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On the Sum and the Maximum of Non-identically Distributed Composite $\eta-\mu$/gamma Variates Using a Mixture Gamma Distribution with Applications to Diversity Receivers

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***Abstract*—In this paper, the statistical characterizations of the sum and the maximum of independent and non-identically distributed ($i.n.d$) composite $\eta-\mu$/gamma variates are derived using a mixture gamma (MG) distribution. The statistical properties, namely, probability density function (PDF), cumulative distribution function (CDF), and moment generating function (MGF) are obtained in general unified exact analytic expressions. These statistical results are then used to analyse outage probability (OP), average bit error rate probability (ABEP), and average channel capacity (C) of maximal ratio combining (MRC) and selection combining (SC) schemes over $i.n.d$ composite $\eta-\mu$/gamma fading channels. The validation of our derived expressions is verified by comparing the numerical and simulation results.**

***Index Terms*—Mixture gamma distribution, independent and non-identically distributed, outage probability, average bit error rate probability, average channel capacity.**

I. Introduction

The so-called $\eta-\mu$ distribution has been widely used to model the non-line-of-sight (NLoS) propagation environment [1]-[6]. This is because it gives results closer to the practical measurements than the conventional fading distributions such as Nakagami-m [2]. Moreover, the $\eta-\mu$ distribution includes the most well-known distributions as special cases [1].

Several efforts have been devoted to study the performance of wireless communications systems with diversity receptions over $\eta-\mu$ fading channels. For example, in [3], the average symbol error probability (ASEP) for different digital modulation schemes over independent and non-identically distributed ($i.n.d$) $\eta-\mu$ fading channels with maximal ratio combining (MRC) receivers is derived by using the moment generating function (MGF) approach. The average channel capacity for different transmission policies of MRC over $\eta-\mu$ fading channels with independent but arbitrarily distributed branches is analysed by [4]. Simple closed-form expressions for both the probability density function (PDF) and the cumulative distribution function (CDF) of the sum of squared $\eta-\mu$ random variables (RVs) are presented in [5] and employed to evaluate the outage probability (OP) and the average bit error probability (ABEP) for MRC scheme. On the other hand, the performance of communications systems over $\eta-\mu$ fading channels with selection combining (SC) system has been investigated by very few works. For instance, the ABEP for different modulation formats and average channel capacity for number of transmission scenarios of a SC diversity with $i.n.d$ receivers in $\eta-\mu$ fading channels are derived in [6].

The wireless channels may undergo multipath fading and shadowing simultaneously. Although many studies have been dedicated to analyse the behaviour of communications systems over composite $\eta-\mu$/gamma fading channels [7]-[10], all these efforts are restricted for single diversity receiver. Hence, there is no work dealing with the performance of MRC and SC over composite $\eta-\mu$/gamma fading channels.

Motivated by the above, this paper analyses the performance of MRC and SC over $i.n.d$ composite $\eta-\mu$/gamma fading channels using a mixture gamma (MG) distribution. This distribution is proposed by [11] as approximate general distribution that can be utilised to model a variety of distributions with high accuracy. Although the MG distribution is employed by [12] to derive the ASEP and the average ergodic channel capacity over independent and identically distributed ($i.i.d$) K_G fading channels with MRC and SC, there are still some issues that are not addressed by the aforementioned reference.

The main contributions of this paper that fill the gaps of [12] are summarized as follows:

- This paper provides general exact analytic expressions for the statistical characterizations of the sum and the maximum of MG RVs. These statistical results can be employed for a variety of distributions (e.g., composite $\eta-\mu$/gamma fading) after substituting the equivalent parameters of a MG distribution. On contrary, the derived results in [12] can only be utilised to K_G fading channels.
- The variates of MG are assumed to be independent but not necessarily identically distributed. Accordingly, the performance of MRC and SC with $i.n.d$ diversity receivers can be analysed. In [12], the derived expressions are for the K_G fading channels with $i.i.d$ branches for MRC (please refer to [12, eq. (9)]) and SC (please see [12, eq. (14)]) diversity schemes. Furthermore, one case which is SC with dual branches over $i.n.d$ K_G fading channels is investigated by [12, eq. (19)].
- In contrast to [12] in which the results are restricted by the integer value of fading parameters, i.e., m should be an integer number, our derived statistics are not limited by the values of fading and shadowing parameters.
- In [12], the expressions of the ABEP are given in integral form whereas this work provides them in closed-form.

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II. Statistical Properties of MG Distribution

The PDF of the instantaneous SNR γ, $f_\gamma(\gamma)$, using the MG distribution is expressed by [11, eq. (1)]

$$f_\gamma(\gamma) = \sum_{i=1}^{N} \alpha_i \gamma^{\beta_i - 1} e^{-\zeta_i \gamma}. \tag{1}$$

where N and α_i, β_i and ζ_i are the number of terms and the parameters of ith gamma component, respectively. The main problem in utilising the MG distribution is how to determine N. In [11], some methods are proposed to compute a minimum N that achieves good approximation with high accuracy. One of these methods is based on evaluating the mean square error (MSE) between the PDF of the exact distribution and the PDF of the approximate distribution using the MG distribution.

The CDF of the MG distribution is given by [11, eq. (2)]

$$F_\gamma(\gamma) = \sum_{i=1}^{N} \alpha_i \zeta_i^{-\beta_i} \mathcal{G}(\beta_i, \zeta_i \gamma). \tag{2}$$

where $\mathcal{G}(.,.)$ is the incomplete lower gamma function that is evaluated by $\mathcal{G}(a,b) = \int_0^b x^{a-1} e^{-x} dx$.

The MGF of the MG distribution, $\mathcal{M}_\gamma(s)$, which is the Laplace transform of $f_\gamma(\gamma)$, i.e., $\mathcal{M}_\gamma(s) = \mathcal{L}[f_\gamma(\gamma); s] = \int_0^\infty e^{-s\gamma} f_\gamma(\gamma) d\gamma$ is expressed by [11, eq. (3)]

$$\mathcal{M}_\gamma(s) = \sum_{i=1}^{N} \frac{\alpha_i \Gamma(\beta_i)}{(s+\zeta_i)^{\beta_i}}. \tag{3}$$

where $\Gamma(x) = \int_0^\infty z^{x-1} e^{-z} dz$ is the gamma function.

In this paper, the validation of our analysis is checked by using $\eta-\mu$ fading channel shadowed by gamma distribution. In $\eta-\mu$ fading [2], μ represents the real extension of the number of multipath clusters whereas the definition of η depends on the type of format. In format 1, η represents the power ratio between the in-phase and quadrature scattered components in each multipath cluster with $0 < \eta < \infty$. The respective H and h are expressed by $H = (\eta^{-1} - \eta)/4$ and $h = (2 + \eta^{-1} + \eta)/4$, respectively. In format 2, η stands for the correlation coefficient between the in-phase and quadrature scattered components in each multipath cluster with $-1 < \eta < 1$. The respective H and h are given by $H = \eta/(1-\eta^2)$ and $h = 1/(1-\eta^2)$, respectively.

The PDF of γ over composite $\eta-\mu$/gamma fading channel can be evaluated by integrating the $\eta-\mu$ fading channel [2, eq. (26)] over gamma distribution as follows [7, eq. (6)],

$$f_\gamma(\gamma) = \frac{2\sqrt{\pi} h^\mu \mu^{\mu+\frac{1}{2}} \gamma^{\mu-\frac{1}{2}}}{\Gamma(\mu)\Gamma(k)\Omega^k H^{\mu-\frac{1}{2}}} \times \int_0^\infty y^{k-\mu-\frac{3}{2}} e^{-\frac{2\mu h\gamma}{y} - \frac{y}{\Omega}} I_{\mu-\frac{1}{2}}\left(\frac{2\mu H \gamma}{y}\right) dy. \tag{4}$$

where k, Ω, and $I_a(.)$ are the shaping parameter, the mean power and the modified Bessel function of the first kind and ath order [13, pp. 919, eq. (8.445)], respectively.

By substituting $x = \frac{2\mu h\gamma}{y}$ in (4), this yields

$$f_\gamma(\gamma) = \frac{\sqrt{\pi} 2^{k-\mu+\frac{1}{2}} h^{k-\frac{1}{2}} \left(\frac{\mu}{\Omega}\right)^k \gamma^{k-1}}{\Gamma(\mu)\Gamma(k) H^{\mu-\frac{1}{2}}} \int_0^\infty e^{-x} g(x) dx. \tag{5}$$

where $g(x) = x^{\mu-k-\frac{1}{2}} e^{-\frac{2\mu h\gamma}{\Omega x}} I_{\mu-\frac{1}{2}}\left(\frac{H}{h} x\right)$. The integration in (5), $\mathcal{S} = \int_0^\infty e^{-x} g(x) dx$, can be approximated as a Gaussian-Laguerre quadrature sum as $\mathcal{S} \approx \sum_{i=1}^N w_i g(x_i)$ where x_i and w_i are the abscissas and weight factors for the Gaussian-Laguerre integration [14]. Therefore, (5) can be expressed by the MG distribution with parameters

$$\alpha_i = \frac{\theta_i}{\sum_{l=1}^N \theta_l \Gamma(\beta_l) \zeta_l^{-\beta_l}}, \quad \beta_i = k, \quad \zeta_i = \frac{2\mu h}{\Omega x_i},$$
$$\theta_i = \frac{\sqrt{\pi} 2^{k-\mu+\frac{1}{2}} h^{k-\frac{1}{2}} \left(\frac{\mu}{\Omega}\right)^k}{\Gamma(\mu)\Gamma(k) H^{\mu-\frac{1}{2}}} w_i x_i^{\mu-k-\frac{1}{2}} I_{\mu-\frac{1}{2}}\left(\frac{H}{h} x_i\right). \tag{6}$$

III. Statistical Characterizations of MG Distribution with I.N.D Random Variables

A. *Statistical Characterizations of the Sum of $i.n.d$ MG Random Variables*

The statistical characterizations of the sum of SNRs in a fading channel are very important in analysing the performance of wireless communications systems with MRC scheme.

Assume γ_j be a MG variate with average $\bar{\gamma}_j$ and shaping parameters α_{i_j}, β_{i_j} and ζ_{i_j} for $j = 1, \cdots, L$, where L is the number of independent but arbitrarily distributed RVs. The MGF of $\gamma^{Sum} = \sum_{j=1}^L \gamma_j$, $\mathcal{M}_{\gamma^{Sum}}(s)$, can be evaluated by

$$\mathcal{M}_{\gamma^{Sum}}(s) = \prod_{j=1}^L \mathcal{M}_{\gamma_j}(s) = \prod_{j=1}^L \left[\sum_{i_j=1}^{N_j} \frac{\alpha_{i_j}\Gamma(\beta_{i_j})}{(s+\zeta_{i_j})^{\beta_{i_j}}}\right]. \tag{7}$$

The right hand side expression in (7) can be rewritten in multiple summations as follows

$$\mathcal{M}_{\gamma^{Sum}}(s) = \sum_{i_1=1}^{N_1} \cdots \sum_{i_L=1}^{N_L} \prod_{j=1}^{L} \frac{\alpha_{i_j}\Gamma(\beta_{i_j})}{(s+\zeta_{i_j})^{\beta_{i_j}}}. \tag{8}$$

By plugging (8) into $f_{\gamma^{Sum}}(\gamma) = \mathcal{L}^{-1}\left[\mathcal{M}_{\gamma^{Sum}}(s); \gamma\right]$[1] where $f_{\gamma^{Sum}}(\gamma)$ is the PDF of γ^{Sum}, this yields

$$f_{\gamma^{Sum}}(\gamma) = \sum_{i_1=1}^{N_1} \cdots \sum_{i_L=1}^{N_L} \left(\prod_{j=1}^{L} \alpha_{i_j}\Gamma(\beta_{i_j})\right)$$
$$\mathcal{L}^{-1}\left[(s+\zeta_{i_1})^{-\beta_{i_1}}(s+\zeta_{i_2})^{-\beta_{i_2}} \cdots (s+\zeta_{i_L})^{-\beta_{i_L}}; \gamma\right]. \tag{9}$$

The inverse Laplace transform in (9) can be computed by [15, pp. 290, eq. (9.4.55)]. Thus, we have

$$f_{\gamma^{Sum}}(\gamma) = \sum_{i_1=1}^{N_1} \cdots \sum_{i_L=1}^{N_L} \left(\prod_{j=1}^{L} \alpha_{i_j}\Gamma(\beta_{i_j})\right) \frac{\gamma^{\Xi-1}}{\Gamma(\Xi)} \times \Phi_2^{(L)}\left(\beta_{i_1}, \cdots, \beta_{i_L}; \Xi; -\zeta_{i_1}\gamma, \cdots, -\zeta_{i_L}\gamma\right). \tag{10}$$

where $\Xi = \sum_{j=1}^L \beta_{i_j}$ and $\Phi_2^{(L)}(.)$ is the confluent multivariate hypergeometric function defined in [15, pp. 34, eq. (1.4.8)]. One can see that $\Phi_2^{(L)}(.)$ is not yet available in Matlab and Mathematica software packages. However, an efficient method to calculate this function is given in [16, Appendix V].

The CDF of γ^{Sum} can be computed by using $F_{\gamma^{Sum}}(\gamma) = \mathcal{L}^{-1}\left[\mathcal{M}_{\gamma^{Sum}}(s)/s; \gamma\right]$ and [15, pp. 290, eq. (9.4.55)]. Accord-

[1]Here $\mathcal{L}^{-1}[.]$ stands for inverse Laplace transform operation.

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ingly, this yields

$$F_{\gamma^{sum}}(\gamma) = \sum_{i_1=1}^{N_1}\cdots\sum_{i_L=1}^{N_L}\left(\prod_{j=1}^{L}\alpha_{i_j}\Gamma(\beta_{i_j})\right)\frac{\gamma^{\Xi}}{\Gamma(1+\Xi)} \times\Phi_2^{(L)}\big(\beta_{i_1},\cdots,\beta_{i_L};1+\Xi;-\zeta_{i_1}\gamma,\cdots,-\zeta_{i_L}\gamma\big). \quad (11)$$

B. Statistical Characterizations of the Maximum of $i.n.d$ MG Random Variables

The statistical characterizations of the maximum of SNRs in a fading channel are utilized in studying the behaviour of wireless communications systems with SC technique.

By using the same assumptions in III.A, the CDF of $\gamma^{Max} = \max\{\gamma_1,\gamma_2,\cdots,\gamma_L\}$ of independent but not necessarily identically distributed MG variates, $F_{\gamma^{Max}}(\gamma)$, is given by

$$F_{\gamma^{Max}}(\gamma) = \prod_{j=1}^{L}F_{\gamma_j}(\gamma) = \prod_{j=1}^{L}\left[\sum_{i_j=1}^{N_j}\alpha_{i_j}\zeta_{i_j}^{-\beta_{i_j}}\mathcal{G}(\beta_{i_j},\zeta_{i_j}\gamma)\right]. \quad (12)$$

Using [14, pp. 262, eq. (6.5.12)] with some mathematical manipulations, $F_{\gamma^{Max}}(\gamma)$ in (12) becomes

$$F_{\gamma^{Max}}(\gamma) = \sum_{i_1=1}^{N_1}\cdots\sum_{i_L=1}^{N_L}\left(\prod_{j=1}^{L}\frac{\alpha_{i_j}}{\beta_{i_j}}\right) \times\left(\prod_{j=1}^{L}\gamma^{\beta_{i_j}}\,{}_1F_1(\beta_{i_j};1+\beta_{i_j};-\zeta_{i_j}\gamma)\right). \quad (13)$$

The MGF of γ^{Max}, $\mathcal{M}_{\gamma^{Max}}(s)$, can be calculated by invoking $\mathcal{M}_{\gamma^{Max}}(s) = s\mathcal{L}[F_{\gamma^{Max}}(\gamma);s]$ with the aid of [15, pp. 285, eq. (9.4.35)]. Accordingly, the desired result is

$$\mathcal{M}_{\gamma^{Max}}(s) = \sum_{i_1=1}^{N_1}...\sum_{i_L=1}^{N_L}\left(\prod_{j=1}^{L}\frac{\alpha_{i_j}}{\beta_{i_j}}\right)\frac{\Gamma(1+\Xi)}{s^{\Xi}}F_A^{(L)}\Big(1+\Xi; \beta_{i_1},\cdots,\beta_{i_L};1+\beta_{i_1},\cdots,1+\beta_{i_L};-\frac{\zeta_{i_1}}{s},\cdots,-\frac{\zeta_{i_L}}{s}\Big). \quad (14)$$

where $F_A^{(L)}(.)$ is the multivariate Lauricella hypergeometric function defined in [15, pp. 33, eq. (1.4.1)]. It can be noted that $F_A^{(L)}(.)$ is not yet implemented in Matlab and Mathematica software packages. Thus, the accurate method that is proposed by [17] is used to compute this function in this paper.

Using $f_{\gamma^{Max}}(\gamma) = \mathcal{L}^{-1}\big[\mathcal{M}_{\gamma^{Max}}(s);\gamma\big]$ and [15, pp. 33, eq. (1.4.1)] with the help of the identity $\mathcal{L}^{-1}\big[1/s^{1+\nu};\gamma\big] = \gamma^{\nu}/\Gamma(1+\nu)$ [12, eq. (12)], the following exact expression of the PDF of γ^{Max}, $f_{\gamma^{Max}}(\gamma)$, is obtained

$$f_{\gamma^{Max}}(\gamma) = \sum_{i_1=1}^{N_1}\cdots\sum_{i_L=1}^{N_L}\left(\prod_{j=1}^{L}\frac{\alpha_{i_j}}{\beta_{i_j}}\right)\frac{\Xi}{\gamma^{1-\Xi}} F^{\overbrace{1:1;\cdots;1}^{L}}_{1:1;\cdots;1}\left[\begin{matrix}1+\Xi: & \beta_{i_1} & ;...; & \beta_{i_L} & ; \\ \Xi & :1+\beta_{i_1} & ;...; & 1+\beta_{i_L}; \end{matrix} \; -\zeta_{i_1}\gamma,...,-\zeta_{i_L}\gamma\right]. \quad (15)$$

where $F^{1:1;\cdots;1}_{1:1;\cdots;1}[.]$ is the Kampé de Fériet function defined in [15, pp. 38, eq. (1.4.24)].

IV. Performance Analysis of Diversity Reception Using MG Channel Model

A. Outage Probability

The outage probability, P_o, that is defined as the probability of crossing the output SNR, γ, for a certain predefined threshold, Υ, can be evaluated as $P_o = F_{\gamma}(\Upsilon)$ [18].

1) MRC: In MRC reception, each diversity branch is weighted via multiplying it by a factor. This factor is relative to the complex fading coefficient of the branch. Thus, the instantaneous SNR at the output of the MRC combiner is $\gamma^{MRC} = \sum_{j=1}^{L}\gamma_j$ [18] where L and γ_j are the total number of diversity branches and the instantaneous SNR at the ith branch, respectively. Accordingly, the outage probability for MRC can be calculated by (11).

The asymptotic expression of the P_o for MRC, $P_{o_{MRC}}$, when $\gamma_j \to 0$ for all $j = 1,\cdots,L$ can be computed by using $\Phi_2^{(L)}(\beta_{i_1},\cdots,\beta_{i_L};1+\Xi;0,\cdots,0) = 1$ [16] as follows

$$P_{o_{MRC}} \sim \sum_{i_1=1}^{N_1}\cdots\sum_{i_L=1}^{N_L}\left(\prod_{j=1}^{L}\alpha_{i_j}\Gamma(\beta_{i_j})\right)\frac{\Upsilon^{\Xi}}{\Gamma(1+\Xi)}. \quad (16)$$

2) SC: In SC, the receiver with maximum SNR among all diversity receivers is chosen by the combiner. Hence, the instantaneous SNR at the output of the SC combiner is given by $\gamma^{SC} = \max\{\gamma_1,\gamma_2,\cdots,\gamma_L\}$ [18]. Consequently, the outage probability for SC reception can be evaluated by (12).

The asymptotic behaviour of the $P_{o_{SC}}$ can be deduced from (13) with the aid of ${}_1F_1(\beta_{i_j};1+\beta_{i_j};0) = 1$, as follows

$$P_{o_{SC}} \sim \sum_{i_1=1}^{N_1}\cdots\sum_{i_L=1}^{N_L}\prod_{j=1}^{L}\frac{\alpha_{i_j}}{\beta_{i_j}}\Upsilon^{\beta_{i_j}}. \quad (17)$$

B. Average Bit Error Probability

The ABEP, P_e, can be expressed as [19, eq. (12)],

$$P_e = \frac{q^p}{2\Gamma(p)}\int_0^{\infty}\gamma^{p-1}e^{-q\gamma}F_{\gamma}(\gamma)d\gamma. \quad (18)$$

The parameters p and q represent the modulation dependent constants. Specifically, p = 0.5 and q = 1 for coherent binary phase-shift keying (BPSK).

1) MRC: The ABEP for MRC, $P_{e_{MRC}}$, can be evaluated by substituting (11) into (18) to give

$$P_{e_{MRC}} = \frac{q^p}{2\Gamma(p)}\sum_{i_1=1}^{N_1}\cdots\sum_{i_L=1}^{N_L}\frac{\big(\prod_{j=1}^{L}\alpha_{i_j}\Gamma(\beta_{i_j})\big)}{\Gamma\big(1+\Xi\big)} \times\int_0^{\infty}\gamma^{p+\Xi-1}e^{-q\gamma}\Phi_2^{(L)}\Big(\beta_{i_1},\cdots,\beta_{i_L};1+\Xi; -\zeta_{i_1}\gamma,\cdots,-\zeta_{i_L}\gamma\Big)d\gamma. \quad (19)$$

The integral in (19) is available in [15, pp. 286, eq. (9.4.43)]. Thus, after some mathematical manipulations, this yields

$$P_{e_{MRC}} = \frac{1}{2\Gamma(p)}\sum_{i_1=1}^{N_1}\cdots\sum_{i_L=1}^{N_L}\frac{\big(\prod_{j=1}^{L}\alpha_{i_j}\Gamma(\beta_{i_j})\big)\Gamma\big(p+\Xi\big)}{\Gamma\big(1+\Xi\big)q^{\Xi}} \times F_D^{(L)}\Big(p+\Xi;\beta_{i_1},\cdots,\beta_{i_L};1+\Xi;-\frac{\zeta_{i_1}}{q},\cdots,-\frac{\zeta_{i_L}}{q}\Big). \quad (20)$$

where $F_D^{(L)}(.)$ is another model of the multivariate Lauricella hypergeometric function defined in [15, pp. 33, eq. (1.4.4)]. Although $F_D^{(L)}(.)$ is not yet performed in Matlab and Mathematica software packages, it can be accurately evaluated by doing the same steps that are given in [16, Appendix V].

By following a similar procedure for [16, eq. (22)], the asymptotic behaviour of $P_{e_{MRC}}$ is given by

$$P_{e_{MRC}} \sim \frac{1}{2\Gamma(p)} \sum_{i_1=1}^{N_1} \cdots \sum_{i_L=1}^{N_L} \frac{\left(\prod_{j=1}^{L} \alpha_{i_j} \Gamma(\beta_{i_j})\right)\Gamma(p+\Xi)}{\Gamma(1+\Xi) q^{\Xi}}. \tag{21}$$

2) SC: The ABEP of SC, $P_{e_{SC}}$, can be calculated by plugging (13) in (18) as follows

$$P_{e_{SC}} = \frac{q^p}{2\Gamma(p)} \sum_{i_1=1}^{N_1} \cdots \sum_{i_L=1}^{N_L} \left(\prod_{j=1}^{L} \frac{\alpha_{i_j}}{\beta_{i_j}}\right) \times \int_0^\infty \gamma^{p+\Xi-1} e^{-q\gamma} \prod_{j=1}^{L} {}_1F_1(\beta_{i_j}; 1+\beta_{i_j}; -\zeta_{i_j}\gamma) d\gamma. \tag{22}$$

Using [15, pp. 285, eq. (9.4.35)] to evaluate (22), this yields

$$P_{e_{SC}} = \frac{1}{2\Gamma(p)} \sum_{i_1=1}^{N_1} \ldots \sum_{i_L=1}^{N_L} \left(\prod_{j=1}^{L} \frac{\alpha_{i_j}}{\beta_{i_j}}\right) \frac{\Gamma(p+\Xi)}{q^{\Xi}} F_A^{(L)}\Big(p+\Xi; \beta_{i_1}, \cdots, \beta_{i_L}; 1+\beta_{i_1}, \cdots, 1+\beta_{i_L}; -\frac{\zeta_{i_1}}{q}, \cdots, -\frac{\zeta_{i_L}}{q}\Big). \tag{23}$$

With the aid of the identity $F_A^{(L)}(p+\Xi; \beta_{i_1}, \cdots, \beta_{i_L}; 1+\beta_{i_1}, \cdots, 1+\beta_{i_L}; 0, \cdots, 0) = 1$, the asymptotic expression for $P_{e_{SC}}$ is given by

$$P_{e_{SC}} \sim \frac{1}{2\Gamma(p)} \sum_{i_1=1}^{N_1} \cdots \sum_{i_L=1}^{N_L} \left(\prod_{j=1}^{L} \frac{\alpha_{i_j}}{\beta_{i_j}}\right) \frac{\Gamma(p+\Xi)}{q^{\Xi}}. \tag{24}$$

C. Average Channel Capacity

According to Shannon's theorem, the average ergodic channel capacity, C, is given by [18]

$$C = B \int_0^\infty \log_2(1+\gamma) f_\gamma(\gamma) d\gamma. \tag{25}$$

where B is the channel bandwidth.

It is observed that (25) can not be utilised in evaluating C for both the MRC and the SC diversity receptions. This is because the expressions of the PDFs in (10) and (15) are not included an exponential function that gives the possibility to calculate the integral in (25) analytically. Therefore, in this paper, we employ an approximation method in computing C.

It can be noted that C can be expressed as [20, eq. (8)],

$$C = \frac{B}{\ln 2} \int_0^\infty \frac{1-\mathcal{M}_\gamma(z)}{z} e^{-z} dz. \tag{26}$$

The integral in (26) can be approximately calculated by using the Gauss Legendre quadrature approach [14] as follows

$$C \approx \frac{B}{\ln 2} \sum_{l=1}^{M} w_l \frac{1-\mathcal{M}_\gamma(x_l)}{x_l}. \tag{27}$$

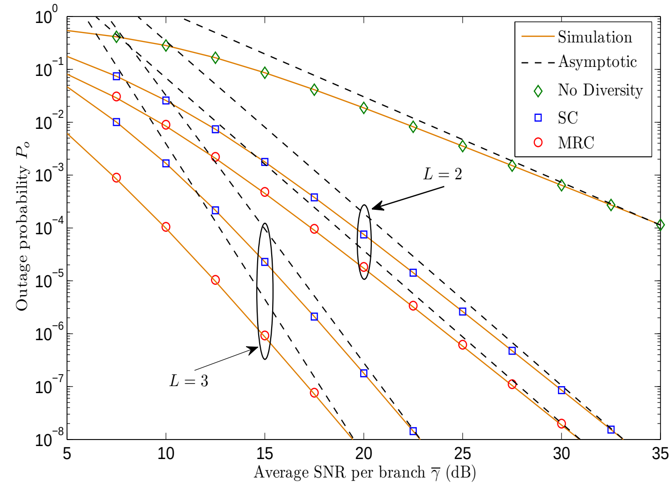

Fig. 1. OP comparison between no-diversity, MRC and SC schemes against $\bar{\gamma}$ with Υ = 5 dB.

V. Numerical and Simulation Results

In this section, the validation of our derived expressions is verified by the numerical and simulation results over $i.n.d$ composite $\eta-\mu$/gamma fading channels[2]. These results are given for single, dual and triple diversity branches. Monte Carlo simulations with 10^6 iterations are utilized to compare with the numerical results. In all figures, the simulation results are represented by solid lines whereas the numerical results and the asymptotic behaviours are shown by marks and dashed lines, respectively. The simulation parameters for each branch are $(\eta_1, \mu_1, k_1, N_1) = (0.1, 0.5, 1.5, 8)$, $(\eta_2, \mu_2, k_2, N_2) = (0.3, 1.5, 1.5, 11)$, and $(\eta_3, \mu_3, k_3, N_3) = (0.9, 2.5, 1.5, 14)$. The number of terms for each branch, namely, N_1, N_2, and N_3, is chosen to achieve MSE $\leq 10^{-6}$ between the exact PDF and the approximate PDF using the MG distribution.

Fig. 1 shows the OP for single, MRC and SC diversity receptions with Υ = 5 dB over $i.n.d$ composite $\eta-\mu$/gamma fading channels. As expected, the OP of the MRC is less than the OP of the SC and no-diversity cases. This is because the total received SNR in MRC scheme is based on summing of the SNR of all branches while in SC, the total received SNR is the largest SNR among all receivers. For example, in Fig. 1, when $\bar{\gamma} = 15$ dB (fixed), the P_o of the MRC with $L = 2$ is nearly 73% and 99% lower than the SC with $L = 2$ and $L = 1$, respectively. Furthermore, one can see that the value of the OP reduces when the number of diversity branches increases.

Fig. 2 illustrates the ABEP for single, MRC and SC diversity receptions with BPSK modulation over $i.n.d$ composite $\eta-\mu$/gamma fading channels. This figure confirms our results that are presented in Fig. 1. For instance, in Fig. 2 at $\bar{\gamma} = 10$ dB (fixed), the ABEP for the MRC with double branches is approximately 48% and 95% less than the corresponding case of the SC and $L = 1$, respectively.

[2]In this paper, Format 1 of the $\eta-\mu$ is employed. The results for Format 2 can be easily obtained from Format 1 by applying the bilinear transformation as described in [2].

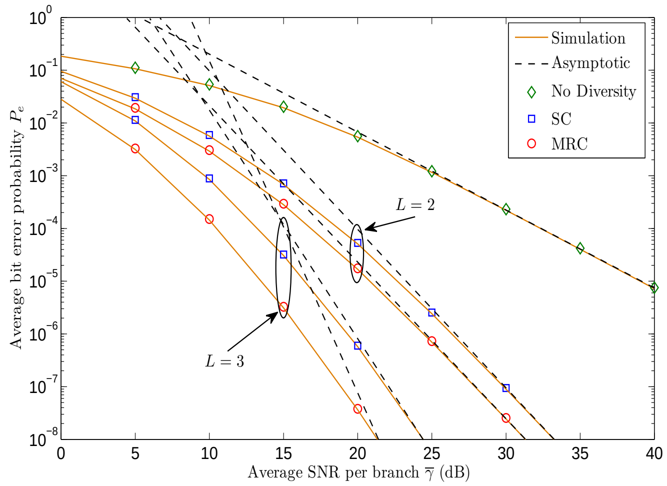

Fig. 2. ABEP comparison between no-diversity, MRC and SC schemes against $\bar{\gamma}$ with BPSK.

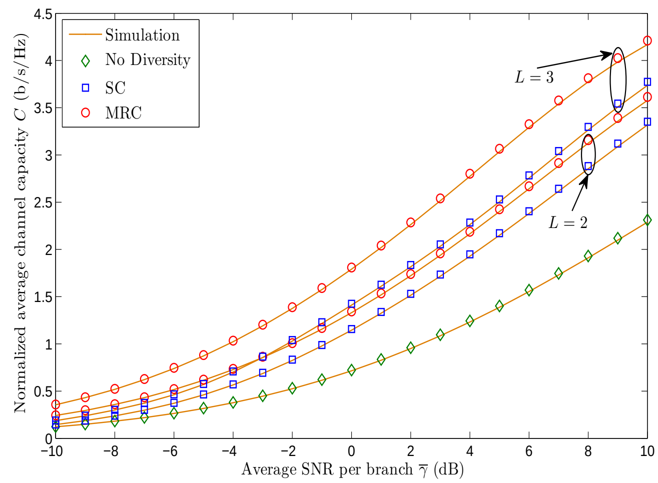

Fig. 3. Average channel capacity comparison between no-diversity, MRC and SC schemes against $\bar{\gamma}$.

Fig. 3 explains the normalized average channel capacity for single, MRC and SC diversity receptions over $i.n.d$ composite $\eta-\mu$/gamma fading channels. The number of terms, M, in (27) is chosen to be 15. The results in this figure show the superiority of the MRC on the SC and no-diversity schemes and for the same reasons that are mentioned for Fig. 1. In Fig. 3, when $\bar{\gamma}=0$ dB (fixed), the C for the MRC with L = 3 is roughly 1.805 b/s/Hz whereas the C for the SC and L = 1 are nearly 1.433 b/s/Hz and 0.6762 b/s/Hz, respectively.

In all provided figures, it is clear that the numerical results match well with their Monte Carlo simulation counterparts, proving the high accuracy of the analysis using the MG distribution.

VI. Conclusions

In this paper, a MG distribution was employed to derive the statistical properties of the sum and the maximum of $i.n.d$ composite $\eta-\mu$/gamma variates in general unified exact analytic expressions. These expressions were used to analyse the performance of MRC and SC over composite $\eta-\mu$/gamma fading channels with independent but not necessarily identically distributed branches. In particular, the outage probability, the average bit error probability and the average channel capacity of MRC and SC schemes were derived. The results of this paper can be employed to study the behaviour of MRC and SC diversity receivers over a variety of composite fading channels that have mathematically intractable statistics.

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