H_∞ Control for Two-Dimensional Fuzzy Systems with Interval Time-Varying Delays and Missing Measurements

Yuqiang Luo, Zidong Wang, Jinling Liang Guoliang Wei and Fuad E. Alsaadi

Abstract—In this paper, we consider the H_∞ control problem for a class of two-dimensional (2-D) Takagi-Sugeno (T-S) fuzzy described by the second Fornasini-Machesini local state-space model with time-delays and missing measurements. The state delays are allowed to be time-varying within a known interval. The measurement output is subject to randomly intermittent packet dropouts governed by a random sequence satisfying the Bernoulli distribution. The purpose of the addressed problem is to design an output-feedback controller such that the closed-loop system is globally asymptotically stable in the mean square and the prescribed H_∞ performance index is satisfied. By employing a combination of the intensive stochastic analysis and the free weighting matrix method, several delay-range-dependent sufficient conditions are presented that guarantee the existence of the desired controllers for all possible time-delays and missing measurements. The explicit expressions of such controllers are derived by means of the solution to a class of convex optimization problems that can be solved via standard software packages. Finally, a numerical simulation example is given to demonstrate the applicability of the proposed control scheme.

Index Terms—Fuzzy systems, Two-dimensional systems, H_∞ control, output feedback, time-varying delays, missing measurements.

I. INTRODUCTION

Nonlinearity is a ubiquitous phenomenon in the natural world that has been receiving ever-increasing research attention from a variety of subject areas. Traditionally, rigorous mathematical analysis on nonlinear systems rely on precise description of the nonlinearities with some stringent assumptions, and this sometimes hinders the nonlinear systems theory from being applied to certain engineering practice, for example, those data-based applications where an exact mathematical

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model is hard to be established [4], [16], [26], [29]–[31], [38]. On the other hand, the fuzzy logic theory has proven to be effective in handling reasoning that is approximate rather than fixed and exact. Furthermore, after a decade of theoretical and practical development, the Takagi-Sugeno (T-S) model has been recognized as an efficient way to approximate certain nonlinear systems. By using a set of local linear models which are smoothly connected by nonlinear fuzzy member ship functions to present a nonlinear plant, the T-S fuzzy model has brought the analysis and synthesis of nonlinear systems into a unified framework. Moreover, owing to the peculiarity of the T-S model, a large portion of existing results for linear systems can be readily extended for some nonlinear systems. As pointed out in [12], [32], the T-S model is able to approximate any smooth non-linear function to any degree of accuracy in any convex compact region.

As is well known, many practical systems can be ideally described by the two-dimensional (2-D) systems that have received tremendous research attention because of their applications in thermal processes, seismic data sections, digitized photographic data, digital filtering and magnetic maps, etc. [11]. For four decades, the theoretical investigations on 2-D systems have been attracting recurring research interests and fruitful research results have been available in the literature. As early as in 1970s, some basic behaviors and modeling issues were thoroughly examined for 2-D systems, see e.g. [11], [18]. Parallel to the rapid research development of the traditional 1-D systems, in the past few years, some important breakthroughs have been reported on the analysis and design issues for the 2-D control systems. For example, the stability analysis and stabilization problems for 2-D systems have been addressed in [7], [13], and the filter/observer design problems have been investigated in [25]. In particular, some new kinds of 2-D models and new techniques have recently been put forward in the literature. In [23], a state estimation problem for 2-D complex networks with randomly occurring nonlinearities and randomly varying sensor delays has been considered, and a new synchronization problem has been dealt with for an array of 2-D coupled dynamical network in [24].

Given the practical importance of 2-D systems in representing two-dimensional evolutions and the technical convenience of T-S fuzzy systems for handling affine nonlinearities, a seemingly natural research issue would be to investigate the dynamical behaviors as well as the estimator/controller design problems of 2-D T-S fuzzy systems. Unfortunately, a literature review has revealed that the corresponding results have been

very few due primarily to the mathematical complexities for 2-D T-S fuzzy systems. On the other hand, time delays are very often a major concern in many practical applications such as communication line, electrical signal processing systems, network transmission systems, seismic wave, urban traffic management systems, etc. Time delays are well recognized as one of the main sources for poor performance or even instability of control systems. In the 1-D settings, the time-delay systems have recently received considerable research attention and a rich body of literature has appeared on this topic, see e.g. [3], [17]. Moreover, among the existing works, plenty of stability conditions are delay-dependent, which pose less conservativeness, see e.g. [19], [20]. With respect to 2-D time-delay systems, over the past few years, some initial results have been reported on the control and filtering problems. For example, the problem of delay-dependent H_∞ control for 2-D discrete state delay systems has been investigated in [22], and a robust H_∞ filter for 2-D discrete systems with time delays has been designed in [34]. It should be noted that most available results have been concerned with time-invariant delays.

As a matter of fact, the phenomenon of missing measurements (packet losses or dropouts) is virtually inevitable in measurement processes (particularly within a networked environment), which is caused by some harsh working conditions and imperfect communications. Many factors have contributed to this kind of less-than-ideal situation. Such factors include, but are not limited to, the limited bandwidth of the communication channel, abnormity of swap device, random network congestion, accidental loss of some collected data in a very noisy environment. So far, the problem of missing measurements has been well studied for 1-D systems and a great deal of literature has been available, see e.g. [28], [33]. Nevertheless, the relevant research for 2-D systems is still in its early stage especially when both the time-varying delays and missing measurements are simultaneously present, not to mention the case when the 2-D systems are further complicated by the T-S fuzzy model. As such, the stability analysis and stabilization problem for 2-D time-delay fuzzy systems with missing measurements remains a challenging issue that motivates our current research, and the main task of this paper is to propose a general framework for handling such a challenge.

In this paper, we endeavor to research into the H_∞ control problem for a class of two-dimensional (2-D) Takagi-Sugeno (T-S) fuzzy systems described by the second Fornasini-Machesini local state-space model with time-delays and missing measurements. The main contributions of this paper can be boiled down as follows. *1) A 2-D T-S fuzzy model is considered, which is comprehensive to include time-varying delays, bounded noises and probabilistic missing measurements, thereby reflecting engineering practice more closely. 2) For the purpose of stabilizing the addressed 2-D fuzzy systems, an energy-like functional is constructed and several delay-range-dependent stability criteria are obtained to accomplish the design goal. 3) The close-loop system has the expected disturbance attenuation level in terms of a prescribed H_∞ performance index.* Compared with the existing delay-independent or delay-dependent results for 1-D delayed systems, the delay-

range-dependent method addressed in this paper is of less conservativeness. The adopted 2-D T-S model can be thought to be a universal approximator for nonlinear 2-D systems, and the corresponding controller can also be used to stabilize the complicated 2-D nonlinear plant. Potential applications of the investigated fuzzy control approach include the 2-D digital systems, image processing and wireless communications, etc. Especially for the case of interval time-varying delays and randomly missing measurements, the proposed fuzzy controller will show its strength.

The rest of this article is organized as follows. Section II is devoted to the formulation of the H_∞ control problem for the addressed 2-D fuzzy systems with interval time-varying delays, where the phenomenon of probabilistic missing measurements is characterized by a Bernoulli distribution model, and some notations and related definitions are also given. In Section III, with the aid of an energy-like functional and the delay-range-dependent method, both the analysis and the synthesis problems of the 2-D fuzzy control system are investigated. In Section IV, an example is given to validate the design approach of the proposed fuzzy control scheme, and some concluding remarks have been drawn in Section V.

Notation. In this paper, $\mathbb{R}^n$, $\mathbb{R}^{n \times m}$ and $\mathbb{Z}$ ($\mathbb{Z}^+, \mathbb{Z}^-$) denote, respectively, the n-dimensional Euclidean space, the set of all $n \times m$ real matrices and the set of all integers (nonnegative integers, negative integers). $\|\cdot\|$ refers to the Euclidean norm in $\mathbb{R}^n$. I_n represents the identity matrix of dimension $n \times n$. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). For a matrix M, M^T and M^{-1} represent its transpose and inverse, respectively. The shorthand diag$\{M_1, M_2, \ldots, M_n\}$ denotes a block diagonal matrix with diagonal blocks being the matrices $M_1, M_2, \ldots, M_n$. In symmetric block matrices, the symbol '$*$' is used as an ellipsis for terms induced by symmetry. Matrices, if they are not explicitly stated, are assumed to have compatible dimensions.

II. PROBLEM FORMULATION

Consider a 2-D discrete-time T-S fuzzy system with state-delays and stochastic perturbations described by the following Fornasini-Marchesini (FM) local state-space (LSS) second model:

Plant Rule i:
IF $\theta_1^{(k,l)}$ is $\mathcal{F}_{i1}$, $\cdots$, $\theta_j^{(k,l)}$ is $\mathcal{F}_{ij}$, $\cdots$ and $\theta_p^{(k,l)}$ is $\mathcal{F}_{ip}$,
THEN

$$\begin{cases} x(k+1, l+1) = A_{1i}x(k, l+1) + A_{2i}x(k+1, l) \\ \qquad\qquad + D_{1i}x(k - \tau_1(k,l), l+1) \\ \qquad\qquad + D_{2i}x(k+1, l - \tau_2(k,l)) \\ \qquad\qquad + B_{1i}u(k, l+1) + B_{2i}u(k+1, l) \quad (1) \\ \qquad\qquad + E_{1i}\omega(k, l+1) + E_{2i}\omega(k+1, l), \\ y(k, l) = C_i x(k, l) + H_i \omega(k, l), \qquad i \in \mathbb{S} \\ z(k, l) = G_i x(k, l) + L_i \omega(k, l), \end{cases}$$

where $k, l \in \mathbb{Z}^+$ and $x(k, l) \in \mathbb{R}^{n_x}$ is the state vector; $y(k, l) \in \mathbb{R}^{n_y}$ is the measured output; $z(k, l) \in \mathbb{R}^{n_z}$ is the

controlled output; $u(k,l) \in \mathbb{R}^{n_u}$ is the control input vector; $\omega(k,l) \in \mathbb{R}^{n_\omega}$ is the disturbance input which belongs to $l_2(\mathbb{Z}^+, \mathbb{Z}^+)$, namely, $\sum\limits_{l=0}^{\infty} \sum\limits_{k=0}^{\infty} \mathbb{E}\{\|\omega(k,l)\|^2\} < \infty$; $\tau_1(k,l)$ and $\tau_2(k,l)$ are interval time-varying delays along the horizontal direction and the vertical direction, respectively, which are subjected to $\underline{\tau}_1 \leq \tau_1(k,l) \leq \overline{\tau}_1$ and $\underline{\tau}_2 \leq \tau_2(k,l) \leq \overline{\tau}_2$; the input vector $\theta_j^{(k,l)} = [\theta_j(k-1,l), \theta_j(k,l-1)]$ ($j = 1, 2, \cdots, p$) represents the spatial premise variable at the location (k,l), which may be states or measurable variables, and $\theta_j(k,l)$ is the corresponding component of the spatial input vector $\theta_j^{(k,l)}$; $\mathcal{F}_{ij}$ is a spatial fuzzy set of rule i corresponding to the spatial input vector $\theta_j^{(k,l)}$; $\mathbb{S} = \{1, 2, \cdots, r\}$ with r being the number of IF-THEN rules; A_{si}, B_{si}, C_i, D_{si}, E_{si}, H_i, G_i and $L_i(s = 1,2)$ are known real constant system matrices with compatible dimensions.

The initial boundary condition is given by

$$x(k,l) = \begin{cases} \psi_1(k,l); & \text{if } (k,l) \in [-\overline{\tau}_1, 0] \times [0, z_1] \\ \psi_2(k,l); & \text{if } (k,l) \in [0, z_2] \times [-\overline{\tau}_2, 0] \\ 0; & \text{if } (k,l) \in [-\overline{\tau}_1, 0] \times (z_1, \infty) \\ 0; & \text{if } (k,l) \in (z_2, \infty) \times [-\overline{\tau}_2, 0] \end{cases} \quad (2)$$

with $\psi_1(0,0) = \psi_2(0,0)$, where z_1 and z_2 are finite positive integers, $\psi_1(k,l)$ and $\psi_2(k,l)$ are given vectors which are independent of the stochastic input sequence $\{\omega(k,l)\}$.

Let $h_i^{(k,l)} = [h_i(k-1,l), h_i(k,l-1)]$, where $h_i(k,l)$ is the normalized membership function defined by

$$h_i(k,l) = \frac{\Psi_i(k,l)}{\sum\limits_{i=1}^{r} \Psi_i(k,l)}, \quad (3)$$

where, $\Psi_i(k,l) = \prod\limits_{j=1}^{p} \mathfrak{F}_{ij}(\theta_j^{(k,l)})$ and $\mathfrak{F}_{ij}(\theta_j^{(k,l)}) \geq 0$ is the grade of membership of $\theta_j^{(k,l)}$ in $\mathcal{F}_{ij}$, which is also called the fuzzy basis function. It can be easily verified that

$$0 \leq h_i(k,l) \leq 1, \quad \sum_{i=1}^{r} h_i(k,l) = 1, \quad \forall k,l \in \mathbb{Z}^+. \quad (4)$$

Remark 1. *Compared with the usual model rule for 1-D systems, the premise variable $\theta_j^{(k,l)}$ of the 2-D systems is actually in a spatial type. According to the FM model, the value of the state at location $(k+1, l+1)$ is related to those at $(k, l+1)$ and $(k+1, l)$. Therefore, it is reasonable to define the normalized membership function of the inferred fuzzy set to be a two-element vector $h_i^{(k,l)}$, where $h_i(k-1,l)$ and $h_i(k,l-1)$ are deemed to be the scalar normalized membership function regarding the premise variables $\theta_j(k-1,l)$ and $\theta_j(k,l-1)$, respectively. As pointed out in [1], [21], the membership function satisfying (3) can be viewed as one of the vertices of a polyhedron. It should be noted that the vector valued normalized membership function comprises the main features of the 2-D T-S fuzzy model, which is totally different from the 1-D case.*

Subsequently, by fuzzy blending, the T-S fuzzy system (1)

can be transformed into

$$\begin{cases} x(k+1, l+1) = \sum\limits_{i=1}^{r} h_i(k, l+1) \sum\limits_{i=1}^{r} h_i(k+1, l) \\ \qquad \times \{A_{1i}x(k, l+1) + A_{2i}x(k+1, l) \\ \qquad + D_{1i}x(k - \tau_1(k,l), l+1) \\ \qquad + D_{2i}x(k+1, l - \tau_2(k,l)) \\ \qquad + B_{1i}u(k, l+1) + B_{2i}u(k+1, l) \\ \qquad + E_{1i}\omega(k, l+1) + E_{2i}\omega(k+1, l)\}, \\ y(k,l) = \sum\limits_{i=1}^{r} h_i(k,l)[C_i x(k,l) + H_i \omega(k,l)], \\ z(k,l) = \sum\limits_{i=1}^{r} h_i(k,l)[G_i x(k,l) + L_i \omega(k,l)]. \end{cases} \quad (5)$$

The actual signal received by the designed controller may contain missing measurements, which can be governed by

$$\vec{y}(k,l) = \eta(k,l)y(k,l), \quad (6)$$

where $\eta(k,l) \in \mathbb{R}$ is a random white sequence taking values of 0 and 1 with

$$\text{Prob}\{\eta(k,l) = 1\} = \alpha, \quad \text{Prob}\{\eta(k,l) = 0\} = 1 - \alpha, \quad (7)$$

and $\alpha \in [0,1]$ is a known scalar. Throughout this paper, we assume that the stochastic variables $\eta(k,l)$ and $\omega(i,j)$ $(k,l,i,j \in \mathbb{Z}^+)$ are mutually independent.

In this paper, we are interested in designing a 2-D fuzzy controller for system (1) of the following form:

Controller Rule i:
IF $\theta_1^{(k,l)}$ is $\mathcal{F}_{i1}$, $\cdots$, $\theta_j^{(k,l)}$ is $\mathcal{F}_{ij}$, $\cdots$ and $\theta_p^{(k,l)}$ is $\mathcal{F}_{ip}$,
THEN

$$u(k,l) = K_i\vec{y}(k,l), \quad (8)$$

where K_i is the gain matrix of the designed controller to be determined. Similar to (5), the proposed 2-D fuzzy controller can also be fuzzily blended as

$$u(k,l) = \sum_{i=1}^{r} h_i(k,l)\eta(k,l)K_i y(k,l). \quad (9)$$

In what follows, for brevity, we define $\acute{h}_i = h_i(k, l+1)$, $\grave{h}_i = h_i(k+1, l)$ and

$$\sum_{i_1, i_2, \cdots, i_s = 1}^{r} h_{i_1} h_{i_2} \cdots h_{i_s} = \sum_{i_1=1}^{r} h_{i_1} \sum_{i_2=1}^{r} h_{i_2} \cdots \sum_{i_s=1}^{r} h_{i_s}$$

for $s \in \mathbb{Z}^+$.

Combining (9) with (5), the closed-loop 2-D fuzzy system with the static output feedback controller is governed by (10), shown at the top of the next page, which can be rewritten as

$$\begin{cases} x(k+1, l+1) = \sum\limits_{\acute{i}, \acute{j}, \acute{m}, \grave{i}, \grave{j}, \grave{m}=1}^{r} \acute{h}_{\acute{i}} \acute{h}_{\acute{j}} \acute{h}_{\acute{m}} \grave{h}_{\grave{i}} \grave{h}_{\grave{j}} \grave{h}_{\grave{m}} \underset{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}{F_1}(k,l), \\ z(k,l) = \sum\limits_{i=1}^{r} h_i(k,l)[G_i x(k,l) + L_i \omega(k,l)], \end{cases} \quad (11)$$

where

$$\underset{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}{F_1}(k,l) = A_{1\acute{i}}x(k, l+1) + A_{2\grave{i}}x(k+1, l)$$

$$
\left\{
\begin{aligned}
x(k+1,l+1) &= \sum_{i=1}^{r} \acute{h}_i \left\{ A_{1i}x(k,l+1) + D_{1i}x(k-\tau_1(k,l),l+1) + E_{1i}\omega(k,l+1) + \sum_{j,m=1}^{r} \acute{h}_j\grave{h}_m B_{1i}\eta(k,l+1) \right. \\
&\quad \left. \times K_j\left[C_m x(k,l+1) + H_m \omega(k,l+1)\right] \right\} + \sum_{i=1}^{r} \grave{h}_i \left\{ A_{2i}x(k+1,l) + D_{2i}x(k+1,l-\tau_2(k,l)) \right. \\
&\quad \left. + E_{2i}\omega(k+1,l) + \sum_{j,m=1}^{r} \grave{h}_j\grave{h}_m B_{2i}\eta(k+1,l) \times K_j\left[C_m x(k+1,l) + H_m \omega(k+1,l)\right] \right\}, \\
z(k,l) &= \sum_{i=1}^{r} h_i(k,l)\left[G_i x(k,l) + L_i \omega(k,l)\right],
\end{aligned}
\right.
\tag{10}
$$

$+D_{1\acute{i}}x(k-\tau_1(k,l),l+1) + D_{2\grave{i}}x(k+1,l-\tau_2(k,l))$
$+\alpha B_{1\acute{i}}K_{\acute{j}}(C_{\acute{m}}x(k,l+1) + H_{\acute{m}}\omega(k,l+1))$
$+(\eta(k,l+1) - \alpha)B_{1\acute{i}}K_{\acute{j}}(C_{\acute{m}}x(k,l+1) + E_{1\acute{i}}\omega(k,l+1)$
$+H_{\acute{m}}\omega(k,l+1)) + \alpha B_{2\grave{i}}K_{\grave{j}}(C_{\grave{m}}x(k+1,l)$
$+H_{\grave{m}}\omega(k+1,l)) + (\eta(k+1,l) - \alpha)B_{2\grave{i}}K_{\grave{j}}$
$\times(C_{\grave{m}}x(k+1,l) + H_{\grave{m}}\omega(k+1,l)) + E_{2\grave{i}}\omega(k+1,l).$

Definition 1. *The 2-D fuzzy system (10) with $\omega(k,l) \equiv 0$ is said to be globally asymptotically stable in the mean-square sense if*

$$
\lim_{k+l\to\infty} \mathbb{E}\left\{\|x(k,l)\|^2\right\} = 0
\tag{12}
$$

holds for all the initial condition in the form of (2). In this case, the 2-D fuzzy controller (8) is said to be a globally asymptotically stable fuzzy controller for the 2-D system (1).

Definition 2. *Given scalar $\gamma > 0$, the controller (8) is said to be an H_∞ fuzzy controller for the 2-D system (1) with output measurements (6) and disturbance attenuation level γ if the dynamics of system (10) is globally asymptotically stable in the mean-square sense and, under the zero initial conditions, the following performance constraint*

$$
\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\mathbb{E}\left\{\left\|\begin{bmatrix} z(k,l+1) \\ z(k+1,l) \end{bmatrix}\right\|^2\right\}
$$
$$
\leq \quad \gamma^2\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\mathbb{E}\left\{\left\|\begin{bmatrix} \omega(k,l+1) \\ \omega(k+1,l) \end{bmatrix}\right\|^2\right\}
\tag{13}
$$

holds for all nonzero sequences $\{\omega(\cdot,\cdot)\}$.

III. MAIN RESULTS

Before proceeding, let us recall the following lemmas which will be used in the sequel.

Lemma 1. *Let $R \in \mathbb{R}^{n\times n}$ be a symmetric positive definite matrix. For any real vectors $X_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}} \in \mathbb{R}^n$ and $X_{\acute{a}\acute{b}\acute{c}\grave{a}\grave{b}\grave{c}} \in \mathbb{R}^n$ with $\acute{i},\acute{j},\acute{m},\grave{i},\grave{j},\grave{m},\acute{a},\acute{b},\acute{c},\grave{a},\grave{b},\grave{c} \in \mathbb{S}$, we have*

$$
\sum_{\acute{i},\acute{j},\acute{m},\acute{a},\acute{b},\acute{c},\grave{i},\grave{j},\grave{m},\grave{a},\grave{b},\grave{c}=1}^{r} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{m}}\acute{h}_{\acute{a}}\acute{h}_{\acute{b}}\acute{h}_{\acute{c}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}}\grave{h}_{\grave{a}}\grave{h}_{\grave{b}}\grave{h}_{\grave{c}}
$$
$$
\times X^T_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}RX_{\acute{a}\acute{b}\acute{c}\grave{a}\grave{b}\grave{c}}
$$
$$
\leq \sum_{\acute{i},\acute{j},\acute{m},\grave{i},\grave{j},\grave{m}=1}^{r} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{m}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}}X^T_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}RX_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}},
$$

where $\acute{h}_t \geq 0$, $\grave{h}_t \geq 0$, $\sum_{t=1}^{r}\acute{h}_t = \sum_{t=1}^{r}\grave{h}_t = 1$ with $t \in \mathbb{S}$.

Proof: Similar to the proof of Lemma 2 in [15], based on the well-known inequality

$$
2X^T RY \leq X^T RX + Y^T RY,
$$

where X and Y are any vectors belonging to $\mathbb{R}^n$, one can easily have

$$
2 \sum_{\acute{i},\acute{j},\acute{m},\acute{a},\acute{b},\acute{c},\grave{i},\grave{j},\grave{m},\grave{a},\grave{b},\grave{c}=1}^{r} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{m}}\acute{h}_{\acute{a}}\acute{h}_{\acute{b}}\acute{h}_{\acute{c}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}}\grave{h}_{\grave{a}}\grave{h}_{\grave{b}}\grave{h}_{\grave{c}}
$$
$$
\times X^T_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}RX_{\acute{a}\acute{b}\acute{c}\grave{a}\grave{b}\grave{c}}
$$
$$
\leq \sum_{\acute{i},\acute{j},\acute{m},\acute{a},\acute{b},\acute{c},\grave{i},\grave{j},\grave{m},\grave{a},\grave{b},\grave{c}=1}^{r} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{m}}\acute{h}_{\acute{a}}\acute{h}_{\acute{b}}\acute{h}_{\acute{c}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}}\grave{h}_{\grave{a}}\grave{h}_{\grave{b}}\grave{h}_{\grave{c}}
$$
$$
\times (X^T_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}RX_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}} + X^T_{\acute{a}\acute{b}\acute{c}\grave{a}\grave{b}\grave{c}}RX_{\acute{a}\acute{b}\acute{c}\grave{a}\grave{b}\grave{c}})
$$
$$
= 2 \sum_{\acute{i},\acute{j},\acute{m},\grave{i},\grave{j},\grave{m}=1}^{r} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{m}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}}X^T_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}RX_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}},
$$

which completes the proof. ∎

In this section, both the stability analysis and the H_∞ performance for the 2-D closed-loop fuzzy system will be discussed and then a detailed process for the design of the proposed H_∞ controller will be further presented. Finally, a cone complementarity linearization (CCL) algorithm is employed to overcome the numerical difficulty caused by the several matrix equality constraints presented in the main results.

A. Stability analysis for the 2-D fuzzy system with missing measurements

We begin with the stability analysis for the 2-D fuzzy system (10) when the controller gain K_i ($i = 1, 2, \cdots, r$) are given. The following theorem presents a sufficient condition under which the closed-loop 2-D fuzzy system (10) is globally asymptotically stable in the mean-square sense.

Theorem 1. *The closed-loop 2-D fuzzy system (10) with the given controller structure (8) is globally asymptotically stable in the mean-square sense if there exist matrices $Q^h > 0$, $Q^v > 0$, $P^h > 0$, $P^v > 0$, $R^h > 0$, $R^v > 0$, J_{kl}, L_{kl} ($k,l = 1,2$) and K_j ($j \in \mathbb{S}$) such that the matrix inequalities (14) and (15) hold,*

$$
R^h > P^h, \quad R^v > P^v,
\tag{14}
$$

$$\Xi = \begin{bmatrix} \Lambda & * \\ \underline{\Xi}_1 & \underline{\Xi}_2 \end{bmatrix} < 0, \qquad (15)$$

where $\acute{i}, \acute{j}, \acute{m}, \grave{i}, \grave{j}, \grave{m} \in \mathbb{S}$, $\bar{\alpha} = \alpha(1 - \alpha)$, $\mathcal{I} =$ diag$\{\widetilde{\tau}_1 I_{n_x}, \ \widetilde{\tau}_2 I_{n_x}\}$, $\widetilde{\tau}_1 = \overline{\tau}_1 - \underline{\tau}_1$, $\widetilde{\tau}_2 = \overline{\tau}_2 - \underline{\tau}_2$, $\Xi_1 = \begin{bmatrix} \Xi_2^T & \Xi_3^T & \Xi_4^T & \Xi_5^T & \Xi_{67}^T & \Xi_8^T \end{bmatrix}^T$, $\underline{\Xi}_2 = $ diag$\{-\mathcal{I}R_2, -\mathcal{I}R_2, -\bar{\alpha}^{-1}\widetilde{Q}^{-1}, -\bar{\alpha}^{-1}\widetilde{Q}^{-1}, -R_1^{-1}, -\widehat{Q}^{-1}\}$,

$$\widehat{R}_1^h = \widetilde{\tau}_1 R^h + P^h, \ \ \widehat{R}_2^h = R^h - P^h, \ \ \widehat{R}_1^v = \widetilde{\tau}_2 R^v + P^v,$$
$$\widehat{R}_2^v = R^v - P^v, \ \ R_2 = \text{diag}\{\widehat{R}_2^h, \ \widehat{R}_2^v\}, \ \ \widehat{Q} = Q^h + Q^v,$$
$$\widetilde{Q} = \widehat{Q} + \widehat{R}_1^h + \widehat{R}_1^v, \ \ \Xi_2 = \mathcal{I}J_\Xi^T, \ \ \Xi_3 = \mathcal{I}L_\Xi^T,$$
$$\Xi_4 = \begin{bmatrix} B_{1\acute{i}}K_{\acute{j}}C_{\acute{m}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\Xi_5 = \begin{bmatrix} 0 & B_{2\grave{i}}K_{\grave{j}}C_{\grave{m}} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\Xi_6 = \begin{bmatrix} \Xi_{6,1} & \Xi_{6,2} & 0 & 0 & D_{1\acute{i}} & D_{2\grave{i}} & 0 & 0 \end{bmatrix},$$
$$\Xi_7 = \begin{bmatrix} \Xi_{7,1} & \Xi_{7,2} & 0 & 0 & D_{1\acute{i}} & D_{2\grave{i}} & 0 & 0 \end{bmatrix},$$
$$\Xi_{67} = \begin{bmatrix} \Xi_6^T & \Xi_7^T \end{bmatrix}^T, \ \ \Lambda_{11} = -Q^h, \ \ \Lambda_{33} = J_{11} + J_{11}^T$$
$$\Xi_8 = \begin{bmatrix} \Xi_{7,1} & \Xi_{6,2} & 0 & 0 & D_{1\acute{i}} & D_{2\grave{i}} & 0 & 0 \end{bmatrix},$$
$$R_1 = \text{diag}\{\widehat{R}_1^h, \ \widehat{R}_1^v\}, \ \ \Lambda_{22} = -Q^v, \ \ \Lambda_{44} = J_{21} + J_{21}^T,$$
$$\Xi_{6,1} = A_{1\acute{i}} - I + \alpha B_{1\acute{i}}K_{\acute{j}}C_{\acute{m}}, \ \ \Lambda_{53} = J_{12} - J_{11}^T,$$
$$\Xi_{6,2} = A_{2\grave{i}} + \alpha B_{2\grave{i}}K_{\grave{j}}C_{\grave{m}}, \ \ \Lambda_{64} = J_{22} - J_{21}^T$$
$$\Xi_{7,1} = A_{1\acute{i}} + \alpha B_{1\acute{i}}K_{\acute{j}}C_{\acute{m}}, \ \ \Lambda_{75} = L_{12} - L_{11}^T,$$
$$\Xi_{7,2} = A_{2\grave{i}} - I + \alpha B_{2\grave{i}}K_{\grave{j}}C_{\grave{m}}, \ \ \Lambda_{77} = -L_{12} - L_{12}^T,$$
$$J_\Xi = \begin{bmatrix} 0 & 0 & 0 & J_{21}^T & 0 & J_{22}^T & 0 & 0 \\ 0 & 0 & J_{11}^T & 0 & J_{12}^T & 0 & 0 & 0 \end{bmatrix}^T,$$
$$L_\Xi = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & L_{21}^T & 0 & L_{22}^T \\ 0 & 0 & 0 & 0 & L_{11}^T & 0 & L_{12}^T & 0 \end{bmatrix}^T,$$
$$\Lambda_{55} = -J_{12} + L_{11} - J_{12}^T + L_{11}^T, \ \ \Lambda_{86} = L_{22} - L_{21}^T,$$
$$\Lambda_{66} = -J_{22} + L_{21} - J_{22}^T + L_{21}^T, \ \ \Lambda_{88} = -L_{22} - L_{22}^T.$$

Proof: In order to establish the globally asymptotic stability of the closed-loop 2-D fuzzy system (10) with $\omega(\cdot, \cdot) = 0$, let us define the following energy-like functional

$$\begin{aligned} V(k, l) &= V_1^h(k, l) + V_2^h(k, l) + V_3^h(k, l) \\ &\quad + V_1^v(k, l) + V_2^v(k, l) + V_3^v(k, l) \end{aligned} \qquad (16)$$

with

$$V_1^h(k, l) = x^T(k, l)Q^h x(k, l), V_1^v(k, l) = x^T(k, l)Q^v x(k, l),$$

$$V_2^h(k, l) = \sum_{\tau = -\tau_1(k,l)}^{-1} f^T(k + \tau, l)P^h f(k + \tau, l),$$

$$V_2^v(k, l) = \sum_{\tau = -\tau_2(k,l)}^{-1} g^T(k, l + \tau)P^v g(k, l + \tau),$$

$$V_3^h(k, l) = \sum_{d = -\overline{\tau}_1}^{-\underline{\tau}_1 - 1} \sum_{\tau = d}^{-1} f^T(k + \tau, l)R^h f(k + \tau, l),$$

$$V_3^v(k, l) = \sum_{d = -\overline{\tau}_2}^{-\underline{\tau}_2 - 1} \sum_{\tau = d}^{-1} g^T(k, l + \tau)R^v g(k, l + \tau),$$

$$f(k + \tau, l) = x(k + \tau + 1, l) - x(k + \tau, l),$$
$$g(k, l + \tau) = x(k, l + \tau + 1) - x(k, l + \tau).$$

Define the following index

$$\mathfrak{I}(k, l) = \mathfrak{I}^h(k, l) + \mathfrak{I}^v(k, l), \qquad (17)$$

where the expressions of $\mathfrak{I}^h(k, l)$ and $\mathfrak{I}^v(k, l)$ are shown in (18). Calculating along the trajectories of system (11), one has (19)-(21). Similarly, it can be obtained (22)-(24) (Notice: (18)-(24) are shown on page 6).

From the inequalities (19)-(21), we have

$$\mathfrak{I}^h(k, l) \leq \mathbb{E}\Bigg\{ \bigg(x^T(k + 1, l + 1)Q^h x(k + 1, l + 1) $$
$$- x^T(k, l + 1)Q^h x(k, l + 1) + f^T(k, l + 1)\widehat{R}_1^h$$
$$\times f(k, l + 1) - \bigg(\sum_{d = -\tau_1(k,l)}^{-\underline{\tau}_1 - 1} + \sum_{d = -\overline{\tau}_1}^{-\tau_1(k,l) - 1} \bigg)$$
$$\times f^T(k + d, l + 1)\widehat{R}_2^h f(k + d, l + 1) \bigg) \bigg| \hbar(k, l) \Bigg\}$$

and, similarly, one has

$$\mathfrak{I}^v(k, l) \leq \mathbb{E}\Bigg\{ \bigg(x^T(k + 1, l + 1)Q^v x(k + 1, l + 1)$$
$$- x^T(k + 1, l)Q^v x(k + 1, l) + g^T(k + 1, l)\widehat{R}_1^v$$
$$\times g(k + 1, l) - \bigg(\sum_{d = -\tau_2(k,l)}^{-\underline{\tau}_2 - 1} + \sum_{d = -\overline{\tau}_2}^{-\tau_2(k,l) - 1} \bigg)$$
$$\times g^T(k + 1, l + d)\widehat{R}_2^v g(k + 1, l + d) \bigg) \bigg| \hbar(k, l) \Bigg\}.$$

Before proceeding further, it is easy to show that the equalities (25) (shown on page 6) hold for any matrixes J_{11}, J_{12}, J_{21}, J_{22}, L_{11}, L_{12}, L_{21} and L_{22}. Letting

$$\bar{J}_1 = \begin{bmatrix} 0 & J_{11}^T & J_{12}^T & 0 \end{bmatrix}^T, \ \ \bar{J}_2 = \begin{bmatrix} 0 & J_{21}^T & J_{22}^T & 0 \end{bmatrix}^T,$$
$$\bar{L}_1 = \begin{bmatrix} 0 & 0 & L_{11}^T & L_{12}^T \end{bmatrix}^T, \ \ \bar{L}_2 = \begin{bmatrix} 0 & 0 & L_{21}^T & L_{22}^T \end{bmatrix}^T,$$
$$\mathscr{I}_1 = \begin{bmatrix} 0 & I_{n_x} & -I_{n_x} & 0 \end{bmatrix}, \ \ \mathscr{I}_2 = \begin{bmatrix} 0 & 0 & I_{n_x} & -I_{n_x} \end{bmatrix},$$
$$\phi = \begin{bmatrix} x(k, l + 1) \\ x(k - \underline{\tau}_1, l + 1) \\ x(k - \tau_1(k, l), l + 1) \\ x(k - \overline{\tau}_1, l + 1) \end{bmatrix}, \ \ \varphi = \begin{bmatrix} x(k + 1, l) \\ x(k + 1, l - \underline{\tau}_2) \\ x(k + 1, l - \tau_2(k, l)) \\ x(k + 1, l - \overline{\tau}_2) \end{bmatrix},$$

one obtains (26) and (27) (shown on page 6), where conditions $\underline{\tau}_i \leq \tau(k, l) \leq \overline{\tau}_i$ $(i = 1, 2)$ and (14) have been utilized in the second step when deriving (26) and (27). It should be noted that inequalities constrains (14) ensures that $\widehat{R}_2^h > 0$ and $\widehat{R}_2^v > 0$.

Combining the inequalities (26) and (27) together and noting that $f(k, l) = x(k + 1, l) - x(k, l)$, $g(k, l) = x(k, l + 1) - x(k, l)$, it can be calculated along the closed-loop system (11) that

$$\mathfrak{I}(k, l) \leq \sum_{\acute{i}, \acute{j}, \acute{m}, \acute{a}, \acute{b}, \acute{c}, \grave{i}, \grave{j}, \grave{m}, \grave{a}, \grave{b}, \grave{c} = 1}^{r} \acute{h}_{\acute{i}} \acute{h}_{\acute{j}} \acute{h}_{\acute{m}} \acute{h}_{\acute{a}} \acute{h}_{\acute{b}} \acute{h}_{\acute{c}} \grave{h}_{\grave{i}} \grave{h}_{\grave{j}} \grave{h}_{\grave{m}}$$
$$\times \grave{h}_{\grave{a}} \grave{h}_{\grave{b}} \grave{h}_{\grave{c}} \mathbb{E}\Bigg\{ \bigg(\overline{F}_{1 \atop \acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}^{\ T}(k, l)\widehat{Q} \ \overline{F}_{1 \atop \acute{a}\acute{b}\acute{c}\grave{a}\grave{b}\grave{c}}(k, l)$$
$$+ \overline{F}_{2 \atop \acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}^{\ T}(k, l)\widehat{R}_1^h \ \overline{F}_{2 \atop \acute{a}\acute{b}\acute{c}\grave{a}\grave{b}\grave{c}}(k, l)$$

$$\begin{cases} \mathfrak{I}^h(k,l) = \mathbb{E}\left\{ \left(\Delta V_1^h(k,l) + \Delta V_2^h(k,l) + \Delta V_3^h(k,l) \right) \Big| \hbar(k,l) \right\}, \\ \mathfrak{I}^v(k,l) = \mathbb{E}\left\{ \left(\Delta V_1^v(k,l) + \Delta V_2^v(k,l) + \Delta V_3^v(k,l) \right) \Big| \hbar(k,l) \right\}, \\ \Delta V_i^h(k,l) = V_i^h(k+1,l+1) - V_i^h(k,l+1), \quad \iota = 1,2,3, \\ \Delta V_j^v(k,l) = V_j^v(k+1,l+1) - V_j^v(k+1,l), \quad \kappa = 1,2,3, \\ \hbar(k,l) = \{ x(k,l+1),\ x(k-1,l+1),\ x(k-2,l+1),\cdots,x(k-\overline{\tau}_1,l+1), \\ \qquad\qquad x(k+1,l),\ x(k+1,l-1),\ x(k+1,l-2),\cdots,x(k+1,l-\overline{\tau}_2) \}, \end{cases} \tag{18}$$

$$\Delta V_1^h(k,l) = x^T(k+1,l+1)Q^h x(k+1,l+1) - x^T(k,l+1)Q^h x(k,l+1), \tag{19}$$

$$\Delta V_2^h(k,l) \le f^T(k,l+1)P^h f(k,l+1) + \left(\sum_{\tau=-\tau_1(k,l)}^{-\underline{\tau}_1-1} + \sum_{\tau=-\overline{\tau}_1}^{-\tau_1(k,l)-1} \right) f^T(k+\tau,l+1)P^h f(k+\tau,l+1), \tag{20}$$

$$\Delta V_3^h(k,l) = \widetilde{\tau}_1 f^T(k,l+1)R^h f(k,l+1) - \left(\sum_{d=-\tau_1(k,l)}^{-\underline{\tau}_1-1} + \sum_{d=-\overline{\tau}_1}^{-\tau_1(k,l)-1} \right) f^T(k+d,l+1)R^h f(k+d,l+1). \tag{21}$$

$$\Delta V_1^v(k,l) = x^T(k+1,l+1)Q^v x(k+1,l+1) - x^T(k+1,l)Q^v x(k+1,l), \tag{22}$$

$$\Delta V_2^v(k,l) \le g^T(k+1,l)P^v g(k+1,l) + \left(\sum_{-\tau_2(k,l)}^{-\underline{\tau}_2-1} + \sum_{\tau=-\overline{\tau}_2}^{-\tau_2(k,l)-1} \right) g^T(k+1,l+\tau)P^v g(k+1,l+\tau), \tag{23}$$

$$\Delta V_3^v(k,l) = \widetilde{\tau}_2 g^T(k+1,l)R^v g(k+1,l) - \left(\sum_{d=-\tau_2(k,l)}^{-\underline{\tau}_2-1} + \sum_{d=-\overline{\tau}_2}^{-\tau_2(k,l)-1} \right) g^T(k+1,l+d)R^v g(k+1,l+d). \tag{24}$$

$$\left[x^T(k-\underline{\tau}_1,l+1)J_{11} + x^T(k-\tau_1(k,l),l+1)J_{12} \right]\!\left[x(k-\underline{\tau}_1,l+1) - x(k-\tau_1(k,l),l+1) - \sum_{d=-\tau_1(k,l)}^{-\underline{\tau}_1-1} f(k+d,l+1) \right] = 0,$$

$$\left[x^T(k-\tau_1(k,l),l+1)L_{11} + x^T(k-\overline{\tau}_1,l+1)L_{12} \right]\!\left[x(k-\tau_1(k,l),l+1) - x(k-\overline{\tau}_1,l+1) - \sum_{d=-\overline{\tau}_1}^{-\tau_1(k,l)-1} f(k+d,l+1) \right] = 0,$$

$$\tag{25}$$

$$\left[x^T(k+1,l-\underline{\tau}_2)J_{21} + x^T(k+1,l-\tau_2(k,l))J_{22} \right]\!\left[x(k+1,l-\underline{\tau}_2) - x(k+1,l-\tau_2(k,l)) - \sum_{d=-\tau_2(k,l)}^{-\underline{\tau}_2-1} g(k+1,l+d) \right] = 0,$$

$$\left[x^T(k+1,l-\tau_2(k,l))L_{21} + x^T(k+1,l-\overline{\tau}_2)L_{22} \right]\!\left[x(k+1,l-\tau_2(k,l)) - x(k+1,l-\overline{\tau}_2) - \sum_{d=-\overline{\tau}_2}^{-\tau_2(k,l)-1} g(k+1,l+d) \right] = 0.$$

$$\mathfrak{I}^h(k,l) \le \mathbb{E}\Bigg\{ \Bigg(x^T(k+1,l+1)Q^h x(k+1,l+1) - x^T(k,l+1)Q^h x(k,l+1) + f^T(k,l+1)\widehat{R}_1^h f(k,l+1)$$
$$+ 2\phi^T \bar{J}_1 \mathscr{I}_1 \phi + 2\phi^T \bar{L}_1 \mathscr{I}_2 \phi + \widetilde{\tau}_1\ \phi^T\left(\bar{J}_1 \left(\widehat{R}_2^h\right)^{-1} \bar{J}_1^T + \bar{L}_1\left(\widehat{R}_2^h\right)^{-1}\bar{L}_1^T \right)\phi\ \Bigg) \Bigg| \hbar(k,l) \Bigg\}, \tag{26}$$

$$\mathfrak{I}^v(k,l) \le \mathbb{E}\Bigg\{ \Bigg(x^T(k+1,l+1)Q^v x(k+1,l+1) - x^T(k+1,l)Q^v x(k+1,l) + g^T(k+1,l)\widehat{R}_1^v g(k+1,l)$$
$$+ 2\varphi^T \bar{J}_2 \mathscr{I}_1 \varphi + 2\varphi^T \bar{L}_2 \mathscr{I}_2 \varphi + \widetilde{\tau}_2\ \varphi^T\left(\bar{J}_2 \left(\widehat{R}_2^v\right)^{-1} \bar{J}_2^T + \bar{L}_2\left(\widehat{R}_2^v\right)^{-1}\bar{L}_2^T \varphi \right) \Bigg) \Bigg| \hbar(k,l) \Bigg\}. \tag{27}$$

$$+\widetilde{\tau}_1\phi^T\Big(\bar{J}_1(\widehat{R}_2^h)^{-1}\bar{J}_1^T+\bar{L}_1(\widehat{R}_2^h)^{-1}\bar{L}_1^T\Big)\phi$$

$$+\widetilde{\tau}_2\varphi^T\Big(\bar{J}_2(\widehat{R}_2^v)^{-1}\bar{J}_2^T+\bar{L}_2(\widehat{R}_2^v)^{-1}\bar{L}_2^T\Big)\varphi$$

$$+\overline{F}_3{}^T_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)\widehat{R}_1^v\,\overline{F}_3{}_{\acute{a}\acute{b}\acute{c}\grave{a}\grave{b}\grave{c}}(k,l)+\beth(k,l)\Big)\Big|\hbar(k,l)\Big\}, \quad (28)$$

where $\beth(k,l)=-x^T(k,l+1)Q^hx(k,l+1)-x^T(k+1,l)Q^vx(k+1,l)+2\phi^T\Big(\bar{J}_1\mathscr{I}_1+\bar{L}_1\mathscr{I}_2\Big)\phi+2\varphi^T\Big(\bar{J}_2\mathscr{I}_1+\bar{L}_2\mathscr{I}_2\Big)\varphi$, $\overline{F}_2{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)=\overline{F}_1{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)-x(k,l+1)$,

$$\overline{F}_1{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)=\Big(A_{1\acute{i}}+\alpha B_{1\acute{i}}K_{\acute{j}}C_{\acute{m}}\Big)x(k,l+1)$$

$$+\Big(A_{2\grave{i}}+\alpha B_{2\grave{i}}K_{\grave{j}}C_{\grave{m}}\Big)x(k+1,l)$$
$$+D_{1\acute{i}}x(k-\tau_1(k,l),l+1)$$
$$+D_{2\grave{i}}x(k+1,l-\tau_2(k,l))$$
$$+(\eta(k,l+1)-\alpha)B_{1\acute{i}}K_{\acute{j}}C_{\acute{m}}x(k,l+1)$$
$$+(\eta(k+1,l)-\alpha)B_{2\grave{i}}K_{\grave{j}}C_{\grave{m}}x(k+1,l),$$

$$\overline{F}_3{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)=\overline{F}_1{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)-x(k+1,l),$$

and $\overline{F}_i{}_{\acute{a}\acute{b}\acute{c}\grave{a}\grave{b}\grave{c}}(k,l)$ (i=1,2,3) have the similar representations as $\overline{F}_1{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)$ (i=1,2,3) by just substituting $\acute{i}$, $\acute{j}$, $\acute{m}$, $\grave{i}$, $\grave{j}$ and $\grave{m}$ with $\acute{a}$, $\acute{b}$, $\acute{c}$, $\grave{a}$, $\grave{b}$ and $\grave{c}$, respectively.

Subsequently, it follows from Lemma 1 that

$$\Im(k,l)\leq$$
$$\sum_{\acute{i},\acute{j},\acute{m},\grave{i},\grave{j},\grave{m}=1}^{r}\acute{h}_i\acute{h}_j\acute{h}_{\acute{m}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}}\mathbb{E}\Bigg\{\Bigg(\Big(\beth(k,l)+\widetilde{\tau}_1\phi^T\Big(\bar{J}_1(\widehat{R}_2^h)^{-1}\bar{J}_1^T$$
$$+\bar{L}_1(\widehat{R}_2^h)^{-1}\bar{L}_1^T\Big)\phi+\widetilde{\tau}_2\varphi^T\Big(\bar{J}_2(\widehat{R}_2^v)^{-1}\bar{J}_2^T+\bar{L}_2(\widehat{R}_2^v)^{-1}\bar{L}_2^T\Big)\varphi$$
$$+\bar{\alpha}\Big[B_{1\acute{i}}K_{\acute{j}}C_{\acute{m}}x(k,l+1)\Big]^T\widetilde{Q}\Big[B_{1\acute{i}}K_{\acute{j}}C_{\acute{m}}x(k,l+1)\Big]$$
$$+\bar{\alpha}\Big[B_{2\grave{i}}K_{\grave{j}}C_{\grave{m}}x(k+1,l)\Big]^T\widetilde{Q}\Big[B_{2\grave{i}}K_{\grave{j}}C_{\grave{m}}x(k+1,l)\Big]$$
$$+\widetilde{F}_2{}^T_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)\,\widehat{R}_1^h\,\widetilde{F}_2{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)+\widetilde{F}_3{}^T_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)\widehat{R}_1^v$$
$$\times\widetilde{F}_3{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)+\widetilde{F}_1{}^T_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)\widehat{Q}\,\widetilde{F}_1{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)\Bigg)\Bigg|\hbar(k,l)\Bigg\}.$$

where the relationship (7) is utilized in the last step when deriving the above inequality, in which

$$\widetilde{F}_1{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)=\Big(A_{1\acute{i}}+\alpha B_{1\acute{i}}K_{\acute{j}}C_{\acute{m}}\Big)x(k,l+1)$$
$$+\Big(A_{2\grave{i}}+\alpha B_{2\grave{i}}K_{\grave{j}}C_{\grave{m}}\Big)x(k+1,l)$$
$$+D_{1\acute{i}}x(k-\tau_1(k,l),l+1)+D_{2\grave{i}}x(k+1,l-\tau_2(k,l)),$$
$$\widetilde{F}_2{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)=\widetilde{F}_1{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)-x(k,l+1),$$
$$\widetilde{F}_3{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)=\widetilde{F}_1{}_{\acute{i}\acute{j}\acute{m}\grave{i}\grave{j}\grave{m}}(k,l)-x(k+1,l).$$

Defining $\bar{\zeta}(k,l)=[x^T(k,l+1)\ x^T(k+1,l)\ x^T(k-\underline{\tau}_1,l+1)\ x^T(k+1,l-\underline{\tau}_2)\ x^T(k-\tau_1(k,l),l+1)\ x^T(k+1,l-$

$\tau_2(k,l))\ x^T(k-\overline{\tau}_1,l+1)\ x^T(k+1,l-\overline{\tau}_2)]^T$, it can be observed from (29) that

$$\Im(k,l)$$
$$\leq\lambda_{\max}\Big(\widetilde{\Xi}\Big)\sum_{\acute{i},\acute{j},\acute{m},\grave{i},\grave{j},\grave{m}=1}^{r}\acute{h}_i\acute{h}_j\acute{h}_{\acute{m}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}}\mathbb{E}\Big\{\big\|\bar{x}(k,l)\big\|^2\Big|\hbar(k,l)\Big\}(29)$$

where $\widetilde{\Xi}=\Lambda+\Xi_2^TR_2{}^{-1}\mathcal{I}^{-1}\Xi_2+\Xi_3^TR_2{}^{-1}\mathcal{I}^{-1}\Xi_3+\bar{\alpha}\Xi_4^T\widetilde{Q}\Xi_4+\bar{\alpha}\Xi_5^T\widetilde{Q}\Xi_5+\Xi_{67}^TR_1\Xi_{67}+\Xi_8^T\widehat{Q}\Xi_8$, $\bar{x}(k,l)=[x^T(k,l+1)\ x^T(k+1,l)]^T$ and $\lambda_{\max}\Big(\widetilde{\Xi}\Big)$ is the maximal eigenvalue of $\widetilde{\Xi}$. It should be noted that the Schur Complement Lemma has been utilized to get $\widetilde{\Xi}<0$ from the inequality condition $\Xi<0$ in (15).

For any integer $M>1$ and $N>1$, summing up both sides of the inequality (29) for k and l varying from 0 to, respectively, M and N, we obtain

$$\sum_{l=0}^{N}\sum_{k=0}^{M}\Im(k,l)\leq\lambda_{\max}\Big(\widetilde{\Xi}\Big)\sum_{l=0}^{N}\sum_{k=0}^{M}\sum_{\acute{i},\acute{j},\acute{m},\grave{i},\grave{j},\grave{m}=1}^{r}\acute{h}_i\acute{h}_j\acute{h}_{\acute{m}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}}$$
$$\times\mathbb{E}\Big\{\big\|\bar{x}(k,l)\big\|^2\Big|\hbar(k,l)\Big\}, \quad (30)$$

which immediately infers the following inequality by further considering the nonnegativeness of the energy-like functional $V(k,l)$:

$$\sum_{l=0}^{N}\sum_{k=0}^{M}\sum_{\acute{i},\acute{j},\acute{m},\grave{i},\grave{j},\grave{m}=1}^{r}\acute{h}_i\acute{h}_j\acute{h}_{\acute{m}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}}\mathbb{E}\Big\{\big\|\bar{x}(k,l)\big\|^2\Big|\hbar(k,l)\Big\}\leq$$
$$\frac{-1}{\lambda_{\max}\Big(\widetilde{\Xi}\Big)}\Bigg\{\sum_{l=0}^{N}\mathbb{E}\Big\{\big(V_1^h(0,l+1)+V_2^h(0,l+1)$$
$$+V_3^h(0,l+1)\big)\Big|\hbar(k,l)\Big\}+\sum_{k=0}^{M}\mathbb{E}\Big\{\big(V_1^v(k+1,0)$$
$$+V_2^v(k+1,0)+V_3^v(k+1,0)\big)\Big|\hbar(k,l)\Big\}\Bigg\}. \quad (31)$$

Taking the mathematical expectation on both sides of (31) and further considering the finite initial boundary condition (2), we conclude that

$$\sum_{l=0}^{N}\sum_{k=0}^{M}\sum_{\acute{i},\acute{j},\acute{m},\grave{i},\grave{j},\grave{m}=1}^{r}\acute{h}_i\acute{h}_j\acute{h}_{\acute{m}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}}\mathbb{E}\Big\{\big\|\bar{x}(k,l)\big\|^2\Big\}<\infty, \quad (32)$$

which implies that $\lim_{k+l\to\infty}\mathbb{E}\Big\{\big\|x(k,l)\big\|^2\Big\}=0$. According to Definition 1, it is concluded that the closed-loop 2-D fuzzy system (10) is globally asymptotically stable in the mean-square sense. The proof is complete. ∎

Remark 2. *As is well known, delay-independent condition will inevitably bring conservativeness, especially when the delay is very large. Here, we choose a delay-dependent Lyapunov functional associated with both the lower and the upper bound of the time-varying delay $\tau_i(k,l)$ $(i=1,2)$, which will undoubtedly lead to less conservativeness. Moreover, the matrix inequality constraint (15) derived by introducing the free-weighting matrix method is actually a delay-range-dependent stability criterion, which has proven to be an*

effective approach in stability analysis of time-delay systems, see e.g. [35].

B. H_∞ performance analysis for the 2-D fuzzy system with missing measurements

In Theorem 1, a general condition has been derived to guarantee the globally asymptotic stability of the closed-loop 2-D fuzzy system (10). Based upon the above stability analysis, we now deal with the capacity of disturbance attenuation and rejection as well as the tolerance level to the missing measurements with the designed 2-D controller (8). The sufficient condition obtained in the following theorem is to make sure that the fuzzy system (10) is asymptotically stable with a prescribed H_∞ performance index for non-zero exogenous disturbances and possible missing measurements under the zero-initial condition.

Theorem 2. *The closed-loop 2-D fuzzy system (10) with the designed controller structure (8) is globally asymptotically stable with disturbance attenuation level γ if there exist matrices $Q^h > 0$, $Q^v > 0$, $P^h > 0$, $P^v > 0$, $R^h > 0$, $R^v > 0$, J_{kl}, L_{kl} $(k, l = 1, 2)$ and K_j $(j \in \mathbb{S})$ such that the matrix inequality (14) and the matrix inequalities (33) hold,*

$$\Omega = \begin{bmatrix} \bar{\Lambda} & * \\ \underline{\Omega}_1 & \underline{\Omega}_2 \end{bmatrix} < 0, \qquad (33)$$

where $\acute{i}, \acute{j}, \acute{m}, \grave{i}, \grave{j}, \grave{m} \in \mathbb{S}$, $\underline{\Omega}_1 = \begin{bmatrix} \Omega_2^T & \Omega_3^T & \Omega_4^T & \Omega_5^T & \Omega_{67}^T & \Omega_8^T & \Omega_{910}^T \end{bmatrix}^T$, $\underline{\Omega}_2 = \mathrm{diag}\{-\mathcal{I}R_2, -\mathcal{I}R_2, -\bar{\alpha}^{-1}\widetilde{Q}^{-1}, -\bar{\alpha}^{-1}\widetilde{Q}^{-1}, -R_1^{-1}, -\widehat{Q}^{-1}, -I_{2n_z}\}$, $\Omega_2 = \mathcal{I}J^T$,

$$\Omega_4 = \begin{bmatrix} \Xi_4 & B_{1\acute{i}}K_{\acute{j}}H_{\acute{m}} & 0 \end{bmatrix}, \quad \Omega_5 = \begin{bmatrix} \Xi_5 & 0 & B_{2\grave{i}}K_{\grave{j}}H_{\grave{m}} \end{bmatrix},$$

$$\Omega_6 = \begin{bmatrix} \Xi_6 & \Omega_{6,9} & \Omega_{6,10} \end{bmatrix}, \quad \Omega_7 = \begin{bmatrix} \Xi_7 & \Omega_{6,9} & \Omega_{6,10} \end{bmatrix},$$

$$\Omega_{67} = \begin{bmatrix} \Omega_6^T & \Omega_7^T \end{bmatrix}^T, \quad \Omega_8 = \begin{bmatrix} \Xi_8 & \Omega_{6,9} & \Omega_{6,10} \end{bmatrix},$$

$$\Omega_9 = \begin{bmatrix} G_{\acute{i}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{\acute{i}} & 0 \end{bmatrix},$$

$$\Omega_{10} = \begin{bmatrix} 0 & G_{\grave{i}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_{\grave{i}} \end{bmatrix},$$

$$\Omega_{910} = \begin{bmatrix} \Omega_9^T & \Omega_{10}^T \end{bmatrix}^T, \quad \Omega_{6,9} = \alpha B_{1\acute{i}}K_{\acute{j}}H_{\acute{m}} + E_{1\acute{i}},$$

$$\Omega_{6,10} = \alpha B_{2\grave{i}}K_{\grave{j}}H_{\grave{m}} + E_{2\grave{i}}, \quad J = \begin{bmatrix} J_\Xi \\ 0_{2n_\omega \times 2n_\omega} \end{bmatrix},$$

$$L = \begin{bmatrix} L_\Xi \\ 0_{2n_\omega \times 2n_\omega} \end{bmatrix}, \quad \bar{\Lambda} = \begin{bmatrix} \Lambda & * \\ 0 & -\gamma^2 I_{2n_\omega} \end{bmatrix}, \quad \Omega_3 = \mathcal{I}L^T,$$

and other symbols are the same as defined in Theorem 1.

Proof: First of all, let us recall the relationship exposed in (30). In view of the zero initial condition, one can readily know

$$\sum_{l=0}^{N}\sum_{k=0}^{M} \mathfrak{I}(k,l) \geq 0. \qquad (34)$$

Now, define the following index function

$$\mathcal{J} = \sum_{l=0}^{N}\sum_{k=0}^{M} \left\{ \mathfrak{I}(k,l) + \mathbb{E}\left\{ \left(\tilde{z}^T(k,l)\tilde{z}(k,l) - \gamma^2 \tilde{\omega}^T(k,l)\tilde{\omega}(k,l) \right) \big| \hbar(k,l) \right\} \right\}. \qquad (35)$$

Substituting $\tilde{z}(\cdot, \cdot)$ and $\tilde{\omega}(\cdot, \cdot)$ into the equality (35) and using Lemma 1, we obtain

$$\mathcal{J} \leq \sum_{l=0}^{N}\sum_{k=0}^{M} \Bigg\{ \mathfrak{I}(k,l) + \sum_{\acute{i},\grave{i}=1}^{r} \acute{h}_{\acute{i}}\grave{h}_{\grave{i}} \mathbb{E}\Bigg\{ \Big[-\gamma^2\omega^T(k,l+1) \\ \times \omega(k,l+1) - \gamma^2\omega^T(k+1,l)\omega(k+1,l) \\ + (G_{\acute{i}}x(k,l+1) + L_{\acute{i}}\omega(k,l+1))^T (G_{\acute{i}}x(k,l+1) \\ + L_{\acute{i}}\omega(k,l+1)) + (G_{\grave{i}}x(k+1,l) + L_{\grave{i}}\omega(k+1,l))^T \\ \times (G_{\grave{i}}x(k+1,l) + L_{\grave{i}}\omega(k+1,l)) \Big] \Big| \hbar(k,l) \Bigg\} \Bigg\}.$$

Letting $\zeta(k,l) = [\bar{\zeta}^T(k,l) \quad \omega^T(k,l+1) \quad \omega^T(k+1,l)]^T$, along the similar lines for the proof of Theorem 1, it can be derived that

$$\mathbb{E}\{\mathcal{J}\} \leq \sum_{l=0}^{N}\sum_{k=0}^{M} \sum_{\acute{i},\acute{j},\acute{m},\grave{i},\grave{j},\grave{m}=1}^{r} \acute{h}_{\acute{i}}\acute{h}_{\acute{j}}\acute{h}_{\acute{m}}\grave{h}_{\grave{i}}\grave{h}_{\grave{j}}\grave{h}_{\grave{m}} \\ \times \mathbb{E}\left\{ \zeta^T(k,l)\widetilde{\Omega}\zeta(k,l) \right\}, \qquad (36)$$

where

$$\widetilde{\Omega} = \bar{\Lambda} + \Omega_2^T R_2^{-1}\mathcal{I}^{-1}\Omega_2 + \Omega_3^T R_2^{-1}\mathcal{I}^{-1}\Omega_3 \\ + \bar{\alpha}\Omega_4^T\widetilde{Q}\Omega_4 + \bar{\alpha}\Omega_5^T\widetilde{Q}\Omega_5 \\ + \Omega_{67}^T R_1\Omega_{67} + \Omega_8^T\widehat{Q}\Omega_8 + \Omega_9^T\Omega_9 + \Omega_{10}^T\Omega_{10}. \qquad (37)$$

Taking advantage of the Schur Complement again, one knows that $\mathbb{E}\{\mathcal{J}\} \leq 0$ can be inferred by the condition $\Omega < 0$ in (33), which immediately implies that inequality (13) in Definition (2) holds by further utilizing (34). Moreover, since Ξ is a principal submatrix of Ω, $\Xi < 0$ can also be inferred by $\Omega < 0$ in the inequalities (33). To this end, it follows from Theorem 1 that the closed-loop 2-D system (10) is globally asymptotically stable in the mean-square sense. The proof is now complete. ∎

C. H_∞ fuzzy controller Design for the 2-D system with missing measurements

In this subsection, we are in the position to seek the parameters of the concerned static output feedback controllers for the discussed 2-D T-S fuzzy system (10). By the aid of Matlab tool box, gain matrices in (8) can be readily obtained by solving the LMIs in the following theorem.

Theorem 3. *For the closed-loop system (10) with missing measurements (6), the 2-D controller (8) is a globally asymptotically stable H_∞ fuzzy controller if there exist matrices $\mathscr{Q}^h > 0$, $\mathscr{Q}^v > 0$, $\mathscr{P}^h > 0$, $\mathscr{P}^v > 0$, $\mathscr{R}^h > 0$, $\mathscr{R}^v > 0$, $Q^h > 0$, $Q^v > 0$, $P^h > 0$, $P^v > 0$, $R^h > 0$, $R^v > 0$, J_{kl}, L_{kl} $(k, l = 1, 2)$ and K_j $(j \in \mathbb{S})$ such that the linear matrix inequality (14) and the inequalities/equalities (38)-(40) hold,*

$$\mathscr{Q}^h Q^h = I, \quad \mathscr{P}^h P^h = I, \quad \mathscr{R}^h R^h = I, \qquad (38)$$

$$\mathscr{Q}^v Q^v = I, \quad \mathscr{P}^v P^v = I, \quad \mathscr{R}^v R^v = I, \qquad (39)$$

$$\begin{bmatrix} \bar{\Lambda} & * \\ \underline{\Gamma}_1 & \underline{\Gamma}_2 \end{bmatrix} < 0, \qquad (40)$$

where $\acute{i}, \acute{j}, \acute{m}, \grave{i}, \grave{j}, \grave{m} \in \mathbb{S}$, $\underline{\Gamma}_1 = \begin{bmatrix} \Omega_2^T & \Omega_3^T & (\mathbf{1}_6 \otimes \Omega_4)^T \end{bmatrix}$
$(\mathbf{1}_6 \otimes \Omega_5)^T \; \Omega_{67}^T \; \Omega_{67}^T \; (\mathbf{1}_2 \otimes \Omega_8)^T \; \Omega_{910}^T \end{bmatrix}^T$, $\underline{\Gamma}_2 = \text{diag}\{$
$-\mathcal{I}R_2, \; -\mathcal{I}R_2, \; -\bar{\alpha}^{-1}\widetilde{\mathscr{Q}}, \; -\bar{\alpha}^{-1}\widetilde{\mathscr{Q}}, \; -\mathscr{P}, \; -\mathscr{R}\mathcal{I}^{-1}, \; -\mathscr{Q},$
$-I_{2n_z}\}$, $\mathbf{1}_6 = [1\ 1\ 1\ 1\ 1\ 1]^T$, $\widetilde{\mathscr{Q}} = \text{diag}\{\mathscr{Q}, \; \mathscr{P}, \; \mathscr{R}\mathcal{I}^{-1}\}$,

$\mathscr{Q} = \text{diag}\{\mathscr{Q}^h, \; \mathscr{Q}^v\}$, $\mathscr{P} = \text{diag}\{\mathscr{P}^h, \; \mathscr{P}^v\}$, $\mathbf{1}_2 = [1\ 1]^T$, $\mathscr{R} = \text{diag}\{\mathscr{R}^h, \; \mathscr{R}^v\}$.

Proof: Rewrite $\widetilde{\Omega}$ in (37) as follows

$$\widehat{\Omega} = \bar{\Lambda} + \Omega_2^T R_2^{-1}\mathcal{I}^{-1}\Omega_2 + \Omega_3^T R_2^{-1}\mathcal{I}^{-1}\Omega_3$$
$$+ \bar{\alpha}(\mathbf{1}_6 \otimes \Omega_4)^T \text{diag}\{Q^h, \; Q^v, \; P^h, \; P^v, \; \widetilde{\tau}_1 R^h, \; \widetilde{\tau}_2 R^v\}$$
$$\times (\mathbf{1}_6 \otimes \Omega_4) + \bar{\alpha}(\mathbf{1}_6 \otimes \Omega_5)^T \text{diag}\{Q^h, \; Q^v, \; P^h, \; P^v,$$
$$\widetilde{\tau}_1 R^h, \; \widetilde{\tau}_2 R^v\}(\mathbf{1}_6 \otimes \Omega_5) + \Omega_{67}^T \text{diag}\{P^h, \; P^v\}\Omega_{67}$$
$$+ \Omega_{67}^T\mathcal{I}\,\text{diag}\{R^h, \; R^v\}\Omega_{67} + (\mathbf{1}_2 \otimes \Omega_8)^T$$
$$\times \text{diag}\{Q^h, \; Q^v\}(\mathbf{1}_2 \otimes \Omega_8) + \Omega_{910}^T\Omega_{910} < 0.$$

Noting that equalities (38)-(39) are equivalent to $\mathscr{Q}^h = (Q^h)^{-1}$, $\mathscr{Q}^v = (Q^v)^{-1}$, $\mathscr{P}^h = (P^h)^{-1}$, $\mathscr{P}^v = (P^v)^{-1}$, $\mathscr{R}^h = (R^h)^{-1}$, $\mathscr{R}^v = (R^v)^{-1}$, and by using the Schur Complement again, it can be observed that $\Omega < 0$ holds if and only if the LMIs in the inequalities (38)-(40) hold, and the proof is then complete. ∎

Remark 3. *It is worth pointing out that there are six matrix equality constraints in Theorem 3, and this gives rise to considerable difficulty for numerical computation. Fortunately, the cone complementarity linearization (CCL) algorithm is capable of resolving this problem tactically. The CCL algorithm has been well developed and applied to solve non-strict linear matrix inequalities, see e.g. [8], [10], [36]. Similar to the approaches used in [9], [14], [37], the non-convex feasibility problem in Theorem 3 can be converted to the following nonlinear minimization problem*

$$\min \text{tr}\big(\mathscr{Q}^h Q^h + \mathscr{Q}^v Q^v + \mathscr{P}^h P^h$$
$$+ \mathscr{P}^v P^v + \mathscr{R}^h R^h + \mathscr{R}^v R^v\big)$$

$$s.t. \begin{cases} \begin{bmatrix} \mathscr{Q}^h & I \\ I & Q^h \end{bmatrix} \geq 0, \begin{bmatrix} \mathscr{P}^h & I \\ I & P^h \end{bmatrix} \geq 0, \begin{bmatrix} \mathscr{R}^h & I \\ I & R^h \end{bmatrix} \geq 0, \\ \begin{bmatrix} \mathscr{Q}^v & I \\ I & Q^v \end{bmatrix} \geq 0, \begin{bmatrix} \mathscr{P}^v & I \\ I & P^v \end{bmatrix} \geq 0, \begin{bmatrix} \mathscr{R}^v & I \\ I & R^v \end{bmatrix} \geq 0, \\ \qquad\qquad (14) \text{ and } (40). \end{cases}$$

According to the idea of CCL algorithm, we know that the conditions in Theorem 3 are satisfied when $\min \text{tr}(\mathscr{Q}^h Q^h + \mathscr{Q}^v Q^v + \mathscr{P}^h P^h + \mathscr{P}^v P^v + \mathscr{R}^h R^h + \mathscr{R}^h R^v) = 6n_x$ (named C_{min}). For more discussions on the nonlinear minimization problem, we refer the readers to [8], [9], [14], [37] and references listed therein. It can be seen that the LMIs (40) with the equality constraints (38)-(39) have been successfully transformed into a set of strict LMIs by the adopted CCL procedure, where a minimization problem with extra LMI constraints is induced. Fortunately, the newly formed optimization problem can be easily solved via mincx solver in the standard MATLAB LMI toolbox. The resulting computation burden is mainly dependent on the iteration precision of CCL and the prescribed maximum iteration times, and the impact from the dimension of the 2-D system on the computational complexity is relatively small. Note that research on LMI optimization is a very active area in communities of applied mathematics, optimization and operation research, and substantial speed-ups can be expected in the near future.

Remark 4. *For the H_∞ control problem for a class of 2-D T-S fuzzy systems described by the second Fornasini-Machesini local state-space model with time-delays and missing measurements, there are four main aspects which complicate the design of the output controller, i.e. interval time-varying delays, randomly missing measurements, two-dimensional dynamics evolutions and T-S model-based fuzzifications. In our main results (Theorems 1-3), sufficient conditions, which include all of the information on these four aspects, are established for an output feedback controller to satisfy the prescribed H_∞ performance requirement. The corresponding solvability conditions for the desired controller gains are expressed in terms of the feasibility of a series of LMIs with equality constraints that can be solved by the well-known CCL algorithm. It should be pointed out that an energy-like functional is constructed to derive several delay-range-dependent stability criteria and our developed algorithm would enjoy the advantage of less conservatism since more information about the delays is employed.*

IV. ILLUSTRATIVE EXAMPLE

In this section, an example is presented to illustrate the feasibility of the proposed control algorithm for the discussed fuzzy system.

Consider the 2-D discrete T-S fuzzy system (1) with the interval time-varying delays $\tau_1(k,l) = \underline{\tau}_1 + (\overline{\tau}_1 - \underline{\tau}_1 - 2)|\cos(kl)|$ and $\tau_2(k,l) = \underline{\tau}_2 + (\overline{\tau}_2 - \underline{\tau}_2 - 2)|\sin(kl)|$, where $\underline{\tau}_1 = 2$, $\overline{\tau}_1 = 16$, $\underline{\tau}_2 = 4$ and $\overline{\tau}_2 = 18$. Take the number of IF-THEN rules $r = 2$ and the other parameters are given as follows (to conserve space, here only part of the matrices are listed out):

$$A_{11} = \begin{bmatrix} -0.6754 & -0.3021 \\ -0.1511 & 0.1003 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0.0024 & 0.00251 \\ 0.0001 & 0.0145 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} -0.2001 & -0.2861 \\ -0.1230 & -0.0123 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0.0254 & -0.0040 \\ 0.0044 & 0.0541 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} -0.4601 & -0.3032 \\ -0.4241 & 0.0205 \end{bmatrix}, \quad E_{11} = \begin{bmatrix} -0.3244 & 0.210 \\ 0.1824 & 0.1023 \end{bmatrix}.$$

Taking the probability of the missing measurements as $\alpha = 0.42$ and the disturbance attenuation level as $\gamma = 0.28$, the feasible solution of the LMIs in Theorem 3 can be obtained readily by employing the LMI toolbox, where the gain matrices of the desired 2-D fuzzy controller are obtained as follows

$$K_1 = \begin{bmatrix} -0.0269 & 0.0818 \\ 0.1127 & -0.3068 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.0267 & 0.0822 \\ 0.1121 & -0.3095 \end{bmatrix}. \tag{41}$$

Choose the fuzzy basis functions as

$$h_1(k,l) = \frac{\sin^2(k+l+x_1(k,l))}{2+\cos(k+l+x_1(k,l))}$$

and $h_2(k, l) = 1 - h_1(k, l)$, and the exogenous disturbance as $\omega(k, l) = \begin{bmatrix} \sin(kl)e^{-\frac{\sqrt[5]{kl}}{\cos^2(kl)}} & \cos(kl)e^{-\frac{\sqrt[5]{kl}}{\sin^2(kl)}} \end{bmatrix}^T$. The boundary condition is assumed to be

$$\psi_1(k, l) = \begin{bmatrix} \sin^3(\frac{k}{l}) \\ \sin^3(\frac{l}{k}) \end{bmatrix}, \quad \psi_2(k, l) = \begin{bmatrix} \cos^3(\frac{k}{l}) \\ \cos^3(\frac{l}{k}) \end{bmatrix},$$

with $z_1 = 35$ and $z_2 = 30$.

In the simulation study, the corresponding state evolutions of the closed-loop 2-D fuzzy system are shown in Fig. 1 and Fig. 2, respectively. Fig. 3 and Fig. 4 depict the states of the uncontrolled system, from which we can see that the original system is clearly unstable. As is indicated in Fig. 1 and Fig. 2, the designed fuzzy controller can stabilize the target 2-D system very well. Additionally, it is also demonstrated that the expected performance of disturbance attenuation and rejection has been achieved. Furthermore, in spite of a high probability for the missing measurements, the fuzzy rule-based controller can perform satisfactorily. However, if we ignore the fuzzy rules and just impose the controller with the same gains as in (41), the corresponding results in Fig. 5 and Fig. 6 imply the invalidity of the control law. To check the sensitivity of the design parameters, we intentionally adjust the initial conditions and the exogenous disturbance, and the results have shown that the controlled system can also be stabilized well by the proposed fuzzy controller.

V. CONCLUSIONS

In this paper, the static output feedback control problem has been studied for a class of 2-D fuzzy system with bounded disturbances. The system model contains time-varying delays in both horizontal- and the vertical directions. Considering the realistic situation of sensors and complicated transmission condition, a mathematic model reflecting the practical measurement outputs has been proposed to depict the phenomena of missing measurements. By constructing an energy-like quadratic functional, a set of delay-range-dependent criteria in the form of matrix inequalities has been derived to make the 2-D fuzzy system asymptotically stable in the mean-square sense. With the help of the CCL algorithm, parameters of the desired fuzzy controller have been obtained readily. Finally, the stabilization of the discussed closed-loop system can be realized by the IF-THEN rules based fuzzy controller. The feasibility of the suggested design method has been checked by the simulation results. It should be pointed out that the main results in this paper can be extended to the 2-D networked systems. Our most promising direction for future research concerns the estimation problem for 2-D networked systems with a variety of incomplete measurements such as sensor quantization, sensor saturation and fading channels [5], [6]. Moreover, due to the superiority of the piecewise Lyapunov function in terms of reducing conservativeness, see e.g. [27], we will discuss the performance tracking control of complicated networked systems based on the piecewise Lyapunov functions in the future.
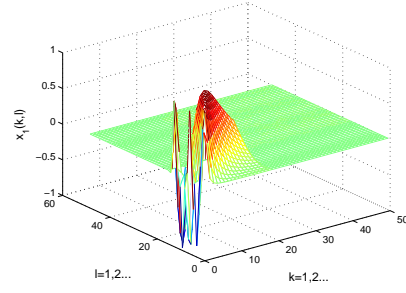

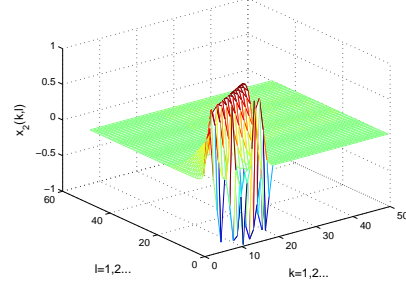Fig. 1. Closed-loop state evolution $x_1(k, l)$ of the fuzzy controlled system

Fig. 2. Closed-loop state evolution $x_2(k, l)$ of the fuzzy controlled system
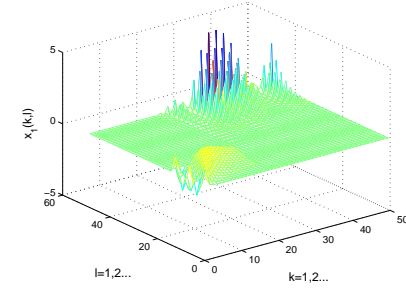

Fig. 3. State evolution $x_1(k, l)$ of the uncontrolled system

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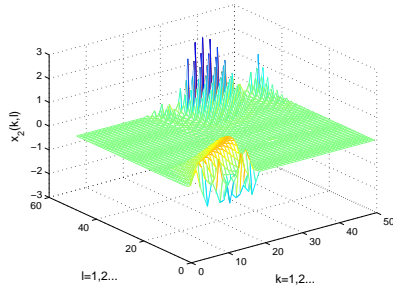
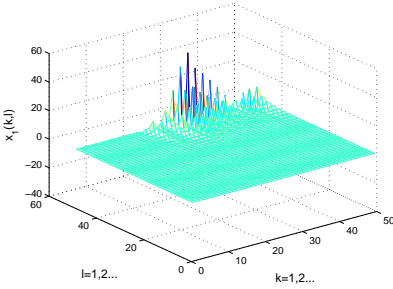Fig. 4. State evolution $x_2(k,l)$ of the uncontrolled system

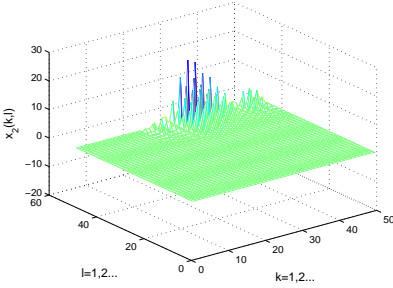Fig. 5. State evolution $x_1(k,l)$ of the controlled system without fuzzy rules

Fig. 6. State evolution $x_2(k,l)$ of the controlled system without fuzzy rules

Fuzzy Systems, Vol. 17, No. 2, pp. 440-450, 2009.

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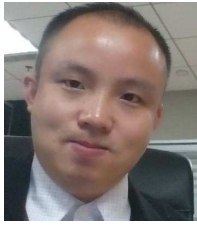
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