What knowledge is necessary to encourage mathematical talk?

Gwen Ineson

Brunel University London, UK; gwen.ineson@brunel.ac.uk

This paper presents the results from a follow-up study to a London Mayor's funded project which aimed to support teachers of KS1 pupils (age 4-6) to encourage mathematical talk in their classrooms. The original study was an eighteen-month intervention programme to support teachers to develop their mathematics subject knowledge and their use of mathematical talk by providing readings and activities to use in school with their pupils. Findings from follow-up interviews with eight teachers suggest that the impact of developing subject knowledge gave teachers the confidence to try different pedagogical approaches related to talk.

Keywords: Mathematics subject knowledge, pedagogic subject knowledge, talk.

Introduction and Context

The mathematics knowledge that teachers *should* have has long been researched and debated. Is it enough for teachers to be one step ahead of their pupils, or does having a degree in mathematics necessarily make for a successful teacher? And what do we mean by 'successful teacher'? Is it to do with exam results, conceptual understanding, or the way that pupils engage with mathematics? This paper attempts to address some of these questions, by analysing teachers' conversations about their professional growth during an intervention programme to develop their mathematics subject knowledge and talk pedagogy.

Mathematics subject knowledge

Until the 1970s, researchers largely believed that secure knowledge of the subject was sufficient for effective teaching. During this time, research tended to focus more closely on the effects of different teacher behaviour in the classroom, rather than the type and level of subject knowledge a teacher had. However, this focus was challenged by researchers who suggested that all pedagogic choices were made because of teachers' subject knowledge (Ball & Feiman-Nemser, 1988; Leinhardt & Smith, 1985; Thompson, 1984). It was not until the emergence of Shulman's work on the conceptualisation of the knowledge required for teaching that attention once again became directed at teachers' knowledge of the discipline (Shulman, 1986; Rowland, 2007; Ball, 1990). He suggested that broadly, there were two main types of teacher knowledge: that which related to the specific content of the subject being taught (content knowledge), and that which is unrelated to the subject, but necessary for teachers to know, for example about how children think and learn (pedagogic knowledge). The focus of this study is content knowledge and attention is now turned to this. Shulman identifies three categories within content knowledge: subject matter content knowledge (SMK), pedagogical content knowledge (PCK) and curriculum knowledge (CK). SMK "refers to the amount and organization of knowledge per se in the mind of teachers" (Shulman, 1986, pp. 9). It includes substantive content in the form of facts and concepts together with syntactical content which relate to how knowledge is generated and structured and what makes facts and concepts true. PCK "goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching" (Shulman, 1986, pp. 9). Shulman (1986) used the term to encompass elements of both subject knowledge and pedagogic knowledge. For example, it included the way in which a teacher understood a discipline's concepts and could explain them, or create situations for pupils to understand the concepts. It was 'the ways of representing and formulating the subject that make it comprehensible to others' (Shulman, 1986, pp. 9). It described what teachers knew about the topics that typically caused students difficulty, what they knew about how different ideas were integrated and organised in curricula, and how particular examples or explanations could be useful in teaching specific concepts. Finally, CK involved an awareness of the ordering of topics over time, and within programmes of study.

Rowland, Turner, Thwaites and Huckstep (2009) built on this work to develop the knowledge Quartet, encompassing four dimensions of mathematics subject knowledge for teaching. The first of these is defined as Foundation Knowledge, which incorporates Shulman's SMK and is described by Rowland et al as the mathematical knowledge gained, before being 'put to use' in the classroom (Rowland et al, 2009. p.29). The following three dimensions rest on Foundation Knowledge, and are related to Shulman's PCK. Transformation Knowledge is described as 'knowledge-in-action' and relates to how teachers transform their knowledge of mathematics to enable pupils to make sense of this knowledge. Connection Knowledge relates to the coherence of decisions about planning and teaching lessons and finally Contingency Knowledge is the ability to respond to unanticipated pupil responses and questions. This work was developed through observation and analaysis of mathematics lessons as a framework to identify and discuss mathematical subject knowledge.

Mathematics Talk

There is little doubt that when children are engaged in their learning through talk, their learning is deeper (Mercer, 2000; Alexander, 2012). Within mathematics lessons, approaches taken to encourage talk are varied. On the one hand, it is used to explain "correct mathematical structures" (Murphy, 2015, pp. 62) whereas other approaches use it as a 'mediating tool for meaning-making through the negotiation of ideas' (Murphy, 2015, pp. 62). Mercer and Littleton (2007) describe this as 'Exploratory Talk', where:

partners engage critically but constructively with each other's ideas. Statements and suggestions are offered for joint consideration. These may be challenged and counter-challenged, but challenges are justified and alternative hypotheses are offered. Partners all actively participate, and opinions are sought and considered before decisions are jointly made. (Mercer and Littleton, 2007, pp. 51)

Incorporating exploratory talk in classrooms requires teachers to develop a new pedagogy (Goos, 2004; Hunter, 2005). Murphy (2015) attempted to investigate this by designing an intervention programme for teachers to use with their pupils. She found that despite considerable information being provided on the features of exploratory talk, the teachers in the project interpreted this in different ways. For example, one teacher continued to dominate classroom talk and another failed to encourage pupils to discuss their developing ideas, and instead focused on talking about their final solutions.

This required change in pedagogy is not necessarily straight forward. Alexander (2012) suggests that a stronger focus on talk exposes two of teachers' greatest concerns which leaves them feeling vulnerable; subject knowledge and classroom behaviour, and that this, therefore, has serious implications for continuing professional development.

Theoretical Framework

This paper draws on Rowland et al's work (2009) on the Knowledge Quartet to identify the dimensions that were influential in teachers' use of talk in their classrooms. The illustration in figure 1 below shows the interaction between the four dimensions of the Knowledge Quartet, indicating that foundation knowledge impacts on all three other dimensions, and that contingency knowledge relies on each of the other three dimensions. Alexander's proposal above, about the relationship between subject knowledge and pedagogy involving talk, suggests that each of these dimensions may have a role to play in enabling teachers to use more mathematical talk in their classrooms.

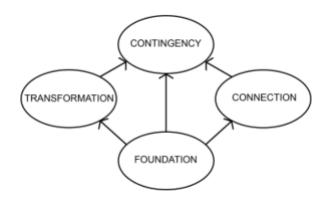


Figure 1: The relationships of the four dimensions that comprise The Knowledge Quartet (Rowland, Turner, Thwaites and Huckstep, 2009).

This paper uses the framework above to explore the following questions:

- What support do teachers feel helps to enhance their mathematics subject knowledge?
- How is mathematics subject knowledge related to teachers' use of mathematical talk?

Methodology

This paper attempts to address the questions outlined above, by interviewing teachers who had been involved in a project entitled 'Talk Maths Talk: Enhancing Mathematical Learning Through Talk'. The aim of the project was to:

Raise the quality of mathematics teaching through developing teachers' subject knowledge and classroom practice with an emphasis on understanding the importance of talk in teaching and learning mathematics.

This was a London Mayor's funded project, which involved Key Stage One (ages five to seven) teachers from thirty-three schools across two south west London Boroughs and ran for eighteen

months. Supply cover was provided to ensure that teachers could attend six full day sessions, resources were provided to support the development of the project, activities were set for participants to use in their own classrooms, and teachers kept a learning journal. Booster sessions were also held to support the development of subject knowledge. During the first meeting the teachers took an audit of subject knowledge which included a range of questions, modelled on those found in the Year 6 (ten and eleven year olds) standardised tests. As well as solving each problem, they were required to state how confident they felt about their solution, and how confident they would be to work with pupils on the problem. These audits were marked by the project team and returned to the participants. Follow up boosters were offered throughout the course of the project and a second audit was conducted at the end of the project. The purpose of this paper is to present the teachers' own reflections about how their subject knowledge had developed over the course of the project.

Ethical approval for the follow-up study was provided by the host institution and participants were selected based on their involvement in the entirety of the previous project. Twelve teachers were emailed to invite participation in this follow-up study, eight responded and all were interviewed (there were two pairs of teachers in the same school who requested to be interviewed together, resulting in a total of six interviews). They were all experienced teachers and had some responsibility for the mathematics teaching in their schools, either as mathematics coordinators or on the mathematics team. A phenomenological approach was taken to the analysis of the data, to ensure that teachers' experiences were the focus throughout the project. This focused on identifying aspects that participants felt had enhanced their subject knowledge, and how it had impacted on their classroom pedagogy relating to talk.

Results

A pre-and post audit was used in the earlier project to identify any changes in confidence and competence. Questions were adapted from a Level 6 SATs paper (one which would have been given to only the very highest attaining pupils in their final year of primary school) and as well as completing each problem, members of the project were asked to rate their confidence in their responses. Although the results of these audits are not presented here, these *tests*, as participants saw them, were a popular topic during the interviews.

All participants indicated that they felt that their subject knowledge had developed during the project and many linked this to an increase in confidence. Throughout the project there were 'booster' sessions which were run to support teachers in their understanding of the items used on the audit. Bernie explained that working through each example with other teachers involved in the project enabled her to identify that there were a range of approaches to solving the problems and that these gave her confidence in her own approaches. Working with others had highlighted the connections between a range of calculation methods. In fact, Bernie's views about teaching had completely changed, because by the end of the project she explained that she felt that it wasn't possible to teach if you didn't have confidence in your subject knowledge. Janice, who is a mathematics coordinator, said that the booster sessions had given her confidence to help her own colleagues to develop their subject knowledge because she had learnt methods she was not aware of

previously, again indicating the importance of making connections. She also felt that talking each problem through was an important aspect to her learning.

Janice: I've learnt methods that maybe I didn't have before. And the element of talking about it and seeing how somebody else works it out, has been quite a learning process I think.

Another aspect that teachers identified as a trigger for supporting the development of their subject knowledge was opportunities to unpick children's understanding. Teachers in the original project had specific tasks to try out with pupils, which they discussed at subsequent sessions in the project. Teachers explained that these activities resulted in an increase in their subject knowledge because they "unpicked the language" (Bernie). Netty explained that it was the opportunity to 'play around with a few things', and not be pushed to find solutions quickly, that caused the increase in confidence, and ultimately, in her subject knowledge. This is further evidence that the opportunity to talk about connections in differing approaches supports the development of subject knowledge.

Involvement in activities such as these was a regular feature of the original project and discussion that teachers engaged in about these, both within the project and back in their own schools, impacted on their developing subject knowledge. For example, this is a response from Janice about the factors which she felt contributed to the development of her subject knowledge:

Janice: We were provided with a lot of examples and things that we could come back to school straight away and use, but we were also given plenty of time for discussion with other colleagues to share good practice.... And I think one of the strengths of it is was a lot of shared ideas and being able to come back and then share those in your own school.

This suggests that by 'unpicking' the approaches that pupils had used, teachers may have developed their contingency dimension of subject knowledge because they were discussing a range of approaches to a task, and possible responses to expect.

A further indication about the change in attitude by the end of the project was Bernie's comment on how she felt about other teachers completing the audit before her.

Bernie: Actually, I felt far more confident, and I didn't mind that other people had finished before me.... because I knew I was plodding through it and I would get through it. It was much easier.

This indicates that she was less concerned about the speed at which she was working, and instead was more interested in finding the solutions, the result of which was that she found it easier. Her earlier comments indicated that she was anxious about the first audit, indeed she described this as 'fear, absolute fear' even though she started the project slightly later and completed the audit at home. Janice explained that by the end of the project, her approach to completing the audit was to stop and consider what her approach would be for each problem, something she did not do during the first audit.

Confidence was also a theme that emerged in response to the second research question, relating to how teachers perceived that development in subject knowledge impacted on their talk pedagogy.

Bernie explained that she felt confident to explain and model the mathematical language in her classroom, which resulted in her developing confidence to unpick the language that pupils were using. An interesting comment came from a mathematics coordinator, who explained that when observing another teacher in her school, who was also involved in the project, she had noticed that the teacher now 'released control'. She explained that because the focus of the teaching was more on the process, rather than the solution, pupils seemed more confident to 'have a go' and no longer became upset if they gave an incorrect solution. Netty also indicated a belief that her increased confidence had affected her pedagogy, as she now focused more closely on helping her pupils to develop greater confidence themselves. She explained that she felt this led to them developing a deeper understanding.

Bernie explained that a significant change in her pedagogy was that she teaches in a more open way, where she tends to focus on approaches taken, rather than solutions to problems. She believed that this is a result of her increased confidence in her own subject knowledge, but she went further, to explain that it has also given her confidence to adapt the curriculum. She explained that she felt confident to allow children to take their time to talk about their learning, rather than being concerned to move on to the next topic. This is further evidence that the teachers' increased subject knowledge, in this case relating to the foundation dimension, resulted in a changed pedagogy, one which allowed more freedom because teachers' were not so concerned about the contingency dimension of their subject knowledge.

Val felt that changes in her pedagogic approach was partly due to the collaborative aspect of the project. She identified the benefits of having to talk and work through problems with other colleagues on the course, as well as the opportunities to try out ideas in school and come back together to talk about how it had gone.

One further change that several teachers mentioned was that their involvement in the project had resulted in them being less concerned with getting through the mathematics curriculum. Prior to the project many teachers had planned in advance and would move on regardless of the progress that their pupils made. They reported that they are now less concerned about rushing through the curriculum, because they felt confident that the children would 'deeply' understand the concepts by engaging in talking about it, so they would not have to revisit the concept a few weeks later.

Finally, several teachers identified that a pedagogic change that they put down to their increase in subject knowledge was the focus on asking more questions. Bernie explained that the approach had 'really changed my way of thinking' as she is more open in her teaching. She explained that her focus is much more about method than solution, and then feels confident to 'probe' their understanding when they give an incorrect solution. Jemima also explained that she constantly asks different type of questions which 'delve' deeper into her pupils' understanding, and that this had been noted during observations by senior management colleagues. This confidence to 'probe' and 'delve' may be a result of the impact that the increased foundation dimension of subject knowledge had on the connection and transformation dimensions, and ultimately, on the contingency domain.

Conclusions

This project did not aim to explore whether specific dimensions of the Knowledge Quartet increased through the original project, or indeed whether teachers used more talk in their classrooms, but rather the focus was on exploring the teachers' perceived changes in their teaching. The comments from teachers about the effect that working through examples together had on their confidence suggests that whilst the purpose of these activities was to develop the foundation dimension of subject knowledge, through engaging in discussion about alternative approaches, an additional result was an increase in the transformation domain as they identified different ways of approaching the teaching of these problems. It also developed the connection domain as they became aware of a variety of approaches to solving the problems.

These different perspectives, and opportunities to take their time talking and working through problems was helpful for these teachers and therefore they recognized that they needed to provide these learning opportunities for their pupils. So, experiencing this themselves had a direct impact on their changing pedagogy.

The results showed that teachers in this study believed that their subject knowledge developed through being involved in the original project. It is also clear that confidence plays an important role, both in terms of how they feel about their subject knowledge, and in terms of the transformation choices that they make in the classroom. The aim of the original project was to develop both subject knowledge and talk pedagogy. Evidence from this follow up study suggests that a result of attempting to develop the teachers' SMK was that they felt more confident about the contingency dimension of subject knowledge and were therefore more confident to make pedagogic changes to their teaching.

Acknowledgment

My thanks go to Professor Valsa Koshy and her team on the *TalkMathsTalk* project, for the opportunity to work with the teachers involved.

References

Alexander, R. (2012) *Improving oracy and classroom talk in English schools: achievements and challenges*. Extended and referenced version of a presentation given at the DfE seminar on Oracy, the National Curriculum and Educational Standards, 20 February 2012

Ball, D. L., & Feiman-Nemser, S. (1988). Using textbooks and teachers' guides: A dilemma for beginning teachers and teacher educators. *Curriculum Inquiry*, 18(4), 401-423.

Ball, D. L. (1990). Prospective Elementary and Secondary Teachers' Understanding of Division. *Journal for Research in Mathematics Education*, 21(2), 132-144.

Ball, D. L., Hoover Thames, M., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education*, 59 (5), 389-407.

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education*, 35(4), 258-291.

Hunter, R, (2005). Reforming communication in the classroom: One teacher's journey of change. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Research, Theory and Practice*. (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia. 451-458). Sydney: MERGA.

Leinhardt, G., & Smith, D. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology*, 77, 247-271.

Mercer, N. (2000). Words and Minds: how we use language to think together. London: Routledge.

Mercer, N and Littleton, K. (2007). *Dialogue and the Development of Children's Thinking: A Sociocultural Approach*. London: Routledge.

Murphy, C. (2015). Changing teachers' practices through exploratory talk in mathematics: A discursive pedagogical perspective. *Australian Journal of Teacher Education*, 40(5), 61-84.

Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2009). Developing primary mathematics teaching: Reflecting on practice with the Knowledge Quartet. London: Sage.

Rowland, T. (2007). Auditing the Mathematics Subject Matter Knowledge of Pre-Service Elementary School Teachers. Paper Presented at *Seminar 3: Auditing and assessing mathematical knowledge in teaching*. Nuffield seminar series.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15 (2), 4-14.

Thompson, A. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, *15*, 105-127.