Quantifying the influence of elliptical ring geometry on the degree of restraint in a ring test

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Abstract

A novel elliptical ring test method has been proposed to replace the traditional circular ring test method for the purpose of shortening test duration for assessing cracking tendency of concrete and more conveniently examining crack evolution in concrete rings. To further explore the mechanism of this new test method, a numerical model was established to analyse stress development and crack initiation in concrete rings in which a fictitious temperature field simulated the externally applied mechanical effects of concrete shrinkage on concrete causing the same strain as shrinkage does. The proposed numerical model was validated against experimental results and was then used to evaluate the effects of ring geometry on the degree of restraint to concrete. Moreover, a series of numerical experiments were conducted to derive the expression of the degree of restraint to elliptical rings. Through the error analysis of the degree of restraint on concrete properties and ring geometry, the expression was verified so that it can be used to calculate the degree of restraint provided by the internal steel ring. According to the derived expression, the restrained elliptical ring test can be designed to quantitatively simulate the service condition of concrete in an engineering structure more accurately.

Keywords: ring test, restraint shrinkage, elliptical ring, cracking, degree of restraint, numerical simulation.

1 Introduction

When changes in the volume of concrete from autogenous, drying or thermal shrinkage are restrained, residual stress will develop and cracking may occur once the residual tensile stress exceeds the tensile strength of concrete. This type of shrinkage cracking is a major problem for flat concrete elements/structures with a large exposed surface area-to-volume (A/V) ratio such as industrial floors, concrete pavements and bridge decks. Shrinkage cracking is especially severe for concrete at early ages, which possesses very low fracture/cracking resistance but high shrinkage. Cracks in concrete can reduce load carrying capacity and accelerate deterioration, shortening the service life of structures and increasing maintenance costs. Therefore, researchers are seeking to develop simple tests to assess how susceptible a concrete mixture may be cracking during its service life. As a standard test method, the circular ring test was first approved by the American Association of State Highway and Transportation (AASHTO), i.e. AASHTO PP34-99: *Standard Practice for Estimating the Cracking Tendency of Concrete.* This test consists of a concrete ring surrounding a restraining steel ring. The concrete ring has an inner diameter of 305 mm (12 inches), a wall thickness of 76 mm (3 inches) and a height of 152 mm (6 inches). The steel ring has an outer diameter of 305 mm (12 inches) and a wall thickness of 12.7 mm (0.5 inches). Later, the circular ring test was also recommended by ASTM (ASTM C1581/C1581M-09a: *Standard Test Method for Determining Age at Cracking and Induced Tensile Stress Characteristics of Mortar and Concrete under Restrained Shrinkage*). Differing from AASHTO, ASTM recommends thin concrete rings with a reduced wall thickness of 38 mm (1.5 inches) to enable cracking to initiate at an earlier age to shorten the duration of ring test.

Due to the simplicity and versatility of the circular ring test, it has been the most widely used method for decades for assessing the cracking potential of concrete. Research regarding the cracking of concrete with the circular ring test has primarily focused on the influence of concrete mixture compositions [1-6], specimen size/geometry [7-9], moisture gradients and drying condition [8-10]. However, it has been reported that a circular steel ring provides a relatively low degree of restraint thus making the duration for crack initiation fairly long [11]. Meanwhile, due to the symmetrical geometry of a circular ring, the first crack can appear anywhere along the circumference when subjected to restrained shrinkage. Therefore, considering the long duration before the first crack can be detected and its random location that is not easy to observe, novel elliptical ring geometries were adopted to replace circular ring geometries for assessing shrinkage cracking of mortar and concrete [12-15]. The experimental results have demonstrated that elliptical rings with an appropriate geometry enable crack initiation much faster than circular rings, which can consequently accelerate the ring test [16, 17]. Further, employing the stress- [16] and fracture mechanics-based criteria [17], a numerical method, which simulates the effect of concrete shrinkage by using an externally applied fictitious temperature field, has been developed to predict the cracking age and position and explores the mechanism of the elliptical ring test.

In the restrained ring tests, both the circular and elliptical rings measure the material's response under a particular restraint condition, e.g. specimen size, drying direction and degree of restraint. The cracking potential of concrete in a ring test depends not only on material properties (i.e. internal factors) but also on the degree of restraint provided by the central restraining steel core (i.e. external factors). With regard to the concrete mixtures with different cracking resistance, it is, therefore, necessary to find a reasonable estimation of the degree of restraint, requiring the selection of appropriate ring geometries for assessing the cracking potential of concrete structures in the field. With the latter aim in mind, research has been carried out to investigate the influence of specimen size/geometry [7, 8, 10] on the cracking of concrete or other cement-based materials using the circular ring test. However, for the elliptical ring geometry, very limited research has focused on exploring the appropriate geometrical ratio of the major to minor radius for an elliptical shape, in which the ratio of 2 has previously been recommended for thin concrete rings [16, 17]. It should be noted that for the research conducted on elliptical ring geometries, both the experimental and numerical results were based only on the recommended specimen thickness by ASTM, i.e. concrete ring wall thickness of 37.5 mm and steel ring wall thickness of 12.5 mm. In this case, for a specified concrete mixture, the assessment of cracking potential is just based on one degree of restraint while in practical engineering, concrete may be under various degrees of restraint. Furthermore, for elliptical ring specimens, the effects of the central restraining ring and concrete on the degree of restraint are not reported in literature. Therefore, it is significant to quantify the effects of elliptical ring geometry and material properties on the degree of restraint in the elliptical ring test.

In line with this, numerical simulations are carried out to investigate the influence of material (i.e. concrete and steel) properties and ring geometry on the degree of restraint. Based on the numerical results, the degrees of restraint provided by circular and elliptical rings are calculated and compared. Some interesting findings are presented and lead towards a better understanding of the mechanism of restrained circular and elliptical ring tests and guide on determining ring geometries to simulate the restraint condition in the field. This study is based on the proceedings of conference paper by Dong et al. [18], but with a major revision and extension of results. The changes made are summarised as following:

(1) In this paper, a series of numerical experiments have been carried out to study the degree of restraint of elliptical rings. By investigating the free shrinkage displacement $U'_{SH|R_{IC}}(t)$ and the actual displacement $U'_{S|R_{OS}}(t)$ at the inner surface of an elliptical concrete ring under various restraint conditions, the empirical formula for the degree of restraint in the elliptical ring was derived. The empirical formula is a function of the elastic modulus of concrete, concrete ring thickness and the geometry of the steel ring (i.e. shape and thickness).

(2) An error analysis was carried out to verify the empirical formula used in quantifying the degree of restraint for elliptical rings. By comparison of the degree of restraint obtained from numerical results and the derived empirical formula, it is concluded that the derived empirical formula can calculate the degrees of restraint of the elliptical ring specimens corresponding to various concrete modulus, concrete ring thicknesses and geometries of inner steel ring.

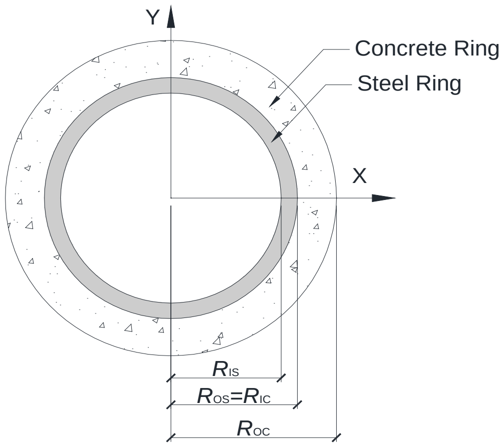
(3) Two examples were taken to illustrate how to determine the geometry of an elliptical ring for assessing the cracking potential of concrete in the field. The derived empirical formula was used to design the elliptical ring test appropriately with respect to the actual restraint condition, which concrete experiences in an engineering structure. In the case of a high degree of restraint in practical engineering, it is more convenient for the elliptical ring than the circular ring to assess the likelihood of cracking of the concrete.

Due to geometrical effects of an elliptical ring, the distribution of stress in concrete is complex. It is almost impossible to derive an analytical expression for the degree of restraint in an elliptical ring test. Through conducting a series of numerical experiments, the expression for the degree of restraint for an elliptical ring was proposed in this paper. It is expected that the discussion of the degree of restraint in this study will be helpful in exploring the mechanism of the elliptical ring test. Particularly, through the quantitative analysis of the degree of restraint for the elliptical ring, the appropriate geometry of elliptical ring specimens can be determined to more accurately simulate the service condition of concrete in an engineering structure in the field.

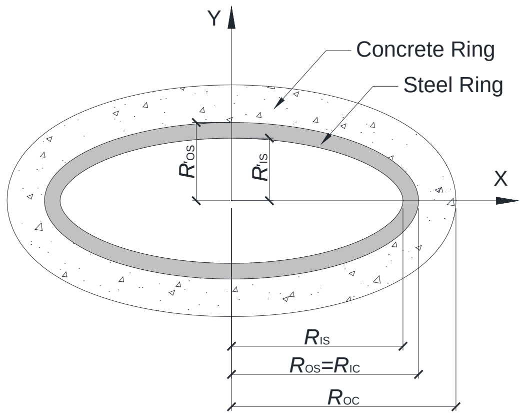
2 Ring geometries and degree of restraint

2.1 Circular and elliptical ring geometries

In this study, two typical geometries, i.e. circular and elliptical, are investigated for restrained ring test. Figure 1(a) and (b) illustrates the geometries of the circular and elliptical rings, respectively. R_{IS}, R_{OS}, and R_{OC} are the radii of the inner surface, the outer surface of the steel ring, and the outer surface of the concrete ring respectively for a circular ring specimen. They also correspond to the major radii for an elliptical ring specimen (see Figure 1(b)). R'_{OS} is the minor radius of the inner surface for an elliptical concrete ring specimen and is a constant 150 mm throughout the research. The ratio of R_{IS}/R_{OS} ranges from 0 to 1, and R_{OC}-R_{OS} is designed as 37.5 mm, 75 mm, and 150 mm, respectively. Moreover, according to previous studies [16, 17], thin elliptical rings with R_{OS}/R'_{OS} between 2 and 3 can provide a higher degree of restraint that leads to a shorter cracking period in the restrained shrinkage ring test. Therefore, in this study, $R_{OS}/R'_{OS} = 2$ is chosen for all elliptical rings.



(a) Circular ring



(b) Elliptical ring

Figure 1: Notation of geometrical parameters of ring specimens.

2.2 Degree of restraint

For a concrete ring without an inner restraining steel ring, the free shrinkage of concrete will occur under a drying condition. Once a steel ring is inserted, the drying shrinkage will be restrained. In this case, the steel and concrete have to deform together, which results in the concrete shrinkage displacement under restrained conditions being less than that without restraint. To ensure the cracking of the concrete mixture can occur in a reasonable duration, the thicknesses of concrete and steel rings should be determined in an appropriate range. Therefore, the degree of restraint was defined for rings [3, 10], which can be written as Equation 1:

$$\psi = \frac{U_{SH|R_{IC}}(t) - U_{S|R_{OS}}(t)}{U_{SH|R_{IC}}(t)} \tag{1}$$

where $U_{SH|R_{IC}}(t)$ and $U_{S|R_{OS}}(t)$ are the free shrinkage and restraint shrinkage displacements of the concrete at the inner surface of the concrete ring, respectively. In the case of an elliptical ring, experimental results have demonstrated that the crack position is close to the vertex on the major axis [16]. Therefore, the study about the degree of restraint for the elliptical ring aims at being provided by the steel ring in the major axis direction. Correspondingly, for the elliptical ring, $U'_{SH|R_{IC}}(t)$ and $U'_{S|R_{OS}}(t)$ denote the free shrinkage and restraint shrinkage displacements of concrete along the major axis of an elliptical geometry.

3 Numerical Model

In this study, finite element analyses were conducted using ANSYS code to simulate displacement field in concrete ring specimens under restrained shrinkage. The numerical process included thermal and structural analyses. In thermal analysis, the 2-D 8-node thermal elements (PLANE77) were used for modelling concrete, which have compatible temperature shapes and are well suited to model curved boundaries such as the circular and elliptical rings investigated in this study. From the thermal analysis, the temperature field in a concrete ring is obtained. In the following structural analysis, the elements for modelling concrete were replaced by the equivalent structural elements, i.e. PLANE183, which is a type of 2-D 8-node element with quadratic displacement behaviour. PLANE183 elements are also well suited to modelling irregular meshes and support plasticity, creep, stress stiffening, large deflection and large strain. In numerical analyses, contact elements with zero friction between the contacting pairs were utilised to eliminate the effect of friction between concrete and steel. The material parameters employed in numerical analyses were as following: the elastic modulus and Poisson's ratio of steel both remain constant as 210 GPa and 0.3, respectively; elastic modulus of concrete is determined based on its age with the consideration of the creep effect while Poisson's ratio of concrete is taken constant as 0.2 in all the ages investigated. With the implementation of the fictitious temperature field, concrete shrinkage resulting from the fictitious temperature field is restrained by the inner steel ring, resulting in a compressive stress developing in the steel ring and tensile stress in the concrete ring. Meanwhile, the displacements of $U_{SH|R_{IC}}(t)$ and $U_{S|R_{OS}}(t)$ can be obtained through numerical analyses.

It should be noted that the moisture gradient across a concrete ring wall can affect the position of crack initiation. The assumption of uniform shrinkage across ring section is acceptable when the influence of a moisture gradient is less pronounced [8, 19], for example, when the ring specimens are drying from the top and bottom surfaces. On the other hand, it may be approximately assumed that the shrinkage is uniform along the cross-section for a thin ring specimen and the concrete wall thickness is not greater than 37.5 mm [3, 16, 17]. In this study, the assumption of uniform shrinkage, i.e. a uniform fictitious temperature field, across a concrete ring wall is adopted.

According to See et al. [3] and Moon et al. [10], the degree of restraint in a circular ring is determined by the specimen geometry, e.g. the wall thickness of the restraining steel ring and materials properties, e.g. the elastic modulus and Poisson's ratios of concrete and steel. The effects of concrete age on the degree of restraint are reflected by the increasing elastic modulus and Poisson’s ratio of concrete. Therefore, simplify the analyses, a fictitious temperature drop of -20 ℃ is chosen in all the numerical examples presented in this paper for simulation of the restrained shrinkage behaviour of concrete subjected to drying. The age-dependent effective elastic modulus of concrete is reduced to be 60% of the measured instant elastic modulus at the same age to account for creep effects in numerical analyses. Such an approximation for the consideration of the concrete creep effect at early ages has been used by Weiss [7], and Zhou et al. [16].

4 Results and discussion

Considering the effects of various concrete stiffness, i.e. E_C, and its wall thickness on the degree of restraint, values of E_C = 1, 10, 30 GPa and R_{OC}/R_{IC} = 1.25, 1.5, 2 are used in the numerical studies.

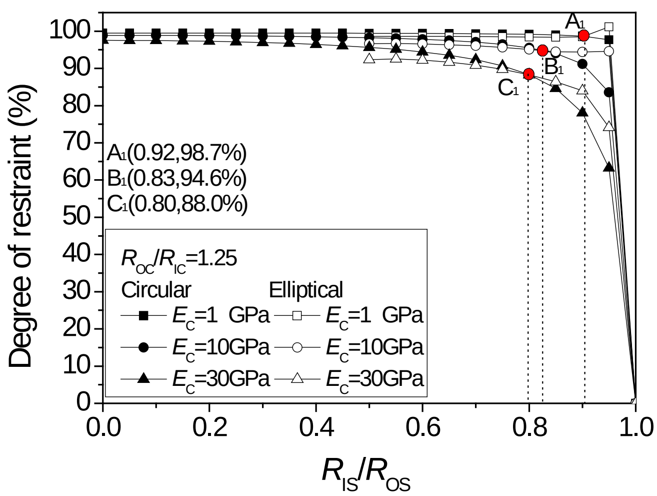
It should be noted that E'_C can be regarded as the effective elastic modulus that has been reduced to 60% of the corresponding instant value E_C to account for creep effects as elaborated earlier. Figure 2 illustrates the influence of ring geometry on the degree of restraint from numerical analyses. It can be seen from Figure 2, for the circular and elliptical rings, a stiffer (higher E_C) and thicker concrete wall (higher R_{OC}/R_{IC}) can result in a lower degree of restraint. Meanwhile, for the circular ring, the degree of restraint reduces with the decrease in steel ring thickness (higher R_{IS}/R_{OS}). However, there is no significant increase (i.e. by less than 5%) in the degree of restraint when R_{IS}/R_{OS} is less than 0.6. In the case of an elliptical ring, the influence of steel ring thickness on the degree of restraint is less significant than the scenario of a circular ring. When concrete wall thickness is 37.5 mm (see Figure 2(a)), and 75 mm (see Figure 2(b)), the decrease in the degree of restraint is less than 10% with R_{IS}/R_{OS} ranging from 0.5 to 0.9 in the case of an elliptical ring. In comparison, the degree of restraint in the case of a circular ring decreases up to 20%.

It is also worthy to point out that the degree of restraint in an elliptical ring is greater than in a circular ring only if R_{IS}/R_{OS} exceeds a certain value. The data points A (representing A_1, A_2 and A_3), B (representing B_1, B_2 and B_3)and C (representing C_1, C_2 and C_3) in Figure 2 present the same degree of restraint in a circular and an elliptical ring. The corresponding steel ring thickness at the data points A, B and C are listed in Table 1. It can be seen that, for concrete rings of the same wall thickness, both the circular and elliptical geometries have their respective advantages on providing a higher degree of restraint, which depends on concrete stiffness, i.e. elastic modulus, and steel ring wall thickness. For the 12.5mm-thick steel ring recommended by ASTM and AASHTO, the elliptical ring is more appropriate for assessing cracking tendency of stiffer concretes, which may not crack at the restrained shrinkage test when using an inner circular ring. In addition, it can be predicted that for the ring geometry recommended by ASTM (R_{OC}/R_{IC} = 1.25, R_{IS}/R_{OS} = 0.92), the development of the maximum circumferential tensile stress in concrete with an elliptical geometry is less than in a circular geometry when concrete is of low stiffness i.e. with a low elastic modulus.

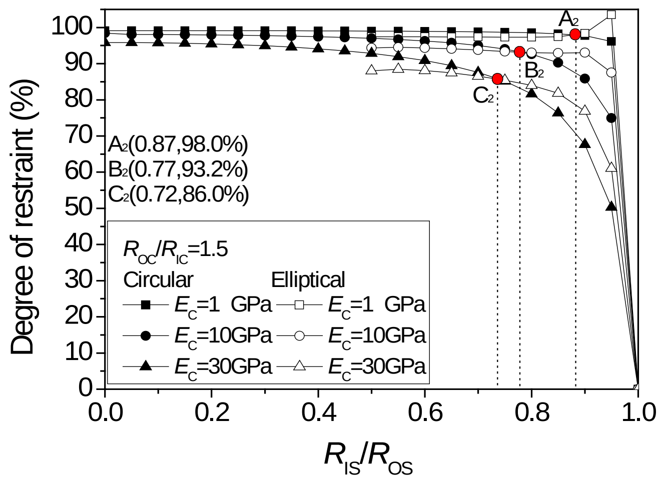
Table 1. Corresponding steel ring thicknesses of points A, B and C in Figure 2.

No.	R_{OC}/R_{IC}	Steel ring thickness at		
		Point A	Point B	Point C
1	1.25	12 mm	25.5 mm	30 mm
2	1.5	19.5 mm	34.5 mm	42 mm
3	2	19.5 mm	39 mm	49.5 mm

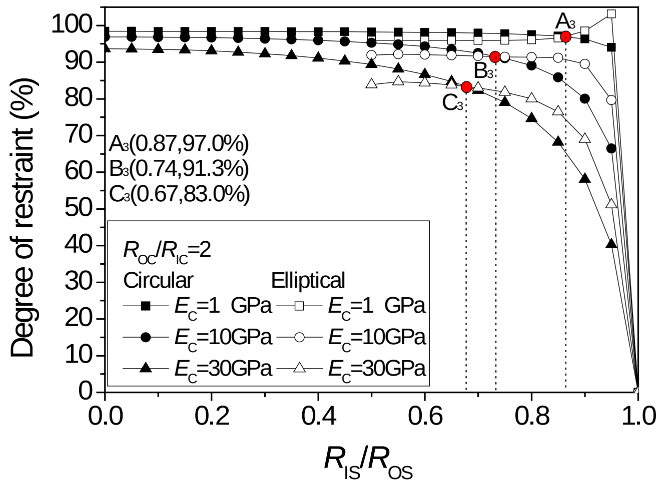
In general, for E_C = 1 GPa which is the typical value of elastic modulus for concrete at very early ages of around 1 day, the elliptical and circular steel rings provide a similar degree of restraint. In this case, the degree of restraint in an elliptical ring is more than 100% when R_{IS}/R_{OS} is close to 1 (see Figure 2). It can be explained that the displacement at the two vertices on the minor axis of the inner elliptical circumference results in the extension of displacement at the two vertices on the major axis. In the case of E_C = 10 GPa which is a typical value of the elastic modulus of normal strength concrete at the ages of around 2 weeks, the elliptical rings provide an almost constant degree of restraint, in which the increase in steel ring thickness does not increase the degree of restraint significantly. In the case of E_C = 30 GPa which is the elastic modulus of mature normal strength concrete, the increase in the steel ring thickness does increase the degree of restraint for both elliptical and circular rings when R_{IS}/R_{OS} is between 0.6 and 1.0.



(a) R_{OC}/R_{IC} = 1.25



(b) $R_{OC}/R_{IC} = 1.5$



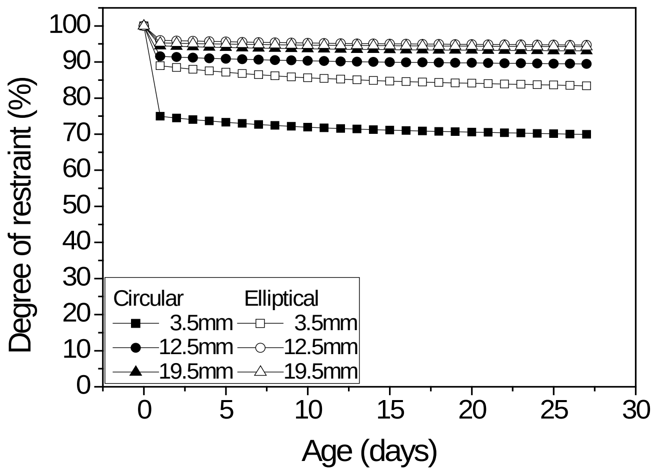
(c) $R_{OC}/R_{IC} = 2$

Figure 2: Relationship between the degree of restraint and ring geometry

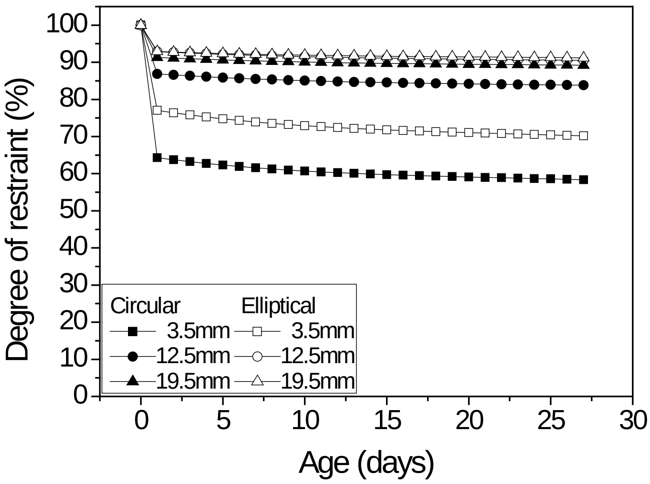
Also, two types of concretes with different elastic modulus at 28 days, i.e. E_C = 17 GPa and E_C = 28GPa, are chosen in numerical examples to compare the degree of restraint provided by elliptical and circular rings. In these analyses, R_{OS} and R'_{OS} are fixed as 150 and 75 mm, respectively, the thicknesses of the steel ring is chosen as 3.5, 12.5 and 19.5 mm, respectively, and the concrete wall thickness as 37.5 and 75 mm, respectively.

For the concrete with E_C = 17 GPa and a concrete ring wall thickness of 37.5 mm, it can be seen from Figure 3(a) that the degree of restraint can be significantly increased (i.e. by more than 15%) for a circular ring when the steel ring wall thickness increased from 3.5 to 12.5 mm. However, the increase is marginal (i.e. by around 5%) for a circular ring when the steel ring wall thickness increased from 12.5 to 19.5 mm. In the case of an elliptical ring, the degree of restraint is increased by less than 10% when the steel ring wall thickness is increased from 3.5 to 12.5 mm. However, the increase in the degree of restraint is very small when the steel ring wall thickness increased from 12.5 to 19.5 mm. Moreover, the results show that the elliptical ring with the thickness of 12.5 mm can provide a higher degree of restraint than the circular one with the steel ring wall thickness of 19.5 mm suggesting the advantage of choosing elliptical geometry than circular one for faster assessing cracking potential of concrete.

For the concrete with higher elastic modulus, i.e. E_C = 28 GPa, the degree of restraint provided by a restraining steel ring is lower than the case with the concrete having a lower elastic modulus, i.e. E_C = 17 GPa, at the same ring geometry (see Figure 3 (a) and (b)). This can be explained that the increase in concrete stiffness can reduce the degree of restraint, which has been indicated in Figure 2. Moreover, comparing the degree of restraint for a 12.5 mm-thick steel ring between the circular and elliptical geometries, the increase in the degree of restraint is more significant for concrete with higher elastic modulus by replacing a circular ring with an elliptical one. It is worthy to point out that, for the elliptical geometry, the degree of restraint is even slightly higher in the case of a 12.5 mm-thick restraining steel ring than in the case of a 19.5 mm-thick ring (see Figure 3 (b)).



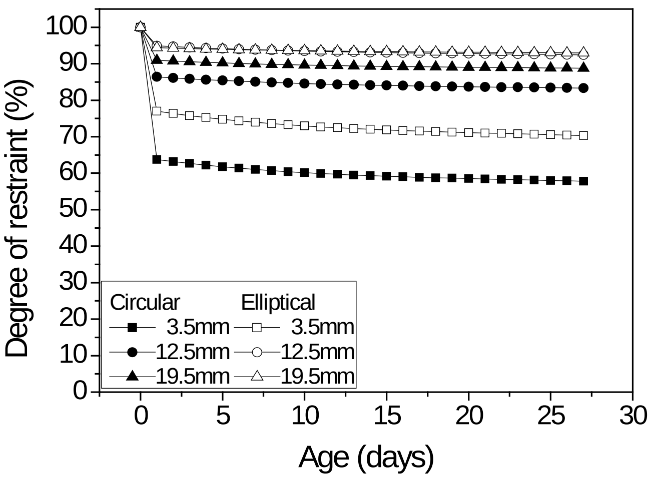
(a) Concrete with E_C of 17 GPa up to 28 days age



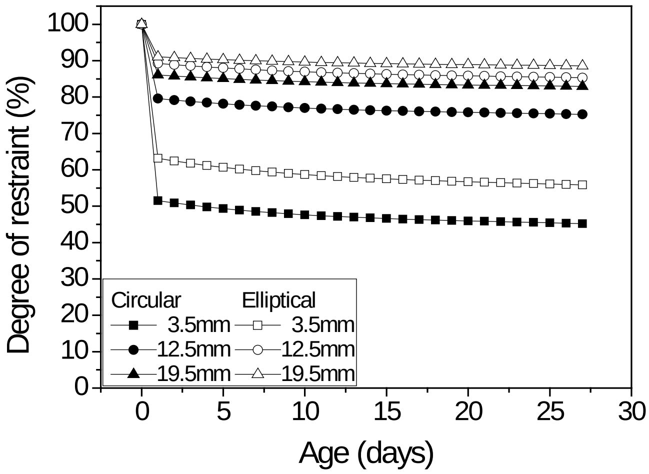
(b) Concrete with E_C = 28 GPa up to 28 days age

Figure 3: Degree of restraint with respect to concrete age for a 37.5 mm-thick concrete wall.

Figure 4 shows the variation of the degree of restraint for a 75 mm-thick concrete ring. In Figure 4(a), the degrees of restraint in an elliptical concrete ring restrained by a 12.5 mm and 19.5 mm-thick steel ring are almost identical in case of softer concrete with E_C = 17 GPa, which shows similar tendency as in Figure 3. However, the results are different for the concrete rings with a wall thickness of 75 mm and E_C of 28 GPa (See Figure 4(b)), which is the stiffer concrete investigated in this study. In this case, the increase of steel ring thickness in an elliptical ring from 12.5 to 19.5 mm also slightly increases the degree of restraint but by less than 5%. In comparison, the circular ring can increase the degree of restraint slightly higher, in this case by around 10%.



(a) Concrete with E_C = 17 GPa up to 28 days age



(b) Concrete with E_C = 28 GPa up to 28 days age

Figure 4: Degree of restraint with respect to age for a 75 mm-thick concrete ring.

In general, for the specimen with a concrete ring wall thickness of 37.5 mm and a steel ring wall thickness of 12.5 mm, i.e. the ring geometries recommended by ASTM, further increasing the restraining steel ring thickness does not increase the degree of restraint for elliptical rings, but can increase the degree of restraint for circular rings. Therefore, it can be concluded that an elliptical concrete ring restrained by a 12.5 mm-thick steel ring seems to be the most optimum scenario if a 37.5 mm-thick concrete ring is chosen for the restrained shrinkage test as per ASTM. Based on the calculated degree of restraint results from Figure 2, it can also be established that under the same size combination (steel and concrete ring wall thickness), the elliptical shape proves to always be more advantageous over the circular one. In particular, this advantage is more evident in the case of thicker concrete rings and/or concrete with a higher elastic modulus. Therefore, for concrete with a high cracking resistance, the elliptical ring test is more superior to the circular ring test for assessing the potential for early-age cracking under restrained shrinkage.

5 Derivation of the degree of restraint formula for elliptical rings

5.1 Expression for the degree of restraint

It has been verified that the ring test measures a concrete's response to a specific stimulus subject to a specific boundary condition (i.e. restrained condition), rather than reflecting the material property

[10]. In fact, the restraint to concrete varies in engineering structures, so that in a restrained ring test the ring specimen geometry and external stimulus should also vary to better simulate the service condition of concrete in an engineering structure. Therefore, it is significant to quantify the degree of restraint to concrete in the ring test, which can help tailor the ring geometry. In line with this, an analytical solution for the circular ring test has been proposed by Moon et al. [10] for the case of uniform shrinkage, which can be expressed by Equation 2.

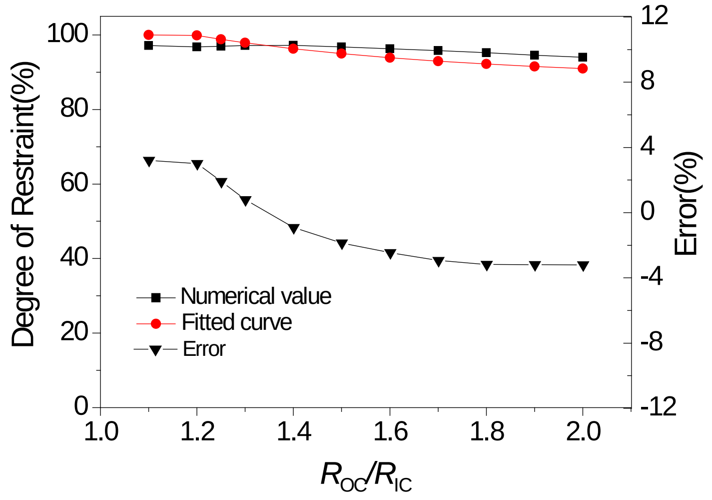
$$\psi = 1 - \frac{E_C^{'}}{E_S} \frac{1}{\frac{E_C^{'}}{E_S} - \frac{1-(\frac{R_{IS}}{R_{OS}})^2}{1-(\frac{R_{OC}}{R_{OS}})^2} \frac{[(1+v_c)(\frac{R_{OC}}{R_{OS}})^2 + (1-v_c)}{(1+v_s)(\frac{R_{IS}}{R_{OS}})^2 + (1-v_s)}} \tag{2}$$

Where, v_c and v_s are the Poisson's ratios of concrete and steel, respectively; E'_C is the effective elastic modulus of concrete (after considering the creep effect), E_S is the elastic modulus of steel; and R_{IS}, R_{OS}, and R_{OC} refer to the same geometry as defined earlier in this paper shown in Figure 1. It can be seen from Equation 2 that the degree of restraint is only a function of ring geometry and material properties (in this case modulus of elasticity and Poisson's ratio of steel and concrete). Therefore, the ring test can be tailored to specific concrete properties and degree of restraint, which concrete experiences in an engineering structure in the field. It should be noted that the above derivation of the degree of restraint formula is based on the uniform interfacial pressure between the concrete and steel rings. However, the pressure in an elliptical ring is non-uniform along its circumference. In this case, the major and minor semi-axes have a significant effect on the stress distribution. Thus, it is difficult to quantify the degree of restraint provided by an elliptical steel ring by means of the analytical method. Alternatively, a numerical method is utilized when analyzing the effects of various factors, including E'_C/E_S, R_{IS}/R_{OS}, and R_{OC}/R_{OS}, on the degree of restraint in an elliptical ring. As the advantage and effectiveness of the elliptical ring with the ratio of the major to the minor radius of 2 has been verified in previous research [16, 17], the ratio of R_{OS} /R'_{OS} =2 with R_{OS} =150 mm was adopted in the following numerical analyses. Meanwhile, by considering feasibility and applicability in a laboratory test, the geometrical ratios of R_{IS} /R_{OS} and R_{OC}/R_{IC} were chosen to lie between 0.525 and 0.95 and between 1.1 and 2, respectively, in the numerical analyses. Regarding concrete properties, the elastic modulus E_C ranging from 5 to 35 GPa was studied, which can approximately cover concrete from the initial setting to maturation. The expression for the degree of restraint for an elliptical ring was derived based on Equation 2 for a circular ring. By introducing a function $R(R_{IS}/R_{OS}, R_{OC}/R_{IC}, E'_C/E_S)$, geometrical effects (i.e. thicknesses of the concrete and steel rings) and material properties (i.e. effective elastic modulus of concrete), the degree of restraint ψ' in an elliptical ring can be expressed by Equation 3. Based on the results of a series of numerical analyses, the function R can be derived by curve fitting, which is expressed by Equation 4.

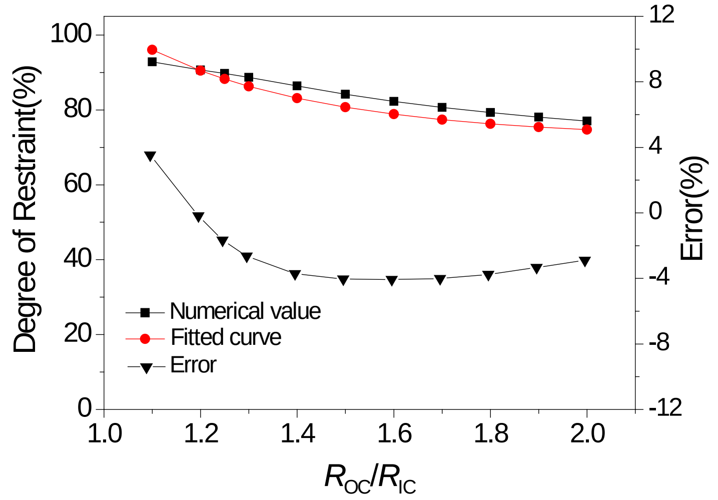
$$\psi' = \psi \cdot R(R_{IS} / R_{OS}, R_{OC} / R_{IC}, E'_C / E_S) \quad (3)$$

$$\begin{aligned} & R(R_{IS} / R_{OS}, R_{OC} / R_{IC}, E'_C / E_S) \\ & = (0.9+0.2R_{IS} / R_{OS} - 0.025R_{OC} / R_{IC}) \\ & -\left\{[-4(R_{IS} / R_{OS})^2 + 2.57(R_{IS} / R_{OS}) - 0.42]\left(R_{OC} / R_{IC}\right) + 9.15(R_{IS} / R_{OS})^2 - 9(R_{IS} / R_{OS}) + 2.23]\right\}(E'_C / E_S) \end{aligned} \quad (4)$$

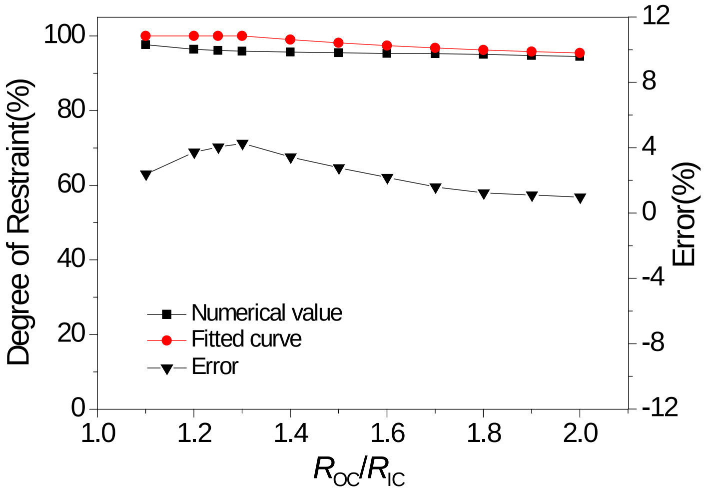
In order to verify the fitted expression, error analyses were carried out with respect to R_{IS}/R_{OS}, R_{OC}/R_{IC} and E'_C/E_S. For the cases where the steel ring thickness was 12.5 and 19 mm, concrete wall thickness was 37.5 and 75 mm, the concrete elastic modulus of 10 and 30 GP were chosen as examples to illustrate verification process. Corresponding to the latter conditions, the difference in ψ' between numerical results and the fitted expression with respect to R_{OC}/R_{IC}, E'_C/E_S, and R_{IS}/R_{OS} are illustrated in Figures 5, 6, and 7, respectively. Based on the error analysis, it can be seen that most of the errors keep within $\pm$5%, which indicates that the fitted expression shows good agreement with the numerical results. Meanwhile, it can be seen that the calculated degree of restraint provided by the elliptical geometry for various restrained conditions and material properties is fairly constant at 80% to 100%. This indicates that the elliptical ring test is appropriate for assessing the potential of cracking for stiffer concretes or under higher restraint in an engineering structure. In general, the derived expression for the degree of restraint in an elliptical ring can be used to design the restrained shrinkage test for any specific type of concrete under any specific degree of restraint in an engineering structure.



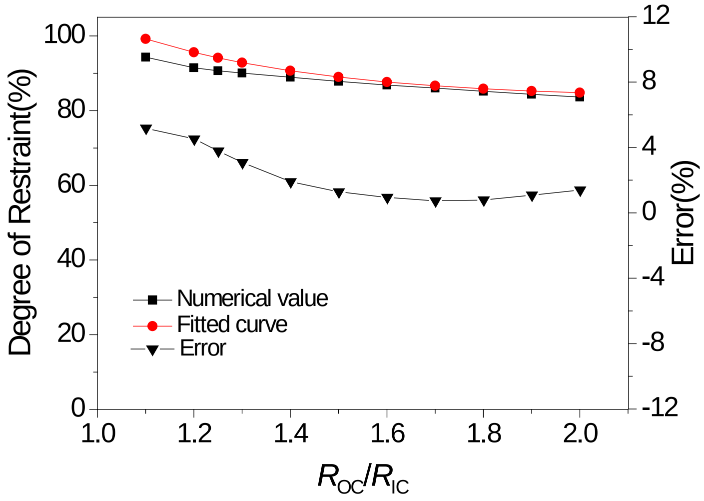
(a) R_{OS}-R_{IS} = 12.5mm, E_C = 10GPa



(b) R_{OS}-R_{IS} = 12.5mm, E_C = 30GPa

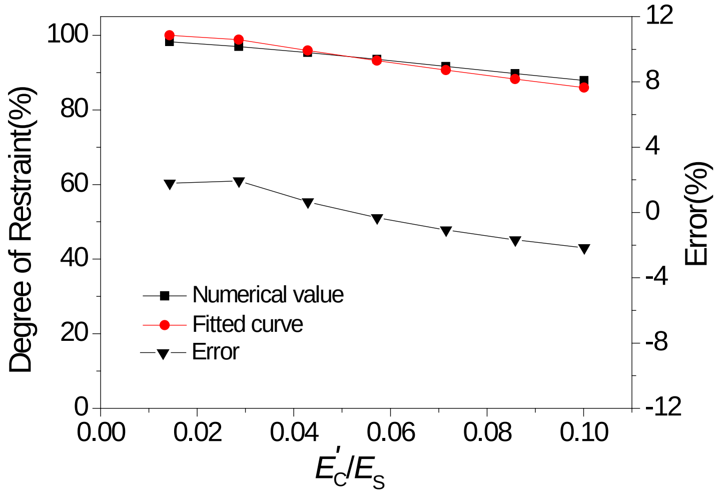


(c) R_{OS}-R_{IS} = 19.5mm, E_C = 10GPa

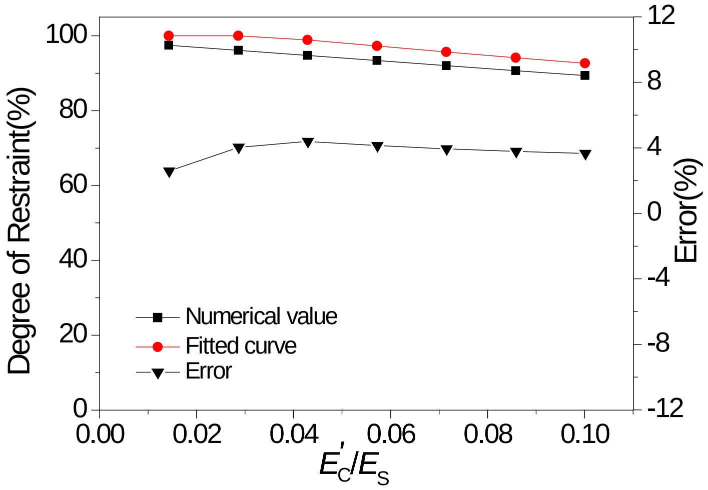


(d) R_{OS}-R_{IS} = 19.5mm, E_C = 30GPa

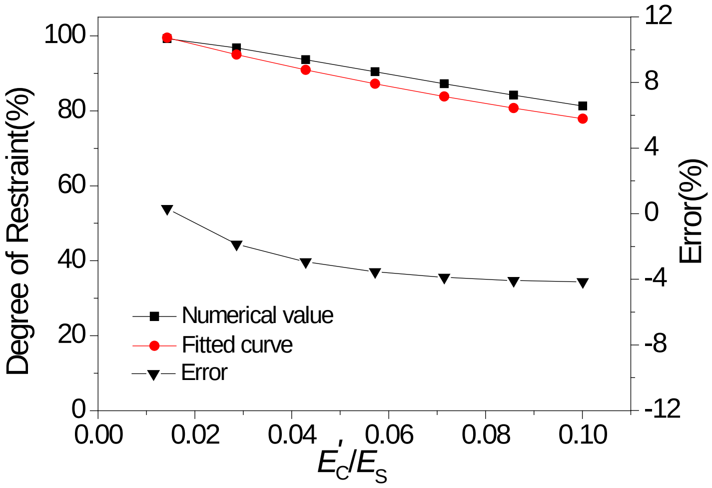
Figure 5: Error analysis for the degree of restraint with respect to R_{OC}/R_{IC}



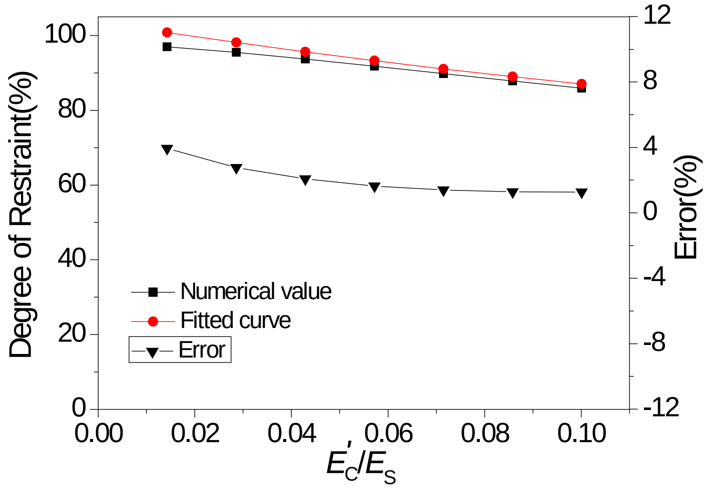
(a) R_{OC}-R_{IC} = 37.5mm, R_{OS}-R_{IS} = 12.5mm



(b) R_{OC}-R_{IC} = 37.5mm, R_{OS}-R_{IS} = 19.5mm

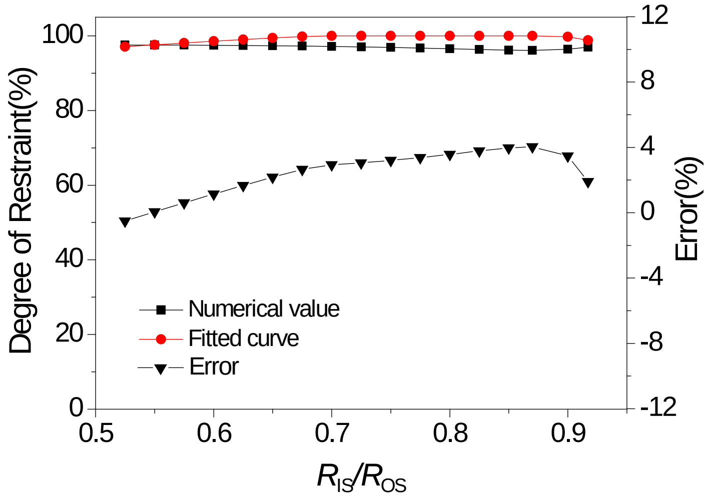


(c) R_{OC}-R_{IC} = 75mm, R_{OS}-R_{IS} = 12.5mm

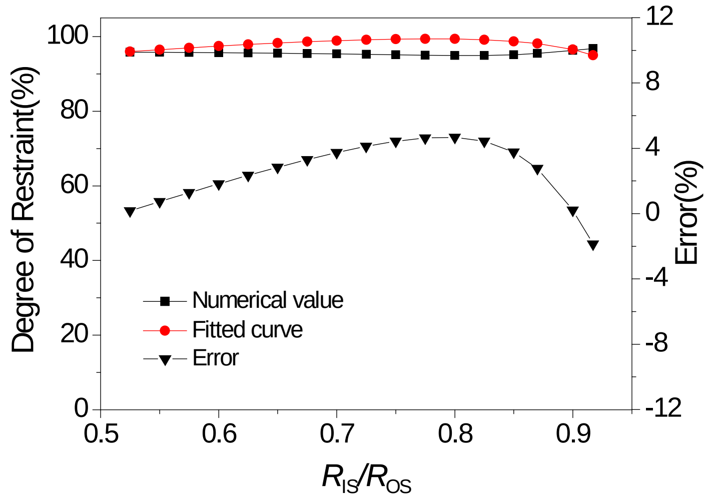


(d) R_{OC}-R_{IC} = 75mm, R_{OS}-R_{IS} = 19.5mm

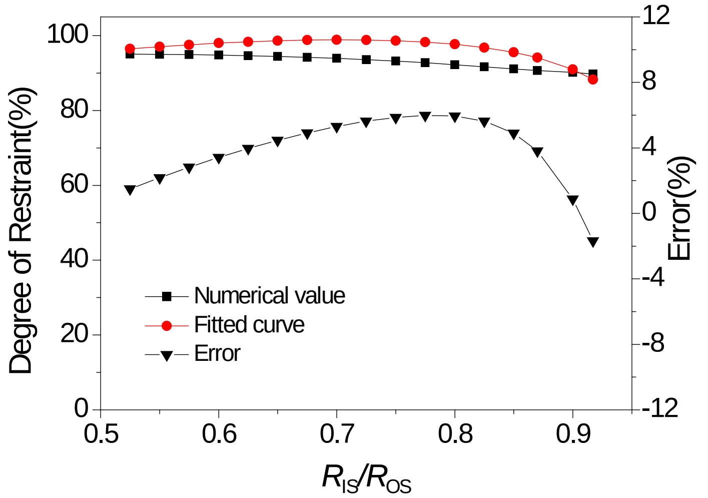
Figure 6: Error analysis for the degree of restraint with respect to E'_C/E_S



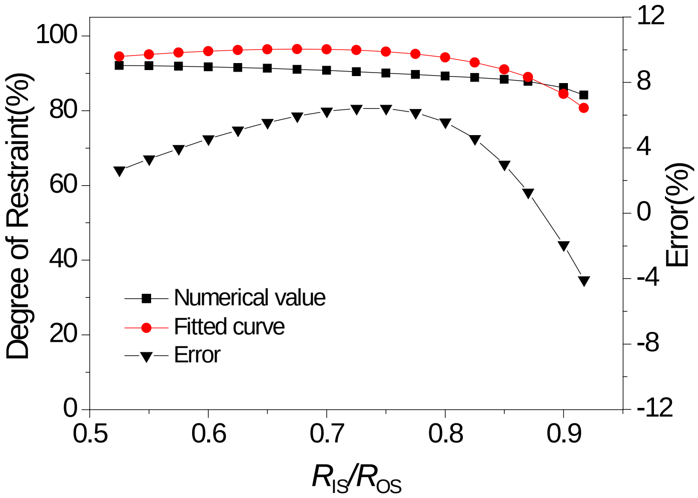
(a) E_C = 10 GPa， R_{OC}-R_{IC} = 37.5 mm



(b) E_C = 10GPa， R_{OC}-R_{IC} = 75mm



(c) E_C = 30GPa， R_{OC}-R_{IC} = 37.5mm



(d) E_C = 30GPa， R_{OC}-R_{IC} = 75mm

Figure 7: Error analysis for the degree of restraint with respect to R_{IS}/R_{OS}

5.2 Practical examples of the determination of ring geometry for the restrained elliptical ring test

In order to demonstrate the effectiveness of the derived formula (i.e. Equation 4) for the degree of restraint of an elliptical ring, two examples were taken to illustrate how to determine the geometry of an elliptical ring for assessing the cracking potential of concrete in the field. In the first example, it is assumed that the concrete in the field is a pavement repair material, which is 37.5 mm-thick and experiences drying from its top surface. The properties of the concrete are assumed as following: elastic modulus of 30 GPa and Poisson's ratio of 0.2 at the age of 28 days. Considering the effect of creep, the effective elastic modulus is reduced to 18 GPa (i.e. 60%×30 GPa). The degree of restraint is approximately assumed as 90% in order to simulate a thin but fully bonded surface repair. Meanwhile, the elastic modulus and Poisson's ratio of steel are taken as 210 GPa and 0.3, respectively.

Accordingly, the elliptical ring geometry should be 37.5 mm-thick to simulate the actual thickness of the pavement. Drying from the circumferential surface should be used to ensure a similar moisture condition and shrinkage as that experienced by the pavement. In this example, the height of the rings has little effect on the test design as long as it can be greater than 3 times the maximum aggregate size as specified in concrete engineering codes. Therefore, the height of 75 mm as recommended by ASTM is used in this study. However, it should be noted that the height must be appropriately calculated if drying from the top and bottom surfaces are used to simulate the shrinkage of the floor concrete with the same drying condition. In this case, the height of elliptical ring should be the same as the actual floor thickness.

Using the derived formula of Equation 3, the curves presenting the relationship between R_{OC}/R_{IC} and R_{IS}/R_{OS}, at various targeted degrees of restraint ranging from 60% to 90% are illustrated in Figure 8 in terms of possible elliptical ring geometries. In the case of $R_{OC}/R_{IC} = 1.25$ (i.e. $R_{IC} = 150$ mm and $R_{OC} = 187.5$ mm), the ring geometry with $R_{IS}/R_{OS} = 0.90$ should be used to achieve the targeted 90% degree of restraint, in which case the steel ring has a thickness of 15.7 mm. By considering commercial availability, elliptical ring geometry with $R_{IS}/R_{OS} = 0.9$ and steel ring thickness of 15 mm can be chosen resulting in the actual degree of restraint of 91% which is close to the targeted value of 90% (see Point A in Figure 8). In this case, this ring geometry is acceptable from the point of view of practical applications.

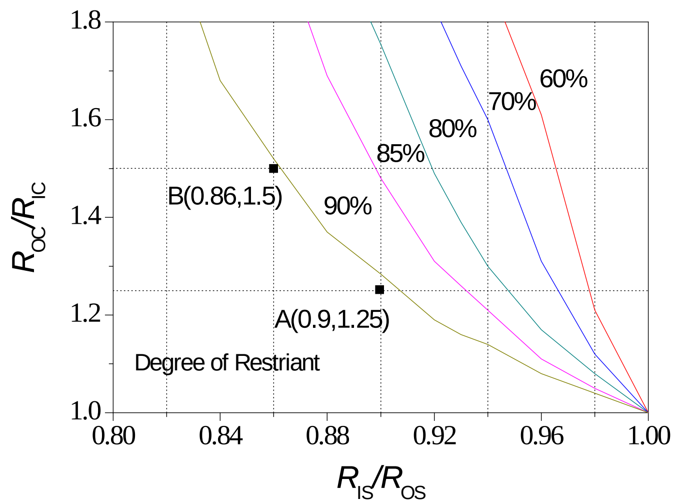


Figure 8: Degrees of restraint for various combinations of elliptical ring geometries.

In the second example, it is assumed that concrete in the field is a floor slab with the thickness of 75 mm and experiencing drying from the top and bottom surfaces. The properties of the concrete and steel are the same as in the first example, except that the maximum aggregate size in this example is 25 mm. The degree of restraint is also targeted at 90%. Since drying from the top and bottom surfaces of a 75 mm-thick concrete floor is similar to the scenario of concrete drying from one surface (top or bottom) from a 37.5 mm-thick concrete floor, the uniform shrinkage assumption still applies, i.e., Equation 3 can be employed to calculate the degree of restraint in this example as well.

Accordingly, the elliptical rings should be 75 mm-high to simulate the actual thickness of the floor slab. Drying from the top and bottom surfaces should be used to ensure similar moisture condition and shrinkage of concrete as experienced in the floor slab. As the thickness of the ring should be no less than 3 times the maximum aggregate size, the height of 75 mm is chosen in this study.

After R_{OC}/R_{IC} = 1.5, i.e. R_{IC} = 150 mm, and R_{OC} = 225 mm, is determined for this numerical example, R_{IS}/R_{OS} = 0.862 should be used in order to achieve the 90% degree of restraint target, in which case the steel ring should have a thickness of 20.8 mm. For practical purposes, steel ring geometries with R_{IS}/R_{OS} = 0.86 and thickness of 21 mm can be chosen, resulting in the actual degree of restraint of 90.2% (see Point B in Figure 8).

Finally, a comparison of the degrees of restraint for circular and elliptical rings with different possible geometrical combinations is illustrated in Figure 9. It can be seen that, in the case of a 60% degree of restraint, both the circular and elliptical rings can be used to assess the cracking potential of concrete. In contrast, in the case of a 90% degree of restraint, a wider selection of geometrical combinations is available for elliptical rings than for circular rings. Particularly, the elliptical ring is more appropriate for assessing the cracking potential of stiffer concrete (i.e. with higher elastic modulus) and concrete in thicker elements. For example, in the case of a 24 mm and 37.5 mm steel and concrete ring thickness respectively, both the circular and elliptical rings can meet the requirement of 90% degree of restraint (see Point A in Figure 9). However, when the concrete ring thickness increases to 75 mm, the circular ring can no longer provide enough restraint (see Point B in Figure 9) while the elliptical ring can. This is a positive indication that the elliptical ring test can complement the circular ring test as an improved method for assessing the cracking potential of concrete serving thicker concrete elements.

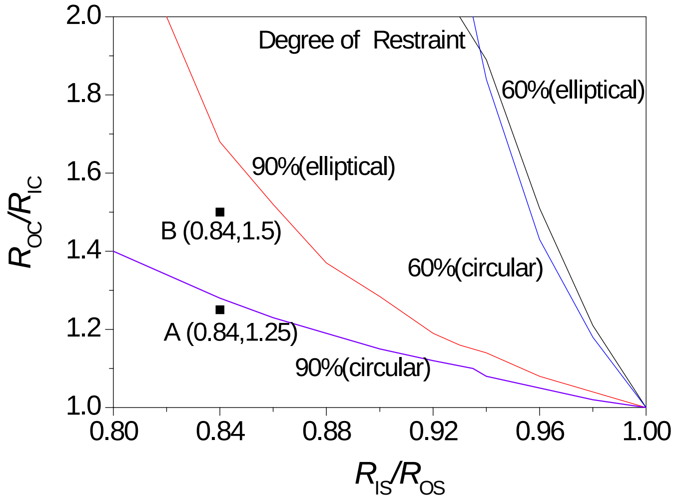


Figure 9: Degrees of restraint in terms of possible combinations of circular and elliptical ring geometries.

6 Conclusions

In this study, a fictitious temperature drop is applied to concrete to simulate the mechanical effects of shrinkage strain in both circular and elliptical restrained shrinkage ring tests. By comparing the deformation caused by free shrinkage and that by restrained shrinkage, the degree of restraint to concrete subject to a uniform shrinkage in the restrained shrinkage ring test can be obtained for both circular and elliptical rings. The effects of specimen sizes and concrete properties on the degree of restraint are investigated with respect to both the circular and elliptical geometries. Furthermore, an expression for the degree of restraint for elliptical rings was derived based on the results from a series of numerical analyses. Through the error analysis for a series of concretes and restraint conditions, the derived expression for the degree of restraint was verified, allowing the elliptical ring test to be tailored to map the restraint conditions that a particular concrete may experience in engineering structures. Based on the results presented in this paper, the following conclusions can be drawn.

(1) The degree of restraint to concrete in the restrained ring test depends on the ring geometry and concrete material properties. It decreases with the increase of concrete elastic modulus and concrete ring wall thickness. For normal strength concrete (with an elastic modulus of around 30 GPa at 28 days), the degree of restraint to concrete in a restrained circular ring increases with the increase of steel ring thickness. In comparison, it remains almost constant in the case of an elliptical ring. In this case, a 12.5 mm-thick steel ring seems to be an appropriate size in a restrained elliptical ring test.

(2) For a specimen of 37.5 mm-thick concrete and 12.5 mm-thick steel ring (as recommend by ASTM) and a 75 mm-thick concrete and 12.5 mm-thick steel ring (as recommended by AASHTO), the elliptical ring can increase the degree of restraint by around 10% more than a circular ring for normal strength concrete. For stiffer concrete which may not crack in the restrained circular ring test because of the high cracking resistance, the elliptical ring test is a better method to assess the cracking tendency of concretes by providing a higher degree of restraint to concrete than a circular ring.

(3) Through numerical analyses, an expression for the degree of restraint of an elliptical ring was obtained, which clearly demonstrated how the concrete mixture and restraint condition affect the degree of restraint. It is helpful for a better understanding of the mechanism of the circular and elliptical ring tests so that an appropriate test method (including associated ring geometries) can be chosen for the assessment of the cracking potential of concrete under restrained shrinkage as described in AASHTO PP 34-99 and ASTM C1581/C1581M-09a.

(4) According to the derived formula for the degree of restraint, the elliptical ring test can be tailored by adjusting the ring geometry to simulate the actual restraint condition that concrete experiences

in engineering structures in the field. Particularly, in the case of a high degree of restraint, which cannot be achieved by means of a circular ring, the formula can provide quantitative information for the determination of the appropriate elliptical ring geometries so that the elliptical ring test can complement the circular ring test as an improved method for assessing cracking potential of concrete.

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