SMOOTHED PARTICLE HYDRODYNAMICS MODELLING OF DYNAMIC FRACTURE AND FRAGMENTATION PROBLEMS

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Outline

- SPH Formulations
- Constitutive Model
- Modelling Failure in SPH
- Applications
  - > Electro-Magnetically driven rings
  - > Mock Holt Experiments
  - > Explosively driven cylinder with end cap
- Conclusion and Future Work

Introduction

To study dynamic fracture and fragmentation problems a numerical method must be capable of dealing with:

- Crack formation and propagation.
- Crack branching and crack joining, leading to fragmentation.
- Large Deformations.

A meshless method such as the SPH method has the potential to do this

SPH Formulations

Based on convolution integral

Approximation (

$$\langle F(x_i) \rangle = \int F(x) W(|x_i - x|, h) dV$$

Approximatic

$$\langle F(x_i) \rangle \approx \sum F(x_j) W(|x_i - x_j|, h) \frac{m_j}{\rho_j}$$

$$\langle \nabla F(x_i) \rangle \approx \sum F(x_j) \nabla W(|x_i - x_j|, h) \frac{m_j}{\rho_j}$$

SPH Formulations

Normalised kernels to ensure 0-th order consistency

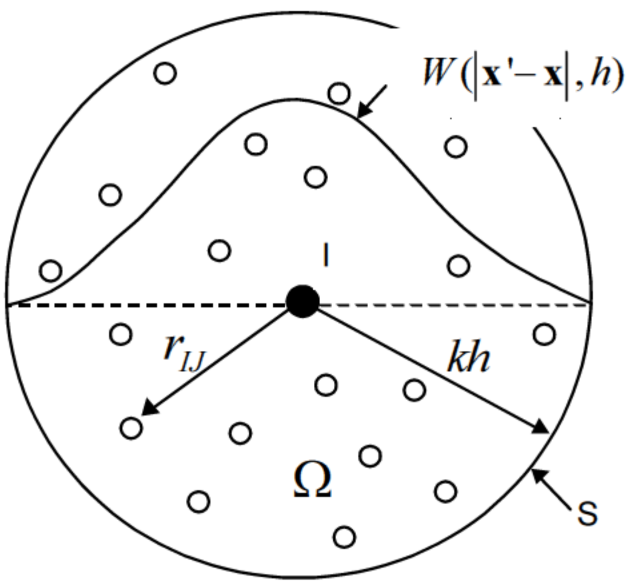
$$W\left(\left|x_i - x_j\right|, h\right) = \frac{\tilde{W}\left(\left|x_i - x_j\right|, h\right)}{\sum_{j=1}^{np} \frac{m_j}{\rho_j} \tilde{W}\left(\left|x_i - x_j\right|, h\right)}$$

Higher order possible when using MLS interpolation

Eulerian Kernel SPH

Eulerian kernel formulation

- Evaluated in current configuration (particle positions)
- Particles can move within, and in and out of kernel



Eulerian Kernel SPH

Vignjevic, Campbell, Jaric, Powell (2009) Comp Methods Appl Mech Eng, 198, 2403-2411

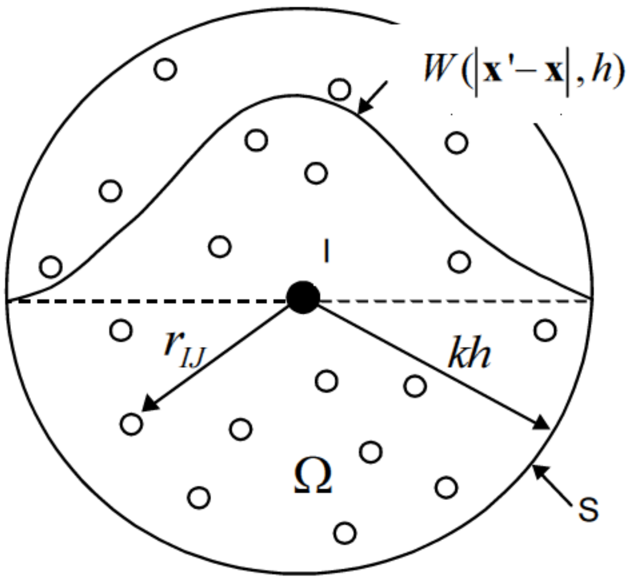
Conservation Equations	
Continuous	Discretised (Eulerian SPH, moving reference frame)
$\dot{\rho} = -\rho \nabla \cdot v$	$\langle \dot{\rho}_i \rangle = -\rho_i \sum_{j=1}^{np} \frac{m_j}{\rho_j} (v_i - v_j) \cdot \nabla_{x_i} W(\lvert x_i - x_j \rvert, h)$
$\rho \ddot{u} = \nabla \cdot \sigma + b$	$\langle \ddot{u}_i \rangle = \sum_{j=1}^{np} \frac{m_j}{\rho_j} (v_j - v_i)(v_R \cdot \nabla_{x_i} W(\lvert x_i - x_j \rvert, h)) - \sum_{j=1}^{np} m_j \left(\frac{\sigma_i}{\rho_i^2} + \frac{\sigma_j}{\rho_j^2} \right) \nabla_{x_i} W(\lvert x_i - x_j \rvert, h) + b$
$\rho \dot{e} = -\sigma : \dot{\varepsilon}$	$\langle \dot{e}_i \rangle = \frac{\sigma_i}{\rho_i} : \sum_{j=1}^{np} \frac{m_j}{\rho_j} (v_i - v_j) \nabla_{x_i} W(\lvert x_i - x_j \rvert, h)$

Total Lagrangian Kernel SPH

Total Lagrangian kernel formulation

> Gradients evaluated using initial particle positions

> Neighbourhood is fixed



Total Lagrangian Kernel SPH

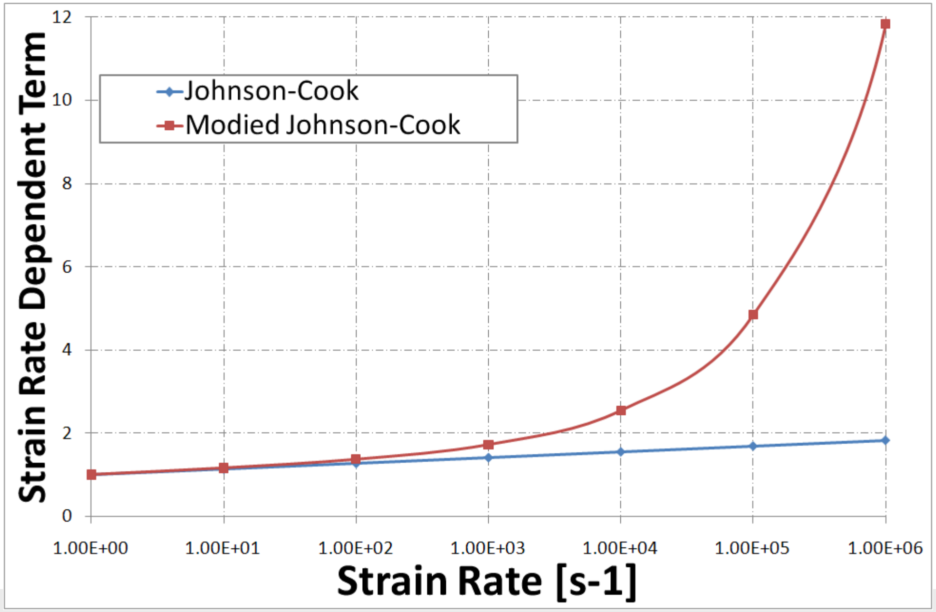
Vignjevic, Reveles, Campbell (2006) Comput Model Eng Sci, 14 (3), 181-198

Conservation Equations	
Continuous	Discretised (TL SPH, reference configuration)
$\rho J = \rho^0$	$\rho = J^{-1}\rho^0$
$\rho^0 \ddot{u} = \nabla_0 \cdot P + b$	$\langle \ddot{u}_i \rangle = -\sum_{j=1}^{np} m_j \left(\frac{P_i}{\rho_i^{0\,2}} + \frac{P_j}{\rho_j^{0\,2}} \right) \nabla_{x_i^0} W \left(\left\| x_i^0 - x_j^0 \right\|, h^0 \right) + b$
$\rho^0 \dot{e} = P : \dot{F}$	$\langle \dot{e}_i \rangle = \frac{P_i}{\rho_i} : \sum_{j=1}^{np} \frac{m_j}{\rho_j^0} \left(v_i - v_j \right) \nabla_{x_i^0} W \left(\left\| x_i^0 - x_j^0 \right\|, h^0 \right)$

Constitutive Model

Modified Johnson-Cook (with equation of state)

$$\sigma_Y = \left(A + B\bar{\varepsilon}_{pl}^{n}\right)\left(1 + \ln\left(\frac{\dot{\bar{\varepsilon}}}{\dot{\bar{\varepsilon}}_0}\right) + \left(\frac{\dot{\bar{\varepsilon}}}{D}\right)^{E}\right)\left(1 - T^{*m}\right)$$



Constitutive Model

Lemaitre Damage Model:

$$\dot{D} = \left(-\frac{Y}{S}\right)^{t} \dot{\bar{\varepsilon}}_{pl} \quad \text{if} \quad \bar{\varepsilon}_{pl} \geq \bar{\varepsilon}_{threshold}$$
$$\dot{D} = 0 \quad \text{if} \quad \bar{\varepsilon}_{pl} < \bar{\varepsilon}_{threshold}$$

with

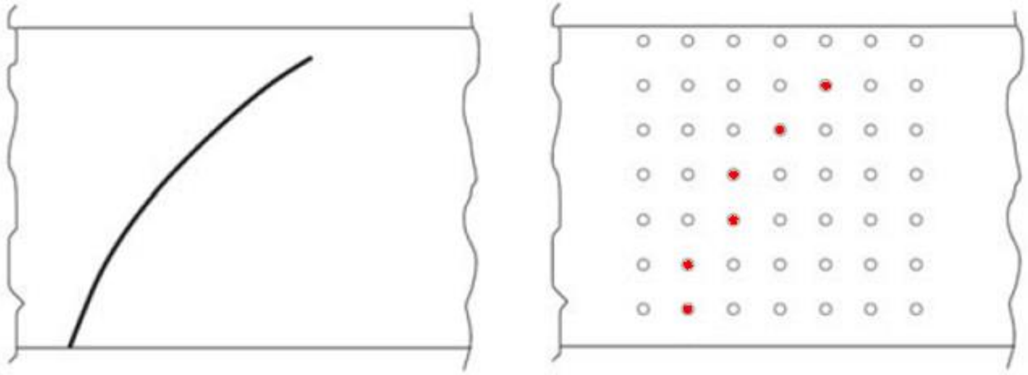
$$-Y = \frac{\sigma_{eq}^2}{2E\left(1-D\right)^2}\left(\frac{2}{3}\left(1+\nu\right)+3\left(1-2\nu\right)\left(\frac{-p}{\sigma_{eq}}\right)^2\right)$$

$$0 < D_c \leq 1$$

Modelling Failure

Randles P.W., Libersky L.D., Smoothed Particle Hydrodynamics: Some recent improvements and applications. Comput. Methods Appl. Mech. Engrg. 139, 1996.

'Stress to zero concept' – Upon failure a particle will be prevented from transferring tensile loads by setting its stress to zero. Equivalent to basic failure models implemented in FE codes, but mass and momentum conserved.



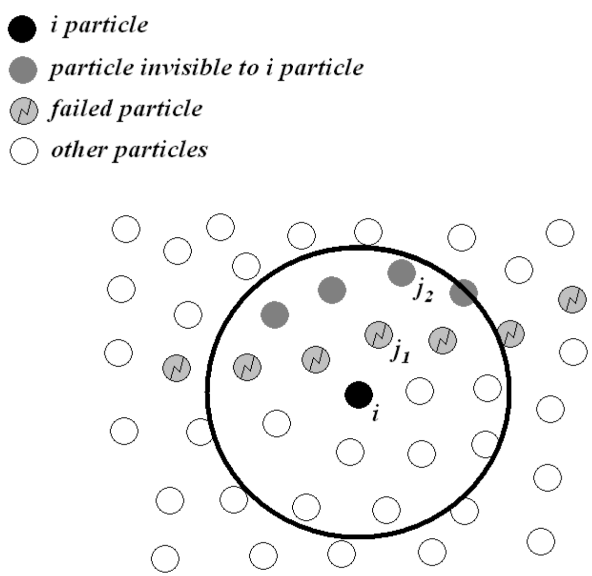
Modelling Failure

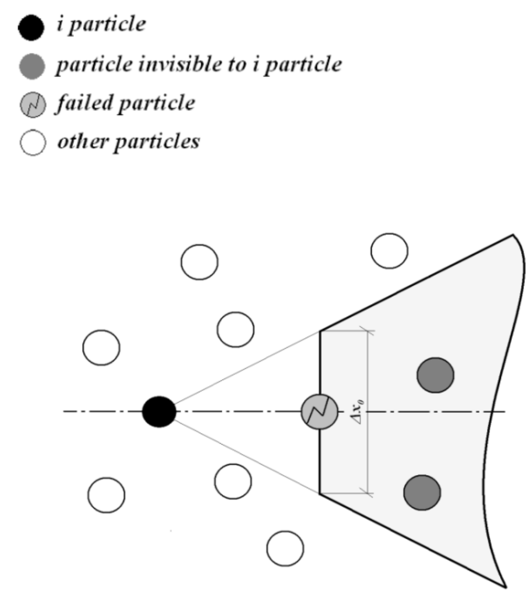
Problem – Treatment of Fracture

Solution – Particle visibility criterion

> Truncated cone

> Invisible particles are removed from neighbourhood





De Vuyst, Vignjevic, International Journal of Fracture, 2013

Fragment Calculation

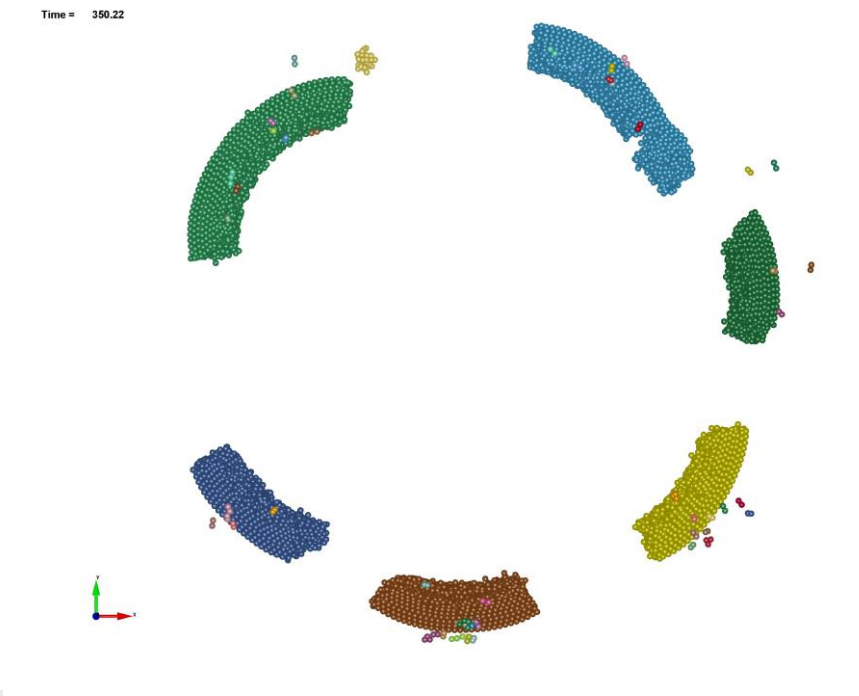
Post Processing Calculation

Same fragment if $| x_j - x_i | < C * \Delta x$

with $1.2 \leq C \leq 1.5$

Output Data:

- Fragment Number
- Mass
- Centre of Gravity
- Velocity

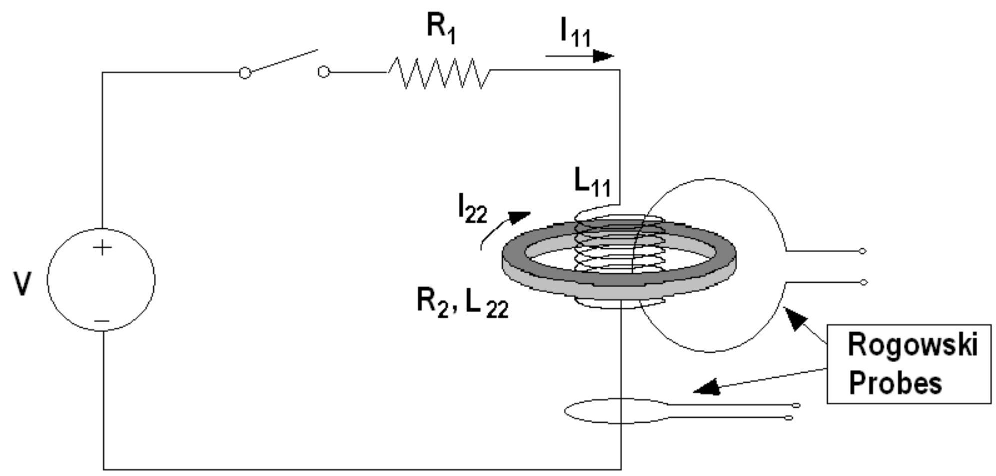


Electromagnetically driven rings

Zhang, Ravi-Chandar (2006) Int J Fract, 142, 183-217

AA6061-O rings

1.0x0.5mm cross section, radius 15.25mm

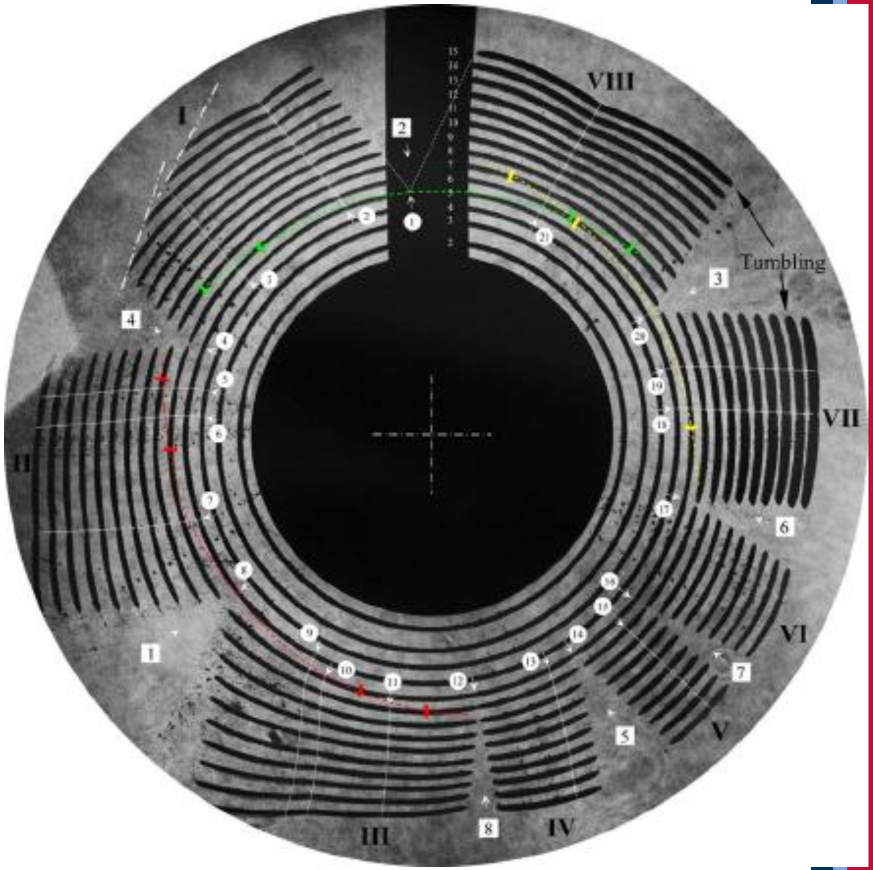


Electromagnetically driven rings

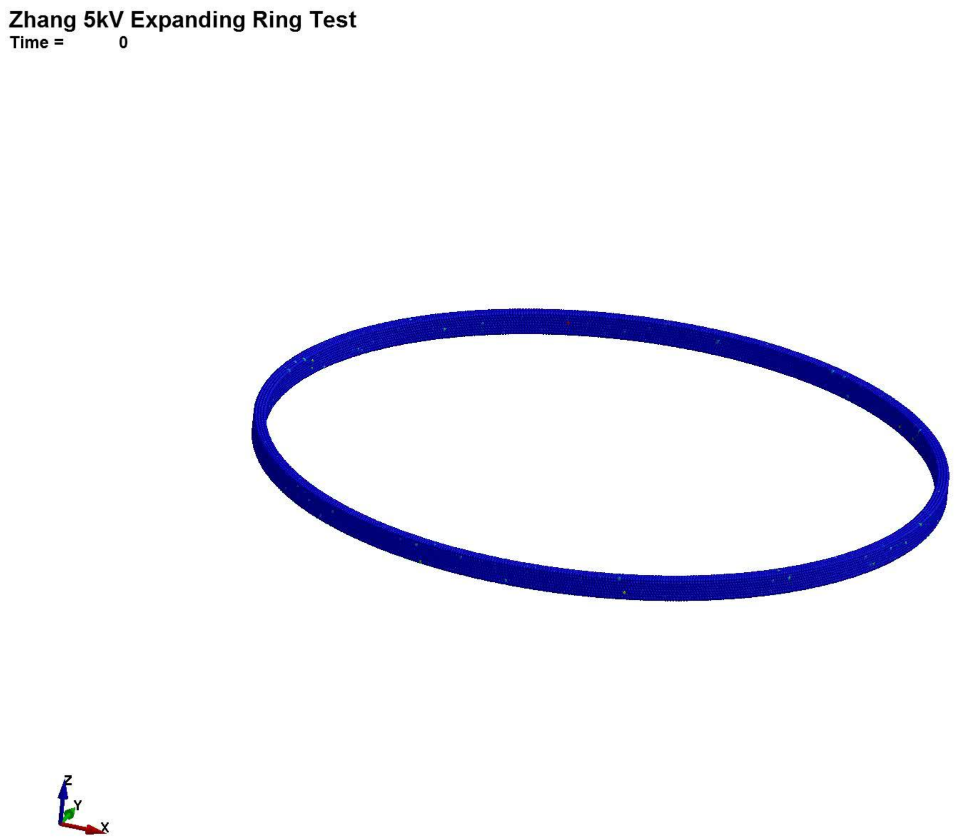
Zhang, Ravi-Chandar (2006) Int J Fract, 142, 183-217

Observations:

- Number of fragments
- Number of necks
- Time of first fracture
- Bending and rotation

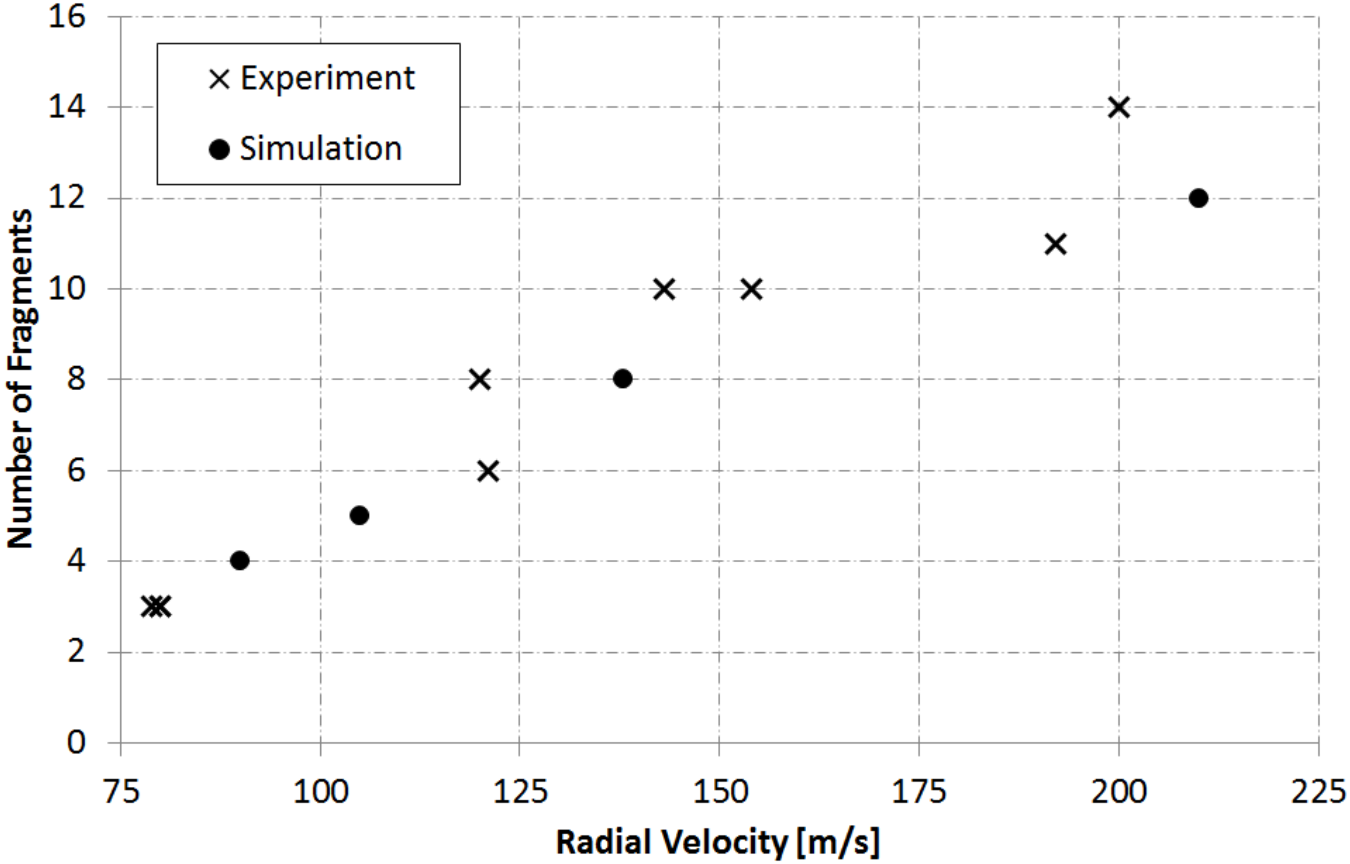


EM driven ring model



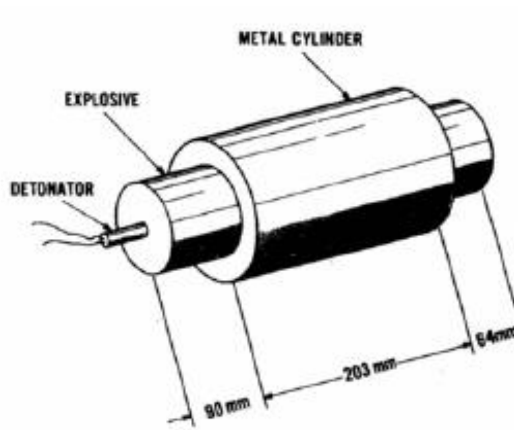
EM driven ring model

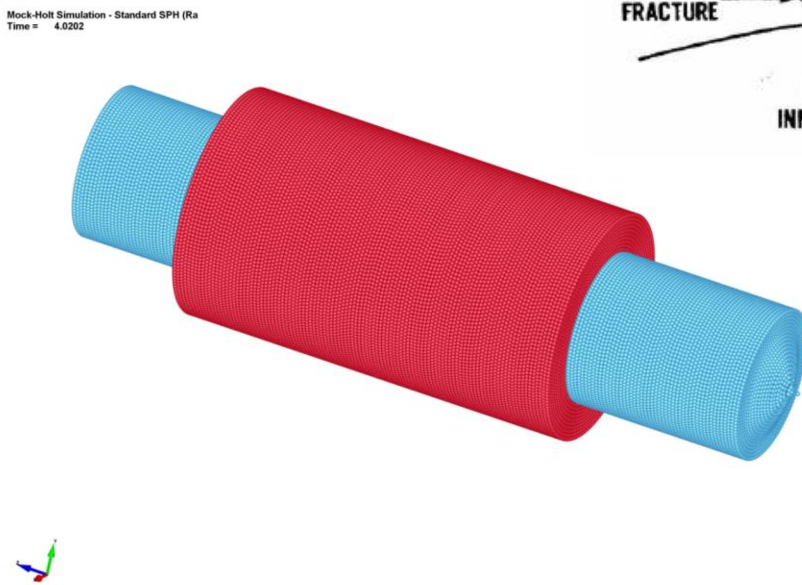
Number of Fragments

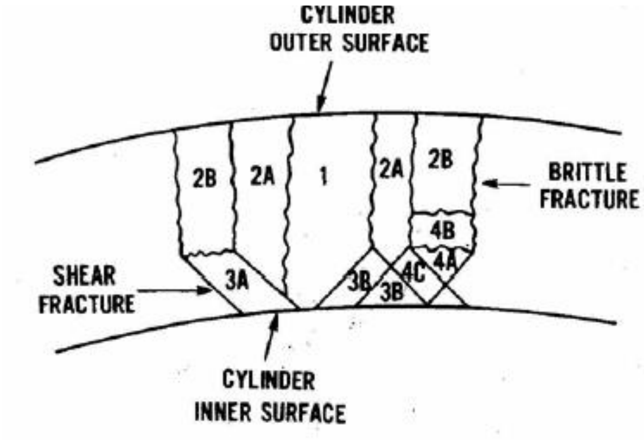


Mock-Holt Experiments

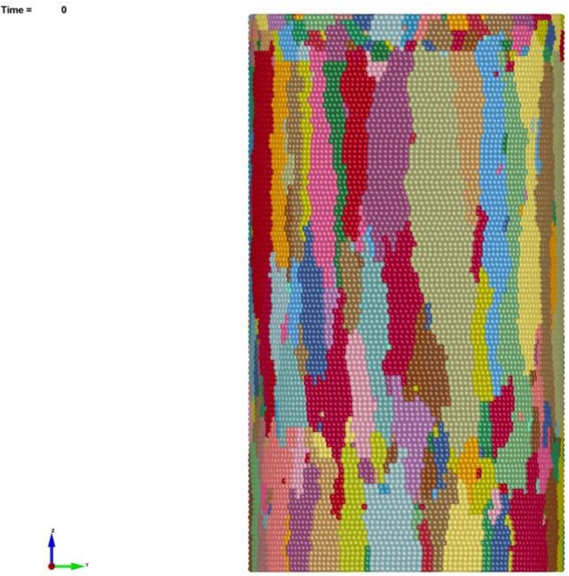
Experimental setup Model Fragment types



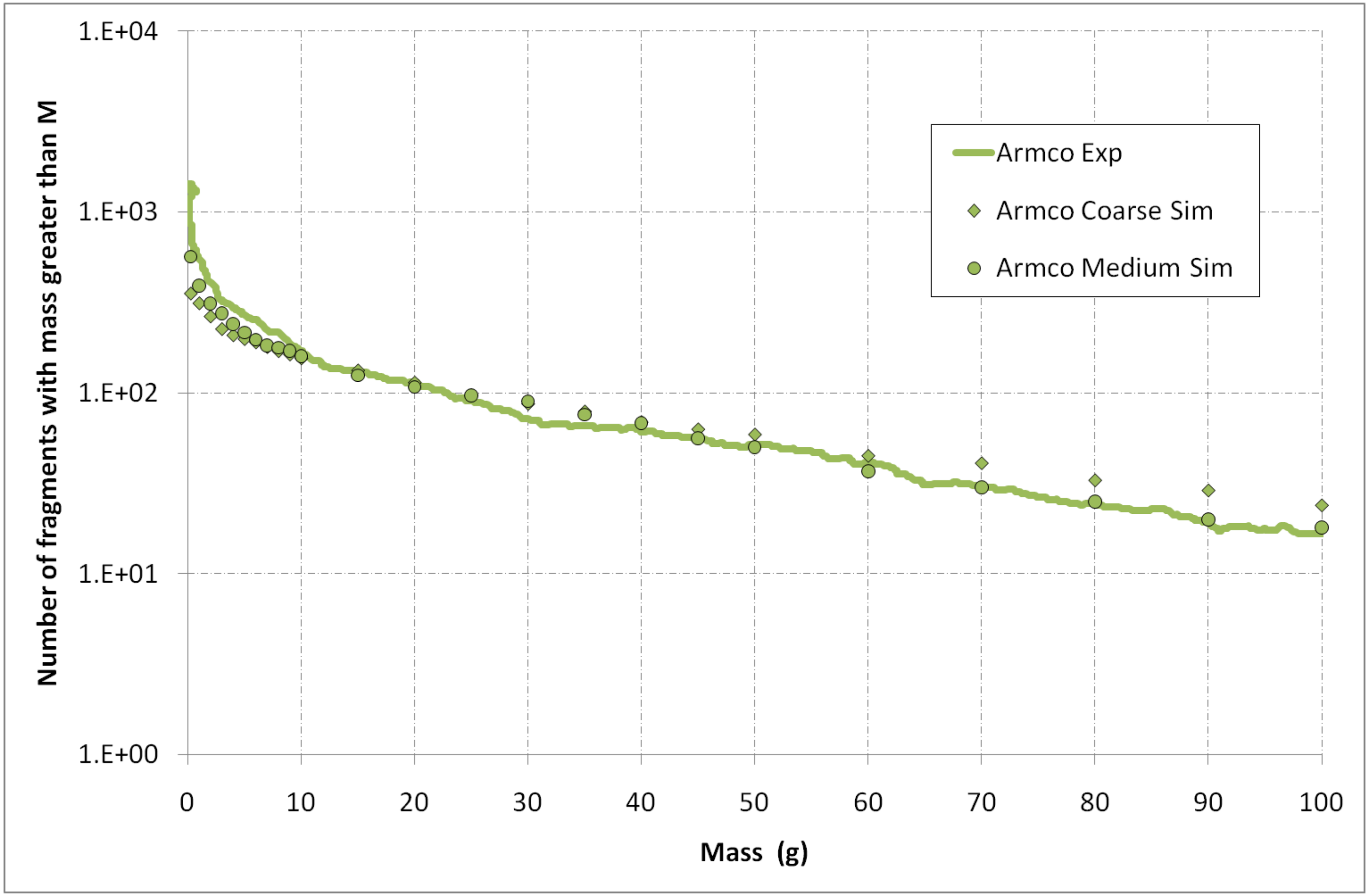




Mock-Holt Experiments



Mock-Holt Experiments



Conclusion and Future work

02 February 2017

SPH method has been successfully used to model high strain rate fracture and fragmentation problems

Model can be used as starting point to validate more physically based constitutive models

Ability to predict failure of brittle materials can be investigated