Analysis of energy detection with diversity receivers over non-identically distributed $\kappa-\mu$ shadowed fading channels

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The performance of energy detection (ED) over $\kappa-\mu$ shadowed fading channel is analysed. The analysis is then extended to include the maximal ratio combining (MRC) and square law combining (SLC) schemes with non-identically distributed branches. Moreover, the analysis over $\kappa-\mu$ extreme shadowed fading channel, that is utilised to model the wireless communications scenarios in enclosed areas, is also investigated. To this end, exact closed-form analytic expressions of the average area under the receiver operating characteristics curve (AUC) are derived.

Introduction: Energy detection (ED) technique has been employed to provide spectrum sensing in cognitive radio. This is because the unlicensed user can detect any type of unknown signals of the licensed user with low computational and implementation complexities [1]. Hence, in the technical literature, several efforts have been dedicated to analyse the performance of ED over multipath fading channels. For example, the average probability of detection, P_d, and the probability of false alarm, P_f, over Rayleigh, Nakagami-m and Rician fading channels with maximal ratio combining (MRC) and square law combining (SLC) schemes are derived in [1, 2]. The behaviours of ED over generalised multipath fading channels such us $\eta-\mu$ and $\kappa-\mu$ fading channels that give better fitting to the practical data than the conventional models are given in [3] and [4, 5], respectively.

In addition to the impact of the multipath fading, the wireless signals may be also affected by the shadowing. Thus, several works have been devoted to study the aforementioned scenario. For instance, the performance of ED in K_G fading channels with diversity reception and extended generalised K fading channels with single branch are given in [6] and [7, 8], respectively. The behaviour of ED over $\kappa-\mu$ shadowed fading which is recently proposed by [9] as a composite $\kappa-\mu$/gamma fading is studied in [10-13] by using different approaches for the performance metrics. Although these works are investigated for single diversity receiver, they are included mathematically intractable expressions. Furthermore, all these expressions are included an infinite series which requires convergence for a specific number of terms.

Motivated by the above, this letter analyses the performance of ED in $\kappa-\mu$ shadowed fading using simple tractable closed-form analytic expression of the average area under the receiver operating characteristics curve (AUC). The analysis is then extended to derive the average AUC for the MRC and SLC diversity with independent and non-identically (i.n.d.) branches. To the best of the authors' knowledge, the performance of ED with diversity reception over non-identically distributed $\kappa-\mu$ shadowed fading channels has not been presented in the open technical literature.

Energy detection model: To decide whether the primary user (PU) is present or absent using the ED, the decision statistic, Υ, that is computed via filtering the received signal by an ideal band-pass filter with bandwidth, W, and then squaring and integrating the result over time interval, T, is compared with a threshold value, λ. The distribution of Υ is either central chi-square or non-central chi-square which represent the hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. The former means the received signal is noise only whereas the latter indicates the received signal is the PU's signal and noise. Accordingly, in additive white Gaussian noise (AWGN) [1], $P_d = Pr(\Upsilon > \lambda|\mathcal{H}_1) = Q_u(\sqrt{2\gamma}, \sqrt{\lambda})$ and $P_f = Pr(\Upsilon > \lambda|\mathcal{H}_0) = \Gamma(u, \lambda/2)/\Gamma(u)$ where $u = TW$, $Pr(.)$, stands for the probability, and γ, $\Gamma(.,.)$, $\Gamma(.)$, and $Q_u(.,.)$ are the received instantaneous SNR, the upper incomplete Gamma function, the Gamma function and the uth order generalized Marcum-Q function, respectively.

The AUC over AWGN for the aforementioned ED model can be computed by [5, eq. (9)]

$$A(\gamma) = 1 - \sum_{a=0}^{u-1} \sum_{b=0}^{a} \binom{a+u-1}{a-b} \left(\frac{1}{2}\right)^{a+b+u} \frac{\gamma^b e^{-\frac{\gamma}{2}}}{b!} \qquad (1)$$

where $\binom{y}{x} \triangleq \frac{y!}{(y-x)!}$ is the binomial coefficients.

The $\kappa-\mu$ shadowed fading: In the $\kappa-\mu$ shadowed fading, κ, μ and m stand for the ratio between the total power of the dominant components and the total power of the scattered waves, the number of the multipath clusters and the shadowing severity index, respectively. The probability density function (PDF) of the received instantaneous SNR, γ, over $\kappa-\mu$ shadowed fading is given by [9, eq. (4)]

$$f_\gamma(\gamma) = \frac{1}{\Gamma(\mu)} \left(\frac{m}{\theta}\right)^m \left(\frac{\vartheta}{\bar{\gamma}}\right)^\mu \gamma^{\mu-1} e^{-\frac{\mu(1+\kappa)}{\bar{\gamma}}\gamma} {}_1F_1\left(m; \mu; \frac{\mu\kappa\vartheta}{\theta\bar{\gamma}}\gamma\right) \qquad (2)$$

where $\theta = \mu\kappa + m$, $\vartheta = \mu(1+\kappa)$, $\bar{\gamma}$ is the average SNR and ${}_1F_1(.;.;.)$ is the confluent hypergeometric function [14, eq. (1.3.3), p. 36].

The PDF of the received SNR over i.n.d. $\kappa-\mu$ shadowed fading channels with L MRC diversity branches is given by [9, eq. (7)]

$$f_\gamma^{MRC}(\gamma) = \frac{\Psi}{\Gamma(\Omega)} \gamma^{\Omega-1} \Phi_2^{(2L)}\big(\mu_1 - m_1, ..., \mu_L - m_L, m_1, ..., m_L; \Omega; -\rho_1\gamma, ..., -\rho_L\gamma, -\alpha_1\gamma, ..., -\alpha_L\gamma\big) \qquad (3)$$

where $\Psi = \prod_{i=1}^{L} \frac{\mu_i^{\mu_i} m_i^{m_i} (1+\kappa_i)^{\mu_i}}{(\mu_i\kappa_i + m_i)^{m_i} \bar{\gamma}_i^{\mu_i}}$, $\Omega = \sum_{i=1}^{L} \mu_i$, $\rho_i = \frac{\mu_i(1+\kappa_i)}{\bar{\gamma}_i}$, $\alpha_i = \frac{m_i}{(\mu_i\kappa_i + m_i)}\rho_i$, and $\Phi_2^{(2L)}(.)$ is the confluent multivariate hypergeometric function [14, eq. (1.7.10), p. 62].

ED with single receiver: The average AUC, $\bar{A}$, can be evaluated by [3, eq. (7)]

$$\bar{A} = \int_0^\infty A(\gamma) f_\gamma(\gamma) d\gamma \qquad (4)$$

Substitute (1) and (2) in (4) to yield

$$\bar{A} = 1 - \frac{1}{\Gamma(\mu)} \left(\frac{m}{\theta}\right)^m \left(\frac{\vartheta}{\bar{\gamma}}\right)^\mu \sum_{a=0}^{u-1}\sum_{b=0}^{a} \binom{a+u-1}{a-b} \left(\frac{1}{2}\right)^{a+b+u} \frac{1}{b!} \times \int_0^\infty \gamma^{\mu+b-1} e^{-\frac{2\mu(1+\kappa)+\bar{\gamma}}{2\bar{\gamma}}\gamma} {}_1F_1\left(m; \mu; \frac{\mu\kappa\vartheta}{\theta\bar{\gamma}}\gamma\right) d\gamma \qquad (5)$$

Using $\int_0^\infty f_\gamma(\gamma) d\gamma \triangleq 1$ and [14, 4.1.6, p. 219] to compute the integral in (5). Thus, the following expression is obtained

$$\bar{A} = 1 - \left(\frac{m}{\theta}\right)^m \left(\frac{\vartheta}{\beta}\right)^\mu \sum_{a=0}^{u-1}\sum_{b=0}^{a} \binom{a+u-1}{a-b} \left(\frac{1}{2}\right)^{a+u-\mu} \left(\frac{\bar{\gamma}}{\beta}\right)^b \frac{(\mu)_b}{b!} \times {}_2F_1\left(m, \mu+b; \mu; \frac{2\mu\kappa\vartheta}{\beta\theta}\right) \qquad (6)$$

where $\beta = \bar{\gamma} + 2\mu(1+\kappa)$, $(a)_b$ is the Pochhammer symbol, and ${}_2F_1(.,.;.;.)$ is the Gauss confluent hypergeometric function defined in [14, eq. (1.2.4), p. 29].

It is noted that, when $m \to \infty$ and with some mathematical manipulations, (6) is equivalent to [5, eq. (14)], i.e., $\kappa-\mu$ fading channel. Although the average AUC over $\kappa-\mu$ shadowed fading is derived in [11, eq. (13)], our expression in (6) is simpler and given in closed-form.

ED with MRC receivers: In the MRC, the combining is performed before energy calculation. The average AUC for ED in non-identical $\kappa-\mu$ shadowed fading channels with MRC scheme can be evaluated by plugging (1) and (3) into (4). Thus, we have

$$\bar{A} = 1 - \sum_{a=0}^{u-1}\sum_{b=0}^{a} \binom{a+u-1}{a-b} \left(\frac{1}{2}\right)^{a+b+u} \frac{\Psi}{\Gamma(\Omega) b!} \times \int_0^\infty \gamma^{\Omega+b-1} e^{-\frac{\gamma}{2}} \Phi_2^{(2L)}\big(\mu_1 - m_1, ..., \mu_L - m_L, m_1, ..., m_L; \Omega; -\rho_1\gamma, ..., -\rho_L\gamma, -\alpha_1\gamma, ..., -\alpha_L\gamma\big) d\gamma \qquad (7)$$

With the help of [14, eq. (1.i), p. 259], the integral in (7) can be evaluated. Accordingly, this yields

$$\bar{A} = 1 - \sum_{a=0}^{u-1}\sum_{b=0}^{a} \binom{a+u-1}{a-b} \frac{\Psi}{2^{a+u-\Omega}} \frac{(\Omega)_b}{b!} F_D^{(2L)}\big(b+\Omega; \mu_1 - m_1, ..., \mu_L - m_L, m_1, ..., m_L; \Omega; -2\rho_1\gamma, ..., -2\rho_L\gamma, -2\alpha_1\gamma, ..., -2\alpha_L\gamma\big) \qquad (8)$$

where $F_D^{(2L)}(.)$ is the multivariate Lauricella hypergeometric function presented in [14, eq. (1.7.4), p. 60]. To the best of our knowledge, (8) has not been previously presented in the open technical literature. It can be observed that $F_D^{(2L)}(.)$ is not yet designed as a built-in function in the popular mathematical software packages such as MATLAB and MATHEMATICA. Thus, the highly accurate numerical method that is suggested by [9, Appendix E] is utilised to calculate this function.

ED with SLC receivers: In contrast to the MRC, the SLC is a non coherent combining technique, therefore, it does not require the channel state information (CSI). The principle work of SLC is based on evaluating the energy of the received signal by each branch before the combining process. Hence, the difference between the MRC and SLC is the time-bandwidth product [1]. Consequently, the average AUC for ED with SLC over i.n.d. $\kappa\mu$ shadowed fading channels can be easily evaluated by (8) after replacing u by Lu.

ED in $\kappa-\mu$ extreme shadowed fading: In sometimes, the total power of the dominant components is much higher than the total power of the scattered waves, i.e., $\kappa \to \infty$ and the number of the multipath is too small, i.e., $\mu \to 0$ [15]. In this case, the $\kappa-\mu$ shadowed fading is called the $\kappa-\mu$ extreme shadowed fading which is used to model the wireless communications scenarios in enclosed areas such as buildings. The average AUC for the ED in $\kappa-\mu$ extreme shadowed fading channels can be deduced by inserting $\kappa \to \infty$, $\mu \to 0$ and $\kappa\mu \approx \mathbf{m}$ into (6) and (8) where $\mathbf{m}$ is the fading severity index.

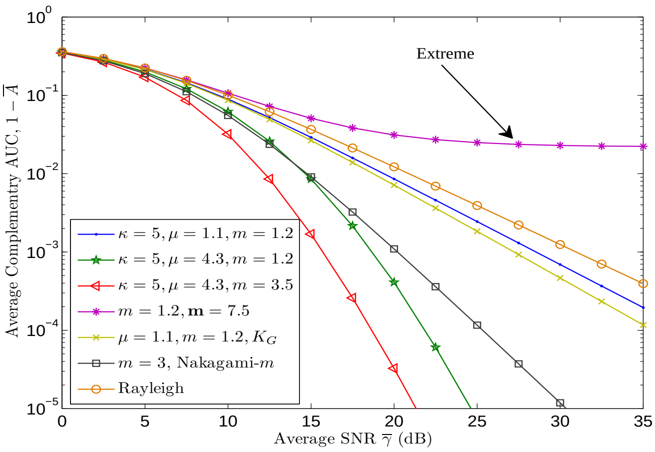

Fig. 1 *Average complementary AUC against average SNR $\bar{\gamma}$ for $u = 2$.*

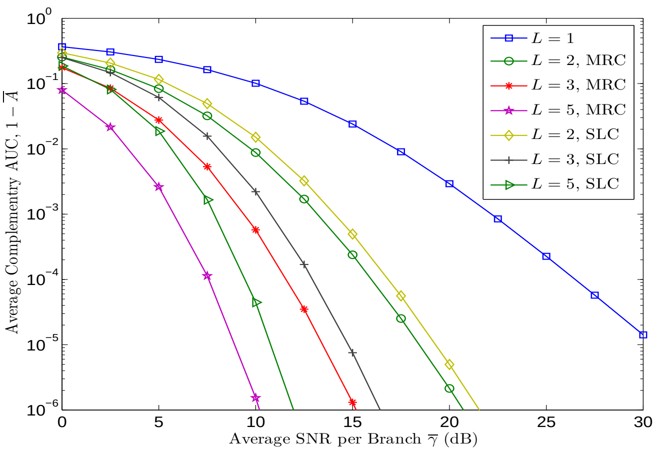

Fig. 2 *Average complementary AUC against average SNR $\bar{\gamma}$ for MRC and SLC schemes with $\kappa_1 = \kappa_2 = \kappa_3 = \kappa_4 = \kappa_5 = 3.6$, $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = 2.5$, $m_1 = 0.5$, $m_2 = 2.4$, $m_3 = 3.9$, $m_4 = 5.4$, $m_5 = 7.1$, and $u = 2$.*

Numerical results: Fig. 1 and Fig. 2 show the average complementary AUC (CAUC) which is expressed by 1-$\bar{A}$ against average SNR for single receiver and diversity combining schemes with i.n.d. branches, respectively. In both figures, the simulation and numerical results are represented by markers and solid lines, respectively. As expected, when μ or/and m increase, the behaviour of ED becomes better. This is because higher μ and m correspond to large number of the multipath clusters and less shadowing effects at the receiver side, respectively. Although the SLC has lower implementation than the MRC, the ED with MRC has better performance than the SLC. This refers to a higher received instantaneous SNR in MRC in comparison with SLC and no diversity.

Conclusion: In this letter, we have analysed the performance of ED over $\kappa-\mu$ shadowed fading channels with no diversity, MRC, and SLC reception schemes. The diversity branches are assumed to be independent but not necessarily identically distributed. To this effect, simple tractable closed-form analytic expressions of the average AUC were derived. From the results, it can be seen that the detection capability improves when μ or/and m increase. The scenario of reaching κ and μ for their extreme values is also explained. The provided results in this paper can be utilised to obtain on wide insight about the behaviour of ED over different composite fading channels with i.n.d. diversity receivers.

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