ESSAYS ON MODELLING HOUSE PRICES

A thesis submitted for the degree of Doctor of Philosophy

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Abstract

Housing prices are of crucial importance in financial stability management. The severe financial crises that originated in the housing market in the US and subsequently spread throughout the world highlighted the crucial role that the housing market plays in preserving financial stability.

After the severe housing market crash, many financial institutions in the US suffered from high default rates, severe liquidity shortages, and even bankruptcy. Against this background, researchers have sought to use econometric models to capture and forecast prices of homes. Available empirical research indicates that nonlinear models may be suitable for modelling price cycles. Accordingly, this thesis focuses primarily on using nonlinear models to empirically investigate cyclical patterns in housing prices. More specifically, the content of this thesis can be summarised in three essays which complement the existing literature on price modelling by using nonlinear models. The first essay contributes to the literature by testing the ability of regime switching models to capture and forecast house prices. The second essay examines the impact of banking factors on house price fluctuations. To account for house price characteristics, the regime switching model and generalised autoregressive conditionally heteroscedastic (GARCH) in-mean model have been used. The final essay investigates the effect of structural breaks on the unit root test and shows that a time-varying GARCH in-mean model can be used to estimate the housing price cycle in the UK.

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January 2018

DECLARATION

I declare that this thesis is the result of my own work and includes nothing which is the outcome of work done in collaboration except as declared in the Reference and specified in the text.

I declare that this thesis is not substantially the same as any that I have submitted, or, is being concurrently submitted for a degree or diploma or other qualification at the Brunel University London or any other University or similar institution except as declared in the Reference and specified in the text. I further state that no substantial part of my dissertation has already been submitted, or, is being concurrently submitted for any such degree, diploma or other qualification at the Brunel University London or any other University of similar institution except as declared in the Reference and specified in the text.

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Chapter One

Introduction

It is common knowledge that business cycles are constituted by expansion and contraction phases. For policymakers and investors, accurate modelling of business cycles is crucially important to preserving financial stability and managing financial risk. Historically, a burst housing bubble has resulted in regional or global recessions. Therefore, an important issue for econometricians is accurately modelling and forecasting business cycles in order to inform policymakers and investors and hopefully prevent unsustainable fluctuations.

In the literature, nonlinearity has been found in business cycles (see, for example, Terasvirta and Anderson, 1992). Accordingly, researchers have developed nonlinear models for modelling business cycle fluctuation. For example, Hamilton (1989) developed a Markov switching model for the US GNP, and Terasvirta, Dijk, and Medeiros (2005) applied a smooth transition autoregressive (STAR) model to forecasting macroeconomic variables in G7 countries. This thesis contributes to the investigation of the usefulness of several types of nonlinear models and their implementation in modelling the housing market.

Academics are interested in modelling the housing market due to the fact that housing markets play an important role in financial stability. However, most previous empirical works have used linear models to estimate house prices, although nonlinearity in house prices has been identified in numerous studies (e.g., see Muellbauer and Murphy, 1997; Miles, 2008; and Balcilar et al., 2015). Furthermore, house price cycles also exhibit asymmetrical properties. Igan and Loungani (2012) found that house price cycles have longer lasting expansion than contraction phases in most countries. Accordingly, we believe that nonlinear models may exhibit more capacity in modelling house prices. More specifically, this thesis examines a range of regime switching models to capturing the cyclical patterns of house prices, and we test the volatility patterns in housing markets by GARCH in-mean models. We believe that the cyclical modelling provides useful information for macro-economic researcher and policy makers. Meanwhile, the volatility modelling shows lots of econometric evidence for investors who try to understand on house price returns.

Nonlinear time series mainly cater to three types of nonlinearity, namely nonlinearity in mean, nonlinearity in variance and discrete time series[1]. In relation to linear models, the nonlinear counterpart augments linear models with nonlinear terms in a conditional mean equation specification so that the conditional mean becomes nonlinear in the lagged dependent variables and disturbances. These models have a better ability to capture jumps and asymmetrical adjustments across cycles. Against this background, this thesis tests three types of regime switching models. Namely, the momentum threshold autoregressive (M-TAR) model used by Enders and Granger (1998), the Markov switching time-varying transition probability (MS-TVTP) model proposed by Kim and Nelson (1999) and Filardo (1994), and the STAR model developed by Terasvirta (1994).

We believe that the regime switching models, broadly defined, are well fitted to capture house price cycles due to their mechanism, which is consistent with the characteristics of house price cycles. However, only a few types of research focus on applying nonlinear models to the housing market. For example, Kim and Chung (2016) investigated the linkage between house prices and business cycles in the UK and US using a Markov switching model. Nneji, Brooks, and Ward (2015) also used the Markov switching model to investigate the US housing market. Further, Chen, Cheng, and Mao (2014) studied housing returns in the US with an application of the Markov switching model. In addition, Simo-Kengne et al. (2013) examined the impact of monetary policy on the South African housing market by employing a Markov switching vector autoregressive model. Finally, Ricci-Risquete, Ramajo, and

[1] Discrete time series situation is not considered in this thesis because the house price is a continuous random variable.

de Castro (2016) investigated the effect of house prices on fiscal policy in Spain using an adjusted Markov switching model. The above research all estimates house price cycles within a Markov switching framework; however, as Crawford and Fratantoni (2003) point out, the Markov regime model is suitable for modelling house prices in regions with high volatility but fails to capture house price characteristics in more stable markets. Also, an important issue with the aforementioned studies is that they do not give suggestions as to which model is the best fit for modelling housing markets.

In addition to nonlinear models in the conditional mean equation, nonlinearities can be considered when modelling the conditional variance equation.

Nonlinearity in variance is a crucial aspect of many financial areas, such as portfolio construction, risk management, and derivatives pricing. In this thesis, the particular nonlinear variance specification investigated is the autoregressive conditionally heteroscedastic (ARCH) model. The ARCH was introduced by Engle (1982), and it is widely used for financial modelling. In 1986, Bollerslev developed the GARCH model. The GARCH model captures the autocorrelation properties in the variance, which simulates a more real-world context for financial modelling.

In previous studies, some researchers also tested GARCH-class models on the housing market. Dolde and Tirtiroglu (1997) examined changes in price volatility with a GARCH in-mean (GARCH-M) model. Further, Miller and Peng (2006) used GARCH models to analyse the volatility in US home prices. Miles (2008) tested GARCH models for all fifty states in the US and found that more than half of the states had GARCH effects. Miles (2011) further examined GARCH models for forecasting prices in US metropolitan areas. Crawford and Fratantoni (2003) tested the GARCH model at the regional level in the US and compared the forecasting abilities of the GARCH, autoregressive integrated moving average (ARIMA), and regime switching models. However, since house prices may have a GARCH effect, it is important to note that researchers have found that a strong GARCH effect leads to error estimation in the unit root test. Therefore, it is crucial to investigate the power of the unit root test under the GARCH effect in house price modelling.

Our research, which is in line with the aforementioned problems, includes three empirical studies. Specifically, the first essay tests nonlinearity in the conditional mean equation of house price series in the UK. In particular, the first essay compares two types of regime switching models: the MS-TVTP and STAR models. The second essay investigates the relationship between house price and bank lending by estimating an LSTAR model and between house price and the loan-to-value (LTV) ratio by estimating a GARCH model. In the third essay, we investigate nonlinearity in the conditional variance equation of a first order autoregressive - GARCH-M (AR (1)-GARCH-M) model, and consider the size and power of commonly used unit root tests in the case of structural breaks. An outline of the thesis is given below.

Chapter 2 adds to the literature that examines cyclical fluctuations in real estate prices. In the extant studies, researchers found that nonlinear models are better fitted for capturing house price cycles. For example, Crawford and Fratantoni (2003) and Miles (2008) examined several nonlinear models for US regional house prices. However, they concurred that nonlinear models, such as the Markov switching model, are ill-suited for stable markets. Moreover, nonlinear models' forecasting abilities are generally inferior to more standard forecasting techniques. Against this background, a critical problem still exists in terms of which model is most fit for capturing stable housing market cycles and forecasting prices. Given this, in this chapter, we contribute to the literature by comparing two nonlinear models in order to investigate performance for the housing market in the UK. We will first test the MS-TVTP model. We expect that this approach will capture the asymmetric dynamic of house price cycles more accurately than the traditional Markov switching model. Moreover, the Markov switching model shows changes from one regime to another as just a sudden transition without capturing the often gradual nature of such changes, such as those of stable house prices. Consequently, we also contemplate using the STAR model to capture the dynamic behaviour of regime change.

As previously mentioned, in Chapter 2, we will use data from the UK housing market. In the literature, Muellbauer and Murphy (1997) indicated that the UK housing price data contains non-linearity in house price dynamics. Further, Tsai, Chen and Ma (2010) found that the UK house prices are relatively stable. From our empirical tests, we find that the STAR model is more suitable for capturing house price cycles. In addition, we test the STAR model out-of-sample forecasting ability against a linear AR model. Unfortunately, our result shows that the STAR model does not outperform its linear model counterpart.

Chapter 3 innovates with respect to the literature by investigating the determinants of house price fluctuations, with a special focus on the effect of the banking sector on housing markets. More specifically, we investigate how bank lending and the LTV ratio affect house prices. Over-borrowing and loose loan policies in good times can lead to bank failure when housing market bubbles burst. In this context, it is critically important to investigate the relationship among house prices, bank lending and the LTV.

A common methodology used to investigate the relationship between economic variables is the co-integration test and Granger causality. In Chapter 3, we raise three hypotheses and empirically test them using nonlinear econometric models. First, we use the STAR model to investigate whether or not house price fluctuations are affected by bank lending. We chose the STAR model due to the fact that this model has a strong ability to capture house price cycles. A standard STAR model specifies the transition variable as part of the explanatory variables or trend. We consider using an exogenous variable as the transition variable in the transition function in order to investigate the impact of the credit market on the housing market. Second, we take the volatility of house price cycles into account and use a GARCH model to examine the linkage between house prices and the LTV. In our estimation, we allow the LTV to be an exogenous variable. Finally, we test the long-run equilibrium relationship between house prices and bank lending, and house prices and the LTV ratio. Due to house prices exhibiting nonlinear behaviour, our co-integration test is based on the threshold autoregressive (TAR) and M-TAR models.

For the empirical application, we use the housing market in Hong Kong. We chose the Hong Kong housing market as it has experienced two severe bubble bursts in recent years. Also, since the 1990s, the regulation of the credit market via the LTV ratio has played a crucial role in the Hong Kong housing market.

The empirical findings of Chapter 3 suggest that bank lending significantly affects house price fluctuations and that LTV strongly affects house price behaviour. With respect to the co-integration test, we find empirical evidence that illustrates the long-run relationship between house prices and bank lending and between house prices and the LTV.

In **Chapter 4,** the results of a Monte Carlo experiment to investigate the size and power properties of commonly used unit root test statistics in the presence of structural breaks are presented. It is found that the location and magnitude of the breaks strongly affect the size and power properties of the test statistics. Also, the presence of an in-mean effect severely affects the performance of the test statistics. In particular, when the magnitude of the in-mean term becomes large, conventional unit root tests tend to indicate falsely that the underlying process is integrated of order 1 (I (1)).

To illustrate the empirical relevance of the results, an example of a structural break in house price cycles is also presented. In particular, accurate modelling of house price volatility is crucially important, especially after the subprime mortgage crisis in 2008. After 2008, housing markets in many countries experienced large and frequent swings. In Chapter 4, it is shown that house price can be modelled using a time-varying AR (1)-GARCH-M model, where house price volatility is transmitted to house price levels.

Chapter 5 provides the conclusions of this thesis. We analyse the results and contribution. We also provide suggestions for future research.

Chapter Two

Nonlinear Modelling of UK House Prices

2.1 Introduction:

The profound crisis in the US sub-prime mortgage market highlighted the crucial role that the housing market plays in undermining financial stability. Starting in the late 1990s, there was a sharp increase in subprime mortgages fuelled by low-interest rates and lax lending standards. However, while the quality of bank loan portfolios was deteriorating due to the constant growth in the subprime mortgages, default rates remained artificially low due to the large appreciation in house prices. The housing market boom and low default rates encouraged banks to invest heavily in the real estate market. A substantial increase in real estate lending by banks led to the creation of a real estate market bubble which eventually burst in 2005. The collapse of the housing market was the primary cause of the onset of financial instability in the US, and instability rapidly spread throughout the world. As were many other countries, the UK was hit heavily by the shockwave generated by the US as the economy experienced one of the worst recessions seen in recent times. In the light of this, it is clear that understanding how real estate prices affect the quality of loan portfolios is of crucial importance for financial institutions and financial regulators interested in maintaining financial stability.

This paper adds to the literature that examines cyclical fluctuations in real estate prices. Accurate econometric estimation and forecasting of house price cycles are crucial for housing market regulators and market participants interested in an early warning of an impending financial crisis. Available empirical works mainly use linear models. For example, Abraham and Hendershott (1996) described an equilibrium price level to which the housing market tends to adjust. The authors divided the determinants of house price appreciation into two groups: one that explains changes in the equilibrium price and another that accounts for the adjustment mechanism in the equilibrium process. Slow adjustment toward the equilibrium can be regarded as an indication of asymmetries in real estate cycles. Muellbauer and Murphy (1997) explored the behaviour of house prices in the UK. The authors suggested that the presence of transaction costs associated with the housing market cause important nonlinearity in house price dynamics. Further, Holly, Peseran, and Yamagata (2010) (see also Holly et al. (2011)) extended the analysis to the spatiotemporal diffusion of shocks in the housing market.

House prices modelling has long been the object of interest in applied and theoretical research. It is therefore not surprising to find a significant body of literature on this topic. For example, Crawford and Fratantoni (2003) compared the performances of the Markov switching and ARIMA models to test the dynamic behaviour of home price growth rate. However, as the authors pointed out, although the Markov switching model is useful for characterising house price volatility patterns, it fails to beat the linear models' accuracy when it comes to forecasting. This case may be associated with the misclassification of future regimes, as Bessec and Bouabdallah pointed out (2005). Other researchers extended the work by Crawford and Fratantoni. For example, Miles (2008) adopted the generalised autoregressive (GAR) model. The GAR model performs better in forecasting house price cycles with high home-price volatility, but it does not add much forecasting ability when the housing market is stable. A possible reason could be that the discrete changes of the Markov switching process are not consistent with the realities of house price movement.

This chapter contributes to the existing literature on house price modelling in two ways. First, we compare several types of nonlinear models in order to investigate the cyclical patterns of the UK housing market. In particular, the MS-TVTP model proposed by Kim and Nelson (1999) and Filardo (1994) has been used to estimate the time series of house price changes in the UK. With respect to the Markov switching model developed by Hamilton (1989), the innovation of MS-TVTP is that the transition probabilities evolve over time, and, therefore, it may be more accurate in terms of capturing expansion and contraction phases of the housing market than Hamilton's model. Although the MS-TVTP can provide extra flexibility and information than the traditional MS model. However, like the traditional Markov switching model, the MS-TVTP model assumes sudden transitions between one state and the next and may not be suitable for capturing the gradual nature of regime shifts in the UK housing market. Consequently, we compare the performance of the MS-TVTP model to the STAR model introduced by Teräsvirta (1994). The STAR model is considered due to its smooth transition between regimes has the good fitness to capture house price dynamics. In order to compare the goodness of fit of the two models, the Akaike information criterion (AIC), the Schwarz criterion (SC), and the Hannan–Quinn information criterion (HQ) are used. Using these criteria, the STAR model is identified as the most suitable model for capturing house price cycles in the UK.

Having identified the STAR model as suitable, it is of interest to investigate the forecasting properties of the selected model versus a simple linear autoregressive model. Accordingly, the second aim of this chapter is to investigate the forecasting performance of regime switching models compared to a linear model. Previous studies, such as Crawford and Fratantoni (2003), Miles (2008), and so on, have found the forecasting performance of nonlinear models to be poorer than that of linear models. Therefore, we compare the STAR model with an AR model in their out-of-sample forecast. In order to investigate the forecasting ability, our comparison relies on the following criterion: mean forecast error (MFE) and root mean square forecast error (RMSFE). We also consider the median relative absolute error (mRAE) and symmetric mean absolute percentage error (sMAPE) because they are associated more closely with nonlinearity (Tashman, 2000). In a similar fashion, we consider four scoring rules, such as the logarithmic (Logs), for the same forecasting exercise. More details can be found in the forecasting experiment section on this thesis. Our results indicate that the STAR model may not add much forecastability in relatively stable markets, such as the UK's.

The remainder of this chapter is organised as follows. Section 2 reviews literature concerning UK house price cycles. Next, Section 3 provides the fundamental theoretical framework of all the models examined. Further, Section 4 discusses the details of our modelling process, including model selection, estimation and diagnosis, as well as analysing empirical results based on the estimations. Finally, Section 5 provides the concluding remarks.

2.2 Modelling house price cycles in the UK

Early house price models were mainly developed using a linear framework. For example, Giussani and Hadjimathrou (1990), Mayes (1979), and Hendry (1984) developed linear models to investigate the determinants of house prices, such as supply-demand for housing. Darke and Leigh (1993) analysed the long-run equilibrium of UK house prices using the Johansen co-integration test. In addition, Holly and Jones (1997) found that real income is the most important determinant of house prices. Meen (1996) claimed that the UK regional market might be better characterised than national data, and the author tested the nature of spatial interactions in the UK regional housing market. Similar studies, such as Alexander and Barrow (1994) and Drake (1992), also analysed the UK housing market at the regional level relying on co-integration and causality tests. However, as Brown, Song, and McGillivray (1997) pointed out, UK housing data contains structural changes. Therefore, the supply-demand based models and the asset market approach proved to be poor forecasters. Accordingly, recent empirical studies have turned to nonlinear models for house prices modelling.

Given the asymmetric nature of housing prices, nonlinear models seem intuitively well suited for capturing house prices and forecasting future trends. Therefore, it seems natural that some researchers have turned to nonlinear models to explain recent developments in the housing market. For example, Muellbauer and Murphy (1997), using UK annual house price data, found that there have been substantial shifts in housing price dynamics over time and important nonlinearities in house price dynamics. More recently, Ihlanfeldt and Mayock (2014) found a strong correlation between house price movements and new construction during the prior US housing market bust in 2005.

In a similar vein, Cook and Vougas (2009) applied a smooth transition momentum threshold autoregressive model to test the unit root of the UK housing market. In general, consensus evidence suggests that models that take into account nonlinear, asymmetric house price cycles perform better than their linear counterparts.

The relationship between features of the housing market and business cycles has been empirically discussed in a vast amount of literature. For example, Leamer (2007) identified housing as an important precursor of the US business cycle. Moreover, Dufrénot and Malik (2012) suggested that house prices can provide significant informational content for modelling business cycles and that house prices develop asymmetrically when using UK data. To compare with prices with business cycles, Cunningham and Kolet (2011) tested the duration of housing market cycles in North America, and they found that house prices have a shorter expansion period than that of business cycles, while contractions are substantially longer. Kim and Chung (2016) investigated the role of UK house prices in business cycles using a Markov switching model, and they found that house prices significantly affect business cycles.

Another strand of the literature has related house price cycles to consumption, and credit and monetary policies. For example, Attanasio et al. (2009) found evidence that common causality is the most significant explanation for the co-movement of house prices and consumption growth in the UK from 1970 to 2006. Furthermore, Attanasio, Leicester, and Wakefield (2011) indicated the synchronisation between house price cycles and consumption in the last 30 years in the UK has been relatively strong. In a similar vein, Gerlach and Peng (2005) identified that bank lending has been closely correlated with the real estate market. Some countries' governments also regulate the real estate market by using bank lending policies.

Against this background, we introduce two types of regime switching models below that have been used to model the UK housing market.

2.3 Econometric model

In this section, we briefly introduce the two nonlinear models used in this chapter, namely the MS-TVTP and STAR models. Below, we analyse them in turn.

2.3.1 The MS-TVTP model

The traditional Markov switching model is well known and widely used to model business cycles. However, it is too restrictive for many empirical settings due to its transition probability matrix remaining constant over time. Hence, we consider, instead, an MS-TVTP model, which allows the transition probabilities to vary over time. Let the house price y_t be defined as:

$$y_t = \begin{cases} \mu_1 + \epsilon_1 , & for\ Regime\ 1 \\ \mu_2 + \epsilon_2 , & for\ Regime\ 2 \end{cases} \tag{2.1}$$

where $\epsilon \sim N(0, \sigma_t^2)$, and the subscripts 1 and 2 represent contractionary and expansionary phases of the house price cycle, respectively.

In the MS-TVTP model, the transition between regimes is stochastic. In general, MS-TVTP (i.e., more than two regimes) uses a recursive time-varying probability generating function for each probability cell. Mathematically, for a k-state system, we have $(k-1)k$ independent time varying probability components that need to be estimated, given by:

$$Q_t = \begin{Bmatrix} q_{11,t} & q_{12,t} & \cdots & q_{1k,t} \\ q_{21,t} & q_{22,t} & \cdots & q_{2k,t} \\ \vdots & \vdots & \ddots & \vdots \\ q_{k-1,t} & q_{k-2,t} & \cdots & q_{k-1,t} \\ 1 & 1 & \cdots & 1 \end{Bmatrix}. \tag{2.2}$$

For each probability cell (i, j) where $(\mathrm{i} = 1, 2 \ldots \mathrm{k}-1, \mathrm{j} = 1, 2 \ldots \mathrm{k})$ we specify a probability generating function as follows:

$$Q_{ij,t} = \phi(X_{ij,t} b_{ij}), \tag{2.3}$$

where $\phi(\cdot)$ is the cumulative normal density function, $X_{ij,t}$ is the state variable vector for cell (i, j), and b_{ij} is the parameters to be estimated. The state variables can be different for different cells. We generate an auxiliary matrix R_t based on Q_t given by:

$$R_t = \begin{Bmatrix} 1 & 1 & \cdots & 1 \\ 1-q_{11,t} & 1-q_{12,t} & \cdots & 1-q_{1k,t} \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{i=1}^{k-2}(1-q_{i1,t}) & \prod_{i=1}^{k-2}(1-q_{i2,t}) & \cdots & \prod_{i=1}^{k-2}(1-q_{ik,t}) \\ \prod_{i=1}^{k-1}(1-q_{i1,t}) & \prod_{i=1}^{k-1}(1-q_{i1,t}) & \cdots & \prod_{i=1}^{k-1}(1-q_{ik,t}) \end{Bmatrix}. \tag{2.4}$$

The final time-varying transition probability matrix can be constructed as follows:

$$P_t = Q_t \circ R_t = \begin{Bmatrix} p_{11,t} & p_{12,t} & \cdots & p_{1k,t} \\ p_{21,t} & p_{22,t} & \cdots & p_{2k,t} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k-1,t} & p_{k-2,t} & \cdots & p_{k-1,t} \end{Bmatrix}, \tag{2.5}$$

where $\circ$ stands for the element-wise matrix Hadamard product. The matrix (2.5) can be expressed as:

$$
\begin{aligned}
p_{1j,t} &= q_{1j,t}, \\
p_{2j,t} &= (1-q_{1j,t})q_{2j,t}, \\
&\vdots \\
p_{k-1,j,t} &= (1-q_{1j,t})(1-q_{2j,t})\cdots(1-q_{k-2,j,t})q_{k-1,j,t}, \\
p_{kj,t} &= (1-q_{1j,t})(1-q_{2j,t})\cdots(1-q_{k-2,j,t})(1-q_{k-1,j,t}),
\end{aligned}
\tag{2.6}
$$

for j $= 1,2,\dots,$k. Each column will sum to 1 by construction. The matrix P_t is the transition probability from regime j to regime i. These transition probabilities are allowed it vary over time.

In this paper, the MS-TVTP model is estimated by Maximum Likelihood. Considering equation (2.1), the log likelihood is given by:

$$
\ln L = \sum_{t=1}^{T} \ln\left(\frac{1}{\sqrt{2\pi\sigma_{S_t}^2}} exp\left(-\frac{y_t-\mu_{S_t}}{2\sigma_{S_t}^2}\right)\right), \tag{2.7}
$$

where $S_t = (1,\dots,k)$ is the numbers of regimes. However, equation (2.7) represents the case where the states are known. When the states of the MS-TVTP model are unknown, consider $f(y_t|S_t = j,\theta)$ as the likelihood function for state j conditional on a set of parameters(θ), then the log likelihood function is given by:

$$
\ln L = \sum_{t=1}^{T} \ln \sum_{j=1}^{2} (f(y_t|S_t = j,\theta))\Pr(S_t = j)). \tag{2.8}
$$

As the state's probabilities are not observed, we use Hamilton's filter (1989) to make an inference on the probabilities in (2.8). Considering φ_{t-1} as the matrix at time $t\text{-}1$, the Hamilton's filter for the estimation of $Pr(S_t = j)$ is available using the following iterative algorithm:

1. Make an assumption for the starting probabilities at $t = 0$ of each regime, $\Pr(S_0 = j)$ for$j = 1{,}2$. For example, assume the initial $Pr(S_0 = j) = 0.5$, then the probabilities of S_t:

$$Pr(S_0 = 1|\varphi_0) = \frac{1 - p_{11}}{2 - p_{11} - p_{22}}, \tag{2.9}$$

$$Pr(S_0 = 2|\varphi_0) = \frac{1 - p_{22}}{2 - p_{22} - p_{11}}. \tag{2.10}$$

2. Set $t = 1$ and estimate the probabilities of each regime up to time $t - 1$:

$$Pr(S_t = j|\varphi_{t-1}) = \sum_{i=1}^{2} p_{ij}(\Pr(S_{t-1} = i|\varphi_{t-1}), \tag{2.11}$$

where the p_{ij} are the transition probabilities from the Markov chain in equation (2.5).

3. For time t, we have parameters for each regime. Based on this new information from time t, we can update the probability of each regime. This can be accomplished by:

$$Pr(S_t = j|\varphi_t) = \frac{f(y_t|S_t = j, \varphi_{t-1})Pr(S_t = j|\varphi_{t-1})}{\sum_{j=1}^{2} f(y_t|S_t = j, \varphi_{t-1})Pr(S_t = j|\varphi_{t-1})}. \tag{2.12}$$

4. Set $t = t + 1$ and re-apply stages 2 and 3 until $t = T$. Then we have all the estimation observations

Finally, we obtain the probabilities by estimating the log likelihood of the model as a function of the set of parameters:

$$\ln L = \sum_{t=1}^{T} \ln \sum_{j=1}^{2} (f(y_t|S_t = j, \theta))Pr(S_t = j|\varphi_t)). \tag{2.13}$$

The estimation of the MS-TVTP model is obtained by maximizing equation (2.13). For further details, see Hamilton (1994), Perlin (2014), and Bazzi et al. (2017).

2.3.2 The STAR model

The STAR model allows house price to move according to a smooth transition between regimes. Teräsvirta (1994) specified the estimation of the STAR model. The author suggested that the modelling procedure be started by considering a linear model, then nonlinear extensions should be considered, if necessary. In a housing price study, movements between two regimes are smooth. Accordingly, for modified stationary housing price index y_t, we specify the following STAR model of order p to capture the nonlinearities that are characterized by asymmetries in price growth dynamics:

$$\begin{aligned} y_t &= [\varphi_0 + \textstyle\sum_{i=1}^{P} \varphi_i y_{t-i}] + [\rho_0 + \textstyle\sum_{i=1}^{P} \rho_i y_{t-i}] \cdot F(y_{t-d}) + \varepsilon_t \\ &= [\varphi_0 + \varphi(L) y_t] + [\rho_0 + \rho(L) y_t] \cdot F(y_{t-d}) + \varepsilon_t, \end{aligned} \tag{2.14}$$

where φ and ρ are the parameter vectors, $F(y_{t-d})$ is the transition function that determines the change between regimes, $\rho(L)$ is an autoregressive coefficient that controls change smoothly along with lagged house prices, the past-realized house price index y_{t-d} is the transition variable, d is the delay parameter that indicates the number of the state that the transition variable causes to be switched, and ε_t is a sequence of independent identically distributed (iid) errors.

The STAR model has different specifications depending on the form of the transition function. Namely, it can be specified as the logistic smooth transition autoregressive (LSTAR) model or the exponential smooth transition autoregressive (ESTAR) model. In this chapter, we will consider the LSTAR model to cater to the fact that the UK house price cycle appears to be asymmetric with long expansions followed by short, steep contraction phases. The ESTAR model would not be able to capture this behaviour as this model assumes a symmetric cycle. The transition function F $(\cdot)$ of the LSTAR model is specified as follows:

$$F(y_{t-d}) = (1 + \exp\{-\gamma(y_{t-d} - \mathrm{c})\})^{-1}, \gamma > 0, \tag{2.15}$$

where γ is the slope parameter which measures the speed of transition between regimes.

In particular, the larger the magnitude of the parameter γ, the faster the transition between two regimes. When $\gamma \rightarrow \infty$, the STAR model will converge to a Markov switching type of model. On the other hand, as $\gamma \rightarrow 0$, the STAR model degenerates into a linear model. The vector c is the vector of location parameters. Before presenting the estimation results, we discuss the model specification, estimation, and evaluation in detail below.

Model Specification:

The model selection procedure used to estimate the LSTAR model involves two steps:

i. Testing linearity against STAR. If the test does not reject the null hypothesis of linearity, it is not necessary to estimate a nonlinear model. However, if the linearity test rejects the null against the alternative hypothesis of STAR or another type of transition function, then step (ii) below follows.

ii. Choosing the best transition function according to the strongest rejection of the null hypothesis and selecting the suitable type of nonlinear model, such as the LSTAR or ESTAR model, based on auxiliary regression with the appropriate transition function.

In order to test for linearity against a STAR model, we start by considering the AR model with the maximum lag order. Then, to select the most parsimonious model that still describes the data, we rely on the Bayesian information criterion (BIC).

Following Teräsvirta (1994), we use the Lagrange Multiplier (LM) test for testing linearity. The auxiliary regression for the UK house price index y_t is:

$$y_t = \varphi_0 + \sum_{i=1}^{P} \varphi_{1,i} \cdot y_{t-i} + \sum_{i=1}^{P} \varphi_{2,i} \cdot y_{t-i} y_{t-d} + \sum_{i=1}^{P} \varphi_{3,i} \cdot y_{t-i} y_{t-d}^2 + \sum_{i=1}^{P} \varphi_{4,i} \cdot y_{t-i} y_{t-d}^3 + \varepsilon_t. \quad (2.16)$$

The null hypothesis is:

$$H_{01}: \varphi_{2i} = \varphi_{3i} = \varphi_{4i} = 0 \text{ for all } i.$$

For example, if the BIC indicates the maximum lag is $S \in N$, then for transition function $F(t-1) \dots F(t-S)$, the linearity test is repeated for each predetermined transition variable. If linearity is rejected against a STAR model by a unique variable, then this variable will certainly be used for the following estimation. However, if there are two or more transition variables that reject linearity against a STAR model, then we should first consider the variable with the lowest $p-$ value, which indicates the strongest rejection of the null hypothesis. In practice, predetermined variables may have similar $p-$ values. If this is the case, then we need to test each of them.

As noted in the transition function (2.15), the difference between LSTAR and ESTAR lies in the transition function. The parameters change monotonically for the LSTAR model and symmetrically for the ESTAR model. One of the most commonly used tests for selecting either an LSTAR or ESTAR model is the sequence of ordinary F-test introduced by Teräsvirta (1994). The following decision rule is based on the nested sequence of the null hypothesis for the order of the polynomial in the auxiliary regression (2.16):

1. Test $H_{04}: \varphi_{\mathbf{3}i} = 0$
2. Test $H_{03}: \varphi_{\mathbf{2}i} = 0 | \varphi_{\mathbf{3}i} = 0$
3. Test $H_{02}: \varphi_{\mathbf{1}i} = 0 | \varphi_{\mathbf{2}i} = \varphi_{\mathbf{3}i} = 0$

where i=1,…, P.

Here, we should choose the model with the strongest rejection of the null hypothesis by the F-test. Accordingly, if the calculated value of the test statistic under H_{04} has the lowest p-value, then the LSTAR model should be selected. Otherwise, if the calculated value of the test statistic under H_{03} has the smallest p –value, one should choose the ESTAR model. Also, if the calculated value of the test statistic under H_{02} has the smallest p-value, then the LSTAR model should be selected. In case none of the tests gives a clear indication, one may fit more than one model. Teräsvirta (1994) recommends postponing the choice between models until a later stage and proceeding by estimating both models. Then he suggests using diagnostic tests to evaluate the selected model.

Model Estimation

The conditional maximum likelihood method is used to estimate the STAR model. As Teräsvirta (1998) points out, it maximises numerically the log-likelihood and provides the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm with numerical derivatives. For the BFGS algorithm, it is essential to find a good starting value; this can be done by grid-search.

Model Evaluation

The estimated model needs to be evaluated before it can be used for forecasting prices. Furthermore, as noted previously, testing is also helpful when making a choice between variables when the *p*-values of the linearity tests are similar.

Terasvirta (1998) suggested several types of misspecification tests for STAR models. In this chapter, we use the LM test of no error autocorrelation, which involves testing for residual correlation after estimating the STAR model by regressing the residuals and partial derivatives of the log-likelihood function with the model's parameters. Terasvirta (1998) assumes that $M(y_{t-i};\Psi)$ is at least twice continuously differentiable with respect to the parameters everywhere in the series and that:

$$y_t = M(y_{t-i};\Psi) + u_t, \qquad t = 1,\dots,T, \tag{2.17}$$

where $u_t = \alpha' v_t + \varepsilon_t$ with $\alpha = \left(\alpha_1,\dots,\alpha_q\right)'$, $v_t = (u_{t-1},\dots,u_{t-q})'$, and $\varepsilon_t \sim iid\ N(0,\sigma^2)$. The null hypothesis of the test is H_0: no error autocorrelation against the alternative of H_1: autocorrelation of at most order q in u_t in (2.17) is $\alpha = 0$.

Furthermore, we also test no additive nonlinearity, which measures the model characterising most of the nonlinearity. According to Terasvirta (1998), the $F(\gamma_2, c_2, y_{2(t-d)})$ is defined as another transition function of equation (2.14). Then the null hypothesis H_0 of no additive nonlinearity, which is the same as the linearity test that $\varphi_{2i} = \varphi_{3i} = \varphi_{4i} = 0,$ is tested against the alternative H_1: paremeters of $F(\gamma_2, c_2, y_{2(t-d)})$ are identified.

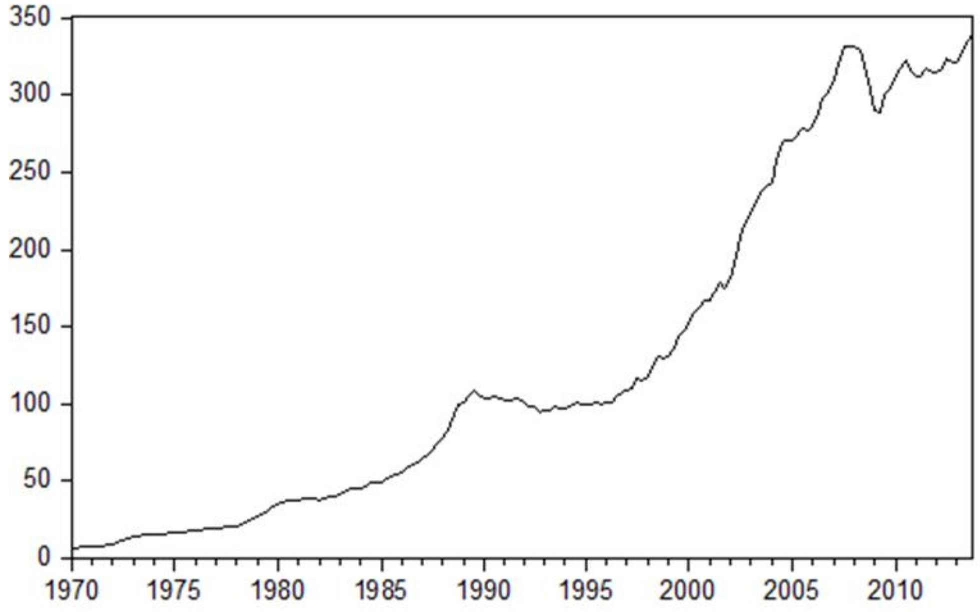
2.4 Data and empirical result

We begin the empirical investigation with a preliminary analysis of the UK quarterly house price index from the year 1970 to 2013[2]. From Figure 2.1, it appears that the house price index tends to increase, as a whole, from 7.05 in 1970 to 338.88 in 2013. From the plot of the data, a typical characteristic of house price cycles, i.e.,

[2] Source: Office for National Statistics.

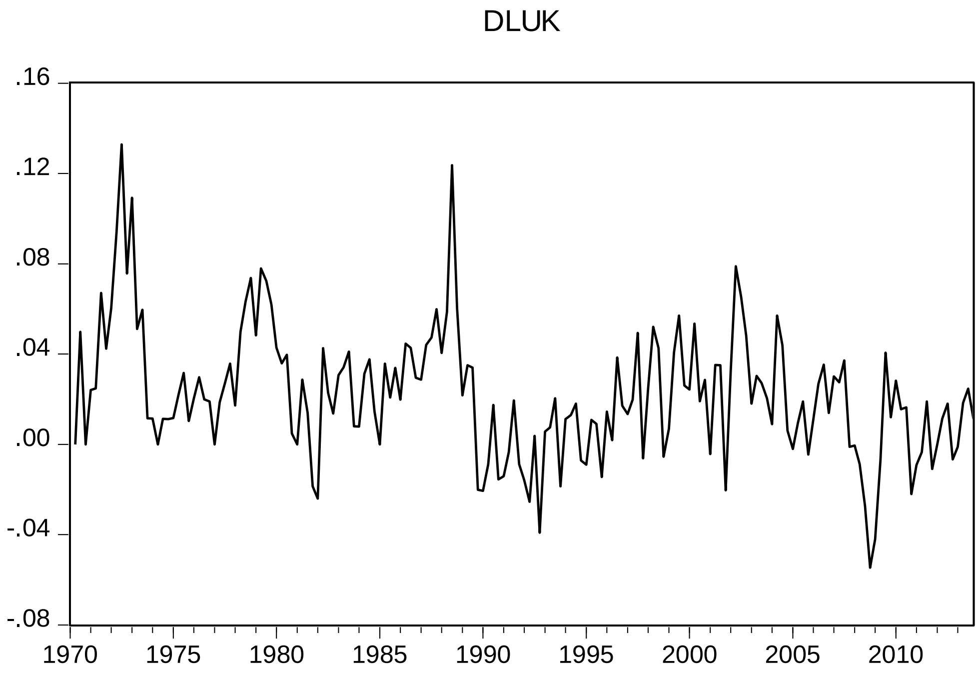
long-term expansions and short-term contractions, can be noted. It should be noted as well that, during the 90s, house prices had several phases of minor contractions and expansions.

Figure 2.1 Index of all dwelling residential property prices from the year 1970 to 2013 of the UK



Note: Index 1995=100

Figure 2.2 First difference and log logarithmic transformation



To test to see if the series is stationary, we use a logarithmic transformation of the first order for the preliminary analysis of the variable in levels suggesting that the series contains a unit root. Table 2.1 shows the basic statistics and results of the ADF test, and the lower part of the table illustrates that the series is stationary after modified. The ADF test shows that the unit root test rejects the null hypothesis by less than a 1% significant level. The top section of Table 2.1 provides the mean, median and standard deviation. It appears that the mean and median are very close, and the small standard deviation indicates that the data points tend to be close to the mean of the series. Below we describe the estimation results in detail for each model, then give a more detailed comparison between the MS-TVTP and STAR models.

Table 2.1 Basic statistics and Augmented Dickey-Fuller test statistic

Mean	0.022
Median	0.019
Std.Dev.	0.029
ADF statistic	-5.37***
Prob.	0.0001

The ADF test contains trend and constant. *** indicates smaller than the critical values at the 1% levels

Table 2.2 provides the result of AR model based on AIC and BIC. We examined the maximum lag of 6 for the AR term. The AR result will help us to select lag terms for MS-TVTP and STAR model. The result shows lag at 5 is the minimum value but it is not significant enough in our MS-TVTP and STAL estimation. The optimization will be described in following estimation.

Table 2.2 AR model result of the AIC and BIC

	Delay					
	1	2	3	4	5	6
AIC	-4.700	-4.715	-4.726	-4.762	-4.930	-4.927
BIC	-4.664	-4.661	-4.653	-4.671	-4.819	-4.798

2.4.1 The Markov switching model

Below we consider the estimation of the MS-TVTP model with two regimes ($S_t = 1, 2$) given by:

$$y_t = \mu_{S_t} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2). \tag{2.18}$$

Table 2.3 reports the estimated parameters. The top section of Table 2.3 describes the coefficients for the specific regime. It appears that the house price series y_t can be classified into two regimes. However, the coefficients of the switching parameters μ_1 and μ_2 are 0.014 and 0.062, respectively, with both being very small and positive. The average index in the expansionary phase of the house price cycles is $\mu_1 + \mu_2 = 0.076$ per quarter, compared to the average index in the co ntractionary phase of the cycles, which is $\mu_1 = 0.014$ per quarter. The distribution parameter σ^2 is not switching, and the value is constant at 0.0004.

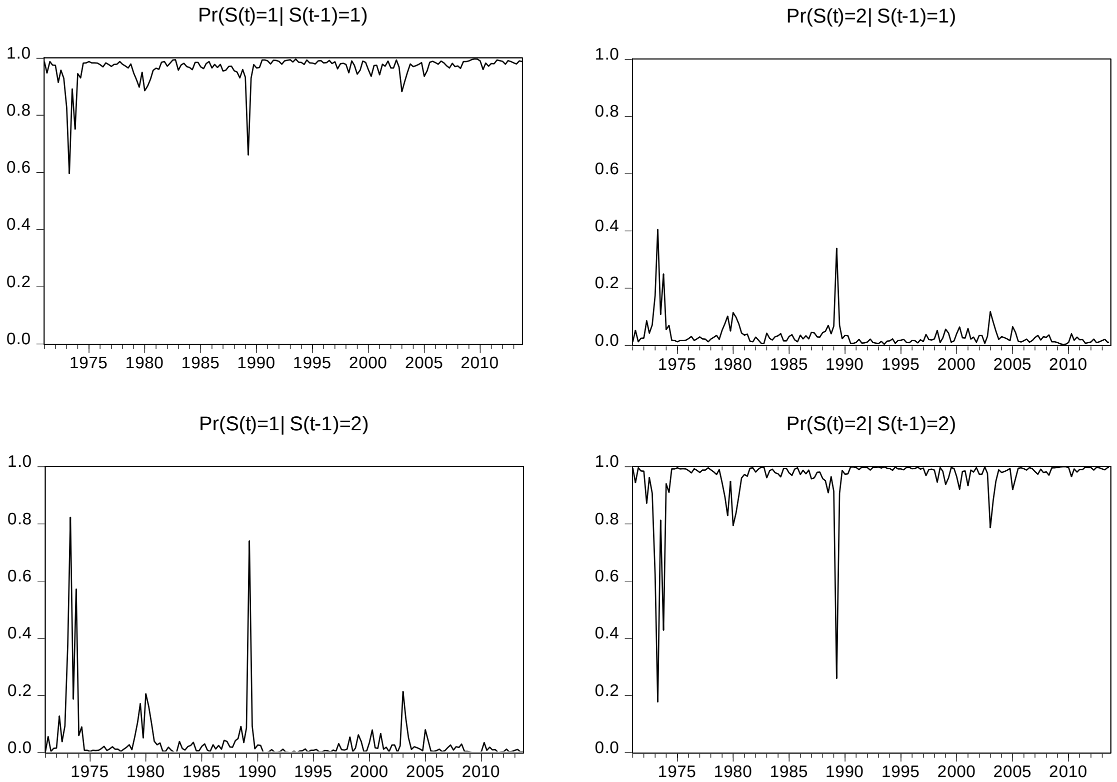
Table 2.3 MS-TVTP model estimation results

Variable	Coefficient
μ_1	0.014*** (0.002)
μ_2	0.062*** (0.005)
σ^2	4.843e-4*** (0.058)
P_{11}, μ	4.404*** (0.991)
$P_{11}, y_{(t-3)}$	-30.214 (27.883)
P_{21}, μ	-5.441** (2.546)
$P_{21}, y_{(t-3)}$	52.474* (32.667)

Note: ***), **), *) indicate statistical significance at 1%, 5% and 10%, respectively. ( ) is Standard Error.

The bottom panel of Table 2.3 shows the estimated results for the TVTP matrix parameters. We see that the transition probabilities' parameters change over time. This is also evident from the plot in Figure 2.3. However, it should be noted that $y_{(t-3)}$ at P_{11} is not statistically significant, while the other p-values from the TVTP matrix estimation are all statistically significant. Moreover, all terms have large standard errors. Therefore, the goodness of fit of the MS-TVTP model for the UK house price series is questionable.

Figure 2.3 Time-varying Markov Transition Probabilities

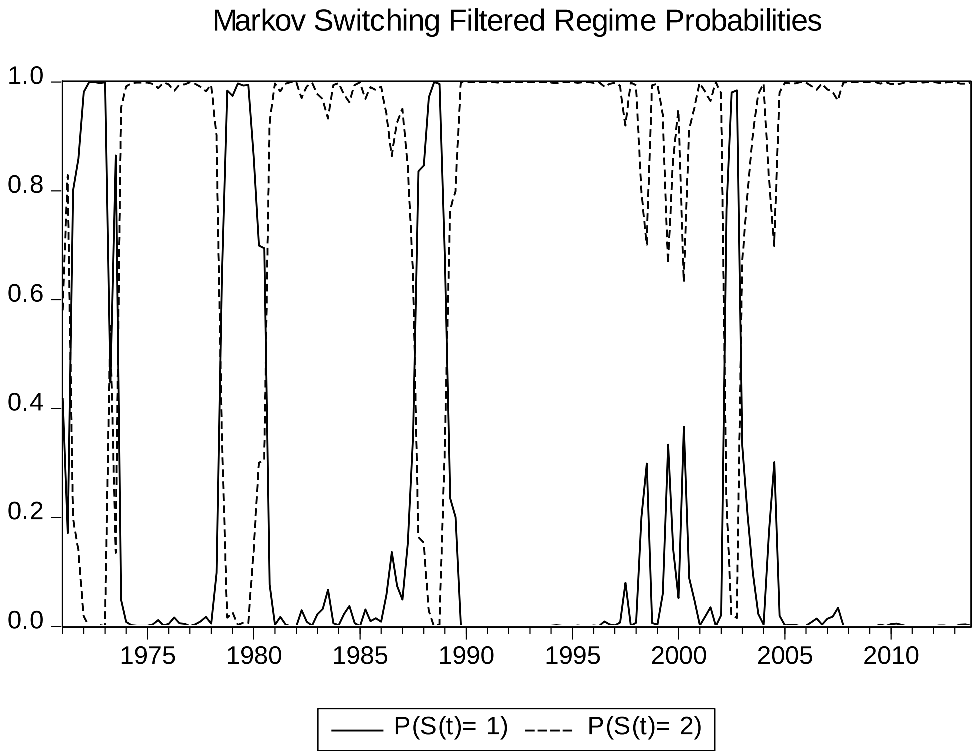


Base on the values in Table 2.3, we can obtain the matrix of MS-TVTP as follows:

$$\text{mean: } \begin{bmatrix} 0.966 & 0.034 \\ 0.040 & 0.960 \end{bmatrix} \qquad \text{Std. Dev: } \begin{bmatrix} 0.047 & 0.047 \\ 0.102 & 0.102 \end{bmatrix}$$

The time-varying expected durations of the mean are 59.39 quarters and 190.52 quarters for regimes one and two, respectively, and the standard deviations are 53.46 quarters and 398.19 quarters, respectively. Even these estimates suggest that there is significant evidence of regimes one and two, but the MS-TVTP model over-estimates the asymmetry of UK house price cycles. Figure 2.4 plots the filtered and smoothed regime probabilities for each state. We can see that regime one has a short period and that regime two is of long duration.

Figure 2.4 Filtered and Smoothed States Probabilities of MS-TVTP



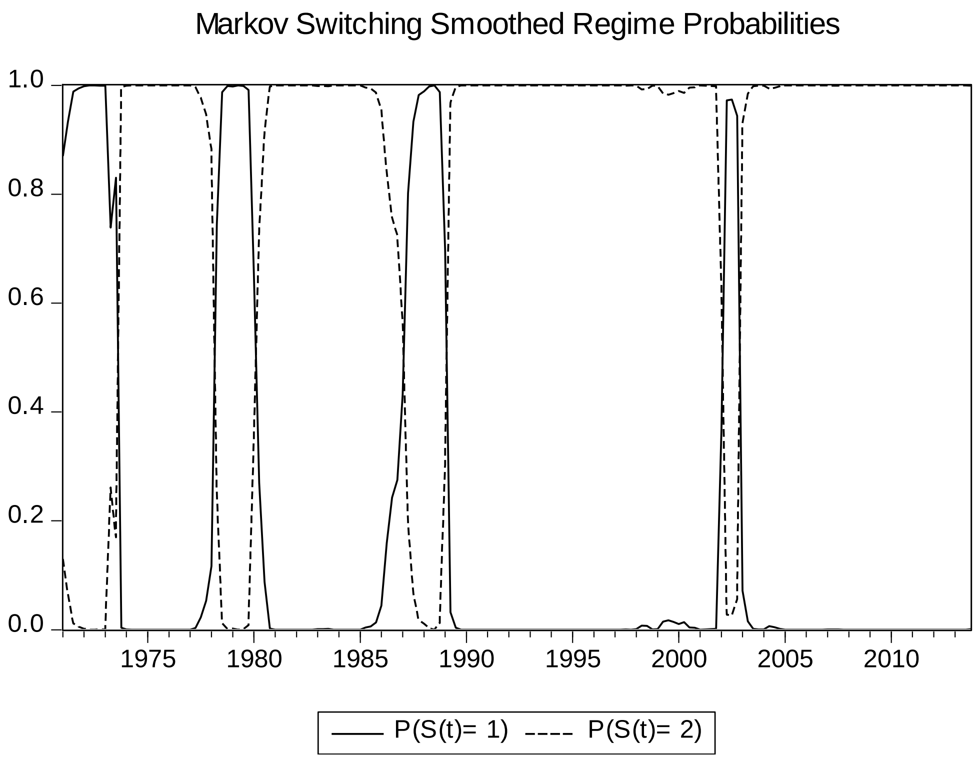
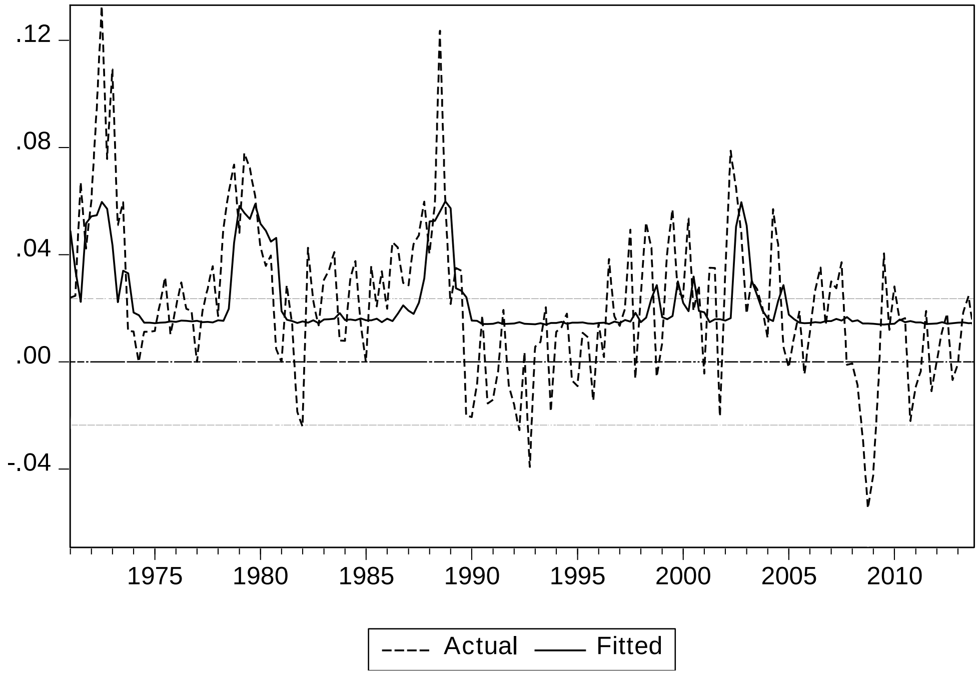


Figure 2.5 shows the actual and fitted series. We can see that the MS-TVTP model captures some periods with large swings, for example, around the years 1970, 80 and 90. However, it cannot estimate small fluctuation phases.

Figure 2.5 Actual and fitted series



2.4.2 The STAR model

We use the logarithmic and first order difference transformation for the UK house price index. Table 2.4 shows the estimates of the *p*-values for the F test in auxiliary regression. In the table, we find that the linearity hypothesis is rejected for the AR (1) and AR (3) models at 5% significance. In order to select the type of transition function, we see in the first row that the hypothesis H_{04} is associated with the lowest *p*-value. LSTAR appears to be the model that best fit the data. Looking at the row for t-3, the H_{03} has the smallest p-value corresponding to the transition function of the ESTAR model. Despite the lag 3 with the strongest linearity rejection suggesting an ESTAR model, we prefer to estimate a LSTAR model with lag 1. Because the UK house prices exhibit asymmetric character, the LSTAR model is preferred to the ESTAR model. Moreover, preliminary estimation results found that the ESTAR model did not fit the data well.

Table 2.4 Result of testing linearity and selection of model

Transition variable	H_0	H_{04}	H_{03}	H_{02}
t-1	0.047	0.016	0.112	0.906
t-2	0.218	0.033	0.881	0.499
t-3	0.014	0.071	0.006	0.744

Note that, in the estimation process, the starting values were generated by grid-search. As far as the results are concerned, Table 2.5 shows the estimated parameters and model diagnostic tests. We set restrictions making the linear part φ_2 and nonlinear part ρ_3 be zero. In the top part of Table 2.5, the estimation results for the LSTAR model appear. We can see that most of the coefficients are statistically significant and the estimated parameter for γ is small. Following the literature, we investigate the validity of the estimated model using the misspecification tests described in the previous section. Namely, the tests for no remaining autocorrelation and no remaining nonlinearity. The middle part of Table 2.5 reports the calculated F-value and relative p-value of the autocorrelation test. As already specified, under the null hypothesis there is no error autocorrelation in the residuals. We find that the p-values for the columns lag 1 and 2 are not sufficiently small enough to cause misspecification.

The results of testing for no additive nonlinearity and Jacques-Bera tests can be found in the bottom part of Table 2.5. The results in the second column do not indicate substantially significant rejection of linearity. Therefore, we can conclude that the LSTAR model captured the nonlinearity in the series well. However, we do need to mention that, in row t-3, nonlinearity is rejected at around the 5% significant level, but this is not strong enough justification to add more STAR components to the model. Considering that we selected the LSTAR instead of the ESTAR model above, we consider just one rejection acceptable. The Jacques-Bera test shows that it fails to reject of null hypothesis of residuals are normal distribution. We also give the skewness and kurtosis value with 0.126 and 3.329, which also support the normal distribution.

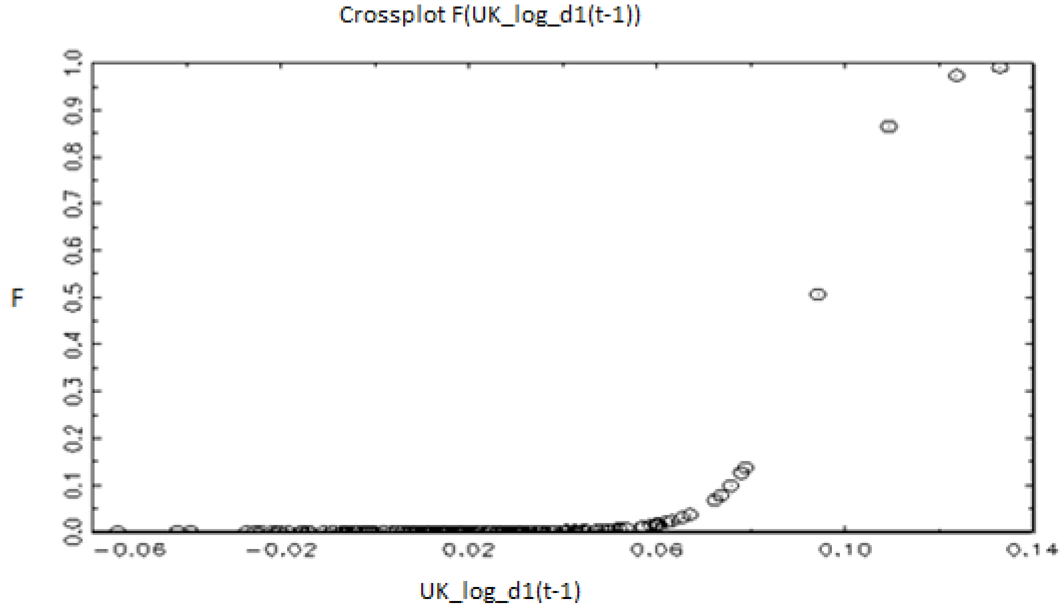
Table 2.5 Result of LSTAR and model diagnoses

variable	estimate	SE		
φ_0	0.007***	0.002		
φ_1	0.543***	0.096		
φ_3	0.111*	0.068		
ρ_0	0.274**	0.113		
ρ_1	-2.725***	1.067		
ρ_2	0.660	0.938		
γ	3.490***	0.000		
c	0.094***	0.000		
lag	1	2		
F-value	0.397	0.325		
`p-value	0.530	0.723		
Transition variable	H_0	H_{04}	H_{03}	H_{02}
t-1	0.686	0.580	0.251	0.726
t-2	0.698	0.194	0.693	0.793
t-3	0.057	0.123	0.014	0.074
Jacques-Bera tests	Statistic	p-Value	skewness	kurtosis
	1.231	0.540	0.126	3.329

Note: ***), **), *) indicate statistical significance at 1%, 5% and 10%, respectively.

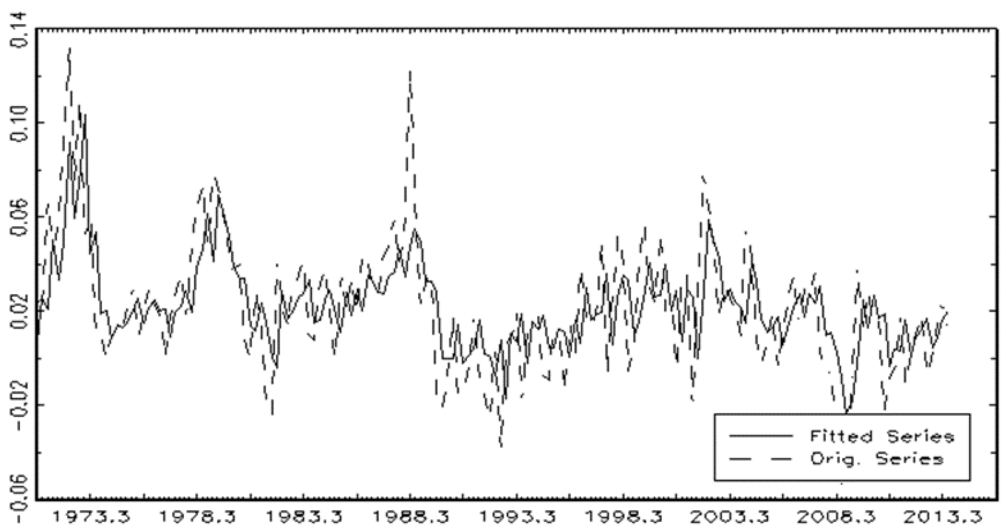
From Figure 2.1, it is clear that house price change in the UK from 1970 to 2013 was quite smooth. Therefore, the magnitude of the estimated transition parameter γ is justified. In Figure 2.6, we can see the transition function in equation (2.15) as a function of observations, where each dot corresponds to an observation. The transition variable is y_{t-1}. As seen from Figure 2.6, the transition is indeed smooth. Note that most observations are located near the x-axis due to small estimates of the location parameter c and a small standard deviation.

Figure 2.6 LSTAR model transition function versus transition variable



In term of the goodness of fit, comparing the fitted and original data from Figure 2.7, it appears that the LSTAR model captures the feature of house price changes well during the period under consideration. However, we should notice that around large swing periods, such as 1988, the LSTAR model cannot capture the peak point.

Figure 2.7 Plot of estimated series and original series



Overall, we can conclude that the LSTAR model has no error autocorrelation and no remaining nonlinearity, confirming that the LSTAR model describes the data well. The following section discusses the comparison of estimated models as the main contribution of this paper.

2.4.3 Model comparison

Our preliminary analysis suggests that the MS-TVTP model shows some ability to capture the expansionary and contractionary phases of the UK housing price cycles. From Figure 2.3, it appears that the transition probabilities are time-varying. However, the *p*-values of the standard errors of the transition probabilities are high and non-significant. In this respect, the LSTAR model appears to fit the data better. Comparing Figures 2.5 and 2.7, it is clear that the estimated LSTAR model fits the data better than the MS-TVTP model. In order to further compare the goodness of fit of the two models, the AIC, SC, and HQ criterion have been calculated, and the results reported in Table 2.6.

A comparison of AIC, SC, and HQ levels across the MS-TVTP, LSTAR and AR models clearly suggest that the LSTAR model better fits the data since, no matter the criteria under consideration, the LSTAR model should be preferred.

Table 2.6 Summaries of the parameters of the estimation result

	MS-TVTP	*LSTAR*	*AR*
AIC	-4.455	-7.556	-4.921
SC	-4.328	-7.409	-4.483
HQ	-4.403	-7.496	-4.488

2.4.4 Forecasting result

In the previous sections, the models are estimated for capturing house price cycles. In this section, we investigate the forecasting ability of the estimated models. Our experiment is done in the spirit of the work by Crawford and Fratantoni (2003) and Miles (2008). As the LSTAR model fits the data better than the Markov switching model, we consider the former and discard the latter below.

The out-of-sample predictive properties of the estimated models are investigated via a rolling forecast experiment. In the forecasting exercise, the performance of the LSTAR model is compared with the simple AR model. In particular, in our forecasting experiment of series y_t, we compared the h-steps-ahead forecast with the test period, which is going from time T^s to T where $T^s = t + h$. This design allows us to quantify $T - h - T^s + 1$ out-of-sample forecasts. Accordingly, we denote the corresponding realization of the series as y_t , $y_{T,}^s$ and y_T , and the corresponding density forecasts as f_t , $f_{T,}^s$ and f_T . The forecasting horizon considered is h $=$ $\{1, 2, 4, 8\}$. The forecasting performances of STAR and AR models are investigated with the following measures: MFE, sMAPE, mRAE, and RMSFE.

$$MFE_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} \left(y_{t+h} - \hat{y}^j_{t+h|t} \right), \tag{2.19}$$

$$sMAPE_{j,h} = \frac{100|y_{t+h} - \hat{y}^j_{t+h}|}{0.5(y_{t+h} - \hat{y}^j_{t+h|t})}, \tag{2.20}$$

$$mRAE_{j,h} = \frac{|y_{t+h} - \hat{y}^j_{t+h}|}{y_{t+h} - \hat{y}^{(1)}_{t+h}}, \tag{2.21}$$

$$RMSFE_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} \left(y_{t+h} - \hat{y}^j_{t+h|t} \right)^2. \tag{2.22}$$

We also considered four scoring rules for the forecasting period of $T - h - T^s + 1$, which are explained as follows:

The logarithmic score (LogS) (Good, 1952):

$$LogS_{j,h} = \frac{1}{T - h - T^s + 1} \sum_{t=T^s}^{T-h} log\hat{f}^j_{t+h|t}. \tag{2.23}$$

It corresponds to a Kullback-Liebler distance from the true density. This scoring rule is preferred for models with a higher value LogS.

The quadratic score (QS) (Brier, 1950), defined as:

$$QS_{j,h} = \frac{1}{T-h-T^s+1}\sum_{t=T^s}^{T-h}\sum_{k=i}^{K}(f_{t+h|t}^{j} - d_{kt})^2, \qquad (2.24)$$

where $d_{kt} = 1$ if k=t and 0 otherwise. This score rule is preferred for models with lower QS.

The (aggregate) continuous-ranked probability score (CRPS) (Epstein, 1969), defined as:

$$CRPS_{j,h} = \frac{1}{T-h-T^s+1}\sum_{t=T^s}^{T-h}\left(\left|f_{t+h} - \hat{f}_{t+h|t}^{j}\right| - 0.5|f_{t+h} - f'_{t+h}|\right), \qquad (2.25)$$

where f and f' are independent random draws from the predictive density and $f_{t+h|t}$, the observed value. This score rule is preferred for models with lower CRPS.

The quantile score (qS) (Cervera and Munoz, 1996), defined as:

$$qS_{j,h} = \frac{1}{T-h-T^s+1}\sum_{t=T^s}^{T-h} q_{t+h|t}^{\alpha}. \qquad (2.26)$$

This score is used in risk analysis because it provides information about deviations from the true tail of the distribution.

Tables 2.7 and 2.8 reports the results of the forecasting exercise. From Table 2.7, it appears that the AR model has better point forecasting properties than the nonlinear model for most of the forecast horizon, according to the sMAPE and RMSFE criteria. The LSTAR model only has better forecasting performance at a short horizon, according to the one horizon of the MFE. This result is entirely in line with the literature (see Balcilarm, Gupta, and Miller, 2015), as is well known that nonlinear models do not outperform their linear counterpart when it comes to forecasting.

Table 2.7 Point Forecast Accuracy

point predictive performances			
Forecast horizon	Forecast Error Measure	AR	STAR
	MFE		
1		-0.1777	0.1431
2		-0.1954	0.2066
4		-0.2068	-0.2182
8		-0.2111	-0.2533
	sMAPE		
1		0.0035	0.0126
2		0.0035	0.0128
4		0.0036	0.0132
8		0.0038	0.0137
	mRAE		
1		1.0000	0.4419
2		1.0000	0.4373
4		1.0000	0.5495
8		1.0000	0.7219
	RMSFE		
1		0.3147	0.8385
2		0.3166	0.8473
4		0.4213	0.8671
8		0.9319	0.9067

The density forecasting results are shown in Table 2.8. The results are mixed. According to LogS and QS, the LSTAR shows higher forecasting accuracy. However, according to the CRPS and qS, the AR model outperforms the LSTAR model for all horizons. This result is also consistent with the work done by Balcilarm, Gupta, and Miller (2015). The authors found the AR and LSTAR models produce similar performances in density forecasting of US house prices.

Table 2.8 Density Forecast Accuracy

Density predictive performances			
Forecast horizon	Scoring Rule	AR	STAR
	LogS		
1		0.0168	0.0176
2		0.0169	0.0177
4		0.0175	0.0178
8		0.0173	0.0179
	QS		
1		0.1646	0.1709
2		0.1620	0.1709
4		0.1621	0.1690
8		0.1655	0.1657
	CRPS		
1		21.1533	18.4129
2		21.3889	18.5984
4		25.4515	18.9910
8		26.4635	19.6639
	qS		
1		0.5077	0.4870
2		0.5096	0.4910
4		0.5000	0.4992
8		0.5328	0.5315

2.5 Conclusion

A large number of recent papers have highlighted the crucial importance of modelling house prices. Models that capture house price behaviour can give market regulators and investors insights into house price direction and inform decision making processes.

This chapter considers several issues. First, a model that captures the UK housing market cycles is estimated by using a nonlinear specification. In particular, two different types of nonlinear models are considered. Namely, the MS-TVTP and LSTAR models. The empirical investigation reveals that the LSTAR model better fits the data.

Second, the forecasting properties of the LSTAR model are also investigated. The empirical analysis reveals that the LSTAR model does not outperform the AR model for point prediction in out-of-sample forecasting. However, the LSTAR and AR models produce similar evidence with regard to density prediction. Nevertheless, we still believe the STAR-type models can be used to accurately forecast house price cycles due to the fact that we have examined the mechanism of STAR model, and it does well in capturing stable house price cycles. Also, work done by Canepa and Chini (2016) indicates that the generalised STAR model performs well in forecasting exercises.

Since downturns in house prices have consequences for the economy, understanding the asymmetric cycles of UK house prices alerts market participants to the possibility that a downturn in the housing market may happen. Against this background, our results have significant consequences for housing market regulators and researchers. In particular, our findings in this chapter empirically illustrate that the STAR model we have fitted to the UK house prices does well in stable housing market research.

Chapter Three

Bank Lending and House Price: The Hong Kong Experience

3.1 Introduction

It is now common knowledge that the subprime mortgage crisis had a major role in, but was not the sole cause of, the worldwide financial crisis during the last decade. Loose bank lending and the over-borrowing of households played a significant role in the collapse of the housing market. As a result, central banks and financial authorities in many countries have introduced more stringent risk management standards. For example, the Third Basel Accord imposed regulations in order to strengthen bank capital requirements, leverage ratios, and liquidity requirements.

The fact that the collapse of the housing market leads to bank failures has been observed several times in the past. For example, since 2008, the US subprime mortgage crisis has led to the failure of more than 400 banks across the US.[3] These banks had housing loans and mortgages as their primary business, but underestimated the risk of falling house prices. As a consequence, high default rates on subprime mortgages and loans and the collapse of the secondary market for mortgage-backed securities led to severe liquidity shortages.

Against this background, it is crucially important to investigate the relationship between banking sectors and house prices. Accordingly, this chapter complements the existing literature by investigating the effect on house prices by financial institutions' lending policies by empirical testing three hypotheses formulated as follows.

[3] The Federal Deposit Insurance Corporation (FDIC) is often appointed as receiver for failed banks in the US, and the following page shows that 465 banks failed from 2008 to 2012: https://www.fdic.gov/bank/individual/failed/banklist.html

Hypothesis 1. House prices are severely affected by bank lending swings.

Available empirical work focuses primarily on the linear framework to investigate the relationship between house prices and banking sectors. For example, Gimeno and Martínez-Carrascal (2010) used a vector error-correction model to investigate the relationship between Spanish housing loans and house prices. Their results illustrate the strong relation between house prices and housing loans in Spain. Oikarinen (2009) found similar results when considering house prices and household borrowing in Finland. However, numerous empirical studies have found that nonlinearity exists in house price cycles. For example, Kim and Bhattacharya (2009) contributed to the literature by using a STAR-type model for US house prices. The authors found that the STAR-type models estimated the house prices for markets characterised by large price fluctuations well. Consequently, we empirically examine the relationship between house price movement and bank lending with a STAR model.

The STAR model developed by Teräsvirta (1994) has been widely used in nonlinear time series, see, for example, Skalin and Teräsvirta (1999, 2002). We believe the STAR model can be used to test Hypothesis 1 for three main reasons. First, the STAR model allows for switching between regimes with the transition function. Therefore, by using bank lending as the transition variable in the transition function of the STAR model, one can investigate the impact of the credit market on house price cycles. If bank lending has a strong impact on the housing market cycle, one should observe a switch from one phase to the other in relation to the sign and magnitude of the change in the credit market. If, on the other hand, the statistical significance of bank lending as a transition variable is not supported by the data, then one should not observe such a switch in relation to changes in the credit market. Second, the STAR model describes a smooth transition process which can well characterise the movement of house prices cycles. Finally, the lag of the transition variable in the transition function in the STAR model can provide information on the speed of house price change in response to variation in bank lending, and this information is of interest to policymakers and financial regulators, alike.

Hypothesis 2. The loan-to-value (LTV) ratio can be used to reduce large house prices swings and, therefore, foster financial stability.

In the aftermath of the 2008-09 global financial crisis, there is a growing consensus that monetary policy is not effective in preventing systemic risks. Central banks around the world became aware the LTV ratio is a crucial factor in lending risk assessment (e.g., Hungary, Norway, Sweden, and the UK). In spite of the fact that lowering the LTV ratio could raise liquidity constraints for property buyers, several empirical studies show that the LTV is significantly associated with the default risk (see, e.g., Otero-Gonzalez et al., 2016; Gete and Reher, 2016; Qi and Yang, 2009). However, a critical issue remains: How effective is the LTV ratio in avoiding large swings in house prices?

Accordingly, the objective of this chapter is to investigate this issue. In particular, we are interested in investigating if, and to what extent, the LTV ratio can be used to prevent large house price volatility. In order to address this issue, we estimate an AR-GARCH-M model with the LTV ratio as an explanatory variable. In a related paper, Tillmann and Peter (2015), using a structural VAR model, found that lowering the LTV ratio reduces the appreciation of house prices (see also Guirguis et al., 2005; Miles and William, 2008, 2011; Tsai et al., 2010).

Hypothesis 3. House prices have a long-run equilibrium relationship with bank lending and the LTV ratios.

In relation to Hypothesis 1 and 2, it is essential to investigate if house price, bank lending, and the LTV are related by a long-run equilibrium relationship. If house prices and bank lending or house prices and LTV ratios have a long-run equilibrium, then the error correction parameter may provide information on the speed of adjustment towards the equilibrium, and this information may be useful to policymakers.

The traditional way to test for long-run equilibrium relationships is by testing for co-integration between variables. For example, Gerlach and Peng (2003) analyzed the long-run relationship of Hong Kong house price and bank credit by a VAR cointegration. They found Granger causality evidence of the direction of influence goes from property prices to bank credit. Wong, Tsang and Kong (2014) also found that a weak direct pass-through of LTV policy to the property market. However, traditional co-integration tests are well known to have low power when structural breaks occur (see Banerjee, 1999). To overcome this problem, Enders and Siklos (2012) introduced the co-integration and TAR adjustment approach to extend the Engle-Granger test in the case of structural breaks in time series processes. The authors point out that this nonlinear approach out-performed the Engle-Granger test when there were asymmetric departures from equilibrium. In this context, considering that the house prices also exhibit asymmetric, nonlinear behaviour, we apply the co-integration test based on the TAR and M-TAR models in order to investigate Hypothesis 3.

The above-mentioned hypotheses were empirically tested using data from Hong Kong. As a major financial centre, Hong Kong has house prices that have undergone large and frequent swings over the last three decades. In particular, the Hong Kong housing market experienced two major collapses that led to a banking system crisis in the past three decades. Therefore, it is a significant case study with which to investigate the relationship between bank lending policy and house price cycles. In our empirical test, we use data for total lending rather than just mortgage lending. The reasons are as follows: First, bank lending reflects lending policy changes and the economic situation more directly than mortgages. Second, the majority of loans are used in the housing market directly and measures other participants in the real estate market as well, such as developers and corporate investors, who use a lot of loans that effect property prices. In contrast, the demand for mortgages is caused mainly by the demand for properties. Kim and Bhattacharya (2001), for example, found strong Granger causality from mortgages to house prices. However, this does not provide enough information to prove how bank lending policy affects the housing market. Gerlach and Peng (2005) found that the direction of influence in Hong Kong goes from house prices to bank credit rather than the converse, whereas, after 2008, Hong Kong house prices and bank lending are much more closely related to each other than they were in the 80's and 90's. We believe that cyclical bank lending is the cause of the house price cycles. Meanwhile, in the past 25 years, introducing the maximum LTV ratio is the main tool used in Hong Kong for housing market regulation.

The empirical results of this chapter reveal several insights into the relationship between house prices, bank lending, and the LTV ratio. More specifically, the empirical results show that house prices are driven by bank lending, which narrows the gap in knowledge concerning the influence of bank lending on house prices (Gerlach and Peng, 2005). We also discover the impact of the LTV ratio on house price volatility from the estimation of an AR-GARCH-M model. In fact, the conditional heteroscedasticity effect may imply higher risk to financial stability. In this context, it is important to consider the volatility of house prices when we investigate the relationship between house prices and the LTV ratio. Furthermore, unlike prior studies, our co-integration test is based on the TAR model framework. More specifically, it has been shown that nonlinear models have a stronger ability to capture house price cycles and, therefore, more comprehensively describe the long-run relationship between house prices, bank lending, and the LTV ratio.

This chapter is organised as follows. Section 2 reviews the past trends in house price cycles, bank lending policy, and the LTV policy in Hong Kong from 1980 to 2014. Section 3 introduces the proposed models, namely the STAR model, GARCH-M model, and the co-integration test with TAR adjustment. Section 4 provides the empirical results. Finally, Section 5 provides some concluding remarks.

3.2 House price cycles and bank lending policy in Hong Kong

The experience of the Hong Kong housing market in the last three decades offers a useful case study for house prices, bank lending, and LTV ratios. In particular, the house prices and growth of bank lending underwent extraordinarily large swings around 1997 in relation to the Asian financial crisis. Hong Kong is particularly prone to large fluctuations in house prices due to severe supply-demand mismatches. More specifically, Hong Kong covers a land area of 1,104 square kilometres that includes Hong Kong Island, Lantau Island, the Kowloon Peninsula, the New Territories, and 262 other outlying islands. However, more than 75% of this land is too hilly for residential purposes. Thus, a very limited land area is home to more than 7 million citizens. This, in turn, leads to frequent housing market overheating.

Besides the severe supply-demand unbalance in real estate market, the Hong Kong housing market is subject to investment for speculative purposes. As a result, mortgage lending is extremely risky. The official figures show that most bank loans in Hong Kong are used directly in the real estate market and that more than half of these loans are mortgages. Mortgage lending has never been lower than 20% of all loans issued in Hong Kong since 1991, and it peaked at 37% in September 2002. In general, the increases in house prices in the last thirty years have been accompanied by increases in bank credit, but this left banks exposed to adverse conditions in the housing market (IMF Report, 2000). For example, Hong Kong had a major housing market crash in the early 80's. Mortgage lending grew 34% of total loans in 1979 and increased to 56% in 1980. However, in 1981 the property market collapsed. This event caused great financial instability in the following years.

To prevent a recurrence of the over-heating of the housing market and to lower the financial risk level, the government set a restriction that all bank's residential mortgages were to have an LTV ratio of no more than 70% in 1991. International experience shows that limiting the LTV ratio in bank lending can be a macro-prudential policy with which to address systemic risk: consider, for example, Hungary, Norway, Sweden, and the UK. Theoretically, the LTV ratio should have a significant effect on the housing market because the demand for mortgage loans is associated with the demand for properties. The LTV ratio restriction has been the main regulatory measure for Hong Kong's for the past two decades.

The LTV ratio limit has been adjusted four times since the 1990s. The first time the Hong Kong government reduced the maximum LTV ratio in 1991. It was lowered from 90% to 70%. Nevertheless, lending in the real estate market continued rising rapidly between 1991 and 1994. To counter this situation, a 40% benchmark property lending policy was introduced to Hong Kong banks. Around 1995, property prices dropped, and the maximum LTV ratio of 70% was confirmed as a long-term regulatory policy. Prior to 1997, the Hong Kong housing market experienced another major expansion, with both bank lending and house prices increasing rapidly in a couple of months. The Hong Kong Monetary Authority (HKMA) then reduced the LTV ratio a second time, to 60% for properties with a market price of 12 million HKD and above. At the same time, it was required that borrowers be assessed for their repayment ability for a residential mortgage. Furthermore, borrowers' monthly repayments could not exceed 50%-60% of their monthly income. In 1997, when the Asian financial crisis hit, properties prices fell more than 20%. Although the Hong Kong government removed the 40% benchmark for bank lending for properties in 1998 and withdrew the 60% LTV ratio limit for luxury properties in 2001 to stimulate the housing market, the housing market did not recover from the crash. The bank lending amount also did not go up. Until 2003, Hong Kong house prices remained at the 1991 level, which was 60% lower than the peak in 1997. Likewise, bank lending was down 15% in the period between 1997 and 2003. House prices and bank lending recovered in the years following 2003, but both did not reach levels seen before 1997. The third lowering of the LTV ratio occurred in 2009, along with quantitative easing by major central banks. In October, the LTV ratio limit was reduced from 70% to 60% for properties priced at HKD 20 million and above. After 2010, both house prices and bank lending increased rapidly and achieved another relative peak. On this occasion, regulators reacted by introducing an LTV ratio limit of 60% for properties valued at HKD 12 million and above that are not intended for personal use. Banks were also instructed to introduce stress tests for borrowers' repayment abilities. In November 2010, the maximum LTV ratio allowed for properties priced at HKD 12 million and above was reduced from 60% to 50%. For properties priced at HKD between 8 million and 12 million, the maximum allowed LTV ratio was reduced from 70% to 60%, and the maximum loan amount set at HKD 6 million. For properties priced lower than HKD 8 million, the maximum loan amount was set at 4.8 million. For non-owner-occupied properties, company-held properties, and industrial and commercial properties, the LTV ratio limit was set at 50%. Finally, after February 2015, a new supervisory measure was introduced by the HKMA; there were fewer restrictions for first-time borrowers than for other borrowers. At the same time, different LTV ratio limits were introduced for different value properties and non-owner-occupied properties. All of these new measures were used by the government to try to tamp down the boom in the housing market to prevent the banking risks.

The LTV ratio is the main tool used by the HKMA to regulate the housing market. In this respect, the Hong Kong experience provides a significant case for studying the relationship between house prices and bank lending or LTV ratios. However, most of the previous research on the Hong Kong housing market does not pay much attention to this issue. For example, Monkkonen, Wong, and Begley (2012) examined house price response to economic and population changes in Hong Kong and found spatial dynamics in the Hong Kong housing market. Further, Mak, Choy, and Ho (2010) used a quantile regression to estimate Hong Kong house prices and provide a relationship between housing characteristics and prices. In addition, Hui and Zheng (2012) found that the correlations between house prices and rental are time-varying. Funke and Paetz (2013) tested the house price and business cycles in Hong Kong, and showed that property prices are mainly driven by intratemporal preference perturbations. Finally, Chan, Lee, and Woo (2001) examined the misspecification errors and rational bubbles in the Hong Kong housing market. To fill the gap, we attempt to provide insight into the relationships between house prices, bank lending, and the LTV ratios through econometrics. In the following section, we introduced the models used for our empirical study.

3.3 The Model

In this chapter, we consider three models, which are: the STAR model for Hypothesis 1, the AR-GARCH-M model for Hypothesis 2, and the TAR models co-integration test for Hypothesis 3. The economic motivation for these three models can be concluded as follow: First, the house prices and bank lending exhibit evident economic cyclic patterns and the regime switching model has good fitness. Second, the volatility also found in the level of returns of first order differenced date series. Thus, this chapter also uses GARCH in mean model to estimate house prices by LTV's mean returns and its conditional variance. Finally, using a TAR model to examine the cyclic long run relationship for house prices, bank lending and the LTV. We introduce the three models in turn below.

3.3.1 The STAR model

The STAR model was discussed in Chapter 2. In this chapter, we use the model in equation (2.14) but replace the transition variable y_{t-d} with the bank lending series S_{t-d}. Let y_t be a realization of the house price series, then the STAR model is specified as:

$$y_t = [\varphi_0 + \varphi(L)y_t] + [\rho_0 + \rho(L)y_t] \cdot F(S_{t-d}) + \varepsilon_t, \qquad (3.1)$$

and the transition function for an LSTAR is specified as:

$$F(S_{t-d}) = [1 + \exp\{-\gamma(S_{t-d} - c)\}]^{-1}, \gamma > 0. \qquad (3.2)$$

Similarly, for the ESTAR model, the transition function is:

$$F(S_{t-d}) = [1 - \exp\{-\gamma(S_{t-d} - c)^2\}], \gamma > 0. \tag{3.3}$$

The STAR model used in this chapter has been described in section 2.3.2.

Following Teräsvirta (1994), we use the Lagrange Multiplier (LM)-type tests for testing linearity and follow the steps described in the previous chapter for the choice of transition function. However, in the auxiliary regression, the transition variable (S_t) is now a proxy for bank lending. Hence, the auxiliary regression in now given by:

$$y_t = \varphi_0 + \sum_{i=1}^{P} \varphi_{1,i} \cdot y_{t-i} + \sum_{i=1}^{P} \varphi_{2,i} \cdot y_{t-i} S_{t-d} + \sum_{i=1}^{P} \varphi_{3,i} \cdot y_{t-i} S_{t-d}^2 + \sum_{i=1}^{P} \varphi_{4,i} \cdot y_{t-i} S_{t-d}^3 + \varepsilon_t. \tag{3.4}$$

where S_{t-d} is the time series for "bank lending" in Hong Kong.

The null hypothesis is:

$$H_{01}: \varphi_{2i} = \varphi_{3i} = \varphi_{4i} = 0 \text{ for all } i.$$

The decision rule for the transition function to decide between LSTAR and ESTAR models follows that in Section 2.3.2. We evaluate the goodness of fit of the model by testing for no error autocorrelation and no remaining nonlinearity.

3.3.2 The GARCH-M model

The basic GARCH model was proposed by Bollerslev (1986) and Taylor (1986) and this model is often interpreted in various branches of econometrics, especially in financial time series where the study of volatility is of primary interest. For example, Baillie and DeGennaro (1990) estimated the GARCH in mean model for examining the stock returns and volatility. Baillie and Chung (2001) found that the minimum distance estimator methodology does a good job of estimating a GARCH model for exchange rate returns. The GARCH model expresses a conditional mean equation and a conditional variance equation. This model is commonly used for capturing autocorrelation structure of the variance. In house price modelling, Crawford and Fratantoni (2003) argue that there is strong evidence of persistent volatility dynamics for US home price series.

This study considers the GARCH in-mean model which measures conditional volatility have an impact on the level of the time series variable. For example, in financial modelling, the relationship between the risk and the expected return of an asset depends upon the attitudes toward risk of asset holders.

In our house price study, we use the LTV ratio as an exogenous variable, and it is included in the conditional mean equation. This indicates that the estimated coefficient on the LTV ratio is a measure of house prices. Given that house prices are highly persistent, we include an autoregressive term in the conditional mean equation. Also, since we are interested in investigating if house price volatility affects the conditional mean equation, we estimate an AR(1)-GARCH(1,1)-M model given by:

$$y_t = \vartheta X_t' + \delta y_{t-1} + \lambda \sigma_t + \varepsilon_t, \tag{3.5}$$

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{3.6}$$

where X_t' is the LTV ratio on return level, y_{t-1} is the lagged measure of first order difference and log transformed house price changes, ω is a constant term, σ_t^2 is the conditional variance, ϵ_{t-1}^2 is the ARCH term that measured as the lag of the squared residual from the mean equation, and σ_{t-1}^2 is the GARCH term.

Notably, prior to estimating the GARCH model and afterwards, we use an LM test for checking whether or not the standardised residuals exhibit additional ARCH. The LM test is commonly used for testing for the ARCH effect. If equations (3.5) and (3.6) are correctly specified, there should be no ARCH left in the residuals (Engle, 1982). The null hypothesis is no ARCH up to order q in the residuals, and the LM test auxiliary regression is given by:

$$e_t^2 = \beta_0 + \left(\sum_{s=1}^{q} \beta_s e_{t-s}^2 \right) + v_t, \tag{3.7}$$

where e is the residual. The ARCH test result is reported by an F-statistic.

3.3.3 Co-integration test

In order to investigate the long-run relationship among bank lending, the LTV ratio, and house prices, we consider testing for co-integration in this section. Co-integration implies a set of dynamic long-run equilibria. Considering that house prices and bank lending in Hong Kong are closely related, it is interesting to test if there is an equilibrium relationship between these two variables so that any short-run deviation would be corrected over time. Moreover, since the LTV ratio is the key tool used in housing market policy, it is also of interest to test for co-integration between house prices and the LTV ratio.

Standard models of co-integrated variables assume linearity and symmetric adjustment. For example, the methodologies developed by Johansen (1996) and Stock and Watson (1998) and a similar alternative hypothesis in the Engle and Granger (1987) test assumes symmetric adjustment. In a simple case, the error-correction co-integration test and its extensions are misspecified if adjustment is asymmetric. Considering that the Hong Kong house price series are nonlinear and asymmetric, we use an alternative nonlinear model to test for co-integration. Enders and Siklos (2001) suggested testing co-integration with the threshold autoregressive model. They modified the basic TAR as below:

$$\Delta\mu_t = I_t\rho_1\mu_{t-1} + (1 - I_t)\rho_2\mu_{t-1} + \sum_{i=1}^{p-1}\gamma_i\Delta\mu_{t-1} + \varepsilon_t, \tag{3.8}$$

where μ_t is the serially correlated disturbance term, ρ represents the estimated parameters, and I_t is the Heaviside indicator function:

$$I_t = \begin{cases} 1 & if\ \mu_{t-1} \geq \tau, \\ 0 & if\ \mu_{t-1} < \tau, \end{cases} \tag{3.9}$$

where τ is the value of the threshold, and ε_t~IID(0,σ^2).

However, the TAR has low power when the adjustment is asymmetric. Enders and Siklos (2001) also provide an alternative adjustment specification which is called momentum-threshold autoregressive models. The authors suggested that the M-TAR adjustment is very useful for attempting to smooth out large structural breaks in the series. According to Enders and Siklos (2001), the M-TAR is given by:

$$x_{1t} = \beta_0 + \beta_2 x_{2t} + \beta_3 x_{3t} + \cdots + \beta_n x_{nt} + \mu_t, \tag{3.10}$$

$$\Delta\mu_t = I_t \rho_1 \mu_{t-1} + (1 - I_t)\rho_2 \mu_{t-1} + \varepsilon_t, \tag{3.11}$$

$$M_t = \begin{cases} 1 & \text{if } \Delta\mu_{t-1} \geq \tau \\ 0 & \text{if } \Delta\mu_{t-1} < \tau, \end{cases} \tag{3.12}$$

where β_i are the estimated parameters, and x_{it} are the individual $I(1)$ components of x_i.

In such circumstances, it is necessary to estimate the threshold value τ along with the values of ρ_1 and ρ_2, two types of t-statistics for the null hypotheses $\rho_1 = 0$ and $\rho_2 = 0$ also used in the F statistic for the joint hypothesis $\rho_1 = \rho_2 = 0$. The largest of the individual t-statistics is called t-Max, the smallest is denoted t-Min, and the F statistic is called F-joint.

To use the statistics, we follow three steps:

1. Regress one of the variables on a constant and the other variables and save the residuals in the sequence $\{\hat{\mu}_t\}$. Next, depending on the type of asymmetry under consideration, set the indicator function I_t using the threshold value. Estimate a regression equation in the form of (3.8) and record the larger of the t-statistics for the null hypothesis $\rho_{\mathrm{i}} = 0$ along with the F-statistic for the null hypothesis $\rho_1 = \rho_2 = 0$. Compare these sample statistics with the appropriate critical values.

2. If the alternative hypothesis of stationery is accepted, it is possible to test for asymmetric adjustment. For example, $\rho_1 = \rho_2$.

3. Diagnostic checking of the residuals should be undertaken to ascertain whether or not the $\hat{\varepsilon}_t$ series can reasonably be characterized by a white-noise process.

3.4 Empirical result

In this section, we empirically test the three hypothesises described in section

3.1. We first test Hypothesis 1 using the STAR model, and then we test Hypotheses 2 and 3 using GARCH-M model and TAR (M-TAR) models, respectively. First, we provide some basic descriptions of the data. Second, we report on the full estimation procedures and results.

3.4.1 Data and descriptive statistics

The empirical data were collected from the HKMA. Due to the fact that house prices, and bank lending and LTV ratio data are released in different times and formats, we construct two datasets to reconcile quarterly and monthly data. The first dataset for investigating house prices and bank lending consists of quarterly indexes for all types of properties in Hong Kong and all loans used in Hong Kong over the period 1980: q1 to 2014: q4. The second dataset for investigating house prices and the LTV ratio consists of monthly indexes from June 1998 to November 2016.

Looking at the data in Figure 3.1, it appears that both the housing market and bank lending in Hong Kong have experienced significant cyclical volatility over the last thirty-four years. Before 1997, the housing market was subject to cyclical variations, but the market suffered a major contraction in 1997 when house prices entered a six years' long recession that resulted in an average loss of more than 60% compared with 1997. A second major contraction occurred in 2007 after the subprime crisis in the US market and the European sovereign debt crisis. The house prices in the Hong Kong market decreased by approximately 20 percent in one year. Similarly, bank lending contracted approximately 20 percent from 1997 to 2003 and 10 percent in 2008.

Figure 3.2 shows the LTV ratio and house prices. The LTV ratio does not have a similar trend with house prices but is inversely related. In particular, around 2000 the house prices fall to the bottom, and the LTV ratio achieves its peak. In addition, after 2008, higher house prices are associated with a lower LTV ratio.

Figure 3.1 House price and bank lending in Hong Kong

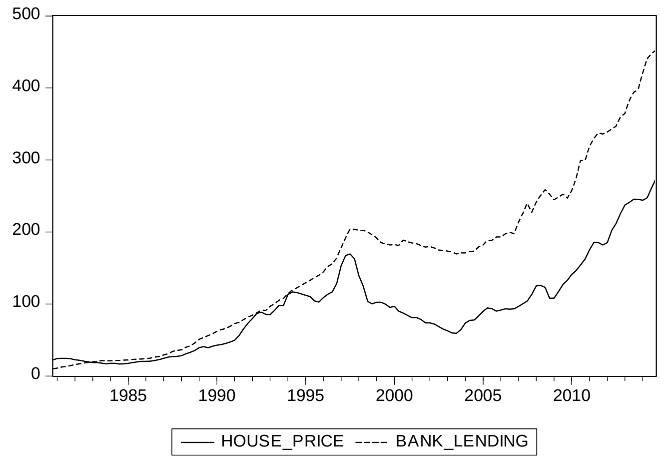
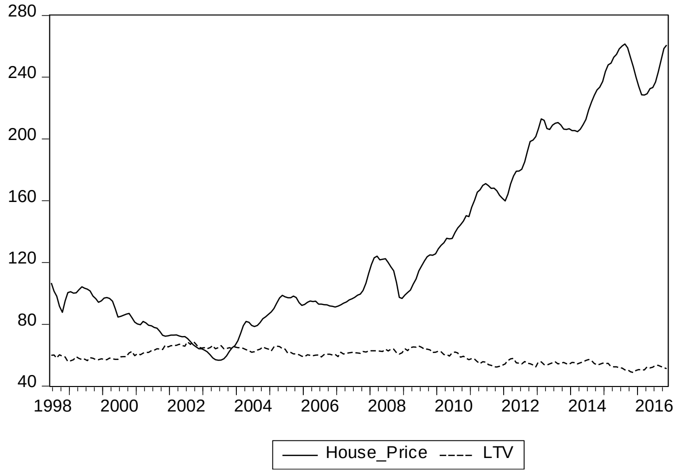


Figure 3.2 House price and LTV in Hong Kong



Prior to estimating the STAR model, it is necessary to test for stationary house prices, bank lending and LTV series. First, we transform the series using first-order difference. The house price dataset for Hypothesis 1 and 2 are denoted as House price-1 and House price-2, respectively. For testing to see if the data is stationary, we use the ADF unit root test, and the results are reported in Table 3.1. From the ADF unit root results, it can be seen that the values of the t-Statistic are lower than the 1% critical value with p-value near zero. This means that the series is stationary.

Table 3.1 ADF Unit Root test

	t-Statistic	Prob.
House price-1	-5.179***	0.0000
House price-2	-7.305***	0.0000
Bank lending	-3.898***	0.0027
LTV	-17.003***	0.0000

Note: *** indicate statistical significance at 1%

3.4.2 STAR model result

The modelling procedure adopted follows the steps described in Section 3.3.1. Table 3.2 reports the results of the linearity test and corresponding F-statistics of the null hypothesis based on equation (3.4). The dependent variable y_t represents Hong Kong house prices, and the auxiliary regression is applied in the case of S_t, which is bank lending. The maximum lags are determined by an initial analysis of the AIC of VAR model estimation. We set the maximum lags at 10 for both endogenous and exogenous, and the result indicates that the maximum lag is 2 for y_{t-i} and 9 for transition variables S_{t-d}. However, our test indicates that the optimal lag for transition variable S_{t-d} is zero. We denote the dependent variable house price as HP, and the transition variable bank lending as BL. The null hypothesis of linearity is $H_0\!: \beta_1 = \beta_2 = \beta_3 = 0$ in equation (3.4).

Table 3.2 Linearity test in equation (3.4)

	HP-BL
	$H_0\!: \beta_1 = \beta_2 = \beta_3 = 0$
Transition Variable	
S_t	0.000
S_{t-1}	0.027
TREND	0.220

Table 3.2 indicates that the strongest linearity rejection happened at S_t at the 1%

significance level, and Table 3.3 shows the estimated p-values of the auxiliary regression equation (3.4).

Table 3.3 Choosing the Type of Model

	HP-BL		
	H_{04}	H_{03}	H_{02}
Transition variable			
S_t	0.001	0.997	0.004
S_{t-1}	0.163	0.009	0.553
TREND	0.354	0.170	0.306

Table 3.3 shows the test result for transition function selection. The hypotheses H_{04}, H_{03}, and H_{02} are the null hypotheses for the order of the polynomial in the auxiliary regression (3.4). The null hypotheses and decision rules are as same as the sequence given in Section 2.3.2. The test results are reported by p-values of the F-statistics. Looking at the second column of Table 3.3, at lag 0, the H_{04} is rejected at the 1% level of significance, which indicates a logistic transition function. Accordingly, we will estimate an LSTAR model with S_t as the transition variable.

The start values are created using the grid-search method. The grid search creates a linear grid in c and a log-linear grid in γ. We set the grid for c between 0.1 and 10.0, and for γ in [0.5, 100.0].

Table 3.4 presents the estimation results for Hypothesis 1. The top panel reports estimated parameters of the LSTAR models, and R squared and adjusted R squared values. The models have been reduced in size by eliminating redundant variables, more specifically, y_{t-2} in the linear part and y_{t-1} in the nonlinear part. The middle and bottom sections of Table 3.4 report the results of the LSTAR model diagnostic tests, i.e., the tests for "no error autocorrelation" and "no remaining nonlinearity," respectively.

Looking at the top section of Table 3.4, it appears that the LSTAR model captures the house price cycles well when using bank lending as the transition variable. All parameters are statistically significant, except for the constant in the linear part. The estimated γ parameter indicates a slow speed of regime switching between contraction and expansion phases.

Looking at the middle section of Table 3.4, which reports the results for the test for error autocorrelation (the null hypothesis for this test is that there is “no error autocorrelation”), we see that we do not reject the null hypothesis. This indicates that there is no misspecification in our estimated model. The bottom section of Table 3.4 shows the results of tests with the null hypothesis of “no remaining nonlinearity,” and there is no rejection at 1% significance of the null hypothesis in rows S_t and S_{t-1}, which means that there is no need to add another STR component to the model. However, rejection occurs at 5% and 10% significance, respectively, which is not very strong rejection. Hence, we think the nonlinearities are captured by our models.

Table 3.4 LSTAR model estimation statistics

	HP-BL
Variable	Coefficient
Linear Part	
Constant	-1.918 (1.316)
φ_1	0.587*** (0.079)
Nonlinear Part	
Constant	7.713*** (2.259)
ρ_2	-0.418*** (0.155)
γ	2.874* (1.577)
c	3.193** (1.374)
R2	0.515
Adjusted R2	0.518

lag	F-value	df1	df2	p-value
1	1.113	1	126	0.294
2	0.701	2	124	0.498

Transition variable	H_0	H_{04}	H_{03}	H_{02}
S_t	0.071	0.120	0.622	0.005
S_{t-1}	0.049	0.147	0.125	0.011

Note: ***), **), *) indicate statistical significance at 1%, 5% and 10%, respectively. ( ) is standard deviations.

Figure 3.3 provides the graph of the transition function versus the function of the transition variable $F(S_t)$. In Figure (3.3), each dot corresponds to an observation. We can see that the transition process between the two regimes is indeed smooth. Figure 3.3 also shows the speed of transition between states and illustrates that the LSTAR model captures the cycles well.

Figure 3.3 Plot of transition function versus function of transition variable

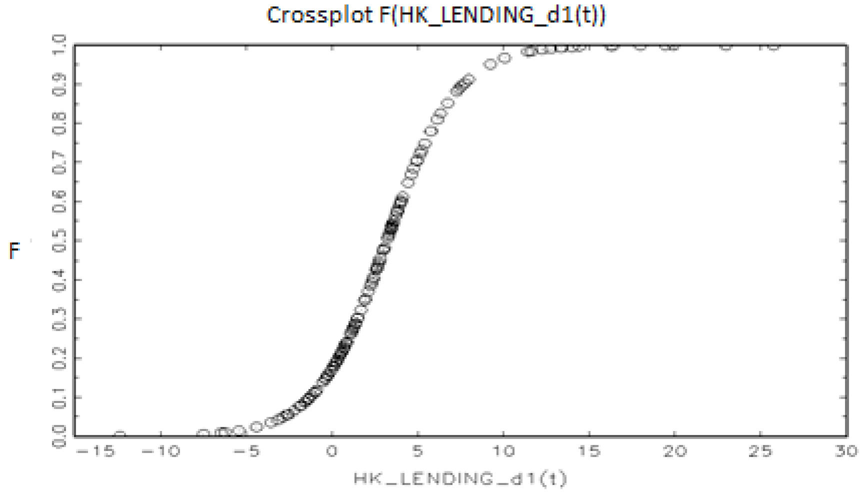


Figure 3.4 plots the transition function (s_t, γ, c) . From Figure 3.4, it appears that the regimes change with high frequency in the period 1989 to 1993 and after 2005. This corresponds to large fluctuation in the Hong Kong housing market and bank lending in those periods.

Figure 3.4 Plot of transition function

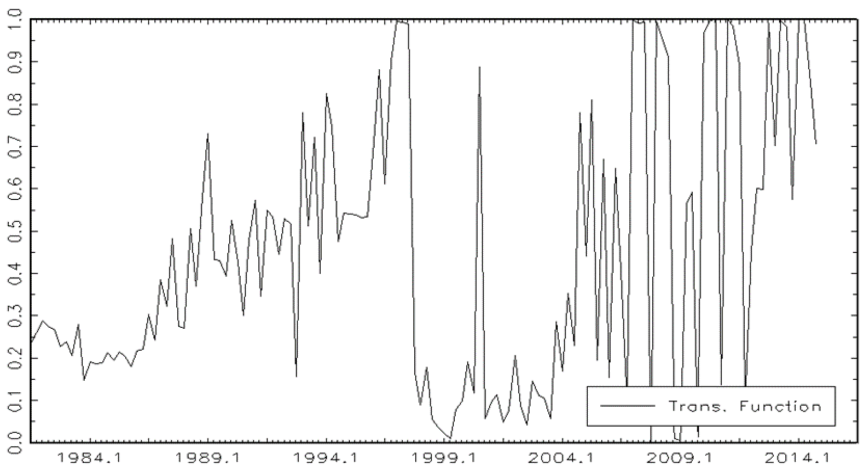


Figure 3.4 shows the estimated and original series. From the figure, we can see that the estimated series is close to the original series and that most fluctuations have been estimated. However, for big swing periods, the estimated series does not reach the peaks and valleys of the original series, such as around the years 1997 and 1998.

Figure 3.5 Plot of LSTAR estimated series and original series

Overall, our estimation indicates that the LSTAR model supports Hypothesis 1. In other words, credit cycles have a significant impact on house price cycles. In the estimated model, it is also clear that the reaction is quite fast, as the adjustment takes place within one quarter.

3.4.3 GARCH model result

In order to test Hypothesis 2, we specify an AR (1)-GARCH-M model, which is given in equations (3.5) and (3.6).

We first check the suitability of the GARCH model by estimating the conditional mean equation only using the OLS method and testing for heteroscedasticity of the residuals using the LM test. Note that for all series under consideration, the log transformation of the first differences has been considered in the estimation process.

The null hypothesis of the LM-ARCH test is that there is no ARCH up to a specified order in the residuals. The top part of Table 3.5 reports on the LM heteroscedasticity test. We specify the lag order as 1, 4, 8, and 12. The reported F-statistic is for an omitted variable test for the joint significance of all lagged squared residuals. In the OLS row of the top panel, we can see that all of the lags reject the null hypothesis. This indicates that the data series has a strong ARCH effect. In the row GARCH-M, all the considered lags fail to reject the null hypothesis that the AR (1)-GARCH (1, 1)-M model captures the volatility in house prices well. The middle and bottom panels of Table 3.5 report the estimated parameters for the AR (1)-GARCH-M model. From Table 3.5, it appears that all coefficients are statistically significant. Our estimation result indicates that the LTV ratio affects house prices, which is consistent with Hypothesis 2.

Table 3.5 ARCH model estimation statistics

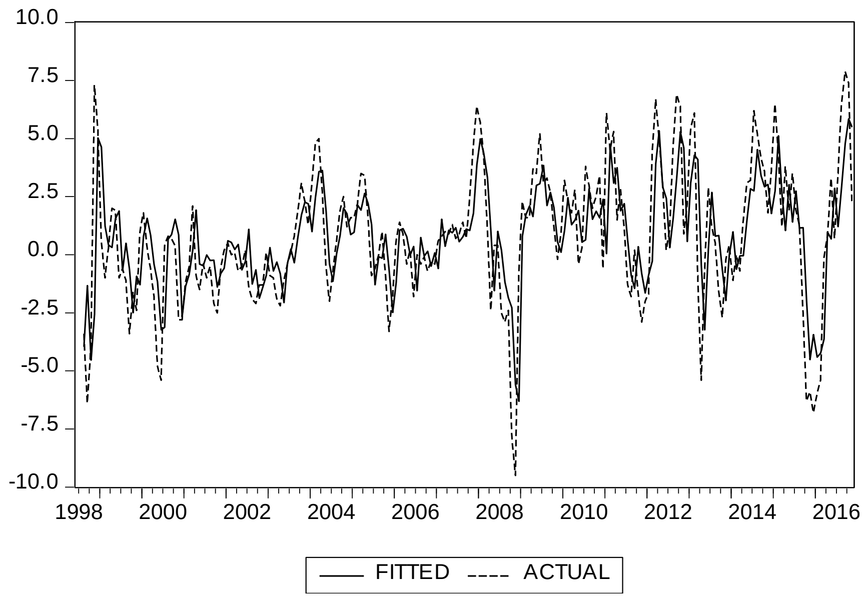
	lag			
	1	4	8	12
	F-statistic			
OLS	79.862***	21.172***	10.988***	7.897***
GARCH-M	0.068	0.579	1.012	1.116

Variable	Coefficient
λ	0.128** (0.066)
ϑ	19.856*** (6.962)
δ	0.732*** (0.043)
ω	0.254* (0.152)
α	0.175*** (0.065)
β	0.762*** (0.073)
R-squared	0.520
Adjusted R-squared	0.515

Note: ***), **), *) indicate statistical significance at 1%, 5% and 10%, respectively.

Figure 3.6 plots the fitted and actual series for the conditional mean equation. We can see the AR (1)-GARCH-M model accurately captures the characteristics of house prices in Hong Kong. All in all, our empirical test strongly supports Hypothesis 2, which postulated that house price changes are affected by the LTV ratios imposed by financial authorities.

Figure 3.6 Plot of GARCH estimated series and actual series



3.4.4 TAR co-integration test for long-run relationship

The series of the house prices, LTV ratios, and bank lending are logarithms transferred. Endogenous variables are denoted as LHP, LLTV and LBL. The threshold and lags are determined by the data we use. According to Enders and Siklos (2001), the length of lag is determined by AIC and BIC, and the threshold value follows the methodology of Chan (1993). The 5% significance level of critical values are from 10,000 Monte Carlo simulations. Base on Section 3.3.3, we set the hypothesis as H_0: $\rho_1 = \rho_2 = 0$ in equations (3.8) and (3.11).

The co-integration with TAR and M-TAR results are shown in Table 3.6. Looking at the results in Column 2, the estimated parameters ρ_1 and ρ_2 are negative suggest faster convergence for negative than for positive discrepancies from long-run equilibrium. The positive value of the μ_{t-1}indicates a random-walk behavior occurs. The threshold value is estimated by the Chan's (1993) methodology. From the bottom panel of Table 3.6 it appears that the calculated F-equal for the null hypothesis of symmetric adjustment ($\rho_1 = \rho_2$) given in the second column exceeds the 5% critical value (given in the []). Hence, at conventional significance level, one can reject the null hypothesis of symmetric adjustment. This is consistent with the asymmetric behaviours of house prices and bank lending in Hong Kong. The estimates' F-joint-statistic at the bottom of Table 3.6 indicates that the null hypothesis that ρ_1=ρ_2=0 can be rejected near the 5% level (note that the critical values of the test are given in []), meaning that house prices and bank lending in Hong Kong are co-integrated. Note that the point estimate for T-Max is only -0.861. Hence, we cannot reject the null hypothesis of no co-integration, according to the T-max test. Therefore, the two test statistics produce contradictory results. However, according to Enders and Siklos (2001), the T-Max statistic has lower power than the F-joint-statistic. Accordingly, we rely on the latter test and discard the former.

Column 3 of Table 3.6 reports the results of estimated M-TAR co-integration test for house prices and the LTV ratio. The M-TAR model uses the consistently estimated threshold of 0.056. The AIC and BIC selected a model using three-lagged changes of $\{\Delta\mu_t\}$. Note that the $|\rho_1| < |\rho_2|$ and $\Delta\mu_{t-1} < 0$. According to Enders and Siklos (2001), the M-TAR model exhibits substantial decay. Once again the test rejects the symmetric hypothesis at the 5% level, and the T-max statistic fails to reject the null hypothesis that $\rho_1 = \rho_2 = 0$, whereas the F-joint-statistic rejects it. Under these circumstances, we conclude that the house prices and LTV ratio in Hong Kong also have a long-run relationship.

Table 3.6 TAR and M-TAR co-integration results

	LHP-LBL	LHP-LLTV
Variable	**Coefficient**	**Coefficient**
ρ_1	-0.094 (0.025)	0.064 (0.052)
ρ_2	-0.016 (0.019)	-0.122 (0.032)
$\Delta\mu_{t-1}$	0.567 (0.068)	-0.066 (0.068)
$\Delta\mu_{t-2}$	-	-0.076 (0.067)
$\Delta\mu_{t-3}$	-	-0.035 (0.067)
τ	0.302	0.056
F-equal	6.420 [6.302]*	9.865 [8.284]*
T-Max	-0.861 [-1.887]*	1.243 [-1.809]*
F-joint	7.699 [7.101]*	8.275 [8.041]*

*() are the standard errors; [] are the 10,000 Monte Carlo simulated critical values for the 5% significance level.

3.4.5 Empirical test result analysis

In this section, we empirically tested the three hypotheses of interest in this chapter. Regarding Hypothesis 1, we find that bank lending from financial institutions heavily affects house price cycles. The magnitude of the transition parameter reflects the smooth swings between boom and bust phases. The fact that the transition variable has no lags implies that changes in bank lending policy are promptly transmitted to the housing market. The LSTAR model captures all of these characteristics. With respect to Hypothesis 2, we found a strong ARCH effect in the conditional variance equation of house prices, and the in-mean parameter in the conditional mean equation implies that the volatility of house prices also affect the conditional mean equation. Furthermore, the house prices also react changes in the LTV ratio. Finally, in our co-integration test for Hypothesis 3, we also found the long-run equilibrium relationship between house prices and bank lending as well as house prices and the LTV ratio. In light of these results, we conclude that bank lending fluctuations and LTV ratios significantly affect house prices.

3.5 Conclusion

This chapter considers the relationship between house prices, bank lending, and the LTV ratio in Hong Kong. As the Hong Kong housing market experienced two major collapses, it is a significantly useful case to study. In particular, we have empirically tested three hypotheses. Hypothesis 1 aims to investigate if the house price changes are affected by bank lending. We estimate an LSTAR model that uses the bank lending as transition variable. The rationale of the model specification is that the re-specified LSTAR model is capable of capturing the smooth and asymmetric nonlinear behaviours in house prices and bank lending. Our results highlight that house prices are severely affected by bank lending policies. According to the estimation results, when the lending standard is relaxed, house prices will experience a boom phase. Additionally, when financial institutions tighten their lending, the house price cycles will enter a contraction phase. Also, the speed of adjustment of the housing market to changes in lending policies is rapid as house prices will fall within one-quarter of a tightening.

The purpose of Hypothesis 2 is to investigate the linkage between house prices and the LTV ratios. We find the ARCH effect in the house price series, and, therefore, we estimate a GARCH-M model. In order to test the LTV ratio's impact on house prices, we allow the LTV ratio as an explanatory variable in our GARCH-M model. Our empirical test found that a strong effect directly from the LTV ratio to house prices. In addition, we also found that the GARCH-M model describes the volatility in our data well.

Finally, the results for Hypothesis 3 indicates that house prices and bank lending are co-integrated. Similarly, house prices and the LTV ratio have a long-run equilibrium relationship based on the M-TAR co-integration test.

Our research has several important implications. Firstly, in the aftermath of the global crisis, house prices play a more important role in financial stability. Our finding can be a useful reference for market regulators and investors. They should be aware that bank lending and the LTV ratio are capable of regulating house prices in order to promote financial stability effectively. For example, when the regulator anticipates that the housing market is facing a recession or accumulated bubbles, an effective way to head such an event off is to change the bank lending policy or introduce a new LTV ratio standard. In the meantime, with regard to the signs of these policy changes, investors can predict the rough trend.

Secondly, research in the future may consider testing more factors that may be associated with house prices. In spite of this chapter illustrating that the banking sector strongly affects house prices, we believe other factors are also associated with house price swings. For example, personal income, consumption, and different regions or countries may influence house price swings.

Finally, we are interested in proposing a STAR-GARCH type model in house prices modelling in the future. Previous research in the housing market has found that regime switching models capture structural changes well, and the GARCH model describes the volatility well. Therefore, we think the combination of these two mechanisms could have more capacity in house price modelling and forecasting.

Chapter Four

Investigating the performance of the unit root test in the presence of the in-mean term: the Monte Carlo Experiment

4.1 Introduction

Unit root tests have been widely used to classify time series as being either stationary or nonstationary. However, it is well known that these tests tend to over-reject the null hypothesis in the presence of heteroscedastic errors. For example, an early study by Kim and Schmidt (1993) is one of the first works to investigate this issue, and they found that the unit root tests are generally not robust in the face of non-normal innovations. In particular, the authors conduct a Monte Carlo experiment to investigate the Dickey-Fuller (DF) test when the innovations admit a GARCH-type process. According to their experimental results, the DF test over-rejects the null hypothesis frequently in the presence of conditional heteroscedasticity. In addition, the DF tests also appear to be seriously inaccurate when the ratio of the GARCH intercept to the initial variance is near zero and the volatility parameter is larger. Haldrup (1994), Ling et al. (2003), and Valkanov (2005) considered the DF test and also found that this commonly used inference procedure is oversized when GARCH innovations are included in the data generating process.

To understand this problem, Seo (1999) studied the asymptotic distribution of the AR unit root test when the error term follows a GARCH process. The author found that the power of the unit root test increased when the GARCH effect increases. Ling and Li (1998) compare the maximum likelihood and least squares estimators for various types of random walk processes. They indicated that the maximum likelihood estimator of unit roots is significantly more efficient than ordinary least squares estimation. Against this background, Cook (2008) further explored the work done by Seo (1999) and considered the local-to-unity detrending of the unit root testing procedure to increase the power of unit root test with GARCH process. The author concluded that the proposed test increases the power compared to the traditional DF test. Similar results are found in Li and Shukur (2011). Here, the authors used the wavelet method to address the over-rejection issue and indicated that the proposed method improves the unit root test in finite samples. According to Perron and Ng (1996), the two main problems of unit root tests are: i) low power when the root of the AR polynomial is close to one; ii) severe size distortions when the MA polynomial of the first-differenced series has a large negative root.

This chapter contributes to the literature by investigating the performance of commonly used unit root tests in the presence of non-normal innovations. However, it is not easily observed in real world and not been widely investigated. Thus, we hope that our research will fill this gap. In particular, we focus on an AR (1)-GARCH-M process and consider the effects of structural breaks on the size and power properties of these tests. The inference procedures under consideration are the DF test proposed by Dickey and Fuller (1979) and the M-test proposed by Stock (1999) and Perron and Ng (1996). The M-test is a kind of modified unit root test which ought to be more robust in the presence of structural breaks in time series processes.

In order to investigate the properties of these tests in the presence of structural breaks, an extensive Monte Carlo experiment has been undertaken. The Monte Carlo experimental design allows for structural breaks in the conditional variance equation. For the DF test and M-test, we consider using both the ordinary least squared method and the generalized least squared method to estimate the autoregressive parameter. In addition, we consider an empirical application of the AR-GARCH-M model to UK house prices. The empirical test will investigate how the in-mean term affects house prices estimation.

Summarizing our results, we find that the in-mean parameter significantly affects the size and power properties of the test. More specifically, the size and power of two considered unit root tests perform well when the in-mean parameter is fixed at a small value and severely deviated when the in-mean parameter increased. Furthermore, our empirical findings indicate that the in-mean term significantly increases the estimation. In particular, the GARCH model is severe inaccurate, but the GARCH in-mean model captures house price volatility well.

This chapter proceeds as follows. Section 4.2 introduces the unit root test and the over-rejection problem. Next, Section 4.3 introduces the data generation process used for the Monte Carlo analysis. Further, Section 4.4 reports on the Monte Carlo experiment and our estimation results. In addition, Section 4.5 provides an implication for housing price. The final section, Section 4.6, is the conclusion.

4.2 Unit root tests

To test for a unit root in y_t, consider the following two equations:

$$y_t = \varphi + \gamma t + u_t, \tag{4.1}$$

$$u_t = \rho u_{t-1} + v_t, \tag{4.2}$$

where $v_t \sim iid(0, \sigma^2)$. The $\varphi + \gamma t$ is a linear trend used for capturing the deterministic trend of y_t.

Equation (4.2) captures the stochastic trend properties of y_t , and $\rho = 1$ corresponds to u_t having a unit root. In other words, to verify that y_t is nonstationary, it is necessary to verify that u_t contains a unit root. The null hypothesis and alternative hypothesis of the unit root tests are:

$$H_0 : \ \rho = 1 \ \Rightarrow \text{Non stationary}$$

$$H_1 : \ \ \rho < 1 \Rightarrow \text{Stationary} \tag{4.3}$$

To implement the unit root test, we use a two-step approach, which includes detrending and testing

4.2.1 Detrending

The detrending method is considered using both OLS and GLS. Applying a lag operator (1 - ρL) to both sides of (4.1) and using (4.2) to replace u_t, y_t is:

$$(1 - \rho L)y_t = \varphi(1 - \rho) + \gamma(1 - \rho L)t + u_t. \tag{4.4}$$

Let $\rho = \rho^*$, where ρ^* is a constant, then estimate the remaining parameters with the full sample of $t = 1,2, \dots, T$. Then (4.4) can be written by using the filter $(1 - \rho^* L)$ as follows:

$$y_t - \rho^* y_{t-1} = \varphi(1 - \rho^*) + \gamma\big(t - \rho^*(t-1)\big) + u_t - \rho^* u_{t-1}. \qquad (4.5)$$

Using matrices, we can re-express equation (4.5) as:

$$y^* = x^* \beta^* + u^*, \qquad (4.6)$$

where $\beta^* = [\varphi\ \gamma]'$ is a (2 x 1) vector, u^* is the disturbance term in (4.6), and the y^* and x^* are:

$$y^* = \begin{bmatrix} y_1 \\ y_2 - \rho^* y_1 \\ y_3 - \rho^* y_2 \\ \vdots \\ y_r - \rho^* y_{T-1} \end{bmatrix}, \quad x^* = \begin{bmatrix} 1 & 1 \\ 1 - \rho^* & 2 - \rho^* \\ 1 - \rho^* & 3 - 2\rho^* \\ \vdots & \vdots \\ 1 - \rho^* & T - (T-1)\rho^* \end{bmatrix}. \qquad (4.7)$$

The choice of ϕ^* in (4.6) is considered in two methods as follows:

1. OLS: $\rho^* = 0$;
2. GLS: $\rho^* = 1 + \frac{\bar{c}}{T}$,

where $\bar{c} < 0$ is fixed by the deterministic variable in (4.7).

4.2.2 Testing

The OLS estimator of ρ in (4.2) can be obtained by:

$$\hat{\rho} = \frac{\sum_{t=2}^{T} \hat{u}_t \hat{u}_{t-1}}{\sum_{t=2}^{T} \hat{u}_{t-1}^2}. \qquad (4.8)$$

A Dickey-Fuller test is given by:

$$DF_t = \frac{\hat{\rho}-1}{se(\hat{\rho})}. \tag{4.9}$$

The M-tests are given by:

$$MZ_\alpha = \frac{T^{-1}\hat{u}_T^2-\sigma^2}{2T^{-2}\sum_{t=2}^{T}\hat{u}_{t-1}^2}, \tag{4.10}$$

$$MSB = \left(T^{-2}\sum_{t=2}^{T}\frac{\hat{u}_{t-1}^2}{\hat{\sigma}^2}\right)^{\frac{1}{2}}, \tag{4.11}$$

$$MZ_t = MZ_\alpha \times MSB = \frac{\frac{1}{2}\left(T^{-1}\hat{u}_T^2-\hat{\sigma}^2\right)}{\hat{\sigma}\left(T^{-2}\sum_{t=2}^{T}\hat{u}_{t-1}^2\right)^{1/2}}, \tag{4.12}$$

where $\hat{\sigma}^2$ is an autoregressive estimator of the spectral density of μ_t, given by:

$$\hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=2}^{T}\hat{v}_t^2, \tag{4.13}$$

where $\hat{v}_t$ are the residuals.

The M-tests in (4.10) and (4.11) are very closely related to the DF test in (4.9).

4.2.3 The over-rejection problem of the unit root tests

In the next section, a comprehensive study of the DF test and M-test is undertaken by Monte Carlo experiment in order to investigate the performance of these inference procedures in the presence of the in-mean term in the conditional mean equation and structural breaks in the conditional variance equation.

The DGP under consideration is an AR-(1)-GARCH in-mean model. We first consider the size properties of the tests statistics under consideration and then the analysis is extended to consider the power.

4.3 The data generating process

In order to investigate the performance of unit root tests in the presence of the in-mean term and structural break, we consider the following model:

$$y_t = \varphi + \phi y_{t-1} + \vartheta h_t^{\frac{\delta}{2}} + \varepsilon_t, \tag{4.14}$$

where $\varphi = 1$, $\phi = 1$ are the AR parameters measuring the intrinsic persistence in the level of y_{t-1}. The $h_t^{\frac{\delta}{2}}$ is the power transformed conditional variance. The ε_t and $h_t^{\frac{\delta}{2}}$ follows the GARCH (1, 1) process:

$$\varepsilon_t = \eta_t\sqrt{h_t}\,, \tag{4.15}$$

$$h_t^{\frac{\delta}{2}} = \omega + \alpha\varepsilon_{t-1} + \beta h_{t-1}^{\frac{\delta}{2}}\,, \tag{4.16}$$

where $\delta = 1$, $\omega = 1 - \alpha - \beta$, η_t is a sequence of i.i.d ~ N (0, 1). The $h_t^{\frac{\delta}{2}}$ in the mean equation and the lagged y_{t-1} in the conditional variance equation are the two variables in our test.

In this model, the term $h_t^{\frac{\delta}{2}}$ is referred to as the in-mean effect in the unit root test. In order to investigate the performance of the unit root test when there is an unknown break point in the lagged innovation term ε_{t-1}, the conditional variance is augmented as:

$$h_t^{\frac{\delta}{2}} = \omega + (\alpha + \gamma\Delta_t)\varepsilon_{t-1} + \beta h_{t-1}^{\frac{\delta}{2}}, \tag{4.17}$$

where γ is a constant denoting the magnitude of the break and Δ_t is a break dummy variable standing for the timing of the break, given by:

$$\Delta_t = \begin{Bmatrix} 0, & t \le \tau T_B \\ 1, & t > \tau T_B \end{Bmatrix}, \tag{4.18}$$

where τ is the unknown fraction of the simulated series which indicates the location of the break point.

Similarly, we investigate the effect of the unknown breakpoint in the transformed conditional variance $h_{t-1}^{\frac{\delta}{2}}$ and augment the β in euquation (4.16) as follow:

$$h_t^{\frac{\delta}{2}} = \omega + \alpha\varepsilon_{t-1} + (\beta + \gamma\Delta_t)h_{t-1}^{\frac{\delta}{2}}. \tag{4.19}$$

In the Monte Carlo experiment, we considered the DGP with different values of α and β to test the performance of the unit root tests. For example, we extend the work by Kim and Schmidt (1993) by considering the case where the GARCH variance process is being near integrated ($\alpha + \beta \to 1$).

4.4 Monte Carlo Experiment

The DGP for the Monte Carlo simulation analysis is specified by equations (4.14), (4.15), and (4.16), and the simulation experiment is conducted using GAUSS 9 software. In the estimation, the autoregressive parameter is estimated with both OLS and GLS. In total, four tests are considered, which are denoted as DF_{OLS}, DF_{GLS}, $M_{OLS,}$, and M_{GLS}, respectively.

All simulations are based on 10,000 replications and use sample size T=1,000. We also created further 50 initial observations and discarded, in order to avoid the influence of the initial values. The GARCH in-mean parameters are specifically set as $\tau = \{0.25,\ 0.5, 0.75\}$ and $\gamma = \{0, 0.05, 0.07, 0.1\}$. For each τ, we test with the full set of γ. At the same time, both τ and γ are tested under different values of $\vartheta = \{0, 0.3, 0.5, 0.7\}$. Test results are shown in Tables 4.1 and 4.2. In particular, Table 4.1 reports the simulations with the parameters α and β set in equation (4.16) as (0.85, 0.1), (0.5, 0.1), and (0.1, 0.1), and Table 4.2 reports the result of the Monte Carlo when α and β are set as (0.1, 0.5), (0.45, 0.5), and (0.1, 0.85). We summarize the results for empirical sizes of the tests for the 5% nominal significant level.

Firstly, from Table 4.1, it appears that with the in-mean parameter fixed at $\vartheta = 0$, both the DF test and M-test have good size properties. However, when the magnitude of ϑ increases, the empirical sizes of all statistics under consideration are severely affected. Among the four statistics, the DF_{OLS} has the best size properties.

Secondly, for any given ϑ, the values of the α and β have an effect on the performance of the test statistics. In particular, when $\alpha + \beta$ is close to 1, the test statistics become severely undersized. Table 4.1 reports empirical sizes for ($\alpha + \beta$) as 0.95, 0.60, and 0.25, respectively. It can be seen that the empirical size of the test for 0.95 is much less than the nominal 5% significant level. In addition, we cannot find large differences in the influence of the parameters α and β, but it is still indicated that the small value of α performs better than the small value of β, see Table 4.1, rows 3-6. For different values of a and β, the four statistics do not appear to have large differences. Finally, it is of interest to note that the over-rejection problem is affected by break size γ and break point τ. Our results indicate that increasing the value of the parameter γ greatly impacts the performance of the test statistics, as the size distortion of the tests become larger and larger. While, for a given γ, increasing the value of τ has a positive effect on the performance of the unit root test.

Table 4.1 Size result of the Monte Carlo simulation

	DF-OLS	DF-GLS	MZt-OLS	MZt-GLS	DF-OLS	DF-GLS	MZt-OLS	MZt-GLS	DF-OLS	DF-GLS	MZt-OLS	MZt-GLS
Conditional Variance	$h_t^{\frac{\delta}{2}} = \omega + 0.85\varepsilon_{t-1} + 0.1h_{t-1}^{\frac{\delta}{2}}$				$h_t^{\frac{\delta}{2}} = \omega + 0.5\varepsilon_{t-1} + 0.1h_{t-1}^{\frac{\delta}{2}}$				$h_t^{\frac{\delta}{2}} = \omega + 0.15\varepsilon_{t-1} + 0.1h_{t-1}^{\frac{\delta}{2}}$			
Conditional Mean	$y_t = 1 + y_{t-1} + \varepsilon_t$											
τ=0.25 γ=0.00	0.070	0.080	0.078	0.078	0.052	0.056	0.058	0.054	0.052	0.054	0.046	0.052
τ=0.25 γ=0.05	0.114	0.088	0.122	0.086	0.072	0.048	0.052	0.048	0.058	0.042	0.048	0.042
τ=0.25 γ=0.07	0.104	0.092	0.122	0.092	0.058	0.050	0.058	0.050	0.054	0.058	0.046	0.058
τ=0.25 γ=0.10	0.086	0.092	0.116	0.092	0.048	0.054	0.032	0.054	0.042	0.046	0.038	0.044
τ=0.50 γ=0.00	0.070	0.080	0.078	0.078	0.052	0.056	0.058	0.054	0.052	0.054	0.046	0.052
τ=0.50 γ=0.05	0.120	0.096	0.118	0.096	0.074	0.046	0.050	0.044	0.058	0.042	0.048	0.042
τ=0.50 γ=0.07	0.102	0.092	0.114	0.090	0.052	0.052	0.060	0.052	0.052	0.058	0.050	0.056
τ=0.50 γ=0.10	0.090	0.076	0.112	0.074	0.048	0.062	0.040	0.062	0.040	0.046	0.036	0.044
τ=0.75 γ=0.00	0.070	0.080	0.078	0.078	0.052	0.056	0.058	0.054	0.052	0.054	0.046	0.052
τ=0.75 γ=0.05	0.118	0.090	0.114	0.088	0.072	0.044	0.046	0.044	0.058	0.044	0.050	0.042
τ=0.75 γ=0.07	0.100	0.088	0.118	0.088	0.058	0.048	0.052	0.046	0.052	0.058	0.046	0.056
τ=0.75 γ=0.10	0.080	0.070	0.092	0.070	0.046	0.058	0.042	0.058	0.046	0.048	0.036	0.048
Conditional Mean	$y_t = 1 + y_{t-1} + 0.3h_t + \varepsilon_t$											
τ=0.25 γ=0.00	0.080	0.058	0.060	0.058	0.042	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.25 γ=0.05	0.080	0.034	0.050	0.034	0.026	0.000	0.000	0.000	0.004	0.000	0.000	0.000
τ=0.25 γ=0.07	0.084	0.048	0.046	0.048	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.25 γ=0.10	0.050	0.040	0.050	0.040	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.50 γ=0.00	0.080	0.058	0.060	0.058	0.042	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.50 γ=0.05	0.092	0.030	0.062	0.030	0.026	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.50 γ=0.07	0.094	0.042	0.054	0.042	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.50 γ=0.10	0.046	0.046	0.046	0.046	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.75 γ=0.00	0.080	0.058	0.060	0.058	0.042	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.75 γ=0.05	0.100	0.042	0.066	0.042	0.028	0.000	0.000	0.000	0.004	0.000	0.000	0.000
τ=0.75 γ=0.07	0.100	0.044	0.058	0.044	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.75 γ=0.10	0.078	0.042	0.060	0.042	0.020	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Conditional Mean	$y_t = 1 + y_{t-1} + 0.5h_t + \varepsilon_t$											
τ=0.25 γ=0.00	0.070	0.038	0.050	0.036	0.042	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.25 γ=0.05	0.086	0.044	0.044	0.042	0.024	0.000	0.000	0.000	0.004	0.000	0.000	0.000
τ=0.25 γ=0.07	0.076	0.032	0.026	0.030	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.25 γ=0.10	0.048	0.024	0.032	0.024	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.50 γ=0.00	0.070	0.038	0.050	0.036	0.042	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.50 γ=0.05	0.096	0.042	0.046	0.042	0.026	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.50 γ=0.07	0.086	0.040	0.030	0.040	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.50 γ=0.10	0.052	0.034	0.036	0.034	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.75 γ=0.00	0.070	0.038	0.050	0.036	0.042	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.75 γ=0.05	0.100	0.046	0.044	0.044	0.032	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.75 γ=0.07	0.102	0.046	0.046	0.046	0.022	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.75 γ=0.10	0.064	0.046	0.050	0.044	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Conditional Mean	$y_t = 1 + y_{t-1} + 0.7h_t + \varepsilon_t$											
τ=0.25 γ=0.00	0.076	0.036	0.046	0.034	0.044	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.25 γ=0.05	0.082	0.032	0.026	0.032	0.026	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.25 γ=0.07	0.074	0.024	0.026	0.024	0.016	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.25 γ=0.10	0.040	0.028	0.026	0.028	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.50 γ=0.00	0.076	0.036	0.046	0.034	0.044	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.50 γ=0.05	0.094	0.038	0.038	0.038	0.028	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.50 γ=0.07	0.076	0.032	0.030	0.032	0.018	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.50 γ=0.10	0.050	0.028	0.032	0.028	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.75 γ=0.00	0.076	0.036	0.046	0.034	0.044	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.75 γ=0.05	0.102	0.038	0.040	0.036	0.034	0.000	0.000	0.000	0.002	0.000	0.000	0.000
τ=0.75 γ=0.07	0.086	0.036	0.038	0.036	0.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000
τ=0.75 γ=0.10	0.062	0.038	0.038	0.036	0.014	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 4.2 shows more Monte Carlo experimental results. Once again, it appears that the size properties of the unit root tests are significantly affected by the in-mean term ϑ. It can be seen that when the ϑ increases to 0.3, all the empirical sizes are far away from the nominal 5% size. Comparing Table 4.2, rows 11 to 14, with Table 4.1, rows 3 to 6, while both show the results of $\alpha + \beta = 0.95$, it can clearly be seen that the smaller value of α=0.1 in Table 4.2 has slightly better size properties than the case when the $\beta = 0.1$ in the Table 4.1. Another set of simulation results for $\alpha + \beta = 0.95$ are shown in Table 4.2, rows 7 to 10, but this time $\alpha = 0.45$ and $\beta = 0.5$. From the simulation results, it appears that the test statistics have similar size properties as the case when $\alpha = 0.85$ and $\beta = 0.1$ in Table 4.1.

Table 4.2 Size result of the Monte Carlo simulation

	DF-OLS	DF-GLS	MZt-OLS	MZt-GLS	DF-OLS	DF-GLS	MZt-OLS	MZt-GLS	DF-OLS	DF-GLS	MZt-OLS	MZt-GLS
Conditional Variance	$h_t^{\frac{\delta}{2}} = \omega + 0.1\varepsilon_{t-1} + 0.5h_{t-1}^{\frac{\delta}{2}}$				$h_t^{\frac{\delta}{2}} = \omega + 0.45\varepsilon_{t-1} + 0.5h_{t-1}^{\frac{\delta}{2}}$				$h_t^{\frac{\delta}{2}} = \omega + 0.1\varepsilon_{t-1} + 0.85h_{t-1}^{\frac{\delta}{2}}$			
Conditional Mean	$y_t = 1 + y_{t-1} + \varepsilon_t$											
τ=0.25 γ=0.00	0.056	0.054	0.048	0.052	0.070	0.080	0.090	0.080	0.056	0.060	0.052	0.056
τ=0.25 γ=0.05	0.056	0.044	0.054	0.044	0.096	0.096	0.116	0.096	0.056	0.094	0.088	0.094
τ=0.25 γ=0.07	0.044	0.046	0.046	0.046	0.094	0.086	0.114	0.084	0.100	0.178	0.152	0.178
τ=0.25 γ=0.10	0.040	0.058	0.044	0.056	0.072	0.116	0.114	0.116	0.252	0.372	0.298	0.372
τ=0.50 γ=0.00	0.056	0.054	0.048	0.052	0.070	0.080	0.090	0.080	0.056	0.060	0.052	0.056
τ=0.50 γ=0.05	0.058	0.048	0.056	0.048	0.110	0.088	0.120	0.084	0.076	0.102	0.102	0.100
τ=0.50 γ=0.07	0.048	0.044	0.050	0.044	0.108	0.086	0.110	0.084	0.114	0.172	0.158	0.172
τ=0.50 γ=0.10	0.042	0.050	0.046	0.050	0.082	0.108	0.112	0.108	0.248	0.372	0.304	0.370
τ=0.75 γ=0.00	0.056	0.054	0.048	0.052	0.070	0.080	0.090	0.080	0.056	0.060	0.052	0.056
τ=0.75 γ=0.05	0.062	0.044	0.050	0.044	0.110	0.076	0.110	0.076	0.078	0.072	0.084	0.072
τ=0.75 γ=0.07	0.052	0.046	0.042	0.044	0.112	0.078	0.112	0.078	0.124	0.158	0.168	0.156
τ=0.75 γ=0.10	0.042	0.054	0.046	0.054	0.084	0.088	0.096	0.086	0.248	0.324	0.290	0.320
Conditional Mean	$y_t = 1 + y_{t-1} + 0.3h_t + \varepsilon_t$											
τ=0.25 γ=0.00	0.008	0.002	0.002	0.002	0.060	0.054	0.046	0.054	0.050	0.056	0.054	0.054
τ=0.25 γ=0.05	0.004	0.002	0.000	0.002	0.052	0.026	0.024	0.024	0.006	0.008	0.002	0.008
τ=0.25 γ=0.07	0.004	0.002	0.000	0.002	0.042	0.024	0.028	0.022	0.002	0.000	0.000	0.000
τ=0.25 γ=0.10	0.000	0.000	0.000	0.000	0.024	0.010	0.016	0.010	0.000	0.000	0.000	0.000
τ=0.50 γ=0.00	0.008	0.002	0.002	0.002	0.060	0.054	0.046	0.054	0.050	0.056	0.054	0.054
τ=0.50 γ=0.05	0.006	0.002	0.000	0.002	0.056	0.022	0.028	0.022	0.016	0.022	0.020	0.022
τ=0.50 γ=0.07	0.006	0.000	0.000	0.000	0.056	0.026	0.048	0.026	0.002	0.000	0.004	0.000
τ=0.50 γ=0.10	0.000	0.000	0.000	0.000	0.030	0.020	0.016	0.018	0.000	0.000	0.000	0.000
τ=0.75 γ=0.00	0.008	0.002	0.002	0.002	0.060	0.054	0.046	0.054	0.050	0.056	0.054	0.054
τ=0.75 γ=0.05	0.008	0.000	0.000	0.000	0.070	0.020	0.036	0.020	0.028	0.028	0.030	0.028
τ=0.75 γ=0.07	0.010	0.000	0.002	0.000	0.082	0.032	0.066	0.032	0.014	0.014	0.016	0.014
τ=0.75 γ=0.10	0.004	0.000	0.000	0.000	0.052	0.038	0.044	0.038	0.002	0.002	0.002	0.002
Conditional Mean	$y_t = 1 + y_{t-1} + 0.5h_t + \varepsilon_t$											
τ=0.25 γ=0.00	0.002	0.000	0.000	0.000	0.078	0.046	0.048	0.046	0.058	0.066	0.056	0.066
τ=0.25 γ=0.05	0.004	0.000	0.000	0.000	0.044	0.014	0.014	0.014	0.010	0.004	0.008	0.004
τ=0.25 γ=0.07	0.000	0.000	0.000	0.000	0.036	0.010	0.014	0.010	0.002	0.000	0.000	0.000
τ=0.25 γ=0.10	0.000	0.000	0.000	0.000	0.016	0.000	0.004	0.000	0.000	0.000	0.000	0.000
τ=0.50 γ=0.00	0.002	0.000	0.000	0.000	0.078	0.046	0.048	0.046	0.058	0.066	0.056	0.066
τ=0.50 γ=0.05	0.002	0.000	0.000	0.000	0.048	0.016	0.018	0.016	0.008	0.006	0.006	0.006
τ=0.50 γ=0.07	0.000	0.000	0.000	0.000	0.048	0.020	0.024	0.020	0.000	0.000	0.000	0.000
τ=0.50 γ=0.10	0.000	0.000	0.000	0.000	0.028	0.016	0.012	0.016	0.000	0.000	0.000	0.000
τ=0.75 γ=0.00	0.002	0.000	0.000	0.000	0.078	0.046	0.048	0.046	0.058	0.066	0.056	0.066
τ=0.75 γ=0.05	0.004	0.000	0.000	0.000	0.066	0.022	0.026	0.020	0.024	0.026	0.028	0.026
τ=0.75 γ=0.07	0.000	0.000	0.000	0.000	0.080	0.032	0.048	0.032	0.006	0.004	0.006	0.004
τ=0.75 γ=0.10	0.000	0.000	0.000	0.000	0.048	0.030	0.030	0.030	0.002	0.002	0.002	0.002
Conditional Mean	$y_t = 1 + y_{t-1} + 0.7h_t + \varepsilon_t$											
τ=0.25 γ=0.00	0.002	0.000	0.000	0.000	0.078	0.038	0.048	0.038	0.056	0.060	0.054	0.058
τ=0.25 γ=0.05	0.002	0.000	0.000	0.000	0.044	0.008	0.012	0.008	0.006	0.004	0.004	0.004
τ=0.25 γ=0.07	0.000	0.000	0.000	0.000	0.028	0.006	0.008	0.006	0.002	0.000	0.000	0.000
τ=0.25 γ=0.10	0.000	0.000	0.000	0.000	0.014	0.002	0.002	0.002	0.000	0.000	0.000	0.000
τ=0.50 γ=0.00	0.002	0.000	0.000	0.000	0.078	0.038	0.048	0.038	0.056	0.060	0.054	0.058
τ=0.50 γ=0.05	0.002	0.000	0.000	0.000	0.052	0.020	0.016	0.018	0.006	0.004	0.006	0.004
τ=0.50 γ=0.07	0.000	0.000	0.000	0.000	0.048	0.018	0.024	0.018	0.000	0.000	0.000	0.000
τ=0.50 γ=0.10	0.000	0.000	0.000	0.000	0.024	0.008	0.006	0.008	0.000	0.000	0.000	0.000
τ=0.75 γ=0.00	0.002	0.000	0.000	0.000	0.078	0.038	0.048	0.038	0.056	0.060	0.054	0.058
τ=0.75 γ=0.05	0.004	0.000	0.000	0.000	0.068	0.018	0.016	0.018	0.016	0.022	0.018	0.022
τ=0.75 γ=0.07	0.000	0.000	0.000	0.000	0.068	0.034	0.032	0.034	0.004	0.004	0.002	0.004
τ=0.75 γ=0.10	0.000	0.000	0.000	0.000	0.046	0.014	0.020	0.014	0.000	0.000	0.000	0.000

Figures 4.1-4.4 show the empirical power of the unit root tests to reject the null hypothesis when it is not correct. In particular, Figure 4.1- 4.4 focus on examining the power of the inference procedures to reject the null hypothesis of $\phi = 1$ when, in fact, the process is stationary, i.e., $-1 < \phi < 1$.

The tests investigate the power properties of the test statistics, across the range of local breaks of magnitude $\gamma = \{0.05, 0.07, 0.1\}$ with the set of $\alpha = \{0.1, 0.25, 0.5, 0.75\}$, $\beta = \{0.1, 0.25, 0.5, 0.75\}$, and $\vartheta = \{0, 0.3, 0.5, 0.7\}$ for asymptotic local power functions at the 0.05 nominal level.

Moreover, in order to model the sequence of stationary alternatives that near the null hypothesis of two unit root tests, we also consider the DGP for equation (4.14) to (4.16) with $\phi = 1 - \frac{c}{T}$ in equation (4.15), where c = −30, −29, -28,...,-2, −1, 0 is controlling the size of the departure from a unit root.

In Figures 4.1-4.4, columns 1-2 report the in-mean term ϑ with values of 0, 0.3, 0.5, and 0.7, respectively. From rows 1 to 3 in Figure 4.1 and Figure 4.2, the α is fixed with value of 0.1 and β increases from 0.25 to 0.5 to 0.75. Similarly, in rows 1 to 3 of Figure 4.5 and Figure 4.6, the β is fixed at 0.1 and α is 0.25, 0.5 and 0.75. From the simulated results, it can be seen easily that the ability of the test to reject the null hypothesis is significantly sensitive to the in-mean parameter ϑ, and the statistics DF_{GLS} and M_{GLS} are more affected by the parameter ϑ. Among the four statistics, the DF_{OLS} has the best power properties and the M_{OLS} performs most stable in our test. Now, looking at the Figures 4.1 and 4.2, in the top and bottom panel, for given α and ϑ, increasing β has a negative impact on the power in all of inference procedures taken into consideration. But comparing the effect of an increase in α in Figures 4.3 and 4.4 with fixed β (top and bottom panel), it is clear that the power of the tests is more sensitive to the increasing values of the parameter α.

Figure 4.1 Power simulation results of Monte Carlo simulation

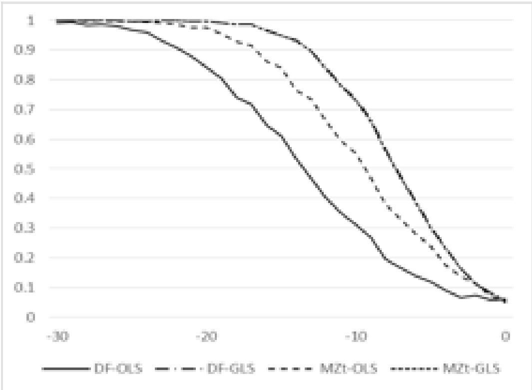
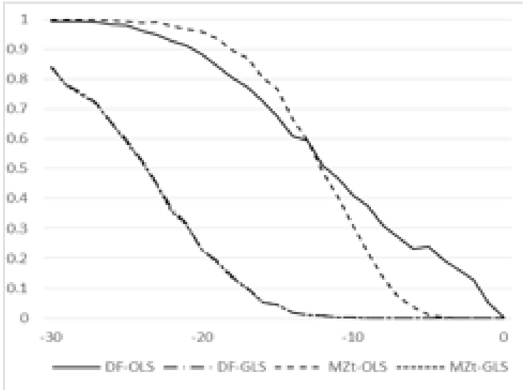
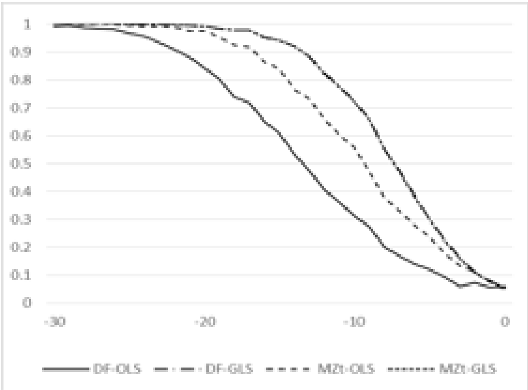
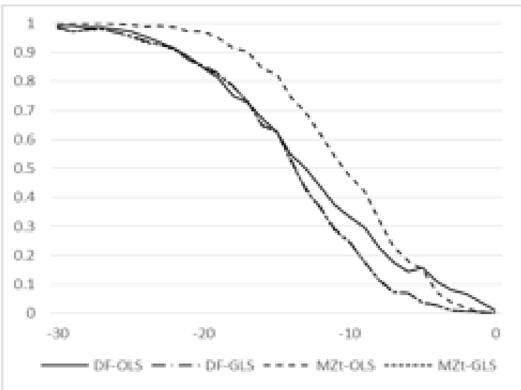
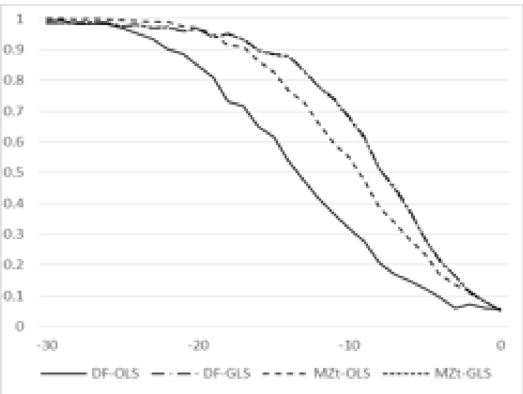
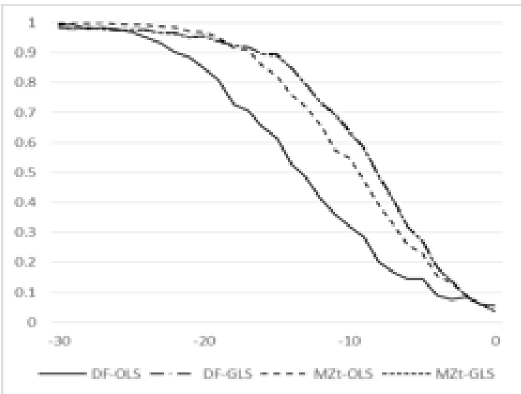
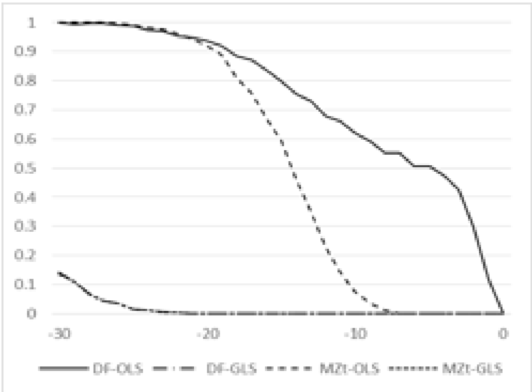
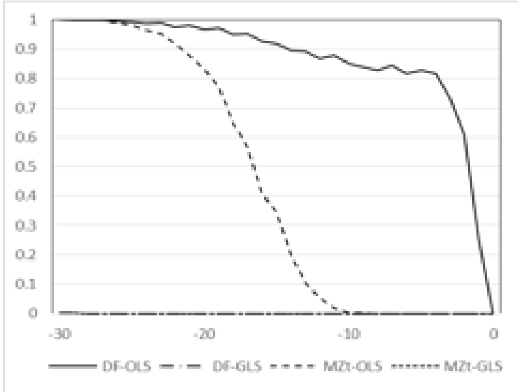
<table>
<tr><td>$y_t = 1 + y_{t-1} + \varepsilon_t$</td><td>$y_t = 1 + y_{t-1} + 0.3h_t + \varepsilon_t$</td></tr>
<tr><td colspan="2">$h_t^{\frac{\delta}{2}} = \omega + 0.1\varepsilon_{t-1} + 0.25h_{t-1}^{\frac{\delta}{2}}$</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">$h_t^{\frac{\delta}{2}} = \omega + 0.1\varepsilon_{t-1} + 0.5h_{t-1}^{\frac{\delta}{2}}$</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">$h_t^{\frac{\delta}{2}} = \omega + 0.1\varepsilon_{t-1} + 0.75h_{t-1}^{\frac{\delta}{2}}$</td></tr>
<tr><td></td><td></td></tr>
</table>

Figure 4.2 Empirical power function

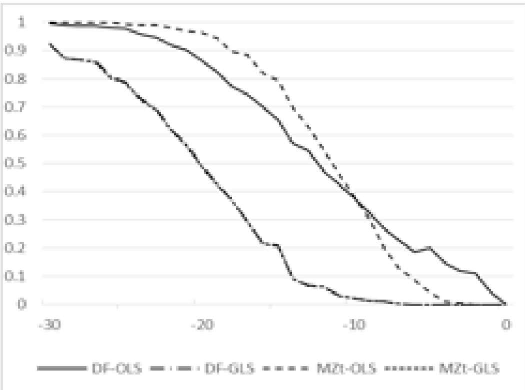
$y_t = 1 + y_{t-1} + 0.5h_t + \varepsilon_t$	$y_t = 1 + y_{t-1} + 0.7h_t + \varepsilon_t$

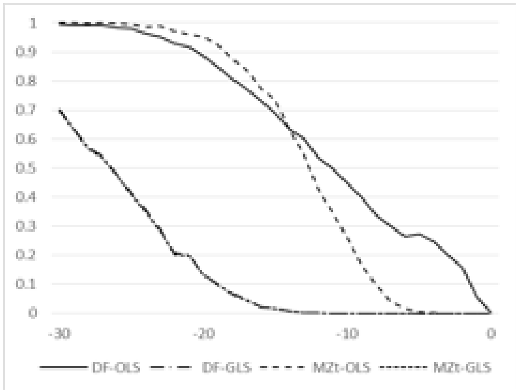
$$h_t^{\frac{\delta}{2}} = \omega + 0.1\varepsilon_{t-1} + 0.25h_{t-1}^{\frac{\delta}{2}}$$



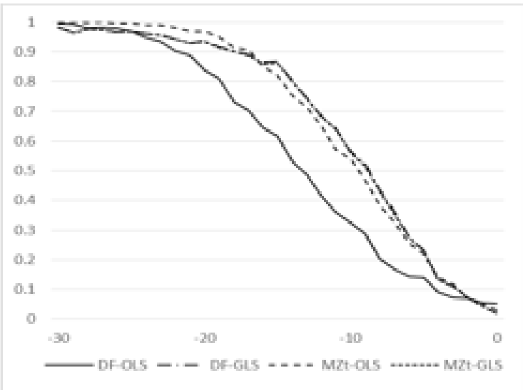


$$h_t^{\frac{\delta}{2}} = \omega + 0.1\varepsilon_{t-1} + 0.5h_{t-1}^{\frac{\delta}{2}}$$





$$h_t^{\frac{\delta}{2}} = \omega + 0.1\varepsilon_{t-1} + 0.75h_{t-1}^{\frac{\delta}{2}}$$



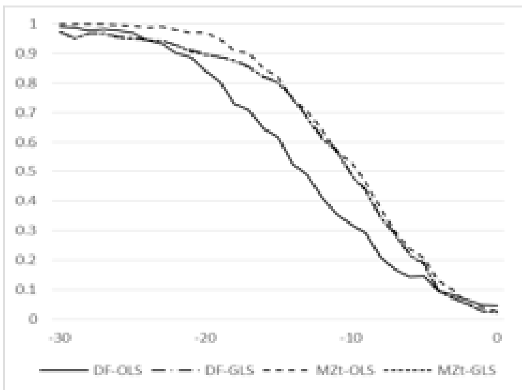
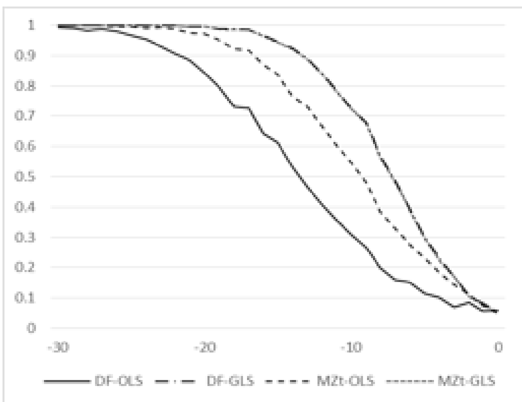
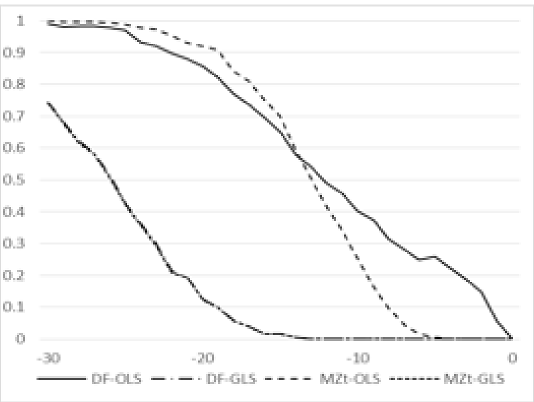


Figure 4.3 Power simulation results of Monte Carlo experiment

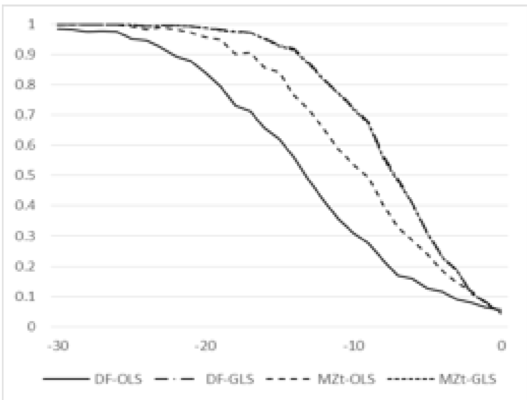
$y_t = 1 + y_{t-1} + \varepsilon_t$	$y_t = 1 + y_{t-1} + 0.3h_t + \varepsilon_t$

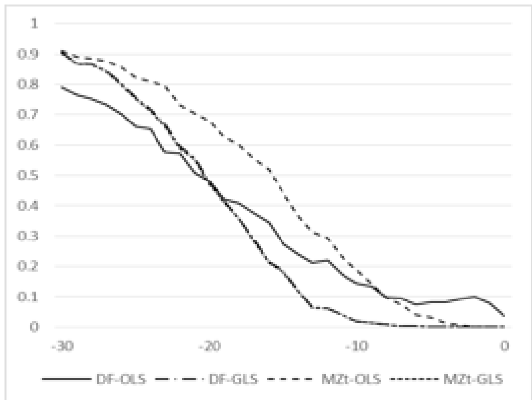
$$h_t^{\frac{\delta}{2}} = \omega + 0.25\varepsilon_{t-1} + 0.1h_{t-1}^{\frac{\delta}{2}}$$



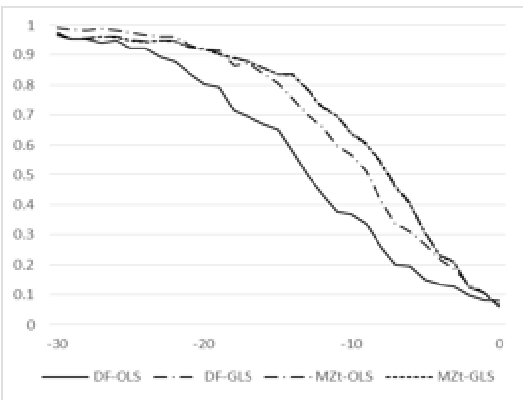


$$h_t^{\frac{\delta}{2}} = \omega + 0.5\varepsilon_{t-1} + 0.1h_{t-1}^{\frac{\delta}{2}}$$





$$h_t^{\frac{\delta}{2}} = \omega + 0.75\varepsilon_{t-1} + 0.1h_{t-1}^{\frac{\delta}{2}}$$



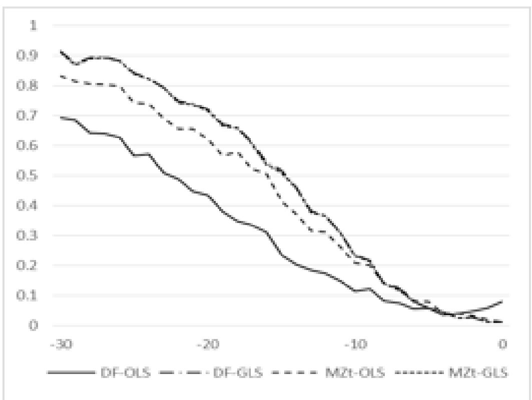
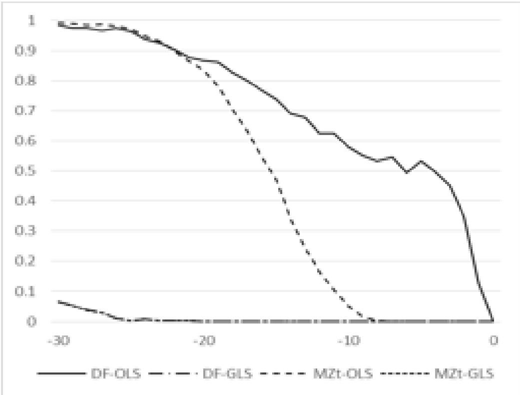
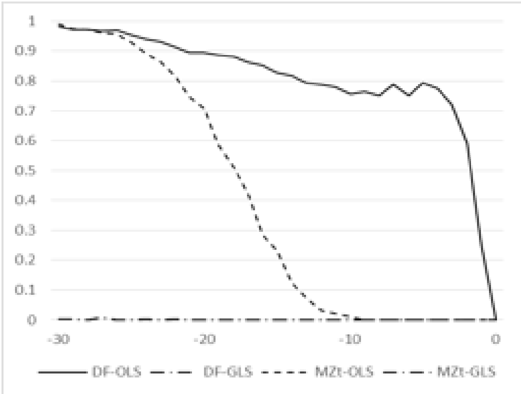


Figure 4.4 Power simulation results

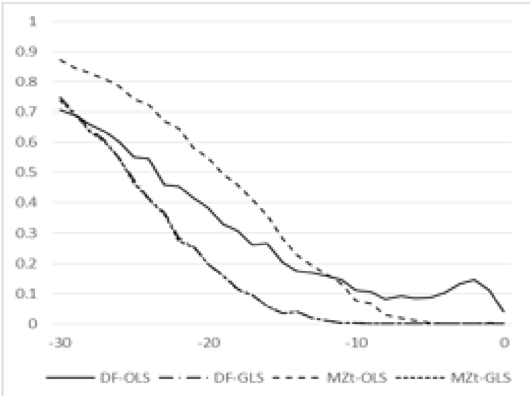
$y_t = 1 + y_{t-1} + 0.5h_t + \varepsilon_t$	$y_t = 1 + y_{t-1} + 0.7h_t + \varepsilon_t$

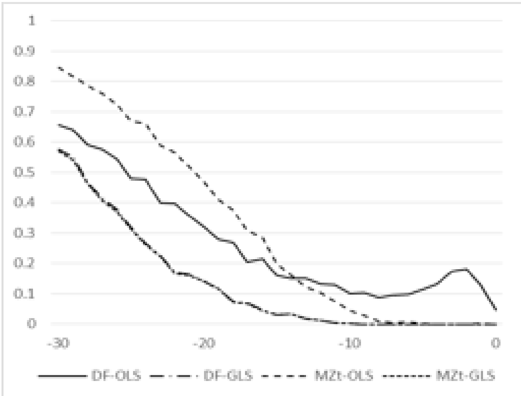
$$h_t^{\frac{\delta}{2}} = \omega + 0.25\varepsilon_{t-1} + 0.1h_{t-1}^{\frac{\delta}{2}}$$



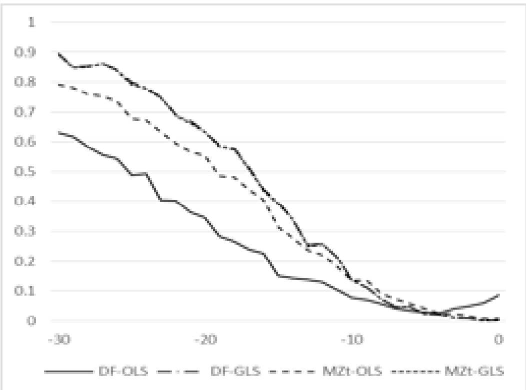


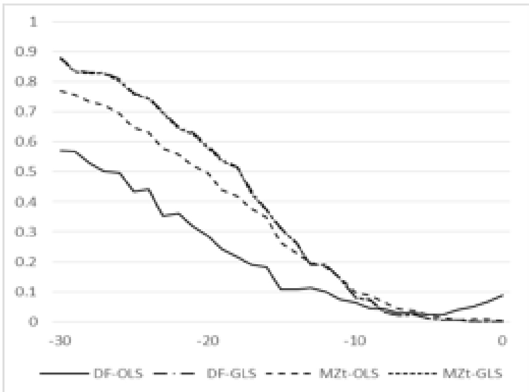
$$h_t^{\frac{\delta}{2}} = \omega + 0.5\varepsilon_{t-1} + 0.1h_{t-1}^{\frac{\delta}{2}}$$





$$h_t^{\frac{\delta}{2}} = \omega + 0.75\varepsilon_{t-1} + 0.1h_{t-1}^{\frac{\delta}{2}}$$





4.5 An empirical application

In this section, we consider an empirical application to UK house prices. The house price fluctuations may have a negative impact on indicators of financial stability, such as defaults, foreclosures, the value of mortgage-backed securities, and the value of derivatives related to house prices. Therefore, investigating the volatility of the housing market is essential for investors, municipalities, and policymakers to manage risk and stabilize the economy. We consider using an AR (1)-GARCH-M model with structural breaks to estimate UK house prices. The data are from a quarterly all-transaction index from 1970: Q1 to 2013: Q4 presented by the Bank for International Settlements (BIS). The log transformation of the first order differences for the house prices is considered. We identify one break for the series, and the breakpoints were determined by using the procedure of Bai and Perron (1998). Accordingly, the breakpoint for UK housing prices was set at 1989: Q3 using a dummy variable taking value zero before the breakpoint and one otherwise.

Let y_t denote the log house price index changes, according to equations (4.14) and (4.16), then the mean equation and the conditional variance can be written as:

$$y_t = \varphi + \phi y_{t-1} + (\vartheta + \Delta_\tau) h_t^{\frac{\delta}{2}} + \varepsilon_t, \tag{4.20}$$

$$h_t^{\frac{\delta}{2}} = \omega + (\alpha + \Delta_\tau)\varepsilon_{t-1} + (\beta + \Delta_\tau) h_{t-1}^{\frac{\delta}{2}}, \tag{4.21}$$

where $\delta = 1$ and Δ_τ are dummy variables that take value 1 for break point and 0 for the rest.

Table 4.3 reports the empirical result for an AR (1)-GARCH-M model and an AR (1)-GARCH model. The top part of the table reports the estimated coefficients for the mean equation (4.20), whereas the estimated coefficients for the conditional variance are reported in the bottom panel of Table 4.3. Looking at the estimated parameters for the AR (1)-GARCH-M model, in the second column of the table, the estimated coefficients are all significant, which indicates the data can well be modelled using the model at hand. The dummy variable for the in-mean parameter is statistically significant at the 1% level. In addition, the in-mean parameter is positive, which means a positive shock to house price volatility will also increase the conditional mean of house prices. The lower part of the table reports the estimated coefficients for the conditional variance in equation (4.21). It appears that all the parameters for the break in the conditional variance are significantly different from zero. The sum of the ARCH and GARCH coefficients is approximately 0.92; this indicates that the volatility shocks are quite persistent.

Now looking at the third column where the estimated parameters of the AR (1)-GARCH model, it appears that the parameters estimated in the mean equation are all statistically significant. However, none of the parameters in the variance equation are statistically significant. Comparing the result of AR (1)-GARCH-M and AR (1)-GARCH, the in-mean term highly improved the performance of the GARCH model for house price estimation. All in all, there is strong evidence of persistent volatility dynamics in UK housing prices, and the in-mean terms, especially, strongly affect house prices.

Table 4.3 Results of AR (1)-GARCH in-mean with a single break

	AR(1)-GARCH-M	AR(1)-GARCH
Mean Equation		
φ	-0.043** (0.021)	0.011** (0.005)
ϕ	0.452*** (0.069)	0.664*** (0.102)
ϑ	2.499*** (0.953)	-
Variance Equation		
ω	0.000** (0.000)	0.000 (0.001)
α	-0.026** (0.012)	-0.015 (0.136)
β	0.945*** (0.028)	0.584 (0.835)
Δ_τ	-0.000* (0.000)	-0.001 (0.002)

() indicates standard error; ***, **, * indicate statistical significance at 1%, 5%, and 10%, respectively.

4.6 Conclusion

This chapter contributes to the literature by investigating the performance of the unit root test in the presence of the GARCH in-mean term and structural breaks. We have focused on the DF and M-type unit root tests. Results of a Monte Carlo simulation experiment have shown that the strong presence of structural breaks in the time series leads to over-rejection of the unit root when inference procedures are taken into consideration. In order to further investigate this issue, we extend the GARCH model with the in-mean parameter. The Monte Carlo simulation evidence for the case of a single break in volatility indicates that the in-mean term significantly affects the size and power properties of unit root tests. More specifically, the size and power properties severely deteriorate when the in-mean parameter increases. In addition, we also test the near-integrated unit root process in the mean equation. Our results indicate that the size and power properties of the unit root test perform badly when increasing the parameters of the ARCH and GARCH terms and that the ARCH term has a stronger effect than the GARCH term.

Finally, in this chapter, the validity of the AR (1)-GARCH in-mean process in describing the UK housing market is investigated. It is found that the model with time-varying parameters is better suited to describe the data at hand.

Chapter Five

Concluding Remarks

Nonlinear time series modelling has been receiving more and more attention in both academic and industry spheres, especially after the global financial crisis of 2008, when the market experienced large swings. In recent years, regulators have established more comprehensive regulations to improve risk management. The major markets recovered from the negative effects of the financial crisis gradually, but sharply swings remain and are the cause of financial instability. Around 2016, the Chinese equity market lost nearly 50% of its index and the market value of most listed companies. Against this background, a model which can capture and forecast market swings well would help policymakers to define financial stability and protect investors. In this respect, a lot of empirical work has been done in the literature. For example, Peel and Speight (1998) proposed a bilinear generalized-quadratic ARCH model for several countries' business cycles. Garcia and Perron (1996) applied a Markov switching model with three regimes for the US real interest rate, McKay and Reis (2016) constructed a business-cycle model for the US, and Miles (2011) tested the C-GARCH model for estimating US home price volatility.

In spite of numerous researchers making significant contributions to empirical studies, gaps still exist. For example, the 2008 US sub-mortgage crisis once again gave us a lesson on the importance of the housing market and banking policy. Therefore, econometricians are keen to model the housing market. For example, Baldi (2014) investigated the impact of a central bank on house prices; Leung (2014) tested error correction dynamics of house prices; Nneji, Brooks, and Ward (2013) investigated house price dynamics and their reaction to macroeconomic changes; Park and Bae (2015) used machine learning algorithms for house price forecasting; and Yang, Liu, and Leatham (2013) examined dynamic relationships among housing prices.

In essence, asymmetric nonlinearity has been found in house price cycles.

Therefore, a large body of studies focused on using nonlinear models to estimate and forecast house prices. The nonlinear research themes may be summarized as nonlinearity in mean and nonlinearity in variance. Researchers have applied both groups of nonlinear models to estimating house prices. For example, Cabrera, Wang, and Yang (2011), Huang (2012), and Tsai, Lee, and Chiang (2012) investigated house prices by using nonlinear in-mean models. Miles and William (2011), and Chang and Liang (2010) applied nonlinear in-variance models, such as the GARCH model, to house prices. However, some problems have still not been addressed. For example, modelling and forecasting house prices in a steady market, the effect of bank lending policy on the housing market, and the over-rejection problem of unit root test when modelling house prices. Accordingly, this thesis aims to address the abovementioned problems. Notably, regime switching models are used to capture and forecast house prices in the UK. In particular, the LSTAR model is used to investigate the linkage between house price and bank lending policy, and the AR(1)-GARCH-M model is used to investigate the unit root test and estimate house price volatility.

Chapter 2 contributes to the literature by using a regime switching model to estimate house prices in a stable housing market. In the previous research, Crawford and Fratantoni (2003) found that the regime switching model has a good performance in the US housing market. Miles (2008) tested models that were complementary to Crawford and Fratantoni's. However, both indicated that the regime switching model captures large swings in house prices well, but failed to capture more stable markets. In addition, their research also found that the forecasting ability of the considered regime switching model is lower than expected. In contrast, the linear model shows more accuracy than nonlinear models. Therefore, in Chapter 2, we consider two regime switching models, the STAR and MS-TVTP models, for UK house prices as a complement to the above literature.

The empirical study in Chapter 2 uses UK housing market data due to the fact that UK house prices were moving gently most of the time in the last forty years (a stable market). In this chapter, we found that UK house prices exhibited asymmetric nonlinearity. We empirically compared the capacity of the MS-TVTP and STAR models to capture house price cycles. The results indicate that the LSTAR model is better fitted to capturing UK house price cycles than the MS-TVTP model. Furthermore, we test the LSTAR forecasting ability against an AR model. However, we found that the forecasting ability of LSTAR model does not outperform than the

AR model.

The findings of Chapter 2 have several important implications for house price researchers. First, the nonlinearity test suggests nonlinear models are better fitted to estimate house prices than a linear model. Second, we found the STAR model captures smooth house prices well. This, in turn, indicates that the regime switching model has a high capacity to estimate house prices. Last, the STAR model may not add much more forecastability in stable markets than linear models. We believe that the form of the STAR model provides a more real-world context than other forms. Therefore, econometricians may consider some adjustments when trying to predict house price cycles based on a STAR frame.

In Chapter 3, we empirically tested the linkage between the banking sector and house prices. Historically, house price collapse has linked to several financial crises, such as when the Hong Kong housing market bubble bust in 1997 following the start of the Asian financial crisis, and when the US sub-mortgage crisis led to a worldwide financial crisis. Since the housing market is extremely crucial to managing financial stability, housing market risk management has become an important function for many central banks. By way of illustration, Hungary, Norway, Sweden, the UK, and China have all introduced bank lending policies, such as imposing a maximum on the LTV ratio, to reduce the risk to housing markets and banking systems. In spite of the LTV ratio help in limiting housing bubble growth, econometric studies still leave some gaps that require further investigation of house prices, bank lending, and the LTV ratio. In the literature, some studies investigated the house price and bank lending by focusing on integration, or ,alternatively, by using linear models. Examples of this can be found in Koetter and Poghosyan (2010); Landier, Sraer, and Thesmar (2017); Mandell and Wilhelmsson (2015); Milcheva and Zhu (2016); and Tajik, Aliakbari, Ghalia, and Kaffash (2015). Some researchers also found that house prices influenced bank lending (see Gerlach and Peng, 2005). Considering that numerous studies and the last chapter of this thesis have found nonlinearity in house prices, we investigate the relationship between house prices and bank lending with a STAR model and between the LTV ratio and house prices with a GARCH in-mean model.

Firstly, the STAR model, introduced in Chapter 2, is considered once again in Chapter 3 due to the transition function illustrating that the transition variable leads to changes in the dependent variable. In other words, we can verify whether or not bank lending impacts house prices by estimating an adjusted STAR model. Secondly, the

GARCH in-mean model estimated with the LTV ratio as an explanatory variable and house price as the dependent variable measures the LTV ratio effect on house prices. Lastly, in order to verify the long-run equilibrium relationship between bank lending and house prices, we used a nonlinear TAR model to test co-integration.

The empirical test is applied to a dataset of Hong Kong house prices, bank lending, and the LTV ratio. The Hong Kong housing market experienced high frequency and large swings in the last three decades due to a severe supply-demand mismatch in the housing market and low financial immunity. On the other hand, bank lending moved closely with house prices in Hong Kong due to the large amount of bank lending in the real estate market. Meanwhile, the LTV ratio has been used as a key tool in housing market regulation. Against this background, the Hong Kong market is an ideal case for studying the linkage between bank lending and house prices.

Chapter 3 provides several important insights of cyclical patterns in house prices. Our main findings indicate that bank lending strongly impacts house prices in Hong Kong. More specifically, the LSTAR model illustrates that bank lending affects house price fluctuations. Furthermore, we find the strong ARCH effect in house prices and the GARCH-M model well describes the volatility of house prices. Our estimation also illustrates that the LTV ratio is significantly associated with house price fluctuation. In the TAR co-integration test, we find a long-run equilibrium relationship between house prices and bank lending as well as between house prices and the LTV ratio. This chapter provides important indications for academic researchers and policymakers. Since we found econometric support for the direction of influence going from bank lending and the LTV ratio to house prices, house prices studies or regulations can be set by more banking sectors.

Chapter 4 tests the nonlinear in-variance model. We used the GARCH in-mean model for investigating the robustness of unit root test in the presence of the in-mean term and structural breaks. The unit root test is a common test used in time series analysis. However, the unit root test over-rejects the null hypothesis, which has been indicated in a great deal of previous research. Kim and Schmidt (1993) first found the problem, and recent research, for example, that of Su (2011); Gospodinov and Tao (2011); Conrad and Karanasos (2015); Harvey, Leybourne, and Taylor (2011&2012); Narayan and Popp (2013); Atil, Fellag, and Sipols (2014); examined the size and power of unit root tests in structural breaks. This chapter follows the research in line with these studies, and we contribute to testing the size and power properties of two unit root tests in the case of the GARCH in-mean model. Our study shows how the in-mean parameters affect the unit root tests based on a Monte Carlo simulation. Furthermore, in the empirical test, we apply an AR(1)-GARCH-M model to estimate UK house prices with structural breaks.

The estimation results of this chapter indicate that the in-mean parameter severely affects the size and power properties of the DF unit root test and M-test. Specifically, when the in-mean parameter increased, the size and power properties heavily deviate from nominal significant level. From the empirical study, we find that the AR-GARCH-M model captures UK house prices well in the case of one break point. In addition, the in-mean parameter strongly affects the house prices. Overall, this chapter suggests that attention needs to be paid to the GARCH in-mean effect when analysing time series. Otherwise, unit root tests may lead to biased estimates.

All in all, this thesis investigates several nonlinear models and their application to the housing market. This research area mainly has three problems. Firstly, the forecasting accuracy of nonlinear models is lower than that of linear models. Secondly, some linkages between economics and finance can be verified in the real world, but it is difficult to find econometric support. Lastly, some mechanisms may be ignored during the estimation that may lead to biased estimates. Therefore, future work can be developed in several directions. For instance, the forecasting mechanism of nonlinear models can be reconsidered. Since the nonlinear model has a high capacity for time series modelling, we have a strong belief that nonlinear models could produce a better performance in forecasting. At the same time, the exploration and development of econometric models will continue, and the future will have more models and technologies to push social and academic research forward.

References

Abraham, J.M. and Hendershott, P.H. (1996) 'Bubbles in metropolitan housing markets', *Journal of Housing Research*, 7, pp. 191-206.

ALEXANDER, C. and BARROW, M. (1994) 'Seasonality and Cointegration of Regional House Prices in the Uk', *Urban Studies,* 31(10), pp. 1667-1689.

Atil, L., Fellag, H. and Garcia Sipols, A. (2014) 'Behavior of the Size in the Unit Root Testing Under Contamination', *Communications in Statistics-Simulation and Computation,* 43(4), pp. 790-803.

Attanasio, O.P., Blow, L., Hamiltonww, R. and Leicester, A. (2009) 'Booms and Busts: Consumption, House Prices and Expectations', *Economica,* 76(301), pp. 20-50.

Attanasio, O., Leicester, A. and Wakefield, M. (2011) 'Do House Prices Drive Consumption Growth? the Coincident Cycles of House Prices and Consumption in the Uk', *Journal of the European Economic Association*, 9(3), pp. 399-435.

Bai, J.S. and Perron, P. (1998) 'Estimating and testing linear models with multiple structural changes', *Econometrica*, 66(1), pp. 47-78.

Baille, R. and Chung, H. (2001) 'Estimation of larch models from the autocorrelations of the squares of a process', *Journal of Time Series Analysis,* 22(6), pp. 631-650.

Baillie, R.T. and Degennaro, R.P. (1990) 'Stock Returns and Volatility', *Journal of Financial and Quantitative Analysis*, 25(2), pp. 203-214.

Balcilar, M., Gupta, R. and Miller, S.M. (2015) 'The out-of-sample forecasting performance of nonlinear models of regional housing prices in the US', *Applied Economics,* 47(22), pp. 2259-2277.

Baldi, G. (2014) 'The economic effects of a central bank reacting to house price inflation', *Journal of Housing Economics,* 26, pp. 119-125.

Banerjee, A. (1999) 'Panel data unit roots and cointegration: An overview', *Oxford Bulletin of Economics and Statistics*, 61, pp. 607-629.

Bazzi, M., Blasques, F., Koopman, S.J. and Lucas, A. (2017) 'Time-Varying Transition Probabilities for Markov Regime Switching Models', *Journal of Time Series Analysis,* 38(3), pp. 458-478.

Bessec, M. and Bouabdallah, O. (2005) 'What causes the forecasting failure of Markov-switching models? A Monte Carlo study', *Studies in Nonlinear Dynamics and Econometrics,* 9(2), pp. 6.

Bollerslev, T. (1986) 'Generalized Autoregressive Conditional Heteroskedasticity', *Journal of Econometrics,* 31(3), pp. 307-327.

Brier, G. (1950) 'Verification of the forecasts Expressed in Terms of Probability', *Monthly Weather Review,* 78, pp. 1-3.

Brown, J., Song, H. and McGillivray, A. (1997) 'Forecasting UK house prices: a time varying coefficient approach', *Economic Modelling,* 14(4), pp. 529-548.

Cabrera, J., Wang, T. and Yang, J. (2011) 'Linear and Nonlinear Predictability of International Securitized Real Estate Returns: A Reality Check', *Journal of Real Estate Research,* 33(4), pp. 565-594.

Canepa, A. and Chini, E.Z. (2016) 'Dynamic asymmetries in house price cycles: A generalized smooth transition model', *Journal of Empirical Finance,* 37, pp. 91-103.

Cervera, J. and Munoz, J. (1996) 'Proper Scoring Rules for Fractiles', in Bernardo, J., Berger, J., Dawid, A. and Smith, A. (eds.) *Bayesian Statistics.* 5th edn. Oxford, UK: Oxford University Press, pp. 513-519.

Chan, H.L., Lee, S.K. and Woo, K.Y. (2001) 'Detecting rational bubbles in the residential housing markets of Hong Kong', *Economic Modelling,* 18(1), pp. 61-73.

CHAN, K. (1993) 'Consistency and Limiting Distribution of the Least-Squares Estimator of a Threshold Autoregressive Model', *Annals of Statistics,* 21(1), pp. 520-533.

Chang, K. (2010) 'House price dynamics, conditional higher-order moments, and density forecasts', *Economic Modelling,* 27(5), pp. 1029-1039.

Chen, N., Cheng, H. and Mao, C. (2014) 'Identifying and forecasting house prices: a macroeconomic perspective', *Quantitative Finance,* 14(12), pp. 2105-2120.

Conrad, C. and Karanasos, M. (2015) 'On the Transmission of Memory in Garch-in-Mean Models', *Journal of Time Series Analysis,* 36(5), pp. 706-720.

Cook, S. (2008) 'Maximum likelihood unit root testing in the presence of GARCH: A new test with increased power', *Communications in Statistics-Simulation and Computation,* 37(4), pp. 756-765.

Cook, S. and Vougas, D. (2009) 'Unit root testing against an ST-MTAR alternative: finite-sample properties and an application to the UK housing market', *Applied Economics,* 41(11), pp. 1397-1404.

Crawford, G. and Fratantoni, M. (2003) 'Assessing the forecasting performance of regime-switching, ARIMA and GARCH models of house prices', *Real Estate Economics,* 31(2), pp. 223-243.

Cunningham, R. and Kolet, I. (2011) 'Housing market cycles and duration dependence in the United States and Canada', *Applied Economics,* 43(5), pp. 569-586.

Dickey, D.A. and Fuller, W.A. (1979) 'Distribution of the Estimators for Autoregressive Time-Series with a Unit Root', *Journal of the American Statistical Association,* 74(366), pp. 427-431.

Dolde, W. and Tirtiroglu, D. (1997) 'Temporal and spatial information diffusion in real estate price changes and variances', *Real Estate Economics,* 25(4), pp. 539-565.

DRAKE, L. (1993) 'Modeling Uk House Prices using Cointegration - an Application of the Johansen Technique', *Applied Economics,* 25(9), pp. 1225-1228.

Dufrenot, G. and Malik, S. (2012) 'The changing role of house price dynamics over the business cycle', *Economic Modelling,* 29(5), pp. 1960-1967.

Enders, W. and Granger, C.W.J. (1998) 'Unit-root tests and asymmetric adjustment with an example using the term structure of interest rates', *Journal of Business & Economic Statistics,* 16(3), pp. 304-311.

Enders, W. and Siklos, P. (2001) 'Cointegration and threshold adjustment', *Journal of Business & Economic Statistics,* 19(2), pp. 166-176.

ENGLE, R. (1982) 'Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United-Kingdom Inflation', *Econometrica,* 50(4), pp. 987-1007.

Epstein, E. (1969) 'A scoring system for probability forecasts of ranked categories', *Journal of Applied Meteorology,* 8, pp. 985-987.

Filardo, A.J. (1994) 'Business-Cycle Phases and their Transitional Dynamics', *Journal of Business & Economic Statistics,* 12(3), pp. 299-308.

Funke, M. and Paetz, M. (2013) 'Housing prices and the business cycle: An empirical application to Hong Kong', *Journal of Housing Economics,* 22(1), pp. 62-76.

Garcia, R. and Perron, P. (1996) 'An analysis of the real interest rate under regime shifts', *Review of Economics and Statistics,* 78(1), pp. 111-125.

Gerlach, S. and Peng, W. (2005) 'Bank lending and property prices in Hong Kong', *Journal of Banking & Finance,* 29(2), pp. 461-481.

Gete, P. and Reher, M. (2016) 'Two Extensive Margins of Credit and Loan-to-Value Policies', *Journal of Money Credit and Banking,* 48(7), pp. 1397-1438.

Gimeno, R. and Martinez-Carrascal, C. (2010) 'The relationship between house prices and house purchase loans: The Spanish case', *Journal of Banking & Finance,* 34(8), pp. 1849-1855.

Giussani, B. and Hadjimatheou, G. (1991) 'Modeling regional house prices in the UK', *Regional Science,* 70(2), pp. 201-219.

Good, I. (1952) 'Rational Decisions', *Journal of Royal Statistical Society,* B(14), pp. 107-114.

Gospodinov, N. and Tao, Y. (2011) 'Bootstrap Unit Root Tests in Models with GARCH(1,1) Errors', *Econometric Reviews*, 30(4), pp. 379-405.

Guirguis, H., Giannikos, C. and Anderson, R. (2005) 'The US housing market: Asset pricing forecasts using time varying coefficients', *Journal of Real Estate Finance and Economics,* 30(1), pp. 33-53.

Haldrup, N. (1994) 'Heteroscedasticity in Nonstationary Time-Series, some Monte-Carlo Evidence', *Statistical Papers,* 35(4), pp. 287-307.

Hamilton, J.D. (1994) *Time Series Analysis.* Princeton, New Jersey.: Princeton University Press.

Hamilton, J.D. (1989) 'A New Approach to the Economic-Analysis of Nonstationary Time-Series and the Business-Cycle', *Econometrica,* 57(2), pp. 357-384.

Harvey, D.I., Leybourne, S.J. and Taylor, A.M.R. (2014) 'On infimum Dickey-Fuller unit root tests allowing for a trend break under the null', *Computational Statistics & Data Analysis,* 78, pp. 235-242.

Harvey, D.I., Leybourne, S.J. and Taylor, A.M.R. (2012) 'Unit root testing under a local break in trend', *Journal of Econometrics,* 167(1), pp. 140-167.

Hendry, D. (1984) 'Econometric modelling of house prices in the United Kingdom', in Hendry, D. and Wallis, K. (eds.) *Econometrics and Quantitative Economics.* Basil Blackwell, pp. 211-252.

Holly, S. and Jones, N. (1997) 'House prices since the 1940s: cointegration, demography and asymmetries', *Economic Modelling,* 14(4), pp. 549-565.

Holly, S., Pesaran, M.H. and Yamagata, T. (2011) 'The spatial and temporal diffusion of house prices in the UK', *Journal of Urban Economics,* 69(1), pp. 2-23.

Holly, S., Pesaran, M.H. and Yamagata, T. (2010) 'A spatio-temporal model of house prices in the USA', *Journal of Econometrics,* 158(1), pp. 160-173.

Huang, M. (2012) 'Forecasts and implications of the current housing crisis: switching regimes in a threshold framework', *Applied Economics Letters,* 19(6), pp. 557-568.

Hui, E.C. and Zheng, X. (2012) 'The dynamic correlation and volatility of real estate price and rental: an application of MSV model', *Applied Economics,* 44(23), pp. 2985-2995.

Igan, D. and Loungani, P. (2012) *Global Housing Cycles.* Washington, DC: International Monetary Fund.

Ihlanfeldt, K. and Mayock, T. (2014) 'Housing Bubbles and Busts: The Role of Supply Elasticity', *Land Economics,* 90(1), pp. 79-99.

Johansen, S. (1996) *Likelihood-based inference in cointegrated vector autore-gressive models.* Oxford: Oxford University Press.

Kim, C. and Nelson, C.R. *State-Space Models With Regime Switching.* Cambridge: The MIT Press.

Kim, J.R. and Chung, K. (2016) 'House prices and business cycles: The case of the UK', *International Area Studies Review,* 19(2), pp. 131-146.

Kim, K.W. and Schmidt, P. (1993) 'Unit-Root Tests with Conditional Heteroskedasticity', *Journal of Econometrics,* 59(3), pp. 287-300.

Kim, S. and Bhattacharya, R. (2009) 'Regional Housing Prices in the USA: An Empirical Investigation of Nonlinearity', *Journal of Real Estate Finance and Economics,* 38(4), pp. 443-460.

Koetter, M. and Poghosyan, T. (2010) 'Real estate prices and bank stability', *Journal of Banking & Finance,* 34(6), pp. 1129-1138.

Landier, A., Sraer, D. and Thesmar, D. (2017) 'Banking integration and house price co-movement', *Journal of Financial Economics,* 125(1), pp. 1-25.

Leamer, E.E. (2007) 'Houseing IS the Business Cycle', *NBER Working Paper,* 13428.

Leung, C.K.Y. (2014) 'Error correction dynamics of house prices: An equilibrium benchmark', *Journal of Housing Economics,* 25, pp. 75-95.

Li, Y. and Shukur, G. (2011) 'Wavelet Improvement of the Over-Rejection of Unit Root Test Under GARCH Errors: An Application to Swedish Immigration Data', *Communications in Statistics-Theory and Methods,* 40(13), pp. 2385-2396.

Ling, S. and Li, W. (1998) 'Limiting distributions of maximum likelihood estimators for unstable autoregressive moving-average time series with general autoregressive heteroscedastic errors', *Annals of Statistics,* 26(1), pp. 84-125.

Mak, S., Choy, L. and Ho, W. (2010) 'Quantile Regression Estimates of Hong Kong Real Estate Prices', *Urban Studies,* 47(11), pp. 2461-2472.

Mandell, S. and Wilhelmsson, M. (2015) 'Financial infrastructure and house prices', *Applied Economics,* 47(30), pp. 3175-3188.

Mayes, D. (1979) *The Property Boom: The Effects of Building Society Behaviour on House Prices.* Martin Robertson.

McKay, A. and Reis, R. (2016) 'The Role of Automatic Stabilizers in the US Business Cycle', *Econometrica,* 84(1), pp. 141-194.

Meen, G. (1996) 'Spatial aggregation, spatial dependence and predictability in the UK housing market', *Housing Studies,* 11(3), pp. 345-372.

Milcheva, S. and Zhu, B. (2016) 'Bank integration and co-movements across housing markets', *Journal of Banking & Finance,* 72, pp. S148-S171.

Miles, W. (2008) 'Boom-bust cycles and the forecasting performance of linear and non-linear models of house prices', *Journal of Real Estate Finance and Economics,* 36(3), pp. 249-264.

Miles, W. (2011) 'Long-Range Dependence in U.S. Home Price Volatility', *Journal of Real Estate Finance and Economics,* 42(3), pp. 329-347.

Miles, W. (2008) 'Volatility clustering in US home prices', *Journal of Real Estate Research,* 30(1), pp. 73-90.

Miller, N. and Peng, L. (2006) 'Exploring metropolitan housing price volatility', *Journal of Real Estate Finance and Economics,* 33(1), pp. 5-18.

Monkkonen, P., Wong, K. and Begley, J. (2012) 'Economic restructuring, urban growth, and short-term trading: The spatial dynamics of the Hong Kong housing market, 1992-2008', *Regional Science and Urban Economics,* 42(3), pp. 396-406.

Muellbauer, J. and Murphy, A. (1997) 'Booms and busts in the UK housing market', *Economic Journal,* 107(445), pp. 1701-1727.

Narayan, P.K. and Popp, S. (2013) 'Size and power properties of structural break unit root tests', *Applied Economics,* 45(6), pp. 721-728.

Nneji, O., Brooks, C. and Ward, C.W.R. (2015) 'Speculative Bubble Spillovers across Regional Housing Markets', *Land Economics,* 91(3), pp. 516-535.

Nneji, O., Brooks, C. and Ward, C.W.R. (2013) 'House price dynamics and their reaction to macroeconomic changes', *Economic Modelling,* 32, pp. 172-178.

Oikarinen, E. (2009) 'Interaction between housing prices and household borrowing: The Finnish case', *Journal of Banking & Finance,* 33(4), pp. 747-756.

Otero-Gonzalez, L., Duran-Santomil, P., Lado-Sestayo, R. and Vivel-Bua, M. (2016) 'The impact of loan-to-value on the default rate of residential mortgage-backed securities', *Journal of Credit Risk,* 12(3), pp. 1-13.

Park, B. and Bae, J.K. (2015) 'Using machine learning algorithms for housing price prediction: The case of Fairfax County, Virginia housing data', *Expert Systems with Applications,* 42(6), pp. 2928-2934.

Peel, D. and Speight, A. (1998) 'Modelling business cycle nonlinearity in conditional mean and conditional variance: Some international and sectoral evidence', *Economica,* 65(258), pp. 211-229.

Perlin, M. *MS Regress - The MATLAB Package for Markov Regime Switching Models.* Available at: http://ssrn.com/abstract=1714016; http://dx.doi.org/10.2139/ssrn.1714016 .

Perron, P. and Ng, S. (1996) 'Useful modifications to some unit root tests with dependent errors and their local asymptotic properties', *Review of Economic Studies,* 63(3), pp. 435-463.

Qi, M. and Yang, X. (2009) 'Loss given default of high loan-to-value residential mortgages', *Journal of Banking & Finance,* 33(5), pp. 788-799.

Ricci-Risquete, A., Ramajo, J. and de Castro, F. (2016) 'Do Spanish fiscal regimes follow the euro-area trends? Evidence from Markov-Switching fiscal rules', *Economic Modelling,* 59, pp. 484-494.

Seo, B. (1999) 'Distribution theory for unit root tests with conditional heteroskedasticity', *Journal of Econometrics,* 91(1), pp. 113-144.

Simo-Kengne, B.D., Balcilar, M., Gupta, R., Reid, M. and Aye, G.C. (2013) 'Is the relationship between monetary policy and house prices asymmetric across bull and bear markets in South Africa? Evidence from a Markov-switching vector autoregressive model', *Economic Modelling,* 32, pp. 161-171.

Skalin, J. and Terasvirta, T. (2002) 'Modeling asymmetries and moving equilibria in unemployment rates', *Macroeconomic Dynamics,* 6(2), pp. 202-241.

Skalin, J. and Terasvirta, T. (1999) 'Another look at Swedish business cycles, 1861-1988', *Journal of Applied Econometrics,* 14(4), pp. 359-378.

Stock, J.H. and Watson, M.W. 'A comparison of linear and nonlinear univariate models for forecasting macroeconomic time series', *NBER Working Paper*, (6607).

Su, J. (2011) 'On the Oversized Problem of Dickey-Fuller-Type Tests with GARCH Errors', *Communications in Statistics-Simulation and Computation,* 40(9), pp. 1364-1372.

Tajik, M., Aliakbari, S., Ghalia, T. and Kaffash, S. (2015) 'House prices and credit risk: Evidence from the United States', *Economic Modelling,* 51, pp. 123-135.

Tashman, J. (2000) 'Out-of-sample tests of forecasting accuracy: an analysis and review', *International Journal of Forecasting,* 16(4), pp. 437-450.

Taylor, S. (1986) *Modelling Financial Time Series.* New York: John Wiley & Sons.

TERASVIRTA, T. (1998) 'Modelling economic relationships with smooth transition regressions', in *Handbook of Applied Economic Statistics.* , pp. 507-552.

TERASVIRTA, T. (1994) 'Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models', *Journal of the American Statistical Association,* 89(425), pp. 208-218.

TERASVIRTA, T. and ANDERSON, H. (1992) 'Characterizing Nonlinearities in Business Cycles using Smooth Transition Autoregressive Models', *Journal of Applied Econometrics,* 7, pp. S119-S136.

Terasvirta, T., van Dijk, D. and Medeiros, M. (2005) 'Linear models, smooth transition autoregressions, and neural networks for forecasting macroeconomic time series: A re-examination', *International Journal of Forecasting,* 21(4), pp. 755-774.

Tillmann, P. (2015) 'Estimating the effects of macroprudential policy shocks: A Qual VAR approach', *Economics Letters,* 135, pp. 1-4.

Tsai, I., Chen, M. and Ma, T. (2010) 'Modelling house price volatility states in the UK by switching ARCH models', *Applied Economics,* 42(9), pp. 1145-1153.

Tsai, I., Lee, C. and Chiang, M. (2012) 'The Asymmetric Wealth Effect in the US Housing and Stock Markets: Evidence from the Threshold Cointegration Model', *Journal of Real Estate Finance and Economics,* 45(4), pp. 1005-1020.

Valkanov, R. (2005) 'Functional Central Limit Theorem approximations and the distribution of the Dickey-Fuller test with strongly heteroskedastic data', *Economics Letters,* 86(3), pp. 427-433.

Wong, E., Tsang, A. and Kong, S. (2014) 'How Does Loan-To-Value Policy Strengthen Banks’ Resilience to Property Price Shocks – Evidence from Hong Kong', *HKIMR Working Paper,* No.03/2014

Yang, J., Liu, H. and Leatham, D.J. (2013) 'The multi-market analysis of a housing price transmission model', *Applied Economics,* 45(27), pp. 3810-3819.