# SPH as Nonlocal Regularisation Method: Solution for Instabilities due to Strain-Softening

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Presentation outline

- Background
- FE modelling
- SPH modelling – the influence of smoothing length (h=λ Δp)
  - Test 1: h=var, λ=1.3, Δp=var
  - Test 2: h=var, λ=var, Δp=cons
  - Test 3: h=2.5mm, λ=var, Δp=cons)
- Summary

# Background

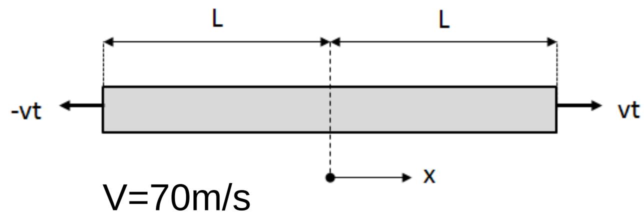
Within the framework of continuum damage mechanics (CDM), mechanical loading leads to material damage and consequent degradation of material properties and potentially to **strain softening** (**negative material stiffness**).

A **local softening material model** in the finite element (FE) method, leads to **an ill posed boundary value problem**, resulting in **mesh sensitivity** and a non-physical solution.

Addition of a **characteristic length scale** to CDM models, a **non-local approach** **(higher order derivative or integral regularisation techniques)**, maintains the character of the governing equations in the damaged zone.

In this presentation, the similarities between **the SPH method and non-local integral regularisation methods** are discussed.

# Background




1D dynamic strain softening problem was used as the test problem for a series of numerical experiments, to investigate the behaviour of SPH.

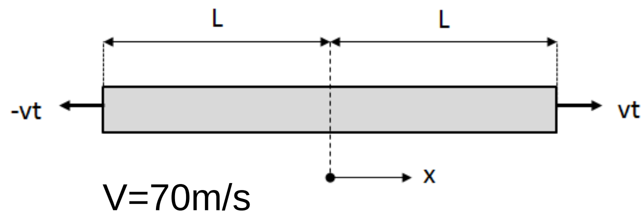
An analytical solution for the test problem is derived, following the solution for a 1D derived by Bažant and Belytschko in 1985.

$$c_e^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \qquad c_e = \sqrt{\frac{E(1-\nu)}{\rho(1-2\nu)(1+\nu)}} \qquad u(x,t) = -v\left\langle t - \frac{x+L}{c_e} \right\rangle + v\left\langle t + \frac{x-L}{c_e} \right\rangle$$

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{v}{c_e}\left[ H\left( t - \frac{x+L}{c_e} \right) + H\left( t + \frac{x-L}{c_e} \right) \right]$$

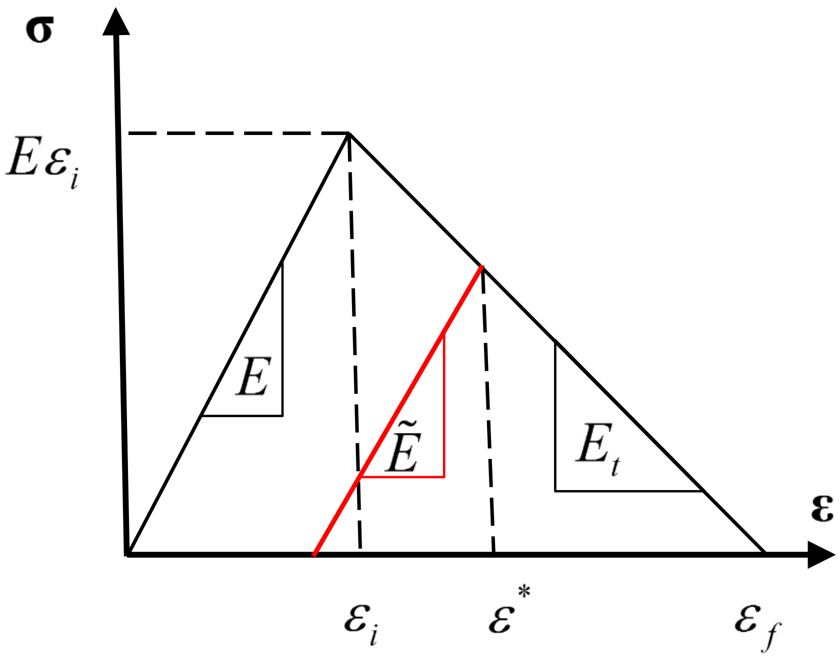
$$\varepsilon_x = \frac{v}{c_e}\left[ H\left( t - \frac{x+L}{c_e} \right) - H\left( t - \frac{L-x}{c_e} \right) + 4c_e t - L\delta(x) \right]$$

# Background




Dynamic strain softening MATERIAL.

$$\sigma_x = \frac{E(1-\nu)}{(1-2\nu)(1+\nu)}\varepsilon_x$$

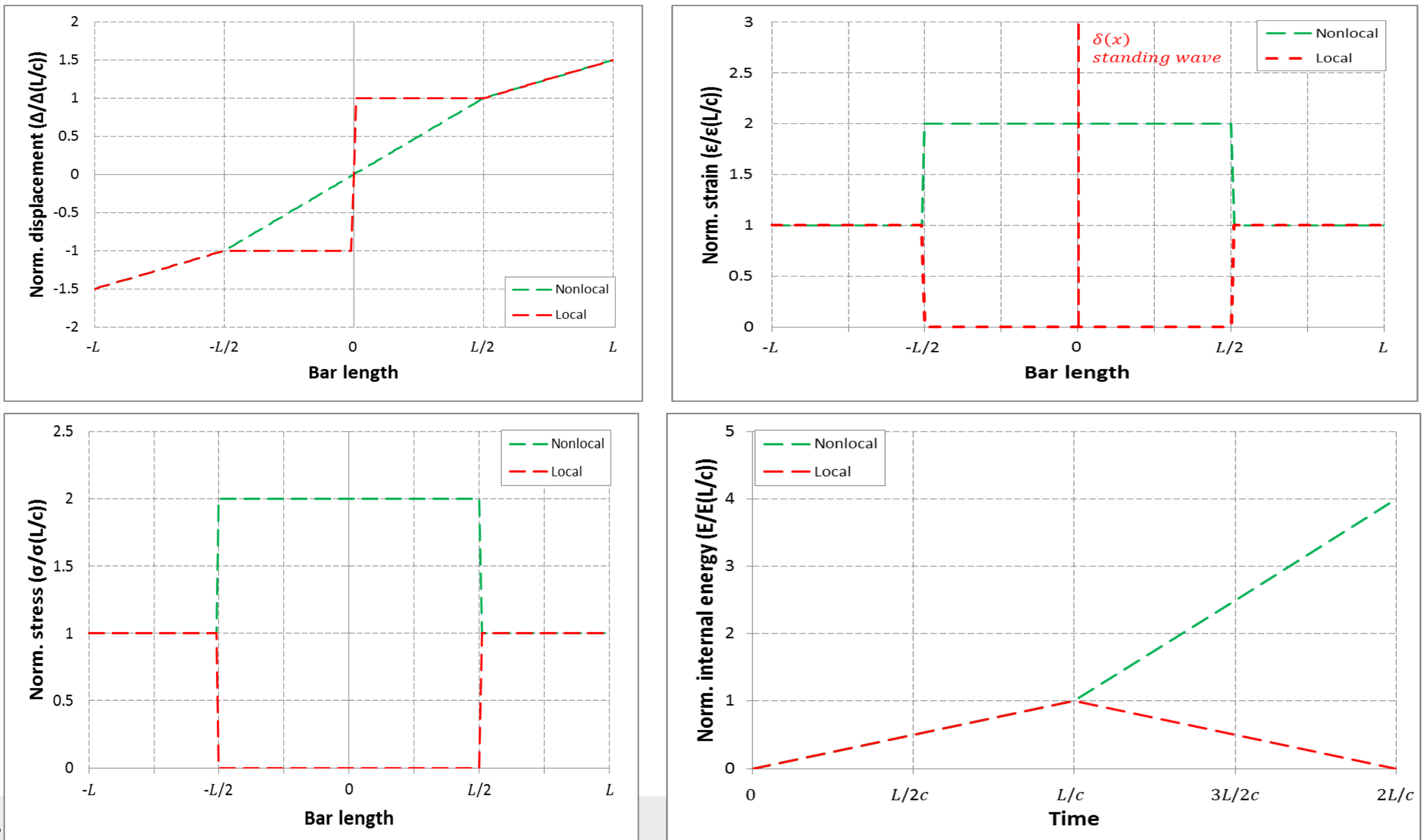



$$\tilde{\sigma} = \sigma\frac{1}{(1-\omega)}$$

$$\omega = \frac{\varepsilon_f\left(\varepsilon^* - \varepsilon_i\right)}{\varepsilon^*\left(\varepsilon_f - \varepsilon_i\right)}$$

# Background

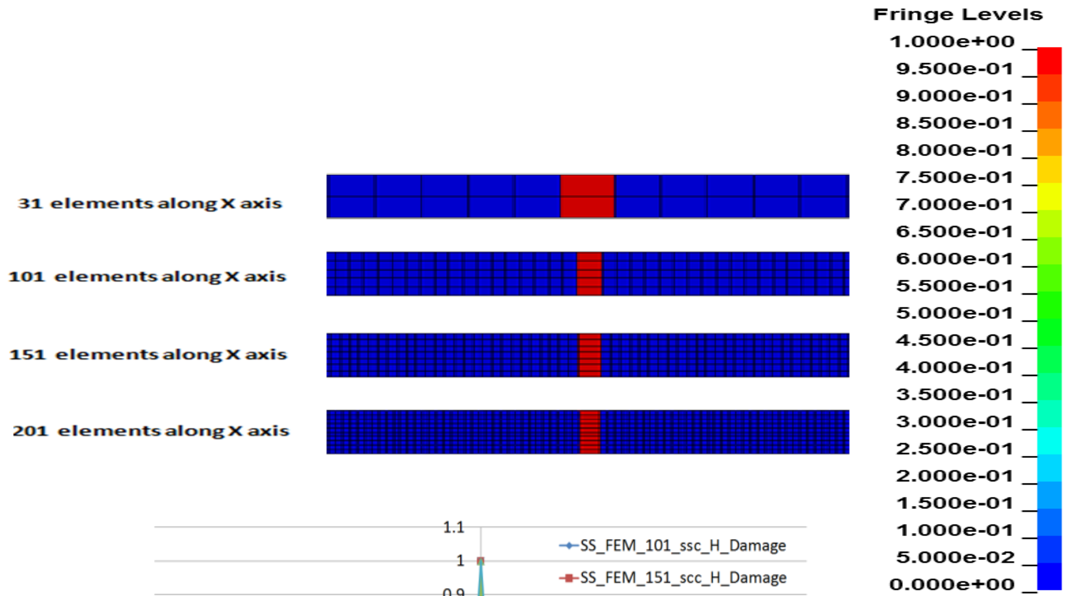
Elastic local and nonlocal analytical solutions for normalised longitudinal displacement, strain, stress and energy at t=3/2·L/c

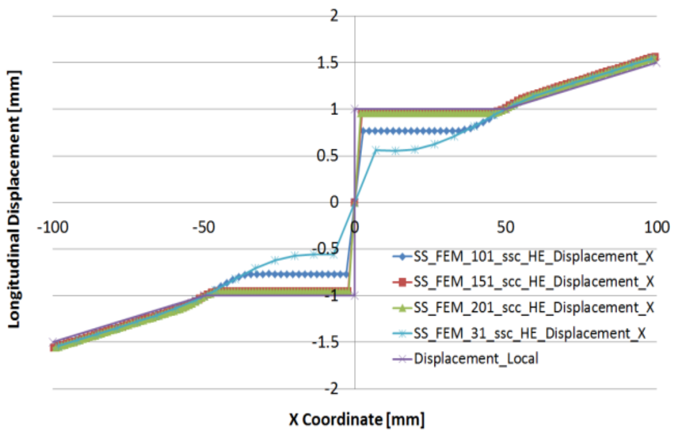


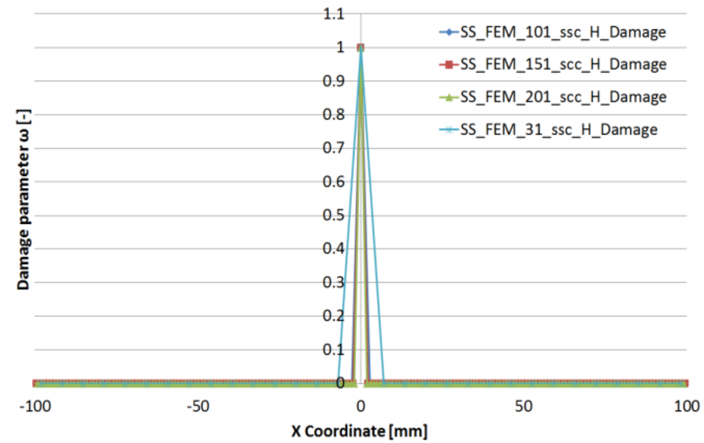
# FE-Modelling

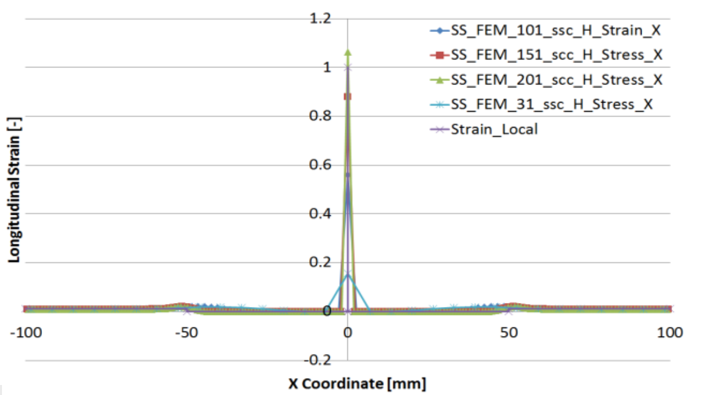
Local analytical solution and FE results

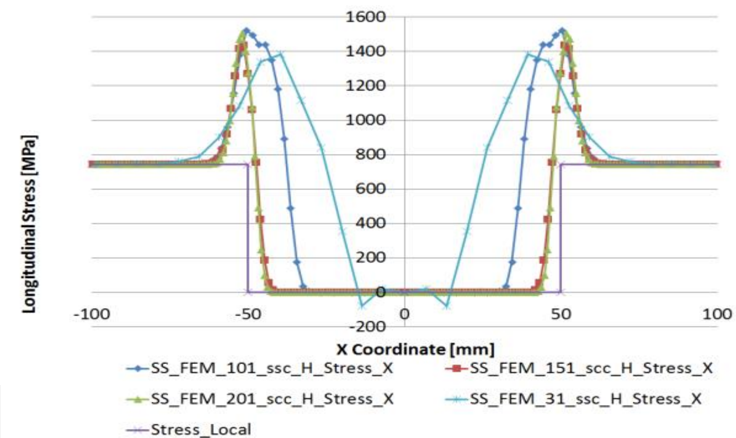
Response time t=3L/(2ce)



# SPH-Modelling

Summary of the three numerical experiments performed with SPH

The influence of **variation of smoothing length h** (h=λ·Δp; Δp=variable, λ =1.3 =constant) on the bar softening . The strain-softening effects were averaged over **the number of neighbouring particles constant** for the three particle densities considered.

The influence of **variation of smoothing length h** (Δp=constant, λ =variable) , when **particle density was constant and number of neighbours variable**. The increase in size of h resulted in reduction of the damage peak value.

The third experiment considered a **fixed size smoothing length h** for **three different particle densities** (Δp=constant, λ =variable, h=2.5 mm). The damage effects propagated the same distance in all simulations, giving (the a constant softening zone of the same size).

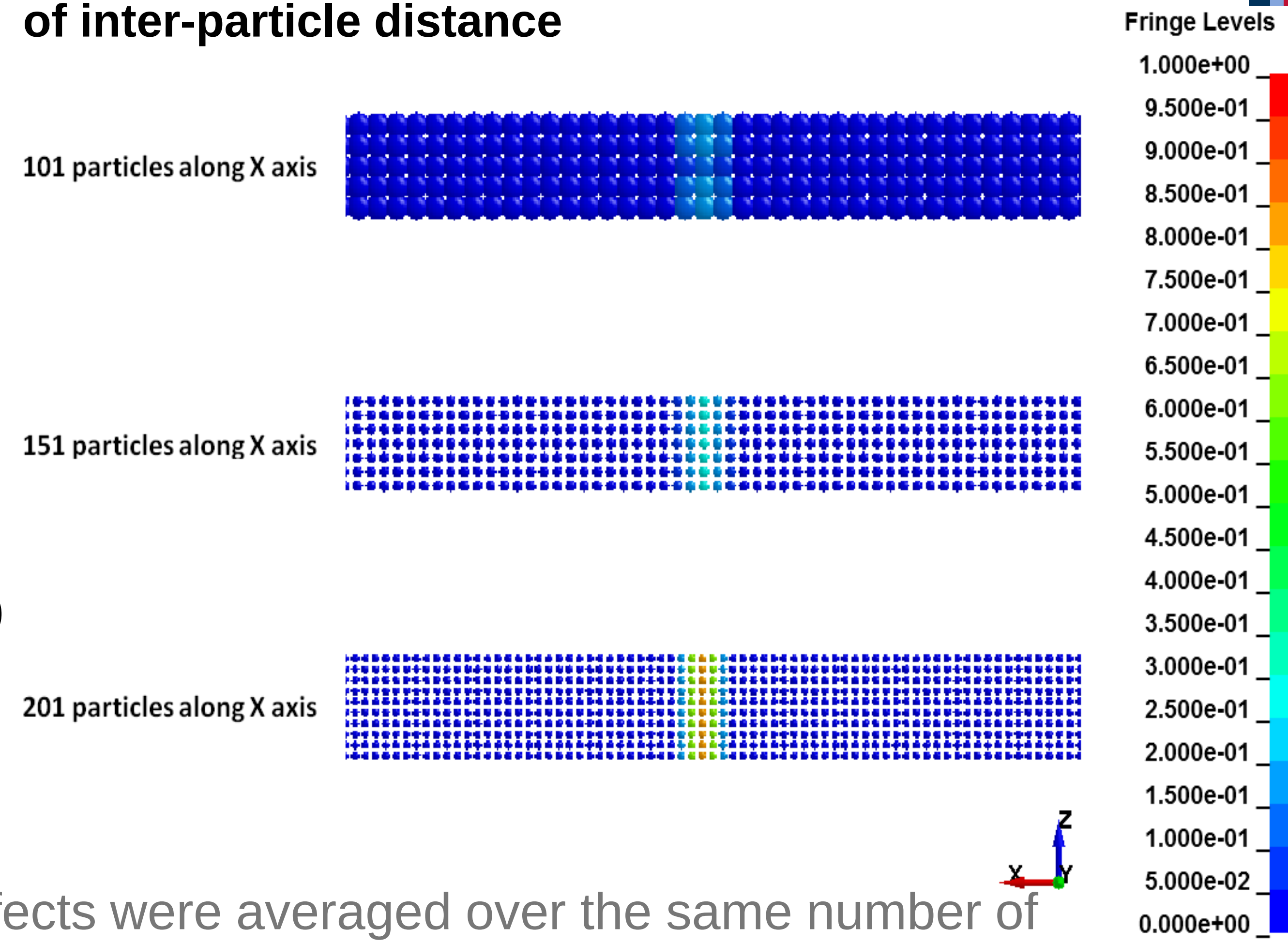
# SPH-Modelling

**Test I** (Δp=variable, λ =constant=1.3, h =variable) **- Influence of inter-particle distance**

Localisation of damage within 4h

Response time $t=3L/(2c_e)$

Fringe level: damage [-]



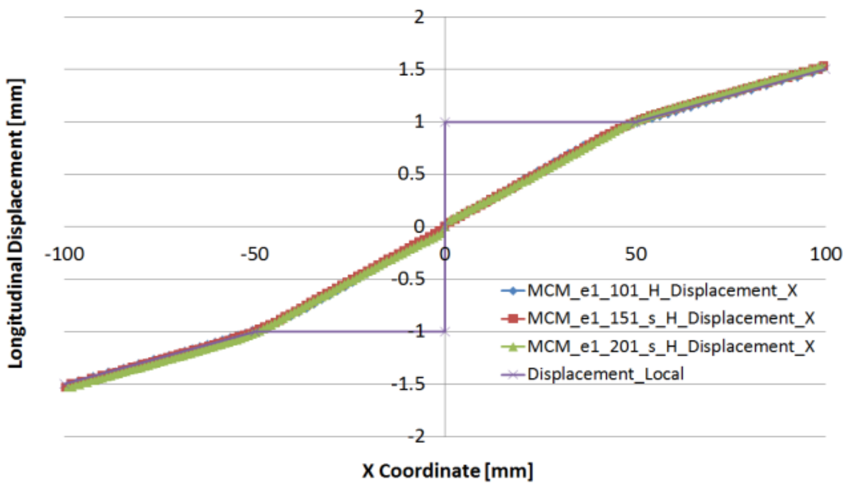

The strain-softening effects were averaged over the same number of neighbouring particles for the three particle densities considered.

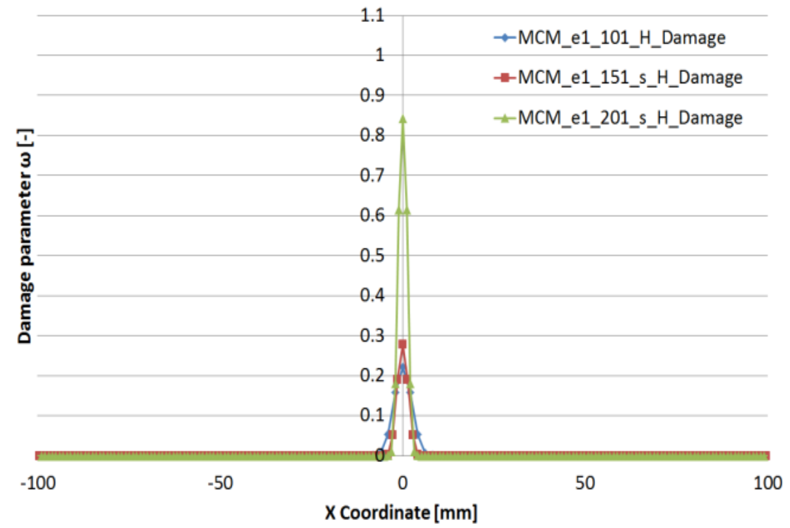
The size of the softening zone was defined by h and the size of the zone increased with the increase in particle size.

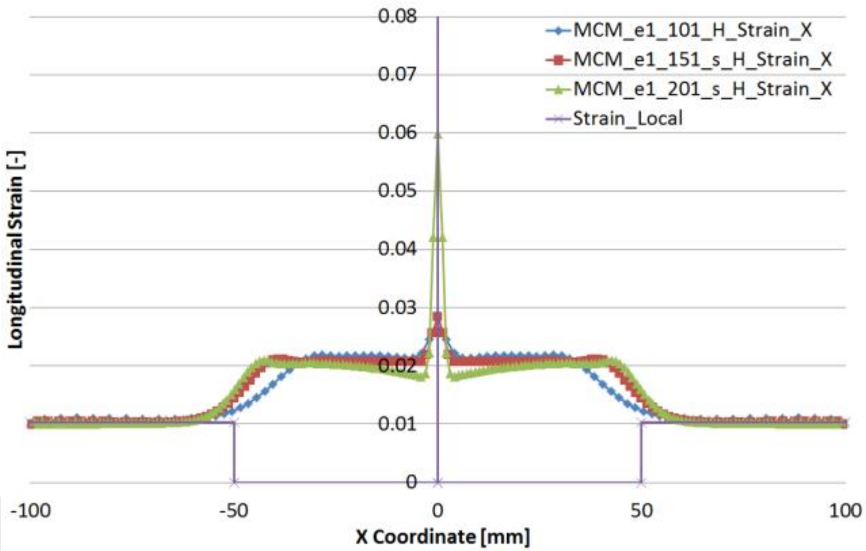
# SPH-Modelling

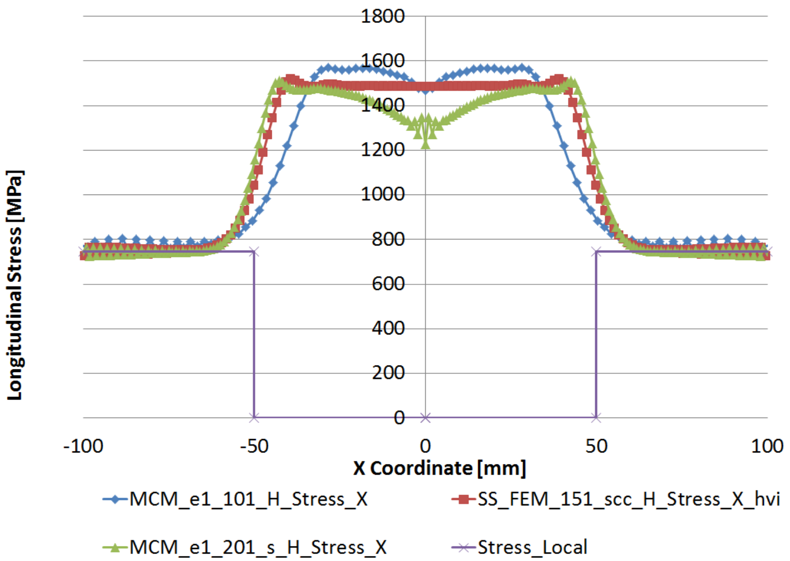
## Test I

Response time t=3L/(2ce)



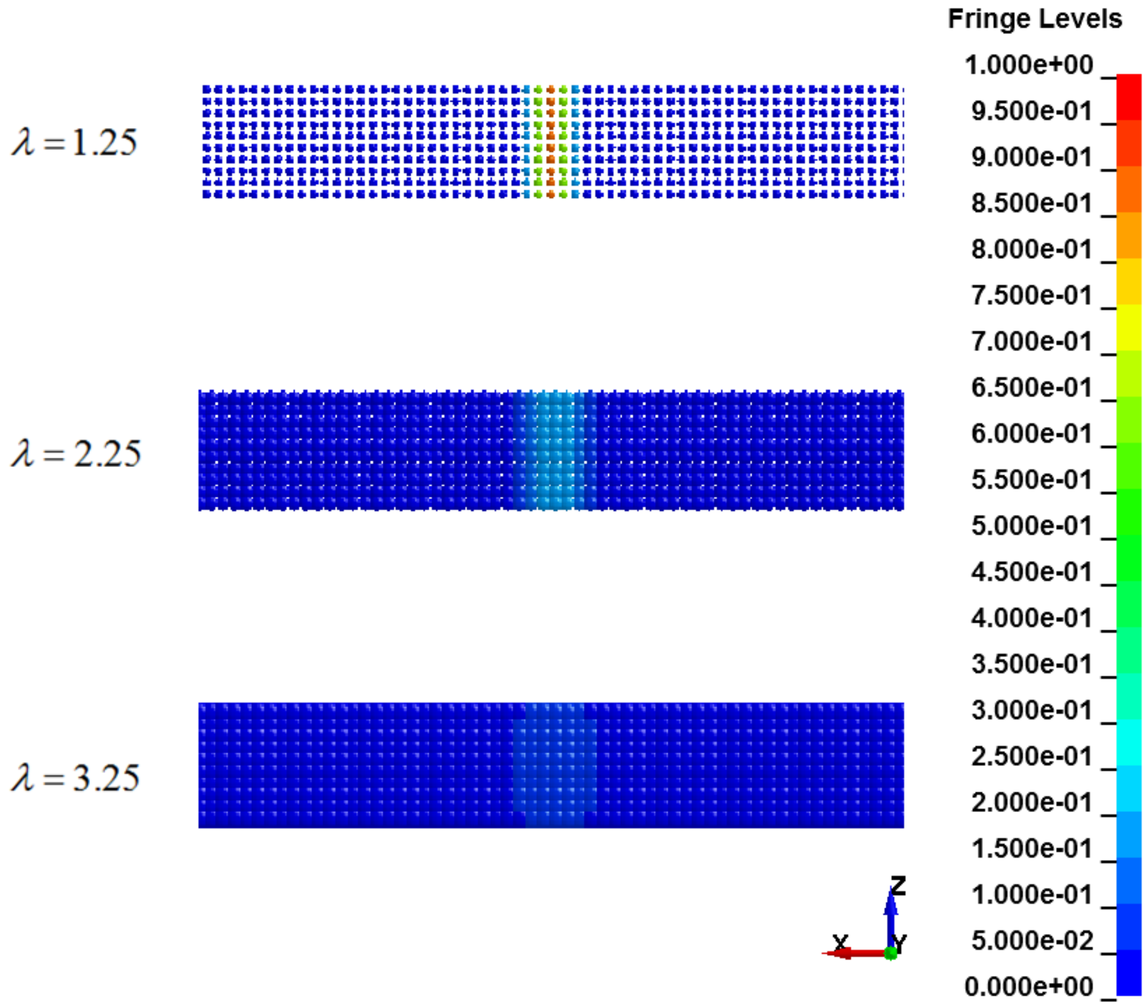



# SPH-Modelling

**Test II** (Δp=constant, λ =1.25, 2.25 & 3.25, h =variable) **- variable smoothing length h, constant inter particle distance**

Localisation of damage within 4h
Response time t=3L/(2ce)
Fringe level: damage [-]

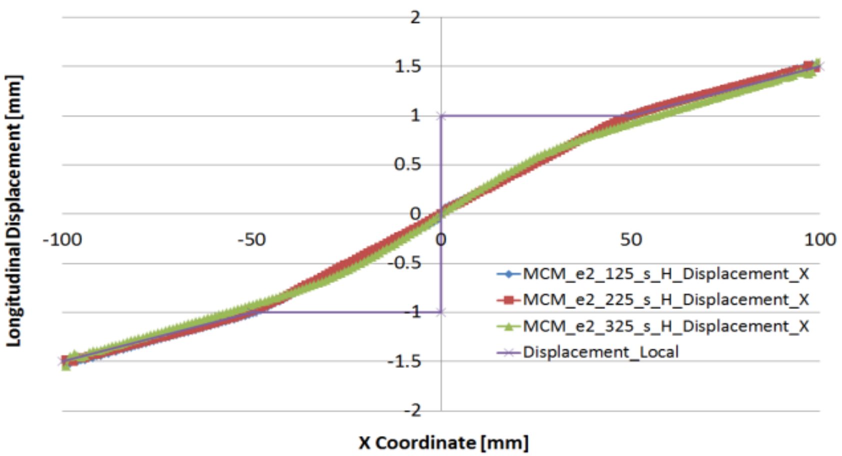


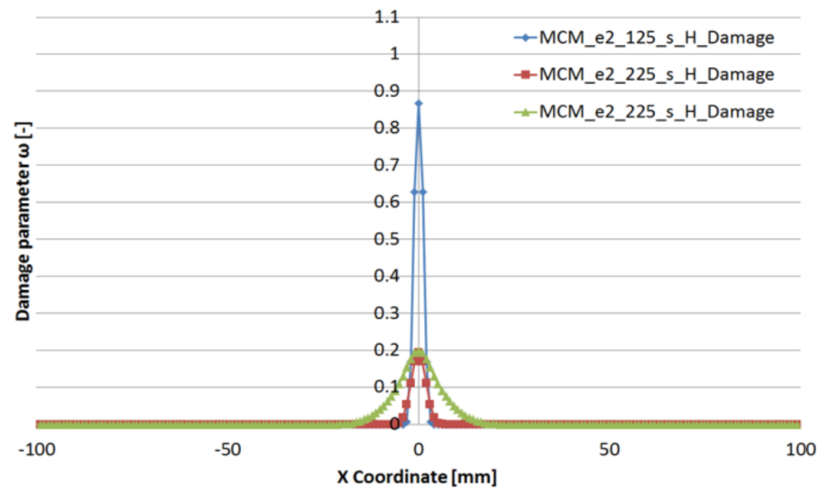

The increase in size of h resulted in reduction of the damage magnitude average and peak values

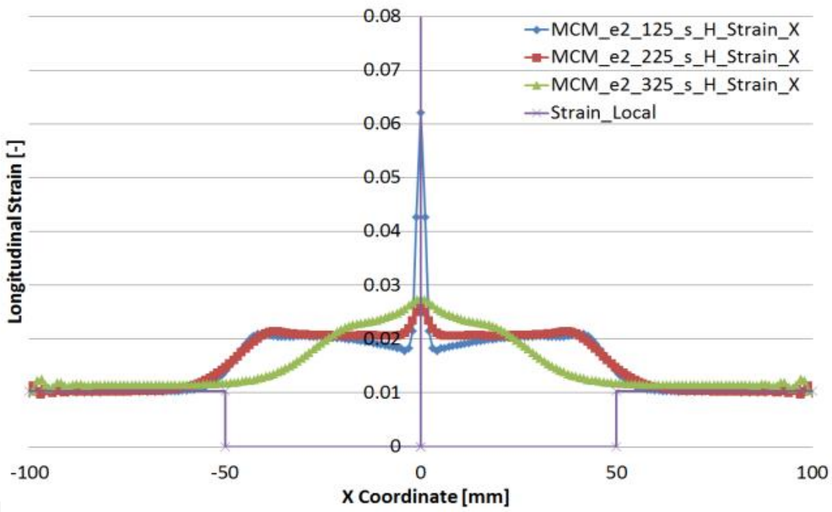
# SPH-Modelling

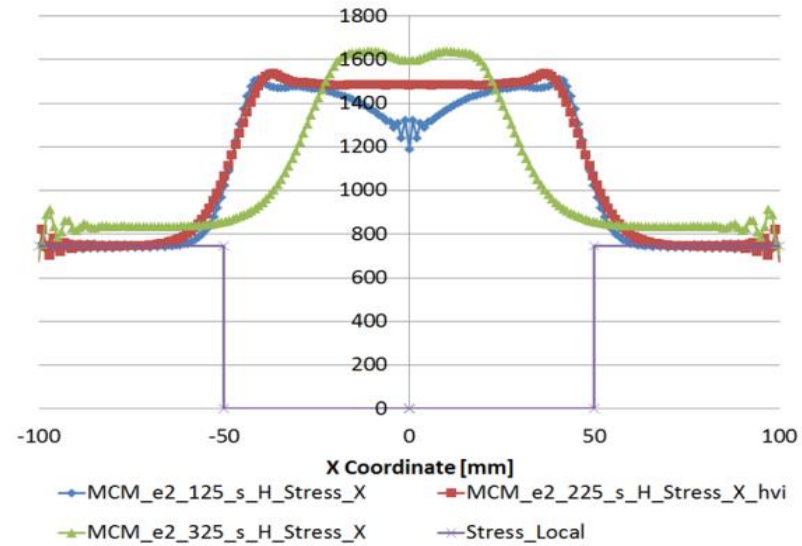
## Test II

Response time t=3L/(2ce)



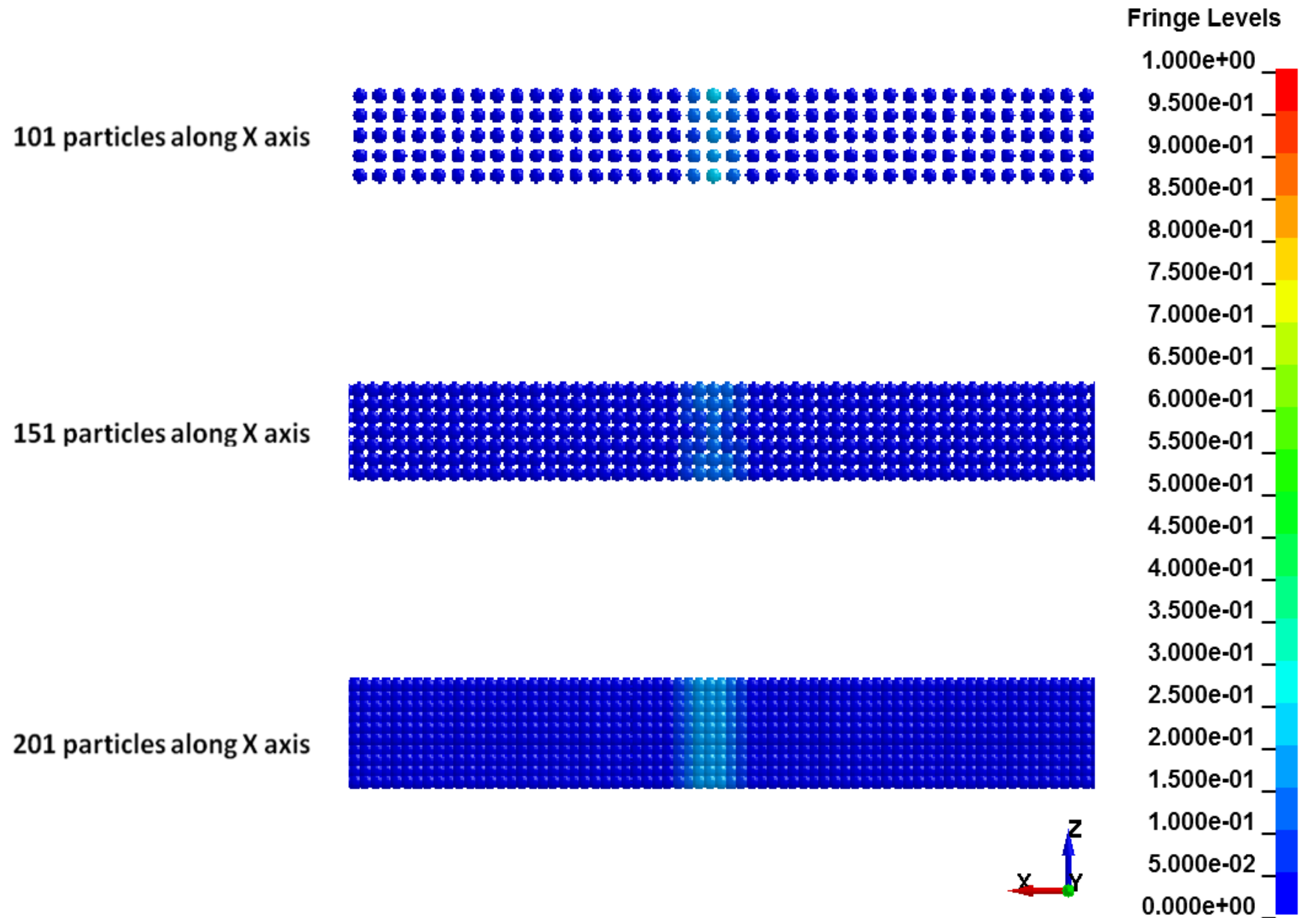



# SPH-Modelling

**Test III** (Δp=variable, λ =variable, h =2.5mm) **- fixed size smoothing length h for three different particle densities**




Localisation of damage within 4h
Response time $t=3L/(2c_e)$
Fringe level: damage [-]

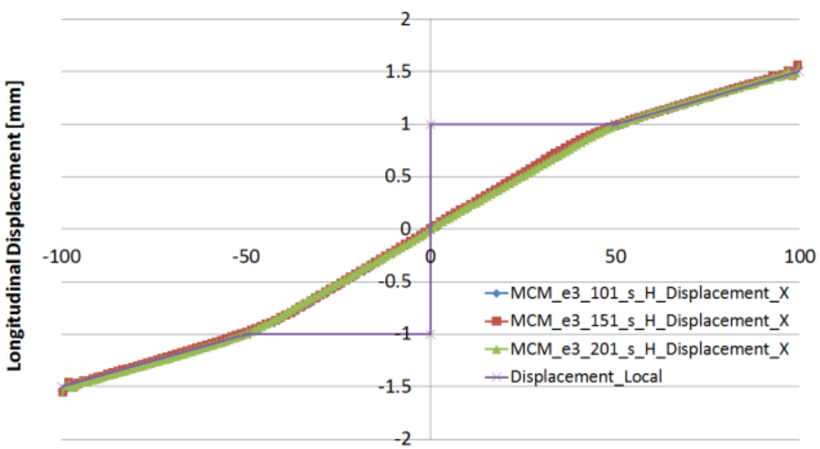
The damage effects propagated the same distance in all simulations, giving a **constant softening zone size**.

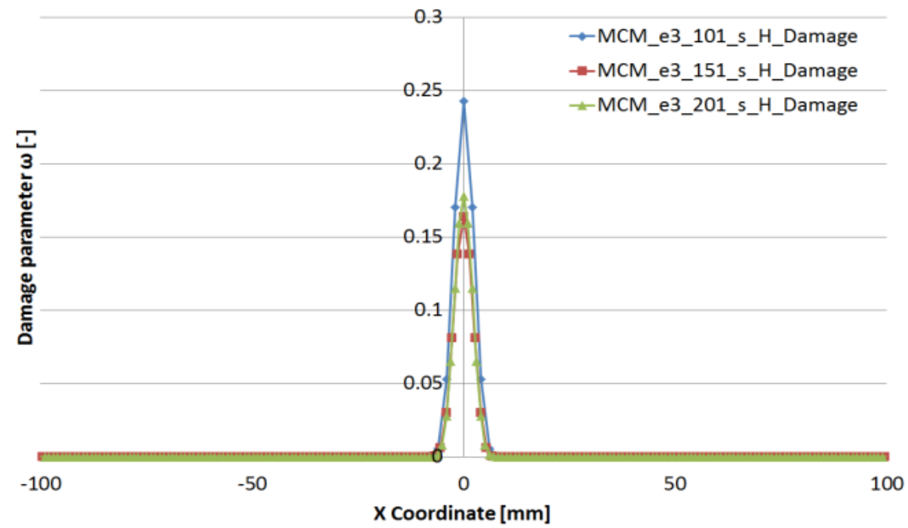
The **peak damage value reduced** with the increase of the number of neighbouring particles.

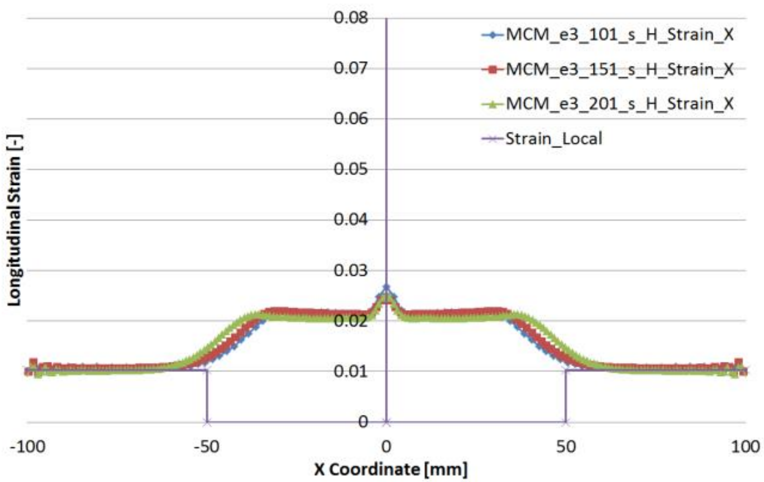
# SPH-Modelling

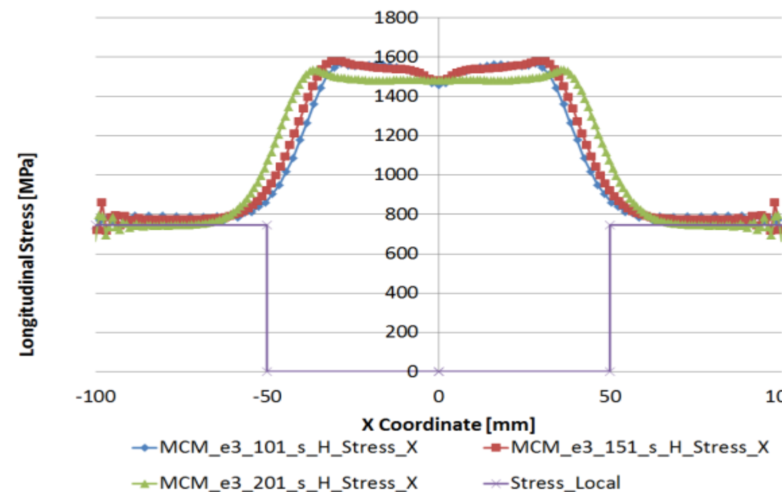
## Test III

Response time t=3L/(2ce)



# Conclusions

- In the SPH simulations damage spread over the smoothing domain used in the kernel interpolation
- The smoothing domain size represents a material characteristic length
- The user can control the localisation process by varying the smoothing length
- Stress waves propagated through the softening zone which is consistent with the physical observations