

# On the Performance Analysis of an Energy Detector over Composite Generalized Multipath/Gamma Fading Channels

Author 1, and Author 2,

**Abstract**—In this letter, the performance of an energy detector (ED) is analysed over different generalized multipath fading channels namely  $\kappa - \mu$ ,  $\eta - \mu$  and  $\alpha - \mu$  shadowed by gamma distribution. The mixture gamma (MG) distribution is used to model the probability density function (PDF) of the signal-to-noise-ratio (SNR) for all channels. General tractable unified closed-form analytic expressions for the performance metrics are obtained. Both the average detection probability,  $\overline{P}_d$ , and the average area under the receiver operating characteristics curve (AUC) which are mainly utilised in evaluating the performance of ED are derived. To validate our analysis, the numerical results are compared with the simulated results.

**Index Terms**—Mixture gamma distribution,  $\kappa - \mu$ ,  $\eta - \mu$ ,  $\alpha - \mu$ , gamma distribution, energy detector, average detection probability, average area under the ROC curve.

## I. INTRODUCTION

DIFFERENT generalized multipath distributions have been proposed by Yacoub [1], [2]. These distributions are the  $\kappa - \mu$ , the  $\eta - \mu$  and the  $\alpha - \mu$  which describe the line-of-sight (LoS), the Non-line-of-sight (NLoS) and the non-homogeneous communications scenarios respectively.

In the open technical literature, these distributions are used extensively in studying the behaviour of different communications systems. In [3] and [4], the performance of energy detector (ED) that is widely used to perform the spectrum sensing (SS) in cognitive radio (CR) systems is investigated over  $\kappa - \mu$  fading channel by deriving analytic expression for the average detection probability,  $\overline{P}_d$ . The performance of ED over  $\eta - \mu$  fading channel using the moment generating function (MGF) of the received instantaneous SNR is analysed in [4] and [5]. In the former, the  $\overline{P}_d$  is derived whereas in the latter, the average area under the receiver operating characteristics curve (AUC) is evaluated. The behaviour of ED over  $\alpha - \mu$  fading channel is introduced in [6].

A part of fading channel is shadowing that may be included in the probability density function (PDF) of the received signal-to-noise-ratio (SNR). Thus, in [7], the log-normal distribution is approximated by a Wald distribution to derive the expression of the  $\overline{P}_d$  over slow fading channels. However, this approximation produces expression with an infinite series. The performance of ED over composite Nakagami- $m$ /gamma fading channels i.e.  $K_G$  is studied in [8]. The average AUC expressions over  $K_G$  and composite Rician/gamma fading channels are given in [9] and [10] respectively. But, these

expressions are complex and include an infinite series that converge slowly and unsteadily. From the previous fast literature review, the performance of ED over composite  $\kappa - \mu$ /gamma,  $\eta - \mu$ /gamma and  $\alpha - \mu$ /gamma fading channels is not yet studied. Furthermore, the most derived expressions of the  $\overline{P}_d$  and the average AUC are either approximated or expressed in non-closed forms. Motivated by these reasons, in this letter, we derive analytic expressions of the  $\overline{P}_d$  and the average AUC over  $\kappa - \mu$ /gamma,  $\eta - \mu$ /gamma and  $\alpha - \mu$ /gamma fading channels. The mixture gamma (MG) distribution that is proposed by [11] as an accurate approximation method to model the SNR of wireless channels is used in representing the PDF of those channels. Although, this approach is utilized by [11] to derive the expressions of the  $\overline{P}_d$  and the average AUC, the  $\overline{P}_d$  is expressed as integral form and it is applicable when the values of the fading parameter and the time-bandwidth product are integer numbers. Moreover, the average AUC expression includes two terms with MG distribution and it is also restricted by the value of the time-bandwidth product that should be an integer number. In contrast, our derived expressions are general and given in closed-form.

## II. ENERGY DETECTION MODEL

The performance of ED is measured by two probabilities which are the probability of detection,  $P_d(\gamma, \lambda)$ , and the probability of false alarm,  $P_f(\lambda)$ . These probabilities are expressed as follows [3]:

$$P_d(\gamma, \lambda) = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) \quad \text{and} \quad P_f(\lambda) = \frac{\Gamma(u, \lambda/2)}{\Gamma(u)} \quad (1)$$

where  $\gamma = |h|^2 E_s / N_0$  is the instantaneous SNR,  $h$  is the channel gain,  $E_s$  is the transmitted signal energy,  $N_0$  is one-side of the noise power spectral density and  $\lambda$  is the pre-defined threshold value. Moreover,  $\Gamma(\cdot, \cdot)$  is the upper incomplete gamma function,  $\Gamma(\cdot)$  is the gamma function, and  $Q_u(\cdot, \cdot)$  is the  $u^{th}$  order generalized Marcum-Q function ( $u$  is the time-bandwidth product).

## III. THE MG DISTRIBUTION FOR COMPOSITE GENERALISED MULTIPATH/GAMMA FADING CHANNELS

The wireless channel can be modelled by the MG distribution. Accordingly, the PDF of the instantaneous SNR,  $\gamma$ , ( $f_\gamma(\gamma)$ ) is given by [11]<sup>1</sup>

$$f_\gamma(\gamma) = \sum_{i=1}^N \alpha_i \gamma^{\beta_i - 1} e^{-\zeta_i \gamma} \quad (2)$$

<sup>1</sup>Bold  $\alpha$  is used here to distinguish between  $\alpha$  of the  $\alpha - \mu$  fading channels and  $\alpha$  of the MG distribution.

where  $N$  and  $\alpha_i$ ,  $\beta_i$  and  $\zeta_i$  are the number of terms and the parameters of  $i$ th Gamma component respectively. It can be observed that the main problem in using the MG is determining  $N$ . In [11], the authors have proposed some methods such as calculating the mean square error (MSE) between the PDF of the exact distribution and the PDF of the approximate distribution by the MG distribution to find minimum  $N$  that gives good approximation with high accuracy. For more details about methods of evaluating  $N$ , the reader can refer to [11]. In the reminder of this section, we derive the parameters of the MG for different composite generalized multipath/gamma fading channels as follows:

#### A. $\kappa - \mu$ /gamma Fading Channel

The  $\kappa - \mu$  contains two parameters which are  $\kappa$  and  $\mu$ . The former represents the ratio between the total power of the dominant components and the scattered waves whereas the latter which is also exist in the  $\eta - \mu$  and the  $\kappa - \mu$  represents the real extension of the number of the multipath clusters respectively. The  $\kappa - \mu$ /gamma fading channel is a composite fading channel from the  $\kappa - \mu$  fading channel and gamma distribution which models the shadowing. Accordingly, the SNR distribution of the  $\kappa - \mu$ /gamma fading channel can be evaluated by averaging [1, eq. (10)] over gamma distribution [8, eq. (4)] as follows

$$f_\gamma(\gamma) = \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}} \gamma^{\frac{\mu-1}{2}}}{\Gamma(k)\Omega^k \kappa^{\frac{\mu-1}{2}} e^{\mu\kappa}} \times \int_0^\infty y^{k-\frac{\mu}{2}-\frac{3}{2}} e^{-\frac{\mu(1+\kappa)\gamma}{y}-\frac{y}{\Omega}} I_{\mu-1} \left( 2\mu \sqrt{\frac{\mu(1+\kappa)\gamma}{y}} \right) dy \quad (3)$$

where  $k$  is the shaping parameter,  $\Omega = \frac{\bar{\gamma}}{kE_s/N_0}$  is the mean power and  $I_a(\cdot)$  is the modified Bessel function of the first kind and  $a$ th order.

By substituting  $x = \frac{\mu(1+\kappa)\gamma}{y}$  in (3), this yields

$$f_\gamma(\gamma) = \vartheta_{\kappa-\mu} \gamma^{k-1} \int_0^\infty e^{-x} g(x) dx \quad (4)$$

where  $\vartheta_{\kappa-\mu} = -\frac{\mu^{k-\frac{\mu-1}{2}}}{\Gamma(k)\kappa^{\frac{\mu-1}{2}} e^{\mu\kappa}} \left(\frac{1+\kappa}{\Omega}\right)^k$  and  $g(x) = x^{\frac{\mu}{2}-k-\frac{1}{2}} e^{-\frac{\mu(1+\kappa)\gamma}{\Omega x}} I_{\mu-1}(2\mu\sqrt{x})$ . The integration in (4),  $\mathcal{S} = \int_0^\infty e^{-x} g(x) dx$ , can be approximated as a Gaussian-Laguerre quadrature sum as  $\mathcal{S} \approx \sum_{i=1}^N w_i g(x_i)$  where  $x_i$  and  $w_i$  are the abscissas and weight factors for the Gaussian-Laguerre integration [12, pp. 923]. Consequently, (4) can be expressed by the MG distribution with parameters

$$\alpha_i = \frac{\theta_i}{\sum_{l=1}^N \theta_l \Gamma(\beta_l) \zeta_l^{-\beta_l}}, \quad \beta_i = k, \quad \zeta_i = \frac{\mu(1+\kappa)}{\Omega x_i},$$

$$\theta_i = \vartheta_{\kappa-\mu} w_i x_i^{\frac{\mu}{2}-k-\frac{1}{2}} I_{\mu-1} \left( 2\mu \sqrt{x_i} \right) \quad (5)$$

#### B. $\eta - \mu$ /gamma Fading Channel

Like the  $\kappa - \mu$  fading channel, the  $\eta - \mu$  includes two parameters which are  $\eta$  and  $\mu$ . The definition of  $\eta$  depends on the type of format. In format 1,  $\eta$  represents the power

ratio between the in-phase and quadrature scattered components in each multipath cluster with  $0 < \eta < \infty$ . The respective  $H$  and  $h$  are expressed by  $H = (\eta^{-1} - \eta)/4$  and  $h = (2 + \eta^{-1} - \eta)/4$  respectively. In format 2,  $\eta$  stands for the correlation coefficient between the in-phase and quadrature scattered components in each multipath cluster with  $-1 < \eta < 1$ . The respective  $H$  and  $h$  are given by  $H = \eta/(1 - \eta^2)$  and  $h = 1/(1 - \eta^2)$  respectively [1].

The SNR distribution of the  $\eta - \mu$ /gamma fading channel can be calculated by integrating the  $\eta - \mu$  fading channel [1, eq. (26)] over [8, eq. (4)] to yield

$$f_\gamma(\gamma) = \frac{2\sqrt{\pi} h^\mu \mu^{\mu+\frac{1}{2}} \gamma^{\mu-\frac{1}{2}}}{\Gamma(\mu)\Gamma(k)\Omega^k H^{\mu-\frac{1}{2}}} \times \int_0^\infty y^{k-\mu-\frac{3}{2}} e^{-\frac{2\mu h \gamma}{y}-\frac{y}{\Omega}} I_{\mu-\frac{1}{2}} \left( \frac{2\mu H \gamma}{y} \right) dy \quad (6)$$

By assuming  $x = \frac{2\mu h \gamma}{y}$  and following the same procedure for the  $\kappa - \mu$ /gamma fading channel, we find

$$\alpha_i = \frac{\theta_i}{\sum_{l=1}^N \theta_l \Gamma(\beta_l) \zeta_l^{-\beta_l}}, \quad \beta_i = k, \quad \zeta_i = \frac{2\mu h}{\Omega x_i},$$

$$\theta_i = \vartheta_{\eta-\mu} w_i x_i^{\mu-k-\frac{1}{2}} I_{\mu-\frac{1}{2}} \left( \frac{H}{h} x_i \right) \quad (7)$$

where  $\vartheta_{\eta-\mu} = -\frac{\sqrt{\pi} 2^{k-\mu+\frac{1}{2}} h^{k-\frac{1}{2}}}{\Gamma(\mu)\Gamma(k)H^{\mu-\frac{1}{2}}} \left(\frac{\mu}{\Omega}\right)^k$ .

#### C. $\alpha - \mu$ /gamma Fading Channel

The  $\alpha - \mu$  distribution is used to model the non-linear environment of wireless communications. Due to limited space, the reader can refer to [2] for further information about this distribution.

The SNR distribution of composite  $\alpha - \mu$  fading channel and gamma distribution i.e.  $\alpha - \mu$ /gamma can be computed by using [2, eq. (1)] and [8, eq. (4)] as follows

$$f_\gamma(\gamma) = \frac{\alpha \mu^\mu \gamma^{\frac{\alpha\mu}{2}-1}}{2\Gamma(\mu)\Gamma(k)\Omega^k} \int_0^\infty y^{k-\frac{\alpha\mu}{2}-1} e^{-\frac{\mu\gamma^{\alpha/2}}{y^{\alpha/2}}-\frac{y}{\Omega}} dy \quad (8)$$

where  $\alpha > 0$  is the non-linear fading parameter.

By using the substitution  $x = \frac{\mu\gamma^{\alpha/2}}{y^{\alpha/2}}$  and following a similar procedure of the  $\kappa - \mu$ /gamma fading channel, the MG distribution parameters for the  $\alpha - \mu$ /gamma fading channel are expressed by

$$\alpha_i = \frac{\theta_i}{\sum_{l=1}^N \theta_l \Gamma(\beta_l) \zeta_l^{-\beta_l}}, \quad \beta_i = k, \quad \zeta_i = \frac{\mu^{2/\alpha}}{\Omega x_i^{2/\alpha}},$$

$$\theta_i = \vartheta_{\alpha-\mu} w_i x_i^{\mu-\frac{2k}{\alpha}-1} \quad (9)$$

where  $\vartheta_{\alpha-\mu} = -\frac{\mu^{2k/\alpha}}{\Gamma(\mu)\Gamma(k)\Omega^k}$ .

## IV. AVERAGE DETECTION PROBABILITY

The average detection probability,  $\bar{P}_d(\lambda)$ , can be evaluated by [7, eq. (9)]

$$\bar{P}_d(\lambda) = \int_0^\infty P_d(\gamma, \lambda) f_\gamma(\gamma) d\gamma \quad (10)$$

When  $u \in \mathbb{R}$  i.e.  $u$  is a real number, the  $\overline{P_d}(\lambda)$  can be expressed as follows [see Appendix A]

$$\overline{P_d}(\lambda) = 1 - \frac{2^{-u} \lambda^u e^{-\frac{\lambda}{2}}}{\Gamma(1+u)} \sum_{i=1}^N \frac{\alpha_i \Gamma(\beta_i)}{(1+\zeta_i)^{\beta_i}} \times \Phi_2\left(\beta_i, 1; 1+u; \frac{\lambda}{2}, \frac{\lambda}{2(1+\zeta_i)}\right) \quad (11)$$

where  $\Phi_2(\cdot, \cdot; \cdot, \cdot; \cdot, \cdot)$  is the bivariate confluent hypergeometric function defined in [13, pp. 1031, eq. (9.261.2)]. It can be noted that,  $\Phi_2(\cdot, \cdot; \cdot, \cdot; \cdot, \cdot)$  is not available in common mathematical packages such as MATLAB and MATHEMATICA software packages. Thus, a series convergence should be assumed by limited number of terms,  $R$ , with truncation error,  $E_R$ .

To evaluate  $R$ ,  $\Phi_2(\cdot, \cdot; \cdot, \cdot; \cdot, \cdot)$  in (11) should be expressed in terms of an infinite series. This can be obtained by invoking [13, pp. 1031, eq. (9.261.2)] and using the identity  $(a+b)_c = (a)_c (a+c)_b$  to yield

$$E_R = \sum_{i=1}^N \frac{\alpha_i \Gamma(\beta_i)}{(1+\zeta_i)^{\beta_i}} \sum_{n=0}^{\infty} \frac{(1)_n}{(1+u)_n n!} \left(\frac{\lambda}{2}\right)^n \times {}_1F_1\left(\beta_i; 1+u+n; \frac{\lambda}{2(1+\zeta_i)}\right) \quad (12)$$

where  $(\cdot)_R$  and  ${}_1F_1(\cdot; \cdot; \cdot)$  are the Pochhammer symbol and the confluent hypergeometric function respectively. It can be noted that in (12),  ${}_1F_1(\cdot; \cdot; \cdot)$  is monotonically decreasing with  $n$ . Accordingly, after doing similar mathematical simplifications to [14], yielding

$$E_R \leq \frac{(1)_R (\frac{\lambda}{2})^R}{(1+u)_R R!} \sum_{i=1}^N \frac{\alpha_i \Gamma(\beta_i)}{(1+\zeta_i)^{\beta_i}} {}_1F_1\left(1; 1+u+R; \frac{\lambda}{2}\right) \times {}_1F_1\left(\beta_i; 1+u+R; \frac{\lambda}{2(1+\zeta_i)}\right) \quad (13)$$

When  $u$  is an integer number i.e.  $u \in \mathbb{Z}$ , the  $\overline{P_d}(\lambda)$  can be computed by substituting the  $P_d(\gamma, \lambda)$  of (1) and (2) into (10) with the help of [15, eq. (9)] and [13, pp. 340, eq. (3.35.3)]. After some mathematical operations, this yields

$$\overline{P_d}(\lambda) = \frac{e^{-\frac{\lambda}{2}}}{\Gamma(2-u)} \sum_{i=1}^N \frac{\alpha_i \Gamma(\beta_i - u + 1)}{(1+\zeta_i)^{\beta_i - u + 1}} \times \Phi_1\left(\beta_i - u + 1, 1; 2-u; \frac{1}{1+\zeta_i}, \frac{\lambda}{2(1+\zeta_i)}\right) \quad (14)$$

where  $\Phi_1(\cdot, \cdot; \cdot; \cdot, \cdot)$  is another form of the bivariate confluent hypergeometric function defined in [13, pp. 1031, eq. (9.261.1)]. This function is not yet implemented in MATLAB and MATHEMATICA software packages. Therefore, a series convergence should be assumed.

Using [13, pp. 1031, eq. (9.261.1)], the series in (14) can be written in terms of  ${}_1F_1(\cdot; \cdot; \cdot)$  as follows

$$E_R = \sum_{i=1}^N \frac{\alpha_i \Gamma(\beta_i - u + 1)}{(1+\zeta_i)^{\beta_i - u + 1}} \sum_{n=0}^{\infty} \frac{(\beta_i - u)_n (1)_n}{(2-u)_n n!} \left(\frac{1}{1+\zeta_i}\right)^n \times {}_1F_1\left(\beta_i - u + n; 2-u+n; \frac{\lambda}{2(1+\zeta_i)}\right) \quad (15)$$

Similar to (12),  ${}_1F_1(\cdot; \cdot; \cdot)$  is monotonically decreasing with  $n$ . Consequently, after some mathematical manipulations, this yields

$$E_R \leq \frac{(\beta_i - u)_R (1)_R}{(2-u)_R R!} \sum_{i=1}^N \frac{\alpha_i \Gamma(\beta_i - u + 1)}{(1+\zeta_i)^{\beta_i - u + 1}} \left(\frac{1}{1+\zeta_i}\right)^R \times {}_2F_1\left(\beta_i - u + R, 1; 2-u+R; \frac{1}{1+\zeta_i}\right) \times {}_1F_1\left(\beta_i - u + R; 2-u+R; \frac{\lambda}{2(1+\zeta_i)}\right) \quad (16)$$

where  ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$  is another model of the confluent hypergeometric function.

## V. AVERAGE AREA UNDER THE ROC CURVE

The average area under the receiver operating characteristics (AUC) curve,  $\bar{A}$ , is given by [10, eq. (18)]

$$\bar{A} = \frac{1}{2^u \Gamma(u)} \int_0^{\infty} \lambda^{u-1} e^{-\frac{\lambda}{2}} \overline{P_d}(\lambda) d\lambda \quad (17)$$

By substituting (11) into (17) with the aid of [13, pp. 1031, eq. (9.261.2)], [13, pp. 340, eq. (3.35.3)] and  $\Gamma(a+b) = (a)_b \Gamma(a)$ ,  $\bar{A}$  for  $u \in \mathbb{R}$  can be expressed in closed-form as follows

$$\bar{A} = 1 - \frac{\Gamma(2u)}{u[2^u \Gamma(u)]^2} \sum_{i=1}^N \frac{\alpha_i \Gamma(\beta_i)}{(1+\zeta_i)^{\beta_i}} F_1\left(2u, 1, \beta_i; 1+u; \frac{1}{2}, \frac{1}{2(1+\zeta_i)}\right) \quad (18)$$

where  $F_1(\cdot, \cdot, \cdot; \cdot; \cdot, \cdot)$  is the double variables Appell hypergeometric function [13, pp. 1018, eq. (9.180.1)] and its a standard built-in function available in MATHEMATICA software.

When  $u \in \mathbb{Z}$ ,  $\bar{A}$  is expressed by

$$\bar{A} = \frac{1}{2^u \Gamma(2-u)} \sum_{i=1}^N \frac{\alpha_i \Gamma(\beta_i - u + 1)}{(1+\zeta_i)^{\beta_i - u + 1}} \times F_1\left(\beta_i - u + 1, 1, u; 2-u; \frac{1}{1+\zeta_i}, \frac{1}{2(1+\zeta_i)}\right) \quad (19)$$

The expression in (19) is evaluated by using (14) and (17) with the help of [13, pp. 1031, eq. (9.261.1)] and [13, pp. 340, eq. (3.35.3)] and doing some mathematical simplifications.

## VI. NUMERICAL RESULTS

In this section, the analytical results of an ED over  $\kappa - \mu/\text{gamma}$ ,  $\eta - \mu/\text{gamma}$  and  $\alpha - \mu/\text{gamma}$  fading channels are compared with the simulated results. Monte Carlo simulations with  $10^6$  iterations are utilized to compare with the numerical results. In all figures, the numerical results are represented by solid lines with marks whereas the simulated results are shown by star mark. To obtain mean squared error (MSE) between the exact PDF and the approximate PDF  $\leq 10^{-6}$ , the number of terms,  $N$ , is chosen to be 15 for all channels.

Fig. 1 and Fig. 2 show the complementary receiver operating characteristics (CROC) which plots the average probability of missed-detection,  $\overline{P_{md}}(\lambda)$ , ( $\overline{P_{md}}(\lambda) = 1 - \overline{P_d}(\lambda)$ ) versus  $P_f(\lambda)$  and the complementary AUC ( $1 - \bar{A}$ ) versus average SNR respectively. In Fig. 1, the numbers of terms,  $R$ , required in

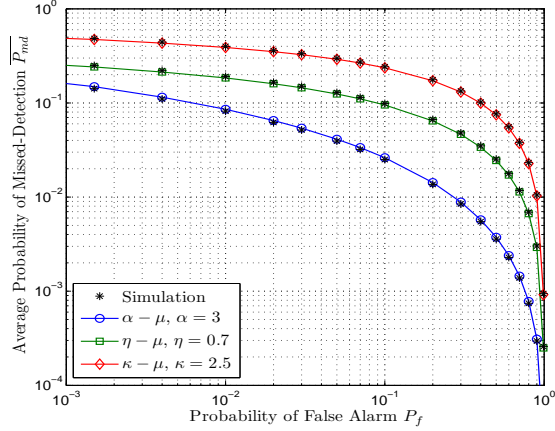


Fig. 1. Complementary ROC curves over  $\kappa - \mu$ /gamma,  $\eta - \mu$ /gamma and  $\alpha - \mu$ /gamma fading channels for  $\mu = 0.5$ ,  $k = 4.5$ ,  $\bar{\gamma} = 15$  dB and  $u = 1.5$ .

evaluating (12) at  $P_f = 0.1$  with seven figure accuracy for  $\alpha - \mu$ /gamma,  $\eta - \mu$ /gamma and  $\kappa - \mu$ /gamma are 20, 21 and 22 respectively. It can be noted that the results in both figures can not be calculated by [11, eq. (29)] and [11, eq. (33)]. This is because the former can be applied when the values of  $\beta_i$  i.e.  $k$  and  $u$  are integer numbers and the latter can be used when  $u$  is an integer number.

## VII. CONCLUSIONS

The performance of ED over composite  $\kappa - \mu$ /gamma,  $\eta - \mu$ /gamma and  $\alpha - \mu$ /gamma fading channels has been analysed in this letter. Both the average  $P_d$  and the average AUC expressions have been derived by using the MG distribution. These expressions are novel, general, unified, exact, and non-limited by any condition. The results show an improvement in the detectability of ED with any increasing in the value of  $k$ . The performance of ED over different composite fading channel models such as Nakagami- $m$ /gamma and Rician/gamma can be deduced from the derived expressions.

## APPENDIX A

### PROOF OF (11)

When  $u \in \mathbb{R}$ , the  $P_d(\gamma, \lambda)$  of (1) can be expressed by [15, eq. (34)] as follows

$$P_d(\gamma, \lambda) = 1 - \left(\frac{\lambda}{2}\right)^u e^{-\frac{2\gamma+\lambda}{2}} \tilde{\Phi}_3\left(1; 1+u; \frac{\lambda}{2}, \frac{\gamma\lambda}{2}\right) \quad (20)$$

where  $\tilde{\Phi}_3(\cdot; \cdot; \cdot, \cdot)$  is the regularized bivariate confluent hypergeometric function defined in [15, eq. (4)].

Substituting (2) and (20) into (10) with the aid of [15, eq. (4)] and  $\int_0^\infty f_\gamma(\gamma) d\gamma \triangleq 1$ , this yields

$$\begin{aligned} \bar{P}_d(\lambda) &= 1 - \frac{\lambda^u e^{-\frac{\lambda}{2}}}{2^u \Gamma(1+u)} \sum_{i=1}^N \alpha_i \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \frac{(1)_l}{(1+u)_{l+m} l! m!} \\ &\quad \left(\frac{\lambda}{2}\right)^l \left(\frac{\lambda}{2}\right)^m \int_0^\infty \gamma^{m+\beta_i-1} e^{-(1+\zeta_i)\gamma} d\gamma \end{aligned} \quad (21)$$

Using [13, pp. 340, eq. (3.35.3)] to evaluate the integral in (21) and invoking the identity  $\Gamma(a+b) = (a)_b \Gamma(a)$ , the desired result in (11) is obtained.

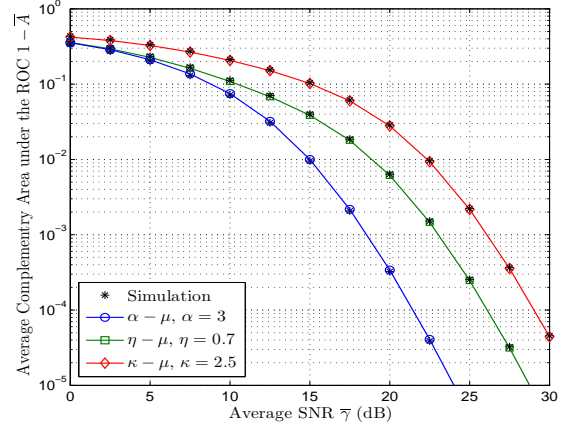


Fig. 2. Complementary AUC curves versus  $\bar{\gamma}$  over  $\kappa - \mu$ /gamma,  $\eta - \mu$ /gamma and  $\alpha - \mu$ /gamma fading channels for  $\mu = 0.5$ ,  $k = 4.5$  and  $u = 1.5$ .

## REFERENCES

- [1] M. D. Yacoub, "The  $\kappa - \mu$  distribution and the  $\eta - \mu$  distribution," *IEEE Antennas Propag. Mag.*, vol. 49, no. 1, pp. 68-81, Feb. 2007.
- [2] M. D. Yacoub, "The  $\alpha - \mu$  distribution: a physical fading model for the Stacy distribution," *IEEE Trans. Veh. Technol.*, vol. 56, no. 1, pp. 27-34, Jan. 2007.
- [3] P. Sofotasios, E. Rebeiz, L. Zhang, T. Tsiftsis, D. Cabric and S. Freear, "Energy detection based spectrum sensing over  $\kappa - \mu$  and  $\kappa - \mu$  extreme fading channels," *IEEE Trans. Veh. Technol.*, vol. 62, no. 3, pp. 1031-1040, Mar. 2013.
- [4] S. Atapattu, C. Tellambura, and H. Jiang, "Energy detection of primary signals over  $\eta - \mu$  fading channels," in *Proc. IEEE Int. Conf. Industrial Inf. Syst. (ICIS)*, Dec. 2009, pp. 118-122.
- [5] S. Atapattu, C. Tellambura, and H. Jiang, "MGF based analysis of area under the ROC curve in energy Detection," *IEEE Commun. Lett.*, vol. 15, no. 12, pp. 1301-1303, Dec. 2011.
- [6] Y. Fathi, and M. H. Tawfik "Versatile performance expression for energy detector over  $\alpha - \mu$  generalised fading channels," *Electronic Lett.*, vol. 48, no. 17, pp. 1081-1082, Aug. 2012.
- [7] H. Sun, D. Laurenson, and C.-X. Wang, "Computationally tractable model of energy detection performance over slow fading channels," *IEEE Commun. Lett.*, vol. 14, no. 10, pp. 924-926, Oct. 2010.
- [8] S. Atapattu, C. Tellambura, and H. Jiang, "Performance of an energy detector over channels with both multipath fading and shadowing," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3662-3670, Dec. 2010.
- [9] S. Alam and A. Annamalai, "Energy detector's performance analysis over the wireless channels with composite multipath fading and shadowing effects using the AUC approach," *Proc. IEEE Consumer Commun. and Net. Conf. (CCNC)*, Jan. 2012, pp. 771-775.
- [10] K. P. Peppas, G. Efthymoglou, V. A. Aalo, M. Alwakeel, and S. Alwakeel, "On the performance analysis of energy detection of unknown signals in Gamma shadowed Ricean fading environments," *Proc. IEEE Pers. Ind. and Mob. Rad. Commun. (PIMRC)*, Sept. 2013, pp. 756-760.
- [11] S. Atapattu, C. Tellambura, and H. Jiang, "A mixture gamma distribution to model the SNR of wireless channels," *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 4193-4203, Dec. 2011.
- [12] M. Abramowitz and I. A. Stegun, editors, *Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables*. Dover Publications, 1965.
- [13] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 7th edition. Academic Press Inc., 2007.
- [14] C. C. Tan, and N. C. Beaulieu, "Infinite series representation of the bivariate Rayleigh and Nakagami- $m$  distributions," *IEEE Trans. Commun.*, vol. 45, no. 10, pp. 1159-1161, Oct. 1997.
- [15] D. Morales-Jimenez, F. J. Lopez-Martinez, E. Martos-Naya, J. F. Paris, and A. Lozano, "Connections between the generalized Marcum-Q function and a class of hypergeometric functions," *IEEE Trans. Inf. Theory*, vol. 60, no. 2, pp. 1077-1082, Feb. 2014.