outperformed statistical techniques (e.g., LDA, LR, NB and MARS) in prediction Accuracy (West, 2000; Baesens *et al*., 2003; Desai *et al*., 1996; Malhorta & Malhorta, 2003; Lee & Chen, 2005; Abdou *et al*., 2008).

2.3.7 SVM

SVM is another powerful machine-learning technique used in classification and credit-scoring problems. It is being widely used in the area of credit-scoring and other fields for its superior results (Huang *et al*., 2007; Lahsasna *et al*., 2010). SVMs first were proposed by Cortes & Vapnik (1995), adopting the form of a linear classifier. SVMs take a set of two classes of given inputs and predict them in order to determine which of the likely two classes have the output. SVMs are used for binary classification in order to make a finest hyperplane (Line) that categorize the input data in two classes (good and bad credit) (Li *et al*., 2006). Figure 2.6 shows the foundation of SVM.

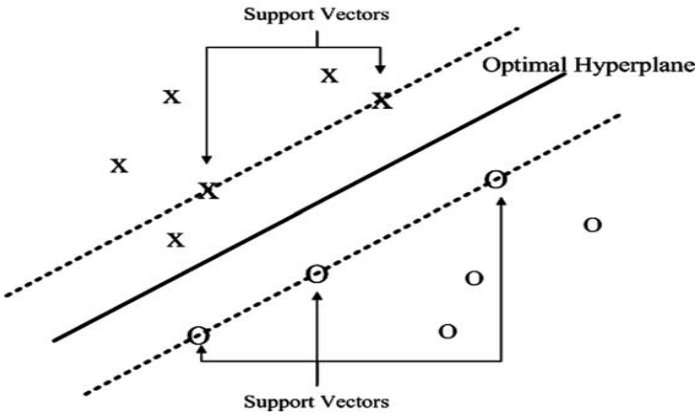

Figure 2.6 The basis of SVM (Li *et al*., 2006)

According to Figure 2.6, an input training vector should be assumed (x_1,y_1), (x_2,y_2),…, (x_n,y_n) exists, where $x \in R^d$, which is a vector in d-dimensional feature space and $y_i \in \{1,+1\}$ is the class label. Next step is to build a dividing hyperplane, which could be described like:

$$w \cdot x - b = 0 \tag{2.14}$$

Vector w is orthogonal to diving hyperplane, and parameter b/w is equal to the distance from the dividing hyperplane to the centre of coordinates. Since the goal is to find for the optimal separation, finding bearing vectors and hyperplanes is needed. However, they can be represented with these equations:

$$w.x - b = 1 \quad (2.15)$$

$$w.x - b = -1 \quad (2.16)$$

where 1 and –1 are coefficients dependent on the normalising. If the data is linearly separable, than the only task left is to find w and b . In such a case that data are non-linear, other types would be proposed in order to improve the Accuracy of the original model. The main difference of the new model, compared to the initial model, is the function used to map the data into a higher dimensional space. New functions were proposed, namely linear, polynomial, radial basis function (RBF) and sigmoid.

SVMs map non-linear data of two classes to a high-dimensional feature space, with a linear model then used to implement the non-linear classes. The linear model in the new space will denote the non-linear decision margin in the original space. Subsequently, SVMs will construct an optimal line (hyperplane) that could perfectly separate the two classes in the space. The advantages of the SVM lie in its capability in model non-linear data, which delivers high prediction Accuracy compared with other techniques. The disadvantages of SVM include that it is difficult to understand, which might lead to hesitation in its use (Martens *et al*., 2007); many studies, on the other hand, have adopted SVMs in building credit-scoring models (Zhou *et al*., 2009; Bellotti & Crook *et al*., 2009; Li *et al*., 2006; Huang *et al*., 2007; Chen & Li, 2009).

2.3.8 RF

RF is considered an advanced technique of DTs, as proposed by Biermann (2001), which consists of a bunch of DTs that are created by generating *n* subsets from the main dataset, with each subset a DT created based on randomly selected variables, which is why it is referred to as RF, since a very large number of trees are generated. After all DTs are generated and trained, the final decision class is based on voting procedure, where the most popular class determined by each tree is selected as a final class for the RF. Figure 2.7 illustrates the RF architecture.

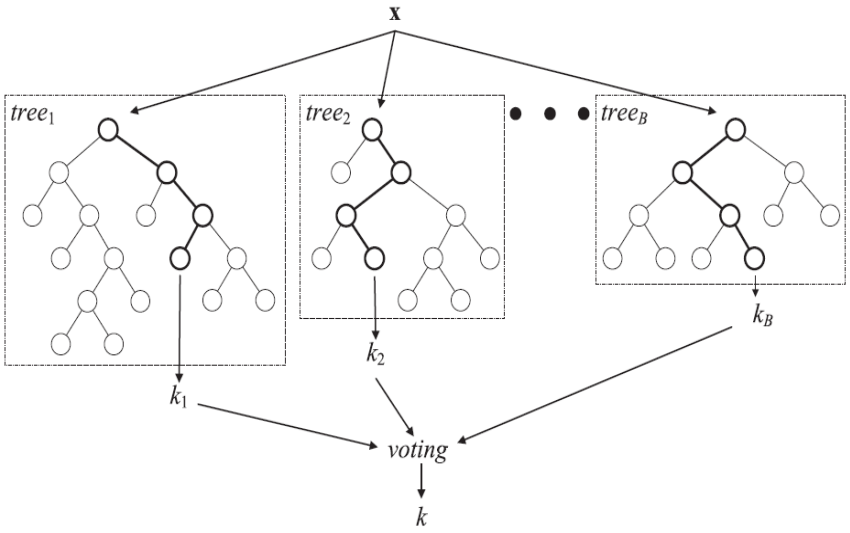

Figure 2.7 RF architecture (Verikas *et al*, 2011)

Regardless of the simplicity of DTs, it has disadvantages that have been addressed by researchers focused on optimising the DT technique, where such efforts lead to the developing of RF (Biermann, 2001; Lariviere & Van den poel, 2005). The main two parameters to be set when building an RF are the number of tress and the number of variables or features needing to grow each DT (Brown & Mues, 2012). According to Breiman (2001), the process of building an RF starts by:

- Creating random bootstraps from the training set with replacement.
- On each bootstrap, building several DTs by selecting random variables from each subset.
- The starting node of each DT is selected with the high information gain.
- The tree will grow up until no more nodes can give more information about the class, there is no pruning.
- A voting procedure then will take place between the several DTs leading to the final class decision of the RF.

According to Miner *et al*. (2009), RFs have many remarkable advantages: 1) it gives relatively high Accuracy amongst other classification techniques; 2) It is robust in handling missing data; 3) It can handle any size data and variables; 4) It can detect the variables with high influence on the classification results; and 5) It is fast to run and easy to use. A limitation in RF is that it cannot handle very large numbers of irrelevant attributes (Grosan & Abraham, 2011).

RF has been afforded much attention since its introduction and has become a very popular technique in classification and other measures, and has been applied in many fields, such as bioinformatics, image processing, chemistry and credit card fraud detection (Verikas *et al*., 2011). However, in the credit-scoring context, RF has not been investigated widely, yet it has shown good classification results (Brown & Mues, 2012; Wang *et al*., 2012; Lessmann *et al*., 2015).

2.4 Credit-Scoring Modelling Approaches

In the previous section, the state-of-the art quantitative approaches from statistical to machine-learning techniques that are mostly used in developing credit-scoring models were overviewed. The reliability of the credit-scoring model is measured in terms of its accuracy. In this section, the approaches and the ways of utilising these techniques in building credit-scoring models is covered.

2.4.1 Individual Classifiers

In order to develop a credit-scoring model, a classifier should be selected to fulfil this task. However, many techniques have been proposed during the last decades, the aim of which was to build a good performing credit-scoring model that serves the purpose for which it was built. The selection of a classifier that could do its best is based on the nature of the problem, dataset size, variables used and the market environment for which it is developed (Hand & Henley, 1997). The early credit-scoring models started using statistical techniques, such as LDA and LG as common methods; however, they have shown various shortcomings that lead to being replaced by other new machine-learning techniques that produce more rigorous models, such as NN, SVM and DTs. Several studies have investigated the use of individual classifiers in constructing credit-scoring models for example (Desai *et al*., 1997; Malhotra & Malhotra, 2003; Zekic-Susac *et al*., 2004; Lee *et al*., 2006).

According to Khashei *et al*. (2009), using individual classifiers in building credit-scoring models might give accurate and precise models, but there are some negatives for both of them; they are considered to use classical logic process in their modelling, they cannot efficiently model complexities and uncertainties. For this reason, researchers have directed their efforts towards finding different ways and approaches to utilising these techniques to get more reliable, credible and accurate results.

2.4.2 Hybrid Techniques

Hybridisation or hybrid models are a new method in building credit-scoring model and have been widely adopted in the literature. The concept of hybrid modelling is to combine one or more classifiers together in order to give higher prediction Accuracy (Khashei *et al*., 2013). The main purpose of hybrid models is to condense the risk of choosing an improper model when solving a problem. However, the basic idea of combining several classifiers is to get benefits of the unique features of each classifier in order to capture different patterns of the characteristics of the dataset, which will lead to an efficient credit-scoring model with highly improved results. Every individual classifier has strong and weak points; however, the notion of hybrid techniques is to overcome weaknesses and exploit strengths of each classifier by integrating techniques together (Khashei *et al*., 2012; Sadatrasoul *et al*., 2013).

The idea behind a hybrid model is to combine two models or techniques in sequential process where one model is trained to serve as an input for the other model to perform the classification task. According to Li & Zhong (2012), there is no clear solution on how to classify hybrid models; hence, in their study, Tsai & Chen (2010) divided the hybrid models structure by combing classification techniques and clustering techniques through all possible ways.

The main aim of the first classifier is either to be trained or pre-processed. Its output then is fused to train the next classifier or clustering technique. The simplest way of building a hybrid model is through three steps: 1) Feature selection as pre-processing step; 2) Model parameters selection; and 3) Classification (Li & Zhong, 2012). The pre-processing step comprises feature selection that selects the most significant features as a new feature subset to the new classifier. Moreover, data pre-processing can also comprise feature extraction, clustering and data-filtering (for more details refer to Verikas *et al*. (2010) and Garcia *et al*. (2012)). More about the data pre-processing step is discussed in detail in the next chapter.

2.4.3 Ensemble Techniques

Alongside the hybrid techniques, another method used by researchers is the ensemble learning method or multiple classifier system, which have become the most recent methods in credit-scoring evaluation (Lin & Zhong, 2012). These techniques have widely attracted the attention of researchers in the field of credit-scoring over the last decade, with many having combined multiple classifier systems in different ways in order to achieve high prediction performance classifiers (Tsai, 2014; Nanni & Lumini, 2009). The ensemble method is an approach that applies multi-classifiers rather than single classifiers in order to achieve higher Accuracy results. Moreover, the difference between ensemble and hybrid methods is that the ensemble output of the multiple classifiers is pooled to give a decision whilst, in hybrid methods, only one classifier gives the final output, whereas the other classifier results are processed as an input to the final classifier (Verikas *et al*., 2010). A central key issue in building an ensemble classifier is to make each single classifier different from the other classifiers as possible; in other words, to be as diverse as possible (Nanni & Lumini, 2009).

The ensemble method in building the credit-scoring models is valued due to its ability to outperform the best single classifier's performance (Kittler *et al*., 1998). The basic idea of ensemble classifiers is to generate multi-classifiers and accordingly combine their information in order to reduce the variance of single classifier estimation errors and increase the overall classification outcomes. In addition, the ensemble method seeks to construct a set of assumptions and combine them to establish the desired results—not as a single classifier, which only learns one assumption from the dataset (Wang & Ma, 2012). Practically, building an ensemble of classifiers involves three main steps: 1) System topology; 2) Classifier generation; and 3) Classifier fusion or combination (Zhang & Duin, 2010; Wozniak *et al*., 2014). Firstly, the topology of building the ensemble model can take two structures: the parallel and serial structures. The parallel structure is the most commonly used structure in building ensemble models in the literature; this is based on training the individual classifiers on the same input data. The serial ensemble structure involves the individual classifiers being applied in sequence order, so that the output of the first classifier is used as new data to the next classifier, classifier after classifier, until the classifier is confident about its output (Wozniak *et al*., 2014). The parallel ensemble structure is adopted in this thesis.

The second step includes the generation of the classifier; practically, there are two common ways of generating an ensemble of classifiers. The first is to build the classifiers using the

same type of learning algorithm that could be with different parameters; this way is homogenous classifier. The other way is to build the ensemble using different types of learning algorithm (Zhang & Duin, 2011). Alternatively, different classification algorithms can be applied to make the ensemble, with the idea being that the different classifiers have different views on the same data and can complement one another (Lessmann *et al*., 2015; Zhang & Duin, 2011). However, these generated classifiers, in either way, are trained on different data or different features of the training dataset. That is, different classifiers are trained on diverse parts of data, meaning each trained classifier will generalise in different ways (Tsai & Wu, 2008; Zhang & Duin, 2011). The most popular approaches for modifying training data are cross-validation, bagging and boosting (West *et al*., 2005).

After all classifiers give their decisions, they are pooled in order to combine and fuse them by some rule or method, here comes the third step. Usually, when combining ensemble outputs two approaches are utilised namely classifier fusion and classifier selection (Canuto *et al*., 2007; Kheradpisheh *et al*., 2013). The classifier fusion considers combining all classifiers predictions that are trained to the whole problem, with a combination rule accordingly applied to combine them. Some common ways of combining classifiers are majority voting (MajVot), weighted average (WAVG), weighted voting (WVOT), mean (AVG), maximum rule (MAX), minimum rule (MIN) and product rule (PROD) (Canuto *et al*., 2007; Zhang & Duin, 2011; Tsai, 2014); besides, these methods also are considered as fixed or static combiners (Zhang & Duin, 2011; Xiao *et al*., 2012). These fusion methods could be the best option for combining multiple classifiers owing to their simplicity and good performance (Zhang & Duin, 2011). On the other hand, classifiers ensemble selection is based on selecting the classifiers that do well on each input data; classifier selection that best fit each input data is made during running phase. A popular method used in this matter is the dynamic classifier selection based on local Accuracy (DCS-LA) where the output of each data is based on its local Accuracy (Woods *et al*., 1997; Canuto *et al*., 2007; Zhang and Duin, 2011; Xiao *et al*., 2012) (For further reading about classifier selection combination methods, please refer to Canuto *et al*. (2007) and Zhang & Duin (2011).

2.5 Model Performance and Significance Measures

After finishing from training the model, it must be validated for its efficiency by applying it to an unseen sample; however, there exist many performance indicators that can designate the efficiency of the developed model. According to Hand (1997), a range of performance measures indicators are available. The most common performance measures adopted in credit-scoring is the Accuracy rate of the model, which shows how many instances where classified correctly. Another measure is the error rate of the model which is 1 – Accuracy; this indicates how many instances are misclassified by the model. However, as the problem in credit-scoring is a binary class problem the decisions made by the model can take the form of a 2×2 confusion matrix as shown in Table 2.1.

	Predicted		
Actual	**Good loans**	**Bad loans**	**Accuracy**
Good applicant	TP	FN (Type II error)	$Accuracy_{good}$
Bad applicant	FP (Type I error)	TN	$Accuracy_{bad}$
	PPV	NPV	$Accuracy_{total}$

Table 2.1 Confusion Matrix for Credit-scoring

It is clear from the above table that loans are in two groups good and bad loan applicant: if the model correctly detected the good loans and the bad loans then it is a TP (true positive) and TN (true negative), respectively, otherwise it is an FN (Type I error) and an FP (Type II error), respectively. The $Accuracy_{good}$ and $Accuracy_{bad}$ indicates the model's accuracy in identifying good and bad loans, whilst the $Accuracy_{total}$ shows the overall Accuracy of the model and how well it performs. Besides the predicted positive value (PPV) and negative predicted value (NPV), the percentages of how many loans were predicted as good and bad loans by the model correspondingly. In other words, it indicates how many applicants are given a loan. Another common measure also used in the area of credit-scoring is the receiving operating characteristic curve (ROC), which is a measure indicating the classification performance of a model through a range of several thresholds, unlike the Accuracy, which the performance of the model is based on pre-determined single threshold. Another measure, known as the area under curve (AUC), is used for comparison across several classifiers. Other uncommon measures are H-measure, Brier score, and Gini coefficient, which also are

addressed in credit-scoring (Lessmann *et al*., 2013). The purpose of investigating different performance measures is centred on assessing the model from different aspects (Lessmann *et al*., 2013).

Another important phase of investigating the efficiency of a credit-scoring model is that it is robust, and its performance is not related to any factors associated during the development process of building the model (Garcia *et al*., 2015). However, in order to achieve this, some kind of hypothesis-testing should be carried out so as to assure that the model performance is not a matter of luck. Many tests' parametrical and non-parametrical significant statistical tests can be utilised to validate the results of the model, such as in the case of the t-test, Friedman rank test and Bonferroni-Dunn test. A greater insight into performance indicator measures and statistical significance tests are discussed in the next chapter.

2.6 Credit-scoring Studies

The area of credit-scoring has been a widely investigated topic in the literature for the last five decades. Significant studies have been adopted on assessing the performance of individual credit customers and corporate credit customers, which also are known as bankruptcy prediction. This thesis will be strict on the area of dealing with individual credit customers and, specifically, on the quantitative approaches used in developing credit-scoring models for these customers. Many approaches have been utilised, from statistical to machine-learning techniques. Such methods were used in different ways, from implementing single classifiers, hybrid models and ensemble models and the main aim were to achieve a reliable and efficient model that can serve the purpose it was developed for. Since adopting the simple statistical approaches such as LDA and LR to score and assess customers' creditworthiness, these approaches have been widely used till the emerging of machine-learning techniques that were believed to fill the shortcomings of the statistical techniques.

In practice, real historical datasets are used in order to develop credit-scoring models; these datasets might differ in size, nature, and the information or characteristics it holds, whilst individual classifiers might not be able to capture different relationships of these datasets characteristics. As a result, researchers have employed hybrid modelling techniques that can exploit the strength and compensate weaknesses of different classifiers in learning the relationships between data. From hybrid-modelling, researchers have inspired the ensemble modelling, which gives classifiers the opportunity express their ability to learn data on

different parts of data and feature space. Almost all studies reported that hybrid and ensemble modelling is superior to individual classifiers; for this reason, this thesis will focus on the studies that applied hybrid and ensemble techniques in the field of customer credit-scoring.

Prior to starting the representing and summarising of related studies, the mechanism of how the related work was collected is explained.

2.6.1 Literature Review Collection Process

The area of credit-scoring is an important topic in finance and data classification, and it has been expected to have received much focus through the completion of a large number of studies related to the development of credit-scoring models. Firstly, the searching process for the relevant work started with the typing the keywords 'credit-scoring', 'credit customer classification', 'hybrid models' and 'ensemble models' in the relevant fields, supported by five most widely known and used academic science databases, namely 'Google Scholar', 'Science Direct', 'IEEE Xplore' and 'Springer'. The search provided results centred on topics related to credit-scoring, credit risk and bankruptcy prediction. The intended search of papers ranged from 2002 up to 2015, and the results included huge resources comprising many journal papers, articles, books and conference papers.

Secondly, only journals papers were included in the further search as these were considered better related to the new developments in the credit-scoring field than books and were believed to contain more in-depth explanations about methods used than conference papers. In the filtering stage, it seems that papers containing credit risk and bankruptcy predictions were related or were seen to have the same concept of credit-scoring in terms of the datasets used in validating the developed model or in the motivation of proposing methods that increase model performance. All the papers that aimed at using individual classifiers only to develop models were excluded as the aim is centred on collecting papers that used hybrid and ensemble models; however, these studies included individual classifiers as a benchmark.

Thirdly, all related studies are selected and organised in sequential order from 2002 till 2015, with all the findings of studies comprehensively summarised and discussed based the experimental design of studies, the features of the datasets used, the classification techniques used, the ways of how data is pre-processed, the modelling approaches, performance indicator measures and hypothesis testing employed.

2.6.2 Literature Discussion and Analysis

A collection of 37 papers was selected from various rigorous scientific journals with the focus on hybrid, ensemble, data-pre-processing studies and studies that focused on improving or proposing new approaches in credit-scoring. Table 2.2 summarises all the collected related studies that contain valuable information and findings that could lead to reliable conclusions.

No.	Study	No. Datasets	Data splitting	Data pre-processing	Data pre-processing Technique	No. Classifiers	Base Classifiers	Hybrid approach	Ensemble approach		Performance measures	Significant test
									Classifiers	**Combination rule**		
1	Lee *et al.* (2002)	1	Hold-out	Feature selection	LDA	3	LDA, LR, NN	Yes	-	-	Acc, Type & II errors	-
2	Hseih (2005)	2	K-fold	Clustering	SOM, K-means	1	NN	Yes	-	-	Acc, Type & II errors	-
3	Lee and Chen (2005)	1	Hold-out	Feature selection	MARS	4	LDA, LR, NN, MARS	Yes	-	-	Acc, Type & II errors, EMC[4]	-
4	West *et al.* (2005)	3	Hold-out	-		4	NN	-	Homogenous	MajVot, WAVG	Acc	Yes
5	Huang *et al.* (2006)	2	Hold-out	Feature selection	GP	6	GP, NN, LR, DT(CART, C4.5), k-NN	Yes	-	-	Acc	-
6	Huang *et al.* (2007)	2	K-fold	Feature selection	Grid search, F-score, GA	4	SVM, NN, DT	Yes	-	-	Acc	-
7	Tsai and Wu (2008)	3	Hold-out	-	-	1	NN ensemble	-	Homogenous	MajVot	Acc, Type & II errors	Yes
8	Yu *et al.* (2008)	3	Hold-out	-	-	7	LR, SVM. NN ensemble	-	Homogenous/Selective ensemble	MajVot, Reliability-based[1]	Acc, Type & II errors	-
9	Nanni and Lumini (2009)	3	Hold-out	-	-	12	Ensemble of LMNC, NN, k-NN, SVM	-	Homogenous	Sum rule	Acc, Type & II errors, AUC	Yes
10	Sustersic *et al.* (2009)	1	K-fold	Feature selection	PCA, GA	2	LR, NN	Yes	-	-	Acc, Type & II errors	-

11	Bellotti and Crook (2009)	1	Hold-out	Feature selection	SVM	4	LR, LDA, k-NN, SVM	-	-	-	AUC	-
12	Chen *et al*. (2009)	1	Hold-out	Feature selection	MARS, DT(CART)	5	SVM, DT (CART), MARS	Yes	-	-	Acc, Type & II errors	-
13	Chunag *et al*. (2009)	1	Hold-out	Feature selection	MARS	5	LDA, LR, DT(CART), NN, CBR	Yes	-	-	Acc, Type & II errors	-
14	Yu *et al*. (2009)	3	Hold-out	-	-	10	LDA, LR, NN, SVM, Ensemble of NN and SVM	-	Heterogeneous	Fuzzy GDM[2]	Acc, Type I & II errors, AUC	Yes
15	Tsai (2009)	5	K-fold	Feature selection	t-test, Stepwise, F-score, CM, PCA	6	NN	-	-	-	Acc, Type I & II error	
16	Chen and Li (2010)	2	Hold-out	Feature selection	LDA, F-score, DT, Rough sets	1	SVM	-	-	-	Acc	Yes
17	Zhou *et al*. (2010)	2	Hold-out	-	-	25	LDA, QDA[19], LR, NN, DT, NB, PR[20]	-	Homogenous/ Selective Ensemble	MajVot, Reliability-based, Weights based on tough samples	Acc, Sn, SP, AUC	
18	Hseih and Hung (2010)	1	K-fold	-	-	4	SVM, NN, NB	-	Heterogeneous	Confidence WAVG	Acc	-
19	Zhang *et al*. (2010)	2	K-fold	Feature selection	Rough sets	11	DT	Yes	Homogenous	MajVot	Acc	-
20	Tsai and Chen (2010)	1	Hold-out	Feature selection/ Clustering	K-means, EM	12	LR, DR, NB, NN	Yes	-	-	Acc	-
21	Yu *et al*. (2010)	1	Hold-out	-	-	8	SVM	-	Homogenous	MajVot, WAVG, ALNN[3]	Acc, Type I & II errors, AUC	Yes
22	Chuang and Hunag. (2011)	2	Hold-out	Feature selection	Rough sets	5	LDA, LR, NN, CBR	Yes	-	-	Acc, Type I & II errors, AUC	-
23	Wang *et al*. (2011)	3	Hold-out	-	-	13	LR, DT, SVM, NN	-	Homogenous/ Heterogenous	MajVot, WAVG, Stacking	Acc, Type I & II errors	-

24	Finlay (2011)	2	K-fold	Feature selection	Stepwise	18	LDA, LR, DT (CART), k-NN, NN, Ensemble of all above	-	Homogenous	MajVot, WAVG, Mean	Classification Error Rate	Yes
25	Akkoc (2012)	1	K-fold	Feature selection	LDA, LR	4	NN	Yes	-	-	Acc, Type I & II errors, AUC, EMC	-
26	Wang and Ma (2012)	1	K-fold	-	-	13	DT, Ensembles of DT	Yes	Homogenous	MajVot	Acc, Type I & II errors	Yes
27	Wang *et al.* (2012)	2	K-fold	-	-	11	LR, NN, DT, SVM, Ensemble of SVM	Yes	Homogenous	MajVot	Acc, Type I & II errors	-
28	Marques *et al.* (2012a)	6	K-fold	-	-	17	Ensembles of k-NN, NB, LR, NNs, SVM, DT (C4.5)	-	Homogenous	MajVot	Type I & II errors, AUC	Yes
29	Marques *et al.* (2012b)	6	K-fold	-	-	35	of k-NN, NB, LR, SVM, DT(C4.5), Ensembles of DT (C4.5)	-	Homogenous	MajVot	Acc, Type I & II errors	Yes
30	Brown and Mues (2012)	5	Hold-out	-	-	10	LR, NN, DT (C4.5), LDA, QDA, RF, k-NN, SVM, GB[21]	-	Homogenous	MajVot	AUC	Yes
31	Tsai and Cheng (2012)	4	K-fold	Data-filtering	K-means	4	NN, DT, SVM, LR	-	-	-	Acc, Type I & II error	-
32	Garcia *et al.* (2012)	8	K-fold	Data-filtering	20 filtering algorithms	20	k-NN	Yes	-	-	Acc	Yes
33	Xiao *et al.* (2012)	2	Hold-out	-	-	8	DT (CART)	-	Selective Ensemble	MajVot, WMajVot	Acc, Sn, Sp, AUC	-
34	Tsai (2014)	5	K-fold	Clustering	SOM, K-means	21	LR, NN, DT (CART)	Yes	Homogenous/ Heterogenous	MajVot, Weighted Vote	Acc, Type I & II errors	Yes
35	Abellan and Mantas (2014)	3	Hold-out	-	-	5	LMNC, DT (C4.5, CDT[22])	-	Homogenous	MajVot	AUC	Yes

							Ensembles of all above					
36	Harris (2014)	2	Hold-out	Clustering	K-means	8	SVM, LR	-	-	-	Acc, AUC, BAC[5], Sn, Sp	Yes
37	Lessmann *et al*. (2015)	8	K-fold	-	-	41	Refer to paper	-	Homogenous/ Heterogenous/ Selective Ensemble	MajVot, WAVG, Stacking	Acc, AUC, Brier Score, KS[6], PG[7], H-measure	Yes
	Total	**102**	**19**	**14/2/4**	**-**	**369**	**-**	**16**	**14/5/4**	**-**	**37**	**16**
	Average	**2.7**	**-**	**-**	**-**	**9.9**	**-**	**-**	**-**	**-**	**-**	**-**

[1]Acc: Accuracy, [2]Sn and Sp: Sensitivity and Specificity, [3] Reliability-based: minimum, maximum, mean, median, and product rules, [4]GDM: group decision making. [5] ALNN: adaptive linear neural network. [6] EMC: expected misclassification cost. [7] BAC: balanced Accuracy.[8] KS: Kolmogorov-Smirnov. [9] PG: Partial Gini index. [10]SOM: [10]Self Organizing Map. [11]GP: Genetic Programming. [12]GA: Genetic Algorithm.[13]Levenberg–Marquardt neural net.[14]CBR: Case Based Reasoning.[15]EM: Expected Minimization algorithm.[16]PCA: Principal Component Analysis. [17]CM: Correlation Matrix. [18]FA: Factor Analysis.[19]QDA: Quadratic Discriminate Analyses. [20]PR: Probit Regression.[21]GB: Gradient Boosting. [22]CDT: Credal Decision Trees.

Table 2.2 Related studies comparison

Table 2.2 summarises all the related studies, taking into consideration various essential factors that make up the development process of any credit-scoring model. The number of datasets used to validate the model with, data-splitting techniques that are responsible for evaluating and assessing the model, the data pre-processing that deals with analysing the data in terms of noise and outliers, and preparing clean data to be trained in order to achieve better performance. Moreover, the number of classifiers used in each study is important as it reflects the extent to which classifiers are used to compete with one another. Moreover, the experimental design approaches, whether hybrid or ensemble approaches, are considered, as well as the performance indication measures used in each study and the significant tests associated to proving the reliability and robustness of the developed models.

Overall, findings that will give us information about what has been done or tackled so far in the field of credit-scoring are summarised, including the procedure of building or designing credit-scoring models, and the extent to which these could help or lead to addressing areas that are not fully investigated or afforded attention in the field of credit-scoring. As a result, we are guided in reaching a systematic comprehensive model that comprises new approaches on different aspects of the model. It is worth noting that this thesis is focused on proposing new approaches in the field of credit-scoring rather than comparing classification results with the related studies.

Various findings and conclusions can be derived from Table 2.2. The first finding is that the majority of the studies (30 out of 37) used between 1 and 3 datasets to evaluate their models.

A total of 2 studies used 8 datasets; however, on average, the number of datasets used in all related studies equated to approximately 3, which are relatively small. Going in-depth into the studies, the vast majority of them shared same common datasets, namely German and Australian credit datasets (is discussed in more details in the next chapter), and according to Lessmann *et al*. (2013), relying on these datasets for developing new models could lead to bias. Moreover, these datasets might not reflect datasets used in industry as different datasets could have different characteristics (Finlay, 2011). However, some studies used one private dataset provided by banks and companies to validate their studies and other studies used combined benchmarked and private datasets to extra validate their models. One main reason for relying on German and Australian credit datasets is that they are publicly available for researchers, with collecting industry datasets not an easy task due to confidential policies by some financial institutions.

The second finding is related to splitting or partitioning the data used to train the model, with the data assessing the model. As can be established, 21 studies used the hold-out splitting technique and 17 studies used the k-fold Cross-validation (CV) splitting technique. Basically, hold-out is cutting the dataset randomly into two parts: one part builds the model and the other part assesses the model. The k-fold CV technique involves dividing the datasets into K subsets (or folds) of equal size (K = 1, 2…, K), although K cannot exceed the size of the dataset. Therefore, the model training is based K-1 folds, and the remaining K folds are saved for model evaluation or testing. The process continues until all K folds are used for evaluation. All the tested K fold predictions are used to estimate the model Accuracy (by taking the average). Despite the attention towards these particular splitting techniques, there are other ways of splitting the data, such as repeated hold-out, leave-one-out and k_1xk_2- fold CV (Alpaydin, 2010; Garcia *et al*., 2015). According to Garcia (2015), the choice of each splitting techniques to be employed depends on researcher preference. However, issues should be considered prior to choosing a splitting technique, such as the stratification of data samples based on their class and the size of the dataset(s) available.

Another important step in model-building is the pre-processing of data that is used to build up the model. Principally, each dataset is made up of samples, where each sample consists of a number of characteristics, attributes or features that vary in size depending on the nature of the data. However, amongst these samples and features, there could exist outliers, noisy, redundant or irrelevant features that may affect the performance of the model. The superiority

of the data is important in order to achieve better model performance, which is highly dependent of the data efficiency in terms of number of data samples, the relevance of its attributes or features and the presence of outliers in the dataset (Garcia *et al*., 2012). Consequently, the cleaning or filtering of the unnecessary and redundant information can consume time and costs, but also increase model performance (Tsai, 2009). Also another way of pre-processing data, according to Tsai & Chen (2010), is clustering, which is another method in data pre-processing used in building hybrid models; here, data are grouped or clustered according to their similarities and dissimilarities. According to Saddatrasoul *et al*. (2013), the clustering technique is done in order to identify and filter the outliers of the data, and then the remaining ones that are not filtered are used to train the classifier in order to improve the classification result. As can be seen from Table 15, feature selection was used as a pre-processing step where only a subset of significant and relevant features was used to train the model. Notably, only 2 and 4 studies embraced data-filtering and clustering, respectively, for data pre-processing. In total, 21 studies applied the processing step for their data, which is almost half of the studies; this reflects the importance of this step for enhancing the model performance. All the studies reported achieving better model predictions than without cleaning data from outliers and noisy information. Many methods are available for feature selection, data-filtering and clustering, such as MARS, Relative neighbourhood graph editing (RNG) and k-means (Lee and Chen, 2005; Garcia *et al*., 2012; Tsai, 2014). In general, data on pre-processing has increased in importance and has become a fundamental step in credit-scoring models development (Garcia *et al*., 2012).

Another major stage in model development is the development of the classifiers and how these classifiers are used for comparison in each study. Well, as can be noticed from the table, the number of classifiers developed and used in the related studies varied from small to large (e.g., from 1 to 41 classifiers), providing an average of 9.9 classifiers. The reason behind the varying numbers of classifiers in each study is that: 1) each study try in proposing new methods and compare it with other methods within the same study (e.g., West *et al*., 2005; Tsai & Wu, 2008; Marques *et al*., 2012a) or to 2) compare new developed methods with other methods from other studies (Yu *et al*., 2008; Zhang *et al*., 2010; Wang *et al*., 2012; Abellan & Mantas. 2014). In general, the number of classifiers used in a study depends on the developed model and how many classifiers or methods are to be compared in order to prove its validation and superiority. It is worth mentioning that Hand (2006) made some remarks on studies, comparing their results with other study results, where the comparison could be fair

enough, as the author making the comparison might be more skilled in the method using and the level to which he can tune it to achieve better results. In other words, there is no universal superiority of a model on another, and reaching fair comparison factors, such as datasets used, data-splitting techniques and performance measures employed, should be taken into consideration in order to prove relative superiority.

In reference to the previous discussion, researchers tend to build classifiers that perform and generalise the data well, whether by proposing novel ideas or creating different modelling structures. One of the approaches used to enhance the model classification results is the hybrid modelling, which is proven to be a superior approach in achieving high performance when applied to credit-scoring problems. As discussed earlier, in Section 2.3.2, the experimental design of this is achieved by integrating or uniting different methods or classifiers together in order to exploit their strengths and overcome the weaknesses of each method. As is clear from Table 2.2, 16 studies adopted hybrid modelling to achieve high prediction performance. It can be noticed that all the hybrid modelling studies adopted feature selection, clustering and data-filtering as a first stage in the hybrid model, where the significant features or representative data to be used as training input for the classifier in the second stage. Majority of the studies focused on applying feature selection, whilst only 4 and 2 studies focused on clustering and data-filtering, respectively.

Clustering is a technique used when data labels are not provided in which the group is data based on their similarities, although credit-scoring datasets are labelled clustering techniques and are applied in credit-scoring studies (e.g., Hseih, 2005; Tsai & Chen, 2010; Tsai, 2014). Regarding feature selection and data-filtering, both are used to select the most appropriate features and data to train the model.

Another experimental approach of modelling, as proposed to achieve better model performance, is the ensemble approach. Unlike hybrid methods, ensemble learning creates several classifiers with different types or parameter, such as several NN classifiers with different structures, and accordingly train different samples of the dataset for several times, with the right classifiers chosen as ensemble members. The results of the members are pooled, with the ensemble strategies employed to get the final results (Lin & Zhong, 2012). In creating the ensemble model, two points should be made in consideration of the selection of the single classifiers. First, the classifiers should be successfully applied in credit-scoring

analysis. Secondly, the design of the classifiers should be based on different theoretical concepts (Hseih & Hung, 2010). The important aspect of ensemble models is to be diverse and accurate (Wang *et al*., 2011).

Table 2.2 shows that there are 19 studies that followed the ensemble modelling in their work. This reflects the importance of the ensemble modelling approach in building a credit-scoring model. Exploring the studies in-depth, all of them have followed the parallel structure, with most of them adopting the bagging method so as to achieve diversity for the data that needs to be trained in the model. After having the data ready, models have to be developed so as to train the subsets of different data. Models can be developed in two ways: 1) building homogenous classifiers where classifiers are all of the same type; or 2) building heterogeneous classifiers where different types of classifiers are used to train the data. After training, all or some of the model predictions are combined into a single classifier so as to give a final answer or decision on the data. A total of 16 studies used homogenous ensemble in their models (e.g., West *et al*., 2015; Tsai & Wu, 2008), whilst 5 studies used heterogeneous ensemble (e.g., Yu *et al*., 2009; Hsieh & Hung, 2010; Tsai, 2014). Some studies did not combine the classifiers' predictions but rather chose the most appropriate and representative ensembles for combining their predictions (Yu *et al*., 2008; Zhou *et al*., 2010; Finlay, 2011; Lessmann *et al*., 2015).

Xiao *et al*. (2012) used dynamic classifiers ensemble using local Accuracy (DCE-LA) where best ensembles are selected based on their ability to classify a certain data sample. DCE-LA is done by computing the local Accuracy of a classifier with respect to output class of the data sample in a region of competence, and classifiers with a high level of competence in classifying the data point is selected for combination (Cruz *et al*., 2014). Another vital step is classifiers combination and regarding the combination rules that are used to fuse all the predictions of classifiers, majority vote (MajVot) is the most popular due to its simplicity; weighted average (Wavg) was also used but with less frequency. Reliability-based methods were only applied in 3 studies and stacking, which is considered a trainable combiner, was employed in 2 studies.

Another major step is the evaluation of the model performance, which assesses how well it will do in consideration to new data. With regards to performance measurements, measures that can be derived from the confusion matrix average Accuracy (Acc) were used in almost

all studies, with Type I and Type II error appearing in 22 studies, and the sensitivity and specificity appearing in 4 studies. The AUC was used in 12 studies. Lessmann *et al*. (2015) investigated the use of new performance measures for credit-scoring, namely the inclusion of Brier Score, KS, PG and the H-measure. Moreover, a new measure was used by Harris (2014), which is the balanced Accuracy measure (BAC). From a practical perspective, each of these methods expresses different views on model performance, meaning the important issue is to use the most appropriate performance measures that fit the model concerns (Garcia *et al*., 2015; Lessmann *et al*., 2015).

Finally, after assessing the model performance the model developers have to check whether the model results are robust and reliable and it is not a pure coincidence. According to Garcia *et al*. (2015) the statistical tests for model results is an important stage in the models development process. However, as can be noticed, less than half of the studies employed statistical significant tests on their proposed methods to determine whether the performance of their proposed models was statistically significant than other compared methods. There are several statistical tests of significance available for comparing and validating performance results of different models, such as the parametric test (e.g., t-test) and non-parametric tests (e.g., Friedman rank test, Bonferroni- Dunn test). Nonetheless, choosing the best test for application depends on different aspects, such as the number of datasets used and the number of classifiers developed and needing to be compared (Garcia *et al*., 2015).
In conclusion, it can be inferred from Table 2.2 that the main steps of general framework for a credit-scoring model comprise the following:

- Collection of the datasets: most of the studies have public benchmark datasets in common, and it is good also to include real industrial datasets in order to have diverse view on different data size, data characteristics and data class distribution.
- Choosing the proper splitting techniques: Choosing the most suitable splitting technique for the data and it important to take in consideration the size of the dataset and the distribution of its classes (e.g., majority and minority classes).
- Modelling approach: This depends on how the developer or researcher looks to solve the problem in hand; however, the main aim is to achieve an effective model with reliable results. Developers could use only single classifiers with raw data, whilst others could analyse the data and clean it before training. Others try to develop novel

ideas either by using hybrid or ensemble modelling. Therefore, in general, this is kept for the researcher's perspective.

- The use of performance measures indicators: Many performance measures are available, and the researcher should choose the most appropriate ones that can reflect all angles of the model performance.
- Statistically testing and validating model results: to reach a reliable conclusion that the developed model is not merely a matter of luck; its results should be statistically validated using an appropriate test.

Moreover, several issues have been found to which little attention has been afforded by the related studies from Table 2.2, as follows:

- Data-filtering is rarely used by studies as data pre-processing step. Furthermore, has not been found any study suggesting combining 2 data pre-processing methods together and determine the extent to which it performs when 1 method is used.
- In ensemble modelling, the vast majority of studies merely focus on creating homogenous classier ensembles in their studies, whilst the heterogeneous classifiers ensemble was not given high attention.
- The area of building a selective ensemble model has not been thoroughly investigated in the related studies, with only three studies applying this with homogenous classifiers and 1 study with homogenous /heterogamous classifiers. It is believed that it is worth investigating; the more classifier members, the better the results, so choosing a selecting strategy for deciding the most appropriate member is quite an interesting trend worth investigating.
- The majority of the studies have developed multiple classifier systems, where each classifier has afforded independent decisions and then combined them into one single output without any collaboration or coordination between the classifiers through the learning process. Contrariwise, in a study by Yu *et al*. (2009), heterogeneous ensembles were developed and combined with those using fuzzy rules based on group decision-making that involves classifiers working in a group to reach a consensus on the final output.
- Statistical tests of significance to prove model results are infrequently used in their model development; however, it is believed that using a proper significant test is an essential part of model validity.

- It has not been really addressed that there has not been a study that has proposed a model that integrated: Public and industry datasets + a combined data pre-processing methods + heterogeneous ensembles + ensemble selection + new combination rule + several performance evaluation measures (for a comprehensive validation of the model) + significant test. Although its sounds very complex and despite the fact it encompasses many phases, here we would like to investigate the extent to which each step of the model can enhance model performance, and answer a question whether complexity is worth investigating in the field of credit-scoring development, as many in the industry consider a simple LR model as s the standard tool for building an efficient credit-scoring models (e.g., Crook *et al*., 2007).

2.7 Summary

In summary, this chapter was divided into two areas: theoretical background of credit-scoring and its related issues; and a review of credit-scoring literature. The first part started by giving a background about credit-scoring in terms of definitions and procedural framework in terms of development and implementation. Also, it discussed why credit-scoring has become an important topic for financial institution and the machine-learning community due to massive credit growth year by year, meaning dealing with credit and trying to discriminate between two groups of credit has become positioned as an important area for developers and researchers.

The general methods used in credit-scoring evaluation techniques starting from the judgmental approach that rely on the experience of credit analysts have been overviewed. Such a form of evaluation has been criticised for various reasons, including the huge increase of credit data and decision bias that could occur, affected by relationships with customers. In order to avoid such issues, researchers tend to identify an easy and systemised way of achieving credit evaluation.

Subsequently, the quantitative tools that were used to develop credit-scoring models that came as a replacement to traditional evaluation techniques were highlighted. An overview of several methods and algorithms, ranging from statistical to machine-learning, as well as their application in credit-scoring field, was provided. After, the relevant algorithms used in credit-

scoring development was considered, shining a light on the different modelling approaches and how these algorithms can be used in such a design so as to achieve a good performance. In credit-scoring model development, approaches begin with the use of individual classifiers, with the motivation then rising to establish efficient modelling ways that utilise most classifiers; this resulted in hybrid classifiers and ensemble classifiers, followed by the proposal of ensemble classifiers. Moreover, an important stage after developing and training the modes, which is the performance evaluation measures and significant tests for the developed model, have been highlighted. The performance measures and significant test that can be used to check models reliability and robustness have been highlighted.

The second fold contained the literature review and the collection for the related studies that utilised the aforementioned algorithms and modelling approaches. The studies in the collected literature was systematically analysed and summarised in terms of their experimental design or procedure that cover several factors, such as the number of datasets used, data-partitioning, data-pre-processing stage, hybrid modelling, ensemble modelling and its approaches, performance evaluation measures and statistical test of significance. Several issues, findings and conclusions were seen to have emerged from the analysis, which is worth further investigation, where studies have not been afforded attention, such as combining more than the data-cleaning method, focus on ensemble selection and introducing new classifiers combination rule.

In the next chapter, the methodological framework of the development process of the experimental design of credit-scoring model and the related issues are presented and discussed.

CHAPTER 3

THE EXPERIMENTAL DESIGN FRAMEWORK FOR THE PROPOSED CREDIT-SCORING MODEL

3.1 Introduction

In this chapter the main phases that make up the experimental design of proposed credit-scoring model and what each phase includes, is explained and discussed. Moreover, an overview of the most common ways in processing is provided, with each phase and the main issues associated to them discussed. Furthermore, the contents of our experimental design of this thesis and the justification behind it are shown. Firstly, this chapter will start with the explanation of the datasets used in this thesis, with the main questions regarding them also detailed. Secondly, focus on the datasets is given in terms of per-processing and normalisation. Thirdly, the ways in which datasets can be partitioned, with the one considered best fitting the datasets, is discussed. Then, the modelling approach adopted is explained. Following, a description of the performance evaluation measurements is employed so as to validate the proposed model. However, extra validation is followed with the use of statistically significant tests.

3.2 Datasets

The main and the first step of the process of building credit-scoring models is the collection of the datasets used to execute the developed models. However, according to Garcia *et al*. (2015), there are two factors that have to be taken into consideration when collecting the datasets. This section, will discuss these factors in addition to the description of the datasets characteristics used in this thesis.

3.2.1 Dataset Size

In the field of credit-scoring, the determination of the ideal size of the dataset is an issue that has been addressed amongst researchers (Abdou & Pointon, 2011). It is believed that the large datasets are beneficial for the model, and small datasets are efficient for the model (Crone & Finlay, 2012). Empirically, the determination of the dataset size is matter to the

data availability, market size and credit environment. Typically, in industry, datasets are very large, whilst datasets used in credit-scoring studies can be either be available public data or a private data provided by a financial institution. In reference to the previous chapter in Table 2.2, there were 31 related studies that used datasets containing 1,000 samples or less (public and private datasets), whereas the majority of them used the available real-world public datasets. Besides, Garcia *et al*. (2015) stated that there are no strict rules about what an adequate dataset size should be. This indicates that dataset size is subject to its availability.

3.2.2 Number of Datasets

Another factor to be taken into consideration is the number of datasets to use to validate the developed model. In reference to Table 2.2, on average, studies used on average three datasets, the majority of which tended to use up to three datasets to validate their models. Generally speaking, datasets vary in their characteristics in terms of number samples, attributes and class distribution, and draw reliable conclusions from the developed model as a preferable way of validating the model by exposing it to different datasets that hold different characteristics. As has been stated earlier, the vast majority of studies have relied on well-known benchmarking public datasets; however, studies also have experimented private collected datasets provided by several financial institutions for extra validation (Garcia *et al*., 2015). Essentially, relying on public datasets only to validate could be not efficient enough as they do not reflect or represent enough the datasets that occur in industry in term of socio-economic conditions and this might lead to out-of-date and insignificant conclusions. However, the main advantage in using public data is to carry comparisons between different studies. Therefore, according to Lessmann *et al*. (2013), along with public datasets, using private datasets from different companies and financial institutions can offer more robustness to the model under different environmental conditions, which, in return, can lead to reliable conclusions on model performance.

3.2.3 Collection of the Datasets

As per our discussions in the above sections, a collection of public and private datasets with different characteristics is employed in the process of empirical model evaluation. In total, seven datasets are obtained where four are public and three are private. The public datasets are well-known real-world credit-scoring datasets that have been widely adopted by researchers in their studies and which are easily accessed and publicly available at the UCI machine-learning repository (Asuncion & Newman, 2007).

The first 3 datasets are German[1], Australian[2] and Japanese[3]. In addition, the fourth and fifth datasets are a corporate and bankruptcy datasets were used for extra validation. The Iranian[4] dataset, which consists of an alteration of a corporate client data from a small private bank in Iran, as investigated by Sabzevari *et al*. (2007), utilised an altered dataset, which will the same as used in other works (Garcia *et al*., 2012; Marques *et al*., 2012a, 2012b). The Polish[5] dataset that is known to contain information on bankrupted polish companies recorded over two years (Pietruszkiewicz, 2008) also was used in several studies (Garcia *et al*., 2012; Marques *et al*., 2012a, 2012b).

The sixth dataset is the Jordanian[6] dataset, which is based on a historical loan dataset, was gathered from one public commercial bank in Jordan. These data are confidential and sensitive; hence, acquiring the data was a detailed and time-consuming process. The dataset consists of 500 loans, 400 of which are good loans and 100 are bad loans. Bad loan cases were more difficult to obtain due to manual storage at the banking institution. Therefore, bad loan data also include current cases rather than historical cases. These are loans that are currently 90 consecutive days past due, which is considered to be in default status in Jordanian banking policy. It is clear that the dataset is biased towards good loans due to the low default rates occurred at that time in the bank. The seventh dataset, which is the UCSD[7]

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[1] https://archive.ics.uci.edu/ml/datasets/Statlog+(German+Credit+Data).

[2] https://archive.ics.uci.edu/ml/datasets/Statlog+(Australian+Credit+Approval).

[3] https://archive.ics.uci.edu/ml/datasets/Japanese0Credit0Screening.

[4] Contact hn_sabzevari@yahoo.com.

[5] Contact wieslaw@)pietruszkiewicz.com.

[6] Contact the author at maher.alaraj@hotmail.com.

[7] Contact the author at maher.alaraj@hotmail.com.

that matches to a reduced version of a database used in the 2007 Data Mining Contest organised by the University of California San Diego and Fair Isaac Corporation.

All the datasets samples differ in their class distribution and number of variables. For example, the datasets vary from high, medium and low imbalance class distribution, and also comprise several amounts of independent variables that make up each loan applicant and one dependent variable which is the status of their application which is either good or bad. A summary of the datasets characteristics is represented in Table 3.1.To access and obtain the full datasets please refer to the footnotes below.

Dataset	#Loans	#Attributes	# Nominal/ Categorical Att.	# Numeric Att.	#Good	#Bad	Good: Bad	Missing values
German	1000	20	13	7	700	300	70:30	No
Australian	690	14	8	6	307	383	44:56	No
Japanese	690	15	10	5	307	383	44:56	Yes
Iranian	1000	27	3	24	950	50	95:5	No
Polish	240	30	0	30	128	112	53:47	No
Jordanian	500	12	7	4	400	100	80:20	No
UCSD	2435	38	6	32	1836	599	75:25	No

Table 3.1 Description of the datasets

3.3 Data Pre-Processing

The quality of the data plays is considered a critical point in enhancing models generalisation performance. This essentially depends on the suitability of the data to be used in relation to the number of samples, the importance of the features used in the analysis and the occurrence of outliers in the dataset. Accordingly, data pre-processing developed to be an essential step in credit-scoring classification problems (Garcia *et al*., 2012). Datasets in general can be collected from different sources and can be in different forms. However, datasets that are collected from the real-world may completely be raw data that is not clean, transformed or changed. Data quality can be measured using three important elements, namely accuracy, completeness and consistency. Contrariwise, this is not the case with real-world datasets as they can be easily sensitive to noise, outliers, missing attribute values and inconsistency (Garcia *et al*., 2015). The confirmation on the data representation and its quality is very important before any further analyses or procedure. If there exists any sample or attribute in

the dataset that is irrelevant, redundant, noisy or unreliable, this could pose a problem in the model training as it makes the knowledge mining and discovery a difficult task (Hall & Holmes, 2003; Kotsiantis *et al*., 2006). Therefore, data pre-processing becomes a very important step to ensuring the quality of the data and hence improving and comforting the knowledge discovery process of the models. Data pre-processing is a very crucial and critical step in the model development that deal with the raw datasets, and it comprises several methods, such as data imputation, normalisation, feature selection and data-filtering or instance selection. Following the processing of the data, a new training dataset is ready for further analysis (Garcia *et al*., 2015). Therefore, this section will present the data pre-processing methods which are opted in the proposed model.

3.3.1 Data Imputation

When a customer is filling in a loan application, he/she might forget or skip some fields in the application. However, when collecting a group of applicants in a dataset and there exists missing or incomplete fields values in the datasets, this can disturb the classifiers' discovery process when training a classifier. In order to overcome this issue, data can go through a stage of pre-processing and cleaning in order to make the data sufficient and easy for knowledge discovery (Luengo *et al*., 2012). The easiest way of dealing with missing values is to delete the instances containing the missing value of the feature; however, there are other ways of handling missing values instead of deleting them, such as by adopting an imputation approach, which means replacing missing values with new values based on some estimation (Acuna *et al*., 2004).

Regarding the collected datasets, only the Japanese dataset was found containing some missing values, and it was deciding to impute them via a simple imputation approach as following (Acuna *et al*., 2004; Lessmann *et al*., 2015):

- Replace missing categorical or nominal data with the most frequent category within the remaining entries, in other words the mode.
- Replace missing quantitative data with the mean value of the features that holds that missing value.

3.3.2 Data Normalisation

Some classifiers, such as NN and SVM, require input values that range from 0 to 1 and in vectors of real number. However, the datasets contain inputs that hold values that are fed to the NN. Each attribute in the dataset contains values that vary in range. In order to avoid bias and accordingly feed the classifiers with data within the same interval, data should be transformed from a different scale of values to a common scale values. In order to achieve this dataset, attributes should be normalised to values in the range of between 0 and 1 using an appropriate way: for example, if a simple normalisation method were used such as taking the highest or the maximum value of an attribute in a dataset and divide all the attribute by this value, here all the normalised values tends to almost 0 which do not reflect the original values, thus leading bias to inefficient classifiers training (Khashman, 2010).

For our datasets, data are normalised using the min-max normalisation procedure (Sustersic *et al*., 2009; Wang & Huang, 2009; Li & Sun, 2009), where the maximum value in an attribute is given a value of 1 (max_new) and the minimum value in an attribute is given value of 0 (min_new) and the values in between are scaled based on the below equation:

$$new\_value = (original - min) / (max - min) * (max\_new - min\_new) + min\_new \quad (3.1)$$

Moreover, in order to provide an example of the normalisation procedure, an example is illustrated on 4 samples of the Australian dataset (Attributes values before and after normalisation)

Attributes	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**Class**
Cus #1	1	22.08	11.46	2	4	4	1.585	0	0	0	1	2	100	1213	0
Cus #2	0	22.67	7	2	8	4	0.165	0	0	0	0	2	160	1	0
Cus #3	0	29.58	1.75	1	4	4	1.25	0	0	0	1	2	280	1	0
Cus #4	0	21.67	11.5	1	5	3	0	1	1	11	1	2	0	1	1

Table 3.2 Attributes values before normalisation

Attribute	**1**	**T2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Max value	1	80.25	28	3	14	9	28.5	1	1	67	1	3	2000	100001
Min value	0	13.75	0	1	1	1	0	0	0	0	0	1	0	1

Table 3.3 The maximum and minimum attributes values

Attributes	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Cus #1	1	0.125	0.409	0.5	0.2307	0.375	0.05	0	0	0	1	0.5	0.05	0.0121
Cus #2	0	0.134	0.25	0.5	0.5384	0.375	0.0057	0	0	0	0	0.5	0.08	0
Cus #3	0	0.238	0.0625	0	0.2307	0.375	0.0438	0	0	0	1	0.5	0.14	0
Cus #4	0	0.119	0.4107	0	0.3076	0.25	0	1	1	0.1641	1	0.5	0	0

Table 3.4 Attributes values after normalisation

3.3.3 Features Selection

As mentioned earlier, the data that are used in building the classification models and each raw data is associated with variables or features. Even though a huge number of features might be available, it is often looked-for to a classification model to be trained on a limited number of features in order to simplify the model and reduce its data requirements (Falangis & Glen, 2010). By developing a classifier with selected features, benefits can be achieved such as: 1) makes data easy to visualise and understand; 2) reduces the data storage requirements; 3) reduces training time; and 4) reduces dimensionality to improve prediction performance (Guyon & Elisseeff, 2003). After replacing the missing variables and normalising the datasets with new entries, the dataset is ready for extra further processing. Datasets in general contain different attributes or features that make them up, and they vary from one dataset to another. However, datasets could include irrelevant and redundant features that make it complex for models to train so leading to models with low performance and Accuracy. As a result, analysing features and investigating its importance has become a necessary and essential task for data pre-processing in data-mining in general and credit-scoring in particular in an effort to enhance the model's prediction performance (Tsai, 2009; Yao, 2009). Feature selection is an important step in selecting the most relevant and appropriate features and accordingly removing the unneeded ones; in other words, it is a process of selection a subset of representative features that can lead to models performance.

In reference to Table 2.2, the vast majority of studies conducted feature selection to their data and train the model with new subset of significant features. Moreover, investigating in-depth inside the studies, the feature selection methods used in the related studies vary from study to study: for example, studies used simple selection approaches, such as stepwise regression, with other incorporated classifiers (e.g., LDA, LR, SVM, NN, DT and MARS), evolutionary algorithms (e.g., GA and GP) and clustering algorithms (e.g., k-means, EM), in an effort to

establish the significance and representative features to be trained. Along with features selection, some studies investigated the use of dimensionality reduction techniques in their studies, such as the feature extraction methods (e.g., PCA), where all features are reduced to fewer features that carry the most information about them. However, these methods are out of the scope of the thesis. Although the many methods that have been used for conducting feature selection process in the related studies, Tsai (2009) states that it is unknown which is the best method to adopt in selecting the best features for a model. Moreover Liu & Schumann (2005) argue that there is no *'economic theory to denote which features are relevant and which are not relevant to creditworthiness, the process of choosing the best set of features in practice is unsystematic and dominated by arbitrary trial'*. In this thesis, MARS will be adopted in order to perform the feature selection task for the developed model.

3.3.4 Data-filtering (Instance Selection)

Studies in credit-scoring have afforded only little high attention to feature selection or elimination in their data pre-processing stage, whilst only a little attention was afforded to that of data-filtering or instance selection as a pre-processing stage for training the data. The purpose of data-filtering or instance selection is to reduce the size of the original dataset and produce a representative training dataset, whilst keeping its integrity (Wilson & Martinez, 2000). Data that are noisy or contain outliers could have as strong effect on model performance as much as redundant and irrelevant features could have on model performance. According to Tsai & Chou (2011), in some cases, removing outliers can increase classifiers' performance and Accuracy by smoothing the decision boundaries between data points or feature space. In general, outliers in a dataset mean that a sample of the dataset appears to be inconsistent within other samples in the same dataset; these data can be atypical data, data without prior class or data that are mislabelled. If all this appears in a dataset, then these outliers must be eliminated by filtering those samples that hold such characterises that could distract the training process, since their occurrence can lead to inefficient data training by classifiers (Tsai & Chou, 2011). In reference to Table 2.1, it may be seen that the data-filtering technique is rarely considered in the area of credit-scoring and here, in this thesis, data-filtering will be considered a part of data-pre-processing steps due to our belief that its important can be found in improving modelling generalisation process along with feature selection.

Practically, assume a training set *TR* that contains *N* data samples, after applying a data-filtering algorithm a *k* number of data *N* is eliminated, so having a new subset of training data *R*, hence *R* is a subset of *TR (R⊆ TR)* whereas the new subset R and the number of data samples from each class that are based on the filtering algorithm used.

Besides, it is believed that training a classifier with the filtered dataset can have several benefits (Garcia *et al*., 2012) such as:

- Decision boundaries are smooth and clear.
- It is easier for classifiers to discriminate between the classes.
- Improve the Accuracy performance of the model.
- Computational costs can be reduced.

Filtering algorithms are widely researched in the field of data-mining (Dasarathy, 1991; Wilson & Martinez, 2000). Garcia *et al*. (2012) have conducted a study using a wide range of filtering algorithms ad applied it on the credit-scoring and assessment problem. They used a total of 20 filtering algorithms, all of which showed superiority over the original training set. From the 20 algorithms used, they reported that the RNG[8] filtering algorithm filtering algorithm which is based on proximity graphs was the most statistically significant to others. For this purpose, the idea of the idea of proximity graphs is adopted in this thesis as the filtering algorithm that will be used to pre-process the training data for the collected datasets of this thesis. The filtering algorithm adopted in this thesis which is based on proximity graphs called Gabriel Neighbourhood Graph editing (GNG). More about the GNG algorithm is summarised in Chapter 5.

3.4 Data Splitting and Partitioning Techniques

After replacing any missing variables in the data and normalising them, data is ready to be partitioned into training and testing sets, which are to be used for building and evaluating the model, respectively. However, after partitioning the dataset, the training set can be further processed through the application of feature and instance selection. Data-splitting, partitioning or resampling is recognised as a fundamental step in the model-building, evaluation and validation processes. Datasets have to be partitioned into two parts, namely

[8] Relative Neighbourhood Graph editing

training and testing, the reason for which is to train the model on the first part, which is the seen data, and then to validate and apply the model on the second part, which is the unseen data, which will communicate how well the model performed and how it would perform on the real-world future cases. Issues related to data consider how many data shall be preserved for training and testing; the more data there is in the training set, the more the model is fitted to the data; on the other hand, the more data there is in the testing set, the more the model is more reliable in its Accuracy estimates, as it is more confident to have good Accuracy on 1,000 testing data than 100 testing data.

Another issue can arise here, which is the size of the available data and the number of data samples associated to each prior class, which makes the use of a particular splitting technique have a great effect on the model performance due to different datasets sizes as well as data class distribution. Also another important thing is the fair distribution of the data of different classes in the training and testing sets, to make sure that data with different classes are trained well so to have a good model generalisation over the testing set. However, different data splitting techniques have been used in the field of credit-scoring, in reference to Table 2.2, it is clear that the majority of the studies focused on just two techniques, which are the hold-out and k-fold techniques, in partitioning the datasets, and can be inferred that the choosing of a particular technique is kept to the authors. Next, both techniques are discussed and one is chosen for application on the collected datasets.

3.4.1 Holdout Technique

This is a technique based on cutting the dataset into two parts: one part for training and learning the model, and the other part for testing and validating the model. This method is very simple, and has been widely adopted in the literature, with the common way involving the partitioning of the dataset in order to randomly preserve 80% of the data for training and 20% for testing. However, the holdout technique might be biased in its Accuracy results, and it could be a matter of luck as data can be poorly used, and both training and testing set might be non-representative (e.g., testing set could have easy or hard data) (Bischl *et al*., 2012).

Nevertheless, this issue can be avoided by repeating the holdout technique several times in order to have randomly selected training and testing sets data each time, meaning the probability of getting a lucky testing set is less, even though the training and testing might be overlap and that may not perfect.

3.4.2 K-Fold Cross-validation

In this technique, the original dataset is partitioned in to k-subsets or folds of approximately equal size, for example consider P_1, P_2, P_3,….., P_k, are number of partitions made from the original dataset. Now, individually each partition must be trained and tested. Practically, the process of training and testing is illustrated in Table 3.5.

Folds/ Partitions	**Training set**				**Testing set**
1	P_2	P_3	P_4	P_k	P_1
2	P_1	P_3	P_4	P_k	P_2
3	P_1	P_2	P_4	P_k	P_3
4	P_1	P_2	P_3	P_k	P_4
5	P_1	P_2	P_3	P_4	P_k

Table 3.5 The k-fold cross-validation process

As can be seen from the table, the process of k-fold cross-validation proposes that, from all the partitions available, one partition is for testing and the rest are for training. The process continues until all partitions are trained and tested. The final Accuracy is estimated by taking the average of all the partitions or folds that were tested. Unlike the holdout technique, k-fold cross-validation ensures the effect use of all data available, hence avoiding any overlapping from occurring, and it would be more robust and efficient to repeat the process multiple times due to having many data trained and tested as much as possible at each repetition.

Further, issues also could arise regarding the consideration of how many folds or partitions to put data in, and whether the folds are too many and the model performance is accurate, but with high variance; on the other hand, there is the question as to whether the folds are small the model performance is biased and the variance is reduced. The ideal number of folds is dependable on the size of the dataset; Garcia *et al*. (2015) state that 5 or 10 folds can be a good choice with data sets with different sizes, with repetitions of the process also desirable in order to ensure switching between training and testing data as much as possible and also to avoid high variances. In this thesis, a 5-fold cross-validation is adopted with the repetition of 50 times in order to achieve reliable and robust conclusions relating to model performance. As a result, in this thesis, a 10 × 5-fold cross-validation is applied on each dataset, and the process is repeated 10 times for each, giving a total of 50 test results that are averaged to give a final result for each dataset.

3.5 Modelling Approach

In this section, the modelling approach and design proposed in this thesis is discussed. In reference to Table 2.2, it may be inferred that the studies followed wither the hybrid or the ensemble modelling due to their advantages on the usage of individual classifiers. Therefore, to justify this experimentally, a comprehensive modelling approach is developed, which includes a single, hybrid and ensemble modelling. The stages of the modelling process as follows:

- Building of the individual classifiers: several heterogeneous classifiers is built and trained on each of the 7 datasets adopted in this thesis.
- Building of the hybrid classifiers: hybridize the built individual classifiers by pre-processing the data to be fed to the individual classifiers as the following:

    - Apply feature selection technique (MARS) separately on the classifiers.
    - Apply data-filtering technique (RNG) separately on the classifiers.
    - Combining both the data-filtering with feature selection on the classifiers.

Several comparisons will carried out to investigate to what extent using such techniques will enhance the classifiers performance in order to be selected for the next modelling stage.

- Building of the ensemble classifiers: After selecting the best technique that performed best on the classifiers, the heterogeneous individual classifiers predictions is pooled all together and be combines using:

    - Traditional combination technique (e.g., majority vote, weighted average, etc.).
    - New D-ENS method based on classifiers ensemble selection (discussed in Chapter 7).
    - New combination methods based on collaboration between classifiers in order to present an outcome which outperform the previous two techniques (discussed in Chapter 7).

All the methods are compared together and to determine the extent to which the new combination technique could outperform all the preceded methods, from D-ENS, traditional

combiner, hybrid classifiers and individual classifiers, as well as to address whether or not complexity in modelling can achieve desirable results. All the models are compared and validated via performance evaluation measures and statistical tests of significance so as to reach a reliable and valid conclusion of the superiority of the proposed approach or method.

3.6 Performance Evaluation Measurement

The performance evaluation of the model is considered the most important stage in the modelling development process. Throughout this stage, the developed model is tested over the collected datasets and the performance evaluation metrics will determine the extent to which the model is well-learned and whether the results are robust and reliable so that it can be ready to predict new real-world data. In order to reach a reliable conclusion on how well the developed model performed, three types of indicator measure, covering all aspects of the model views on the results, should considered (Lessmann *et al*., 2015): firstly, measures that assess the predictive power of the model (e.g., classifying between good and bad loans); Secondly, measures that assess discrimination power of the model; and thirdly, measures that assess the Accuracy of the predictions probabilities of the model. Hence, considering these indicators provides a comprehensive view on the developed model performance. Furthermore, in order to ensure the worth of the model, the suggestion is made to use more than one performance evaluation measure so as to allow the capture of all the important features of the model (Japkowicz & Shah, 2011; Lessmann *et al*., 2015).

As a result, in order to validate our model and reach a reliable and robust conclusion on its predictive Accuracy, eight performance indicator measures are adopted in this thesis, namely the measures that can be integrated from the confusion matrix (Table 2.1), which are the Accuracy, Sensitivity, Specificity, Type I error and Type II error, the area under the curve (AUC), the H-measure and the Brier Score. These were chosen because they are popular in credit-scoring and they cover all aspects of model performance.

3.6.1 Confusion Matrix Measures

In reference to Table 2.1, the confusion matrix describes the prediction performance ability of the developed model in terms of how many data has been correctly classified or has been incorrectly classified. Several indicators can be derived from the confusion matrix and many

of them have been widely used in the literature to assess their developed models. Important measures that can be derived are described below.

3.6.1.1 Accuracy Rate

The Accuracy rate of a model is defined as the proportion of correctly classified cases to the total number of cases. It is very popular measure for assessing models performance and is almost used in every study in credit-scoring literature, and is considered the most important measure to draw conclusions on the models' performance on credit-scoring (Ravi, 2007). Moreover, for good classifiers, its superiority in Accuracy perhaps is considered the most important performance measure in credit-scoring applications (Wang, 2008; Siami *et al*., 2011). This can be clearly seen in Table 2.1. Regardless of the advantages of this measure, there is a shortcoming that can be noticed, which is that it does not take into account or give insight as to how cases with different classes have performed individually. For example, if a dataset is made up of 95% of cases are good loans and 5% are bad loans, if the developed classifier correctly classified the good loans and misclassified all the bad loans a very good classifier performance of 95% is still achieved. Therefore, considering measures that can give an insight on each class is preferable to know how well the developed model is suited to classify different data classes and to see if it is biased toward a particular class. The formula of the Accuracy rate based on Table 2.1 is as follows:

$$Accuracy = (TP + TN) / (TP + FN + TN + FP) \quad (3.2)$$

3.6.1.2 Sensitivity and Specificity

Sensitivity and specificity are measures that deal with loan cases from each class individually. Sensitivity measure is defined as the proportion of good loans cases that correctly predicted as good loans, it's also known as true positive (TP). Specificity is defined as the proportion of bad loans cases that are correctly predicted as bad loans. It is known as true negative (TN). The advantages of these measures is that it can assess the profits gained and losses prevented (Han *et al*., 2011). However, the sensitivity and specificity are measures as follows:

$$Sensitivity = TP / (TP + FN) \quad (3.3)$$

$$Specificity = TN / (TN + FP) \quad (3.4)$$

3.6.1.3 Type I and II Error

Opposite to sensitivity and specificity, Type I also occurs, which is called false negative (FP) and Type II error is false negative (FN). If a bad loan is assigned as a good loan, this is considered a Type I error; conversely, if the good loan is assigned as a bad loan, this is considered a Type II error. This measure was highly adopted in the related studies with Accuracy measures (see Table 2.2).

From the perspective of banks and financial institutions, Type I errors are related to financial loss (bad loan assigned to be good loan), with huge credit risks possibly occurring, and Type II related to the opportunity of profit loss (rejecting a good loan and assign it as bad one). From a financial point of view, risks associated with Type I errors are more costly than Type II (West, 2000; Marques *et al*., 2012); in other words, it can be seen as a 'Financial vs. profit loss'. From Table 2.1, Type I and Type II errors are expressed in following equations:

$$\textit{Type I error} = FP/\ TN+FP\ \textit{(Bad as Good, lose money)} \tag{3.5}$$

$$\textit{Type II error} = FN/\ TP+FN\ \textit{(Good as Bad, lose potential income)} \tag{3.6}$$

Basically, a model that can correctly predict bad loans is more beneficial financially than a model focusing on correctly predicting good loans as financial lost could be prevented, which is more important than missing a profit opportunity. As a general rule, an ideal model is a model that can prevent losses and make profits, which is a challenging task. Moreover, building a balanced model that is not biased to any class is also essential.

3.6.2 Area Under the Curve

The area under curve (AUC) is considered a measure that assesses the discriminatory ability of the developed model. Basically, it represents for the area under the Receiver Operating Characteristics curve (ROC). ROC curve is a graphical tool that represents the possible distributions of the good and the bad loan cases. The ROC curve is a two-dimensional performance classification measurement represented in graphical diagram, which plots the proportion of good loans predicted as good (y-axis) beside the bad loans predicted as good (x-axis) (Baesens *et al*., 2003; Crook *et al*., 2007). The y-axis is the sensitivity (TP), and the x-axis is called '1-specificity' (FP) (Fawcett, 2006; Crook *et al*., 2007). Mainly, the ROC curve provides guidance on setting the cut-off values in addition it describes the classifier property without consideration of factors such as misclassification cost and class distribution, meaning

it will efficiently separate the classification performance from these factors (Baesens *et al*., 2003). Figure 3.1 illustrates the ROC curve.

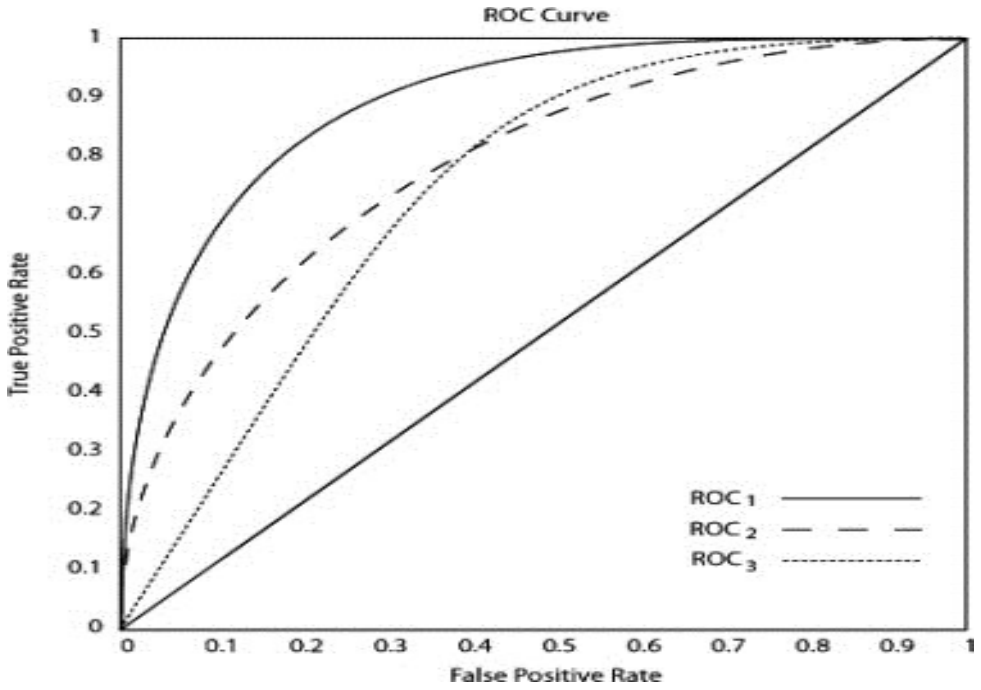


Figure 3.1 ROC curve illustrative example (Brown & Mues, 2012)

The decision of accepting or rejecting a loan relies on a pre-determined cut-off score (*T*): if the score is higher than cut-off (*T*) the loan is rejected, if the score is lower than the cut-off the loan is granted. According to Fawcett (2006) and Crook *et al*., (2007), the ROC curve is unaffected to any change in class distribution (good and bad cases) or any error costs results from misclassification; this depends on only the performance of the classes or cases. In the case of comparing the performance of ROC curve of different classifiers, regularly, AUC curves of the different classifiers are calculated. The classifier with greater AUC is considered as the highest and better performance than the others. For a classifier with good discrimination ability, the ROC curve should be stretched to the upper left corner of the graph. As can be seen from Figure 3.1, it can be inferred that ROC_1 curve indicates the best classifier performance against other ROCs; for the diagonal line, the classifier is considered a random guessing classifier as it correctly classifies good cases as the same rate of misclassifying bad cases, whilst the classifier that lies below the diagonal is a bad classifier.

In other words, AUC can be seen as the average ranking for positive cases, which means that ranking randomly picked positive case higher than a randomly picked negative case (Baesens *et al*., 2003; Fawcett, 2006). Therefore, ranking all positives higher than all negatives will

lead to a perfect classifier. AUC is a good tool in assessing classifiers and measuring their performance; they were given various advantages on other performance measurements due to their ability to work without being affected to class distribution or errors cost (Fawcett, 2006).

3.6.3 H-Measure

As discussed above, AUC measure assess the performance of several classifiers without taking into consideration any prior information of misclassification costs that occur within classifiers, for example if the cost of misclassifying bad case as good case and vice versa are different., the AUC can be incoherent and it works fine only when it assumes costs are equal (Hand, 2009). However, Hand (2009) showed how AUC can derive the misclassification costs and how that they could be providing misleading results about classifiers performance. AUC assumes different costs distribution amongst classifiers depending on their actual score distribution, which prevents them from being compared effectively.

As a result Hand (2009) proposed an alternative measure that can show discriminatory power of a classifier that can fill possible AUC limitations. This measure is called the H-measure, what it basically does is that it assumes different costs distribution between classifiers without depending on their scores. Basically, it includes a beta-distribution that contain 2 parameters alpha (α) and beta (β) these parameters are responsible to deal with different misclassification costs between classifiers in a reliable way (Hand, 2009). Choosing the values for the parameters depends on researcher and how severe they think about the cost of misclassification. H-measure calculates the expected misclassification loss of a classifier and output a classifier that ranges from 0 to 1 where 0 denote for random classifier and 1 denote for perfect classifier, the classifier values are based on the expected minimum misclassification loss (Hand, 2009; Lessmann *et al*., 2013). In the field of credit-scoring H-measure was only used in one study (see Table 2.2).

3.6.4 Brier Score

According to Blochlinger & Leippold (2011) in credit risk, companies and financial institutions risk management decisions are based on accurate and well calibrated estimates of probability of default. Therefore assessing the classifiers based on their probabilities and how accurate are they are considered an important part of a developed model performance, so considering a measure such as brier score would be beneficial for model performance assessment (Lessmann *et al*., 2013). Brier Score which also known as means square error

(Brier, 1950) it measures the Accuracy of the probability predictions of the classifier, by taking the mean squared the error of the probability, or in other words it shows the average quadratic possibility of a mistake, and the main difference between it and Accuracy rate (Acc) is that it directly takes the probabilities into the account, while Accuracy transforms these probabilities into 0 or 1 based on a pre-determined threshold or cut-off score. Brier Score can be expressed as follows:

$$Brier\ Score = 1/N \sum_{i=1}^{N} (P_i - Y_i)^2 \quad (3.7)$$

where N denote for the loan cases, P_i are the probability of the loan case i, and Y_i is the actual class for loan case i. Consequently, the lower the Brier Score the better the predictions are calibrated. Same as H-measure and Brier Score was also not considered in credit-scoring studies as it appears only used in one study (see Table 2.2).

3.7 Statistical Tests of Significance

The final stage of model development is to statistically test its significance. According to Garcia *et al*. (2015), it is not sufficient to prove that a model achieves results better than another, because of the different performance measures or splitting techniques used. For a complete performance evaluation, it would seem appropriate to implement some hypothesis testing to emphasize that the experimental differences in performance are statistically significant, and not just due to random splitting effects. Choosing the right test for specific experiments depends on factors such as the number of data sets and the number of classifiers to be compared. As it can be seen in Table 2.2 more than half of the studies carried out statistical validation tests on their models, this indicates that carrying a statistical validation test is an important step in order to reach a reliable conclusion about the robustness and reliability of the developed model. However, according to McCrum-Gardner (2008) and Garcia *et al*., (2015) choosing the right test depends on several factors such as number of datasets, number of classifiers to be compared and the measurement scale of the data output such as (binary, nominal or interval). Several statistical tests can be used to validate a study, and using an inappropriate test can lead to misleading and unreliable conclusions (McCrum-Gardner, 2008).

According to Demšar (2006) statistical tests can be parametric (e.g., Paired t- test) and non-parametric (e.g., Wilcoxon, Friedman test) However, Demšar advised that using non-

parametric tests are favourable over parametric tests as it can be conceptually inappropriate and statically unsafe. Therefore non-parametric tests can be more appropriate and safer than parametric tests since they don't assume normality of data or homogeneity of variance (Demšar, 2006). Therefore, a normality test for the datasets adopted in this thesis using the statistical software SPSS were tested, and the results showed that the datasets are not normally distributed.

Demšar (2006) suggested using non-parametric test if the comparison is carried out on more than 2 classifiers and over multiple datasets. Accordingly, based on this suggestion and that the proposed method is compared with more than one classifier over 7 datasets. As a result, in this thesis the Friedman (1940) test is used to detect statistical differences in rankings of predictions across multiple classifiers for each dataset separately. The best ranking classifier is given rank of one, the second best classifier ranked second and so on. Friedman test define a null-hypothesis where it tests that all classifiers from to be compared perform identically and all differences are only random fluctuations. The Friedman statistic x_F^2 is calculated and if the output is greater than the Chi-square critical value that correspond to the degree of freedom (number of classifiers (k) -1) and the stated alpha (e.g., 0.05) then the hypothesis is rejected otherwise is accepted (Demšar, 2006). In case the null-hypothesis of the Friedman test is rejected, it's recommended to proceed to a post-hoc test in order to find out whether the proposed model is statistically different from the other classifiers to be compared (Demšar, 2006; Luengo *et al*., 2009).

For instance, the Bonferroni–Dunn (1961) test can be used when all classifiers are compared with the proposed model (Demšar, 2006; Marques *et al*., 2012a; Marques *et al*., 2012b). With this test, the performance of two or more classifiers is significantly different if their average ranks vary by at least the critical difference (CD), as per the following equation (Demšar, 2006; Luengo *et al*., 2009):

$$CD = q_{\propto} \sqrt{\frac{k(k+1)}{6N}} \tag{3.8}$$

where $q_{\propto}$ is calculated as a studentised range statistic with a confidence level $\propto/(k-1) = \propto/k$ divided by $\sqrt{2}$.

3.8 The Proposed Experimental Design Framework

In reference to the all previous discussion, the essential stages of building a comprehensive credit-scoring model were explained. Garcia *et al*. (2015) suggested guidelines that need to be adopted in order to have a rigorous and effective credit-scoring model: 1) Using multiple dataset, with various sizes. 2) Using an appropriate data splitting technique according to datasets characteristics. 3) Choose suitable performance evaluation methods that can well describe the model performance. 4) Carry out appropriate statistical tests in order to validate the model's performance.

Accordingly, the main experimental design stages of the proposed credit-scoring model adopted in this thesis can be illustrated in Figure 3.2 As it can be seem from the figure, several phases and stages make up the experimental design of the proposed model. Therefore, these stages can be summarised as following:

- **Stage I**: Datasets collection (7 datasets are collected to build and validate the model)
- **Stage II**: Pre-processing of the datasets (Cleaning and normalising the dataset).
- **Stage III**: Splitting the datasets (splitting the data for training and testing).
- **Stage IV**: Further processing for the split datasets (feature selection + data-filtering for the split datasets).
- **Stage V**: Classifiers development and modelling approach (Developing of individual hybrid and ensemble classifiers).
- **Stage VI**: Performance evaluation measurement for the developed classifiers (Using different evaluation metrics to assess the classifiers results efficiency).

Stage VII. Statistically test the developed classifiers for their significance (Validate the classifiers predictions performance statistically).

3.9 Summary

In this chapter, the main stages of the experimental design framework of the proposed model of this thesis were overviewed. The development of credit-scoring models is a not a simple practice; it encompasses the collection of datasets and their pre-processing, with the designs validating and implementing the model. Therefore, the experimental design framework of the proposed model comprises several essential methods that will lead to a comprehensive and

reliable experimental modelling design of a credit-scoring model. In summary, each stage of the proposed model is fully demonstrated experimentally in the next coming chapters.

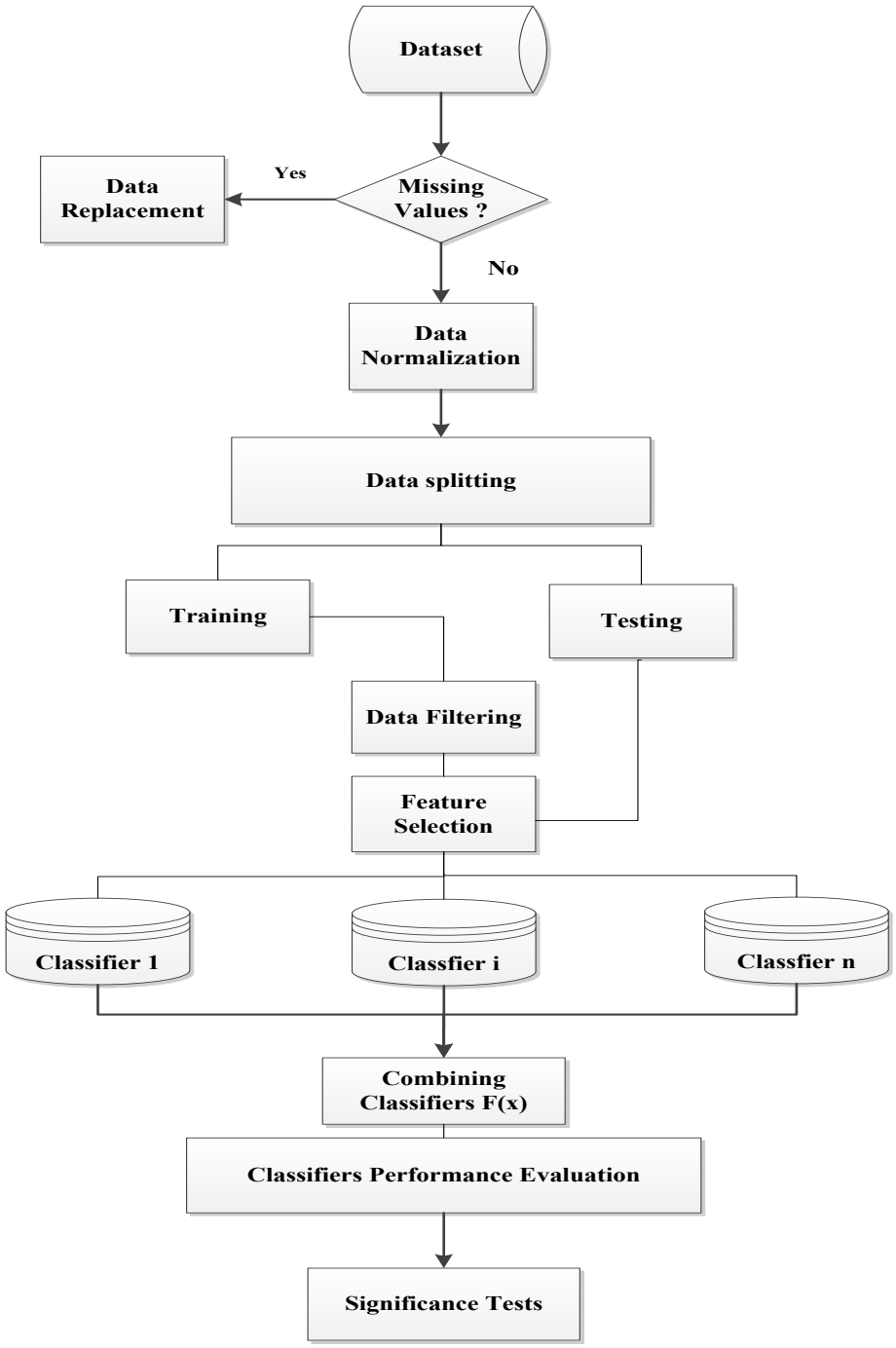


Figure 3.2 The main stages of the experimental design for the proposed model

CHAPTER 4

CREDIT-SCORING MODELS USING INDIVIDUAL CLASSIFIERS TECHNIQUES

4.1 Introduction

In this chapter, several methods are used to generate classifiers in order to evaluate their performance over seven credit datasets. This chapter demonstrates the development of the individual base classifiers and compare their performance amongst each other to assess the extent to which the classifiers can perform on different datasets and by using various evaluation performance metrics. Then, based on the classification results, the need to delve deeper is justified. In reference to the literature review discussion in Chapter 2 and regarding the credit classification problem, the 6 most commonly used classification methods in the literature of credit scoring can be distinguished.

These methods are well-known and are easy to implement, which facilitates banks or credit cards companies in quickly evaluating the creditworthiness of clients. Therefore, 6 base classifiers are analysed, namely, NN, SVM, DT, NB, LR and RF. Each of these classifiers has its own parameters to be set-up based on each dataset. It is worth noting that only the first five classifiers could be considered as individual classifiers: Inherently, RF is a homogenous ensemble of many DT but, due to its significantly high performance in credit-scoring field the decision has been made for it to be included into the group of base classifiers (Lessmann *et al*., 2013). LR is served as a benchmark classifier; the objective is to compare results of the classifiers with performance of LR which serves as the industry standard for credit-scoring modelling. All the experiments of this thesis are performed using Matlab 2014b version, on a PC with 3.4 GHz, Intel CORE i7 and 8 GB RAM, using Microsoft Windows 7 operating system.

4.2 Individual Classifiers Development

The most important step of classifier training and development is the selection of its parameters. In general, credit-scoring datasets vary in their features, and building an effective scoring model that can deal with different datasets features is crucial. Therefore, in the training phase, selecting the best parameters for each classifier is very important in order to evaluate the classifiers and accordingly achieve good performance results. This section will present: 1) input data to the classifiers has to be prepared and the pre-processed to feed the classifiers; and 2) the development and training procedure for each selected classifier which will serve as the base classifiers for the whole proposed developed model in terms of the classifiers parameters selection and tuning is carried out.

4.2.1 Data Pre-processing and Preparation for Training and Evaluation

Prior to fusing data in to classifiers, data have to be prepared and pre-processed in order to guarantee the quality of the data and, as a result, improve and ease the knowledge discovery process of the developed classifiers. In this particular stage, data is pre-processed by:

- Imputation of the missing values if occurred in each dataset.
- Then the normalisation the input data of each dataset.

Subsequently, data is ready to be partitioned into training and testing set with 10 × 5 cross-validation. During the training phase, an important stage of classifier-building and development is conducted, which is selecting and tuning classifiers parameters. Following, the selected and tuned parameters are used to evaluate and test the performance of the developed classifiers. The developed classifiers parameters selection and tuning process is carried out in the next subsection.

4.2.2 Classifiers Parameters Selection and Tuning

All classifiers, with the exception of NB and LR, require parameter selection and tuning. Moreover, in order to reach optimal results performance, each classifier might have different parameters depending on the type of dataset evaluated. Below is the description of the parameters set for each classifier depending on the dataset used. In addition to the description below a summary of all parameters for all classifiers will be also provided (Please refer to Appendix A).

- **NN:** The main issue in developing a NN classifier is finding the most suitable arrangement of learning function, transfer function, training function, learning speed and the structure topology of the network in terms of number of hidden neurons into hidden layers so as to solve the classification problem in hand. The developed NN classifier is based on the back propagation learning algorithm; therefore, various parameters have been selected and tuned, based on the features of input data. The transfer function from the hidden layer is chosen to be $'tansig'$ the hyperbolic tangent. This is the most common transfer function used in NN.

$$tanh(x) = \frac{2}{1-e^{-2x}} + 1 \tag{4.1}$$

As a transfer function from output layer pure linear *'purlin'* function was chosen; this means that single output from a hidden layer does not change but serves as a final decision of NN classifier. Regarding training functions, many functions are available in Matlab NN Toolbox, such as *trainlm*, *traingda and traingdx.* The purpose of these training functions is to train the network by updating inputs weights in order to achieve the optimal output value. Besides, for every particular dataset, it is important to change the way in which the NN is trained: in German, Australian, Polish, UCSD and Jordanian datasets it stay default ($trainlm$), whereas in Japanese and Iranian datasets it change to $\{traingdx, traingda\}$ respectively. For the Japanese dataset a momentum the default parameter of 0.9 was chosen. For all other datasets, momentum was not defined as $trainlm$ and $traingda$ training methods do not require this parameter. Regarding the network structure, one hidden layer is used, and the numbers of neurons in the hidden layers were chosen for reasons of obtained model complexity: the more neurons in the hidden layer are, the more complex evaluated model is, but in this case training become much slower and a risk of overfitting may occur. For datasets German, Australian, Japanese, Iranian, Polish, UCSD and Jordanian the chosen number of neurons in the hidden layer are {4, 10, 3, 10, 10, 10, 10} respectively. For most of the datasets, value '10' shows the best performance, but in case of German dataset, NN with only 4 hidden neurons fits input the best way. Generally, the number of hidden neurons should be chosen relatively to number and complexity of relations between input features for each dataset. In the developed NN classifier, a grid search was carried out to find the optimal number of neurons in the hidden layer for each dataset. The learning rate is default (0.01) in the case of Australian, Japanese, Iranian, UCSD and Jordanian datasets, while in the German and Polish datasets the values

changed to 0.005 and 0.5 respectively. Maximal number of epochs is set to 1000, but almost always training process finishes by not reaching maximum epoch.

- **SVM:** The idea of the SVM lies in the basis that if the input data is not linearly separable lie in moving to a space with higher dimension, in which positive and negative samples would be linearly separable, this is done by using the kernel function. The common kernel functions used by SVM can be:

- **Linear**: Linear kernel, meaning dot product.
- **Quadratic**: Quadratic kernel.
- **Polynomial**: Polynomial kernel
- **RBF**: Gaussian Radial Basis Function kernel with has a hyper parameter scaling factor called sigma.

Throughout the implementing of the SVM classifier, the RBF kernel function was used. For each dataset, different values of kernel scale parameters were provided (German – '1.37', Australian, Japanese, UCSD, Polish and Jordanian: 'auto', Iranian: '1'). Thus, for the majority of datasets, the SVM function automatically chooses the appropriate kernel scale. However, for some datasets, values for the kernel scale parameter that increases SVM Accuracy were found by grid search in comparison to the default ('auto') parameter.

- **RF:** The main parameter of the RF classifier is number of trees N, N was set to 60 based on the best accuracy and computational time on training set. Method used for RF is 'regression', as it gives better results than default Matlab method. Another parameter is number of attributes chosen for growing each DT, the default value was selected (all attributes available in dataset). Another important issue is worth noting is the defining of the categorical variables for each analysed dataset. Number of the features which were define as categorical during the RF evaluation is less than initial number of categorical features because some categorical features is better to consider as numerical. According to (Rhemtulla *et al*., 2012) when the categorical variables have many levels, there is a considerable advantage to treat it as continuous variable. Let's make an example: in some dataset there is 'Education' feature, and '0' means 'No education', '1' – 'ordinary school', '2' – MSc '3' – 'PhD', Here feature can be considered as numerical, the bigger value of this feature is, the smarter is loan

applicant. So during leaf splitting, RF shouldn't iterate over all values of this feature, but can simply define the leaf threshold, which is more efficient in this case.

- **DT:** Since different datasets are assessed, and building a DT requires fine-tuning parameters according to every dataset. For each dataset list of categorical variables is defined, and passed as a parameter into DT algorithm. Categorical variables are being estimated separately from the main program. The impurity evaluation is performed according to Gini's diversity index. Another option is selecting the algorithm for best categorical variables split. For two classes, as it is the case, as an optimal was selected 'Exact' option. That means for categorical variable with C categories, all $2^{C-1}-1$ combinations are considered. After estimating the categorical attributes and defining additional options the only thing left to be done is to build and execute the tree. After building the tree, comes the pruning of the tree, pruning is the process of deletion part of the tree without big decrease of Accuracy. Pruning is used to simplify the model, and avoid the overfitting. As the available datasets do not have a huge amount of parameters, pruning was not used after DT training.

4.3 Experimental Results

This section demonstrates the results obtained from each individual base classifier (The results are evaluated by taking the average of 50 testing sets resulting from the 10 × 5 cross-validation) across the seven datasets adopted in this thesis. Moreover, six performance evaluation metrics were used to assess the classifiers performance. After obtaining the results, all the classifiers across all datasets were compared with the benchmark classifier LR which is considered the industry standard for credit-scoring modelling to determine the extent to which the performance of the individual classifiers can be compared to logistic regression. Table 4.1 show the results of the LR model, Tables 4.2 to 4.6 show the results for the NN, SVM, RF, DT and NB classifiers, respectively, along with a comparison with the LR in Figures 4.1 to 4.5. Subsequently, the classifier results obtained are analysed and discussed. The setup of all classifiers for all datasets is based on the parameters described in previous section.

- **LR**

LR is considered to be a benchmark classifier in this thesis. Table 4.1 demonstrates LR results across all datasets and for various performance metrics. LR Accuracy for the majority of datasets is better than NB, DT and NN, but AUC value of NN may be higher than AUC of LR. On the other hand, LR concedes to RF on all datasets, and sometimes performs worse than SVM. Brier Score and H-measure of LR are changing accordingly to the Accuracy (Brier Score is inversely proportional, H-measure is direct proportional), meaning there is no need to describe them separately. In general, LR is not best, but nonetheless it is a solid classifier that can be used for fast and quite accurate predictions.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7597	0.8641	0.8626	0.9239	0.7246	0.8240	0.8417
Sensitivity	0.8841	0.8585	0.8474	0.9702	07150	0.9698	0.6439
Specificity	0.4715	0.8700	0.8832	0.0305	0.7325	0.2420	0.9064
AUC	0.7798	0.9294	0.9171	0.6227	0.7405	0.7336	0.8824
Brier Score	0.1656	0.0999	0.1039	0.1005	0.2298	0.1354	0.1144
H-measure	0.2725	0.6352	0.6254	0.0623	0.2663	0.2205	0.4417

Table 4.1 LR results

Figure 4.1 demonstrates the ROC curve of LR across all datasets. In general, LRROC curves remain RF ones, albeit a bit lower. The three leaders are the Australian, Japanese and UCSD datasets, whereas other datasets have much lower ROC curves with bad shape.

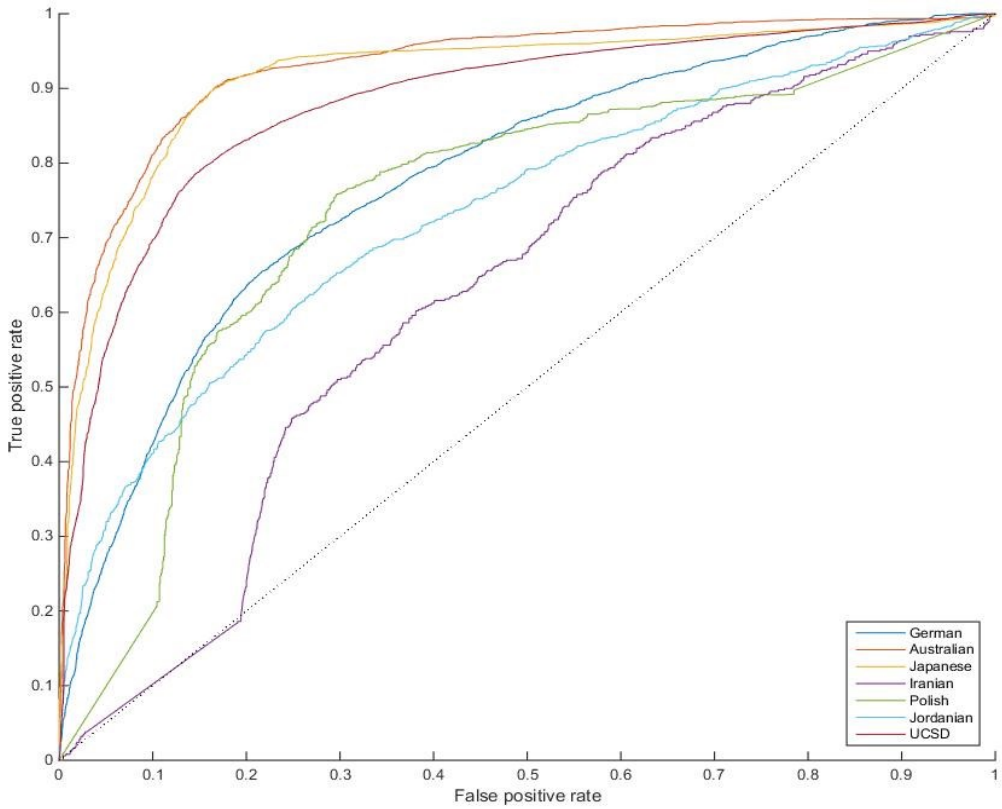


Figure 4.1 LR ROC curve for all datasets

- **NN**

Table 4.2 demonstrates the results of NN across all datasets. In general, the results of the NN are better than DT and NB in all datasets, and it comes behind RF in all datasets, and it rivals SVM in most on datasets.

Moreover, as can be seen from Figure 4.2, compared to Logistic Regression, NN shows worse results across all datasets except in the case of the Iranian dataset (Table 4.2). On the large dataset UCSD, NN shows a performance that is 1% worse than LR and 3% worse than RF, but which beats SVM (not by Accuracy, but by AUC and H-measure). Interestingly, for the Jordanian dataset, NN shows worse results than LR by Accuracy, but better results by specificity and AUC measure. This means that the NN is more preferable than LR when it is more important to achieve stability of results for a variety of thresholds and, besides, it is important to classify bad loans well. This fact describes that the first measure is completely inadequate in terms of making a decision as to which classifier is better; actually, the decision

should be made according to the demands to the predictions and possible threshold and classifying cost permissible set.

The Brier Score of NN shows similar results as SVM and better average results than LR, which shows that the certainty of NN is proportional to its output ranking, which can be used as an additional information during the decision making process.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7476	0.8588	0.8581	0.9499	0.6975	0.8148	0.8311
Sensitivity	0.8784	0.8504	0.8466	0.9980	0.6902	0.9352	0.5661
Specificity	0.4440	0.8679	0.8738	0.0120	0.7083	0.3334	0.9176
AUC	0.7638	0.9153	0.9118	0.6132	0.7673	0.7459	0.8590
Brier Score	0.1724	0.1080	0.1078	0.0476	0.1983	0.1423	0.1233
H-measure	0.2390	0.6144	0.6185	0.0615	0.2504	0.2378	0.3978

Table 4.2 NN results

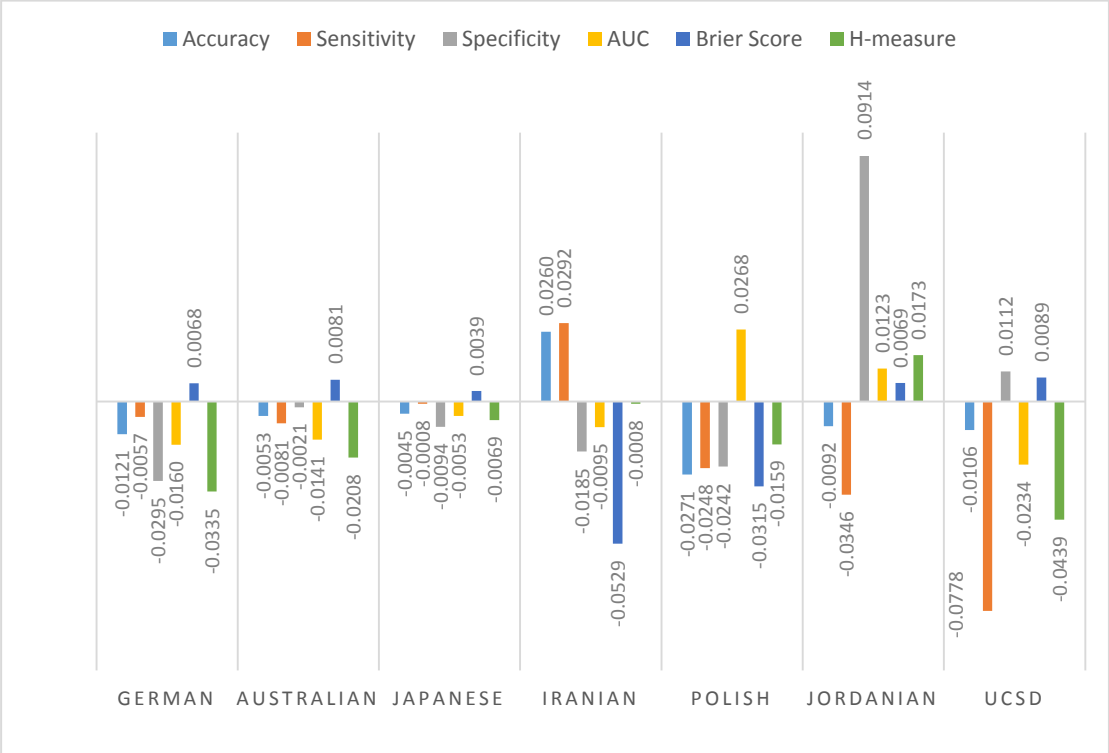


Figure 4.2 NN measures compared to LR

Figure 4.3 shows that NN has the best performance on the Australian and Japanese dataset. On the UCSD dataset, the ROC curve is a bit lower; however, its perfect shape tells that, for this dataset, NN can successfully predict good and bad loans for a wide range of thresholds.

The German, Jordanian, and Polish datasets ROC curves are almost equal by shape, which shows that NN is also quite a good classifier for similar datasets. The worst curve that can be seen is for Iranian dataset.

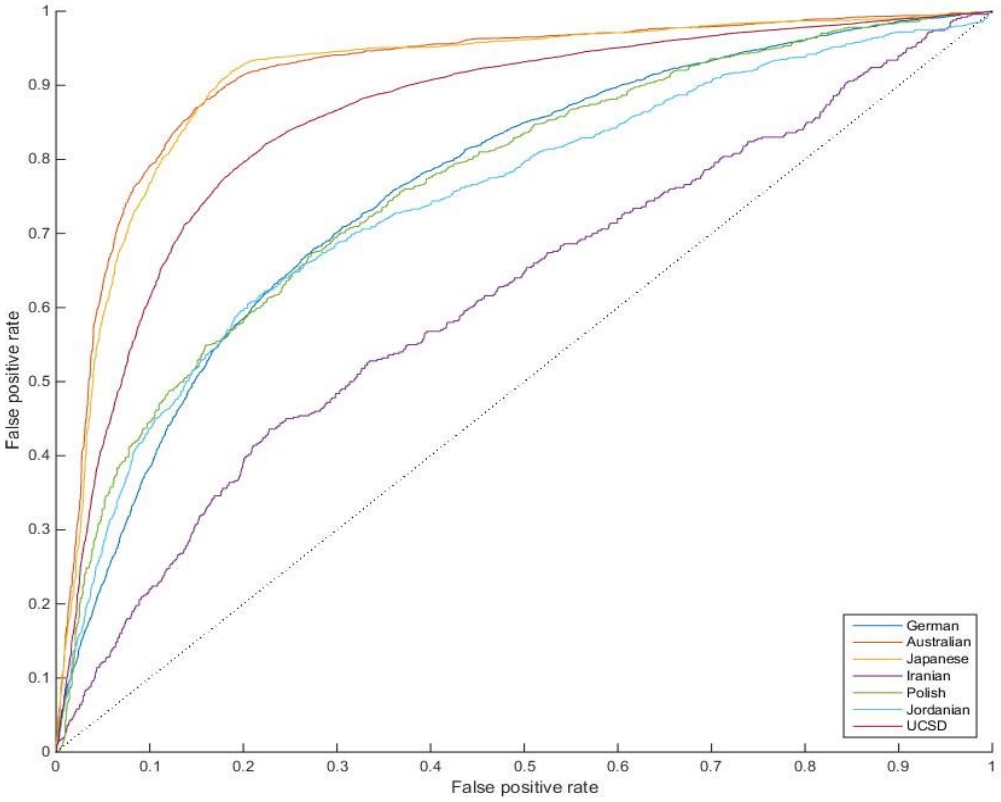


Figure 4.3 NN ROC curve for all datasets

- **SVM**

Table 4.1 demonstrates SVM results across all datasets and for various performance metrics; however, several findings can be summarised. Although SVM shows worse Accuracy results than LR on Australian, Japanese and UCSD datasets, the SVM classifier remains the main competitor of the RF. All the measures, which are almost the closest to the RF, are not described here; only special features and big differences.

The Accuracy of SVM differs from RF not more than by 1.45% on the Australian dataset; however, this difference increases to 2.5% and 3% on the Jordanian and UCSD datasets, respectively. In fact, SVM gives us less qualitative classification across all datasets, if comparing with the RF.

SVM in general is a very good classifying method; however, regarding SVM, there is a risk of over-training and/or mis-training over outliers in training data. This is why it is essential to use filtering, especially with this classifier. SVM performance also decreases in datasets with high-dimensional input (for example, Iranian and Polish datasets). This is why these datasets should use feature selection, where such use is more than justified.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7614	0.8523	0.8581	0.9482	0.7487	0.8298	0.8306
Sensitivity	0.9118	0.8540	0.8661	0.9980	0.6712	0.9655	0.5060
Specificity	0.4119	0.8498	0.8495	0.0029	0.8211	0.2872	0.9365
AUC	0.7826	0.9109	0.9071	0.6032	0.8206	0.7888	0.8431
Brier Score	0.1659	0.1135	0.1139	0.0505	0.1760	0.1284	0.1341
H-measure	0.2721	0.6024	0.6074	0.0733	0.3655	0.3085	0.3864

Table 4.3 SVM results

Looking at Figure 4.4, SVM beats LR on Iranian, Polish and Jordanian datasets, but it cannot be crowned as better regarding to the results of Australian and UCSD datasets. As UCSD dataset is real dataset, it is especially preferably in mind of achieving a good Accuracy on this dataset; with this task LR performing better than SVM.

Regarding Figure 4.5, it can be seen that SVM, like all other classifiers, has the best performance on Australian and Japanese datasets. UCSD datasets also should be outlined as those for which SVM gives pretty good-shaped and high ROC curve. All other datasets are much worse than these three leaders. The worst Datasets in terms of AUC again is the Iranian dataset, which its ROC curve lays close to the diagonal. This means that if it is necessary to significantly increase Specificity for the Iranian dataset, Sensitivity value will drop down till the level, where classifier performance and results became useless. So, for example, if it is needed to obtain 50% Specificity value using SVM, Sensitivity value will drop significantly from 99% to almost 60%, which is considered a completely bad performance.

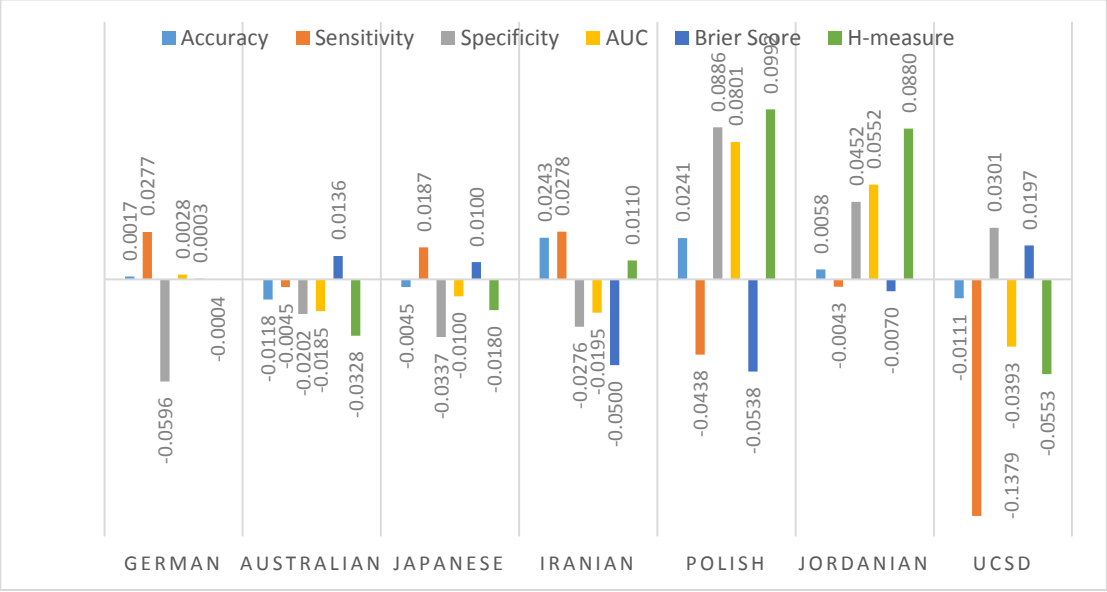


Figure 4.4 SVM measures compared to LR

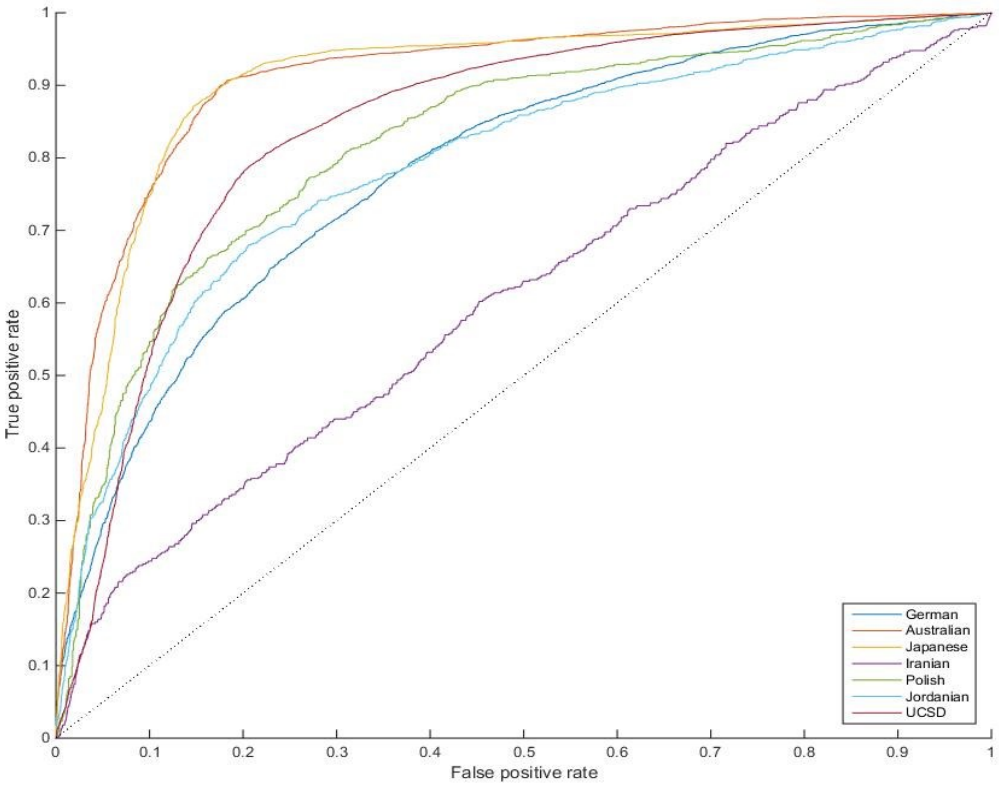


Figure 4.5 SVM ROC curve for all datasets

- **RF**

From Table 4.4 it can be inferred that the RF classifier shows the best Accuracy between all single classifiers on every dataset. Specificity and Sensitivity are the highest when compared to other classifiers in most cases; however, on Jordanian and UCSD datasets, this rivals with SVM classifier. On German and Iranian datasets, the Specificity lowers down to 0.44 and 0.056, respectively. The three highest results RF achieves is on Australian, Japanese and Iranian accordingly. The AUC of the RF classifier, as well as all previous measures, has the biggest and best results. Being just the best on German, Australian and Japanese datasets, it stands out on Iranian dataset. Two other measures are much better than other classifiers.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7669	0.8668	0.8674	0.9510	0.7625	0.8550	0.8619
Sensitivity	0.9078	0.8685	0.8708	0.9982	0.7548	0.9410	0.6690
Specificity	0.4394	0.8638	0.8640	0.0567	0.7768	0.5131	0.9250
AUC	0.7918	0.9360	0.9308	0.7787	0.8373	0.9087	0.9028
Brier Score	0.1616	0.0947	0.0979	0.0433	0.1646	0.0965	0.1005
H-measure	0.2885	0.6608	0.6485	0.2758	0.3840	0.5345	0.5127

Table 4.4 RF results

The best results between all classifiers may be explained in terms of Probability theory and good results of the DT classifier. In fact, RF classifier is just a modification of the DT; the difference lies in the number of decision finding layers. The first layer consists of DT, when the second is analysing the results of the first layer. The Probability theory prompts: the more trees you have in your forest, the more weighted and equitable results you achieve. Of course, the amount of trees should not be infinite; it is easy to get lost with all possible trees. RF is the only homogeneous ensemble analysed in this chapter, and shows stable and robust results, meaning it shows a prospects of using this classifier in credit-scoring models.

Figure 4.6 shows RF in comparison with LR across all performance measures. It is clear that RF beats LR in all measures significantly except the Brier Score and Specificity measure.

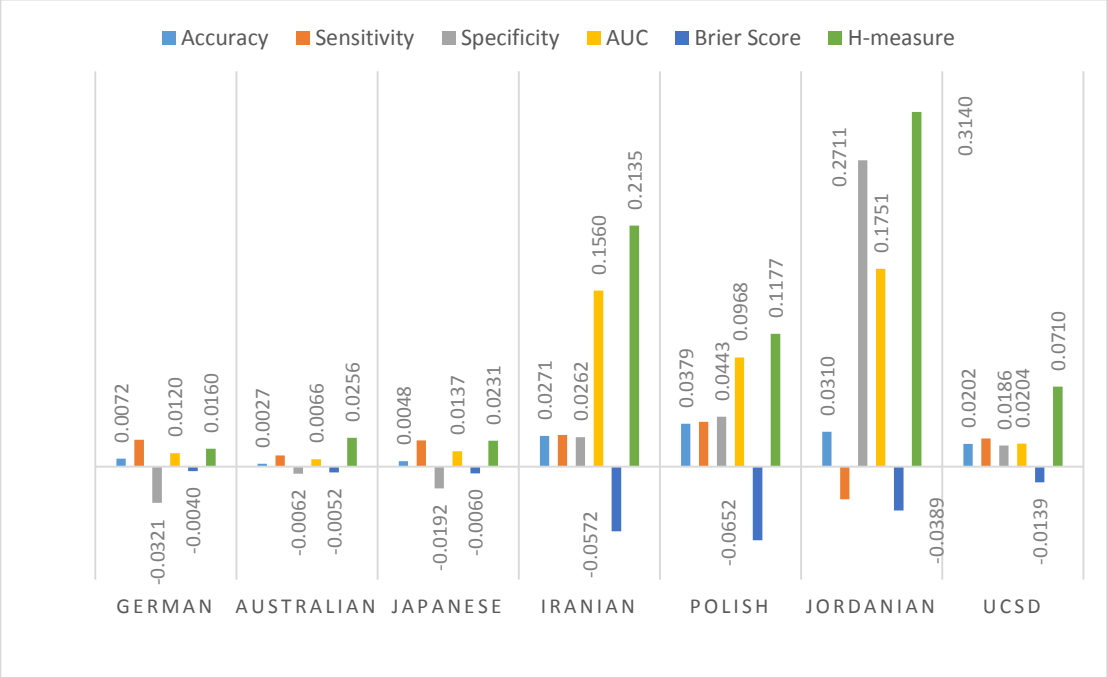


Figure 4.6 RF measures compared to LR

Regarding Figure 4.7, RF has the best performance on Australian and Japanese dataset. In general, RF ROC curves are very similar to those of the DT, but they are shifted more so to the left upper corner, which says that RF is definitely better than the DT. Even for the Iranian dataset, the ROC curve is only slightly worse than the German dataset. For the Polish dataset, despite its lower Accuracy, the ROC curve is much higher than German dataset curve. Therefore, it can be concluded that RF is more efficient (in terms of threshold choosing) for balanced datasets (like Australian, Japanese and Polish), even if there are many noise and outliers in them.

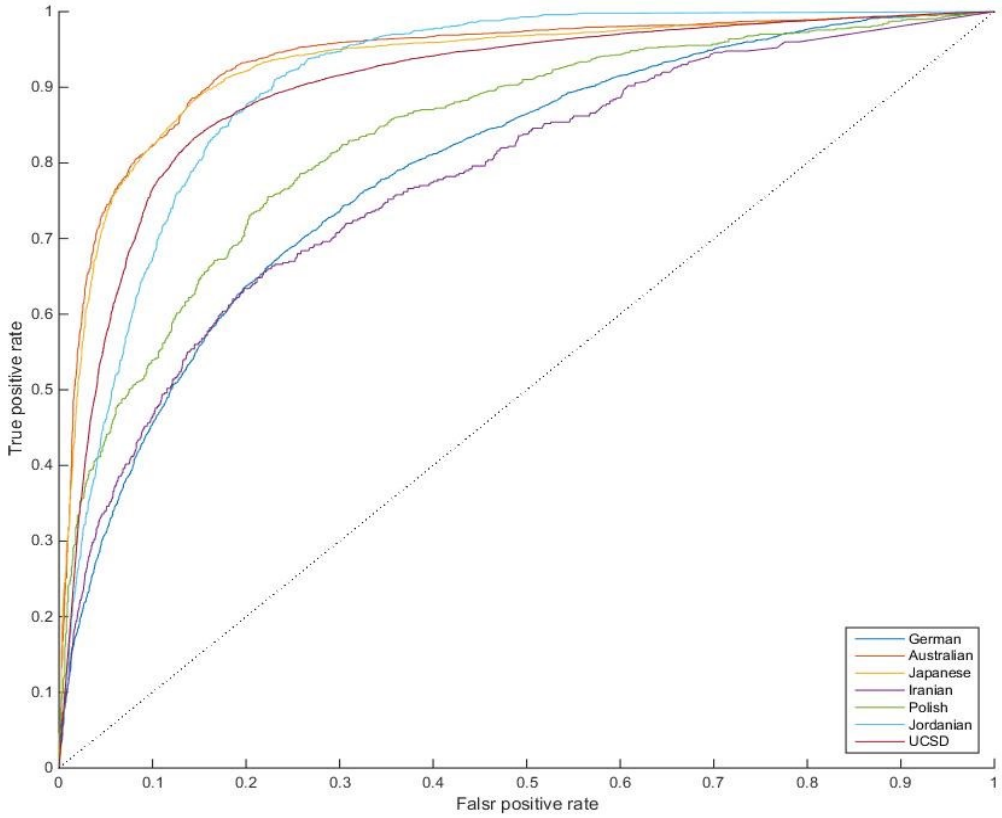


Figure 4.7 RF ROC curve for all datasets

- **DT**

Table 4.5 reveals that the DT classifier is one of the least accurate single classifiers on German, Australian, Japanese, Polish and UCSD datasets. Although it is the worst classifier, results on the Jordanian dataset are amongst the best with SVM and RFs. The reason is that the DT are very sensitive to the number of features, and is inaccurate when it is large. Since the Jordanian dataset has the least number of features, the DT becomes a good choice. In regards the Brier score being stable across all datasets, for this dataset, it lowers it down to 0.2665. This could be explained with a higher Accuracy mark. The only measure left is the H-measure, where the best value has been achieved on datasets but is still the worst compared to other classifiers.

The DT, by its structure, does not show robust results over multiple training-testing iterations. This is why it can be seen as relatively bad results on small datasets as Polish, as well as bad results on complex and big datasets, such as German, Japanese and UCSD. The reason is that

building the optimal DT is a NP-complete task, and existing algorithms sometimes fail to build well-performed DT due to the wrong choice of features. Another reason for the bad performance of this classifier is he lack of bad loans in training data. Figure 4.8 shows DT in comparison with LR across all performance measures it can be seen how worse it is compared to LR except for Brier Score in almost all datasets and Specificity in Jordanian dataset.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7045	0.8258	0.8171	0.9238	0.7013	0.8278	0.8201
Sensitivity	0.8001	0.8562	0.8470	0.9641	0.6862	0.8948	0.6178
Specificity	0.4827	0.7886	0.7814	0.1638	0.7177	0.5617	0.8866
AUC	0.6793	0.8664	0.8556	0.6150	0.7278	0.7950	0.7882
Brier Score	0.2521	0.1470	0.1570	0.0698	0.2665	0.1427	0.1586
H-measure	0.1362	0.5153	0.4893	0.1123	0.2097	0.3852	0.3296

Table 4.5 DT results

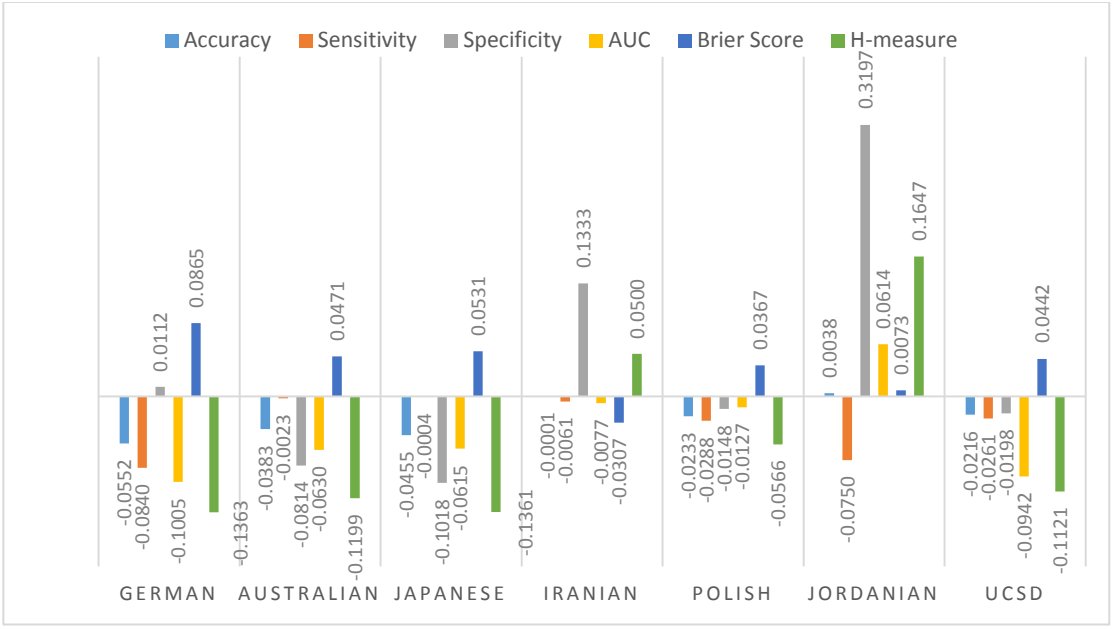


Figure 4.8 DT measures compared to LR

Regarding Figure 4.9, DT show the best performance on Australian and Japanese datasets. On Jordanian and UCSD datasets, the results also are good, but are a bit skewed because of these two datasets and their imbalance. Despite the high Accuracy, the worst curve can be seen is

for Iranian dataset; this may be explained by the fact that Iranian dataset is very skewed.

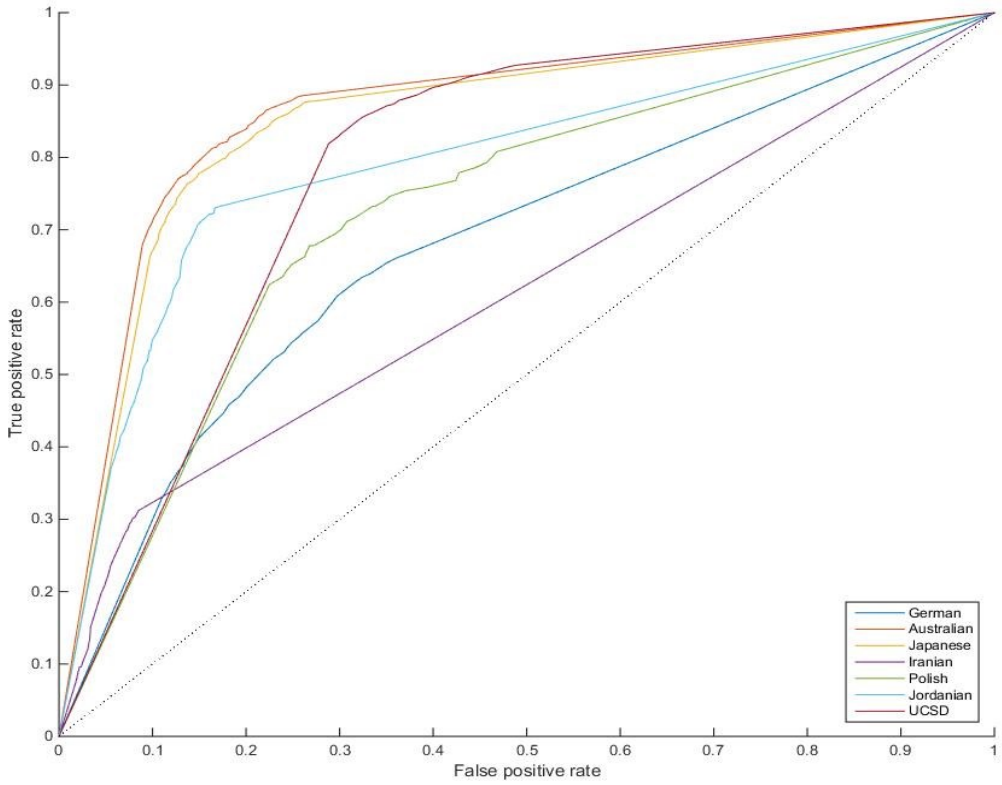


Figure 4.9 DT ROC curve for all datasets

- **NB**

As shown in Table 4.6, the NB classifiers results on German, Australian, Japanese and Iranian datasets can rival the DT by the Accuracy measure. On the Jordanian and UCSD datasets, NB is the least accurate classifier (particularly on UCSD dataset, notably the least accurate dataset ever for NB). It NB is sensitive to it. As far as it is assumed that features are independent, some important information may be lost throughout the construction of NBs model. In fact, NB performance is worse than LR performance in three cases: on large datasets, on unbalanced datasets, and on datasets with large number of features. Moreover, since UCSD dataset satisfies all these three conditions, Accuracy of NB classifier compared to other classifiers drops down very much (see Figure 4.10).

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7250	0.8030	0.7970	0.9262	0.6896	0.8106	0.6140
Sensitivity	0.7639	0.9094	0.9043	0.9622	0.8028	0.9581	0.0553
Specificity	0.6344	0.6707	0.6639	0.2500	0.5880	0.2180	0.7965
Type 1 error	0.2361	0.0906	0.0957	0.0378	0.1972	0.0419	0.9447
Type 2 error	0.3656	0.3293	0.3361	0.7500	0.4120	0.7820	0.2035
AUC	0.7624	0.8962	0.8890	0.7135	0.7401	0.7070	0.5740
Brier Score	0.1994	0.1672	0.1748	0.0755	0.2974	0.1720	0.3143
H-measure	0.2384	0.5802	0.5666	0.1930	0.2346	0.1760	0.0803

Table 4.6 NB results

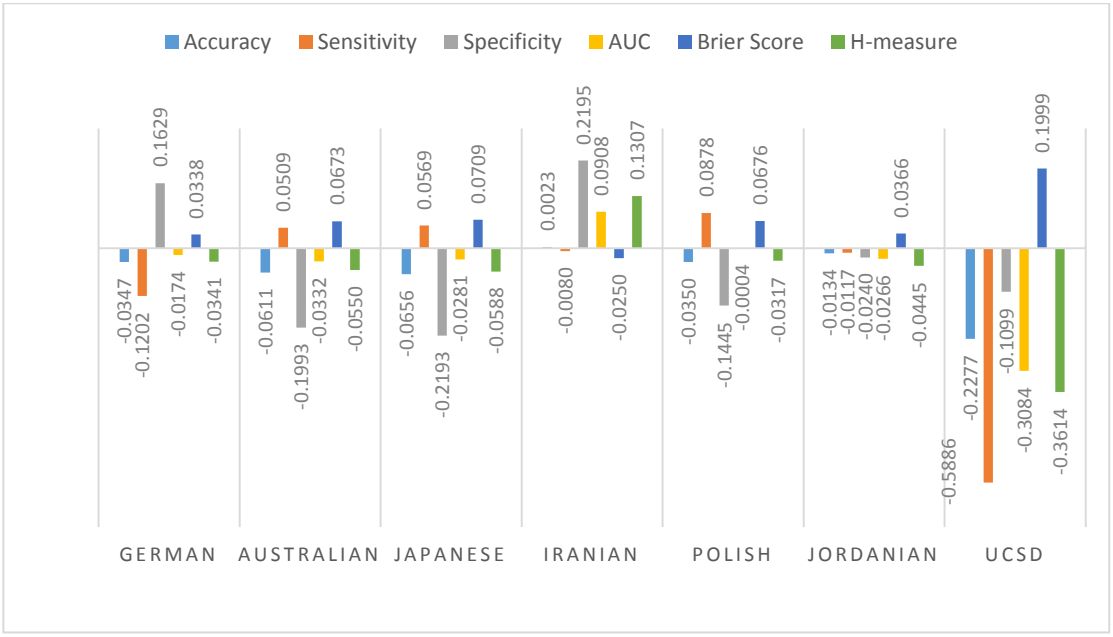


Figure 4.10 NB measures compared to Logistic Regression

Although NB classifier has the worst value of Sensitivity on Iranian dataset, the Specificity value is much greater than every other classifier in the German and Iranian dataset. The Brier Score remains the highest, meaning the worst, but the H-measure concedes only to the NN, SVM and RF.

As has been stated in the method description, NB classifier assumes that all attributes are independent of one another, but still able to give very accurate results. It seems clear that real attributes of a client, whose credit-scoring is being calculated, are never independent. The ideal Bayes classifier would compute the class without that Naive assumption, calculating the sought-for probability with each time for each attribute.

Regarding to Figure 4.11, NB, not surprisingly, has the best performance on the Australian and Japanese datasets. On the German dataset it has very solid ROC curve with a good shape. The ROC curves for all other datasets, with almost all the thresholds lower than ROC curves of three leaders. For the UCSD dataset, it is worth mentioning that, for some threshold values, the curve can be seen under the diagonal, from which the conclusion can be drawn that the Accuracy of NB classifier for the UCSD dataset for some threshold values could be under 50%, which makes no sense, and for these thresholds using NB classifier is completely unwanted, even on the best possible threshold NB gives around 61%, which is not good at all.

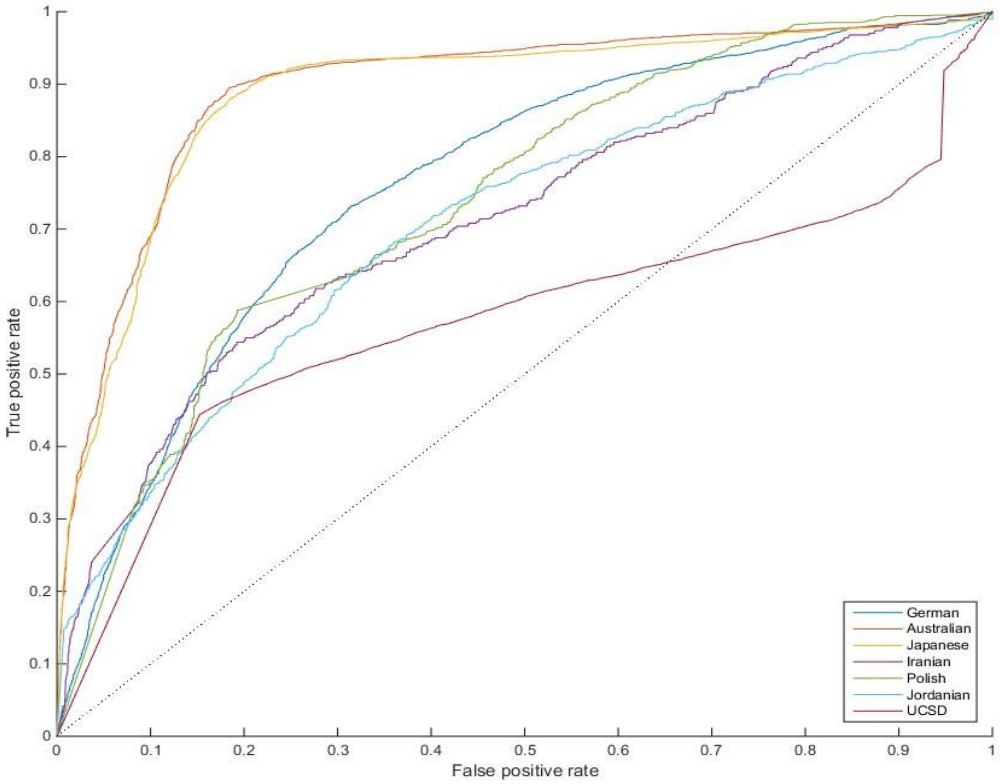


Figure 4.11 NB ROC curve for all datasets

4.4 Analysis and Discussion

Accuracy on the German dataset stands from 70.45% on DT classifier, to 76.76% on RF. Such low Accuracy, compatible with the percentage of good loans throughout the dataset, makes the DT not only the poorest classifier on this dataset, but also almost entirely useless for the classifier, which returns '0' as the prediction in all instances, meaning Accuracy would be 70%—only half of a percent lower. Specificity and sensitivity differ from classifier to classifier, but sensitivity mark is always better; this is caused by the ratio of good and bad

loans. Although sensitivity and specificity reflect the Accuracy, higher Accuracy does not mean that both of these measures will rise with the increase of Accuracy. AUC, Brier score and H-measure also reflect Accuracy, and moreover, this reflection is directly proportional for AUC and H-measure, and inversely proportional for Brier score for all classifiers.

Australian and Japanese datasets are very similar; the same number of good and bad loans has very similar attributes and also has similar results, which is why they are described together. The rating of classifiers differs somewhat to the German dataset: although RF is still the best, the poorest classifier is now NB with 80.3% on Australian dataset and 79.7% on Japanese dataset, unlike on the German dataset, where sensitivity and specificity do not depend on the good/bad loans ratio that much. Although these datasets have more good loans than bad, LR and NN have bigger specificity than sensitivity. AUC, Brier score and H-measure, as well as on German dataset, reflect the Accuracy of a classifier. However, NB does not have that feature, its AUC and H-measure measures are better than some other classifiers, although it has lower Accuracy.

Regarding the Iranian dataset, the rare situation is when every classifier is if not more accurate than LR, at the least almost the same (DT). The Iranian dataset, according to sensitivity and specificity values, shows that the higher Accuracy does not mean that the classifier is implicitly better. When considering the NN and NB classifiers, NN has a higher Accuracy but very low specificity, which means that this Accuracy is reached by setting most of the data to good loans, which provides high accuracy only because of dataset good/bad loans ratio. NB shows a different situation: the total Accuracy is lower; however, the value of specificity is much higher. This fact means that, with lower Accuracy, losses of using this classifier may be less than losses with more accurate classifiers. AUC, Brier score and H-measure only reflect the Accuracy of each classifier. Only the H-measure of NN shows that this classifier is strongly trained for one particular mistake price. In terms of Accuracy values, the Polish dataset is similar to the German dataset; however, the values of the least accurate classifier, with 68.96% Accuracy, are NB, not the DT, which is still less accurate than the Logistic Regression. The most accurate classifier remains the RF with 76.25% Accuracy. In relation to this dataset, there are more bad loans than good ones, which makes the picture of sensitivity and specificity contrast to the German dataset, meaning, in this dataset, specificity exceeds sensitivity. The only odd classifier is the NB as the most inaccurate classifier. This, by far, is the reason behind its in accuracy. AUC, Brier score and H-measure on this dataset,

does not reflect Accuracy values that much as on previous datasets; here, situations are slightly different. On this dataset, the definitions of these measures really matter: for example, less accurate NN is less dependent from threshold values than more accurate DT. In any case, RF exceeds every other classifier in every measure.

Regarding the Jordanian dataset, this dataset has 80% good loans, which means that a good classifier must have an Accuracy equating to higher than 80%. Every classifier on this dataset reaches this goal, and even the least accurate NB, for example, exceed it for more than 1%. The sensitivity and specificity ratio reflects the ratio of good/bad loans in the dataset, being at the same time a good addition to Accuracy. The most accurate classifier on this dataset is RF, although its sensitivity is not the highest; all those classifiers that have higher sensitivity have, at the same time, a much lower specificity; in its turn, this lowers classifier Accuracy. In other words, RF is the most balanced classifier amongst all. In fact, AUC, Brier score and H-measure are also significantly better than their closest rival, which makes RF an absolute leader.

A special feature of this dataset is that it consists of unfiltered real-world data. This fact makes it clear as to why results on this particular dataset differ so much from all obtained earlier. There is no such dataset, where results would make similar pictures. This influences every possible classifier and every results measure. Accuracy, for example, has an average value of 84%, if not counting NB, the accuracy of which constitutes only 61.4%. In the description of NB, it is said that it is assumed that all of the data features are independent from one another. If on other datasets it could work, this assumption here causes the Accuracy, which is even worse if it is said that all of the loans in the dataset are bad. In actual fact, NB is close to that, as far as its sensitivity is ten times lower than the other closest one. Other classifier measures are proportional. Now, with all results obtained, it is desirable to improve these results and decide which classifier can be considered the best.

To decide, let us look at the Accuracy comparison diagrams. They clearly show that the most common classifier Logistic Regression, with no doubt, wins every other classifier on Australian and Japanese datasets, except in the case of the RF. On Polish dataset, it rivals with SVM and RF; however, the difference is less than 0.4 %, so on these datasets the decision of the best classifier comes down to personal preferences. On Australian, Japanese and Iranian, the most desirable classifier seems to be one of SVM, RF or NN. The use of NB

and DT on these three datasets is unwarranted; their Accuracy is much lower than the proposed classifiers. To conclude, the commonness of LR classifier is clearly reasoned with its higher Accuracy on most possible datasets. There is no grounding in changing the classifier until it becomes better than LR on those datasets where it concedes. Looking on the comparing figures for all classifiers, one can conclude that LR is one of the best classifiers, slightly worse than RF, and for some datasets SVM. LR shows convincing results on Australian, Japanese and UCSD datasets Australian, Japanese and UCSD, and slightly worse results on German dataset. LR is a solid classifier, which works well on the data with a lot of features and possible inaccuracies. Table 4.7 summarize the best classifiers in term of their accuracies.

Accuracy	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD	Average Rank
RF	1	1	1	1	1	1	1	1
SVM	2	4	3.5	3	2	2	4	2.9
LR	3	2	2	5	3	4	2	3
NN	4	3	3.5	2	5	5	3	3.6
DT	6	5	5	6	4	3	5	4.9
NB	5	6	6	4	6	6	6	5.6

Table 4.7 Rankings of base classifiers based on their accuracy across all datasets

One trend suggested in this thesis to improve the classifiers all at once is to enable sample or instance filtering and feature selection. Sample filtering is an algorithm which finds the exceptions (outliers) within the training data set. Feature selection dedicates which features do not influence the result that much to be paid attention. These two methods might change the situation in favour of other classifiers. Sample filtering and feature selection Accuracy improving are the topic of the next chapter.

4.5 Summary

In this chapter, six state-of-the art individual base classifiers adopted in this thesis were demonstrated. A total of seven credit datasets were used to validate the classifiers and six performance measures were used to evaluate their performance: first the data was pre-proceeded and normalised; second, they were partitioned into training and testing sets using 10 x 5 cross-validations; and thirdly, classifiers' parameters were tuned on training datasets in

order to achieve the optimal value; and finally, classifiers were ready to be evaluated, with the final results found to be the average of 50 runs.

The classifier results have been developed with LR used as a benchmark classifier to be compared with the rest of the classifiers in order to determine the extent to which classifiers' results perform against the industry standard LR. Results vary from classier to classifier, with the RF showing the best performance across all classifiers and across all datasets, emphasising superiority over logistic regression, with SVM beating LR in some cases.

In the next chapter, the various hybrid approaches are used in order to enhance the performance of the model and to determine the extent to which complex modelling enhances the performance over individual classifiers, as well as over the industry LR.

CHAPTER 5

CREDIT-SCORING MODELS USING HYBRID CLASSIFIERS TECHNIQUES

5.1 Introduction

The target of the previous chapter was centred on the application of the state-of-the art classification techniques individually and the assessment of their performance. This chapter seeks to minimise the risks and accordingly improve the Accuracy over the individual classifiers by implementing the hybrid approaches, such as data-filtering and feature selection for classifiers. The rationale behind the feature selection is the idea that removing unimportant or Accuracy-understating features from the classifier arguments list will improve the classifier's Accuracy. The second method, filtering, is used to improve the results of machine-learning classifiers, which obviously should be trained on some training data before applying to testing entries. Filtering algorithm improves the training set of data by removing the inaccurate samples from the set. Inaccurate data samples are those that stand out from the whole picture: for example, an object of sample data stands all between the good samples, but the label of this sample is ‘bad’. In this case, such an object would be removed from the training set.

This chapter describes the data-filtering and feature selection algorithms and developing hybrid classifiers using these two approaches. After all, using obtained results, classifiers are compared comprehensively in terms of using feature selection and data-filtering separately and by combining them together; up to our best knowledge, combining data-filtering techniques with feature-selection techniques has not been considered before in the area of credit-scoring. Finally, the decision has been made that hybrid classifiers with filtering and feature selection together are better than considering them separately as well as applying classifiers individually. The results are compared to the industry standard LR and see the size of the improvement when used over individual classifiers.

5.2 Data-Filtering

As discussed in Chapter 3, the idea of the Gabriel Neighbourhood Graph editing (GNG) is used as a filtering algorithm, the idea of the GNG algorithm is based on the idea of proximity graphs which consist of a set of vertices and edges such that any two data points are connected if they satisfy some neighbourhood relations (Garcia *et al*., 2012). Hence, the main motivation of selecting GNG algorithm and proximity graphs is as following:

- Proximity graphs are used to avoid incoherentnes of data (when some places are full of points and some places have only few points).
- Proximity graphs find neighbors in each direction, so if some point has two neighbors, one and another just behind the first, the second will not count. This fact is unlike *k*-NN filtering, where directions of neighbors don't count.
- Proximity graph describe the structure of data very well, so for each point the algorithm finds the closest matches to it.

The efficient way to reflect structure of the data and interconnection between training set entries is to represent training data using graph structure. The simplest way is to connect two data points when they are close enough. So x_i and x_j are connected if $d(x_i.x_j) < \epsilon$, where ϵ is chosen manually). This method is easy but using this method in the case of non-uniform data distribution it is impossible to derive coherency of data representation: some graph areas is full of edges, but other areas will have a few edges. Moreover, it is not guaranteed that obtained graph is connected. Thus, another idea is to use proximity graphs, which are building without using fixed distance ϵ, but connects two data points regarding of their neighbours location with respect to them. So, the Gabriel Neighbourhood graphs (GNG) (Garcia *et al*., 2012) were used as a special case of proximity graphs to get a list of neighbours for each point from the training set.

The idea behind data-filtering is the selection of the data outliers, data points which labels are weakly suit to the labels of its neighbours, so it can be assumed that some mistake in data collection or representation was made and this data point may contain an error. Thus, the best way is not to include such data into training process. Now, let $d(\cdot\cdot)$ be the Euclidean distance in R^n. The GNG is defined as follows:

$$(x_i, x_j) \in E \Leftrightarrow d^2(x_i, x_j) \leq d^2(x_i, x_k) + d^2(x_j, x_k), \forall x_k, k \neq i, j \quad (5.1)$$

Figure 5.1 demonstrate the connection between 2 points in the GNG, in simple words, the idea is to connect the points i and j if and only if there is no point k inside the circle with segment $[i.j]$ as a diameter, otherwise the 2 point will not be connected. Euclidian distance is calculated as follows: given two points $x_i = (x_{i1}.x_{i2}.x_{i3}.\dots)$ and $x_j = (x_{j1}.x_{j2}.x_{j3}.\dots)$ Euclidean distance can be calculated as follows:

$$d(xi.xj) = \sqrt{(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2 + \cdots + (x_{in} - x_{jn})^2} \tag{5.2}$$

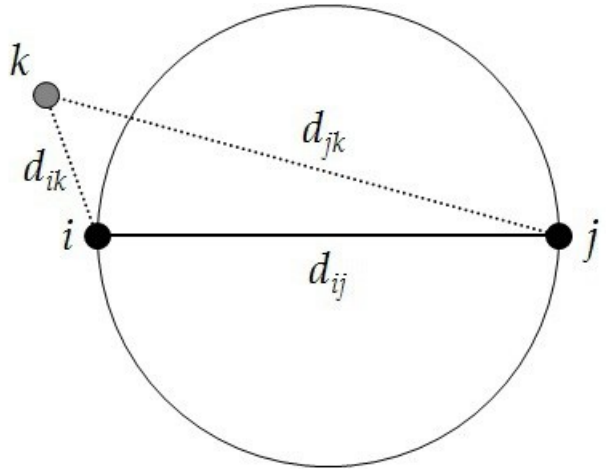


Figure 5.1 Illustration of GNG edge connection (Gabriel & Sokal, 1969)

Figure 5.2 illustrate the construction of a GNG on a 2-D training dataset and show how points are connected and the process of filtering data points depending on meeting certain conditions by the GNG algorithm.

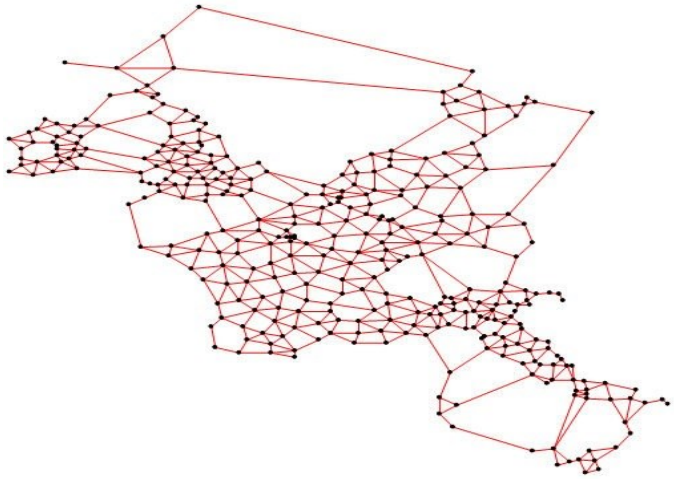


Figure 5.2 Construction of a Gabriel Neighbourhood Graph for a 2-D training dataset (Gabriel & Sokal, 1969)

As it can be seen from Figure 5.2, the GNG is constructed and training data points are connected, now for each sample x_* the weighted average for all neighbour labels to x_* are evaluated. In this case the weights chosen are proportional to the distance from each x_*. Also for every data point two scalar values, T_0 and T_1 are defined that can be interpreted as thresholds. Subsequently, two conditions are checked whether:

- If label of x_* equal to 0, and weighted average of all GNG is greater than T_0, x_* is removed or filtered from training set.
- If label of x_* equal to 1, and weighted average of all GNG is less than T_1, x_* is removed or filtered from training set.
- If neither condition is satisfied, x_*will remain in the training set.

In balanced datasets, where number of '0' labels is approximately equal to number of '1' labels, it is wise to use 0.5 as both T_0 and T_1 thresholds. But in the case of imbalanced datasets, when number of bad loans is far less than number of good loans, values of both thresholds equal to 0.5 leads to excessive filtration for entries with data labelled as '1'. Therefore the proposed new enhancement on the GNG in this thesis is using the following two approaches:

- Using weighted average for GNG instead of simple average in order to find to count far points less than close points for each x_* to more precisely find outliers.
- Using various thresholds to avoid excess filtering of bad loan entries in case the datasets are imbalanced.

For clear understating the steps of the GNG filtering algorithm is summarised in the following pseudo-code:

1. Compute GNG for all entries from training set:

For every pair x_i x_j from training set:

- Check whether to connect them in GNG using Equation (5.1).

End for

2. For each classifier optimal good loans and bad loans thresholds (Tr_0, Tr_1) are evaluated beforehand.

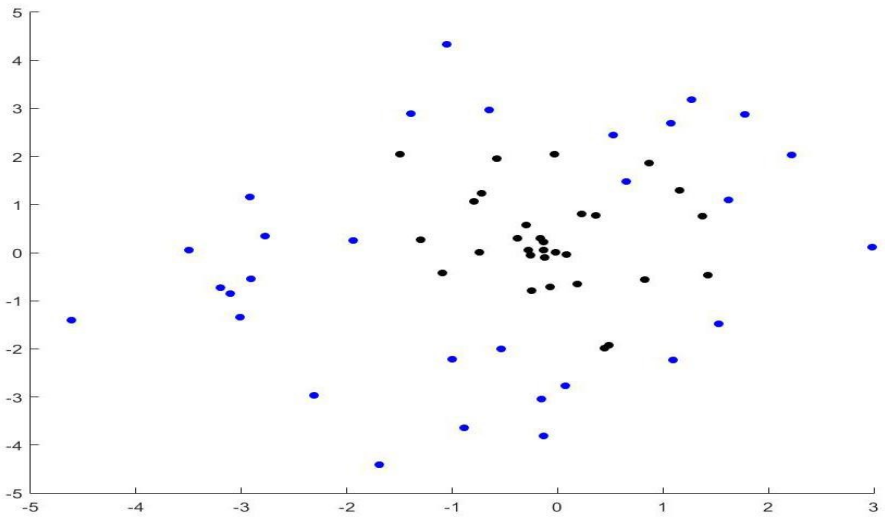
For each entry x_i of training set

- Compute the vector l which consists of actual labels of all x_i Gabriel Graph neighbours.
- Compute the vector w which consists of distances from x_* to its Gabriel Graph neighbours.
- Perform consequently such operations: $w = \max(w) - w$,
- Evaluate$l_* = w \cdot l$, where $\langle \cdot \rangle$ is a scalar product of vectors.
- If label of x_* equal to 0, and l_* is greater than Tr_0, x_* is removed from training set.
- If label of x_* equal to 1, and l_* is less than Tr_1, x_* is removed from training set.

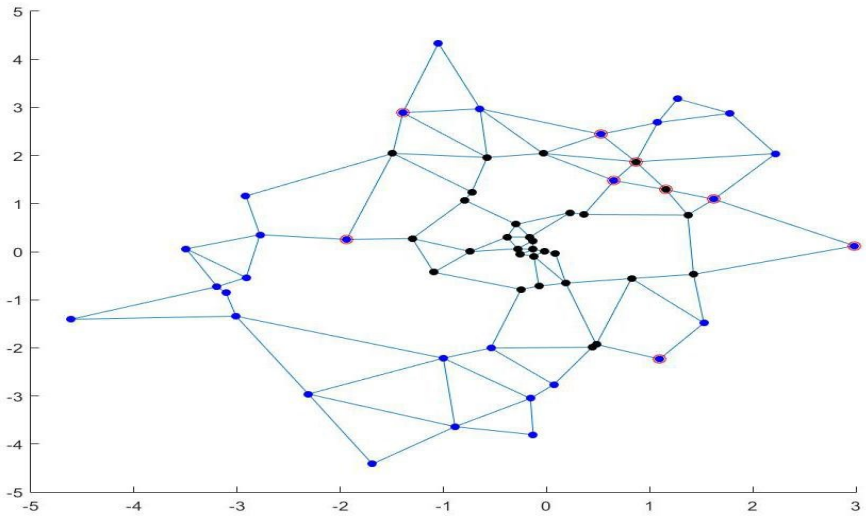
End for

3. Perform training stage of selected classifier over reduced training set.

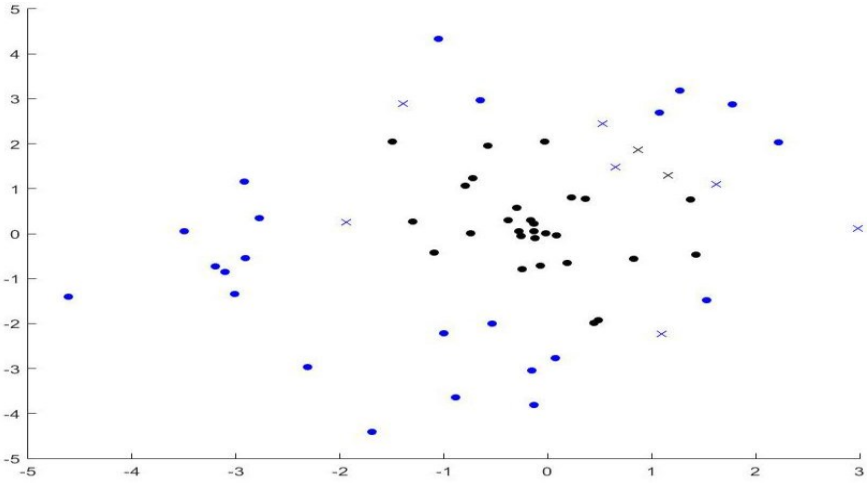
For illustration purposes the steps can be demonstrated in the following 2-D graphs (x-axis: could be any variable in a dataset, y-axis: is the loan decision) presented in Figure 5.3.



a. Initial training data



b. Building GNG using this training Data



c. Filtering entries which satisfy filtering conditions

Figure 5.3 Example of filtering process on a 2-D dataset

Figure 5.3 the process of filtering for a sample 2-D dataset of labelled points is clearly demonstrated. The first sub-graph is the initial training set before filtering. The second sub-graph demonstrates proximity GNG built over the training dataset. The red circles around the dots means that according to the GNG filtering algorithm they is filtered out. The third sub-graph demonstrates the result of filtering algorithm. The black and blue crosses are placed in the positions of the points that were filtered out from the training set and the reaming points are trained. After applying the GNG filtering algorithm on the training set of the 7 datasets of this thesis across the 5 base classification algorithms, the percentages of the filtered training data for each dataset across the several base classifiers are illustrated in Table 5.1.

Dataset	NN	SVM	RF	DT	NB	Average % of filtered data
German	1.4%	3.3%	1.4%	13.3%	14.1%	6.7%
Australian	6.67%	8.7%	12.1%	10%	11.44%	9.782%
Japanese	14.2%	10.43%	4.5%	10.29%	10.43%	9.97%
Iranian	0.5%	0.9%	0.5%	4.9%	0.5%	1.46%
Polish	15.4%	28.7%	1.7%	32.9%	22%	19.74%
Jordanian	4.8%	4.8%	17.8%	17.8%	3%	9.64%
UCSD	5.75%	2.83%	0.45%	5.75%	15.23%	6.00%
Average % of filtered data	6.96%	8.52%	5.49%	13.56%	10.96%	9.04%

Table 5.1 Filtered percentages of training data for all classifiers and datasets

Table 5.1 clearly shows that dataset with the biggest rate of filtered data is the Polish dataset, and dataset with the lowest rate of filtered data is the Iranian dataset. In general, it can be concluded that the more the dataset is balanced, the higher the rate the data could be filtered out (e.g., Australian, Japanese and Polish datasets). However, by choosing thresholds T_0 and T_1 different levels of filtered data can be obtained, but what is interesting, that the most efficient levels are bigger for balanced datasets. This can be explained, by the fact, that for imbalanced datasets it's not efficient to filter out too many data of minor class, as training set can become even more imbalanced, which, in general can badly affect the training process. Regarding the data of major class, this is difficult to choose the right threshold to avoid filtering out any important and valuable data points.

Classifiers which gave the best results with high level of filtered data are DT and NB, which can be easily explained by the structure of these classifiers. Classifiers which need low level of filtering are RF and NN. RF gives good results even without filtering, because this classifier use voting ensemble of DT, and even if some of them make wrong decision due to outlier data points, majority of other DT correct the prediction. Regarding the NN, it is well known that feedforward NN is stable with respect to noisy and incorrect data if the level of the noise and errors isn't too big.

In the next section results of the base classifiers across all datasets using the GNG filtering algorithm is summarised.

5.2.1 Classifiers Results Using the GNG filtering algorithm

Tables 5.2 to 5.6 summarise and discuss the results of the 5 base classifiers across 7 datasets evaluated on six performance measures (The results are evaluated by taking the average of 50 testing sets resulting from the 10 × 5 cross-validation). All the base classifiers is hybridized by applying the GNG filtering algorithm in order to filter the training data from noisy and outlier instances and deliver to the classifiers a new training set.

- **NN**

According to Table 5.2 the changes of the NN are in general are insignificant. NN, on the training phase, adjusts its weights to minimise mean-squared error at the output. Thus, even if some inputs are incorrect or outliers, the majority of good entries will overcome bad entries, thus NN will train properly. But with some datasets filtering makes results of NN much better (e.g., Polish dataset), because this dataset has noisier data than others (19.74% in average, according to Table 5.1). So NN backpropagation procedure fails to deals with such big number of bad entries, and Accuracy of NN became low.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7507	0.8588	0.8654	0.9488	0.7433	0.8198	0.8400
Sensitivity	0.8570	0.8224	0.8626	0.9984	0.7016	0.9317	0.5712
Specificity	0.5025	0.9030	0.8695	0.0065	0.7859	0.3733	0.9273
AUC	0.7663	0.9161	0.9077	0.6132	0.8058	0.7649	0.8573
Brier Score	0.1728	0.1095	0.1091	0.0481	0.1839	0.1368	0.1182
H-measure	0.2461	0.6233	0.6302	0.0703	0.3361	0.2685	0.4207

Table 5.2 NN results using GNG filtering algorithm

- **SVM**

Table 5.3 demonstrates that SVM with data-filtering is better than without data-filtering. However, the reason of the slight increase in Accuracy of the SVM classifier after filtering implementing lay in the problem of impossibility to build the perfect separating hyper plane. During the construction of this hyper plane, some of the samples are being filtered by the classifier itself: If good and bad loans are not linearly separable, the number of ignorable samples depends on the separating-in Accuracy parameter of soft-margin SVM algorithm.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7677	0.8626	0.8533	0.9464	0.7558	0.8342	0.8324
Sensitivity	0.8976	0.8494	0.8066	0.9958	0.6901	0.9603	0.5159
Specificity	0.4667	0.8786	0.9123	0.0095	0.8151	0.3285	0.9356
AUC	0.7956	0.9210	0.9113	0.6491	0.8095	0.8241	0.8442
Brier Score	0.1642	0.1050	0.1112	0.0508	0.1772	0.1191	0.1349
H-measure	0.2961	0.6356	0.6219	0.1166	0.3648	0.3859	0.3971

Table 5.3 SVM results using GNG filtering algorithm

- **RF**

Table 5.4 shows the stability of RF results with and without filtering and this could be explained by the nature of the RF itself. The classifying results of the RF classifier depend not only on the training inputs fed into it, but also on the number and parameters of its members the DT. RF implicitly filters the training input data by itself, and filtering algorithm does not change the result of its work a lot. The best Accuracy RF demonstrates on the Australian and the Iranian dataset, however the best values of AUC and H-measure classifier shows for Japanese dataset. For the UCSD dataset RF demonstrates solid results, only a bit worse than the results for the best two datasets.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7698	0.8677	0.8667	0.9508	0.7521	0.8638	0.8679
Sensitivity	0.8965	0.8734	0.8492	0.9979	0.7632	0.9682	0.6884
Specificity	0.4757	0.8596	0.8893	0.0587	0.7500	0.4486	0.9266
AUC	0.7927	0.9228	0.9292	0.7876	0.8344	0.8892	0.9155
Brier Score	0.1608	0.1006	0.0985	0.0430	0.1669	0.0998	0.0949
H-measure	0.2939	0.6475	0.6493	0.3010	0.3751	0.5138	0.5410

Table 5.4 RF results using GNG filtering algorithm

- **DT**

The results of Table 5.5 absolutely show that all performance measures of DT classifier were improved with filtering, only the Accuracy on Iranian dataset is an exception as it can be explained by the huge imbalance of this dataset (There is a big risk to filter out some bad loan entries that are useful for training process. As there is only 5% of bad loan entries, all filtered-

out bad loan entry increases the imbalance of Iranian dataset even more). The most significant raise amongst all measures can be noticed on AUC and H-measure characteristic. This can be explained by the fact, that filtering process often decreases the imbalance of dataset, so classifier results became more robust with respect to threshold and misclassification cost changing. Regarding to Brier Score, this characteristic also increased a lot: the filtering algorithm allows selecting the most accurate subset of the training data, making the classifier more certain about its correct predictions, which could be considered as useful improvement.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7446	0.8677	0.8635	0.9505	0.7513	0.8526	0.8343
Sensitivity	0.8874	0.8683	0.8425	1.0000	0.7380	0.9414	0.5776
Specificity	0.4143	0.8655	0.8896	0.0117	0.7666	0.4997	0.9182
AUC	0.6890	0.8876	0.8822	0.5303	0.7701	0.7630	0.7796
Brier Score	0.2301	0.1216	0.1274	0.0491	0.2338	0.1323	0.1503
H-measure	0.1815	0.6150	0.6068	0.0363	0.3115	0.3541	0.3444

Table 5.5 DT results using GNG filtering algorithm

- **NB**

As well as DT classifier's results, Table 5.6 show that NB results also are increasing after data-filtering. However, the reason of this improvement is that the filtering improves the distribution of the input training data, so if the normal distribution were used as as a parameter of NB classifier, this assumption became more close to the truth. Moreover, filtering increases the confidence of the classifier that is why the Brier Score also performed very well, indicating the NB is more confidence in its own decisions.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7590	0.8649	0.8630	0.9307	0.7262	0.8134	0.8069
Sensitivity	0.8680	0.8696	0.8579	0.9676	0.8800	0.9614	0.7317
Specificity	0.5052	0.8581	0.8702	0.2376	0.5904	0.2189	0.8314
AUC	0.7747	0.9109	0.9085	0.7224	0.7733	0.7101	0.8231
Brier Score	0.1981	0.1203	0.1221	0.0707	0.2637	0.1746	0.1908
H-measure	0.2675	0.6240	0.6225	0.2107	0.3021	0.1848	0.3730

Table 5.6 NB results using GNG filtering algorithm

5.3 Feature Selection

As discussed in Chapter 3, the main aim of feature selection is to choose an optimal subset of features for improving prediction Accuracy or decreasing the size of the structure without significantly decreasing prediction Accuracy of the classifier built using only the selected features. This type of data pre-processing is important for many classifiers as it increases learning speed and Accuracy over testing set. To fulfil this purpose MARS technique is used to determine the most valuable variables or features over the input data. MARS is widely accepted by researchers and practitioners for the following reasons (Lee et al. 2002) and that is why it is adopted in this thesis:

- MARS is capable of modeling complex non-linear relationship among variables without strong model assumptions.
- MARS can evaluate the relative importance of independent variables to the dependent variable when many potential independent variables are considered.
- MARS does not need long training process and hence can save lots of model building time, especially when working with a large dataset.
- Resulting model can be easily interpreted.

After finishing the training stage, the obtained MARS is as a mathematical generalised linear model, which fits the data well. But this model can be used not only to classify entries, but to analyse the input data and find the most important features, which are highly correlated with target labels. The aim of feature selection is to choose a subset of features for improving prediction Accuracy or decreasing the size of the structure without significantly decreasing prediction Accuracy of the classifier built using only the selected features. This type of pre-processing method is important for many classifiers as it increases learning speed and Accuracy over testing set. ANOVA decomposition is used over MARS mathematical model to determine the most valuable and important features over the input data. The main characteristic which distinguishes MARS from other classifiers is that result of MARS classifier is easily-interpreted model, and then ANOVA can be used with this model to make investigations of input data structure, particularly feature importance. ANOVA, obviously, cannot be used without MARS, and the results depend on how well MARS classifier is trained.

According to (Wood *et al*, 1997) ANOVA decomposition is summarised by one row for each MARS hinge function or product of these functions. Not all columns of ANOVA table are

used, only the second one, which gives the standard deviation of the function. This gives one indication of its relative importance to the overall model and can be interpreted in a manner similar to a standardized regression coefficient in a linear model. Each hinge function inside ANOVA table commonly responds to one feature, so the values of the second column are taken as an importance vector for all features. But if hinge function consists of N features, and there is some value A in the second column, then value to each of the features is added, that constitute this hinge function. The relative importance of each feature is computed by computing MARS for all iterations and afterwards evaluating ANOVA decomposition. For each iteration the second column of the ANOVA table is stored, and then at the end of the whole algorithm all these columns are mixed into the single one by computing the average value of each position for all columns. The steps of MARS feature selection process can be summarised in Pseudo-code as following:

1. Suppose there are N iterations, for each of them dataset is divided to training and testing parts.

For i **from** 1 to N **do**

Evaluate MARS algorithm over training set with given parameters:

- Maximum number of functions in the model allowed. The default value for this parameter is -1 in which case maxFuncs is calculated automatically using formula $min(200. max(20. 2d)) + 1$ (Jekabsons, 2016), where d is the number of input variables.
- Penalty value per knot. Larger values leads to fewer knots being placed (i.e., final models is simpler)
- Perform ANOVA decomposition of obtained model using $aresanova$ function. Store second column of this table as (i) . Thus, $w_k(i)$ denotes importance of k-th input feature during i-th iteration.

End For

2. Assume N_f is total number of features of the data.

For s from 1 to N_f **do**

$$w_s = \frac{\sum_{i=1}^{N} w_s(i)}{N} \tag{5.3}$$

End for

3. **Return** $w = (w_1.w_2.\dots w_{N_f})$
4. Suppose that for each single classifier optimal feature importance threshold T_{imp} was evaluated beforehand. Thus, for training and testing only features i are chosen, for which $w_i > T_{imp}$

As part of analysis of feature selection algorithm, the number of features that are filtered out from each dataset by each classifier is demonstrated in Table 5.7.

Datasets	All features	NN	SVM	RF	DT	NB
German	20	16	18	16	12	12
Australian	14	12	12	13	12	9
Japanese	15	15	15	15	12	15
Iranian	27	21	13	21	18	13
Polish	30	20	20	23	10	22
Jordanian	12	8	8	10	8	7
UCSD	38	15	16	38	15	15

Table 5.7 Number of selected features

The obtained figures in Table 5.7 clearly show that Australian and Japanese datasets are the two datasets that retain most of their features (only 2-3 features were removed). On the other hand, for Iranian and Polish datasets, features reduced massively (up to 20 features, in the for DT for Polish dataset). Some of the classifiers such as RF or NN are more resistant to a large number of features, that is why their results are high enough even with all features selected. Conversely, classifier as DT is very vulnerable to datasets with high amount of features. SVM in general is more stable to redundant features, but computational complexity of building optimal separating hyper plane in case of large-dimensional input space also increases a lot. Another thing worth mentioning is that MARS feature selection algorithm assigned '0' importance to 6 features on Iranian dataset, the reason is that Iranian dataset is highly imbalanced, that is why MARS computed very simple model in that case without these 6 features. Tables 5.7 to 5.13 illustrate weights of each of the dataset features

- **German dataset**

Feature #	1	2	4	5	3	6	10	14	7	9	11
Weight	6.06	3.98	3.58	3.48	3.46	2.74	2.13	1.75	1.65	1.64	1.62
Feature #	**15**	**8**	**20**	**13**	**12**	**19**	**17**	**16**	**18**		
Weight	1.56	1.43	1.42	1.14	1.13	0.45	0.42	0.14	0.02		

Table 5.8 Features importance for German dataset

- **Australian dataset**

Feature`s #	**8**	**10**	**5**	**3**	**14**	**7**	**4**	**13**	**12**
Weight	13.47	3.72	3.66	3.52	3.14	2.59	2.33	2.06	1.49
Feature #	**6**	**2**	**9**	**1, 11**					
Weight	0.36	0.34	0.06	0.00					

Table 5.9 Features importance for Australian dataset

- **Japanese dataset**

Feature #	**5**	**4**	**9**	**11**	**3**	**15**	**6**	**8**	**14**
Weight	17.77	16.08	14.06	3.81	3.35	3.28	2.67	2.46	2.15
Feature #	**7**	**2**	**1**	**10**	**13**	**12**			
Weight	1.10	0.22	0.14	0.09	0.08	0.02			

Table 5.10 Features importance for Japanese dataset

- **Iranian dataset**

Feature #	**1**	**8**	**3**	**19**	**21**	**6**	**9**	**4**	**2**
Weight	2.82	1.64	1.12	1.09	1.06	0.88	0.71	0.62	0.54
Feature #	**24**	**5**	**26**	**14**	**25**	**10**	**23**	**20**	**7**
Weight	0.17	0.15	0.13	0.13	0.07	0.06	0.06	0.06	0.05
Feature #	**13**	**11**	**27**	**12, 15, 16, 17, 18, 22**					
Weight	0.05	0.04	0.04	0.00					

Table 5.11 Features importance for Iranian dataset

- **Polish dataset**

Feature #	**22**	**21**	**30**	**6**	**13**	**18**	**10**	**5**	**23**
Weight	26.01	23.02	12.87	12.34	8.49	7.57	3.72	3.70	3.08
Feature #	**1**	**4**	**24**	**17**	**14**	**15**	**3**	**2**	**19**
Weight	2.96	2.67	2.59	2.52	2.50	2.41	2.31	1.74	1.43
Feature #	**9**	**25**	**8**	**7**	**12**	**27**	**26**	**11, 16, 20, 28, 29**	
Weight	0.93	0.64	0.41	0.23	0.11	0.06	0.06	0.00	

Table 5.12 Features importance for Polish dataset

- **Jordanian dataset**

Feature #	**8**	**9**	**6**	**5**	**10**	**3**	**1**	**7**	**11**	**2**	**4**
Weight	14.69	10.34	7.42	5.00	4.19	4.03	1.07	0.79	0.44	0.37	0

Table 5.13 Features importance for Jordanian dataset

- **UCSD dataset**

Feature #	**32**	**13**	**33**	**28**	**37**	**19**	**24**	**29**	**8**
Weight	7.28	6.07	3.78	3.60	3.28	2.34	2.15	1.98	1.55
Feature #	**11**	**27**	**17**	**26**	**30**	**2**	**18**	**1, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 16, 20, 21, 22, 23, 25, 31, 34, 35, 36, 38**	
Weight	1.52	1.09	0.68	0.25	0.19	0.12	0.03	0.00	

Table 5.14 Features importance for UCSD dataset

Tables 5.8–5.14 illustrate the importance of each feature in each dataset. As it can be seen, many features were weighted high that reflect its importance and some very low which reflect it uselessness. Also it can be noticed that the more features dataset has, the more of them appear to be unnecessary. The best example of it can be observed in the Iranian and the UCSD datasets. These are real world datasets, include a lot of information collected while deciding to loan grant a loan or not and it seems that it only stores space and rarely make difference in decisions.

5.3.1 Classifiers Results Using MARS

Tables 5.15 to 5.19 summarise and discuss the results of the 5 base classifiers across 7 datasets evaluated on 6 performance measures (The results are evaluated by taking the average of 50 testing sets resulting from the 10 × 5 cross-validation). All the base classifiers obtained results are based on applying MARS algorithm to compute the most important and valuable features for each classifier on each dataset.

- **NN**

According to Table 5.15, on the most of datasets, the Accuracy of the NN classifier rises up, but for some of them Accuracy remains the same and even slightly decrease (e.g., German and Iranian datasets). In general, NN results are stable against redundant and unimportant features, so the small rise of NN Accuracy can be considered as a success, even if for some datasets this rise is slight. Other NN measures change accordingly, and often the rise of other measures is even more than Accuracy rise (as for German dataset, where Accuracy remains almost the same, but AUC increases by 0.32%)

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7475	0.8649	0.8617	0.9494	0.6892	0.8382	0.8473
Sensitivity	0.8719	0.8584	0.8515	0.9991	0.6622	0.9326	0.6503
Specificity	0.4602	0.8720	0.8756	0.0037	0.7203	0.4589	0.9120
AUC	0.7670	0.9193	0.9110	0.6180	0.7715	0.8265	0.8889
Brier Score	0.1706	0.1040	0.1079	0.0478	0.1960	0.1209	0.1112
H-measure	0.2479	0.6289	0.6188	0.0609	0.2592	0.3775	0.4659

Table 5.15 NN results using MARS

- **SVM**

In this case, Table 5.16 shows the fact that the SVM classifier becomes better with feature selection is not a surprise. The fewer number of features used means that the points which are used in separating have fewer dimensions to be considered. This in its turn simplifies the process of building a separating hyper plane.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7655	0.8525	0.8575	0.9483	0.7538	0.8368	0.8444
Sensitivity	0.9153	0.8618	0.8692	0.9981	0.6844	0.9447	0.6336
Specificity	0.4178	0.8401	0.8444	0.0029	0.8199	0.4107	0.9134
AUC	0.7827	0.9051	0.9075	0.5655	0.8257	0.7960	0.8742
Brier Score	0.1647	0.1138	0.1143	0.0508	0.1731	0.1238	0.1393
H-measure	0.2807	0.5963	0.6077	0.0379	0.3744	0.3734	0.4558

Table 5.16 SVM results using MARS

- **RF**

As well as the filtering effect, Table 5.17 shows the effect of feature selection algorithm is mostly positive, but insignificant. This can be easily explained by the fact that RF consists of several DT, and each has different features included. So if the number of RF DT members is big enough, we'll definitely have a bunch of DT with the best Accuracy, which have only important features included, so if number of insignificant features is less than 20-30%, these will push DT to make a right decision. The reasons of classifier stability with feature selection are the same as its stability with filtering.

Although the Accuracy for some datasets has dropped down slightly, the drop was not more than tenths of a percent, which can be considered as statistically insignificant. Moreover, with almost the same Accuracy, most of other measures (e.g., AUC and H-measure) show a slight rise.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7667	0.8680	0.8662	0.9507	0.7679	0.8616	0.8664
Sensitivity	0.9031	0.8703	0.8694	0.9976	0.7638	0.9418	0.6793
Specificity	0.4499	0.8647	0.8628	0.0609	0.7776	0.5457	0.9376
AUC	0.7869	0.9403	0.9317	0.7896	0.8417	0.9203	0.9136
Brier Score	0.1628	0.0928	0.0976	0.0434	0.1624	0.0922	0.0960
H-measure	0.2835	0.6678	0.6509	0.2783	0.3950	0.5671	0.5375

Table 5.17 RF results using MARS

- **DT**

Looking at Table 5.18, unlike with the RF, the results of DT change a lot with the implementation of feature selection algorithm. The improvements are mostly positive; the only exception is the Accuracy on the Japanese dataset which drop is little. The reason of the general increment in the DT can be is in computation complexity of building the optimal DT as it grows exponentially with respect to number of features. For example, if the number of features increases by 10, optimal DT will compute 1000 times slower. That is why DT growing Matlab algorithm allows computational simplification and that is why it often builds non-optimal DT. Thus, the reason that results are increased comparing with classical DT classifier is similar to the case of enabling filtering. Each decision which node to divide and how to do that with presence of redundant features have the risk to be incorrect.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7212	0.8275	0.8167	0.9268	0.7254	0.8304	0.8544
Sensitivity	0.8198	0.8553	0.8444	0.9664	0.7179	0.8967	0.6319
Specificity	0.4924	0.7933	0.7837	0.1805	0.7344	0.5690	0.8875
AUC	0.6924	0.8676	0.8568	0.6394	0.7532	0.8092	0.8006
Brier Score	0.2392	0.1462	0.1563	0.0676	0.2412	0.1366	0.1521
H-measure	0.1561	0.5180	0.4929	0.1514	0.2637	0.4023	0.3497

Table 5.18 DT results using MARS

- **NB**

In Table 5.19 and in comparing to the first experiment Chapter 4, where NB was trained without data pre-processing, much better results with feature selection can be seen. The increase is the most obvious on a dataset with large amount of features (e.g., Iranian, Polish and UCSD datasets). In this experiment the features, which their impact on the entry label is minimal, are removed from training and testing data, so they have no opportunity to affect classifier's decision in the bad way. On the other hand, for Japanese dataset the Accuracy does not change, which is explained by the fact, that all features for NB classification were considered as important.

It is worth mentioning, that for UCSD dataset results of NB became better, but it remains the worst classifier with 62.5% Accuracy. So for large real-world datasets using such simple

classifiers with only feature selection and without other powerful pre-processing techniques is not a good idea.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7437	0.7849	0.7970	0.9447	0.7071	0.8210	0.6253
Sensitivity	0.8469	0.9202	0.9043	0.9866	0.7477	0.9875	0.0459
Specificity	0.5043	0.6161	0.6639	0.1555	0.6734	0.1540	0.8147
AUC	0.7678	0.8944	0.8890	0.7397	0.8011	0.7094	0.5930
Brier Score	0.1858	0.1735	0.1748	0.0547	0.2569	0.1692	0.3063
H-measure	0.2463	0.5739	0.5666	0.2181	0.3391	0.1771	0.0967

Table 5.19 NB results using MARS

5.4 Classifiers Results Using GNG + MARS

The previous experiments in this chapter were conducted on applying both GNG filtering algorithm and MARS feature selection on the base classifiers independently. Different results were obtained; this section combines both GNG filtering algorithm and MARS feature selection to see to what extent results can be enhanced. Consequently, Tables 5.20 to 5.24 summarise and discuss the results of the 5 base classifiers across 7 datasets evaluated on 6 performance measures, followed by comparison with LR in Figures 5.4 to 5.8 (The results are evaluated by taking the average of 50 testing sets resulting from the 10 × 5 cross-validation). Results are compared to the results of previous experiments, and assumptions are made of how and why filtering and feature selection affect various measures differently for different classifiers. Moreover, the results are compared to the industry standard LR and assess to what extent classifier results with hybrid modelling can outperform LR.

- **NN**

In NN results as in Table 5.20, the Accuracy for almost all datasets increases with implementing of GNG and MARS methods. The exact method, which causes positive changes, depends on a dataset, but together they always show the best result. Actually, NN does feature selection by itself implicitly: the principal of NN says that while learning, the network assigns the proper weight to each feature. If a feature is not considered as important, in NN, with or without feature selection, it does not influence the result. So external feature selection (what is been conducted in this chapter) on NN should not give significant

advantage. However, looking at the results and by applying feature selection, some datasets results increased (e.g., Jordanian and UCSD datasets). The reason of this that external feature selection removes features completely from the consideration, so if some feature applied to the input of NN, anyway it will have some impact, as weights from this feature are very unlikely to be all '0' at the same time. That is why sometimes feature selection improves the results of NN very much.

Sensitivity and Specificity change on every classifier in the same way. Absolutely the same thing could be said about other three performance measures. The NN algorithm by itself should not be very sensitive to redundant features and noisy outlier data. But indeed, the performance of NN shows advantage and importance of using filtering and feature selection as a pre-processing algorithm. The highest improvements are on Polish, Jordanian and UCSD datasets. Hence, this combination shows improvements on all 7 datasets.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7584	0.8643	0.8694	0.9500	0.7521	0.8454	0.8487
Sensitivity	0.8680	0.8584	0.8689	0.9994	0.7200	0.9325	0.6047
Specificity	0.5069	0.8713	0.8713	0.0127	0.7850	0.4994	0.9285
AUC	0.7717	0.9197	0.9073	0.6289	0.8060	0.8348	0.8825
Brier Score	0.1700	0.1035	0.1092	0.0472	0.1838	0.1193	0.1101
H-measure	0.2580	0.6313	0.6309	0.0747	0.3406	0.4042	0.4634

Table 5.20 NN results using GNG + MARS

As in Figure 5.4, with filtering and feature selection NN becomes a good alternative to the LR, showing better Accuracy on every dataset except German, and on this dataset it concedes on 0.1 percent, which is statistically insignificant difference. On average NN performance is in general better than LR in 4 datasets on the other four measures, So, on the datasets like these, NN shows solid results, stable against threshold and misclassifying cost change.

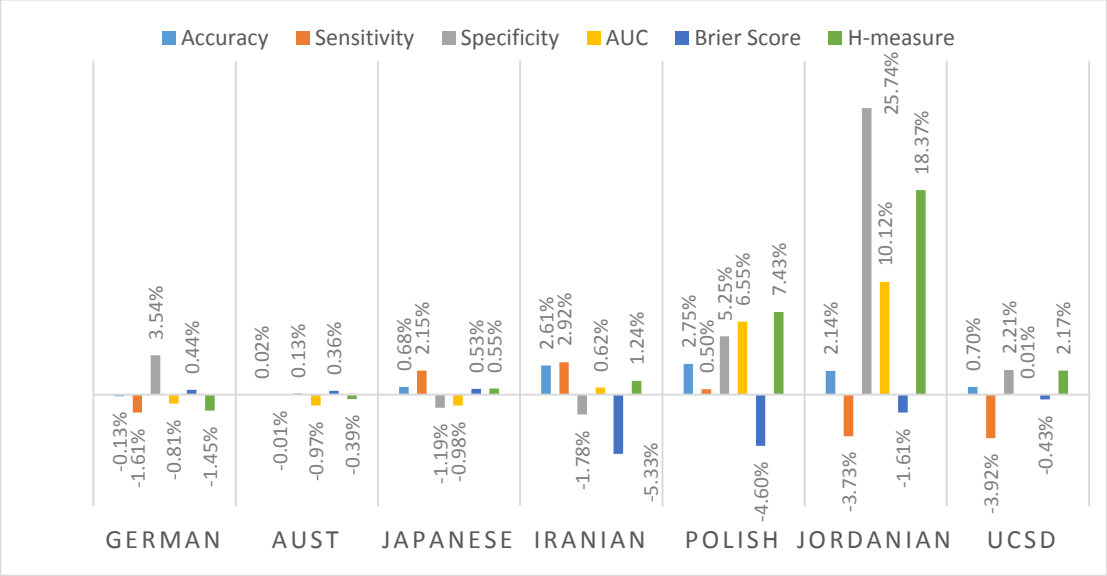


Figure 5.4 NN measures compared to Logistic Regression

The improvement of ROC curves of NN can be seen on all datasets except the Japanese, although this decrease can be caused by random fluctuations, thus it is considered as statistically insignificant. It can be concluded that filtering and feature selection improves the classifiers performance on all datasets (see Figure 5.5).

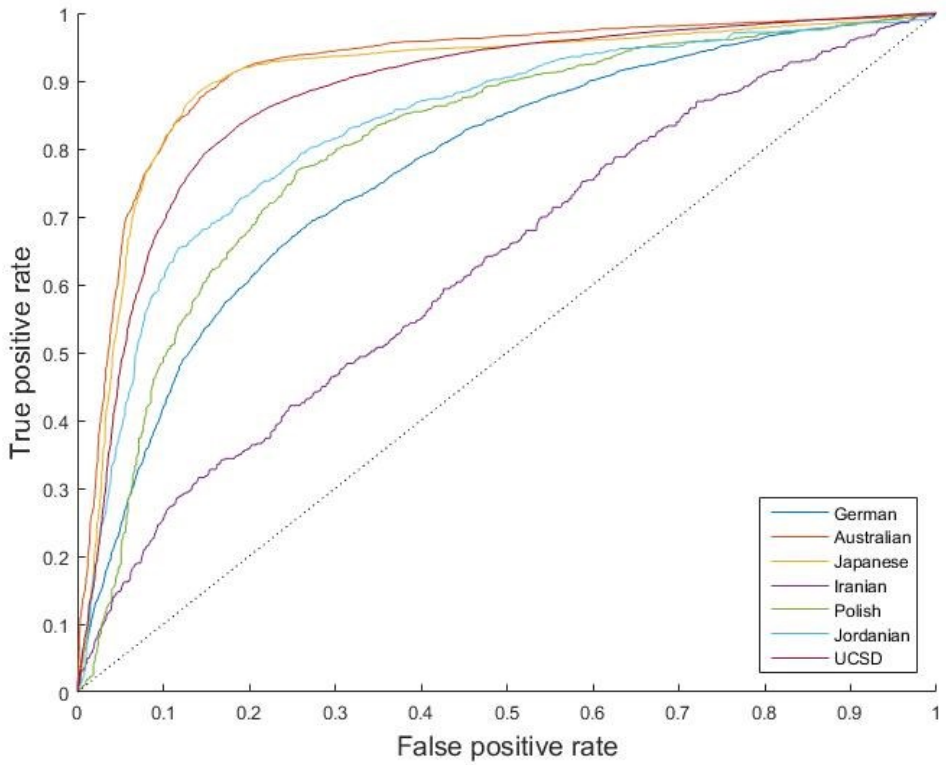


Figure 5.5 NNROC curve for all datasets

- **SVM**

In Table 5.21 and regarding to the filtering and feature selection, the Accuracy of SVM changes contrarily on different datasets. On German, Australian, Polish, Jordanian and UCSD datasets it raises, but on other two datasets it decreases. It is unexpected, because the reduction is caused by filtering, and filtering should conversely improve the Accuracy, simplifying the separating hyper plane. SVM is the only classifier which the impact of feature selection is more than data-filtering.

Sensitivity, Specificity and other measures change according to Accuracy changes for each dataset. The other thing is that feature selection gives more impact on SVM results than filtering, while for all other classifiers situation is exactly opposite. It can be concluded that SVM is very sensitive to the redundant features.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7733	0.8686	0.8538	0.9464	0.7571	0.8474	0.8455
Sensitivity	0.9038	0.8667	0.8069	0.9959	0.7024	0.9424	0.6362
Specificity	0.4703	0.8703	0.9129	0.0082	0.8080	0.4705	0.9140
AUC	0.7942	0.9209	0.9111	0.6123	0.8158	0.8300	0.8683
Brier Score	0.1643	0.1043	0.1112	0.0508	0.1749	0.1133	0.1433
H-measure	0.2985	0.6370	0.6215	0.0784	0.3700	0.4585	0.4548

Table 5.21 SVM results using GNG + MARS

Figure 5.6 clearly shows that SVM got higher results on average than LR on all datasets, except Japanese dataset. For Jordanian dataset, SVM shows much better AUC and H-measure, so for Jordanian dataset from these two measures exactly SVM should be chosen as more robust against threshold and misclassifying cost change. For the Australian dataset all measures of SVM are better, except for AUC.

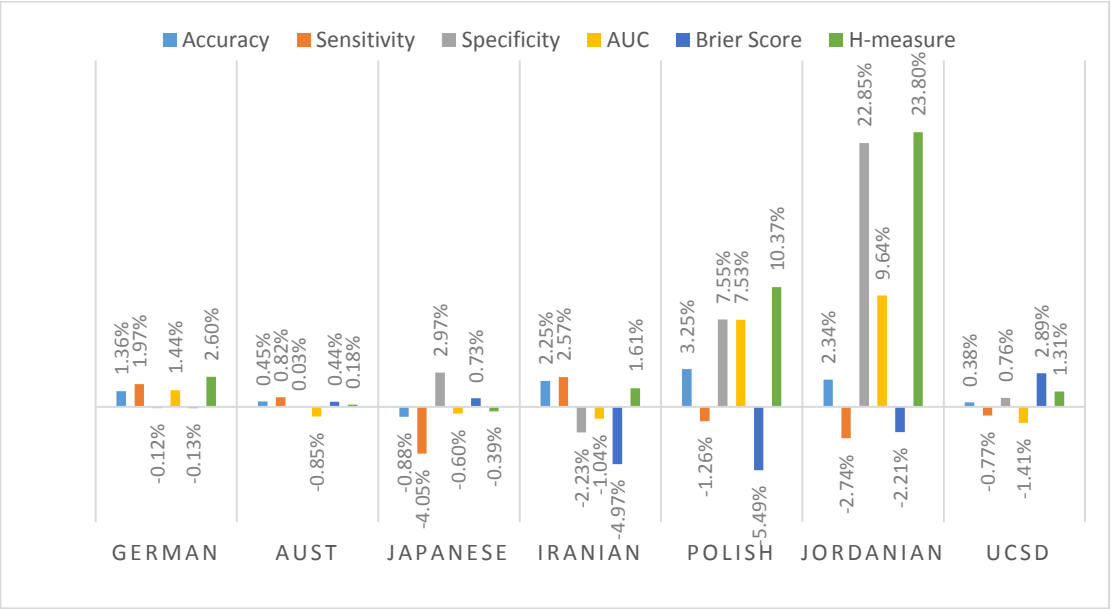


Figure 5.6 SVM measures compared to Logistic Regression

Figure 5.7 shows that SVM is the only classifier whose ROC curves are roughening instead of smoothening, however the overall picture becomes better, all the figures become closer to upper left point, which means becoming better a priori. The ROC curve for the Jordanian dataset has one of the best results, which means that in most cases SVM classifier might work better with the real data than with classical testing data.

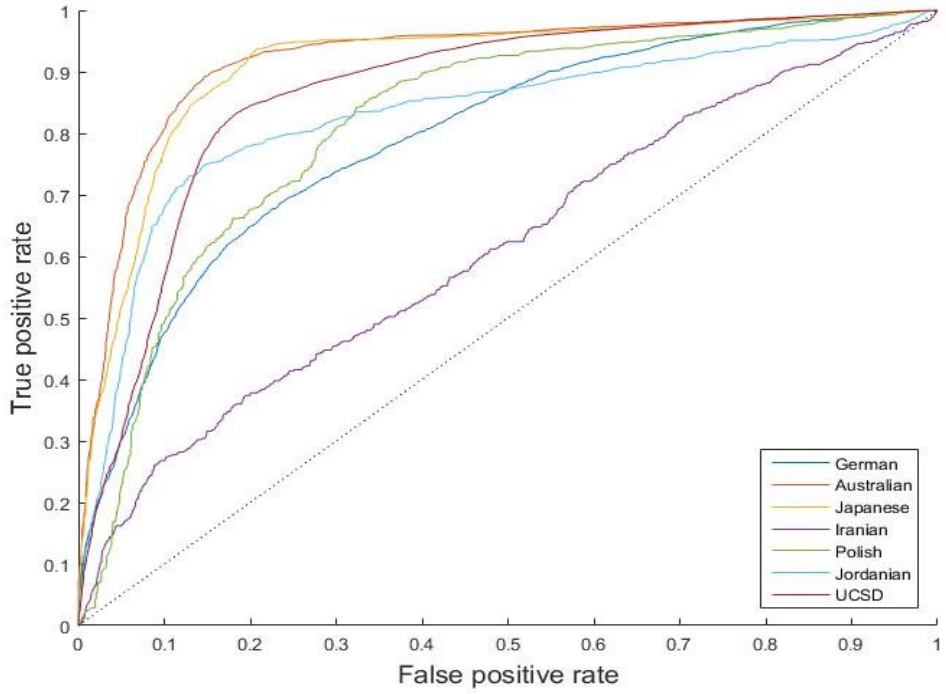


Figure 5.7 SVM ROC curve for all datasets

- **RF**

According to Table 5.22, the results of RF are enhanced by combining both GNG and MARS methods. The improvement of the RF results due to filtering is based on the idea of the filtering itself, for example some of the samples do not help to train the classifier properly as they lie too close to the samples of the opposite class. Without inaccurate samples the error became lower and Accuracy rises. Feature selection also helps to reduce number of features, that's why DT of the RF are built more precisely and optimally. However, the improvement is smaller than for other classifiers, and this fact was explained in previous sections, where analysed results were analysed with filtering and feature selection separately.

Specificity and Sensitivity change differently, the raise of both measures only could be found in the Polish dataset, but on the rest of measures if one has increased, the second has necessarily decreased. These increases and decreases do not exceed 0.2 % and can be explained by the fact, that RF Accuracy changes only a bit, and thus ROC curve changes insignificantly. Here and further, if such tendency reveals itself in other classifiers results, it could be explained by the fact that optimal point of ROC curve. When the ROC curve slightly increases, the optimal point of it changes too, what causes changes in Sensitivity and Specificity.

The reduction of the Brier Score is simply explained by the Accuracy and the H-measure increasing. Although feature selection and filtering are insufficient when applied separately, but together they increase the Accuracy quite noticeably. However, this increase is not enough to say that RF becomes much better with feature selection and filtering.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7725	0.8707	0.8717	0.9513	0.7742	0.8669	0.8690
Sensitivity	0.9066	0.8799	0.8806	0.9985	0.7782	0.9669	0.6924
Specificity	0.4611	0.8585	0.8618	0.0555	0.7774	0.4655	0.9269
AUC	0.7942	0.9286	0.9293	0.7786	0.8408	0.8861	0.9162
Brier Score	0.1603	0.0982	0.1001	0.0430	0.1627	0.1006	0.0946
H-measure	0.2966	0.6491	0.6513	0.2831	0.3945	0.5027	0.5422

Table 5.22 RF results using GNG + MARS

Regarding Figure 5.8, just as it was without filtering and feature selection, RF classifier stays more accurate than LR on every dataset. The Accuracy increases by 3.9% on Australian dataset and by 1.1% on Jordanian dataset. All measures of RF stay higher than corresponding measures of LR The only exception is in Specificity in datasets German, Australian and Japanese datasets, but balance between these two values (Specificity and Sensitivity) can be easily adjusted by changing threshold value. As soon as AUC of RF is bigger than AUC of LR, with changed threshold this advantage is maintained. Also Brier Score for RF is improved across all datasets.

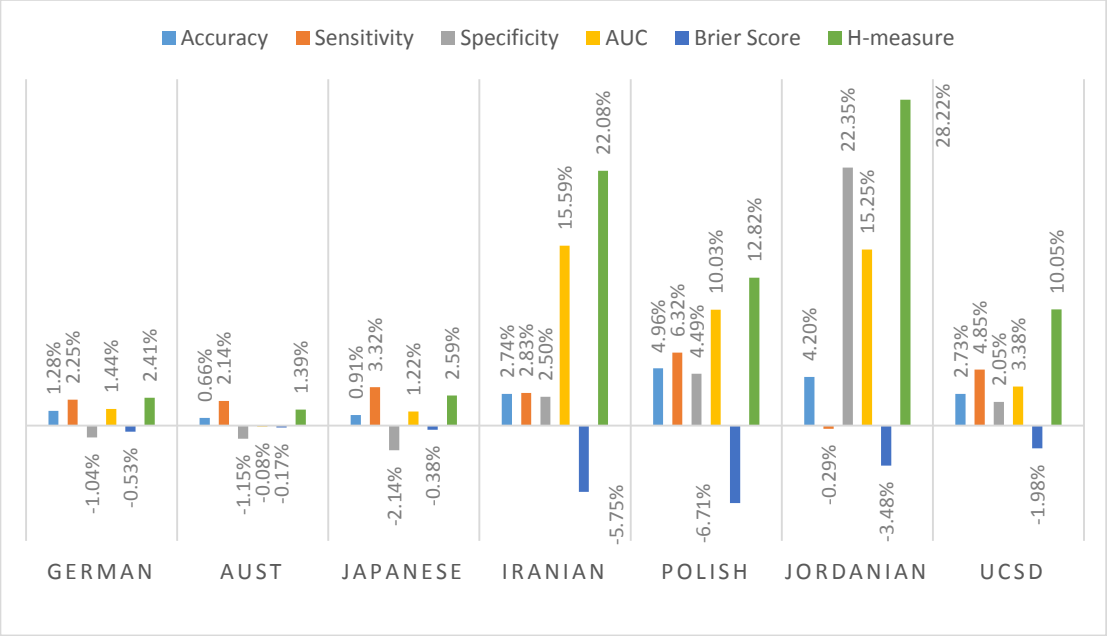


Figure 5.8 RF measures compared to Logistic Regression

Figure 5.9 shows that the ROC curves with feature selection and filtering combined become smoother in comparison with ROC curves of RF from the previous chapter. This means that classifier becomes more balanced, although the AUC values decrease in all cases except Polish and UCSD datasets. The decrease of AUC, as it is known, means decrease the ability of classifier to work with a wide range of thresholds, however if classifier demonstrates high rise of Accuracy, relative small decrease of AUC can be considered as insignificant.

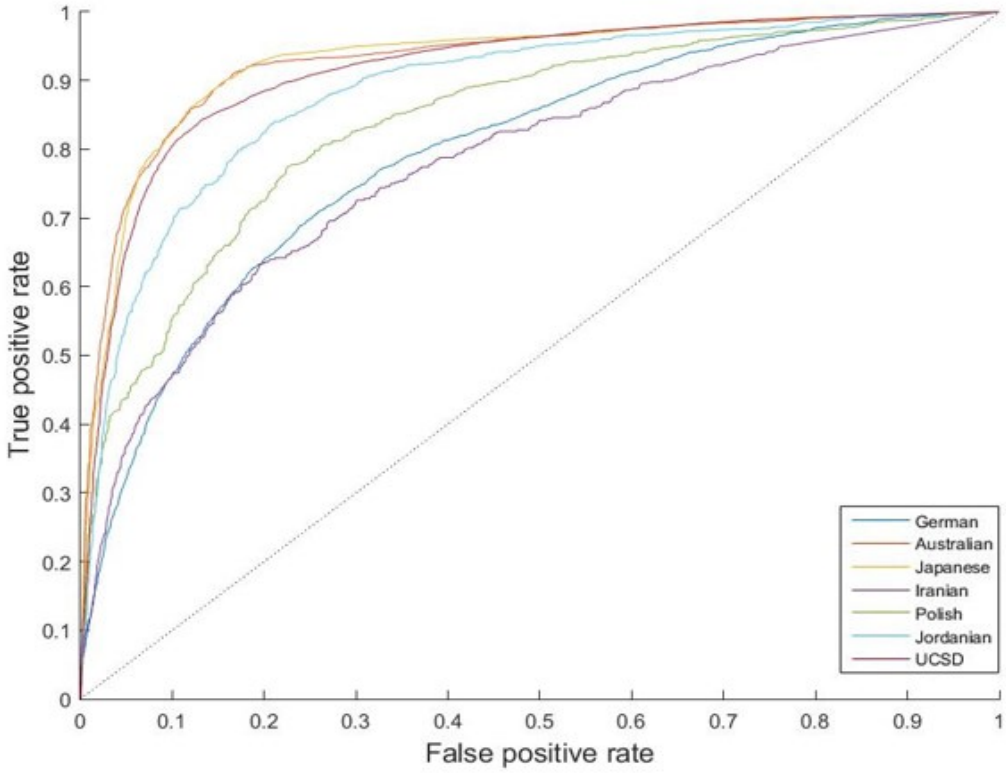


Figure 5.9 RFROC curve for all datasets

- **DT**

It can be clearly seen from Table 5.23 that the results of DT increased comparing to results in the previous chapter; however, according to Figure 5.10 on German and Japanese datasets this raise is not enough to beat the LR. AUC value of DT is also smaller than corresponding value of LR, this can be easily explained by the values of predictions that DT can produce as output, majority of the DT output predictions are either '0' or '1', and only small fraction of these predictions lies between these two values. That's why, its ROC curves have a shape more similar to triangle, than ROC curves of other classifiers, and this is the reason why AUC value is lower than the AUC values of classifiers with similar Accuracy. Brier score and H-measure on majority of datasets are also lower than LR.

As well as on RF, the Accuracy of DT has also increased, and on Polish dataset it improved up to 4%. This increase is caused mostly by data-filtering, and slightly by feature selection. In this case, filtering allows to weight every decision more accurately, when the feature selection only tries to remove the attributes from classification process, that have only slight influence on a label of the data, the set of features is originally are redundant so some of them cannot be

considered as important. The situation with the Sensitivity, Specificity, AUC, Brier Score and H-measure is completely same to RF, as well as its background. Feature selection and filtering improve the results of the DT separately, and together the improve they cause is even better.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7528	0.8688	0.8616	0.9505	0.7900	0.8612	0.8414
Sensitivity	0.8964	0.8653	0.8434	1	0.7516	0.9444	0.5960
Specificity	0.4204	0.8722	0.8847	0.0117	0.8279	0.5295	0.9216
AUC	0.6994	0.8868	0.8795	0.5362	0.7975	0.7809	0.7933
Brier Score	0.2214	0.1216	0.1292	0.0489	0.2027	0.1248	0.1424
H-measure	0.1973	0.6157	0.6025	0.0400	0.3818	0.3994	0.3735

Table 5.23 DT results using GNG + MARS

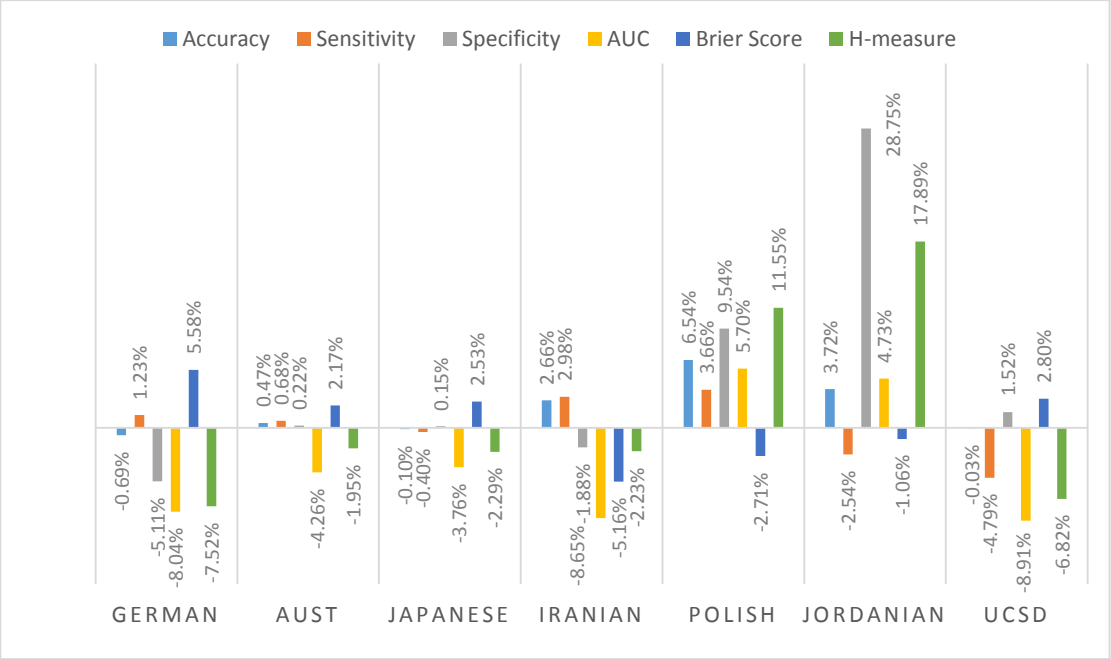


Figure 5.10 DT measures compared to Logistic Regression

In Figure 5.11, ROC curves of the DT with the influence of filtering and feature Selection become more linear. On the Iranian dataset, the curve is parallel to the random line. Other shifts of the curves are due to datasets proportion of good and bad loans in dataset, the more dataset is imbalanced, the more figure shifts, and the Iranian dataset is the best example of this. An opposite conclusion is also possible, the more balanced the dataset is, the better ROC

curve it has, although in this particular case the most balanced dataset the Polish comes after the Australian and Japanese datasets due to its big number of features.

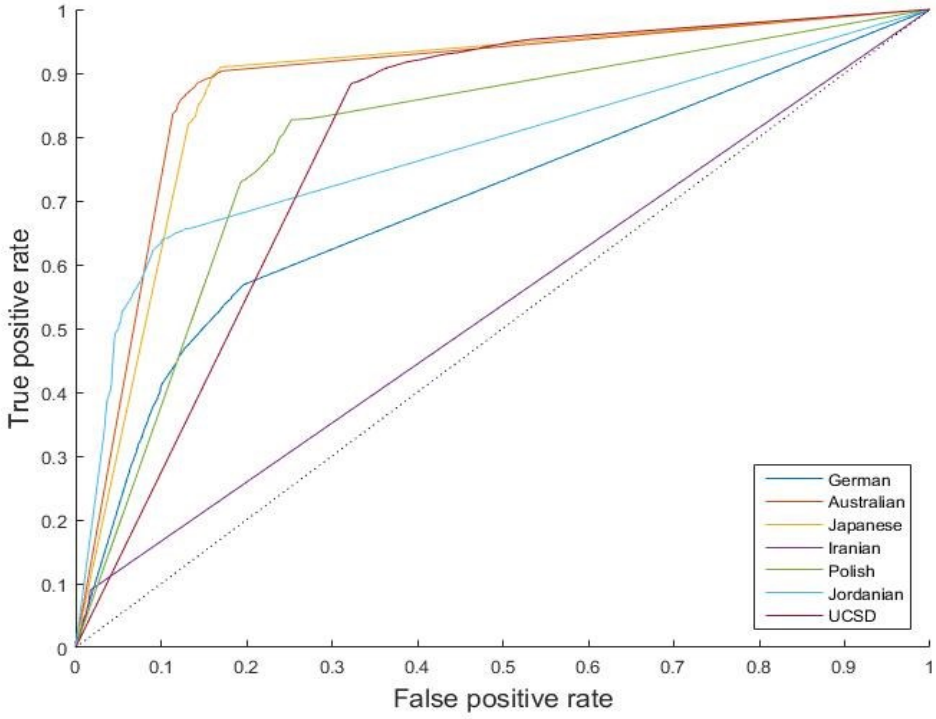


Figure 5.11 DT ROC curve for all datasets

- **NB**

According Table 5.24, the situation with NB Accuracy is a bit different from DT and RFs. In general, the NB performance improved Japanese dataset Accuracy comes in the third place in Accuracy after RF and NN. However, on Iranian dataset the Accuracy stays almost the same comparing with experiment with MARS and without GNG (0.9452 with this experiment and 0.9447 in the experiment with Feature Selection and without filtering).

However, the Accuracy of NB with only filtering is only 0.9307. The possible explanation for this might lie in filtering. Iranian dataset consists of very small number of bad loans, and each bad loan is surrounded by good loan points. So filtering while deleting bad loan entries degrade bad/good loans proportion, the feature selection also has its weak sides; algorithm might have considered some features as unimportant, what, in combination with naive assumption that the attributes are independent, caused reduction of the Accuracy on this dataset. On other measures, the only difference from the already examined measures is the

rise of AUC. This gives a good evidence to say that, for now, only NB becomes more universal.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7638	0.8614	0.8630	0.9452	0.7296	0.8212	0.8083
Sensitivity	0.8861	0.8420	0.8579	0.9881	0.9038	0.9852	0.7787
Specificity	0.4789	0.8844	0.8702	0.1328	0.5775	0.1642	0.8181
AUC	0.7735	0.9093	0.9085	0.7470	0.7996	0.7765	0.8312
Brier Score	0.1927	0.1249	0.1221	0.0537	0.2642	0.1574	0.1908
H-measure	0.2668	0.6149	0.6225	0.2373	0.3449	0.2587	0.3958

Table 5.24 NB results using GNG + MARS

In Figure 5.12 and comparing to LR results, NB shows negative performance. Only on Iranian dataset the performance of this classifier is better. On Polish dataset Accuracy is a bit better, but most of other measures, including AUC is slightly better. On real-life UCSD dataset NB shows lower result than LR, but comparing to the results with GNG and MARS, NB shows tremendous rise in Accuracy (from 61.4% to almost 81%).

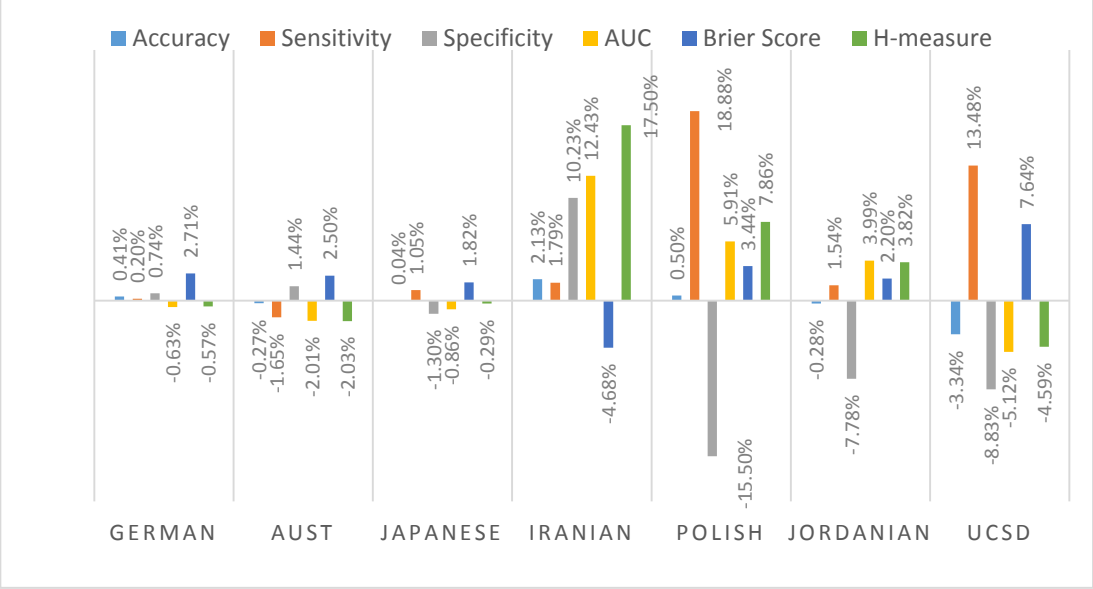


Figure 5.12 NB measures compared to Logistic Regression

According to Figure 5.13, ROC curves give only more evidence to the conclusion that GNG and MARS soften the ROC curve for all datasets. This can be explained by the fact that NB

ROC curves on datasets with lower number of features have good shape (and thus AUC have bigger value). That's why ROC curves on datasets with high number of features improve due to GNG and MARS applied. Thus, the best results are on datasets Australian and Japanese, whereas the best improvement could be seen on the UCSD dataset, which is amongst the most affected datasets by MARS. Other dataset's ROC curves in this case come very close to each other near the optimal value of threshold, which means similar results in the wide range of thresholds for all of them.

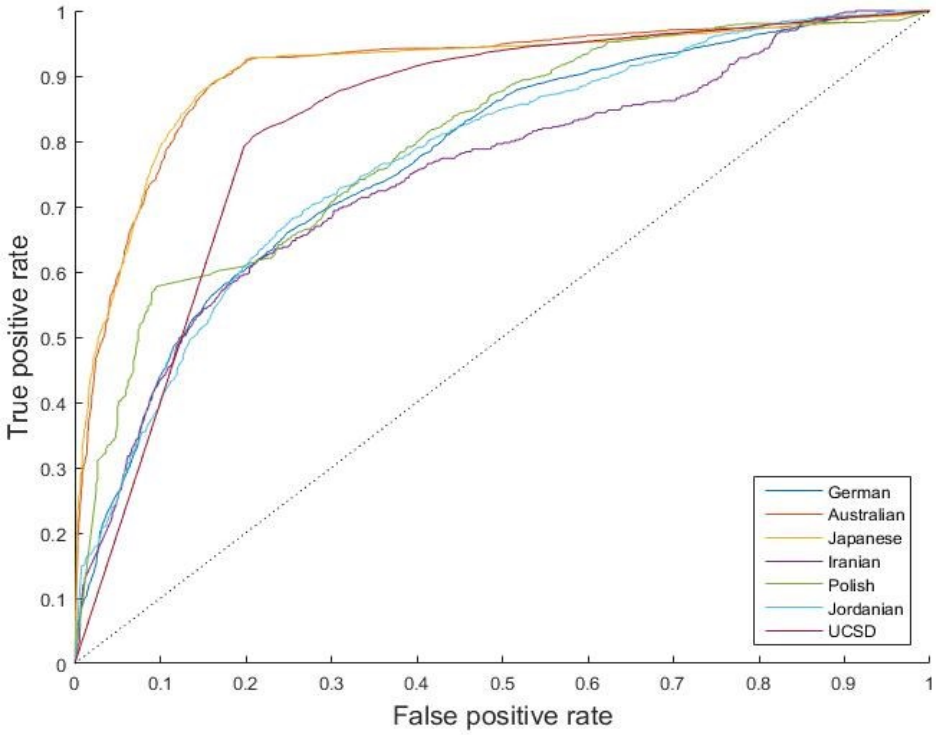


Figure 5.13 NB ROC curve for all datasets

5.5 Comparison of Results and Justification of Combining GNG + MARS

In this section an evaluation is conducted on the Accuracy improvement for all single classifiers depending of pre-processing algorithms being used. For each classifier a table is presented which demonstrates advantage of using GNG + MARS in combination. Tables 5.25 to 5.29 demonstrate an extensive comparison on individual classifiers based on:

- All data and features.
- All features and filtered data by GNG method.
- All data and just important features selected by MARS.

- Filtered data by GNG method and important features selected by MARS.

All increases represent the Accuracy change comparing to individual classification, with all data and features. The advantage of filtering and feature selection pre-processing algorithms combination is clearly seen in row 7 of all tables, when comparing to row 3 and row 5.

- **NN**

Below in Table 5.25 GNG+MARS communicate in very special way. It could be said that GNG and MARS alone work differently. Both methods together improve the Accuracy for all datasets especially from 0.01% on Iranian dataset and to 5.46% on Polish dataset.

	German	**Australian**	**Japanese**	**Iranian**	**Polish**	**Jordanian**	**UCSD**
Individual classification	0.7476	0.8588	0.8581	0.9499	0.6975	0.8148	0.8311
GNG	0.7507	0.8588	0.8654	0.9488	0.7433	0.8198	0.8400
Increment after implementing GNG	**0.31%**	**0%**	**0.73%**	**-0.11%**	**4.58%**	**0.5%**	**0.89%**
MARS	0.7475	0.8649	0.8617	0.9494	0.6892	0.8382	0.8473
Increment after implementing MARS	**-0.01%**	**0.61%**	**0.36%**	**-0.05%**	**-0.83%**	**2.34%**	**1.62%**
GNG + MARS	0.7584	0.8643	0.8694	0.9500	0.7521	0.8454	0.8487
Increment after implementing GNG+MARS	**1.08%**	**0.55%**	**1.13%**	**0.01%**	**5.46%**	**3.06%**	**1.76%**

Table 5.25 NN results comparison of GNG, MARS and GNG+MARS

- **SVM**

According to Table 5.26, shows SVM the most controversial classifier, although GNG+MARS should improve the Accuracy, the result looks different for the Japanese and Iranian datasets. However, MARS on Japanese dataset decrease the Accuracy more than that in GNG because Japanese dataset does not have many features, comparing to the Polish dataset, for example. Finally, the way GNG and MARS work together is good as the overall increase is a little greater than increase of applying pre-processing methods separately.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Individual classification	0.7614	0.8523	0.8581	0.9482	0.7487	0.8298	0.8306
GNG	0.7677	0.8626	0.8533	0.9464	0.7558	0.8342	0.8324
Increment after implementing GNG	**0.63%**	**1.03%**	**-0.48%**	**-0.18%**	**0.71%**	**0.44%**	**0.18%**
MARS	0.7655	0.8525	0.8575	0.9483	0.7538	0.8368	0.8444
Increment after implementing MARS	**0.41%**	**0.02%**	**-0.06%**	**0.01%**	**0.51%**	**0.7%**	**1.38%**
GNG + MARS	0.7733	0.8686	0.8538	0.9464	0.7571	0.8474	0.8455
Increment after implementing GNG+MARS	**1.19%**	**1.63%**	**-0.43%**	**-0.18%**	**0.84%**	**1.76%**	**1.49%**

Table 5.26 SVM Results comparison of GNG, MARS and GNG+MARS

- **RF**

Table 5.27 shows a good example of how GNG+MARS work jointly better than separately. This is clear on the Japanese dataset, where Accuracy decreases by 0.07% and 0.12% with GNG and MARS respectively, but by combining them the Accuracy increases by 0.43%.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Individual classification	0.7669	0.8668	0.8674	0.9510	0.7625	0.8550	0.8619
GNG	0.7698	0.8677	0.8667	0.9508	0.7521	0.8638	0.8679
Increment after implementing GNG	**0.29%**	**0.09%**	**-0.07%**	**-0.02%**	**-1.04%**	**0.88%**	**0.06**
MARS	0.7667	0.8680	0.8662	0.9507	0.7679	0.8616	0.8664
Increment after implementing MARS	**-0.02%**	**0.12%**	**-0.12%**	**-0.03%**	**0.54%**	**0.66%**	**0.45%**
GNG + MARS	0.7725	0.8707	0.8717	0.9513	0.7742	0.8660	0.8690
Increment after implementing GNG+MARS	**0.56%**	**0.39%**	**0.43%**	**0.03%**	**1.17%**	**1.1%**	**0.71%**

Table 5.27 RF Results comparison of GNG, MARS and GNG+MARS

Also a rare case is represented on Polish dataset, when filtering decreases the Accuracy and feature selection increases it. The reason it goes this way is that Polish dataset has the less number of data samples and the large number of features.

- **DT**

Like on the RF, Table 5.28 here demonstrates GNG+MARS work well together. Both of them separately increase the Accuracy, but combining them increase the Accuracy in all datasets accept the Japanese, where GNG alone is better than GNG + MARS, this might be because of MARS performance alone where it was worse than the Individual classification. Also another massive increment for both methods is on Polish dataset where the increment is 8.87% which is better the both methods separately.

	German	**Australian**	**Japanese**	**Iranian**	**Polish**	**Jordanian**	**UCSD**
Individual classification	0.7045	0.8258	0.8171	0.9238	0.7013	0.8278	0.8201
GNG	0.7446	0.8677	0.8635	0.9505	0.7513	0.8526	0.8343
Increment after implementing GNG	**4.01%**	**4.19%**	**4.64%**	**2.67%**	**5.00%**	**2.48%**	**1.42%**
MARS	0.7212	0.8275	0.8167	0.9268	0.7254	0.8304	0.8244
Increment after implementing MARS	**1.67%**	**0.17%**	**-0.04%**	**0.30%**	**2.41%**	**0.26%**	**0.43%**
GNG + MARS	0.7528	0.8688	0.8616	0.9505	0.7900	0.8612	0.8414
Increment after implementing GNG+MARS	**4.83%**	**4.30%**	**4.45%**	**2.67%**	**8.87%**	**3.34%**	**2.13%**

Table 5.28 DT Results comparison of GNG, MARS and GNG+MARS

- **NB**

According Table 5.29, the use of GNG +MARS work the same way as on DT, for all datasets increment is more than using them separately except on the Australian dataset where GNG alone is better. In General, the performance of GNG alone is quite good with NB. But when combining with MARS the performance gets better if the MARS performance is better than Individual classification (e.g., Australian and Japanese datasets).

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Individual classification	0.7250	0.8030	0.7970	0.9262	0.6896	0.8106	0.6140
GNG	0.7590	0.8649	0.8630	0.9307	0.7262	0.8134	0.8069
Increment after implementing GNG	**3.4%**	**6.19%**	**6.4%**	**0.45%**	**3.66%**	**0.28%**	**19.29%**
MARS	0.7437	0.7849	0.7970	0.9447	0.7071	0.8210	0.6253
Increment after implementing MARS	**1.87%**	**-1.81%**	**-0.2%**	**1.85%**	**1.75%**	**1.04%**	**1.13%**
GNG + MARS	0.7638	0.8614	0.8630	0.9452	0.7296	0.8212	0.8083
Increment after implementing GNG+MARS	**3.88%**	**5.84%**	**6.4%**	**1.9%**	**4%**	**1.06%**	**19.43%**

Table 5.29 NB Results comparison of GNG, MARS and GNG+MARS

5.6 Analysis and Discussion

The obtained results can easily change the decision made in the previous chapter. Now it is clear that with a slight modifying of each classifier, LR becomes one of the least accurate classifier from being the most accurate and optimal. Even the worst classifier DT with filtering and feature selection can rival with the Logistic Regression. RF, as was the best comparing to all others, became even better.

Table 5.30 clearly shows that, with filtering and feature selection, if we were to look together on all datasets, every classifier becomes better than LR. The use of it becomes misspend when there is novel hybrid model. Amongst the best classifiers, it can be seen RF, DT and SVM. RF gives the best Accuracy, AUC and H-measure for the most of the datasets.

Using filtering and feature selection improved each classifier, and as for LR filtering and feature Selection were not used, its position drops down from second place from the top (as it was in Chapter 3) to the last place, which it shares with NB.

Accuracy	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD	Average Rank
RF	2	1	1	1	2	1	1	1.29
DT	6	2	5	2	1	2	5	3.29
SVM	1	3	6	4	3	3	3	3.29
NN	5	4	2	3	4	4	2	3.43
NB	3	6	3	5	5	6	6	4.86
LR	4	5	4	6	6	5	4	4.86

Table 5.30 Rankings of base classifiers based on their accuracy across all datasets

Now let us show how much results have increased during applying these GNG and MARS methods together (in average by all datasets):

- NB: 6.07%
- DT: 4.37%
- NN: 2.41%
- SVM: 0.90%
- RF: 0.63%

It can be seen, that the worse results classifier shows without filtering and feature selection, the best improvement can be seen after applying these two pre-processing methods. So let us make a receipt when using filtering and feature selection is extremely useful:

- When the dataset is well-balanced.
- When data have a lot of features and some of them are categorical.
- When any individual classifiers without filtering and feature selection give surprisingly low results, which cannot be explained by other reason than existing of outliers in data.
- When DT or NB used as a part of classification system. Even if one of them can be applied to the data being analysed, using filtering and feature selection in combination is very desirable.

Finally, it is very important to mention and describe the thresholds assigned to GNG + MARS to each classifier for each dataset (Please refer to Appendix B).

5.7 Summary

In Chapter 2, the approaches are introduced namely GNG and MARS methods which are data-filtering and feature selection techniques respectively. Several experiments were carried out in order to build hybrid models with better Accuracy. Data-filtering and feature selection were applied separately and in combination in order to see different effects on classifiers results.

After applying GNG and MARS, relative and absolute performance of all classifiers without an exception changed a lot. The most sensitive classifiers to these techniques are DT and NB classifiers. Accuracy of NB increased by 6% in average, the most change was for the UCSD dataset. Accuracy of DT increased by 4.3%, which also is very good; on the other hand, RF is the least sensitive classifier to filtering and feature selection, the reason of this lies is an in the complex structure of this classifier, which by itself consists of a bunch of random DT, as well as initially high results of RF even without filtering and feature selection. As it can be seen from comparison tables, filtering and feature selection in combination almost gives for classifiers higher result comparing to experiments with applying only one of these techniques.

The main conclusion can be conducted from this chapter is that using filtering and feature selection in combination is justified, and the experiments conducted with these two pre-processing techniques show major improvement for all classifiers, comparing to non-pre-processing and one pre-processing technique. It is worth noting, that filtering is more responsible for Accuracy increase than feature selection. This can be seen when the results of experiments were compared with only filtering and feature selection applied separately. Thus the logical question come up into mind, whether is it feasible to combine the predictions of all five classifiers so the performance of their combination is higher? In the next chapter simple combiners is considered, with the hope that it will help to obtain higher results. (This is next step in the direction of creating a perfect classifier, called the consensus classifiers approach, which is discussed in the Chapter 7). So different experiments is carried out to use several simple combiners, which is described in details in the next chapter and combine them using simple multi-argument functions as MIN, MAX, PROD, AVG etc. amongst all traditional combiners the best and the worse is addressed and suggestions is made on which combiner is better to use in which situations.

CHAPTER 6

CREDIT-SCORING MODELS USING ENSEMBLE CLASSIFIERS TECHNIQUES

6.1 Introduction

Whilst there is a bunch of single classifiers with a good performance, the question on how to use their predictions together in order to obtain the most accurate predictions still remains open. The most common way to merge single classifiers predictions is to use simple function $f(x_1.x_2.x_3.x_4.x_5) = x_*$, which convert all single classifiers predictions x_i into actual output ranking. An important issue in combining classifiers is that it is particularly useful if they are different, see (Kittler *et al*., 1998). This can be achieved using different feature sets or selecting different subsets of training data (Xu *et al*., 1992; Kuncheva, 2004).

In the previous two chapters, the principles of single classifiers and the methods reviewed which improved their Accuracy with data-filtering and feature selection were demonstrated. The next step in their analysis is to examine how they work together, building an ensemble of single classifiers with a more complex ensemble classifier in a result. In fact, the ensemble is just a classifier, whose arguments are the results of all single member classifiers. There are a lot of different ways how the ensemble could work, but here are the ones, which are implemented in this thesis:

- Min Rule (MIN)
- Max Rule (MAX)
- Product Rule (PROD)
- Average Rule (AVG)
- Majority Voting (MajVot)
- Weighted Average (WAVG)
- Weighted Voting (WVOT).

As Chapter 5 has shown, data-filtering and feature selection techniques demonstrated a significant rise on the classifiers accuracies and during the testing phase of the traditional

combiners with different options of filtering and feature selection, it has been discovered, that the traditional combiners achieved highest results when both filtering and feature selection methods are enabled. So the results those are provided only for the experiments with filtering and feature selection ‘on’.

6.2 Traditional Combiners

At this section the main traditional combiners are considered, their mathematical models are analysed and their strengths and weaknesses are demonstrated. Individually, each combiner mathematical model is illustrated by a diagram. Also an assumption is made for what type of data and which combiner is more advisable to use.

6.2.1 Min Rule (MIN)

MIN rule is based of taking the minimal raking of all classifiers to be chosen as final rankings. Figure 6.1 will illustrate the mechanism of how MIN rule operate and followed by the process description.

According to Figure 6.1, the results of MIN ensemble are the minimal ranking from all classifiers' rankings. Although this ensemble looks very simple, it requires some additional setting, namely threshold lowering. With the regular threshold most of the rankings of MIN would be considered as positive (i.e. the majority of data outputs rankings are less than 0.5). To avoid this situation a new threshold is being chosen during the training phase, the value been chosen, gives the highest Accuracy of MIN ensemble over the training set. This classifier mainly is used when most of the loans in training and testing data sets are good, and all classifiers tend to predict good loans better than bad loans. So for each testing set data point with actual output ‘0’, if even one of the classifiers predict this point correctly, the result of MIN would be correct. On the other hand, MIN often predicts bad loans much worse than good loans. This lack of balance, however, can be compensated partially by choosing lower threshold (less than 0.5)

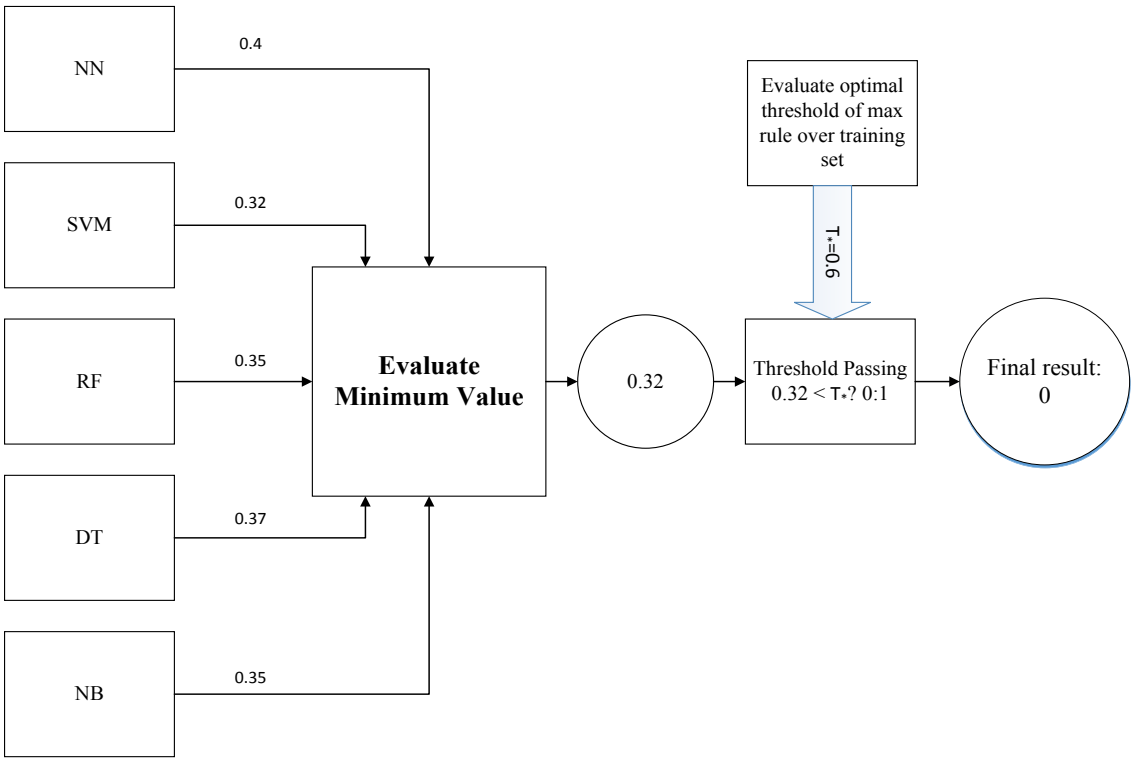


Figure 6.1 MIN ensemble example

6.2.2 Max Rule (MAX)

MAX rule is based of taking the maximal raking of all classifiers to be chosen as final rankings. Figure 6.2 will illustrate the mechanism of how MAX rule operate and followed by the process description.

Based on Figure 6.2, MAX ensemble outputs the highest rankings of all single classifiers. Similar, to MIN ensemble, MAX requires the change of threshold, but in an opposite way. It could be said, that MAX is anti MIN, with the regular threshold most of testing set loans would be predicted as negative (i.e for the majority of queries, output rankings are greater than 0.5). This is being avoided by increasing the threshold while the maximal Accuracy over the training set is obtained. On contrary MAX is mainly being used when most of the loans in training and testing data sets are bad, and all classifiers tend to predict bad loans better then good loans. This two combiners - MIN and MAX ensembles, have one very big disadvantage, which may cause a significant decrease in the Accuracy. If one of the classifiers after training got a bad performance, these two ensembles might choose the minimal or maximal output value, which would be totally wrong. In other words, MIN and MAX combiners require all highly accurate single classifiers to produce good Accuracy.

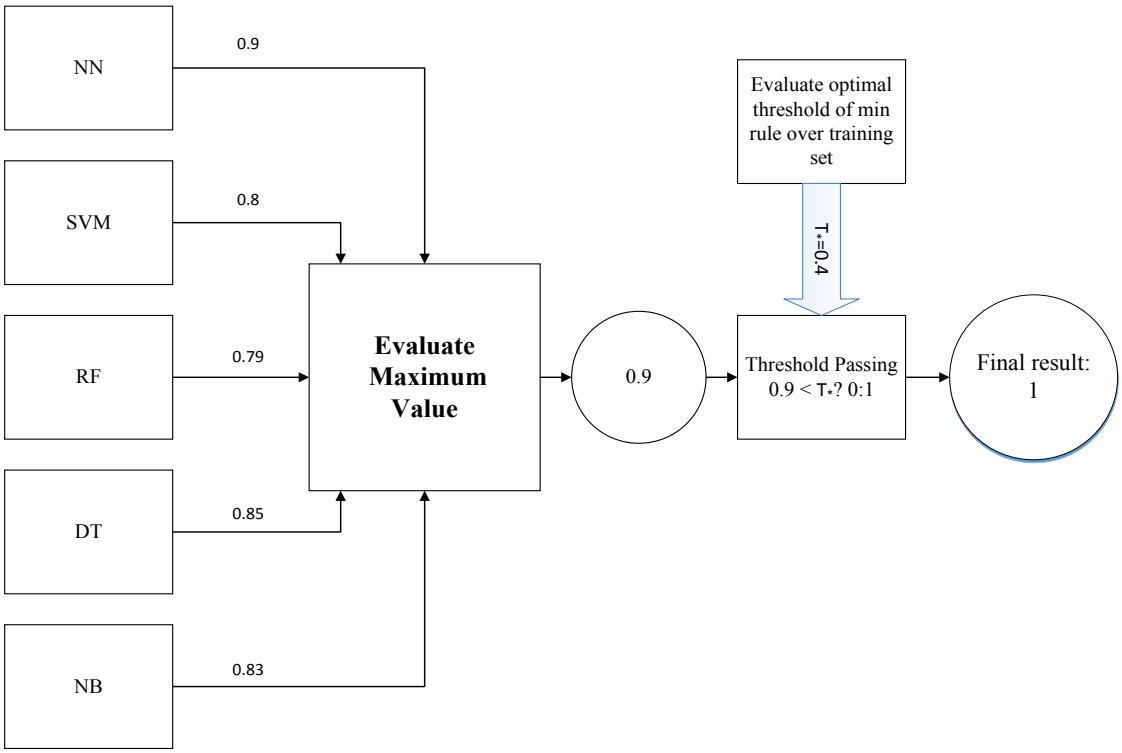


Figure 6.2 MAX ensemble example

6.2.3 Product Rule (PROD)

It can by implied by its name, PROD, which is based on taking the product of all classifiers to be chosen as final rankings. Figure 6.3 illustrates the mechanism of how the PROD rule operates, followed by the process description.

According Figure 6.3, the result of a PROD is a product of rankings of all single classifiers, all single classifiers outputs, are being multiplied. The following result is being compared with the threshold, and the final output depends on comparison result. Obviously, the multiplication result is mostly very low, that imposes the special demands on the threshold. The threshold is being chosen very low (based on the Accuracy over the training set) to match the average multiplication result. PROD is similar to MIN in the terms that output ranking of these two ensembles is low. But PROD has one advantage, unlike Min and MAX combiners; this combiner takes into consideration all single classifiers results.

Sometimes single combiners (e.g., DT or SVM) return a non-floating-point ranking from [0,1] interval, but only ‘0’ or ‘1’ prediction. As a disadvantage for the PROD worth mentioning that this combiner can show bad performance if only one of its components return this type of output. For example, if combiner returns only ‘0’ or ‘1’, and have a bad

Specificity, then PROD will have the same or worse specificity than this combiner, this fact is followed from the PROD mathematical model. However, if all classifiers are well trained, and return a floating-point ranking, PROD should perform better than MIN.

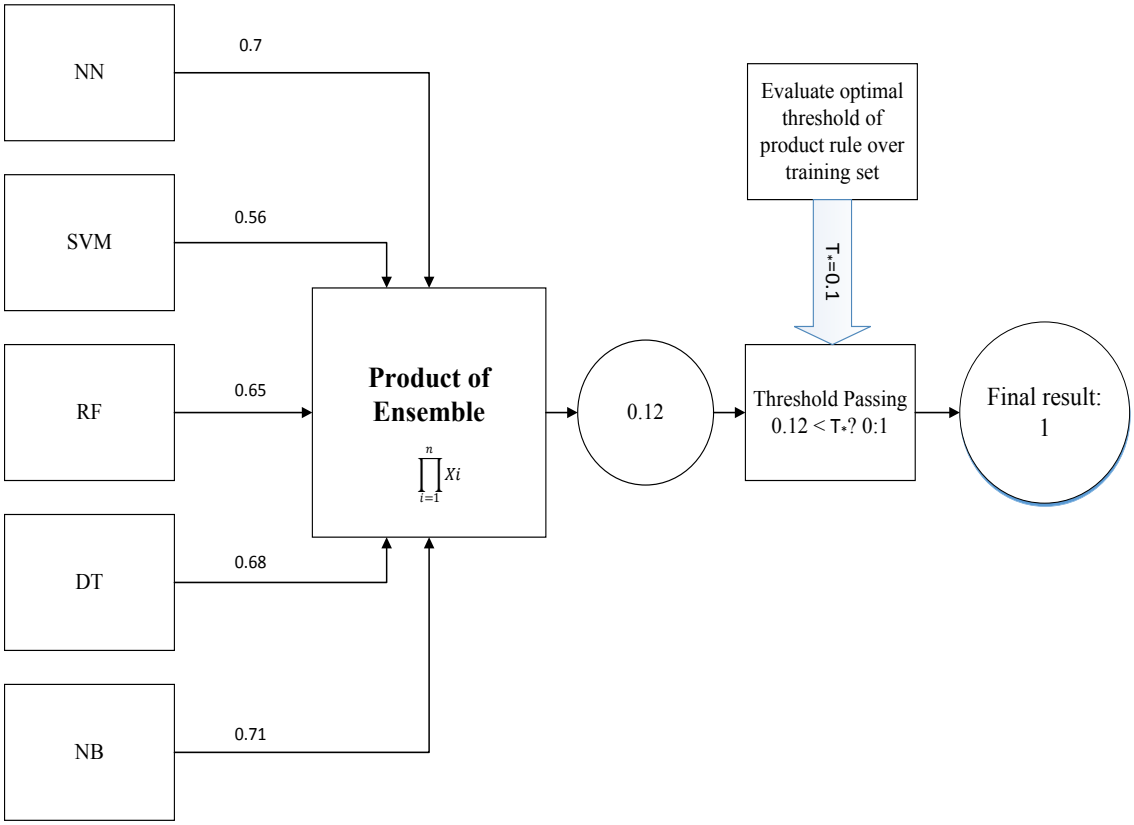


Figure 6.3 PROD ensemble example

6.2.4 Average Rule (AVG)

AVG rule is constructed by taking the average or mean value raking of all classifiers to be chose as final ranking. Figure 6.4 will illustrate the mechanism of how AVG rule operate and followed by the process description.

According Figure 6.4, the AVG ensemble takes the results of the single classifiers and finds its average values, and then compares it with the threshold. Unlike MIN and MAX, the threshold value stays on its default value (0.5). With the AVG calculation procedure threshold change is unnecessary, as if all single classifiers ranking distributions are balanced, the final outputs of AVG are balanced too. The AVG combiner is, more trustworthy than MIN and MAX its outputs are equally based on the output of all classifiers. Another advantage of the AVG is the good balance between Sensitivity and Specificity if the dataset is balanced.

The disadvantage of the AVG rule becomes apparent when some of the single classifier ranking changes are not linearly dependent on the certainty of this classifier on the result. For example, sometimes a classifier gives rankings of 0 and 0.3 which represent almost the same level of certainty in the output result, but from the 0.4 certainty harshly drops down. But the AVG counts all rankings the same, so it may cause the error during the final decision. Another disadvantage shows up when the performance between single classifiers varies a lot, in this case the AVG performance most likely will not be higher than the best single classifier.

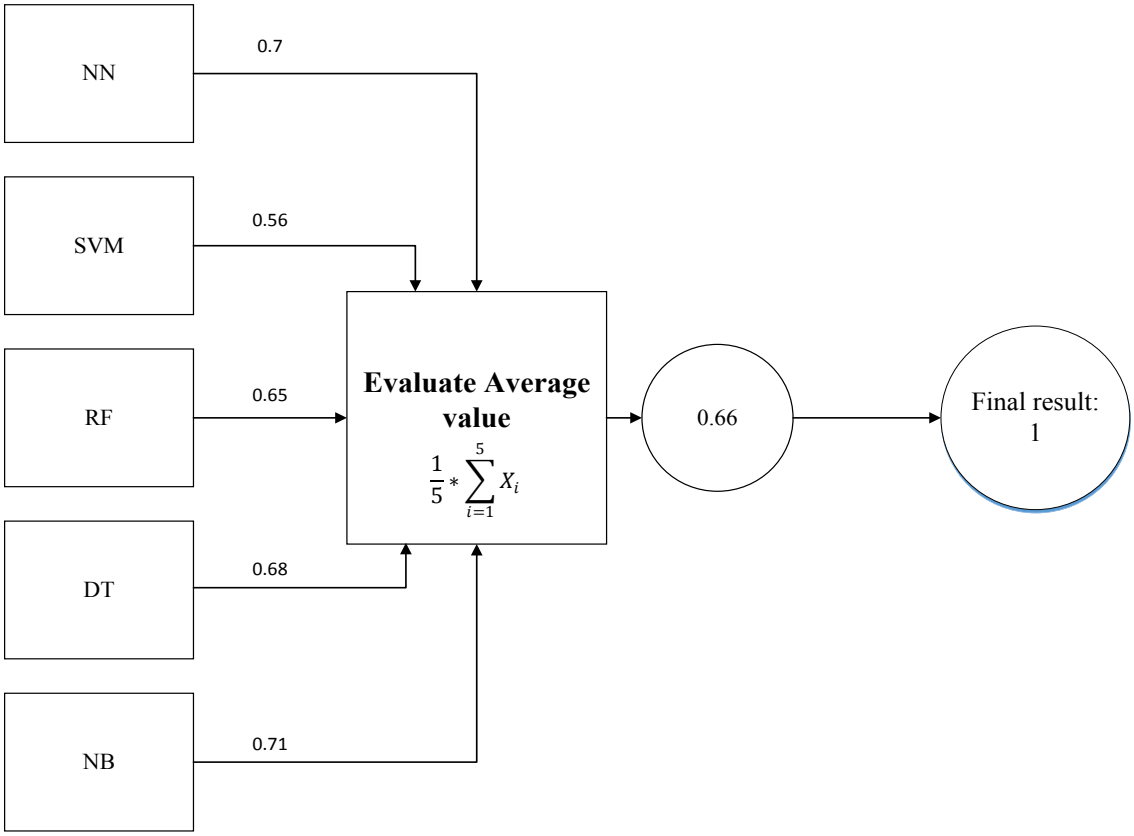


Figure 6.4 AVG ensemble example

6.2.5 Majority Voting Rule (MajVot)

MajVot rule is based on voting procedure, where the final decision is made based on the class or label with which the majority of classifiers agree. Figure 6.5 will illustrate the mechanism of how MajVot rule operate and followed by the process description.

Pointing to Figure 6.5 the idea behind MajVot is very simple where the majority of the single classifiers unlikely have a wrong opinion. In MajVot the output returns that value which got

the most of the classifiers votes. Obviously, with such a hierarchy, there is no need of the threshold, only the numbers of votes given for each class are being compared, thus the final prediction class, is that class, for which a majority of votes is given. In case of equality to tie-in the numbers of the votes, the ensemble would classify the data point as bad loan (but in the current case it is impossible, as there is an odd number of single classifiers).

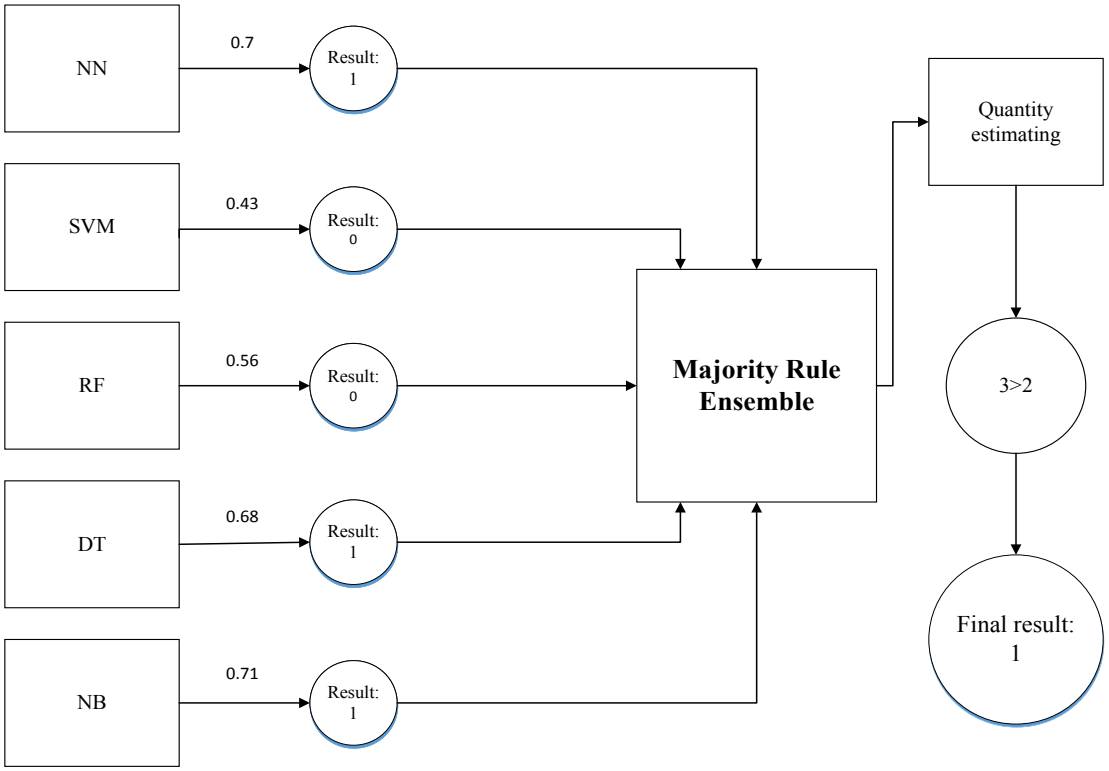


Figure 6.5 MajVot ensemble example

The MajVot avoids the first disadvantage; which AVG classifier has. MajVot is a very solid classifier, which is simple to use and which often gives good performance. The bad efficiency of a single classifier will not matter if all other classifiers performance is good. The Accuracy of classifiers would matter only when number of such classifiers becomes approximately equal to half of the number of all classifiers. However, this ensemble also has a shortcoming, that are connected to simplifying the single classifiers output and sometimes loosing valuable information.

6.2.6 Weighted Average Rule (WAVG)

WAVG rule is based on taking the average or mean value raking of all classifiers with a weight associated to every ranking based on its performance to be chose as final ranking.. Figure 6.6 will illustrate the mechanism of how MajVot rule operate and followed by the process description.

The WAVG ensemble is similar to AVG ensemble. The difference between them lies in the weighting each classifier's output before finding the average value. Weighting coefficients are evaluated according to single classifiers global Accuracy over the training set: the more accurate classifier is on the training set, the bigger is weight coefficient assigned to this classifier. An undeniable advantage of such development is the possibility to make more accurate classifiers decisions more affecting the ensemble result, while less accurate classifiers give less contribution to the final result.

However, WAVG has a very serious shortcoming, which is related to the fact that some single classifiers tend to be over-trained, and give much better results over the training data than over the testing data. The examples of such classifiers are NN, RF and SVM. That is why some good classifiers may get low weights, which will have negative impact on the WAVG result. The method how to overcome this disadvantage lies in increasing training set until training set Accuracy of all classifiers is equally proportional to the test set Accuracy.

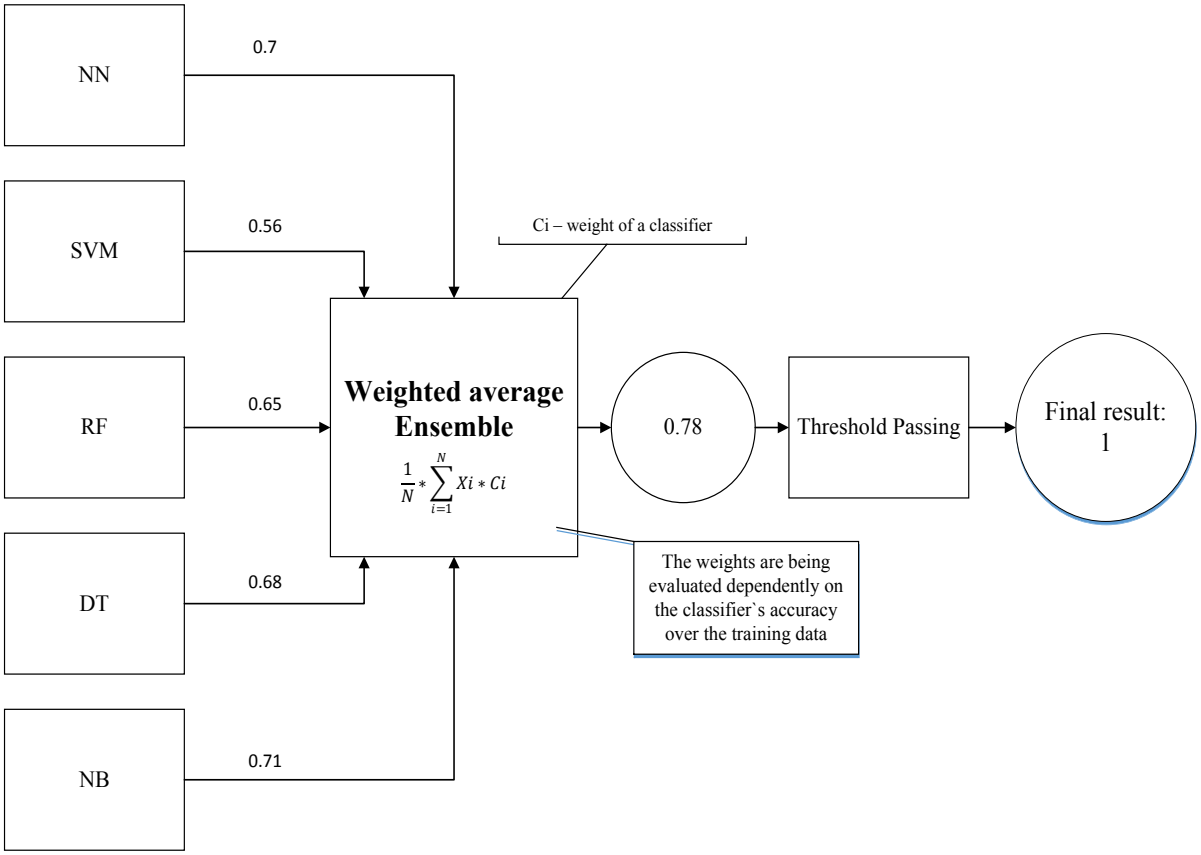


Figure 6.6 WAVG ensemble example

6.2.7 Weighted Voting Rule (WVOT)

WVOT rule is based on the same idea of MajVot but each class is given a weight, the weights of each class are combined and the decision of the final class is based on the highest value after applying a threshold. Figure 6.7 will illustrate the mechanism of how MajVot rule operate and followed by the process description.

WVOT is similar to MajVot ensemble. The difference between them lays in several preparation steps before quantity estimating. The first step is rounding the single classifiers output to the nearest integer. Weighting vector builds according to the same algorithm as in the previous combiner WAVG. In the next step a linear combination of coefficients is done dependently on the absolute Accuracy of each classifier on a training set. Then, the scalar product of vector of weights is being performed and rounded vector of single classifiers rankings. WVOT has the same disadvantage as WAVG, and it can be overcoming in the same way.

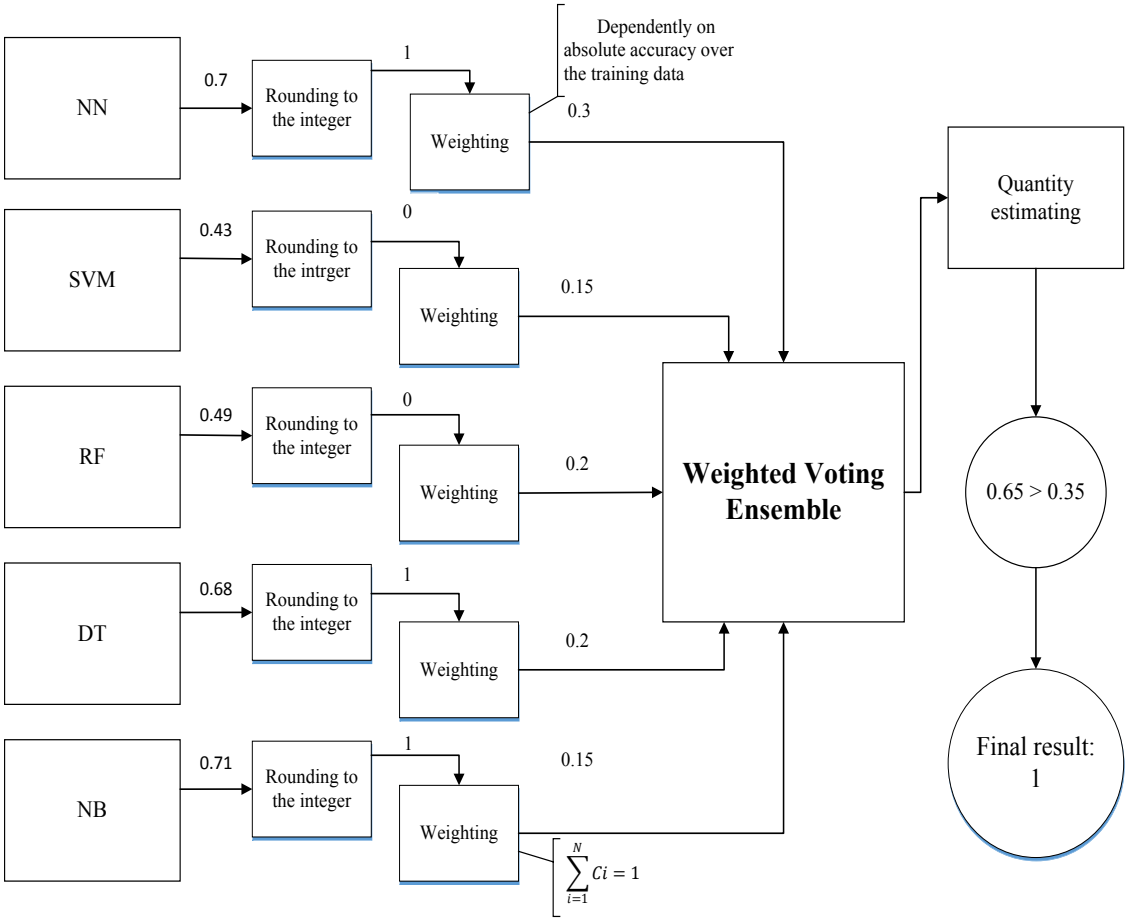


Figure 6.7 WVOT ensemble example

6.3 Experimental Results

In this section results of the seven traditional combiners across seven datasets evaluated on six performance measures are summarised and discussed (The results are evaluated by taking the average of 50 testing sets resulting from the 10 × 5 cross-validation). All the base single classifiers predictions is combined, their results is analysed, discussed and assumptions is made on why these results and what they are. Tables 6.1 to 6.7 will demonstrate the results of each combination method across all dataset on several performance measures.

6.3.1 Min Rule Results

According to Table 6.1, the MIN rule obviously gives good results on data sets for which percentage of good loans is high (e.g., German, Iranian and UCSD datasets), although it's result on the UCSD dataset is still very good. When comparing with MAX, it becomes clear that there is 1% decrease over German dataset and 4% increase over Iranian and UCSD dataset), the reason of this was described in previous section. On Polish dataset, results of MIN are amongst one of the worst, when comparing to all other traditional combiners. Due to threshold adjustment, levels of sensitivity and specificity are more or less balanced (except for Iranian dataset, where specificity is 0).

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7636	0.8662	0.8643	0.9500	0.7204	0.8246	0.8046
Sensitivity	0.9239	0.8927	0.9102	1	0.9052	0.9830	0.8576
Specificity	0.3920	0.8324	0.8087	0	0.5616	0.1906	0.7873
AUC	0.7178	0.9126	0.9130	0.5533	0.8287	0.8059	0.8828
Brier Score	0.2058	0.1153	0.1139	0.0494	0.2637	0.1466	0.1960
H-measure	0.2248	0.6355	0.6336	0.0545	0.3961	0.3837	0.4616

Table 6.1 MIN combination results

In Figure 6.8, the best results MIN provides across the Australian and Japanese datasets. For UCSD dataset performance of MIN is also good. The worst dataset is Iranian, where the plot-ROC curve lies almost on the diagonal. The Jordanian and the German has the same pattern however, Jordanian is better. Very interesting is the Polish dataset ROC curve, which is not convex for some threshold range. It is worth mentioning to describe the thresholds assigned to each classifier for each dataset, Table 6.2 illustrates this:

Dataset	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Threshold	0.326682	0.380086	0.465883	0.258415	0.236384	0.203055	0.428176

Table 6.2 MIN thresholds across all datasets

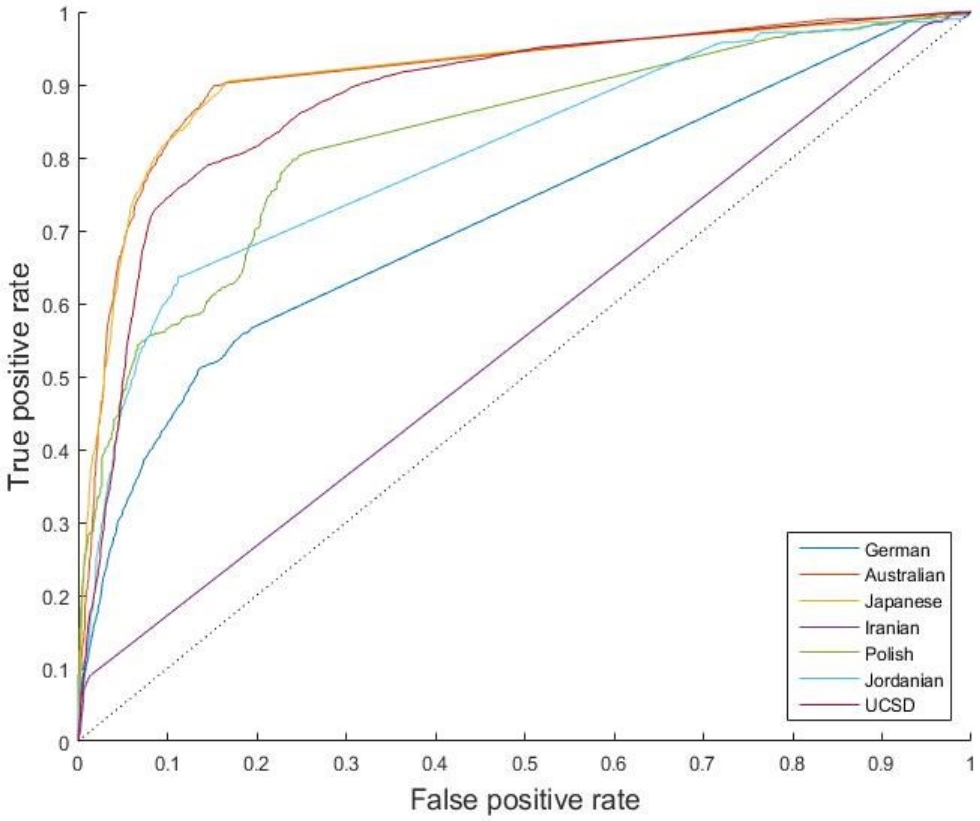


Figure 6.8 MIN ROC curve for all datasets

6.3.2 Max Rule Results

The MIN and MAX combiners are better to describe and analyse together, since they are two extremes of one rule. Table 6.3 reveals that the work of these two combiners is very similar; they both strongly depend on how good classifiers are trained. Also, as well as MIN combiner, MAX works better on datasets with high percentage of bad loans. This could be clearly seen on Polish dataset, comparing to MIN. Another reason of mistakes could be an inappropriate is the choosing of threshold. For example, the MIN combiner outputs only zeros on the Iranian dataset, whereas the threshold of MAX combiner allows it to allocate ones, although the Accuracy rapidly decreases. However, in some cases, like with MIN on the Iranian dataset, it is impossible to set an appropriate threshold. This is happening when one class data prevails over the other class.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7532	0.8662	0.8449	0.9101	0.7679	0.8528	0.8416
Sensitivity	0.7899	0.8235	0.7638	0.9430	0.5826	0.9059	0.4713
Specificity	0.6697	0.9185	0.9458	0.2998	0.9334	0.6431	0.9626
AUC	0.7878	0.9075	0.9028	0.7401	0.8240	0.8607	0.8357
Brier Score	0.1951	0.1108	0.1641	0.0712	0.1881	0.1154	0.1247
H-measure	0.2879	0.6319	0.6137	0.2541	0.4086	0.4693	0.4011

Table 6.3 MAX combination results

According to Figure 6.9 the MAX rule has the highest is Australian and Japanese datasets, the Jordanian comes third. MAX is good on a datasets with high amount of bad loans, so the ROC curve for the Iranian dataset is a lot higher than that in the MIN. UCSD ROC curve is a polyline, which means that for this dataset MAX is not very stable with respect to threshold changings. It means that the threshold cannot be changed in order to obtain a desired sensitivity, with a specificity value decreased very much. It is worth mentioning to describe the thresholds assigned to each classifier for each dataset, Table 6.4 illustrates this:

Dataset	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Threshold	0.510207	0.598982	0.559070	0.396230	0.537883	0.578746	0.599937

Table 6.4 MAX thresholds across all datasets

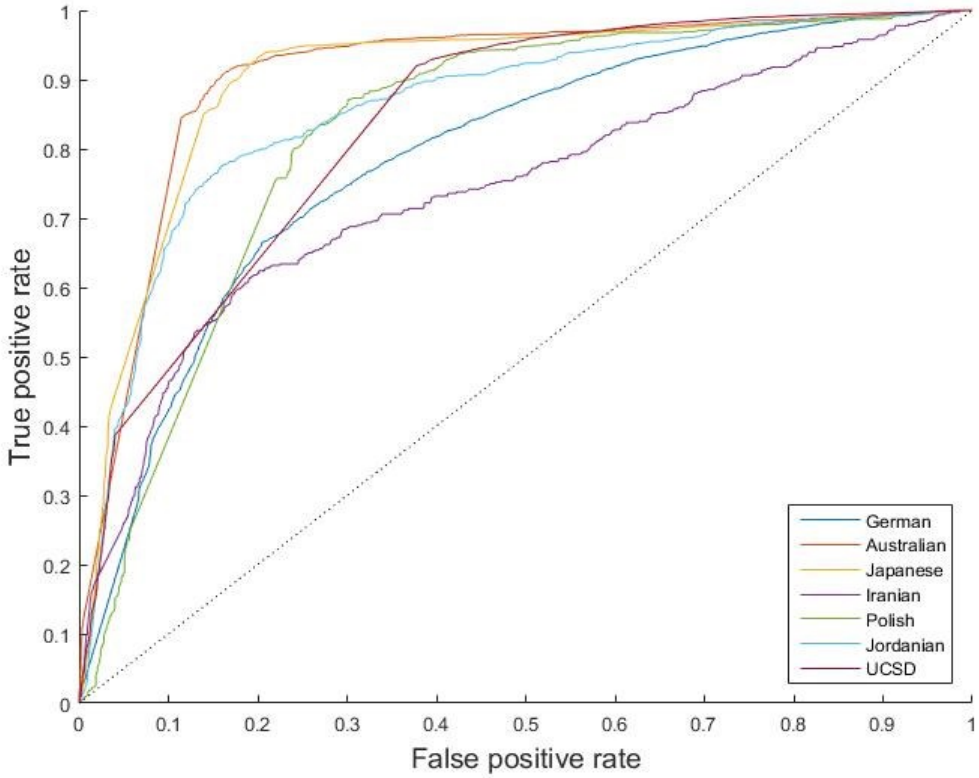


Figure 6.9 MAX ROC curve for all datasets

6.3.3 Product Rule Results

As it has already been mentioned, the PROD is very sensitive to each input value from the classifiers. This particular feature makes the threshold very hard to set perfectly, even the small change in one input value, which makes no real difference, may totally change the result of the combiners work. This can be seen in Table 6.5 results where exactly this feature produces that same situation on the Iranian dataset as the MIN has, and makes it useless on this particular dataset. Unlike Min and MAX combiners, the Accuracy of PROD does not depend on the ratio of good and bad loans. The best results are shown on Australian and Japanese datasets, datasets with the least number of features, although it is not a feature of PROD particularly, but of all the combiners.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7362	0.8583	0.8597	0.9500	0.7187	0.8156	0.8030
Sensitivity	0.9828	0.9149	0.9094	1	0.9437	0.9948	0.8665
Specificity	0.1624	0.7867	0.7991	0	0.5243	0.0986	0.7822
AUC	0.7089	0.9116	0.9100	0.5384	0.8186	0.7727	0.8934
Brier Score	0.2316	0.1234	0.1225	0.0500	0.2687	0.1683	0.1862
H-measure	0.2246	0.6381	0.6332	0.0500	0.3950	0.4099	0.4727

Table 6.5 PROD combination results

Figure 6.10 shows that PROD ROC curves behave similarly as the MIN. The only difference is that for some threshold values. UCSD ROC curve stays behind the two leaders (Australian and Japanese datasets). This can be considered as advantage of PROD against other classifiers. So for real-life datasets using of PROD may be justified. However, on German and Iranian datasets PROD shows bad performance, these two dataset are the worst in terms of AUC and ROC curve shapes. It is worth mentioning to describe the thresholds assigned to each classifier for each dataset, Table 6.6 illustrates this:

Dataset	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Threshold	0.200688	0.201351	0.205318	0.599964	0.241857	0.201298	0.205830

Table 6.6 PROD thresholds across all datasets

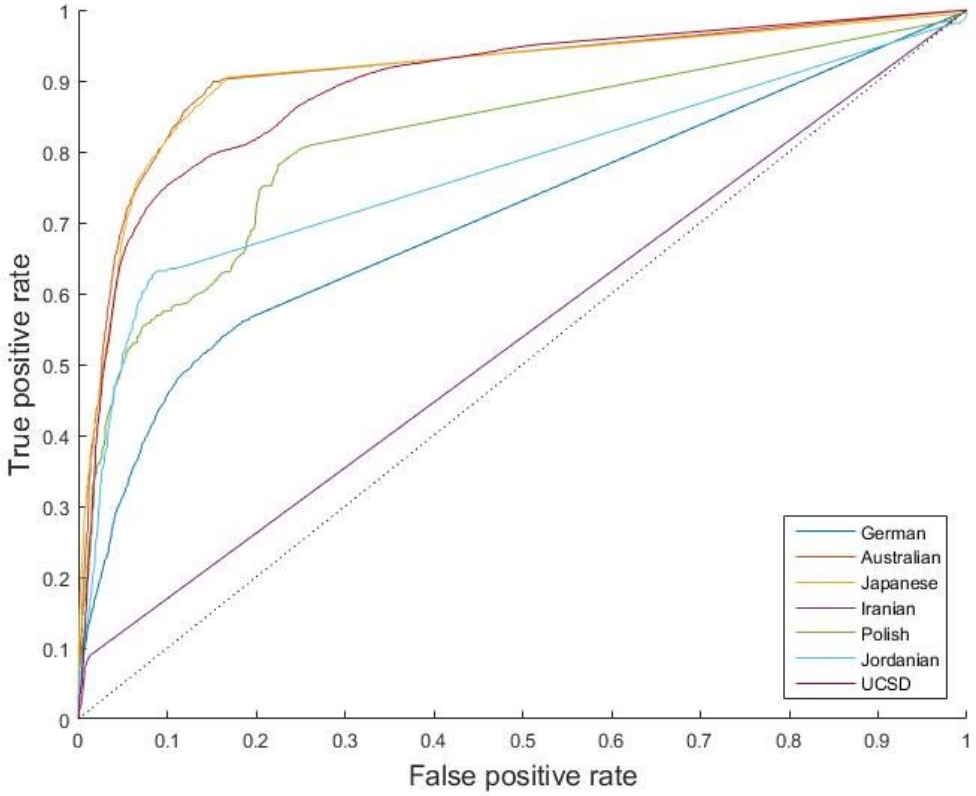


Figure 6.10 PROD ROC curve for all datasets

6.3.4 Average Rule Results

Table 6.7 demonstrates that AVG is one of the best classifiers, the reason has already been mentioned, it relies on every input value from single classifiers. Unlike the PROD, a slight change of one value is not capable of changing the final result. Even approximately, AVG looks like highly improved than the PROD. In Chapter 3 it was mentioned that AUC metric could be explained as the flexibility of the classifier. This classifier shows the best flexibility in pair with very low losses which is represented by the Brier Score. However, this is not enough to prevent only '0' output on the Iranian dataset, which is still the only dataset that so poorly classified by all combiners.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7730	0.8725	0.8648	0.9500	0.7817	0.8566	0.8639
Sensitivity	0.9014	0.8586	0.8437	1	0.7917	0.9607	0.7040
Specificity	0.4759	0.8890	0.8921	0	0.7787	0.4429	0.9158
AUC	0.7996	0.9294	0.9262	0.7770	0.8594	0.8817	0.9082
Brier Score	0.1584	0.0977	0.1005	0.0432	0.1527	0.1036	0.0999
H-measure	0.3063	0.6515	0.6468	0.2931	0.4461	0.4945	0.5160

Table 6.7AVG combination results

AVG is a very good combiner, and this fact is reflected in its ROC curves for all datasets in Figure 6.11. AVG includes all single classifiers rankings equally, that's why the rankings of AVG rule are very versatile and that is why the ROC curves are round-shaped (not similar to polylines, like plot-ROC curves of previous combiners). The leaders are the same (Australian and Japanese datasets) with UCSD and Jordanian datasets in the third and fourth place respectively. Even the ROC curve for Iranian dataset is not extremely low; it is close to the ROC curve of the German dataset. Polish ROC curve is a bit jagged, which can be explained by the small size of the Polish dataset.

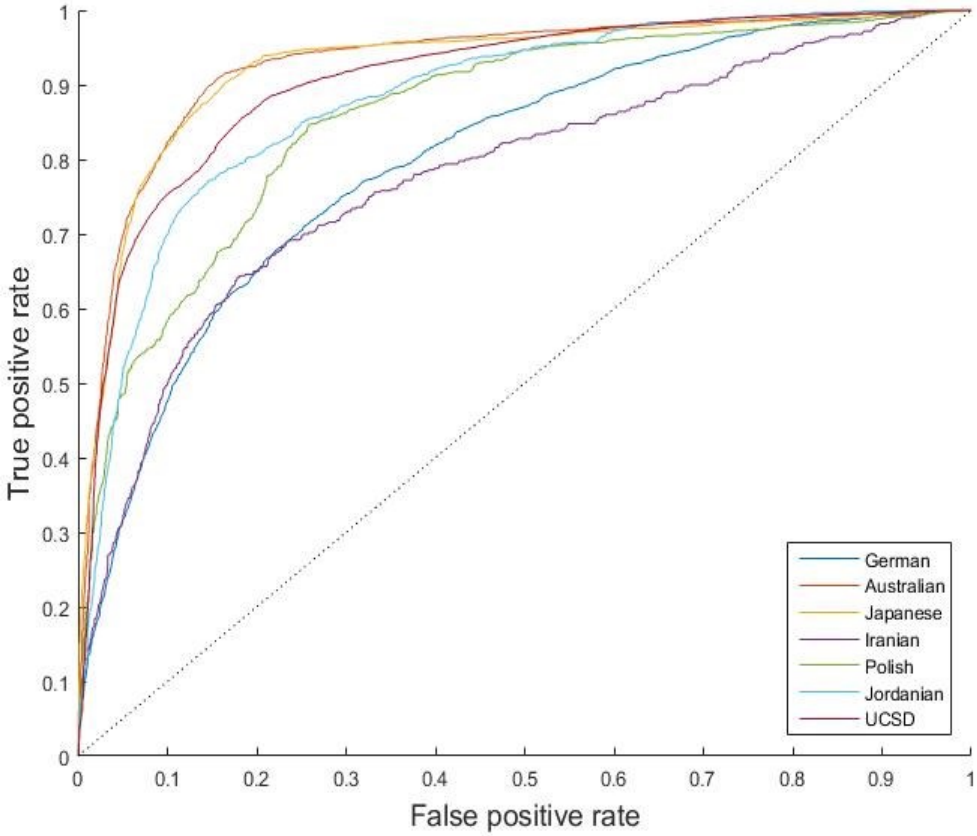


Figure 6.11 AVG ROC curve for all datasets

6.3.5 Majority Voting Rule Results

In Table 6.8, MajVot demonstrates the best performance amongst all combiners. Although it is not so multi-purposed as AVG, but it still has better Accuracy. This classifier shows us the simple truth that there is no need for sophisticated analysis, when working with results of many single classifiers. The result of the most ‘sure’ or ‘confident’ classifier is pretty enough. Unfortunately, there is still a problem with the Iranian dataset where the most 'confident' result would always be close to '0' on this dataset. This fact makes the most optimistic combiner useless on the Iranian dataset.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7776	0.8736	0.8654	0.9500	0.7883	0.8604	0.8649
Sensitivity	0.9059	0.8642	0.8449	1	0.7989	0.9625	0.6859
Specificity	0.4859	0.8848	0.8918	0	0.7847	0.4555	0.9236
AUC	0.7548	0.9031	0.9082	0.5777	0.8578	0.8025	0.8772
Brier Score	0.1837	0.1106	0.1108	0.0471	0.1584	0.1136	0.1068
H-measure	0.2859	0.6382	0.6408	0.1089	0.4415	0.4345	0.4975

Table 6.8 MajVot Rule combination results

As it can be seen from Figure 6.12, MajVot ROC curves are always polylines, as MajVot can take a limited number of values (0, 0.2, 0.4, 0.6, 0.8, 1). That is because the ranking of MajVot is calculated as number of classifiers which vote that loan is bad, divided by the whole number of classifiers. That's why results of MajVot are not flexible with threshold changing. While threshold is changing in (0.4, 0.6) interval, Accuracy remains the same, but as soon as threshold exceeds 0.6, sensitivity dramatically rises, and specificity even more dramatically falls. But for some datasets MajVot gives the best performance over all combiners, so the final decision about which classifier to choose should be made using additional dataset measurements. Three leaders are the same as in AVG rule; however the ROC curve of Iranian dataset is much lower than other ROC curves.

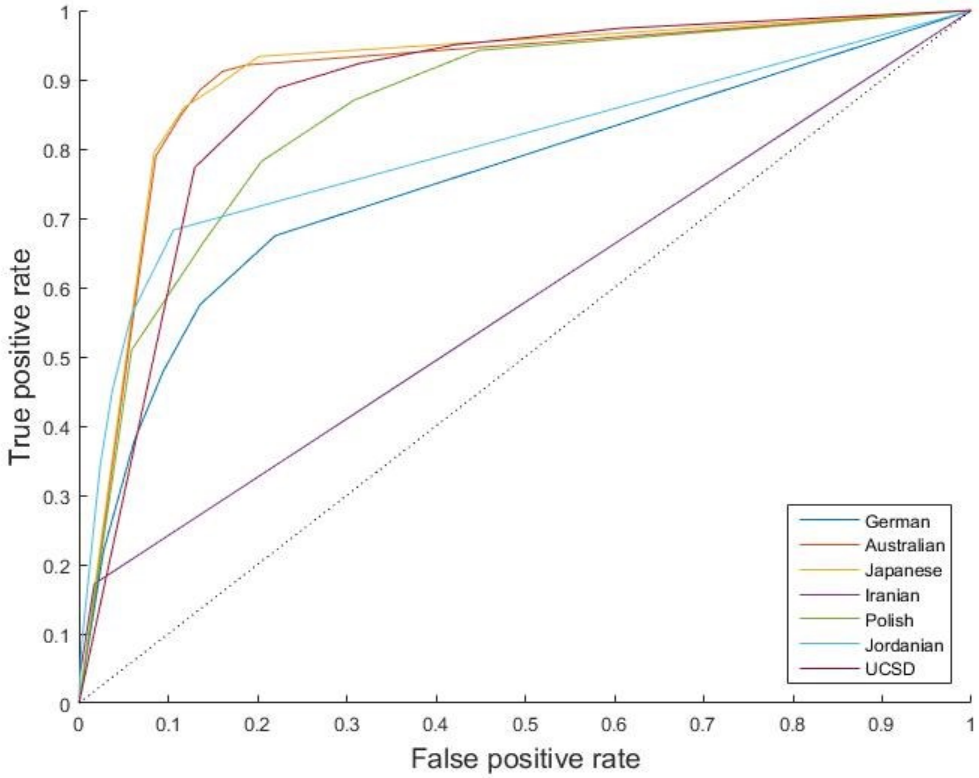


Figure 6.12 MajVot ROC curve for all datasets

6.3.6 Weighted Average Rule Results

According to Table 6.9, this combiner, as a developed version of AVG combiner, takes into consideration Accuracy of single classifiers when computing final ranking. Now classifiers with bad Accuracy on training set have a little weight, and their influence is not enough to change the final result to incorrect. The problem appears if highly weighted classifier makes a mistake. Such situation may cause a mistake of all combiner. Such change instead of increasing Accuracy decreases it. Decrease differs from 0.12% on Australian dataset, to impressive 4.55% on Polish dataset, when compared to AVG. On the Jordanian dataset, on the other hand, WAVG shows the best results. In general, WAVG performs well on the datasets, where training set Accuracy correctly reflects single classifier quality, this became a problem on a small dataset like Polish, where some classifiers on training set can show very high Accuracy (due overtraining), but this has nothing in common with real quality of classifier. It is worth mentioning to describe the weights or coefficients assigned to each classifier for each dataset, Table 6.10 illustrates this:

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7458	0.8713	0.8536	0.9497	0.7362	0.8622	0.8602
Sensitivity	0.8604	0.8789	0.9145	0.9898	0.7434	0.9327	0.7032
Specificity	0.4806	0.8611	0.7798	0.1978	0.7368	0.5842	0.9114
AUC	0.7459	0.9202	0.9091	0.7759	0.8010	0.8791	0.9011
Brier Score	0.1798	0.1021	0.1218	0.0452	0.1874	0.1014	0.1022
H-measure	0.2225	0.6356	0.6012	0.2838	0.3224	0.5064	0.5056

Table 6.9 WAVG combination results

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
RF	0.58	0.18	0.58	0.68	0.63	0.34	0.55
DT	0.21	0.35	0.03	0.08	0.01	0.12	0.30
NB	0.12	0.00	0.20	0.21	0.29	0.00	0.00
NN	0.09	0.28	0.19	0.00	0.07	0.17	0.07
SVM	0.00	0.19	0.00	0.03	0.00	0.38	0.08

Table 6.10 WAVG coefficients for each dataset for all classifiers

WAVG performance in terms of ROC curves is similar to AVG. Figure 6.13 shows that Japanese dataset is slightly worse than Australian dataset. Surprisingly, the Jordanian dataset shows solid ROC curve just after the UCSD ROC curve. The main problem of WAVG, as it was mentioned before, is inconsistency between training set Accuracy and testing set Accuracy, that's why weighted coefficients may not always be optimal. This is the reason why the most accurate classifier RF can receive lower weights than NN, which can show even 100% Accuracy over the training set.

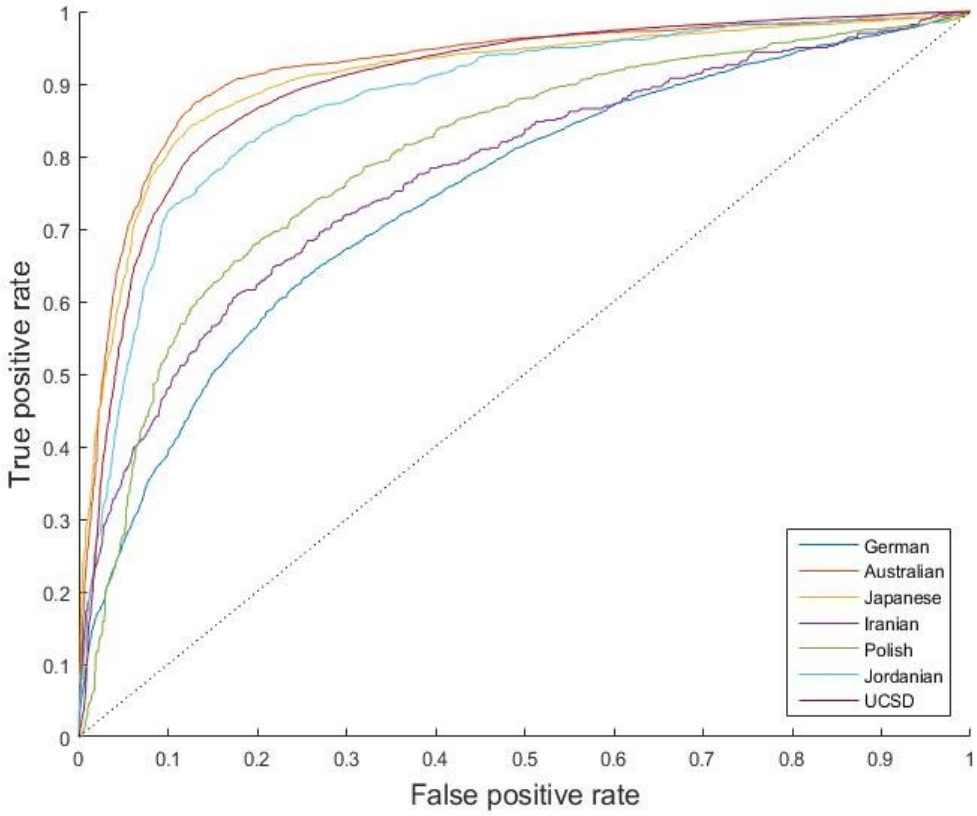


Figure 6.13 WAVG ROC curve for all datasets

6.3.7 Weighted Voting Rule Results

The results of Table 6.11 are seen to be similar to MajVot; however, in this combiner the most effect on the results is caused by the weights. As far as the classifiers outputs are being rounded, the uncertainty of the classifier does not cause any effect, the output could be on the edge with the threshold value, the mistake could be minimal, but rounding has a critical chance to change the combiner's result to wrong prediction. Thus, the possible reason is that the mistake might become a common situation, when classifiers are uncertain about some data. It is worth mentioning to describe the weights or coefficients assigned to each classifier for each dataset, Table 6.12 illustrates this.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Accuracy	0.7725	0.8700	0.8717	0.9460	0.7742	0.8574	0.8690
Sensitivity	0.9066	0.8766	0.8806	0.9875	0.7782	0.9412	0.6924
Specificity	0.4611	0.8607	0.8618	0.1619	0.7774	0.5269	0.9269
AUC	0.6878	0.8908	0.8486	0.5723	0.7990	0.7954	0.8093
Brier Score	0.1933	0.1064	0.1154	0.0477	0.1990	0.1098	0.1145
H-measure	0.2473	0.6308	0.6113	0.1059	0.3843	0.4297	0.4809

Table 6.11 WVOT combination results

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
RF	0.71	0.19	0.46	0.29	0.70	0.20	0.53
DT	0.09	0.57	0.00	0.32	0.00	0.23	0.30
NB	0.00	0.00	0.22	0.26	0.08	0.00	0.00
NN	0.04	0.04	0.19	0.13	0.04	0.18	0.10
SVM	0.16	0.19	0.12	0.00	0.18	0.39	0.06

Table 6.12 WVOT coefficients for each dataset for all classifiers

Looking at Figure 6.14, WVOT ROC curves are similar to MajVot ones, and as that classifier, WVOT ROC curves are polylines, which can be explained by the same way as MajVot. WVOT shows good performance on Australian, Japanese and UCSD datasets, however the ROC curve for the UCSD dataset drastically falls down on a low threshold values. But on the high values of threshold performance of UCSD is even better than Japanese and Australian performance.

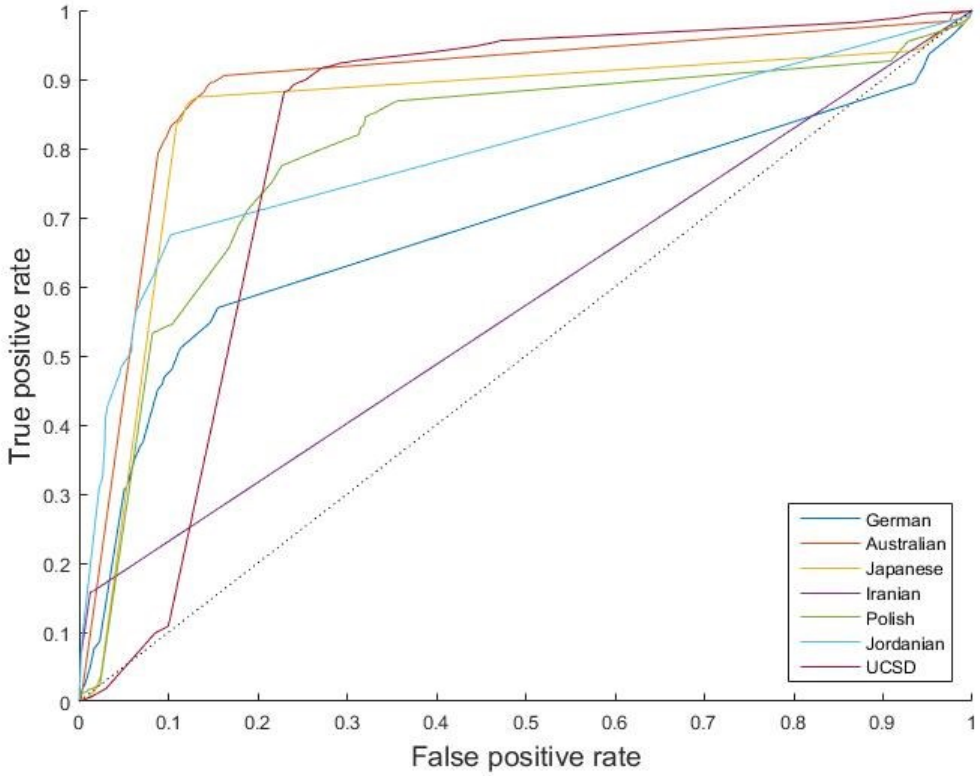


Figure 6.14 WVOT ROC curve for all datasets

6.4 Analysis and Discussion

In this section all traditional combiners are analysed, compared between each other's and with LR. Then, some assumptions are made for which type of datasets; combiner would show the best performance. Firstly, the ranking table of all traditional combiners is given. It would help to analyse, which combiner shows the best Accuracy and for which dataset.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD	Average ranking
MIN	4	5	4	1	6	6	6	4.57
MAX	5	5	7	4	4	5	5	5
PROD	7	6	5	1	7	7	7	5.71
AVG	2	2	3	1	2	4	3	2.43
MajVot	1	1	2	1	1	2	2	1.43
WAVG	6	3	6	2	5	1	4	3.86
WVOT	3	4	1	3	3	3	1	2.57

Table 6.13 Global rating of traditional combiners by Accuracy

Obviously, the best combiner appears to be the MajVot. Its first place can simply be explained by the fact, that the classifiers have quite a high Accuracy by themselves and a possibility that four of classifiers mistake on the same data is low. The same fact is also the reason of the AVG to be second place. WVOT, which is a combination of WAVG and MajVot is third. Moreover, on the Japanese and UCSD datasets it holds the first place. So the final decision about which classifier to choose can be made looking at the structure of the dataset. The worst combiner is a PROD, this can be explained that the result of multiplication of five classifiers less than one values always turns out a very low, that's why the threshold is very hard to choose the output rankings always is much skewed. For example, if all five classifiers have ranking 0.6, then ranking of the combiner is $0.6^5 = 0.078$ which is a very small value, so threshold for the PROD should be chosen far less than this value.

The other interesting thing about combiners and classifiers that appear here, is that every classifier works better when it has less number of features to rely on. Many features unnecessarily tangle the classifier, which in turn reduces the Accuracy and increases the losses.

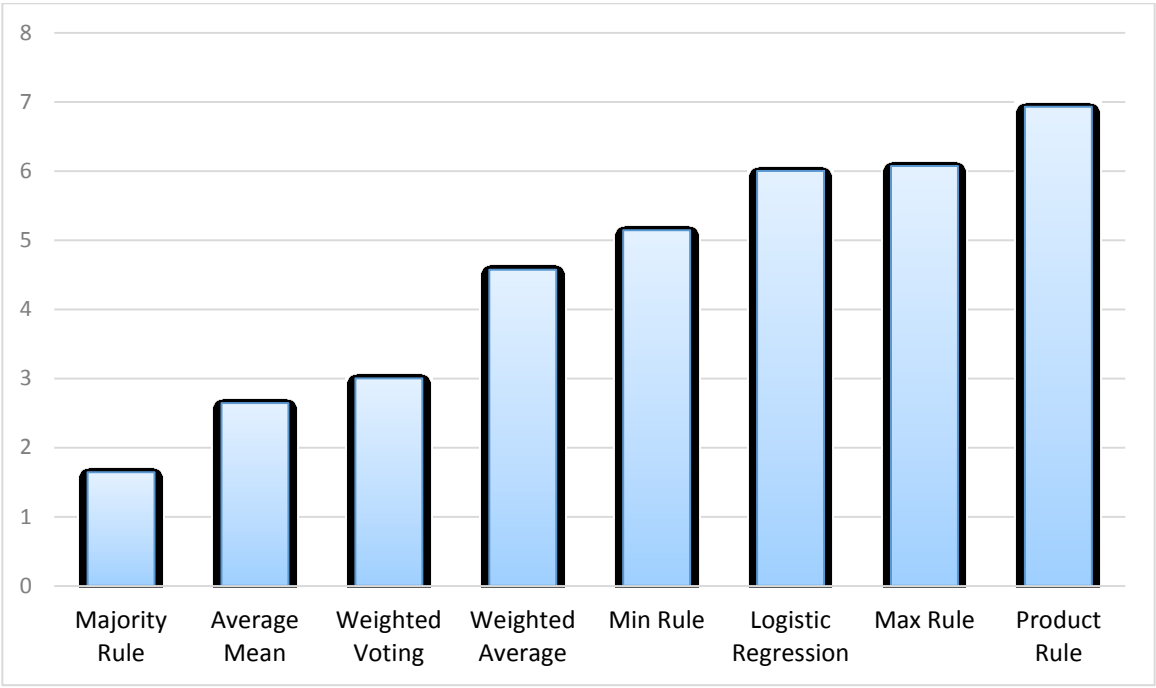


Figure 6.15 Average ranking of the traditional combiners and LR from best to worst

Although Table 6.10 and Figure 6.15 clearly show the best and worst combiners, nonetheless it provides the best rating to MajVot combiner, which is absolutely useless on the Iranian dataset. This fact helps to figure out that on such datasets it might be better to reduce the

Accuracy but be able to find the '1's amongst the ocean of '0's and reduce the losses, as far as all the combiners are being considered for being used in credit-scoring. Finally, the traditional combiners could be used to improve the work of single classifiers, but these could not be used on every dataset with the same productivity, and should be chosen dependently on each dataset. Logistic regression, a benchmark classifier, holds the sixth position, which demonstrates that using novel pre-processing techniques and strong classifiers helps to beat LR even if very simple combiners were used.

One more dataset that requires some extra analysis is the UCSD dataset. This dataset clearly shows, as in Table 6.14, that most of combiners are absolutely capable of giving results better than Logistic Regression. There are two combiners that are very worse than LR which are the MIN and PROD, and both these cases can be easily explained by the fact that the UCSD dataset is a large real-world dataset which is highly imbalanced where it has around 75% of good loans, and MIN and PROD in general have bad performance for such datasets. MAX gives almost the same performance as Logistic Regression, and all other combiners are better up to +2.73% for WVOT. As this dataset is a large real-world dataset, this fact is remarkable and worth mentioning.

Table 6.14 provides the Accuracy of LR which shows how the Accuracy of traditional combiners varies from them. The value is positive if combiner's Accuracy is better, and negative otherwise. Subsequently, the table is illustrated in Figure 6.16.

	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Logistic Regression	0.7597	0.8641	0.8626	0.9239	0.7246	0.8240	0.8417
MIN	0.0039	0.0021	0.0017	0.0261	-0.0042	0.0006	-0.0371
MAX	-0.0065	0.0021	-0.0177	-0.0138	0.0433	0.0288	-0.0001
PROD	-0.0235	-0.0058	-0.0029	0.0261	-0.0059	-0.0084	-0.0387
AVG	0.0133	0.0084	0.0022	0.0261	0.0571	0.0326	0.0220
MajVot	0.0179	0.0095	0.0028	0.0261	0.0637	0.0364	0.0232
WAVG	-0.0139	0.0072	-0.0090	0.0198	0.0116	0.0382	0.0185
WVOT	0.0128	0.0059	0.0091	0.0221	0.0496	0.0334	0.0273

Table 6.14 Difference with Logistic Regression

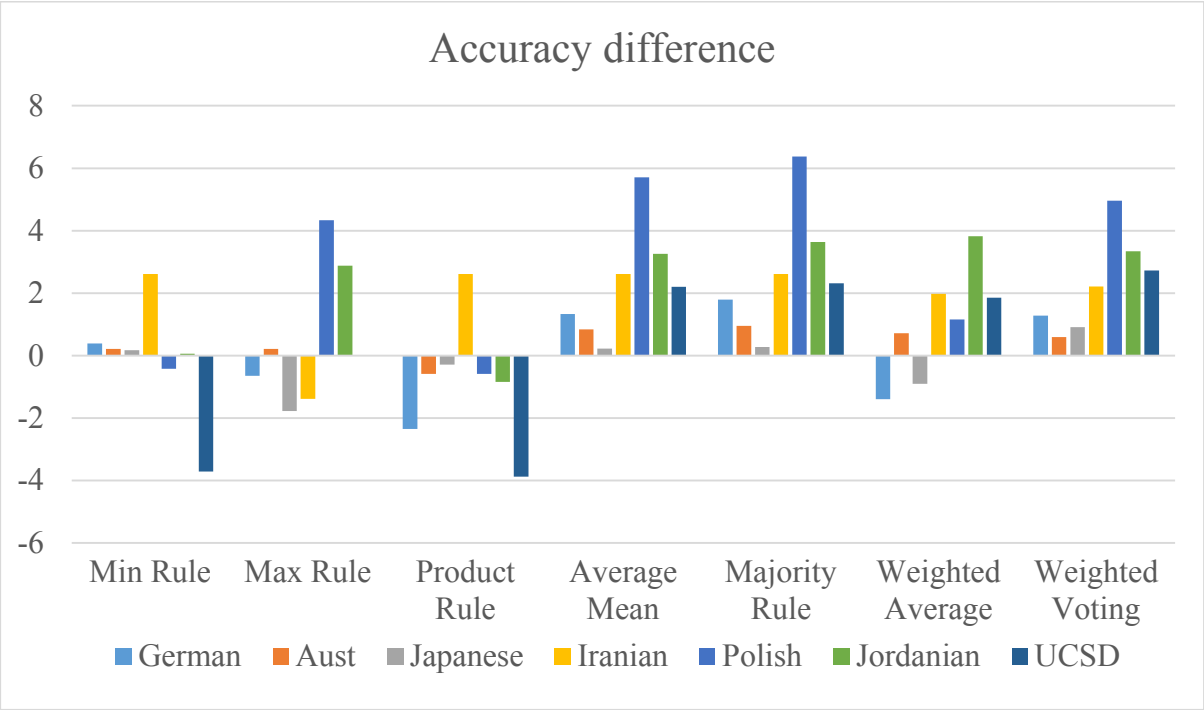


Figure 6.16 Accuracy difference with Logistic Regression

In fact, the table and graph only approve the conclusions made after the ranking table built. The only thing that should be mentioned once more is the Iranian dataset. Although every combiner is better on this dataset, it should not be forgotten, that most of combiners outputs are good loans only. Thus, the superiority of such combiners is very doubtful.

6.5 Summary

The results obtained clearly show that most traditional combiners concede the best of classifiers (RF) for all datasets. Of course, RF stays the best, but actually it is not a single classifier but a homogenous combiner of DT.

However, all traditional combiners could be categorized into two groups. The first one, which gives comparatively good results, that includes the MajVot, AVG and WVOT. These three classifiers compared to LR shows better results in the main measures (e.g., Accuracy, AUC) for all datasets without exception. The second group consists of MIN, MAX, PROD and WAVG, these shows the worse classifiers, which are worse than LR for some datasets. Advantages of traditional combiners from the first group are:

- Simplicity and intuitiveness of the mathematical model of the combiner.
- High stable results over all datasets.

The second advantage should be explained: even though for some datasets single classifiers can have advantage over traditional combiners and other single classifiers, for other datasets for some reason results of single classifiers can become significantly worse than other classifiers results. The example is DT in Polish dataset where it shows Accuracy even better than Accuracy of RF, but for German dataset it shows the worst Accuracy amongst single classifiers. Traditional combiners from the first group, on the other hand, show stable results without significant drops and rises for all datasets.

Therefore, traditional combiners from the first group could be used in real-life applications if they know that input data is varying a lot (input datasets have highly different number of features, percentage of bad loans, number of outliers and noise etc.) so if traditional combiners were used, it can be assured that the results would be good enough for all type of datasets. The selecting of specific combiner between the first group is up to the a priori knowledge about structure and parameters of a majority of input datasets.

On the other hand, the general performance of the traditional combiners has proven insufficient as the Accuracy of the best of traditional combiner concedes the Accuracy of RFs. As a result, in the next chapter an analysis to such complex combiners such as D-ENS and classifier ConsA is undertaken and evaluation of their performance in comparison to the performance of all single and hybrid classifiers and traditional combiners that have been investigated in this and previous chapters.

CHAPTER 7

HYBRID ENSEMBLE CREDIT-SCORING MODEL USING CLASSIFIER CONSENSUS AGGREGATION APPROACH

7.1 Introduction

In Chapter 5, it has been demonstrated that single classifiers performance are increased when applying such pre-training data-selection algorithms as filtering and feature selection were analysed. In Chapter 6 simple heterogeneous combiners in the hope that they show better results than the best of single classifiers were considered. Unfortunately, the results of traditional combiners showed drawbacks of traditional approaches to create ensembles of single classifiers. In this Chapter, 2 ensemble algorithms, which are inherently different, were proposed in the hope of showing high and stable results over all datasets.

In reference to Table 2.2, amongst 37 studies that has been investigated, it can be noticed that only few number of studies focused on using selective ensembles and different classifiers combination methods. For this reason this chapter will focus on developing different combination approached in the hope of increasing the model performance.

Multiple classifiers systems or classifiers ensembles can be combined in 2 ways, either fusion of the classifiers or dynamic classifier(s) selection (Woods *et al*., 1997; Ko *et al*., 2008; Xiao *et al*., 2012). Classifier fusion methods take all classifiers rankings and combine them in one classifier which also can be called static ensemble. On the other hand dynamic selection can be either a selection of one best classifier or best group of classifiers from the pool of classifiers that can classify each test sample; these two types of selection are called dynamic classifier selection or D-ENS selection (Xiao *et al*., 2012). According to Ko *et al*. (2008) the advantage of D-ENS over classifier selection is the distribution of the risk of over-generalisation by choosing a group of classifiers instead of one for a test point. Therefore, D-ENS selection approach is adopted in this work.

In this chapter new ensemble algorithms which belong to dynamic and static combination approaches is investigated. The first algorithm is called the D-ENS classifier; its idea is

inspired from Xiao (2012) which is that in different areas of input N-dimensional space (where N is number of features in dataset) different classifiers can be amongst the leaders in terms of classification Accuracy. So for each point local Accuracy of each single classifier amongst the training set is calculated, and evaluate weighting coefficients w_i proportional to these values. The ranking of the D-ENS classifier would be a linear combination of single classifiers rankings with the weights w_i.

The second approach is called the classifiers ConsA (DeGroot, 1974; Berger, 1981Shaban *et al*., 2002) it simulates a behaviour and team-communication process of real experts group, so that each of single experts can modify its own opinion accordingly to opinions of other experts in the group. The final ranking of ConsA classifier is calculated as a common group decision after equilibrium is reached and opinions do not change anymore. Besides, the D-ENS and ConsA algorithms are connected with the approaches described in the fourth chapter in the hope than if single classifiers shows better results, complex combiners will increase their Accuracy even more.

7.2 The Proposed Approaches

In this section, models of modern and efficient combiners: D-ENS classifiers and classifiers ConsA are considered. Both these models are complex and that is why they show different behaviour in different points of input space (unlike classical combiners, which depends only on single classifiers rankings, and evaluate final ranking using one fixed equation without dynamic additional parameters).

7.2.1 The Dynamic Ensemble Approach

The proposed model of D-ENS classifiers approach is shown in Figure 7.1. In this model, two pre-training phases were used before actually starting single classifiers training. Parameters of single classifiers, including feature selection importance threshold and filtering thresholds Tr_0 and Tr_1 are remaining the same as in Chapter 5.

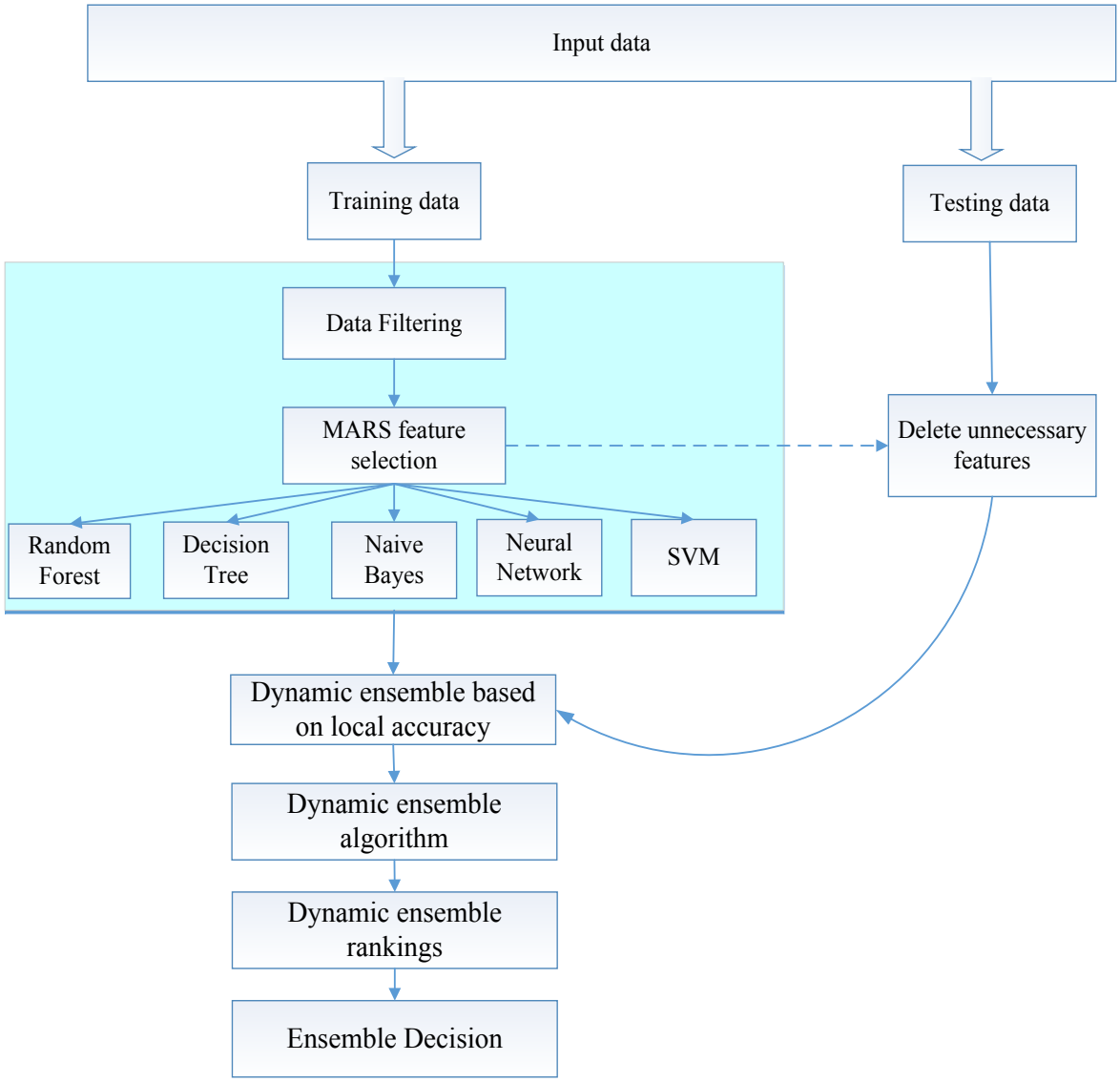


Figure 7.1 The process of the D-ENS Approach

The workflow of the dynamic classifiers ensemble can be summarised in the following steps:

- Divide input data into training and testing set.
- Evaluate MARS model onto training data to obtain list of selected features.
- Build proximity graph and filter training data (the goal of filtering is removing outliers and highly noisy data points)
- Train five classifiers and obtain classifiers rankings for testing set queries.
- Perform D-ENS classifiers algorithm to merge single classifiers rankings into one.

The details of the D-ENS approach are discussed in Section 7.3.

7.2.2 The Classifiers ConsA

The proposed model of classifiers ConsA is shown in Figure 7.2. As in the previous model, filtering and feature selection before training of single classifiers were used.

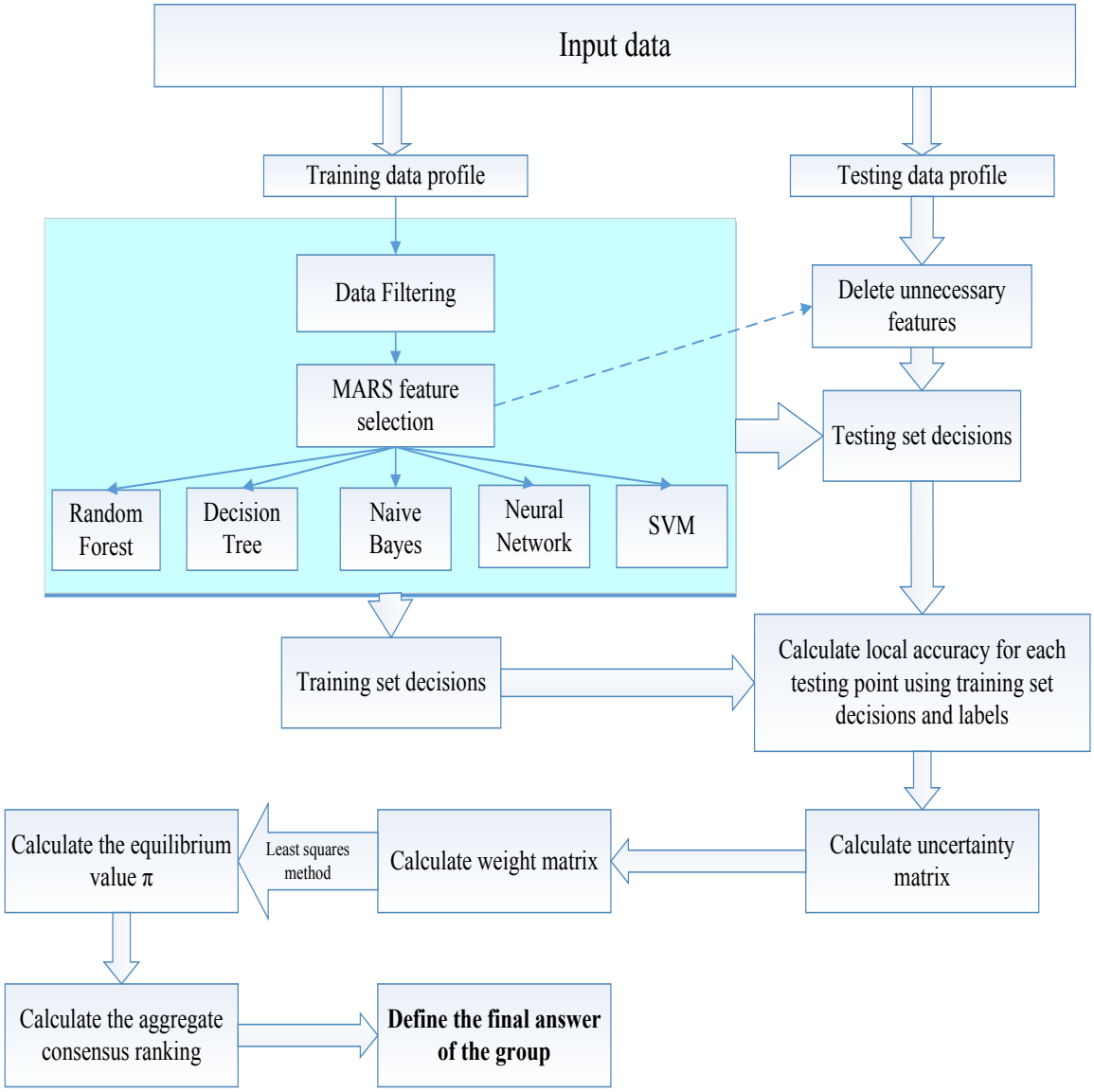


Figure 7.2 The process of the Classifiers ConsA

The workflow of the classifier ConsA can be summarised in the following steps:

- Divide input data into training and testing set
- Evaluate MARS model onto training data to obtain list of selected features
- Build proximity graph and filter training data (the goal of filtering is removing outliers and highly noisy data points)
- Train five classifiers and obtain classifiers rankings for testing set queries.

- For each testing set query:
    - Calculate local Accuracy for each single classifier using training set queries and actual labels for these queries
    - Calculate uncertainty matrix of group of classifiers.
    - Calculate weight matrix.
    - Calculate equilibrium value (group vector of rankings using least square method not iterations method as in DeGroot (1974).
    - Calculate the aggregate ConsA ranking.
    - Using default value of threshold, calculate the final answer (prediction) of the group.
    - The details of the classifiers ConsA is discussed in Section 7.4.

7.3 D-ENS Approach: Description of the Algorithm

In this section the theory which lies behind dynamic classifier ensemble model is demonstrated in addition to the explanation of the approach which is developed and tested in this work. D-ENS is a fusion model, which is based on the idea to combine rankings of single classifiers taking into account their local accuracies. Local Accuracy is estimated by the Accuracy of each classifier in the local region of the feature space surrounding an unknown test point (Woods *et al*., 1997; Oza *et al*., 2005). To evaluate local Accuracy Xiao *et al*. (2012) proposed to put forward two calculation methods: overall local Accuracy (OLA) and local class Accuracy (LCA). The Accuracy of D-ENS strictly depends on the correctness of the local Accuracy estimate, so it is important to choose right local Accuracy evaluation range and choose whether to use overall local Accuracy or simple local class Accuracy. To avoid the problem of choosing these parameters, it is proposed to modify the LCA evaluation method so that it can be taken into consideration the Accuracy in the all points, but with the weight inverse proportional to the distance from the target point. This approach will overcome heterogeneity in the input data, so that for some points the number of local neighbours can be too small, and that's why local Accuracy is not precise.

D-ENS is a combiner which outperforms best classifiers results over the majority of data sets (Woods *et al*., 1997). Initially, Xiao *et al*. (2012) investigated the dynamic classifier selection method, which for each testing set point returns as the output ranking of the best classifier in

that point (in terms of local Accuracy). As an inspiration from their method, the proposed method will take into consideration other classifiers with less local Accuracy (however, their impact is less than impact of classifier with the best local Accuracy). In other words in the proposed approach the final output is linear combination of single classifiers outputs, with weights proportional to local Accuracy for each testing set point rather than taking the final output as the ranking of the most accurate classifier. This allows to build more flexible and robust ensemble, as even the best classifier could be sometimes wrong.

Before applying the ensemble combiner, all single combiners (RF, DT, NB, NN and SVM) are trained and evaluate their predictions over training and testing sets. To combine the decisions of these five classifiers the local Accuracy is evaluated for each entry x_* from a testing set. In Xiao *et al*. (2012) it is proposed to choose non-negative distance d as a local Accuracy area and evaluate Accuracy over all entries from training set that are located at a distance from x_* less than d. As an enhancement, it is proposed to evaluate not a simple mean value, but weighted average for all points from training set with weights are inversely proportional to the distance from training test entries to x_* In other words, lets have a training set entries x_i with target labels p_i, and classifier's predictions over training set $\widetilde{p_i}$, where $i = 1 \ldots N$. Local Accuracy is evaluated as:

$$L(x_*) = \sum_{i=1}^{N} w_i |p_i - \widetilde{p_i}|, \sum_{i=1}^{N} w_i = 1 \tag{7.1}$$

and w_i is inversely proportional to distance from x_* to x_i.

Conditional local accuracies are evaluated as:

$$L(x_*, 0) = \sum_{p_i=0} w_i |p_i - \widetilde{p_i}|, \sum_{p_i=0} w_i = 1 \tag{7.2}$$

$$L(x_*, 1) = \sum_{p_i=1} w_i |p_i - \widetilde{p_i}|, \sum_{p_i=1} w_i = 1 \tag{7.3}$$

For clarity on how local Accuracy for a test point is calculated, Figure 7.5 illustrates an example on the local Accuracy algorithm workflow for a small training and testing sets.

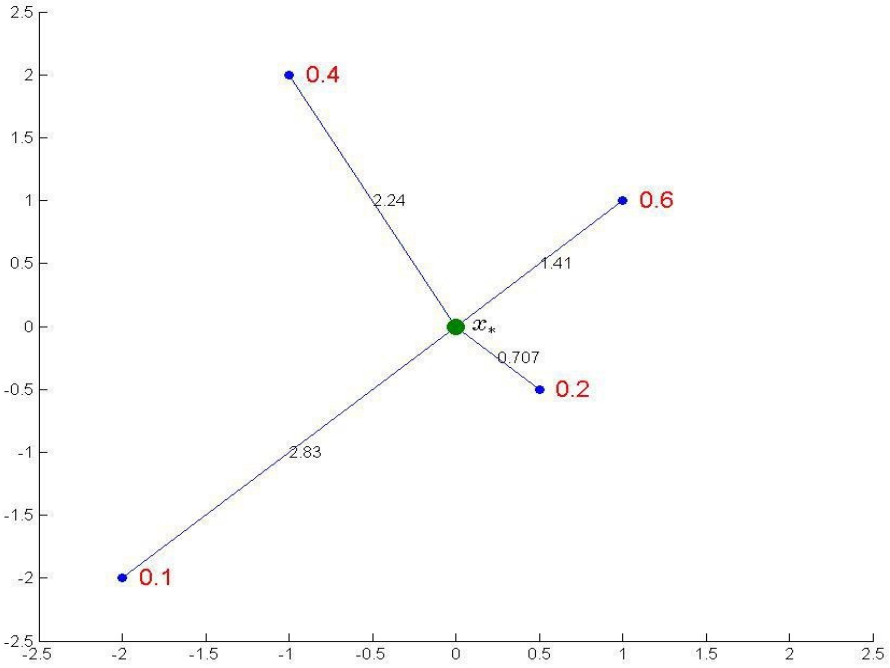


Figure 7.3 Local Accuracy evaluation example

Let's have a training set consists of 4 points with their accuracies in each shown in the Figure 7.3 (Accuracies $= [0.2,0.6,0.4,0.1]$). Standard Accuracy over training set is average of these 4 numbers, so Accuracy $= \frac{0.2+0.6+0.4+0.1}{4} = 0.3250$. The Local Accuracy is calculated in the test point x_* and located in the centre; is calculated as follows:

- Determine vector of distances from this point to all points of the set, which is $D = [0.707,1.41,2.24,2.83]$.
- Perform such operations over D:

$$D = max(D) - D \rightarrow D = [2.12,1.41,0.59,0]$$

$$D = D/sum(D) \rightarrow D = [0.5139,0.3426,0.1435,0]$$

- To evaluate local Accuracy, evaluate $La = A \cdot D = 0.3657$

The aim of evaluating conditional local accuracies is to determine how good each classifier can classify good and bad loans near point x_* As some datasets used int this work are imbalanced, cost of bad loan misclassifying is not equal to cost of good loans misclassifying. After evaluating local accuracies for all entries from the testing set, evaluation of an algorithm were processed.

Let us enumerate local classifiers as:

- RF
- DT
- NB
- NN
- SVM

Now p_i means ranking, and U_i – uncertainty value, $L_i(x_*)$, $L_i(x_*, 0)$, $L_i(x_*, 1)$ – absolute and conditional local accuracies of i-th single classifier. The uncertainty of each classifier using the equation $U_i = -p_i \cdot \log_2(p_i)$ was evaluated. Each ranking coefficient W_i on $(1 - U_i$ were multiplied. The idea behind it is to give lower coefficients to the classifiers with higher uncertainty value. That's why classifier's local coefficient were evaluated as

$$W_i = \begin{cases} L_i(x_*, 0) * (1 - U_i), & 0 < p_{cl} \le a_l, \\ L_i(x_*) * (1 - U_i), & a_l < p_{cl} \le a_h, \ i \in \{1..5\}. \\ L_i(x_*, 1) * (1 - U_i) & a_h < p_{cl} < 1, \end{cases} \quad (7.4)$$

Parameters a_l and a_h are selected for reasons of maximise the training set Accuracy of D-ENS. After this vector $W = (W_1, W_2, W_3, W_4, W_5)$ is normalised:

$$W_{new} = W / \sum_{i=1}^{5} W_i. \quad (7.5)$$

Next point is to evaluate weighted average of classifier's rankings with weights equal to W. In other words, the final ranking of D-ENS classifier is evaluated as

$$P_*(x_*) = \sum_{i=1}^{5} W_{new(i)} \cdot p_i, \quad (7.6)$$

where $p_i, i \in \{1..5\}$ are single classifier predictions.

D-ENS pseudo-code

Train single classifiers (RF, DT, NB, NN and SVM) over the training set and the following is done:

- Evaluate single classifiers predictions of training set entries and testing set entries

For each entry x_* from a testing set

- Evaluate local Accuracy of each classifier in the point x_* using equations (7.1), (7.2) and (7.3).
- Evaluate classifier's local coefficient according to equation (7.4)
- Normalise classifier's local coefficient according to equation (7.5)
- Evaluate prediction value of D-ENS $P_*(x_*)$ according to equation (7.6).

End for

The following example displays the dynamic classifiers ensemble approach workflow for a single testing set point.

- For point x_* from test set the local accuracies for five classifiers were evaluated, results are:
- For RF $L_1(x_*) = 0.8, L_1(x_*, 0) = 0.5,\ L_1(x_*, 1) = 1.0, p_1 = 0.9$
- For DT $L_2(x_*) = 0.8, L_2(x_*, 0) = 0.9,\ L_2(x_*, 1) = 0.7, p_2 = 0.6$
- For NB $L_3(x_*) = 0.7, L_3(x_*, 0) = 0.65, L_3(x_*, 1) = 0.8, p_3 = 1$
- For NN $L_4(x_*) = 0.65, L_4(x_*, 0) = 0.75, L_4(x_*, 1) = 0.6, p_4 = 0.1$
- For SVM $L_5(x_*) = 0.6, L_5(x_*, 0) = 0.8, L_5(x_*, 1) = 0.4, p_5 = 0.5$

Suppose $a_l = 0.25$, $a_h = 0.75$. As a first step, the uncertainties are calculated as follows:

$U_1 = -p_1 \cdot \log_2(p_1) = 0.136803,\quad U_2 = -p_2 \cdot \log_2(p_2) = 0.442179,\quad U_3 = -p_3 \cdot \log_2(p_3) = 0,\ U_4 = -p_4 \cdot \log_2(p_4) = 0.136803,\ U_5 = -p_5 \cdot \log_2(p_6) = 0.5,$

Next step the classifier's local coefficients were calculated:

- As $p_1 > a_h$, calculate W_1 as $L_1(x_*, 1)(1 - U_1) = 0.863197$
- As $a_l < p_2 < a_h$, calculate W_2 as $L_2(x_*)(1 - U_2) = 0.446257$
- As $p_3 > a_h$, calculate W_3 as $L_3(x_*, 1)(1 - U_3) = 0.8$
- As $p_4 > a_l$, calculate W_4 as $L_4(x_*, 0)(1 - U_4) = 0.500855$
- As $a_l < p_5 < a_h$, calculate W_5 as $L_5(x_*, 0)(1 - U_5) = 0.446257$

Then vector W_{new} according to the equation (7.5) is normalised. Then

$$W_{new} = (0.282407, 0.145999, 0.261732, 0.163862, 0.145999)$$

Final ranking is as follows:

$0.9 \times 0.282407 + 0.6 \times 0.145999 + 1 \times 0.261732 + 0.1 \times 0.163862 + 0.5 \times 0.145999 = 0.692884$. **Prediction of ensemble is '1'.**

7.4 Classifiers Consensus Approach: Description of the Algorithm

In this section theory which lies behind ConsA model is demonstrated, along with its process as well as the improvements which were made to adjunct ConsA model to our data. The basic idea behind classifier decisions combination is that, when classifiers make a decision, one should not rely only on a single classifier decision, but, rather, require classifiers to participate in the decision-making process by combining or fusing their individual opinions or decisions. Therefore, the core problem that needs to be addressed when combining different classifiers is resolving conflicts between them. In other words, the problem is how to combine the results of different classifiers to obtain better results (Chitroub, 2010; Xu *et al*., 1992). In this section, a new combination method is introduced in the field of credit-scoring based on classifier Consensus, where those in the ensemble interact in a cooperative manner in order to reach an agreement on the final decision for each data sample.

Tan (1993) emphasized that agents working in partnership can significantly outperform those working independently. The idea of the ConsA is not new as it has been investigated in many studies in different fields, such as statistics, remote sensing, geography, classification, web information retrieval and multi-sensory data (Tan, 1993; DeGroot, 1974; Benediktsson & Swain, 1992; Shaban *et al*., 2002; Basir & Shen, 1993). In this context, the general strategies adopted are those of DeGroot (1974), Berger (1981) and Shaban *et al*. (2002), who proposed a framework that provides a comprehensive and practical set of guidelines on the underpinning constructs of ConsA theory where interactions between classifiers are modelled when an agreement between them is needed. It is believed that their strategies can be useful when adopted for the credit-scoring domain.

ConsA is a modern combiner, which is based on the approach to consider single classifiers as a collaborative society of agents that communicate their estimates of the input entries. After some time passed, they reach a consensus with respect to the best decision possible. By combining decisions made by different agents, more effective decision-making can be achieved. There is a shortcoming of papers analysing performance of ConsA (e.g., DeGroot, 1974; Berger, 1981; Shaban *et al*., 2002). So the task in this work become useful as the only one which compares ConsA performance with the performance of the wide list of other

classifiers and combiners, and do this comparison over seven different datasets, amongst which there is real-life dataset.

ConsA differs from other classifier fusion techniques, as this method uses relationships between single classifiers, and it models a process similar to the process of decision making into the group of real experts. So this method is used in hope that this approach will allow us to build very good and stable classifier.

Practically, ConsA mimics the team-communication processes of a real group of experts, so that each individual expert can modify its own opinion according to the opinions of other experts in the group. The final ranking of ConsA classifier is calculated as a common group decision after equilibrium is reached. When opinions are no longer changing, and in order to reach a consensus between classifiers on each input decision, a set of steps have to be processed. These are discussed in the following sub-sections.

7.4.1 Calculating Classifiers Rankings and Build a Decision Profiles

Consider a group of *N* classifiers or agents, indexed by the a set $A = A_1, A_2, \dots, A_N$. When receiving a an input sample q, A_i chooses an answer from a set of possible answers $\Gamma = (\gamma_1, \dots, \gamma_m)$. For each classifier or agent consider an estimate function R_i, which associates a nonnegative number for every possible answer from Γ. The result of the estimate function R_i is a value in the range of [0, 1] which shows the desirability of the corresponding answer. Prediction of the classifier may be found after finding R_i and applying a threshold to it.

$$\sum_{k=1}^{m} R_i(\gamma_k) = 1 \; \forall i \in \{1..N\} \tag{7.7}$$

After calculating each classifier ranking, the decision profile can be represented in matrix form as:

$$DP = \begin{bmatrix} R_1(e_1) & R_1(e_2) & R_1(e_3) & \dots & R_1(e_n) \\ R_2(e_1) & R_2(e_2) & R_2(e_3) & \dots & R_2(e_n) \\ R_3(e_1) & R_3(e_2) & R_3(e_3) & \dots & R_3(e_n) \\ R_4(e_1) & R_4(e_2) & R_4(e_3) & \dots & R_4(e_n) \\ R_5(e_1) & R_5(e_2) & R_5(e_3) & \dots & R_5(e_n) \end{bmatrix} \tag{7.8}$$

where *n* is the number of queries in the training/testing set, e_i is the i-th input query and Rj (e_i); j ∈1..5 is the j-th classifier ranking for the i-th input query. So, to evaluate the uncertainty between classifiers it is needed to process *n* columns of matrix *DP* for testing the set, input by

input. Therefore, the objective here is to evaluate the common group ranking $R_G: \Gamma \rightarrow [0,1]$ to aggregate the expected rankings for all classifiers. Let Γ_j be a vector of outputs of j-th classifer: $\Gamma_j = R_j(\Gamma)$.

7.4.2 Calculating the Classifiers Uncertainty

After building the decision profile for the classifier rankings, the next stage is about finding a function by which each classifier's uncertainty can be computed. The task here is to assign more weight to classifiers that are less uncertain about their decision, and vice versa. However, the weighting should reflect the contrast in classifiers' decisions. During this stage, uncertainty is divided into two types:

- Local Uncertainty.
- Global Uncertainty.

Here local uncertainty is related to the quality of the classifier's own decision in terms how the classifier is confident about its decision, whereas global uncertainty denotes how much the classifiers are confident about their decisions after knowing each other decisions this emerges as the result of collaboration between classifiers taking place in the form of decision profile exchange. At this stage a classifier is able to review its uncertainty level and modify it given its decision as well as the decisions of others. This shows how a classifier is able to improve its decision when other classifiers' decisions become available.

The uncertainty matrix *U* can be presented as follows:

$$U = \begin{bmatrix} U_{11} & U_{12} & U_{13} & \dots & U_{1N} \\ U_{21} & U_{22} & U_{23} & \dots & U_{2N} \\ U_{31} & U_{32} & U_{33} & \dots & U_{3N} \\ U_{41} & U_{42} & U_{43} & \dots & U_{4N} \\ U_{51} & U_{52} & U_{53} & \dots & U_{5N} \end{bmatrix} \tag{7.9}$$

where U_{ii}; i ∈1…5 is the local uncertainty of the i-th classifier, and U_{ij}, I, j∈1…5; i ≠ j is the global uncertainty of the i-th classifier, when it knows the ranking or the decision of the j-the classifier. To evaluate the classifiers uncertainties, consider $R_i(\gamma_k)$ is i-th agent ranking or decision of answer γ_k, and $R_i(\gamma_k|\Gamma_j)$ is i-th agent ranking or decision of answer γ_k if it knows the ranking or the decision of vector of j-th agent. Matrix *U* can be evaluated via the following equations:

$$U_{ii} = -\sum_{k=1}^{M} R_i(\gamma_k)\log_M(R_i(\gamma_k)) \quad (7.10)$$

$$U_{ij} = -\sum_{k=1}^{M} R_i(\gamma_k|\Gamma_j)\log_M\left(R_i(\gamma_k|\Gamma_j)\right) \quad (7.11)$$

In equation (7.10), U_{ii} is local uncertainty of i-th classifier, and in equation (7.11), U_{ij} is global uncertainty of i-th classifier, knowing the ranking of j-th classifier. Equations (7.10) and (7.11) are applied knowing that equation (7.7) is fulfilled, as well as the following equation:

$$\sum_{k=1}^{m} R_i\left(\gamma_k\middle|\Gamma_j\right) = 1 \; \forall i \in \{1..N\} \quad (7.12)$$

In the case of two possible answers: '0' and '1' which means $M = 2$, and equations (7.7) and (7.12) are converted into:

$$R_i(0) + R_i(1) = 1, R_i(0|\Gamma_j) + R_i(1|\Gamma_j) = 1 \quad (7.13)$$

where $R_i(1)$ is i-th agent ranking of answer '1' and $R_i(0)$ is i-th agent ranking of answer '0'. Denote $R_i = R_i(1)$ and $R_i(\Gamma_j) = R_i(1|\Gamma_j)$. Then $R_i(0) = 1 - R_i$, and $R_i(0|\Gamma_j) = 1 - R_i(\Gamma_j)$. Thus, equations (10), (11) are converted into:

$$U_{ii} = -R_i\log_2(R_i) - (1 - R_i)\log_2(1 - R_i) \quad (7.14)$$

$$U_{ij} = -R_i(\Gamma_j)\log_2(R_i(\Gamma_j)) - (1 - R_i(\Gamma_j))\log_2(1 - R_i(\Gamma_j)) \quad (7.15)$$

It is worth mentioning that the reason the uncertainties in the above two equations are evaluated using a logarithm with base 2 is that this can be demonstrated by plotting equation (7.15) where U_{ii} is a function of parameter R_i. From the plot in Figure 7.4, it is clear that, if the value of the classifier's ranking is close to the edges of the [0,1] interval, uncertainty is near zero (the classifier is certain about its decision). On the other hand, if the ranking is close to the 0.5 point, uncertainty is close to the maximal value, which is one.

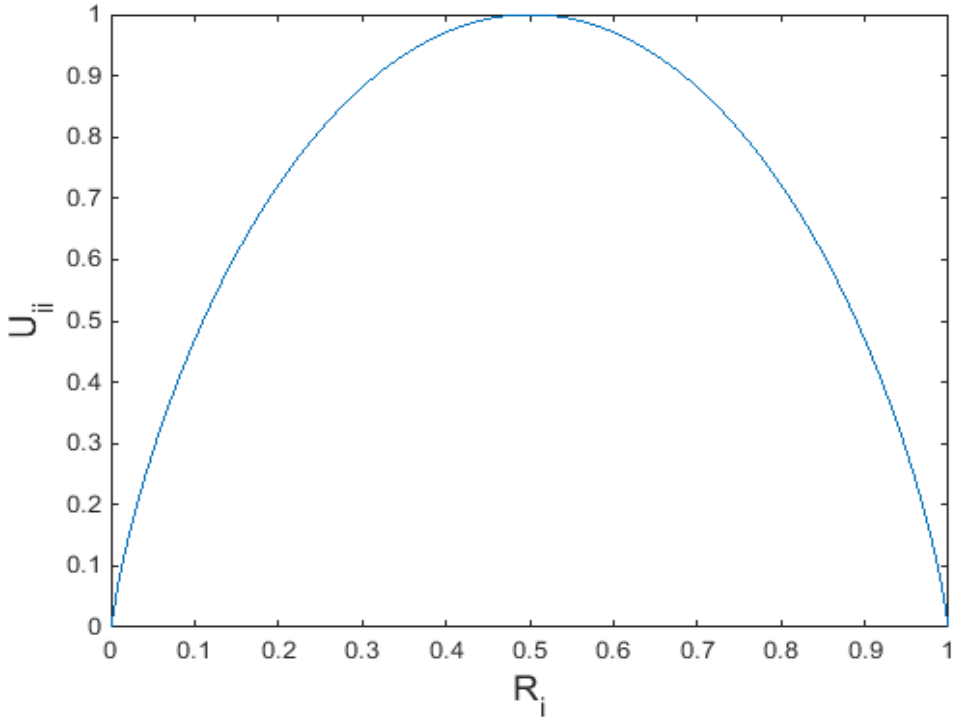


Figure 7.4 Uncertainty value $\boldsymbol{U_{ii}}$ as a function of the parameter $\boldsymbol{R_i}$

In the current situation it is straight forward to calculate the local uncertainties, but when it comes to global uncertainty there is no information available about the rankings of each agent. In (DeGroot, 1974; Berger, 1981; Shaban *et al*., 2002) they investigated convergence conditions and existing of the optimal single decision of the group. To evaluate ConsA rankings, uncertainty approach was proposed. But as no information is available on the global rankings, a solution is proposed which can estimate these global rankings. As the global ranking of classifier i with respect to classifier j in the testing point x_* a linear combinations of i-th and j-th classifer with thee weights proportional to local accuracies of these classifiers in this point were taken.

If $R_i(\Gamma_j)$ values are not avaliable and to estimate them, local Accuracy of each classifier at a given point are calculated using Algorithm (7.1)

Algorithm (7.1)

Calculate $\mathrm{d_i} = \mathrm{R_i} - 0.5$, $\mathrm{d_j} = \mathrm{R_i} - 0.5$

- *If* $d_i > 0$ *and* $d_j > 0$ $d_* = (d_i + d_j) * k_1$
- *If* $d_i < 0$ *and* $d_j < 0$ $d_* = (d_i + d_j) * k_2$

In steps 2 and 3, when k_1 and k_2 are greater than 0.5, the effect of i-th classifier certainty increasing due to similar opinion of j-th classifier. For example, if $R_i = 0.6$, $R_j = 0.7$ and $k = 1$, then $d_* = (0.2 + 0.1) * 1 = 0.3$, and conditional ranking according to equation 16 is $R_i(\Gamma_j) = 0.3 + 0.5 = 0.8$. The logic behind it is that if two experts simultaneously considering a loan as 'good', after communication their certainty in that decision will increase (thus, ranking will decrease). On the other hand, if two experts simultaneously considering a loan as 'bad', after communication their certainty, the decision will increase (thus, ranking is increased)

- *If d_i and d_j have opposite signs, then $d_* = (d_i + d_j) * k_3$*
- $R_i(\Gamma_j) = d_* + 0.5$.

Evaluate U_{ij} according to equation (7.13)

Update $U_{ij} = k_4 \cdot U_{ij}/(LA_i(q, nn) - k_5)$, where $LA_i(q, nn)$ is local Accuracy of i-th classifier upon entry query q, using i-th classifier answer error upon exactly k-neighbour queries from training set. In the current implementation, the parameter $k = 4$. Parameters $k_1 \ldots k_5$ are chosen using Gradient Descent with the objective function, global Accuracy over the training set. For each iteration these parameters were evaluated separately.

The sense behind 7-th step is to take into consideration local Accuracy of classifier, so the classifier with low local Accuracy is more uncertain about its decision (as local Accuracy is in the denominator). Coefficient k_5 is a normalising coefficient which picks lower than the lowest local Accuracy for i-th classifier so denominator stays positive.

7.4.3 Calculating the Classifiers Weights

After having calculated the uncertainties of the classifiers and all values of uncertainties is presented in the uncertainty matrix, at this stage classifiers can assign weights for itself and for other classifiers. The uncertainties weights are evaluated using following equation and it can be presented in a matrix same as the uncertainty matrix and it can be called matrix *W*:

$$W_{ij}cons = \frac{1}{U_{ij}^2 \sum_{k \in A} U_{ki}^{-2}} \tag{7.16}$$

Equation (7.16) is a result of the set of minimisation problems (Shaban et. al, 2002) (One problem for each $i \in 1,2\ldots,N$)

$$\begin{cases} T_i = \sum_{j=1}^{N} w_{ij}^2 \cdot U_{ji} \to min \\ \sum_{j=1}^{N} w_{ij} = 1 \end{cases}, i \in 1,2\ldots,N \tag{7.17}$$

These problems are stated in such form to ensure that each classifier will assign high weights to classifiers with low global uncertainties and low weights to those with high global uncertainties. Solving these N problems via Lagrange method of undetermined coefficients as illustrated in equation (7.16). Detailed process of equation (7.16) derivation is described in (Shaban et. al, 2002).

Weights of matrix W are assigned as transition matrix of Markovian chain with single classifiers as stated in DeGroot (1974). Then stationary distribution π of this chain using system of equations can be evaluated.

$$\begin{pmatrix} \pi \cdot Wcons & = \pi \\ \sum_{i=1}^{N} & \pi = 1 \end{pmatrix} \tag{7.18}$$

Sometimes there is no exact solution of this equation, because number of equations in it is one more than number of variables. In this case Marcovian chain does not converge to stationary distribution. The equation (7.18) can be converted to the form of:

$$\widetilde{W} \cdot \pi = (0,0,0,\ldots,0,1)^T \tag{7.19}$$

where:

$$\widetilde{W} = \begin{pmatrix} (Wcons - E)^T \\ 1,1,\ldots,1 \end{pmatrix} \tag{7.20}$$

Matrix $\widetilde{W}$ is rectangular $N \times (N+1)$ matrix. Sum of elements for each column of matrix$(W-E)^T$ is equal to 0, so at least one row of this matrix is redundant and can be removed. Therefore, if

$$rank((W-E)^T) = rank(W-E) = N-1 \tag{7.21}$$

Then equation (7.19) has single exact solution. To solve equation (7.18) using Matlab the least squares method could be used. It is also a new approach, comparing to articles to DeGroot (1974) and Berger (1981). Using least squares method, it is not needed to worry about vector π convergence, because result of approximate solution of equation (7.19) when equation (7.21) fulfilled is the same as using DeGroot (1974) iterative method $\pi^{i+1} = \pi^i W$

with normalisation on each step until $\left\|\pi^{i+1} - \pi^{i}\right\|$ became very small (ideally, zero). In that scientific paper the final value of π is called 'equilibrium', as this value does not change anymore after reaching it. Generally, equilibrium is a balance between single classifiers opinions so this is common denominator for all classifiers and they all agree with it.

7.4.4 Aggregating the Consensus Rankings and Decision Calculations

This step comes when all classifiers reach a ConsA about their decisions and there is no room for decision updates. Here, the aggregate ConsA ranking is evaluated using the following equation:

$$R_G(\gamma_k) = \sum_{i=1}^{N} R_i(\gamma_k) \cdot \pi_i \qquad \text{b} \qquad (7.22)$$

Vector π is considered as weights importance of each single classifier and sum of all elements of it equals 1. So aggregate ConsA ranking can be evaluated as linear combination of single classifiers rankings. Length of vector R_G is equal to size of the set of the possible answers, and sum of all elements of R_G is equal to 1. Final prediction of the group using ConsA is the answer γ_*, for which $R_G(\gamma_*)$ reaches the maximum value. Thus, using formal language, the final answer of the group can be specified as:

$$\gamma_* = Arg \max_{a \in (\gamma_1, \ldots, \gamma_m)} R_G(a) \qquad (7.23)$$

The below pseudo-code summarises the process of the classifiers ConsA adopted in this work.

The ConsA pseudo-code (generating the common group ranking for one input sample)

Input: R_i – ranking of answer '1' for each agent, $i = 1..5$, A_i– Accuracy of each agent.

Output:

- **For** i = 1 to *N* **do**
- **For** j = 1 to *N* **do**
- **if** (i==j) **then** U_{ii} =(computed by equation (7.14))

Else

- U_{ij} =(computed by algorithm (7.1) and equation (7.15))

End if

End for

End for

- $\forall i, j \in \{1..5\}\ w_{ij} =$ (computed by equation (7.16))
- Compute $\widetilde{W} = (W - E)^T$ (computed by equation (7.20))
- Compute $\pi =$(computed by system (7.19)) In Matlab $lsqnonneg$ function were used.
- Compute aggregate Cons$AR_G(\gamma_k)$ using equation (7.22).
- Define group aggregate answer using equation (7.23).

The ConsA Example

Suppose that five classifiers have such rankings: $R = (0.8, 0.3, 0.4, 0.7, 0.6)$, and such local accuracies (0.77, 0.7, 0.65, 0.75, 0.65):

- During the gradient descent such vector of parameters: $k_1=1,\ k_2=2,\ k_3=0.5,\ k_4=1,\ k_5=0.3$ were obtained which give the best Accuracy over training set
- Calculate uncertainty matrix U (for diagonal elements equation (7.15) is used, for non-diagonal and algorithm (7.1)):
- Calculate weight matrix *W* according to equation (7.16)

$$U = \begin{pmatrix} 0.72 & 2.11 & 2.07 & 0.00 & 1.00 \\ 2.48 & 0.88 & 0.00 & 2.50 & 2.48 \\ 2.77 & 0.00 & 0.97 & 2.84 & 2.86 \\ 0.00 & 2.22 & 2.21 & 0.88 & 1.60 \\ 1.34 & 2.84 & 2,86 & 2.06 & 0.97 \end{pmatrix}$$

Let us show how U_{11} is calculated:

$$U_{11} = -0.8 \times \log_2 0.8 - (1 - 0.8) \times \log_2(1 - 0.8) = 0.72$$

(0.8 is first classifier ranking). Other diagonal elements are calculated by the same way. Algorithm 7.1 is used.

For non-diagonal elements algorithm (7.1) is calculated. Let's show how U_{12} is calculated:

- *Calculate* $d_1 = R_1 - 0.5 = 0.3$, $d_2 = R_2 - 0.5 = -0.2$
- *As* d_1 *and* d_2 *have opposite signs, then*

$$d_* = (d_1 + d_2) \times k_3 = (0.3 - 0.2) \times 0.5 = 0.05$$

$R_1(\Gamma_2) = d_* + 0.5 = 0.55$

- *Evaluate* U_{12} *according to equation (13)*
- $U_{12} = 0.55 \times \log_2(0.55) + (1 - 0.55) \times \log_2(1 - 0.55) = 0.9928$
- *Update* $U_{12} = k_4 \cdot U_{12}/(LA_i(q, nn) - k_5$, where $LA_i(q, nn)$ is local Accuracy.
- $U_{12} = 1 \cdot 0.9928/(0.77 - 0.3) = 2.11$
- Calculate weight matrix *W* according to equation (7.16).

$$W = \begin{pmatrix} 0{,}00 & 0{,}00 & 0{,}00 & 1{,}00 & 0{,}00 \\ 0{,}00 & 0{,}00 & 1{,}00 & 0{,}00 & 0{,}00 \\ 0{,}00 & 1{,}00 & 0{,}00 & 0{,}00 & 0{,}00 \\ 1{,}00 & 0{,}00 & 0{,}00 & 0{,}00 & 0{,}00 \\ 0{,}37 & 0{,}06 & 0{,}04 & 0{,}14 & 0{,}39 \end{pmatrix}$$

In this example four first rows has all zero elements, except one. This fact is because of the equation of w_{ij} evaluation: sum of inverse squares $\sum_{k \in A} U_{ik}^{-2}$ is infinity for $i \in (1,2,3,4,5)$ because in these rows matrix U has zeros. The only one element which is equal one for theze rows are where $U_{ij} = 0$. The last row of matrix *U* has no zeros, so we can evaluate, for example w_{51}. To do this, firstly we evaluate sum of inverse squares of all elements of matrix U for the last row:

$$\sum_{k \in \{1,2,3,4,5\}} U_{5k}^{-2} = \frac{1}{1.34^2} + \frac{1}{2.84^2} + \frac{1}{2.86^2} + \frac{1}{2.06^2} + \frac{1}{0.97^2} = 2.74$$

$w_{51} = \frac{1}{U_{51}^2 * \sum_{k \in A} U_{5k}^{-2}} = \frac{1}{1.34^2 \cdot 2.74} = 0.37$ The same way we calculate all other weights from this row.

- Evaluate matrix $\widetilde{W} = (W - E)^T$

$$\widetilde{W} = \begin{pmatrix} -1.00 & 0.00 & 0.00 & 1.00 & 0.37 \\ 0.00 & -1.00 & 1.00 & 0.00 & 0.06 \\ 0.00 & 1.00 & -1.00 & 0.00 & 0.04 \\ 1.00 & 0.00 & 0.00 & -1.00 & 0.14 \\ 0.00 & 0.00 & 0.00 & 0.00 & -0.61 \end{pmatrix}$$

- Calculate vector π such that $\widetilde{W} \cdot \pi = (0,0,0,\ldots,0,1)^T$

$$\pi \cdot R = (0.3, 0.2, 0.2, 0.3, 0.0) \cdot (0.8, 0.3, 0.4, 0.7, 0.6) = 0.59$$

So ranking of each classifier is multiplied with the constant:
0.8×0.3+0.3×0.2+0.4×0.2+0.7×0.3+0.6×0.0, this is called linear combination, with elements of vector pi as a coefficients of linear combination.

- As global final ranking is greater than 0.5, Consensus consider the loan as 'bad'.

Finally, it is worth mentioning to show the advantage of least squares computation algorithm of vector π, proposed in the ConsA, in comparing with classical iteration computational algorithm. To do this, a simple evaluation test was constructed:

1. Generate random 5×5 matrix with the same properties as matrix $\widetilde{W}$ from equation (7.16) using the code

```
w = rand(5,5);
b = sum(W,1);
for k=1:5
W(:,k) = W(:,k)/b(k);
end
```

2. Evaluate vector π using least squares method

```
T = [W-eye(5);[1,1,1,1,1]];
Pi1 = mldivide (T,[0;0;0;0;0;1]);
```

3. Evaluate vector π using iteration method

```
Pi1 = Pi1/sum(Pi1);
Pi = rand(5,1);
Pi_old = zeros(5,1);
while norm(Pi-Pi_old) > 0.0001 Pi_old = Pi;
Pi = W×Pi;
Pi = Pi/sum(Pi);
```

end [resume]

4. Repeat separately 1 and 2 steps and 1 and 3 steps 10000 times and evaluate processor time to complete the task.

	Processor time, sec.	**Mean Squared Error**
Least squares method	0.173698	1.82×10^{-16}
Iteration method	0.528257	1.43×10^{-5}

This table demonstrates that the proposed method is novel and more efficient than classical iteration method. However, on other computational packages than Matlab, results may be different. On average 8 iterations is needed to achieve desired Accuracy (see the table above). Figure 7.5 shows how Accuracy level increases depending on iteration number. Initial π value is vector (0.2,0.2,0.2,0.2,0.2), so without iterations Accuracy of the ConsA is the same as simple AVG Accuracy. After fourth iteration changes in Accuracy became smaller, as π converges to solution of equation (7.18).

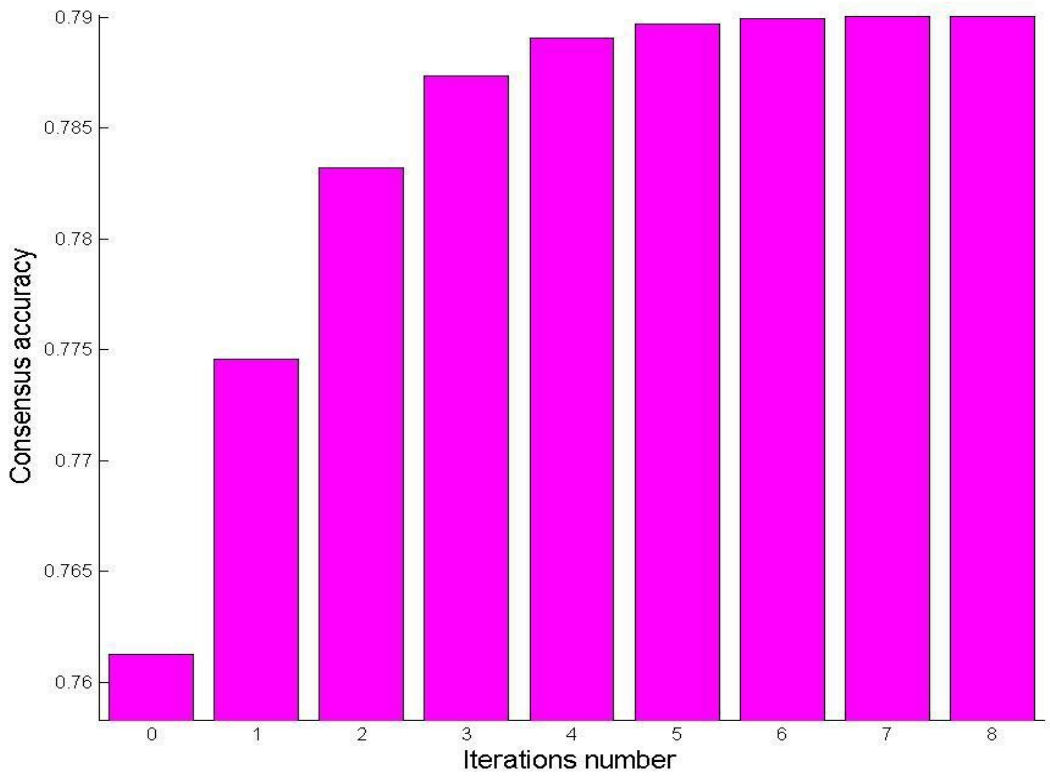


Figure 7.5 The ConsA Accuracy improvements depending on iterations number

7.5 Summary

In summary, this chapter has described, in detail, the mechanism of the proposed methods, the D-ENS and ConsA. Each proposed method has been supported by an illustrative example summarising their process in terms of Local Accuracy evaluation for the classifiers and in terms of classifiers reaching a group consensus for each data sample for D-ENS and ConsA, respectively. In the following chapter, an extensive experimental procedures is conducted using the proposed methods on the 7 datasets investigated, as well as a comprehensive comparison across the proposed methods, traditional combiners, hybrid and single classifiers.

CHAPTER 8

THE EXPERIMENTAL RESULTS FOR THE NEW HYBRID ENSEMBLE CREDIT-SCORING MODEL

8.1 Introduction

In this chapter the experimental design and results of the proposed methods, the D-ENS and ConsA are conducted and analysed in terms of their ability to compete each other as well as their ability in performing better than traditional combiners and hybrid and single classifiers to see to what extent complexity can enhance generalisation of a model. Moreover, an analysis for the computational time of the whole phases of the ConsA will discussed.

Moreover, during the testing phase of the D-ENS and ConsA with different options of filtering and feature selection were analysed, it has been discovered that ConsA and D-ENS achieve highest results when both filtering and feature selection pre-processing methods are enabled. So the results provided only for experiment with filtering and feature selection 'on'.

8.2 Experimental Results

In this section all the results of single, hybrid classifiers, traditional combination methods and along with the D-ENS and classifiers ConsA is compared against each other across seven datasets evaluated on six performance measures is summarised and discussed (The results are evaluated by taking the average of 50 testing sets resulting from the 10 × 5 cross-validation). All the base single classifiers predictions is combined using the two proposed Approaches using filtering and feature selection as stated earlier in this Chapter. The results are analysed, discussed and evaluated. It is worth mentioning to describe the thresholds assigned to each classifier for D-ENS and ConsA (Please refer to Appendix C).

8.2.1 Results of German Dataset

Table 8.1 shows the results of all classifiers and combiners where both data-filtering and feature selection techniques are combined together. During this experiment ConsA shows the Accuracy 1.65% higher than D-ENS (best second classifier in this case). The Accuracy of

ConsA is 0.7903, better than best of traditional combiners by 1.7%. Standard deviation of ConsA Accuracy over all iterations is 0.029. The results clearly shows that using filtering and feature selection in conjunction is advisable, because it increases the performance of almost all of classifiers comparing to experiments, where only feature selection or only filtering are used. AUC value of ConsA is the highest amongst all other classifiers and combiners, and can rival only with D-ENS AUC value. Sensitivity value shows 91.12% testing set good loan entries, where correctly classifier by using ConsA making it practically the highest amongst the others despite that PROD and MIN has better values but when taking Specificity in to consideration, ConsA is definitely better . While almost half of the bad loans entries were classifier correctly using ConsA making it comes second after MAX rule.

	RF	**DT**	**NB**	**NN**	**SVM**	**LR**	**MIN**	**MAX**
Accuracy	0.7725	0.7528	0.7638	0.7584	0.7733	0.7597	0.7636	0.7532
Sensitivity	0.9066	0.8964	0.8861	0.8680	0.9038	0.8841	**0.9239**	0.7899
Specificity	0.4611	0.4204	0.4789	0.5069	0.4703	0.4715	0.3920	**0.6697**
AUC	0.7942	0.6994	0.7735	0.7717	0.7942	0.7798	0.7178	0.7878
Brier Score	0.1603	0.2214	0.1927	0.1700	0.1643	0.1656	0.2058	0.1951
H-measure	0.2966	0.1973	0.2668	0.2580	0.2985	0.2725	0.2248	0.2879
	PROD	**AVG**	**MajVot**	**WAVG**	**WVOT**	**D-ENS**	**ConsA**	
Accuracy	0.7362	0.7730	0.7776	0.7458	0.7725	0.7738	**0.7903**	
Sensitivity	0.9828	0.9014	0.9059	0.8604	0.9066	0.9053	0.9112	
Specificity	0.1624	0.4759	0.4802	0.4806	0.4611	0.4689	0.5090	
AUC	0.7089	0.7996	0.7548	0.7459	0.6878	0.8006	**0.8024**	
Brier Score	0.2316	0.1584	0.1837	0.1798	0.1933	**0.1579**	0.1641	
H-measure	0.2246	0.3063	0.2859	0.2225	0.2473	0.3066	**0.3245**	

Table 8.1 Performance results for German dataset for all single classifiers, hybrid classifiers, traditional combiners and the proposed methods

However, comparing to other classifiers, ConsA good/bad entries loan classification coefficients look good. H-measure of ConsA is the biggest amongst all classifiers; however Brier Score of ConsA holds the third position after D-ENS and AVG. This fact does not change the reached conclusion that ConsA is better than these 2 classifiers, because for other 5 measures ConsA shows its superiority over them.

From Figure 8.1 it can be seen via the ROC curve the advantage of ConsA compared to single classifiers, the best traditional combiner and D-ENS. It can be seen that ConsA has a peak around 0.5 threshold, which gives it an advantage comparing to other classifiers.

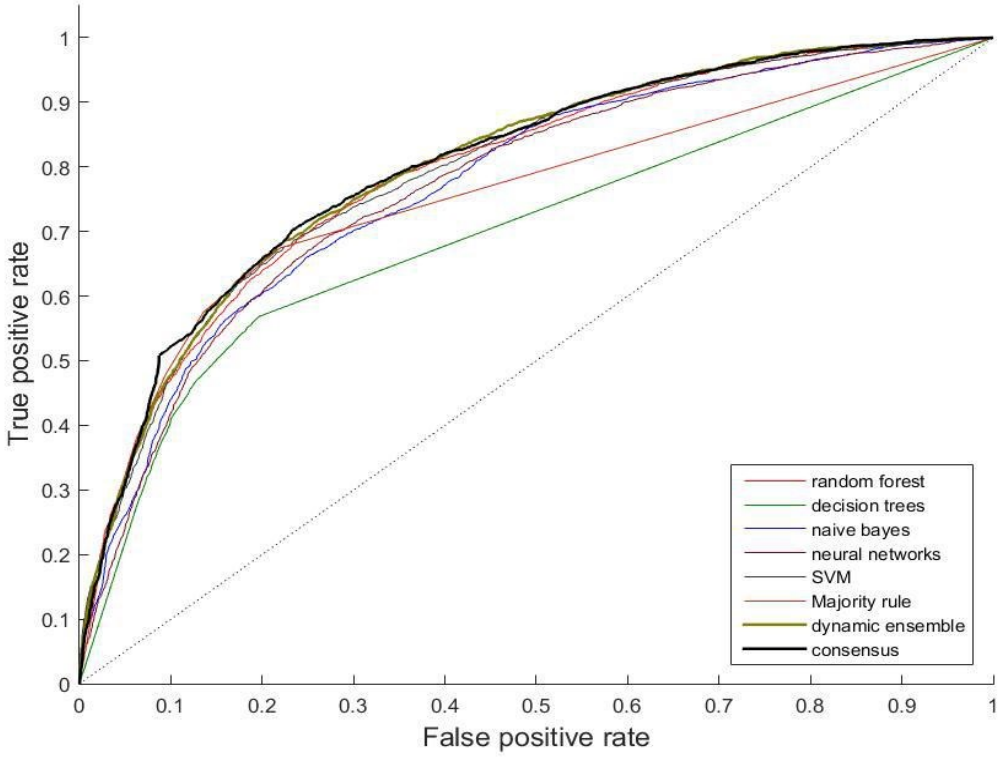


Figure 8.1 ROC curves for all single classifiers, most efficient traditional combiner, D-ENS classifier and ConsA for German dataset

Figure 8.2 shows the conditional $f(R|0)$, $f(R|1)$ and absolute $f(R)$ frequency histogram of predicted values.

$f(R|0)$ Is the predicted values subset where actual target is 0 (black colour).

$f(R|1)$ Is the predicted values subset where actual target is 1 (green colour).

$f(R)$ Is t=he predicted value set (red colour).

From Figure 8.2, it can be concluded that ConsA is much more certain about good loans predictions, than that of bad loans, the highest probability (22%) is that ranking of a random bad loan entry is in the interval [0,0.1]. Bad loans prediction performance of most of other classifiers and combiners is even worse, and the several ones that show higher Accuracy in bad loan prediction have poor good loan prediction and overall Accuracy in general. This

indicates that for German dataset due to its imbalanced structure is very difficult to build over 85% Accuracy combiner.

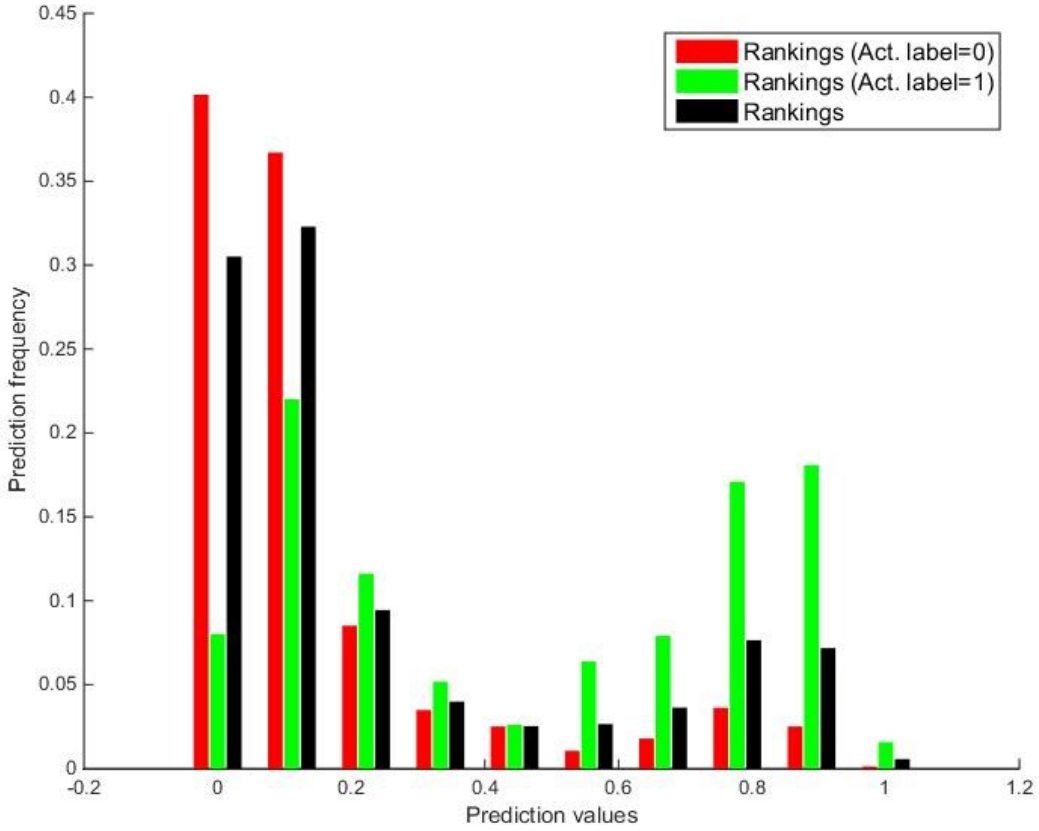


Figure 8.2 Frequency histogram of conditional and absolute values $\boldsymbol{R_G}$ over the test set for German dataset

It is clear from Table 8.2 that ConsA is far away better than LR almost all performance measures. The superiority varies from 2.26% to 5.20%. In Brier Score the negative sign means that ConsA is superior the LR as in Brier Score the less the value the better. It can be concluded that the more complex the model is the better it gets compared to LR.

German dataset	Accuracy	Sensitivity	Specificity	AUC	Brier Score	H-measure
LR	0.7597	0.8841	0.4715	0.7798	0.1656	0.2725
ConsA	3.06%	2.71%	3.75%	2.26%	-0.15%	5.20%

Table 8.2 The improvement of ConsA over LR for German dataset

8.2.2 Results of Australian Dataset

In Table 8.3, given filtering and feature selection methods applied in conjunction, it can be observed that ConsA Accuracy is the highest with 0.881 Accuracy, better than the best second classifier by 0.6%. Standard deviation of ConsA Accuracy over all iterations is 0.0268. Specificity and Sensitivity are almost equally balanced, so ConsA can predict good and bad loans almost with the same Accuracy. ConsA AUC is the highest amongst other classifiers indicating its efficiency through several thresholds. H-measure and Brier score of ConsA are the biggest amongst all classifiers, closest result for Brier score have D-ENS and AVG. Surprisingly; Brier score of LR is also not bad.

	RF	**DT**	**NB**	**NN**	**SVM**	**LR**	**MIN**	**MAX**
Accuracy	0.8707	0.8688	0.8614	0.8643	0.8686	0.8641	0.8662	0.8662
Sensitivity	0.8799	0.8653	0.8420	0.8584	0.8667	0.8585	**0.8927**	0.8235
Specificity	0.8585	0.8722	0.8844	0.8713	0.8703	0.8700	0.8324	**0.9185**
AUC	0.9286	0.8868	0.9093	0.9197	0.9209	0.9294	0.9126	0.9075
Brier Score	0.0982	0.1216	0.1249	0.1035	0.1043	0.0999	0.1153	0.1108
H-measure	0.6491	0.6157	0.6149	0.6313	0.6370	0.6352	0.6355	0.6319
	PROD	**AVG**	**MajVot**	**WAVG**	**WVOT**	**D-ENS**	**ConsA**	
Accuracy	0.8583	0.8725	0.8736	0.8713	0.8700	0.8751	**0.8810**	
Sensitivity	0.9149	0.8586	0.8642	0.8789	0.8766	0.8647	0.8674	
Specificity	0.7867	0.8890	0.8848	0.8611	0.8607	0.8873	0.8968	
AUC	0.9116	0.9294	0.9031	0.9202	0.8908	0.9297	**0.9347**	
Brier Score	0.1234	0.0977	0.1106	0.1021	0.1064	0.0968	**0.0960**	
H-measure	0.6381	0.6515	0.6382	0.6356	0.6308	0.6530	**0.6689**	

Table 8.3 Performance results for Australian dataset for all single classifiers, hybrid classifiers, traditional combiners and the proposed methods

The ROC curves in Figure 8.3 show the advantage of ConsA over single classifiers, the best of traditional combiners and D-ENS. However, its advantage is not beneficial as in German dataset, and this can be explained by overall high level of classification of all single classifiers.

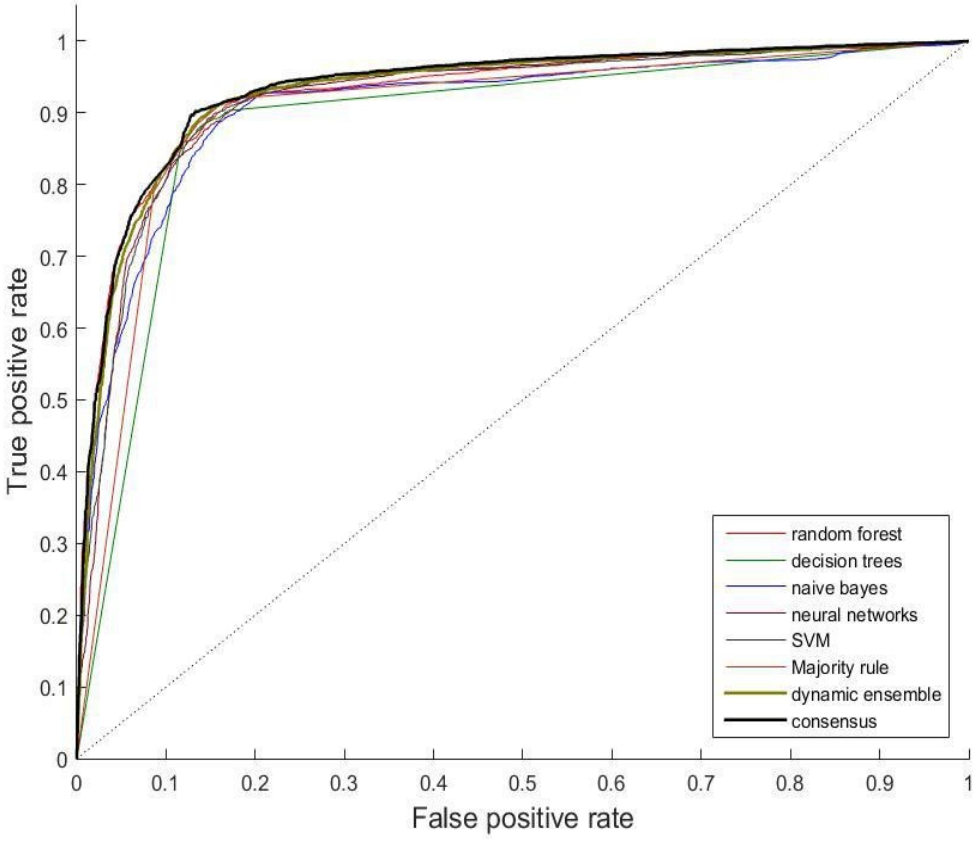


Figure 8.3 ROC curves for all single classifiers, most efficient traditional combiner, D-ENS classifier and ConsA for Australian dataset

From Figure 8.4 it can be concluded that ConsA is very often certain about its decisions (length of bars near 0.4-0.6 points is much less than length of bars on the edges of [0, 1] ranking interval). ConsA often is very certain about good loans (if the loan is good, probability that ConsA will give less than 0.1 prediction value is more than 60%).

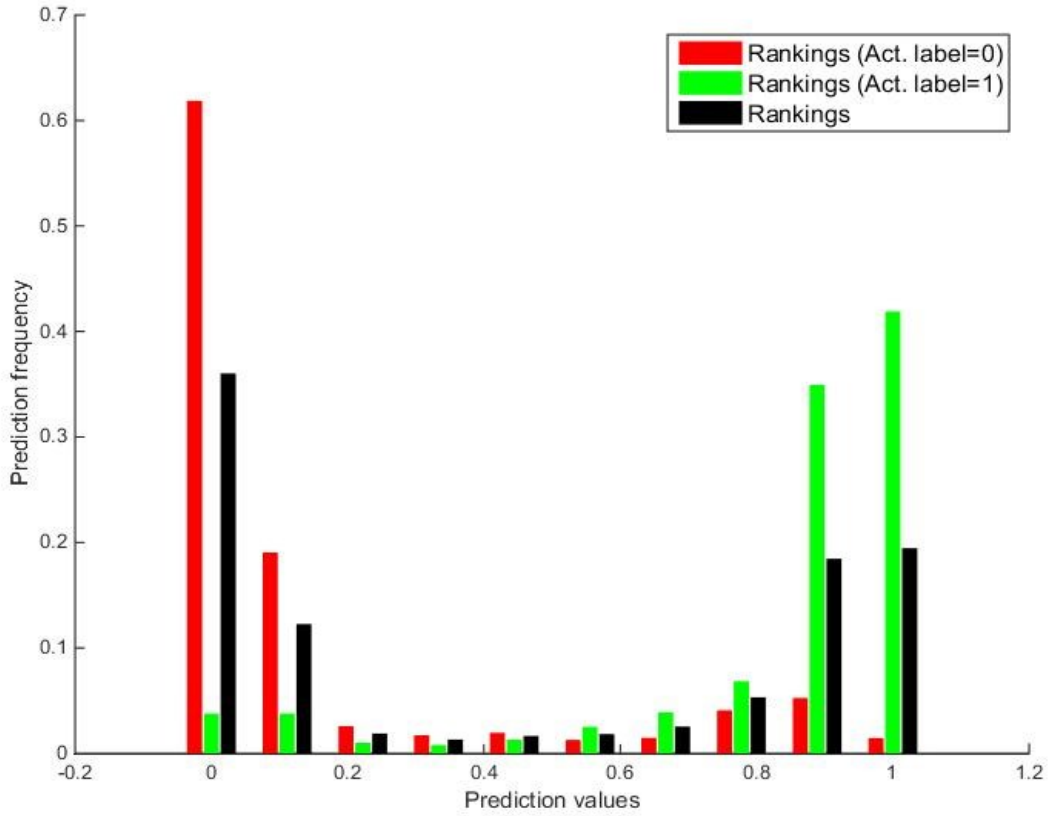


Figure 8.4 Frequency histogram of conditional and absolute values R_G over the test set for Australian dataset

Regarding Table 8.4, the Australian dataset the raise of ConsA Accuracy due to applying pre-processing methods is not big as German dataset. However, ConsA outperforms LR on almost all performance measures.

Australian dataset	Accuracy	Sensitivity	Specificity	AUC	Brier Score	H-measure
LR	0.8641	0.8585	0.8700	0.9294	0.0999	0.6352
ConsA	1.69%	0.89%	2.68%	0.53%	-0.39%	3.37%

Table 8.4 The improvement of ConsA over LR for Australian dataset

8.2.3 Results of Japanese Dataset

According to Table 8.5, the best results, obviously, received when both filtering and feature selection are combined. ConsA Accuracy is 0.8871, better than D-ENS by 0.3%. Standard deviation of ConsA Accuracy over all iterations is 0.0259. D-ENS is very competitive to ConsA as it succeeds it in Specificity, AUC and Brier Score values, hence, not that

significant. H-measure of ConsA is almost 1% higher than D-ENS same measure. In general, so far ConsA is stable over balanced and unbalanced datasets so far.

	RF	DT	NB	NN	SVM	LR	MIN	MAX
Accuracy	0.8717	0.8616	0.8630	0.8694	0.8538	0.8626	0.8643	0.8449
Sensitivity	0.8806	0.8434	0.8579	0.8689	0.8069	0.8474	**0.9102**	0.7638
Specificity	0.8618	0.8847	0.8702	0.8713	0.9129	0.8832	0.8087	**0.9458**
AUC	0.9293	0.8795	0.9085	0.9073	0.9111	0.9171	0.9130	0.9028
Brier Score	0.1001	0.1292	0.1221	0.1092	0.1112	0.1039	0.1139	0.1641
H-measure	0.6513	0.6025	0.6225	0.6309	0.6215	0.6254	0.6336	0.6137
	PROD	**AVG**	**MajVot**	**WAVG**	**WVOT**	**D-ENS**	**ConsA**	
Accuracy	0.8597	0.8648	0.8654	0.8536	0.8717	0.8842	**0.8871**	
Sensitivity	0.9094	0.8437	0.8449	0.9145	0.8806	0.8716	0.8827	
Specificity	0.7991	0.8921	0.8918	0.7798	0.8618	0.9001	0.8925	
AUC	0.9100	0.9262	0.9082	0.9091	0.8486	**0.9353**	0.9330	
Brier Score	0.1225	0.1005	0.1108	0.1218	0.1154	**0.0901**	0.0927	
H-measure	0.6332	0.6468	0.6408	0.6012	0.6113	0.6799	**0.6878**	

Table 8.5 Performance results for Japanese dataset for all single classifiers, hybrid classifiers, traditional combiners and the proposed method

Explanation of the Figure 8.5 is similar to the one in Australian dataset and the figure in general is somehow similar. Despite AUC of ConsA is less than AUC of D-ENS, but ROC curve of ConsA is better than D-ENS ROC curve near 0.5 thresholds.

AUC is area under all curves, for all thresholds. But very often in real life situations it is wise to consider only some interval of thresholds, for example, [0.4, 0.6]. All other thresholds often do not matter, as they give us too low Accuracy. But AUC gather all thresholds using integral, so sometimes classifier with less AUC have better ROC curve near the 0.5 threshold.

This dataset is balanced, so optimal point (a point on ROC curve where Accuracy is the best) of plot ROC curves is situated near the left upper corner of figure, which is good. WVOT shows good results at 0.5 threshold, but if threshold is changed, its performance drops extremely (it can be seen from its AUC curve, which is the worst).

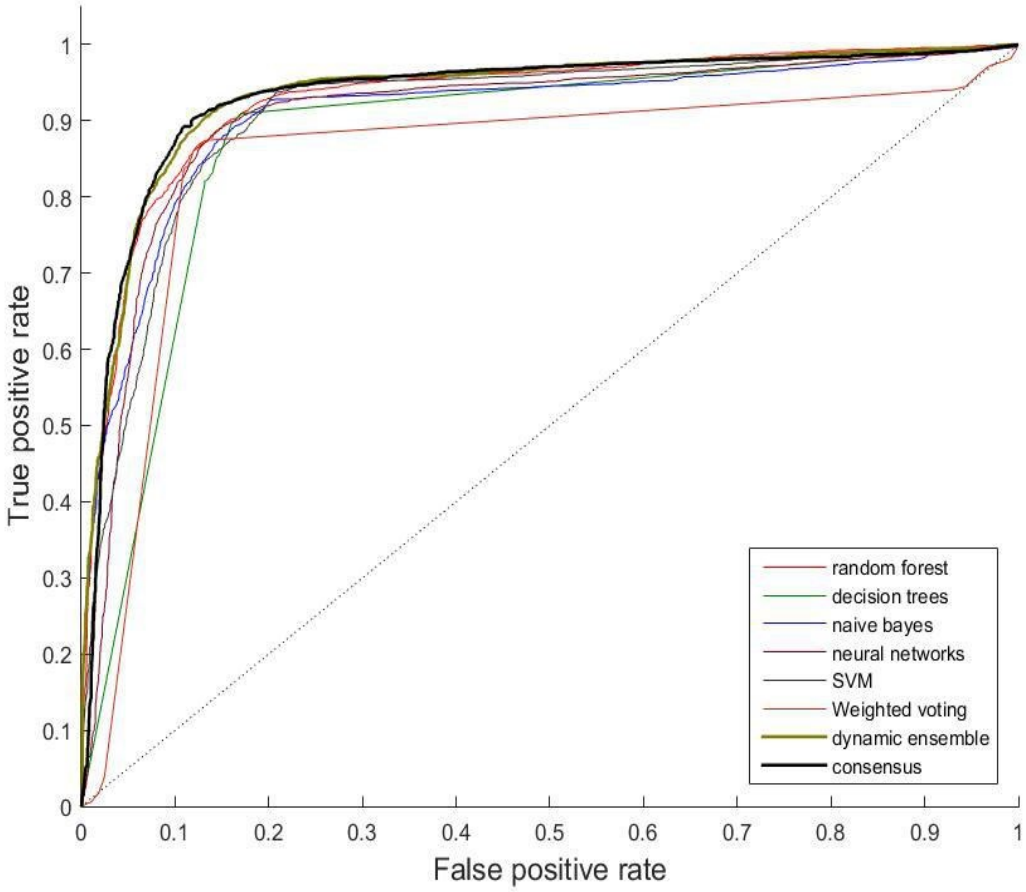


Figure 8.5 ROC curves for all single classifiers, most efficient traditional combiner, D-ENS classifier and ConsA for Japanese dataset

In Figure 8.6 it is clear that ConsA shows very good level of confidence for good and bad loans entries. Most of the rankings of ConsA lies either in [0, 0.1] interval or in [0.9, 1] interval. When the input loan is good, probability that ConsA will give the number near 0.1 or less is almost 80%, and when the input loan is bad, probability that the ConsA will give the number near 0.9 or more is 70%.

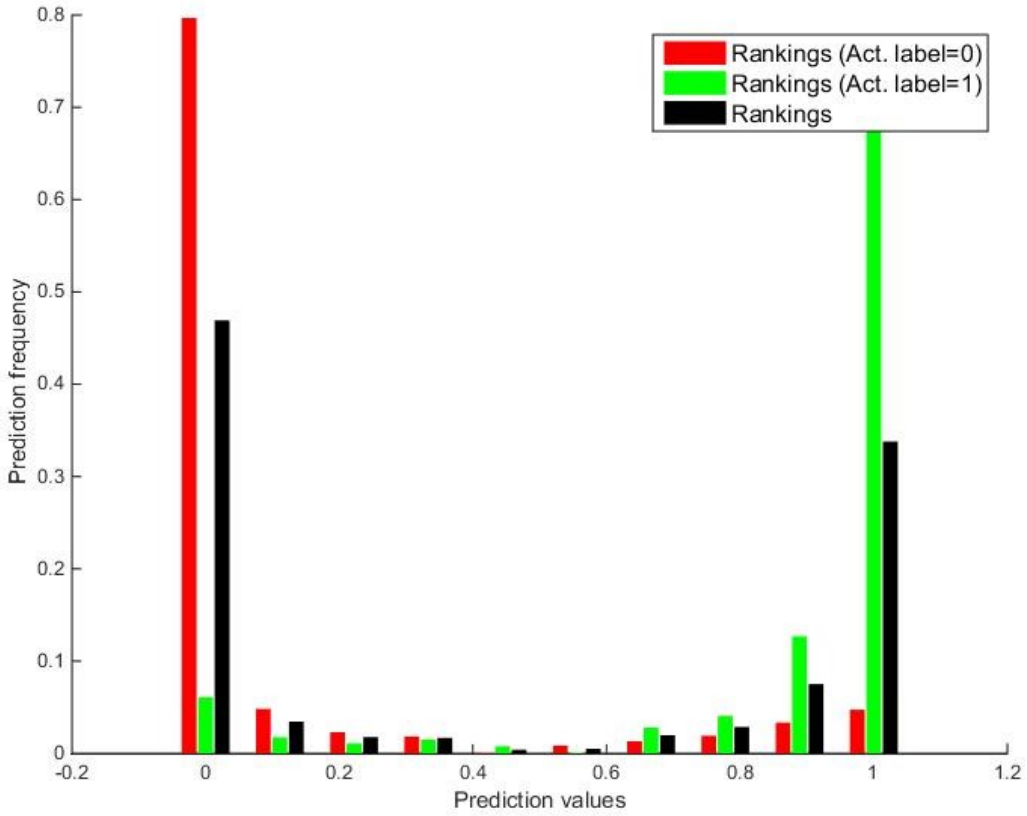


Figure 8.6 Frequency histogram of conditional and absolute values $\boldsymbol{R_G}$ over the test set for Japanese dataset

Table 8.6 shows, ConsA superiority again over LR on all measures, the superiority of the ConsA varies from 1.59% to 6.24%. The highest can be seen in H-measure which is considerably high when compared to Australian and German datasets.

Japanese dataset	Accuracy	Sensitivity	Specificity	AUC	Brier Score	H-measure
LR	0.8626	0.8474	0.8832	0.9171	0.1039	0.6254
ConsA	2.45%	3.53%	0.93%	1.59%	-1.12%	6.24%

Table 8.6 The improvement of ConsA over LR for Japanese dataset

8.2.4 Results of Iranian Dataset

For this severely imbalanced dataset, it can be seen from Table 8.7 that the results of ConsA rises up to 95.75%, better than the best second classifier by 0.05%. Standard deviation of ConsA Accuracy over all iterations is 0.015. The results show that for this dataset ConsA and D-ENS shows almost similar results. But in terms of AUC comparison ConsA is far better,

and amongst these two exactly it should be selected as a desirable classifier. Besides, ConsA has 100% sensitivity, so it can classify correctly all 100% of good entries in the test data. It worth mentioning, that classical combiners MIN, PROD, AVG and PROD shows 100% sensitivity and 0% specificity over this dataset. It means that their prediction for 100% of input entries is '0' and they are useless as they do not give us any information. H-measure and Brier score of ConsA holds the second position after D-ENS. For this dataset D-ENS shows almost the same quality of results as ConsA (however ConsA has much better AUC), so both ConsA and the D-ENS can be placed on the first place of the chart.

	RF	**DT**	**NB**	**NN**	**SVM**	**LR**	**MIN**	**MAX**
Accuracy	0.9513	0.9505	0.9452	0.9500	0.9464	0.9239	0.9500	0.9101
Sensitivity	0.9985	1.0000	0.9881	0.9994	0.9959	0.9702	1.0000	0.9430
Specificity	0.0555	0.0117	0.1328	0.0127	0.0082	0.0305	0	**0.2998**
AUC	0.7786	0.5362	0.7470	0.6289	0.6123	0.6227	0.5533	0.7401
Brier Score	0.0430	0.0489	0.0537	0.0472	0.0508	0.1005	0.0494	0.0712
H-measure	0.2831	0.0400	0.2373	0.0747	0.0784	0.0623	0.0545	0.2541
	PROD	**AVG**	**MajVot**	**WAVG**	**WVOT**	**D-ENS**	**ConsA**	
Accuracy	0.9500	0.9500	0.9500	0.9497	0.9460	0.9569	0.9575	
Sensitivity	1.0000	1.0000	1.0000	0.9898	0.9875	1.0000	**1.0000**	
Specificity	0	0	0	0.1978	0.1619	0.1363	0.1534	
AUC	0.5384	0.7770	0.5777	0.7759	0.5723	0.7815	**0.8420**	
Brier Score	0.0500	0.0432	0.0471	0.0452	0.0477	0.0358	**0.0393**	
H-measure	0.0500	0.2931	0.1089	0.2838	0.1059	0.4423	0.4025	

Table 8.7 Performance results for Iranian dataset for all single classifiers, hybrid classifiers, traditional combiners and the proposed methods

Figure 8.7 the ROC curves show that ConsA ROC curve is lower than D-ENS up to point $TP_{rate} = 0.7$, $FP_{rate} = 0.2$, but after this point, the Accuracy of ConsA becomes much higher than D-ENS Accuracy. So if a preference is given to high Accuracy and high level of good loans recognition, ConsA should be preferred, otherwise (if it is needed to predict bad loans with high Accuracy) it is better to choose D-ENS. However, the price for bad loans high recognition will cause a big drop in Accuracy which is inappropriate in real-life situations.

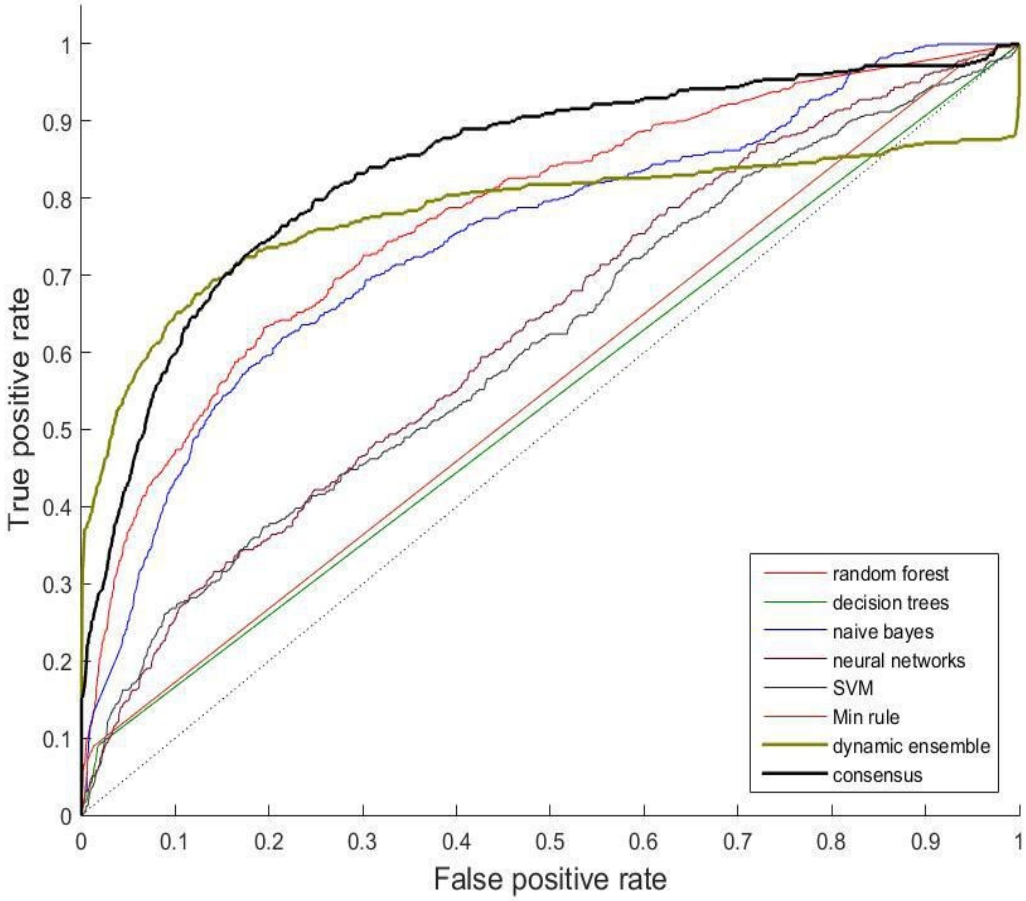


Figure 8.7 ROC curves for all single classifiers, most efficient traditional combiner, D-ENS classifier and ConsA for Iranian dataset

Figure 8.8 proves again the fact that ConsA is very good at good loan recognition, but demonstrates much worse results in bad loans recognition. Most of the times, when input query has 'bad loan' label, ConsA treat this query as good, and its ranking is in [0.1 - 0.3] interval.

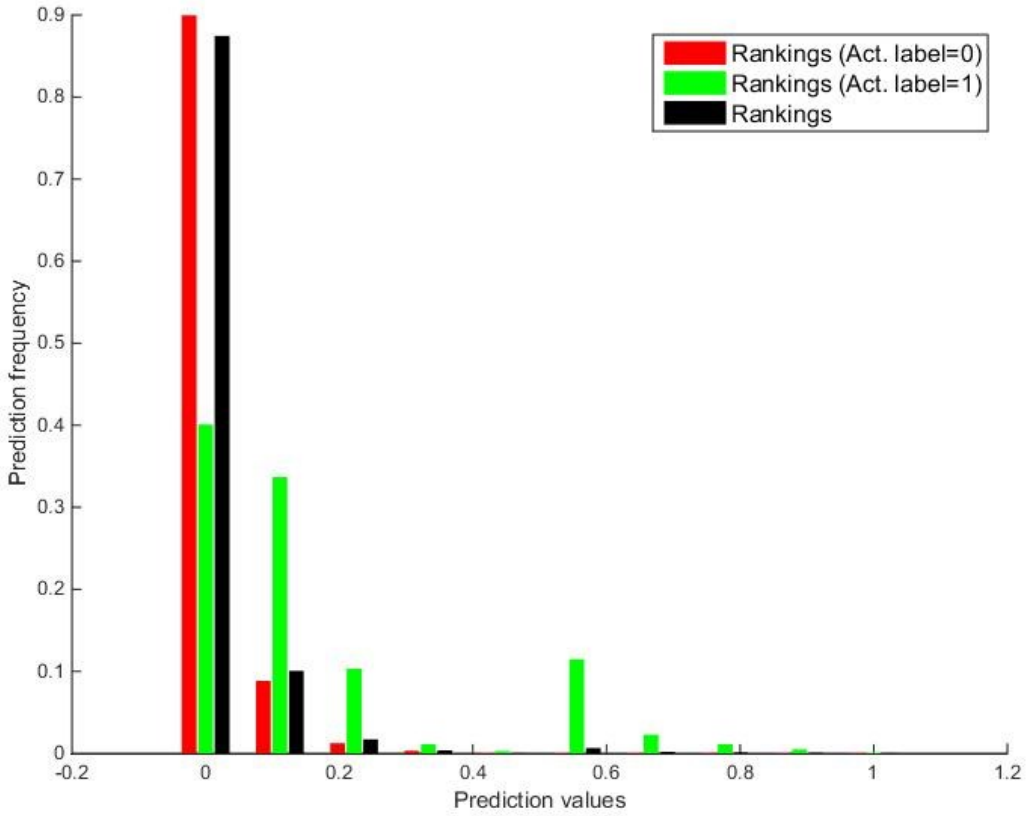


Figure 8.8 Frequency histogram of conditional and absolute values $\boldsymbol{R_G}$ over the test set for Iranian dataset

Iranian dataset is one of the most controversial datasets amongst all. Although this dataset is bad for training, ConsA still is impressively accurate. According Table 8.8 its Accuracy is higher than LR by 3.36%, and besides, other measures also better than every other previously reviewed classifier measures. AUC of ConsA is drastically better, because of the fact that ConsA is much more universal classifier (because it uses several powerful single classifiers as its parts), so for each threshold ConsA will show solid results. Brier score of ConsA is very good because in most of the cases ConsA is sure about its right decisions, and even if it mistakes, difference between actual label and its ranking is not much higher than 0.5. Also H-measure shows massive superiority by 34.02%.

Iranian dataset	Accuracy	Sensitivity	Specificity	AUC	Brier Score	H-measure
LR	0.9239	0.9702	0.0305	0.6227	0.1005	0.0623
ConsA	3.36%	2.98%	12.29%	21.93%	-6.12%	34.02%

Table 8.8 The improvement of ConsA over LR for Iranian dataset

8.2.5 Results of Polish Dataset

According to Table 8.9, Accuracy of ConsA with filtering and feature selection enabled is 81.33%, better than the second best classifier by 2.33%. Standard deviation of ConsA Accuracy over all iterations is 0.0514, which is the highest standard deviation comparing to all other datasets. The AUC value of ConsA remains greater than AUC for all other classifiers. This is the only dataset so far, for which it can be seen as a big advantage of ConsA over other classifiers and combiners.

Filtering and feature selection helped to rise ConsA Accuracy by almost 2.3%, which proves importance of these two pre-processing techniques in classification procedure. D-ENS shows solid results, but interestingly that for this dataset D-ENS and RF shows worse results than DT, which shows 79% Accuracy. The reason of this good performance is that filtering helps this classifier to choose right node splits, and therefore obtained model become quite precise.

ConsA has almost same ability in classifying good and bad loans correctly. H-measure of ConsA is the best, brier score is also the best. For this dataset ConsA shows superiority for almost all measures which evaluated.

	RF	**DT**	**NB**	**NN**	**SVM**	**LR**	**MIN**	**MAX**
Accuracy	0.7742	0.7900	0.7296	0.7521	0.7571	0.7246	0.7204	0.7679
Sensitivity	0.7782	0.7516	0.9038	0.7200	0.7024	0.7150	0.9052	0.5826
Specificity	0.7774	0.8279	0.5775	0.7850	0.8080	0.7325	0.5616	**0.9334**
AUC	0.8408	0.7975	0.7996	0.8060	0.8158	0.7405	0.8287	0.8240
Brier Score	0.1627	0.2027	0.2642	0.1838	0.1749	0.2298	0.2637	0.1881
H-measure	0.3945	0.3818	0.3449	0.3406	0.3700	0.2663	0.3961	0.4086
	PROD	**AVG**	**MajVot**	**WAVG**	**WVOT**	**D-ENS**	**ConsA**	
Accuracy	0.7187	0.7817	0.7883	0.7362	0.7742	0.7896	**0.8133**	
Sensitivity	**0.9437**	0.7917	0.7989	0.7434	0.7782	0.7953	0.8212	
Specificity	0.5243	0.7787	0.7847	0.7368	0.7774	0.7897	0.8092	
AUC	0.8186	0.8594	0.8578	0.8010	0.7990	0.8662	**0.8740**	
Brier Score	0.2687	0.1527	0.1584	0.1874	0.1990	0.1479	**0.1425**	
H-measure	0.3950	0.4461	0.4415	0.3225	0.3843	0.4502	**0.4913**	

Table 8.9 Performance results for Polish dataset for all single classifiers, hybrid classifiers, traditional combiners and the proposed methods

Figure 8.9 shows that ConsA ROC curve have a huge peak near the threshold value 0.5, which shows how better ConsA is comparing to other classifiers and combiners at this point. Also it is important to outline that Polish dataset is the smallest dataset amongst all, so successive training process highly depends on some few crucial entries, which may or may not exist in training set. This is, in particular, the reason of high standard deviation of ConsA on this dataset.

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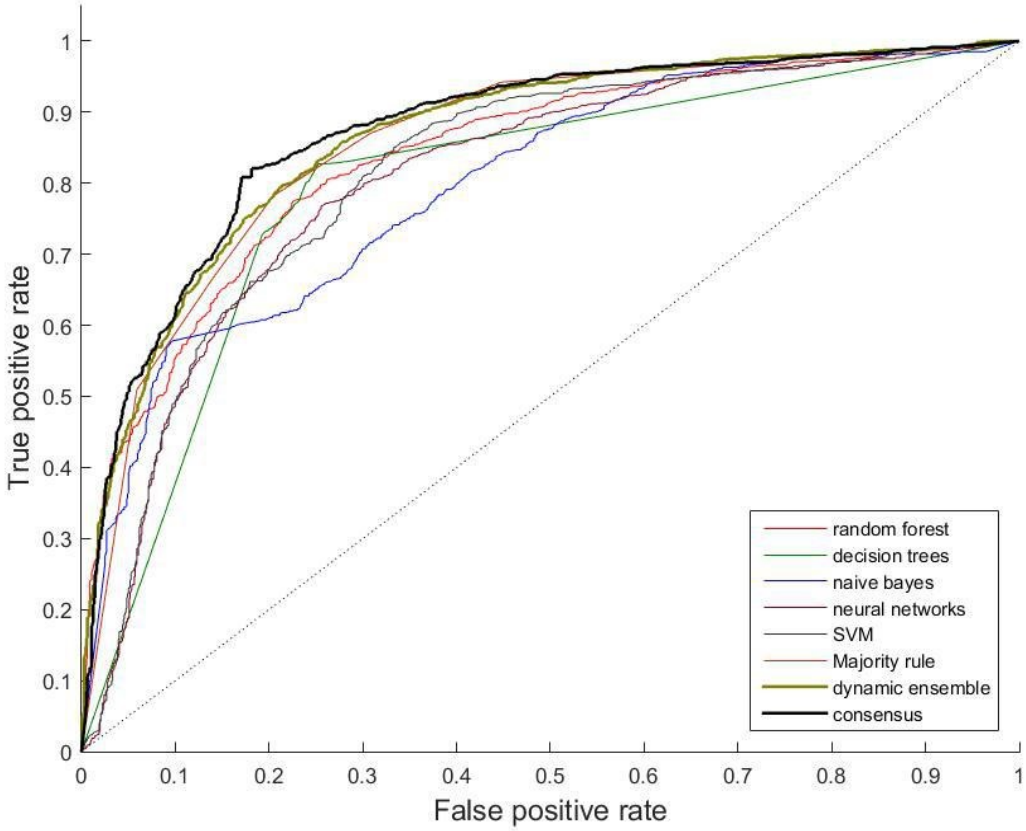

Figure 8.9 ROC curves for all single classifiers, most efficient traditional combiner, D-ENS classifier and ConsA for Polish dataset

As can be seen from Figure 8.10, ConsA is not very certain about its answers, as in several previous datasets. The most likely ranking of good loan entry will lie in [0.1, 0.2] interval (35%). Ranking of bad loans entry can apparently be seen in [0.8- 1] interval. But for 10% of input entries ConsA is not certain at all, as rankings lie in [0.4, 0.6] interval.

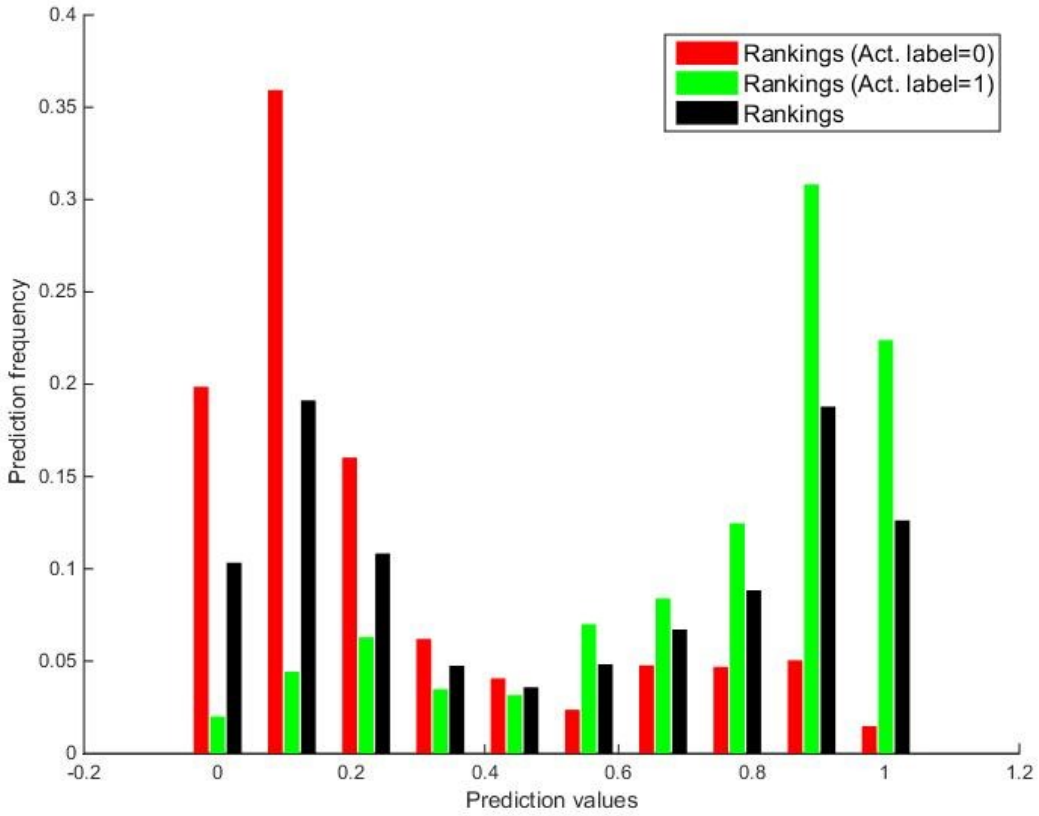

Figure 8.10 Frequency histogram of conditional and absolute values $\boldsymbol{R_G}$ over the test set for Polish dataset

As in the Iranian dataset, Table 8.10 proves the dramatic superiority of ConsA over LR is clear. AUC, Brier's score and H-measure shows very high increase, as it was on Iranian dataset, also Accuracy is very high, almost 9% increase comparing to LR which is the highest increase over the previous investigated datasets. H-measure shows massive increment as Iranian dataset with 22.5%.

Polish dataset	Accuracy	Sensitivity	Specificity	AUC	Brier Score	H-measure
LR	0.7246	0.7150	0.7325	0.7405	0.2298	0.2663
ConsA	8.87%	10.62%	7.67%	13.35%	-8.73%	22.50%

Table 8.10 The improvement of ConsA over LR for Polish dataset

8.2.6 Results of Jordanian Dataset

Table 8.11 reveals that with Filtering and Feature Selection algorithms, ConsA may rightfully be called the best possible option. It overcomes the RF Accuracy by 0.78%, and overcomes it in all other measures; including AUC (almost 3% increase). Regarding Specificity ConsA shows high ability of recognition of good loans, while it has ability to classify half of the bad loans correctly. However this dataset is very imbalanced. D-ENS holds the third position, right after RF. Single combiners shows different results: NN and SVM shows 2% worse results than RF and about 4.5% worse than ConsA. Other classifiers, such as NB and LR, show even worse results.

	RF	**DT**	**NB**	**NN**	**SVM**	**LR**	**MIN**	**MAX**
Accuracy	0.8660	0.8612	0.8212	0.8454	0.8474	0.8240	0.8246	0.8528
Sensitivity	0.9669	0.9444	0.9852	0.9325	0.9424	0.9698	0.9830	0.9059
Specificity	0.4655	0.5295	0.1642	0.4994	0.4705	0.2420	0.1906	**0.6431**
AUC	0.8861	0.7809	0.7735	0.8348	0.8300	0.7336	0.8059	0.8607
Brier Score	0.1006	0.1248	0.1574	0.1193	0.1133	0.1354	0.1466	0.1154
H-measure	0.5027	0.3994	0.2587	0.4042	0.4585	0.2205	0.3837	0.4693
	PROD	**AVG**	**MajVot**	**WAVG**	**WVOT**	**D-ENS**	**ConsA**	
Accuracy	0.8156	0.8566	0.8604	0.8622	0.8574	0.8562	**0.8738**	
Sensitivity	**0.9948**	0.9607	0.9625	0.9327	0.9412	0.9595	0.9699	
Specificity	0.0986	0.4429	0.4555	0.5842	0.5269	0.4453	0.4911	
AUC	0.7727	0.8817	0.8025	0.8791	0.7954	0.8820	**0.9131**	
Brier Score	0.1683	0.1036	0.1136	0.1014	0.1098	0.1028	**0.0960**	
H-measure	0.4099	0.4945	0.4345	0.5064	0.4297	0.4947	**0.5664**	

Table 8.11 Performance results for Jordanian dataset for all single classifiers, hybrid classifiers, traditional combiners and the proposed methods

This fact proves that increasing complexity of classifier will significantly increase benefits of using it. For this dataset complexity of classifier is highly correlated with its Accuracy and other performance metrics. H-measure of ConsA is better than the second place D-ENS by about 0.7%, Brier score is also a bit better (0.6%), so, like for previous dataset, ConsA can be called the best for all measures.

Figure 8.11 show that the AUC characteristic makes it clear to see that in many cases RF rivals ConsA and in some of them even act better than it, although in the end it was concluded that ConsA is the best choice for this dataset. D-ENS holds the third position, but ROC curves of these three classifiers are very close to each other.

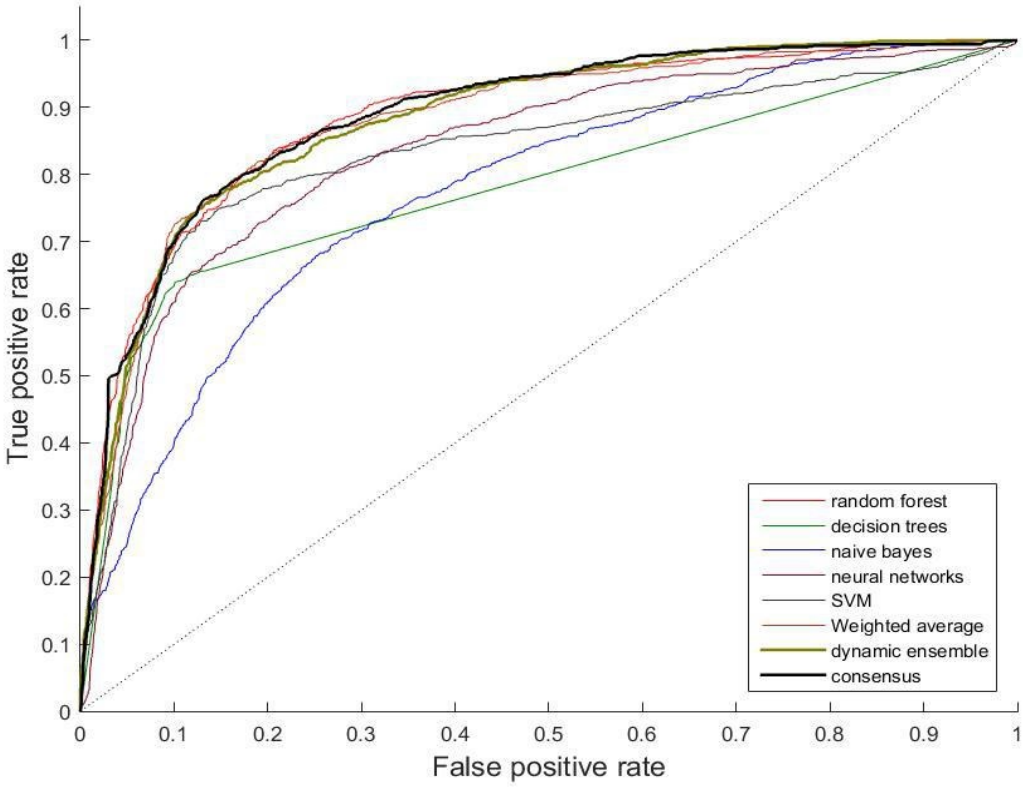

Figure 8.11 ROC curves for all single classifiers, most efficient traditional combiner, D-ENS classifier and ConsA for Jordanian dataset

Figure 8.12 shows that the high red bar on the right of the graph indicates that ConsA is very certain about good loans, whereas for bad loans it cannot be said so. Anyway, ConsA very rarely shows uncertainty (ratings near 0.4-0.6), and in most of the cases if it makes incorrect prediction, its ranking are not completely wrong (so for bad loans, it can make a mistake on 0.2-0.3 raking, but not on 0-0.1). In other words, even when ConsA is wrong and actual class is '1', its ranking is not '0' (completely wrong) but '0.2-0.3', so in case that a 100% guarantee that ConsA will make a correct good loan prediction, a true good loans can only be accepted at 0-0.1 rankings. The same logic is acceptable to bad loan predictions. In the case it is extremely needed to be sure about classifier prediction, a two-threshold system can be recommended as follows:

- If prediction is less than first threshold, it can be accepted that the loan as good with big certainty.
- If prediction is greater than second threshold, it can be accepted that loan as bad with big certainty.
- If prediction is situated between thresholds, this could be interpreted as 'grey zone', so any decision based on that cannot be made.

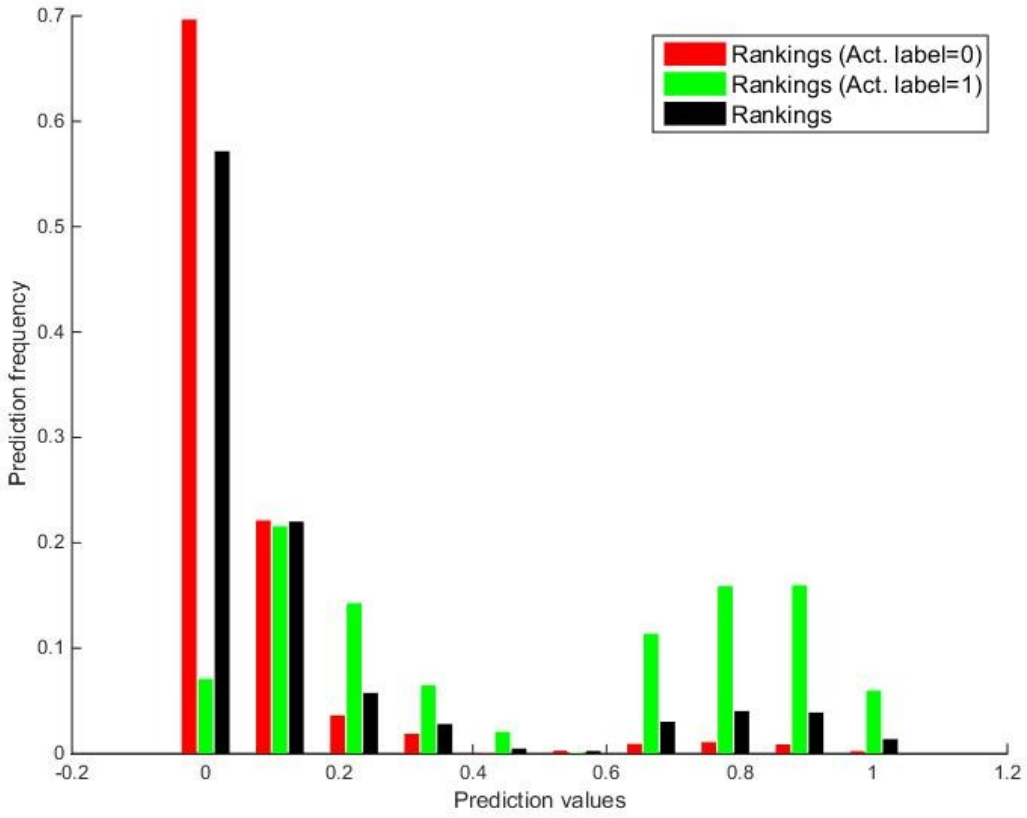

Figure 8.12 Frequency histogram of conditional and absolute values $\boldsymbol{R_G}$ over the test set for Jordanian dataset

For Jordanian dataset shown in Table 8.12, ConsA results exceed the LR results across all measures. The changes vary dramatically from 0.01% to 34.59%. Accuracy of ConsA shows good improvement by 5%. Specificity shows great improvement by 24.91% which is very crucial in highly imbalanced datasets. Sensitivity is almost the same as LR. AUC, Brier's score and H-measure show very high increase, as it is on the Iranian dataset and Polish datasets.

Jordanian dataset	Accuracy	Sensitivity	Specificity	AUC	Brier Score	H-measure
LR	0.8240	0.9698	0.2420	0.7336	0.1354	0.2205
ConsA	4.98%	0.01%	24.91%	17.95%	-3.94%	34.59%

Table 8.12 The improvement of ConsA over LR for Jordanian dataset

8.2.7 Results of UCSD Dataset

In the end it can be seen from Table 8.13 that ConsA classifier is always better than any other classifier. However, the enhancement of ConsA by only 0.59% is not very big. But in large real world datasets this figure may be crucial in question of losses and profits, in which undoubtedly makes ConsA the number one classifier for this dataset.

	RF	DT	NB	NN	SVM	LR	MIN	MAX
Accuracy	0.8690	0.8414	0.8083	0.8487	0.8455	0.8417	0.8046	0.8416
Sensitivity	0.6924	0.5960	0.7787	0.6047	0.6362	0.6439	0.8576	0.4713
Specificity	0.9269	0.9216	0.8181	0.9285	0.9140	0.9064	0.7873	**0.9626**
AUC	0.9162	0.7933	0.8312	0.8825	0.8683	0.8824	0.8828	0.8357
Brier Score	0.0946	0.1424	0.1908	0.1101	0.1433	0.1144	0.1960	0.1247
H-measure	0.5422	0.3735	0.3958	0.4634	0.4548	0.4417	0.4616	0.4011
	PROD	**AVG**	**MajVot**	**WAVG**	**WVOT**	**D-ENS**	**ConsA**	
Accuracy	0.8030	0.8637	0.8649	0.8602	0.8690	0.8662	**0.8749**	
Sensitivity	**0.8665**	0.7040	0.6859	0.7032	0.6924	0.6886	0.7206	
Specificity	0.7822	0.9158	0.9236	0.9114	0.9269	0.9242	0.9255	
AUC	0.8934	0.9082	0.8772	0.9011	0.8093	0.9130	**0.9243**	
Brier Score	0.1862	0.0999	0.1068	0.1022	0.1145	0.0972	**0.0913**	
H-measure	0.4727	0.5160	0.4975	0.5056	0.4809	0.5290	**0.5622**	

Table 8.13 Performance results for UCSD dataset for all single classifiers, hybrid classifiers, traditional combiners and the proposed methods

The UCSD dataset did not show great superiority of ConsA over all other classifiers, but the fact that it is always better on almost all performance metrics, what makes it an unquestionable leader on this dataset. RF shows the second best results, its ROC even sometimes lays higher than ROC curve of ConsA.

However, the region where this happens is far from the optimal point that is why Accuracy in this area is much less. So this region of ROC curve is not so important, and thus there is no

real practical advantage of RF. Moreover, ConsA has the most balanced Sensitivity and Specificity than all other classifiers. Brier score and H-measure of ConsA is the best, the second position for Brier score and H-measure holds RF. AVG also shows good value of Brier Score.

According to Figure 8.13 the ROC curve confirms the results obtained in table above the difference between ConsA and other classifier differ very much, however RF is so close to ConsA, so it might be sometimes used as an alternative of it.

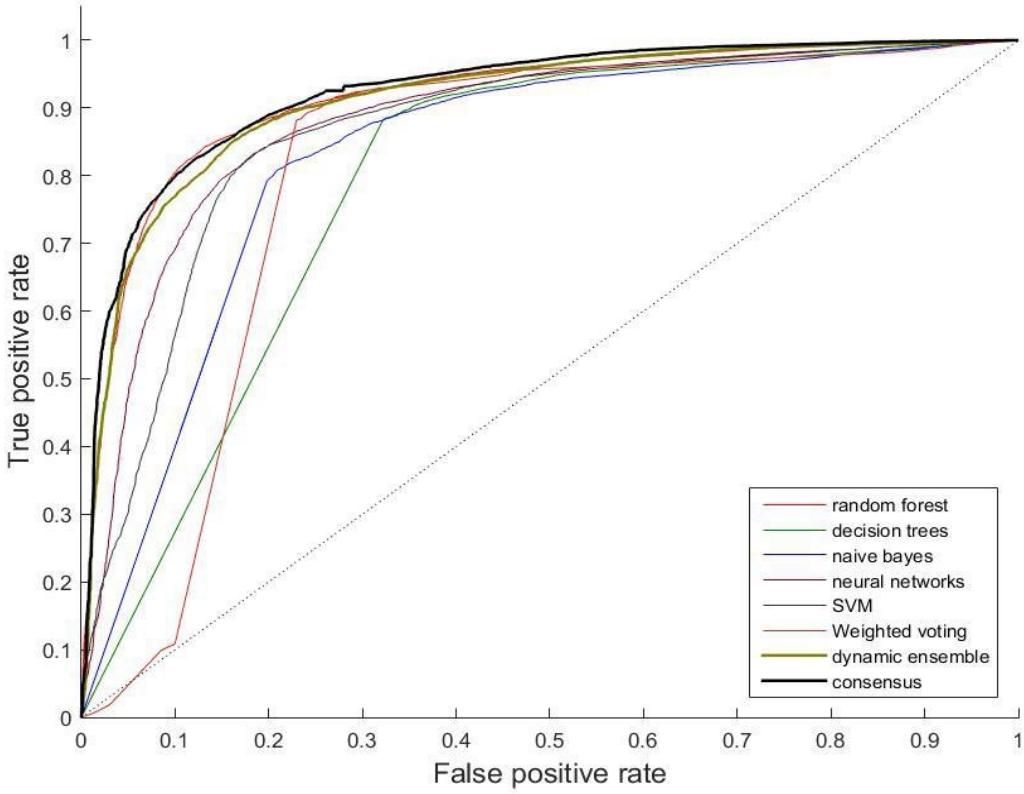

Figure 8.13 ROC curves for all single classifiers, most efficient traditional combiner, D-ENS classifier and ConsA for UCSD dataset

From Figure 8.14, it can be said that ConsA is certain about its good and bad loans predictions, most of the good loans are scored by prediction less than 0.2 and most of the bad loans by prediction value greater than 0.8. This is a big advantage of ConsA as if classifier gives ranking close to the boundary of [0, 1] interval, it can be said almost for sure that ConsA is correct. However, very few bad loans gain prediction value of '1', this may be caused by the fact, that UCSD dataset is skewed.

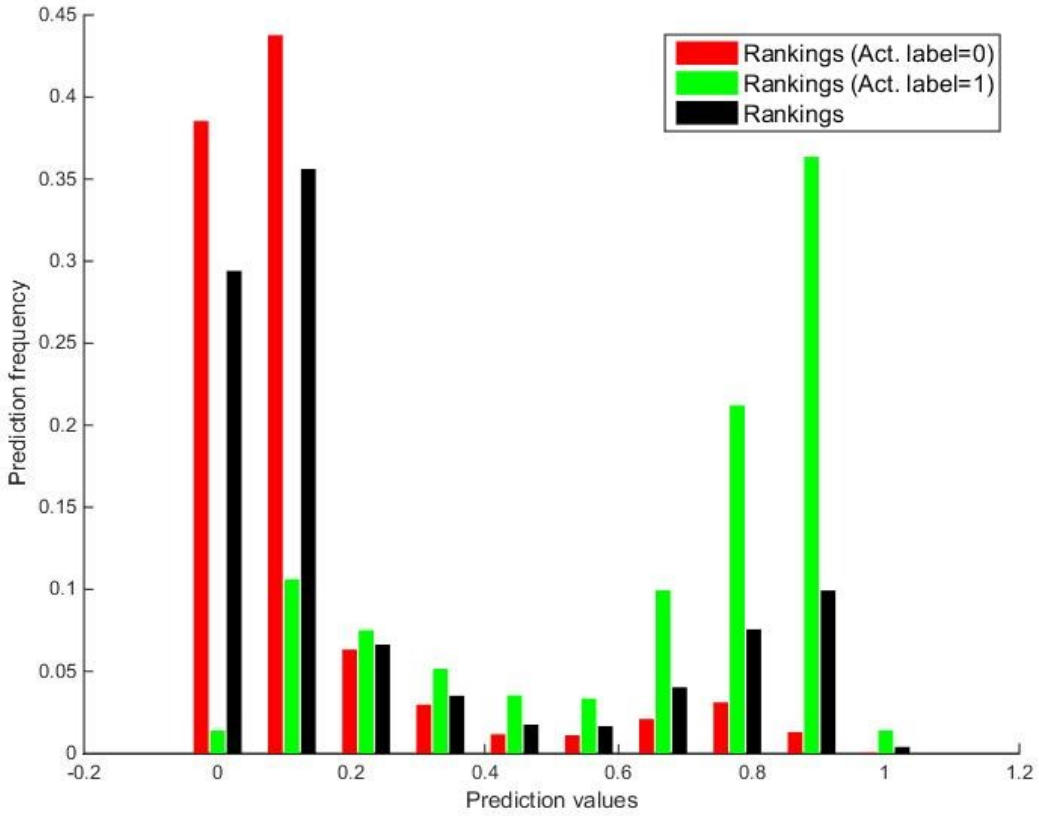

Figure 8.14 Frequency histogram of conditional and absolute values $\boldsymbol{R_G}$ over the test set for UCSD dataset

For the UCSD dataset, Table 8.14 shows that ConsA achieves more than 3% increase in Accuracy comparing to LR. For big real world datasets even 3% increase in good/bad loan classification may result in huge profits for bank funds and ConsA became almost 92.5%, which shows that ConsA can be used effectively even with changed thresholds, so Sensitivity increase will not cause tremendous Specificity decrease and vice versa.

UCSD dataset	Accuracy	Sensitivity	Specificity	AUC	Brier Score	H-measure
LR	0.8417	0.6439	0.9064	0.8824	0.1144	0.4417
ConsA	3.32%	7.67%	1.91%	4.19%	-2.31%	12.05%

Table 8.14 The improvement of ConsA over LR for UCSD dataset

8.3 Statistical Significance Test

In this section Friedman statistical test on all implemented classifiers to prove that ConsA is better not only on the 7 datasets that are being investigated, but with very high probability on all datasets with similar structure to one investigated in this thesis. After this, Bonferroni-

Dunn test is performed to rank all classifiers from the best to the worst, and divide them into two groups, 1) classifiers which under some conditions could rival with ConsA and 2) classifiers that are undoubtedly worse than ConsA.

8.3.1 Friedman Test with Statistical Pairwise Comparison of Best Classifiers

Friedman test is used to detect significance column effects in the classifier prediction matrix (in our case, classifier predictions are columns of matrix), and different classifier's outputs on each input test sample form rows of this matrix. See Table 8.15.

Input	**Classifier 1**	**Classifier 2**	**...**	**Classifier C**
1	x_{11}	x_{12}	...	x_{1C}
2	x_{21}	x_{22}	...	x_{2C}
...				
N	x_{N1}	x_{N2}	...	x_{NC}
Input	**Classifier 1**	**Classifier 2**	**...**	**Classifier C**
1	r_{11}	r_{12}	...	r_{1C}
2	r_{21}	r_{22}	...	r_{2C}
...				
N	r_{N1}	r_{N2}	...	r_{NC}

Table 8.15 Converting table of single classifiers outputs to table of rankings during Friedman test evaluation

First of all, probability is selected, with which can approve or decline Null-hypothesis. The most common values are $p = 0.05$ and $p = 0.1$. It is then needed to convert each row from floating-point outputs of each classifier to ranking row, where $r_{ij} \in \{1,2,\dots,N\},, i \in \{1\dots N\}, j \in \{1\dots C\}$, and $r_{ij} \neq r_{ik}\ \forall i \in \{1\dots N\},. j,k \in \{1..C\}$. So, for example, if initial row is $(0.2, 0.1\ ,0.6, 1, 0.25)$, its convert it to ranking row $(2, 1, 4, 5, 3)$, so that bigger output will receive bigger ranking. After converting classifiers outputs to rankings for each row, proceed with equation (8.1):

$$S = \frac{12}{NC(C+1)} \sum_{i=1}^{C} R_i^2 - 3N(C+1), where\ R_i = \sum_{j=1}^{N} r_{ij} \tag{8.1}$$

If $n > 15$ or $c > 4$ the probability distribution of Q can be approximated by that of a chi-squared distribution. So, when obtained value of S is greater than critical value of chi-squared

distribution $\chi_p^2(c-1)$ for probability p, the Null-hypothesis is rejected, otherwise - accept it. The best 5 classifiers are selected because including all the classifiers results of Friedman test is very large, but it wouldn't give an answer whether ConsA is really better over even the best of single classifiers. To demonstrate, a Friedman test overall and best classifiers across all datasets was performed (see Table 8.16).

Dataset	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Friedmanχ^2(best classifiers)	195.7	64.8	23	563.3	8.9	85.5	220
Friedmanχ^2(all classifiers)	1165	141.6	642.1	2144	85.2	756	1533

Table 8.16 Friedman test for all classifier (1st row) and best classifiers (2nd row)

So to make the conclusions more scientifically solid, analysis of Friedman test is considered for 5 best classifiers, including ConsA. Friedman's test on ConsA, best single classifier, best classical combined classifier, LR and the D-ENS are performed. A null-hypothesis in that case is that the difference between these 5 classifiers rankings is accidental and not caused by the significance of each classifier is random. Null-hypothesis is accepted with 95% probability if Friedman statistics $S < \chi_{0.05}^2(4) = 9.488$. Null-hypothesis is accepted with 90% probability if Friedman statistics $S < \chi_{0.1}^2(4) = 7.779$. Subseqeuntly, to test the hypothesis on 0.05 and 0.1 significance levels, Tables 8.17 to 8.23 demosnstrate pairwise comparison for the 5 best classifiers across all datasets.

Friedman $\chi^2 = 195.6692$	**Accuracy**	**RF**	**Logistic Regression**	**Majority rule**	**D-ENS**
ConsA	0.7903	0	0	0	0
RF	0.7725	-	0.000074	0.035911	0.640542
LR	0.7597	-	-	0	0.000005
MajVot	0.7776	-	-	-	0.064970
D-ENS	0.7738	-	-	-	-

Table 8.17 German dataset pairwise comparison

Friedman $\chi^2 = 64.8358$	Accuracy	RF	Logistic Regression	Majority rule	D-ENS
ConsA	0.8810	0.000016	0	0.000042	0.000063
RF	0.8707	-	0.024102	0.161445	0.052801
LR	0.8641	-	-	0.000303	0.000067
MajVot	0.8736	-	-	-	0.394666
D-ENS	0.8751	-	-	-	-

Table 8.18 Australian dataset pairwise comparison

Friedman $\chi^2 = 23.0763$	Accuracy	RF	Logistic Regression	Weighted voting	D-ENS
ConsA	0.8871	0	0	0	0.043300
RF	0.8717	-	0.005313	1.000000	0.000096
LR	0.8626	-	-	0.005313	0
WVOT	0.8717	-	-	-	0.000096
D-ENS	0.8842	-	-	-	-

Table 8.19 Japanese dataset pairwise comparison

Friedman $\chi^2 = 563.3613$	Accuracy	RF	Logistic Regression	Min rule	D-ENS
ConsA	0.9575	0	0	0	0.010667
RF	0.9513	-	0	0.022160	0.597934
LR	0.9239	-	-	0	0
MIN	0.9500	-	-	-	0.199140
D-ENS	0.9569	-	-	-	-

Table 8.20 Iranian dataset pairwise comparison

Friedman $\chi^2 = 8.9032$	Accuracy	RF	Logistic Regression	Majority rule	D-ENS
ConsA	0.8133	0	0	0	0
RF	0.7742	-	0	0.017259	0.015322
LR	0.7246	-	-	0	0
MajVot	0.7883	-	-	-	0.794822
D-ENS	0.7896	-	-	-	-

Table 8.21 Polish dataset pairwise comparison

Friedman $\chi^2 = 86.544$	Accuracy	RF	Logistic Regression	Weighted average	D-ENS
ConsA	0.8738	0.004763	0	0.000808	0
RF	0.8660	-	0	0.255370	0.000066
LR	0.8240	-	-	0	0
WAVG	0.8622	-	-	-	0.079153
D-ENS	0.8562	-	-	-	-

Table 8.22 Jordanian dataset pairwise comparison

Friedman $\chi^2 = 219.6776$	Accuracy	RF	Logistic Regression	Weighted voting	D-ENS
ConsA	0.8749	0.000015	0	0.000015	0
RF	0.8690	-	0	0.346104	0.050519
LR	0.8417	-	-	0	0
WVOT	0.8690	-	-	-	0.010528
D-ENS	0.8662	-	-	-	-

Table 8.23 UCSD dataset pairwise comparison

At significance level of 0.05, the null-hypothesis for the 4 classifiers are declined, except on the Polish dataset. At significance level 0.1 the null-hyphothesys for all datasets is declined. The reason why Polish dataset is an exception in the first case is its small size, so size of test set is only 60 entries. If Polish dataset had more entries, the Friedman statistics would be much higher. In the table all possible pairwise t-tests for each pair of classifiers to find which classifiers perform in the similar way, and which are not were performed. Obtained data shows us that p-values for all sells in most of datasets are low, so significance of each classifier is proportional to its Accuracy. However, with some low probability ConsA and D-ENS could have similar ranking for some datasets. Results also show similarity in performance of RF and D-ENS, which prove remarks, mentioned in the previous section. So D-ENS and RF performance are similar, but the good thing about D-ENS is that this classifier shows much better performance when filtering is applied.

8.3.2 Bonferroni-Dunn Test for all Classifiers

Friedman ranking test (Accuracy rankings) is calculated over all single classifiers, all classical combiners and ConsA. To evaluate the critical values of significance level $\alpha = 0.05$ and $\alpha = 0.1$, Bonferroni-Dunn two-tailed test is evaluated as in equation 8.2):

$$CD = q_a \sqrt{\frac{k(k+1)}{6N}} \tag{8.2}$$

where $k = 14$ (number of classifiers), $N = 7$ (number of datasets), q_a is calculated as Studentized range statistic with confidence level $\alpha/(k-1) = \alpha/13$, divided by $\sqrt{2}$. So in our case, the Studentised range statistic test is calculated with confidence levels $\alpha = 0.00035$ and $\alpha = 0.00714$. Obtained values $q_{0.05} = 2.9137$, $q_{0.1} = 2.6901$. Obtained results $CD_{0.05} = 6.96$, $CD_{0.1} = 6.43$. The two horizontal lines, which are at height equal to the sum of the lowest rank and the critical difference computed by the Bonferroni–Dunn test, represent the threshold for the best performing method at each significance level ($\alpha = 0.05$ and $\alpha = 0.1$). Obtained results clearly show us that ConsA is obviously the best over all other classifiers and classical combiners. RF shows the good stable results, it holds the second position for all dataset. LR is good, but worse than some of the classical combiners. Based on the evaluated critical value, it can be concluded that PROD, LR, NB, Max and MIN, SVM, WAVG and NN are significantly worse than ConsA Approach at significance levels $\alpha = 0.05$ and $\alpha = 0.1$, and DT is worse only at level $\alpha = 0.1$

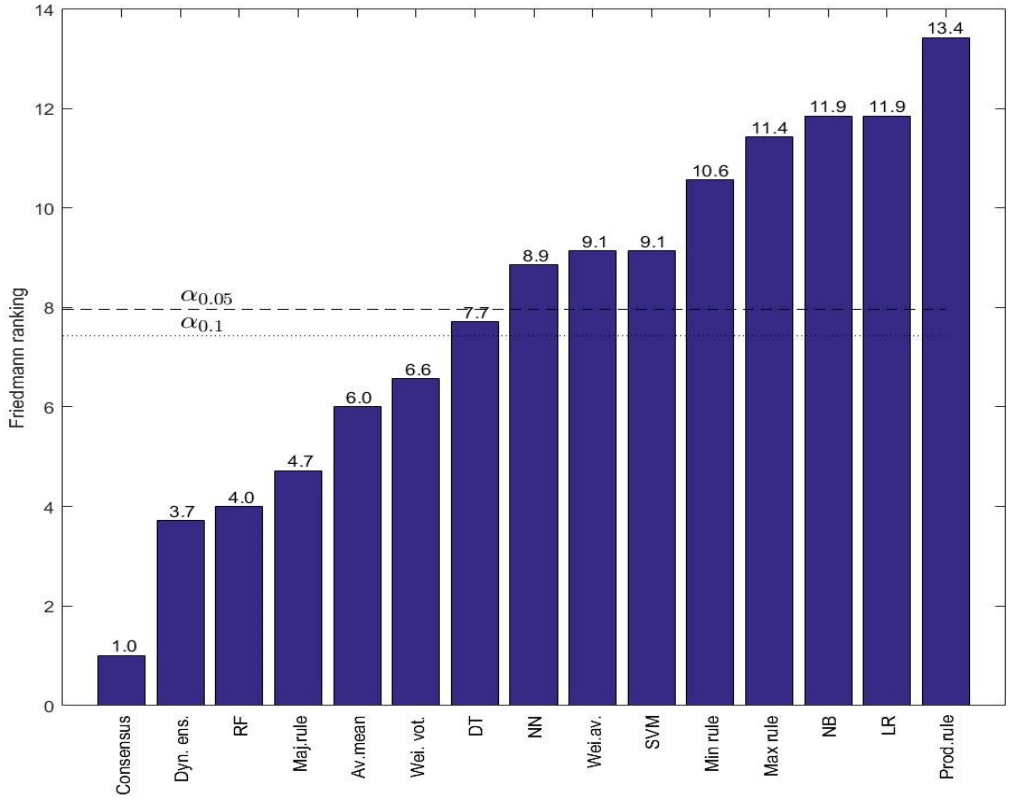

Figure 8.15 Significance ranking for the Bonferroni–Dunn two-tailed test for ConsA Approach, benchmark classifier, base classifiers and traditional combination methods with $\propto = 0.05$ and $\propto = 0.10$

8.3.3. ConsA Computational Time

Dataset	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
RF	17.2	10.3	12.8	13.9	9.0	9.9	33.8
DT	4.3	3.5	3.5	4.3	2.7	3.0	11.1
NB	2.9	1.6	1.7	25.5	8.0	1.1	7.6
NN	23.1	28.0	23.5	13.5	13.3	11.8	35.6
SVM	5.5	2.6	2.4	3.6	1.4	2.1	17.6
LR	6.7	11.1	18.3	84.9	1.3	7.5	2.3
MARS	62.2	48.7	45.4	108.1	38.0	55.9	135.6
GNG	14.6	5.8	5.9	14.5	1.3	2.9	202.2
D-ENS for all testing points for all classifiers	29.6	16.9	17.4	28.7	5.4	11.0	121.9
ConsA	4.6	3.2	0.9	1.0	1.1	1.9	3.0
Total time	170.7	131.7	131.8	298.0	81.5	107.1	570.7

Table 8.24 Computational time for all integral parts of ConsA for all 50 iterations/ second

To evaluate ConsA model firstly it is needed to select most important features from the full list of them. During this procedure MARS model has been built and perform ANOVA analysis of this model to evaluate importance of all features. This procedure takes a lot of time, from 38 seconds for Polish dataset to 135 seconds for the UCSD dataset. In general, the larger dataset is and the more features it has, the longer time it takes to compute this step.

The Next step is GNG for filtering, which takes the most of time in filtering algorithm (other parts of filtering itself takes fraction of seconds)

The following step is training the five single classifiers on the filtered data with selected features. The most time to train the model tis for RF and NN, the least is NB and SVM. Despite of higher complexity and computational time, RF has the highest accuracy, so it is a good idea to keep it in the model.

After computing all integral parts, the final step is to evaluate the ConsA answer. This step takes not a lot of time as at this time was given to compute the data needed for it. This step takes in average only 2.2 seconds as ConsA is not trained in any way only its answers are evaluated using straightforward equations.

In general, timing of model evaluation depends on the size of dataset, for most of the integral parts linearly, but for GNG evaluation it is cubically. For example, German dataset has 1000 entries, and UCSD dataset has 2543 entries, so UCSD dataset is 2.4 times larger than German dataset. If GNG were linearly dependable on the size of dataset, the computational time would be 14.6*2.4=35 seconds, but actually this time is 202.2 seconds, which is 13.84 times more, which is exactly 2.4^3 so the above fact that computational time is cubically-dependable on data size can be easily proved.

The total time for ConsA evaluation can be seen in the last row of the table, not surprisingly to evaluate consensus on the largest UCSD dataset it is needed almost 10 minutes of computer work. But after evaluation all steps, using ConsA on new data is pretty easy and fast: ConsA will give an answer for new several hundred loans in a fraction of second (as we need only 2 last steps to evaluate for this). For German dataset for one iteration it takes 0.5 second, which is very good.

8.4 Analysis and Discussion

In this section the results of each performance measure for the investigated datasets are analysed and discussed, in addition to the ROC curves and the statistical significance tests.

8.4.1 Accuracy, Sensitivity and Specificity

Accuracy is the most obvious and straightforward measure, which gives the percentage of correct predictions that each classifier is able to make. What is obvious, that usage of filtering and feature selection together makes the results of all single classifiers more comparable by their Accuracy. For example, whilst conducting the experiment without including feature selection and data-filtering for German dataset, the difference between the best and the worst of single classifiers when feature selection and filtering are disabled is more than 4.5%, whilst these two methods enabled a difference that decreases to 1.9%. Average performance of all single classifiers increases as well. This fact can describe why performance of complex ensembles like D-ENS or ConsA often grows even more significantly than performance of single classifiers. When single classifiers have comparable performance, complex combiners can use useful information from all 5 classifiers, otherwise the worst classifier drops out of consideration during ConsA or D-ENS method evaluation.

Sensitivity and Specificity values of ConsA is more well-adjusted and steady than other classifiers for all datasets, which means that it can recognize bad loans as well as good loans in a way better than other classifiers. Sensitivity and Specificity is the best for the Australian and Japanese datasets, while for the Iranian dataset ConsA is quite unbalanced as the Iranian itself is very unbalanced.

8.4.2 AUC and ROC Plots

Each classifier gives some ranking value as a respond to the input data. Usually if this value is less than 0.5, it is considered that the prediction of this classifier is ‘0’. And if this value is equal or greater than 0.5, then it can be considered that the prediction of this classifier is ‘1’. Sometimes calculated values of Sensitivity or specificity for this classifier are insufficient for researcher because of a real price of false positive (or false negative) error, which can be expressed in money. One of the ways of increasing one of these parameters is to consider this value (0.5) as a threshold variable, and change it. Increasing this value will lead to sensitivity increasing, but specificity decreasing. Decreasing of threshold has the opposite effect. So, the

price for increasing one of the parameters is decreasing another. Thus, a conclusion can be made based on the ROC plots for ConsA for each dataset:

- **German**: ConsA ROC curve lies above all other curves for all values of threshold. It means that for this dataset ConsA is the best for all needed values of Sensitivty and Specificity. RF curve has also convex circle-like shape with optimal value of threshold near 0.5. But as D-ENS has higher AUC value, and its ROC curve is slightly higher than RF one, this classifier is more preferable to use for such datasets. Maybe it is an option to use D-ENS in a combination with ConsA to achieve highest possible Accuracy.
- **Australian:** Comparing to German dataset ROC curves of ConsA and RF are higher, which means lower rates of false-negative and false positive errors of all classifiers. RF ROC curve lays below ConsA, but almost for all values of threshold above all other classifiers. But surprisingly the second place for AUC holds AVG, which is even not the best amongst classical combiners in the terms of Accuracy.
- **Japanese:** Similar ROC curve to previous dataset. These two datasets are balanced, that's why almost all classifiers show good results. The three leaders are not far away from other classifiers, but their results are surely statistically better. ConsA has the best ROC curve, then D-ENS, and then RF.

- **Iranian:** ROC-curves for all classifiers are skewed, which means that Specificity increasing leads to huge decrease of sensitivity. It can be caused by little amount of bad loan entries in this dataset, so classifiers are not able to learn bad loans patterns. For optimal cut-off most of the classifiers have big value of Sensitivity but relatively small value of Specificity. It means that these classifiers cannot recognize bad loans with sufficient Accuracy. For this dataset ROC curve of D-ENS is even higher than ConsA ROC partially (the first part of ROC curve, which is responsible for higher Specificity than Sensitivity). But for this part of ROC Accuracy is low (because of very few bad loans), so big Sensitivity is definitely better than big Specificity). For this dataset ROC curve of D-ENS is even higher than ConsA ROC curve partially (the first part of ROC curve, which is responsible for higher Specificity than Sensitivity). But for this part of ROC curve Accuracy is low (because of very few bad loans), so big Sensitivity is definitely better than big Specificity.

- **Polish:** ROC curve of RF is not always convex, which means that it is possible to update this classifier a bit. If the ROC curve is not convex in the range from threshold t_0 to threshold t_1, and classifier's ranking lies between t_0 and t_1, this ranking is assigned to t_1. In other words, if ranking $t_0 < ranking \leq t_1$ then assigns $ranking = t_1$. This procedure, made for all entries for dataset changes the classifier's ranging so that ROC curve will have straight line from t_0 to t_1. ROC curve of D-ENS is very close to ROC curve of ConsA, but near the optimal point ConsA is definitely better

- **Jordanian:** For this dataset it can be seen that ROC curve of ConsA mostly is a bit higher than RF ROC curve. But looking at the intervals where ConsA ROC will lie lower than RF ROC, it can be conclude that for some threshold RF could perform slightly better. D-ENS ROC curve is high, but also visually not convex, which means that this classifier is possible to optimise a little bit more.

- **UCSD:** ConsA gives us ROC curve with perfect shape, so for real-time datasets it is the best choice. RF and D-ENS lies below ConsA. However, their ROC curves and AUC values are also good and solid.

The ROC analysis is a powerful tool to analyse classifier's characteristics over all possible values of threshold. Obtained results show us obvious advantage of ConsA over single classifiers and traditional combiners. RF shows also good and stable results. However, as it can be seen from ROC curves, MajVot as the best traditional combiner for German, Australian, Iranian and Polish datasets show good results only in small threshold ranges near 0.5 due to the structure of output rankings of this classifier (MajVot can have as output only values 0, 0.2, 0.4, 0.6, 0.8 or 1).

8.4.3 Brier Score Results

ConsA in average has the smallest brier score, which means that ConsA ranking is highly correlated with actual test set labels. D-ENS holds the second place for German to Polish datasets. For dataset Jordanian and UCSD, at second place holds RF. Amongst traditional combiners, good results show Simple AVG and WVOT rules.

8.4.4 H-measure Results

ConsA is the leader in H-measure evaluation, D-ENS and RF holds second and third place, respectively. The only significant difference is that sometimes AVG shows better results than RF (e.g., German and Australian datasets). H-measure is responsible for stability and robustness of classifier in terms of misclassifying cost changes.

8.4.5 Friedman Test

For each dataset Friedman were performed test on five classifiers (ConsA , RF, Logistic Regression, Best traditional combiner and D-ENS) for 7 datasets with null hypothesis that there are no statistically significant differences between these classifiers, and alternative hypothesis is that at least one classifier performs better than others. Null-hypothesis means that all classifiers from this group perform identically, and all differences are only random fluctuations. It should be noted, that for all datasets with significance level α=10% null-hypothesis was rejected, which means acceptance of an alternative hypothesis. To determine, which classifier perform better than others, a pairwise statistical t-tests were used. Friedman test along with pairwise statistical t-test of the group of five best classifiers shows very high probability that on other similar datasets ConsA will give better Accuracy comparing to other classifiers. In other words, ConsA will remain the best classifier on any other dataset, similar by structure to any of investigated datasets.

8.4.6 Bonferroni-Dunn Test

Obtained results clearly shows a sorted list of classifiers, which could be compared with ConsA by Accuracy under certain conditions, and second sorted list of classifiers, that are undoubtedly worse than ConsA. In the first list the best classifiers are D-ENS, RF, WVOT and MajVot. From the second list, it is worth mentioning the NN, NB, LR and SVM. It can be interpreted that, for such problems (loan quality analysing), with selected datasets using them as single classifiers are doubtful. To divide first group of classifiers from second, the Bonferroni_Dunn test were used, which gives more reliable results, comparing to two-tailed Nemenyi test (Demšar, 2006).

As a conclusion, ConsA shows the best performance, but amongst other classifiers the final decision about which to use should be made based on dataset structure (good/bad loan entries,

number of features, and number of outliers in training set). As of traditional combiners worth mentioning simple and AVG and MajVot which show pretty solid results.

8.5 Summary

In summary, ConsA shows the best performance. On some datasets such as German, Polish and Jordanian datasets, the advantage of ConsA over all other classifiers is impressive. Ranking histograms demonstrate that, in almost all datasets, ConsA is certain about its predictions, which shows that ConsA can be successfully used with various range of thresholds without significant drop of the Accuracy. The most impressive performance ConsA shows on Polish dataset, which can be explained by the fact, that this dataset is balanced. D-ENS holds the second position, competing with RF.

ConsA shows balanced and stable results in Specificity and Sensitivity even on unbalanced datasets in comparison to other classifiers where if a classifier has good Sensitivity it will have bad Specificity and vice versa . The most obvious example is the UCSD dataset, which has four times more good loans entries than bad loan entries; however, ConsA Sensitivity is 72% and Specificity is 92%. For such unbalanced datasets, there is a 20% difference in Specificity and Sensitivity, which is not very much (compare to RF, where difference is 23% and LR with 26%). On balanced datasets sensitivity and specificity values of ConsA are almost equal.

Looking at the ROC curves it can be concluded that ConsA shows good performance with wide range of thresholds, so it is not need to re-train classifiers each time for each needed threshold. ConsA ROC curves often have a peak near the optimal value, this is because of ConsA parameters that are correctly chosen and boost ConsA performance near the 0.5 threshold.

Furthermore, ConsA deals well with imbalanced datasets; its high H-measure shows us that ConsA can be successfully used with different pairs of misclassifying costs (false-positive cost and true-negative cost), so its misclassifying error for all thresholds is lower than for other classifiers. It means that in real life, losses caused by ConsA wrong decisions is smaller than losses caused by decisions of any other classifier being considered.

CHAPTER 9

CONCLUSIONS & FUTURE WORK

9.1 Conclusions

The main aim of this thesis was centred on investigating the benefits of complexity in modelling credit-scoring problems. This aim was achieved by building and accordingly analysing a complex credit-scoring model based on classifiers ConsA. This thesis demonstrated a comprehensive comparison of various credit-scoring models, starting from simple single base classifiers to very complex models based on classifiers ConsA, which is the main proposed method in this thesis. The model started very simply and grew to be more complex, gradually, in order to determine the extent to which complexity affects classification performance; this was investigated by: 1) Implementing the base classifiers; 2) Investigating various data pre-processing methods on single classifiers, hence producing hybrid models; 3) Investigating ensemble classifiers by applying traditional combiners; and 4) Finally, proposing and developing two combination techniques, namely D-ENS and classifiers ConsA, and comparing their performance against one another, along with all previous steps. The proposed model, as well as all classifiers developed, was validated using seven real world-datasets, six performance measurements that reflect different aspects of the classifiers' prediction ability for each classifier, and finally the proposed model was statistically tested for its significance against all classifiers. In general, ConsA showed significant results when compared to other classifiers within the context, and also outperformed the industry standard LR superiorly, which was used as a benchmark mode in this thesis. To sum up, this thesis is made up of seven main chapters.

- **Chapter 2:** This chapter focused on: 1) providing a theoretical background of credit-scoring and its related issues in terms of definitions and procedural framework in terms of development and implementations; and 2) reviewing the related literature in credit-scoring by centring on the different modelling approaches and algorithms how these can be used in such a design in order to achieve a good performance. All the literature were analysed critically, and several findings were drawn, which eventually led to the proposed model.

- **Chapter 3:** Focused on the main stages, including the experimental design of proposed credit-scoring model. These stages consider several issues that should be carefully assessed such as: 1) The datasets to be used in terms (number, size and variation of datasets); 2) Data pre-processing, such as feature selection and data-filtering, for example; 3) Data partitioning techniques; 4) Modelling approach and the way to solve the problem at hand; and finally 5) Performance measurements metrics and statistical significant tests. All the aforementioned stages have their effects on modelling and based on the problem in hand choosing the appropriate elements can help in having a comprehensive credit-scoring model.

- **Chapter 4:** A total of 5 main classifying techniques were implemented and applied for the 7 available datasets. Classifiers analysed are: NN, SVM, RF, DT and NB. Also LR as a benchmark classifier was implemented, against which all other 5 classifiers were compared. As each of classifiers has their own strong and weak sides on different datasets, each classifier behaves differently. In general, the best classifier performed to be RF, but benchmark LR holds third place, only slightly behind SVM classifier. NN also showed solid performance, of some datasets even better than SVM and Logistic Regression, but in average it held the fourth place.

- **Chapter 5:** In this chapter, all 5 classifiers on the same datasets are evaluated; this time, however, a data pre-processing were conducted on the data before training classifiers, namely data-filtering using GNG proximity graphs and feature selection using MARS. Altogether, 3 experiments were carried out: 1) Using data-filtering only; 2) Using feature selection only; and 3) Using both in combination. The idea was to investigate the effect of each experiment on the classification performance and see how complexity can lead to better results. Results clearly prove that having both data-filtering and feature selection combined give effective and better Accuracy results within almost all classifiers. Besides, comparing the first and the second experiment, it can be concluded that, in general, data-filtering is a more effective technique than feature-selection as most of the classifiers are considered much more robust against redundant features than to noisy or outlier input training data. Only RF shows equally high robustness towards these two negative effects. On the other hand, much simple classifiers as DT and NB improve their results tremendously. Not surprisingly,

filtering is more effective on balanced datasets with high amount of entries, and feature selection on datasets with high number of features, some of which are categorical.

- **Chapter 6:** Several experiments based on ensemble classifiers were carried out using the traditional combiners: MIN, MAX, PROD, AVG, MajVot, WAVG and WVOT. Results clearly demonstrate that the best of traditional combiners often concedes to the best of single classifiers of which they consist. Results become better when applying filtering and feature selection, but from the results it can be argued that using single classifiers is unjustifiable in the case where single classifiers performance differs a lot. On the other hand, if all classifiers perform approximately the same, traditional combiners can be used to improve the result. The best of traditional combiners were MajVot and WVOT. The first gives the best results because of the model intuitive simplicity and the inevitable fact that majority of classifiers unlikely to be wrong at the same time. The second combiner uses single classifier Accuracy on the training set, which is why predictions of more accurate classifiers count more. As a result, based on the experiments where complexity was moderate compared to RFs, more complex combiners were intended to be performed.

- **Chapters 7 & 8:** Two complex combiners were considered and analysed: ConsA, which is the proposed method, and D-ENS Selection approach as another complex combiner to be compared with. Chapter 7 mainly provided a theoretical background on both approaches, followed by a practical example on how both were implemented. Chapter 8 carried out all experimental results and comparisons with classifiers—the singles, hybrid, traditional combiners and the D-ENS. Results emphasised the superiority of ConsA against all the classifiers and this was validated at the end by the statistical significance test.

The main advantage of ConsA compared to traditional combiners is the creation of a group ranking as a fusion of individual classifier rankings rather than merging these rankings using arithmetical, logical or other mathematical functions. ConsA simulates the real expert's group behaviour: they continuously interchange their opinions, and change their measurements of

possible answers influenced by other experts. The process continued until they came up with group decision, with which they all agree. Sometimes, however, experts cannot came up with group decision, as well as ConsA do not converge. To prevent these situations, it has been decided to use the least squares method instead of iterations procedure to obtain optimal group ranking. Another problem is unknown conditional ranking values, which has been evaluated as linear combination of two classifier rankings. Moreover, the best Accuracy of classifier is the more impact it has on other classifiers. In other words, $R(i|j)$ is the conditional ranking of i-th classifier know the ranking of j-th classifier is close to $R(j)$ if Accuracy of j-th classifier is greater then Accuracy of i-th classifier, otherwise it is close to $R(i)$. So, the two things new in the investigation compared with (Shaban *et al*, 2002) which are:

- Using local Accuracy algorithm to estimate the performance of single classifiers at a given point and then evaluate conditional rankings.
- Using the least square algorithm instead of iterations to solve the equation (7.19).

ConsA algorithm was tested on 7 datasets with the aim of predicting loan quality of the client (0 – good loan, 1 – bad loan). On every dataset comparing to the single classifiers, hybrid classifiers and traditional combiners ConsA shows the advantage. Worth mentioning, that often the Accuracy of traditional combiners were lower than Accuracy of best single classifier, which means uselessness of blind merging of classifiers results. For example, while merging 2 relatively good classifiers with Accuracy 75% and 3 relatively bad classifiers with Accuracy 72% using traditional combiner method (AVG, MAX, MIN etc.), the Accuracy often achieved is worse than Accuracy of good classifiers and better than Accuracy of bad classifiers (73%-74%). By the contrary, ConsA shows relationship between single classifiers: how each classifier's ranking affects other classifiers, if majority of classifiers at some data entry make wrong prediction, traditional combiners also make wrong predictions with a big chance. However, ConsA, using the relationship between classifiers, is still able to make a correct prediction. ConsA results are more stable (standard deviation test) and often has good specificity values, which means better bad loans recognition. Apart from ConsA, another combination approach has been investigated, which is the D-ENS Selection approach based on local Accuracy. The main essence of this approach is centred on using local Accuracy to select which classifiers can show better performance on a given testing point. The better local

Accuracy is the bigger impact classifier has on a final ranking of D-ENS classifier. However the D-ENS Selection approach was improved as following:

- The weighted average were used to evaluate final ranking instead of simple averaging by picking the result of a best classifier as a final result
- The uncertainty values to modify the result: if the locally best classifier gives ranking close to 0.5, the weight to this classifier is decreased as the given ranking show that at the current point classifier is not certain about its decision.

D-ENS rivals with RF for all dataset, and in average its performance is slightly better than RF performance. However, in real loan classification problem the best thing to be done is to try both of these classifiers and select the one with better Accuracy. As a general conclusion it is worth mentioning that ConsA beats all classifiers for all datasets. By applying data-filtering and feature section combined, superiority of ConsA becomes even more obvious (for example German and Jordanian dataset). Amongst single classifiers, RF shows the best results. This can be explained when considering that RF itself is not actually a single classifier but rather a bunch of DT that produce ranking using voting procedure.

9.2 Limitations

As other classifiers ConsA has limitations, mostly related with processor time needed to:

- Train and evaluate all single classifiers.
- Build GNG and MARS models to process data-filtering and feature selection.
- Adjusting ConsA parameters to fit the data.

Therefore, if the computational abilities of a bank are weak, it is advisable to decrease the number of the single classifiers in ConsA, and also to skip the feature-selection process as it affects performance not so much but data-filtering. ConsA has performance limitations in the case of highly imbalanced data, where it shows not more than half of a percent increase over best of other classifiers. However, financially speaking, a fraction increase in Accuracy can save huge losses. Finally, ConsA is notably reliant on single classifier performance.

9.3 Future Work

As a future work direction, the proposed model could be modified by:

- As a possible extension of this research it can be considered merging top 3 classifiers from this research (ConsA, RF and D-ENS) to achieve even better results. Another option is to include D-ENS into ConsA expert's group, and exclude from it the weakest NB classifier.
- Analysing other approaches to conditional ranking $R_i(\gamma_k|\Gamma_j)$ evaluation ConsA (possibly using rankings of an i-th and j-th classifiers merging combiner). One of the perspective ways for evaluation conditional ranking is using NN with inputs as single rankings and local Accuracy of each classifier, and output as conditional ranking
- Investigate combining homogenous classifiers or different numbers of heterogeneous classifiers to see to what extent ConsA results can change. The possible direction should be centred on including D-ENS as one of the experts into ConsA model.
- Investigate different pre-processing methods for the datasets, such as other feature-selection or data-filtering methods, and accordingly determine how this could reflect on ConsA results. Try to use not pure filtering but filtering-condensing approach, which will remove not only outlier entries, but also non-informative entries, which can interfere training process in a bad way.
- Change the least efficient classifier (NB) in ConsA to D-ENS or other strong combiner.
- Investigate other approaches in defining ConsA parameters k_i other than gradient descent (e.g., search of global optimum using Genetic Algorithms).
- Improve ConsA so it can output no single floating-point ranking, but fuzzy opinion using fuzzy logic. In this regard, consider not floating-point matrix of rankings, but rather fuzzy matrix with fuzzy opinions, and evaluating all algorithms in a similar way.

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APPENDIX A

Below is a summary of all classifiers parameters used to training across all datasets.

- **NN**

Datasets\parameters	Hidden layers = 1/Neurons in hidden layer	Learning rate	Training method	Momentum	Epochs
German	4	0.005	traingda	-	1000
Australian	10	0.01	traingda	-	1000
Japanese	10	0.01	traingdx	0.9	1000
Iranian	3	0.01	traingda	-	1000
Polish	10	0.01	trainlm	-	1000
Jordanian	10	0.01	trainlm	-	1000
UCSD	10	0.01	trainlm	-	1000

- **SVM**

Datasets\parameter	Sigma
German	1.37
Australian	0.7812
Japanese	0.9795
Iranian	1
Polish	0.366
Jordanian	0.2413
UCSD	0.1836

- **RF**

For all datasets the:

- Method used is: **'Regression'**.
- Number of Trees: **60.**
- Attributes used to build the tree: **All attributes**.
- Impurity Evaluation: '**Gini Index'**.
- Categorical variables vectors:

Dataset	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Features #	18,19,20	4,8,9,11,12	1,4,5,6,7,9,10,12,13	5,6	-	2,3,4,5,8,10,11	-

- **DT**

For all datasets the:

- Attributes used to build the tree: **All attributes**.
- Impurity Evaluation: '**Gini Index'**.
- Best categorical variable split: **'Exact' option.**
- Categorical variables vectors:

Dataset	German	Australian	Japanese	Iranian	Polish	Jordanian	UCSD
Features #	18,19,20	4,8,9,11,12	1,4,5,6,7,9,10,12,13	5,6	-	2,3,4,5,8,10,11	-

APPENDIX B

Below is description of the thresholds assigned to GNG + MARS to each classifier across each dataset.

- **NN**

Dataset\parameter	T_0 (good loan threshold)	T_1 (bad loan threshold)	Mars threshold
German	0.5	0	0.5
Australian	0.5	0	0.01
Japanese	0.5	0.5	0.01
Iranian	0.5	0	0.01
Polish	0.6	0.4	0.5
Jordanian	0.5	0.1	0.5
UCSD	0.7	0.3	0.1

- **SVM**

Dataset\parameter	T_0 (good loan threshold)	T_1 (bad loan threshold)	Mars threshold
German	0.5	0.21	0.4
Australian	0.57	0.37	0.05
Japanese	0.5	0.29	0.01
Iranian	0.4	0	0.1
Polish	0.45	0.5	0.5
Jordanian	0.5	0.1	0.5
UCSD	0.8	0.3	0.01

- **RF**

Dataset\parameter	T_0 (good loan threshold)	T_1 (bad loan threshold)	Mars threshold
German	0.5	0	0.5
Australian	0.55	0.45	0.5
Japanese	0.6	0.1	0.01
Iranian	0.5	0	0.01
Polish	1	0.2	0.1
Jordanian	0.35	0.3	0.3
UCSD	1	0.3	0.01

- **DT**

Dataset\parameter	T_0 (good loan threshold)	T_1 (bad loan threshold)	Mars threshold
German	0.5	0.36	1.5
Australian	0.51	0.36	0.01
Japanese	0.51	0.36	0.1
Iranian	0.5	0.38	0.05
Polish	0.48	0.61	2.8
Jordanian	0.35	0.3	0.5
UCSD	0.7	0.3	0.1

- **NB**

Dataset\parameter	T_0 (good loan threshold)	T_1 (bad loan threshold)	Mars threshold
German	0.5	0.37	1.5
Australian	0.48	0.37	0.7
Japanese	0.5	0.29	0.01
Iranian	0.5	0	0.1
Polish	1	0.65	0.5
Jordanian	0.65	0.1	1
UCSD	0.5	0.3	0.1

APPENDIX C

The parameters used for D-ENS and ConsA for all datasets are illustrated in below tables.

- **D-ENS**

For all datasets the values of the parameters a_l and a_h that are responsible for calculating the weights of classifiers local accuracies are 0.3 and 0.7 respectively.

- **ConsA**

Dataset\parameter	k1	k2	k3	k4	k5
German	1.055407	1.179193	0.732077	2	0.5
Australian	1.36913	0.916599	1.133436	4.465645	0.482215
Japanese	1.25724	1.045834	0.389385	8.540678	0.428231
Iranian	1.523186	0.620097	1.302385	2.218118	0.020043
Polish	1.253332	1.129827	0.968988	3.688108	0.3
Jordanian	0.995434	1.179005	0.517154	2.583696	0.171792
UCSD	1.220799	1.538837	0.706372	1.707539	0.070478