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An Asymmetric Duopoly Model of Price Framing

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Abstract:
This note considers an asymmetric duopoly model of price-frame competition in homogeneous product markets. The firms choose simultaneously prices and price formats, and frame differentiation limits price comparability leading to consumer confusion. Here, one firm is more salient than its rival and attracts a larger share of confused consumers. In duopoly equilibrium, the firms randomize on both prices and frames, make strictly positive profits, and pricing is frame-independent. However, the prominent firm sets a higher average price and charges the monopoly price with positive probability. Higher prominence boosts expected profit for both the industry and the salient firm but may harm the rival's expected profit.

Keywords: price framing, price dispersion, imperfect competition

JEL classification: D03, D43, L13

DOI: 10.1515/bejte-2018-0068

1 Introduction

In many homogeneous product markets firms' prices are presented in differentiated formats. In grocery stores, the same fruits or vegetables are priced 'per unit' or 'per kg'. Some service providers quote a total price, while others quote V.A.T. separately. Supermarket promotions are framed as direct price reductions, percentage discounts, or volume discounts. Format differentiation may limit price comparability and lead to consumer confusion or inertia. Confused consumers may uphold their default options, make mistakes, or restrict their consideration sets. Markets with differentiated price formats often exhibit both price frame and price dispersion, despite product homogeneity.

Theoretical models of price-frame competition mainly focus on symmetric settings where firms share confused (or default-biased) consumers equally. Piccione and Spiegler (2012) formulate a duopoly model with a general price frame comparability structure, while Chioveanu and Zhou (2013) analyze price-frame competition in oligopoly markets and its competition policy implications. As an exception, Spiegler (2011, Section 10.4) considers a polar case where all consumers are initially assigned to one firm and uphold their default option when frame differentiation blocks comparisons.[1]

When framing limits price comparability, confused consumers' choices are likely to be affected by firm prominence. Heavily advertised products enjoy higher recognition, which may influence consumers' choices when they cannot compare prices. Incumbent products are more salient and consumers who are familiar with them may be less likely to switch to alternative products. Other sources of prominence include word-of-mouth, social network recommendations, institutional default options, and preferential location in stores.[2]

This analysis allows for arbitrary prominence levels in duopoly markets where homogeneous product sellers compete by choosing simultaneously both prices and one of two price frames. One frame is simple, while the other can be simple or complex. Market composition is determined endogenously. When the firms choose different frames, or when both choose a common complex frame, some consumers get confused. When both firms choose the simple frame, all consumers can compare prices (i.e., are rational). It is assumed that frame differentiation is the main source of confusion, that is, it leads to a larger share of confused consumers than frame complexity. Confused consumers cannot compare prices and make random choices, but are more likely to select the prominent product. Rational consumers buy from the lowest price firm.

There is no equilibrium where the firms choose pure price framing strategies. As the firms face heterogeneous consumers, there is no equilibrium where they choose pure price strategies either.[3] I characterize the mixed strategy duopoly Nash equilibrium and show that both firms randomize on prices and frames. Like in the symmetric duopoly model, where the firms share equally the confused consumers, in equilibrium the firms' price framing strategies are independent of pricing. In my asymmetric model, the pricing strategies and the expected profits depend on the degree of prominence, and so are firm-dependent, but framing strategies

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are firm-independent. The salient firm chooses a higher expected price, has an atom at the highest price equal to consumers' reservation value (so it is less likely to offer price promotions), and makes higher expected profit than the rival.

This asymmetric model highlights how arbitrary levels of prominence affect the firms' equilibrium strategies and market outcomes. Specifically, an increase in prominence leads to increases in the salient firm's probability of choosing the highest price, in both firms' expected prices, in the lower bound of firms' price distributions, and in the expected profits of the salient firm and the industry. The less salient firm's expected profit may decrease in the level of prominence. These insights complement the polar case where the prominent firm attracts all confused consumers presented in Spiegler (2011, Section 10.4).

My analysis is also related to Gu and Wenzel (2014), who study prominence in a duopoly model where the firms commit to complexity levels before competing in prices. This is relevant, for instance, when changes in tariff design take time. They show that in equilibrium the firms choose deterministic complexity levels, but randomize in prices, and that consumer protection policy may have undesired effects by inducing the less salient firm to increase its complexity level. Their two-stage game is a sequential version of Carlin (2009). In these models, price complexity (rather than format differentiation) is the main source of confusion. This is also the case in Chioveanu (2017) where, unlike the current framework, consumer surplus is not monotonic in the degree of prominence. [4]

2 Model

Two firms supply a homogeneous product to a unit mass of consumers. Each consumer demands at most one unit and has reservation value $v = 1$. The marginal production costs are normalized to zero. The firms choose simultaneously and noncooperatively prices, p_1 and p_2, and price frames, z_1 and z_2. The timing reflects the fact that the firms are able to change the price formats as frequently as they change the prices. Differentiation in frames limits price comparability and leads to consumer confusion (or inertia). Each firm can choose one of two frames, A and B. Frame A is simple (e.g., an all-inclusive price), and B is a different and possibly more complex frame (e.g., a partitioned price). A frame is simple if two prices in this frame are perfectly comparable. A frame is complex if some consumers cannot compare two prices presented in this frame. Frame B may also be simple (but different from A): say, A is "price per unit" and B is "price per kilogram".

If firms choose different frames, a share $\alpha(A, B) \equiv \alpha_H > 0$ of the consumers get confused, so cannot compare prices, while a share $1 - \alpha_H$ of the consumers are rational and purchase the cheapest product with positive net surplus. If both firms choose A, all consumers are rational, that is, $\alpha(A, A) \equiv \alpha_0 = 0$. If both firms choose B, a share $\alpha(B, B) \equiv \alpha_L \geq 0$ of consumers get confused, while a share $1 - \alpha_L$ are rational. If $\alpha_L = 0$, all consumers are rational. Simple frame A can cause confusion only when combined with frame B, whereas if $\alpha_L > 0$, B is confusing itself and can limit comparability even if both firms choose it. In this model, frame differentiation is more confusing than frame complexity, i.e. $\alpha_H > \alpha_L$.

This analysis focuses on the interaction between price framing and firm prominence. Let firm 1 be more prominent than its rival: confused consumers are more likely to purchase its product, and so are rational consumers if the firms' prices are equal. A fraction $\sigma \in (1/2, 1)$ of the confused consumers buy from firm 1, while fraction $1 - \sigma$ buy from firm 2. If both firms offer equal prices, a fraction $\sigma \in (1/2, 1)$ of the rational consumers buy from firm 1 and $1 - \sigma$ buy from firm 2.[5]

Firm i's profit is

$$\pi_i(p_i, p_j, z_i, z_j) = p_i \cdot [\sigma_i \alpha(z_i, z_j) + q_i(p_i, p_j)(1 - \alpha(z_i, z_j))] \,,$$

where σ_i is firm i's prominence level (with $\sigma_1 = \sigma$ and $\sigma_2 = 1 - \sigma$), $\alpha(z_i, z_j)$ is the total share of confused consumers, and firm i's share of rational consumers is

$$q_i(p_i, p_j) = \begin{cases} 1, & \text{if } p_i < \min\{p_j, 1\} \\ \sigma_i, & \text{if } p_i = p_j \leq 1 \\ 0, & \text{if } p_i > \min\{p_j, 1\} \end{cases} \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j \,. \tag{1}$$

It is assumed that the confused consumers do not pay more than their reservation price and their choices are independent of prices. Piccione and Spiegler (2012) and Chioveanu and Zhou (2013) discuss these assumptions and the microfoundations of the price-frame competition model.

3 Analysis

The following result characterizes the unique mixed strategy pricing equilibrium in a duopoly model where firm 1 serves a larger fraction ($\sigma > 1/2$) of an exogenously given share of confused consumers ($\alpha \in (0,1)$) and is recalled throughout this analysis.

Proposition 1

(Narasimhan, 1988) *In the unique Nash equilibrium firms randomize on prices in $[p_0, 1]$, with p_0 implicitly defined below, and make expected profits*

$$\pi_1(\alpha) = \sigma\alpha = p_0\left[1-(1-\sigma)\alpha\right] \text{ and } \pi_2(\alpha) = \frac{\sigma\alpha(1-\sigma\alpha)}{1-(1-\sigma)\alpha} = p_0\left(1-\sigma\alpha\right). \tag{2}$$

The firms' pricing c.d.f.s are

$$F_1(p,\alpha) = \frac{1-\sigma\alpha}{1-\alpha} - \frac{\sigma\alpha(1-\sigma\alpha)}{(1-\alpha)\left[1-(1-\sigma)\alpha\right]}\frac{1}{p} \text{ and } F_2(p,\alpha) = 1 - \frac{\sigma\alpha}{1-\alpha}\frac{1-p}{p}. \tag{3}$$

Firm 2's pricing c.d.f. is continuous everywhere, while firm 1's pricing c.d.f. is continuous on $[p_0, 1)$and has an atom at $p = 1$,

$$\phi(\alpha) = 1 - F_1(1) = \frac{(2\sigma-1)\alpha}{1-(1-\sigma)\alpha} \in (0,1). \tag{4}$$

Lemma 1

There is no equilibrium where the duopolists use pure price framing strategies.

Proof

(a) Suppose that both firms choose A. The unique candidate equilibrium entails marginal-cost pricing and zero profit. If firm i unilaterally deviates to B and price $p_i \in (0,1]$, it makes positive profit. A contradiction.

(b) Suppose that both firms choose B. As $\alpha_L < \alpha_H \leq 1$, Proposition 1 applies and the profits in the candidate equilibrium are given by eq. (2) when $\alpha = \alpha_L$.[6] But, if firm 1 deviates to A and price $p_1 = 1$, it makes higher profit $\sigma\alpha_H > \sigma\alpha_L$. A contradiction.

(c) Suppose that firms i and j choose A and B, respectively. Consider two cases. (c1) When $\alpha_H = 1$, in the unique candidate equilibrium $p_1 = p_2 = 1$, $\pi_1 = \sigma$ and $\pi_2 = 1 - \sigma$. But, as $\sigma < 1$, if firm j deviates to A and $p_j = 1 - \varepsilon$, its profit is $1 - \varepsilon > \sigma_j$ for $\forall \varepsilon < 1 - \sigma_j$. A contradiction. (c2) When $\alpha_H < 1$, Proposition 1 applies and profits in the candidate equilibrium are given by eq. (2) when $\alpha = \alpha_H$. But, as $\sigma < 1$, if firm j deviates to A and price $p_j = p_0$, its profit is $p_0 > p_0\left[1-(1-\sigma)\alpha_H\right]$. A contradiction.[7]

By Lemma 1, in any candidate equilibrium at least one firm randomizes on price frames.[8] Therefore, with positive probability the firms face both rational and confused consumers. The next result follows from Proposition 1.

Lemma 2

There is no equilibrium where the duopolists use pure pricing strategies.

By Lemmas 1 and 2, any Nash equilibrium must exhibit dispersion in both prices and frames. Firm i's strategy space is $[0,1] \times \{A,B\}$. Denote by $\xi_i \equiv \xi_i(p_i, z_i)$ firm i's mixed strategy over prices and frames, for $i = 1,2$. ξ_i is a bivariate c.d.f. where $F_i(p_i)$ is the marginal c.d.f. of firm i's random price defined on a support $T_i \subseteq [0,1]$, while $\lambda_i(p_i) = prob\left[z_i = A \mid p_i\right]$ and $1 - \lambda_i(p_i) = prob\left[z_i = B \mid p_i\right]$ are firm i's probabilities of using frame A and, respectively, frame B at price p_i. Firm i's overall probability of using A is $\lambda_i = \int_{p_i \in T_i} \lambda_i(p_i) dF_i(p_i)$. For ξ_i to be well-defined, $F_i(p_i)$ should be weakly increasing on T_i and $\lambda_i(p_i) \in [0,1]$ for all $p_i \in T_i$.

Suppose that firm $j \neq i$ follows ξ_j, so it draws its price from $F_j(p_j)$. Firm i's expected profit at price p_i and frame z_i is

$$\begin{aligned}\pi_i\left(p_i, z_i\right) &= p_i \int_{p_j > p_i} \left[\lambda_j(p_j)(1-\alpha(z_i,A)) + (1-\lambda_j(p_j))(1-\alpha(z_i,B))\right] dF_j(p_j) + \\ &\quad p_i\sigma_i \int_{p_j \in T_j} \left[\lambda_j(p_j)\alpha(z_i,A) + (1-\lambda_j(p_j))\alpha(z_i,B)\right] dF_j(p_j).\end{aligned} \tag{5}$$

The first integral gives firm i's share of rational consumers, while the second gives the overall share of confused consumers. The first term in square brackets represents the expected share of rational consumers for a given realization of p_j. With probability $\lambda_j(p_j)$, firm j uses A and the expected share of rational consumers is $(1-\alpha(z_i, A))$; with probability $1-\lambda_j(p_j)$, firm j uses B and the expected share of rational consumers is $(1-\alpha(z_i, B))$. The second term in square brackets represents the expected share of confused consumers for a given realization of p_j. As j's price p_j is stochastic, these shares are integrated over all p_j's. Firm i serves the rational consumers if its rival's price is higher ($p_j > p_i$) and serves a share σ_i of confused consumers for all p_j's. Equation (5) applies for any price p_i, as $F_j(p_j) = 0$ for $p_j \leq \inf T_j$ and $F_j(p_j) = 1$ for $p_j \geq \sup T_j$.

Lemmata 3 - 6 in the appendix explore properties of the pricing c.d.f.s, and show that both firms choose prices according to c.d.f.s defined on a common interval $T = [p_0, 1]$ and continuous everywhere except possibly at $p = 1$.

Using eq. (5) for $z_i = A$ and $z_i = B$, firm i's incremental profitability of switching from frame A to B is

$$\begin{aligned} \pi_i(p_i, B) - \pi_i(p_i, A) &= p_i \Delta_i(p_i)\text{ , where} \\ \Delta_i(p_i) &= -\int_{p_i}^{1} \Lambda_j(p_j) dF_j(p_j) + \sigma_i \int_{p_0}^{1} \Lambda_j(p_j) dF_j(p_j) \text{ and} \\ \Lambda_j(p_j) &= \left[-(\alpha_H - \alpha_L) + (2\alpha_H - \alpha_L)\lambda_j(p_j)\right] . \end{aligned} \tag{6}$$

Firm i is indifferent between frames A and B at price p_i (i.e., $\lambda_i(p_i) \in (0,1)$) iff $\pi_i(p_i, B) = \pi_i(p_i, A)$. Equation (6) also indicates that if $\Lambda_j(p_j) = 0$ for all p_j's then $\pi_i(p_i, B) = \pi_i(p_i, A)$ for any p_i, where

$$\Lambda_j(p_j) = 0 \Leftrightarrow \lambda_j(p_j) = \frac{\alpha_H - \alpha_L}{2\alpha_H - \alpha_L} \equiv \lambda \in (0,1) . \tag{7}$$

Let $F_i^{z_i}(p_i) = F_i(p_i) \cdot prob\left[z_i \mid p_i\right]$ be firm i's price distribution conditional on using frame z_i. I focus on equilibria where the support of $F_i^{z_i}(p_i)$ is a connected interval, for $i = 1,2$ and $z_i = A, B$. This requires that $prob\left[z_i \mid p_i\right] \in (0,1) \Leftrightarrow \Delta_i(p_i) = 0$ and $d\Delta_i(p_i)/dp_i = 0$ for all $p_i \in T$.

Using eq. (6), Leibnitz's rule implies that

$$\begin{aligned} d\Delta_i(p_i)/dp_i &= \left[-(\alpha_H - \alpha_L) + (2\alpha_H - \alpha_L)\lambda_j(p_i)\right] f_j(p_i) \text{ so} \\ d\Delta_i(p_i)/dp_i &= 0 \Leftrightarrow \lambda_j(p_i) = \lambda \text{ for } all\ p_i \in T. \end{aligned}$$

Proposition 2

In any duopoly equilibrium with connected supports, there is price-frame independence. At any price $p \in T$, firm i (i = 1,2) chooses frame A with probability $\lambda_i(p) = \lambda$ and frame B with probability $(1-\lambda)$, where λ is defined in eq. (7).

Denote the equilibrium expected share of confused consumers by

$$\alpha^e \equiv (1-\lambda)^2 \alpha_L + 2\lambda(1-\lambda)\alpha_H = \frac{\alpha_H^2}{2\alpha_H - \alpha_L} \in \left[\frac{\alpha_H}{2}, \alpha_H\right) . \tag{8}$$

Substituting for λ into eq. (5) and using eq. (8), firm i 's expected profit at price p becomes

$$\pi_i(p) = p\left[(1-\alpha^e)(1-F_j(p)) + \sigma_i \alpha^e\right] . \tag{9}$$

The analysis of firms' pricing strategies echoes Narasimhan (1988). The next result follows from Proposition 1 by using the expected share of confused consumers in eq. (8).

Proposition 3

There exists a unique mixed-strategy equilibrium where each firm adopts frame A with probability λ given in eq. (7) *and frame B with probability $1-\lambda$. The expected share of confused consumers is given by α^e in eq.* (8). *Using eqs.* (2), (3), *and* (4), *the firms' pricing distributions, defined on $[p_0^*, 1]$ with $p_0^* = \sigma\alpha^e / \left[1-(1-\sigma)\alpha^e\right]$, and firm 1's atom at $p = 1$ are*

$$F_1^*(p) = F_1^*(p, \alpha^e), F_2^*(p) = F_2^*(p, \alpha^e), \textit{and } \phi^* = \phi(\alpha^e),$$

while the expected profits are

$$\pi_1^* = \pi_1(\alpha^e) \textit{ and } \pi_2^* = \pi_2(\alpha^e).$$

In the duopoly equilibrium each firm's pricing is frame-independent, but firm-dependent due to prominence. As $\alpha_H > \alpha_L$, when a firm switches from frame A to B, more (less) consumers get confused if the rival uses A (B). Therefore, there is no obvious monotonic relationship between the prices associated with A and B. The value of λ in Proposition 2 equates the expected total number of confused consumers when a firm uses frame A and when it uses frame B, i.e. $(1-\lambda)\alpha_H = \lambda\alpha_H + (1-\lambda)\alpha_L$.

The prominent firm makes higher profits than the rival ($\pi_1^* > \pi_2^*$), puts positive probability on the monopoly price ($\phi^* > 0$), and charges higher average price ($F_2^*(p) > F_1^*(p)$).[9] As it serves a larger fraction of the expected share of confused consumers, it has weaker incentives to offer price promotions. Hence, in markets with price-frame competition, a prominent firm is less likely to offer promotions. Comparative statics results are presented below.

Corollary 1

(i) ϕ^* strictly increases in σ and α^e. (ii) p_0^* strictly increases, so the range of prices strictly decreases, in σ and α^e. (iii) $\exp(p_2) = \exp(p_1 \mid p_1 < 1)$ *and* $\exp(p_1)$ *strictly increase in σ and α^e.*

Total welfare is normalized to one, so an increase in expected industry profit (π^*) corresponds to an equal reduction in expected consumer surplus. Using eq. (2),

$$\pi^* = \pi_1^* + \pi_2^* = \pi^* = \sigma\alpha^e \frac{2-\alpha^e}{1-(1-\sigma)\alpha^e} = \alpha^e + \frac{(2\sigma-1)(1-\alpha^e)\alpha^e}{1-(1-\sigma)\alpha^e}\,.$$

Corollary 2

***Profits and Prominence**. π^* and π_1^* strictly increase in σ. π_2^* may increase or decrease in σ depending on the confusion level.*

When $\alpha^e \leq 0.382$, π_2^* increases in σ; when $\alpha^e > 0.66$, it decreases in σ; and when $\alpha^e \in (0.382, 0.66]$ it increases in σ for $\sigma \in (1/2, \bar{\sigma})$ and it decreases in σ for $\sigma \in (\bar{\sigma}, 1)$, where $\bar{\sigma} = \left[\sqrt{(1-\alpha^e)(2-\alpha^e)} - (1-\alpha^e)\right] / \alpha^e$.

Corollary 3

***Profits and Confusion**. π^* and π_1^* strictly increase in α^e. π_2^* may increase or decrease in α^e depending on the prominence level.*

When $\sigma \in (0.5{,}0.62]$, π_2^* increases in α^e. When $\sigma \in (0.62{,}1)$, π_2^* increases in α^e for $\alpha^e \in (0, \underline{\alpha}^e)$ and decreases in α^e for $\alpha^e \in (\underline{\alpha}^e, 1)$, where $\underline{\alpha}^e = 1/[\sigma + \sqrt{\sigma(2\sigma-1)}]$.

Spiegler (2011, Section 10.4) discusses a default-bias model where all consumers are initially assigned to one firm, which corresponds to $\sigma = 1$. There, the prominent firm serves all confused consumers, while the rival can only sell to rational consumers, and both firms randomize in prices and frames. The case with symmetric firms ($\sigma = 1/2$) is subsumed by Piccione and Spiegler (2012) and Chioveanu and Zhou (2013). These results can be derived from mine by letting $\sigma \to 1$ and $\sigma \to 1/2$, respectively. This note complements the existing literature by characterizing firms' strategies and market outcomes for intermediate prominence levels and by analyzing the related comparative statics.

Appendix

Lemma 3

The pricing supports, T_1 and T_2 are connected intervals (i.e., there are no gaps in either of them).

Proof

Let $T = T_1 \cap T_2$. (a) Suppose that there is a gap G in T. (i) Suppose that firm i does not have mass over G but firm j does. Let $p_a = \sup\{p \in T_i \mid p < \inf G\}$ and $p_b = \inf\{p \in T_i \mid p > \sup G\}$. Then $\pi_j(p, z_j, \xi_i)$ is strictly increasing in p for $p \in G$ and firm j is better off moving mass $F_j(p_b) - F_j(p_a)$ to p_b. A contradiction. (ii) Suppose that neither firm has mass over G. Let $p_c = \sup\{p \in (T_i \cup T_j) \mid p < \inf G\}$. (i) implies that $p_c \in T_i$ for $i = 1{,}2$. But then $F_i(p_c) = F_i(\inf G) = F_i(\sup G)$ and firm j is better off moving mass from p_c to $p' = \sup G$. A contradiction. (b) Suppose that there is a gap $G \subset T_i \setminus T$. There exists either p_a or p_b as defined in (a-i), or both. Firm i is better off moving mass either from p_a to $\sup G$ or from $\inf G$ to p_b. A contradiction.

Lemma 4

(i) Neither firm has an atom in the interior or at the lower bound of the rival's pricing support. (ii) Firm i cannot have an atom at the upper bound of T_j if firm j has an atom there.

Proof

(i) Suppose that firm j has an atom at some $p' \in T_j$ with $p' < \max T_j$. It must be that $p' \in T_i$, otherwise firm j would move the atom to a higher price. Then, if $\alpha(z_i, z_j) < 1$, firm i is better off deviating from (p, z_i) to $(p' - \epsilon, z_i')$ as there is a discrete increase in market share and only a marginal decrease in price. If $\alpha(z_i, z_j) = 1$, firm i is better off deviating from (p, z_i) to $(p' - \epsilon, z_i')$ for $\varepsilon > 0$ and $z_i' \neq z_i$.

(ii) Suppose that firm j has an atom at $p' = \max T_j$. If firm i also has an atom at p', then the deviation argument in (i) applies unchanged. So, both firms cannot have atoms at $\max T_j$.

Lemma 5

In equilibrium, it must be that $T_1 = T_2 = [p_0, p^h]$ *for* $p_0 < p^h \leq 1$.

Proof

Suppose that $\exists p' \in T_i$ s.t. $p' \notin T_j$. Let $A = \{p \in T_j \mid p > p'\}$. Suppose that $A \neq \emptyset$ and let $p'' = \min A$. Then, $\pi_i(p'', z_i, \xi_j) > \pi_i(p', z_i, \xi_j)$ as firm i does not lose market share when deviating from p' to p''. If $A = \emptyset$, it must be that $p' > \max T_j$. If $p' < 1$, a similar argument applies as $\pi_i(1, z_i, \xi_j) > \pi_i(p', z_i, \xi_j)$. If $p' = 1$ (i.e., $\max T_i = 1$), $\max T_j \leq 1$. Then, by Lemma 4, the firms cannot have an atom at $\max T_j$, so at least one firm can profitably deviate to $p' = 1$ from $p = \max T_j$. Therefore, it must be that if $p' \in T_i$ then $p' \in T_j$.

Lemma 6

In equilibrium, $\sup T_1 = \sup T_2 = 1$ *and* $p_0 > 0$.

Proof

Suppose that $\sup T_i < 1$. By Lemma 5, $\sup T_j = \sup T_i = p_h$. By Lemma 4, both firms cannot have atoms at p_h. Then, at least one firm serves only its confused consumer base at p_h and is better off charging $p = 1 > p_h$. A contradiction. By Lemma 1, at least one firm mixes on frames. Thus, the expected confused consumer share and the firms' profits when $p \to 1$ are strictly positive. The constant profit conditions for mixed strategy equilibrium then imply that $p_0 > 0$.

Proof of Corollary 1

(i) $\partial\phi^*/\partial\sigma = \alpha^e(2-\alpha^e)/\,[1-(1-\sigma)\alpha^e]^2 > 0$ and $\partial\phi/\partial\alpha^e = (2\sigma-1)/\,[1-(1-\sigma)\alpha^e]^2 > 0$.

(ii) $\partial p_0/\partial\sigma = \alpha^e(1-\alpha^e)/\,[1-(1-\sigma)\alpha^e]^2 > 0$ and $\partial p_0/\,\partial\alpha^e = \sigma/\,[1-(1-\sigma)\alpha^e]^2 > 0$.

(iii) Consider firm 2. $\exp(p_2) = \int_{p_0}^{1}(1-F_2(p_2))dp_2 + p_0 = -\sigma\alpha^e \ln p_0/(1-\alpha^e) - \sigma\alpha^e/(1-\alpha^e)(1-p_0) + p_0 = -\sigma\alpha^e \ln p_0/(1-\alpha^e)$.

Denoting $\sigma\alpha^e/\,(1-\alpha^e) \equiv \tau$, $\exp(p_2) = \tau\ln(1+1/\tau)$ with $\partial[\exp(p_2)]/\partial\tau > 0$ as $\tau > 0$.

$\partial[\exp(p_2)]/\partial\sigma = (\partial[\exp(p_2)]/\partial\tau)\,(\partial\tau/\partial\sigma) > 0$. $\partial[\exp(p_2)]/\partial\alpha^e = (\partial[\exp(p_2)]/\partial\tau)\,(\partial\tau/\partial\alpha^e) > 0$.

Consider firm 1. Note that $F_1(p)/\,F_1(1) = F_2(p)$ so $\exp\left(p_1 \mid p_1 < 1\right) = \exp\left(p_2\right)$.

$\exp(p_1) = \exp(p_2)F_1(1) + (1-F_1(1)) = \exp(p_2)(1-\phi^*) + \phi^*$. $\partial[\exp(p_1)]/\partial\sigma = (\partial[\exp(p_2)]/\partial\sigma)\,(1-\phi^*) - \exp(p_2)\,(\partial\phi^*/\partial\sigma) + \partial\phi^*/\partial\sigma = (\partial[\exp(p_2)]/\partial\sigma)\,(1-\phi^*) + (1-\exp(p_2))\,(\partial\phi^*/\partial\sigma) > 0$.

$\partial[\exp(p_1)]/\partial\alpha^e = (\partial[\exp(p_2)]/\partial\alpha^e)\,(1-\phi^*) - \exp(p_2)\,(\partial\phi^*/\partial\alpha^e) + \partial\phi^*/\partial\alpha^e = (\partial[\exp(p_2)]/\partial\alpha^e)\,(1-\phi^*) + (1-\exp(p_2))\,(\partial\phi^*/\partial\alpha^e) > 0$.

Proof of Corollary 2

$\partial\pi^*/\partial\sigma = \alpha^e(1-\alpha^e)(2-\alpha^e)/\,[1-(1-\sigma)\alpha^e]^2 > 0$ and $\partial\pi_1^*/\partial\sigma = \alpha^e > 0$.

$\partial\pi_2^*/\partial\sigma = \alpha^e[1-\alpha^e-2\sigma\alpha^e(1-\alpha^e)-\sigma^2\left(\alpha^e\right)^2]/\,[1-(1-\sigma)\alpha^e]^2$.

So $sign(\partial\pi_2^*/\partial\sigma) = sign[1-\alpha^e-2\sigma\alpha^e(1-\alpha^e)-\sigma^2\left(\alpha^e\right)^2]$. The term in square brackets is an inverted-U quadratic, with roots $\underline{\sigma} < 0$ and $\bar{\sigma} = \left[\sqrt{(1-\alpha^e)(2-\alpha^e)}-(1-\alpha^e)\right]/\,\alpha^e > 0$, and positive at the maximum. Then, π_2^* increases for $\sigma \in (\underline{\sigma}, \bar{\sigma})$ and decreases for $\sigma > \bar{\sigma}$. If $\bar{\sigma} \geq 1$, π_2^* increases for $\forall\ \sigma \in (1/2, 1)$. This happens when $\alpha^e \leq 0.382$. If $\bar{\sigma} < 1/2$, then π_2^* decreases for all $\sigma \in (1/2, 1)$. This happens for $\alpha^e > 0.66$. For $\alpha^e \in (0.382, 0.66]$, $\bar{\sigma} \in [1/2, 1)$ and π_2^* first increases and then decreases.

Proof of Corollary 3

$\partial\pi^*/\partial\alpha^e = \sigma\left[2(1-\alpha^e)+(1-\sigma)\left(\alpha^e\right)^2\right]/\,[1-(1-\sigma)\alpha^e]^2 > 0$ and $\partial\pi_1^*/\partial\alpha^e = \sigma > 0$.

$\partial\pi_2^*/\partial\alpha^e = \sigma[\sigma(1-\sigma)\left(\alpha^e\right)^2-2\sigma\alpha^e+1]/\,[1-(1-\sigma)\alpha^e]^2$.

So $sign(\partial\pi_2^*/\partial\alpha^e) = sign[\sigma(1-\sigma)\left(\alpha^e\right)^2-2\sigma\alpha^e+1]$. The quadratic in square brackets is U-shaped, with positive roots $\bar{\alpha}^e = [\sigma-\sqrt{\sigma(2\sigma-1)}]^{-1}$ (where $\bar{\alpha}^e > 2$ for $\sigma \in (1/2, 1)$) and $\underline{\alpha}^e = 1/[\sigma+\sqrt{\sigma(2\sigma-1)}]$. For $\sigma \leq 0.62$, $\underline{\alpha}^e \geq 1$ and $\partial\pi_2^*/\partial\alpha^e > 0$. For $\sigma > 0.62$, $\underline{\alpha}^e \in (0, 1)$ and the result follows.

Notes

1 Spiegler (2016) provides a review of related literature.

2 Armstrong, Vickers, and Zhou (2009) and Armstrong and Zhou (2011) discuss sources of prominence and provide related evidence.

3 See the intuition in Varian (1980) and Narasimhan (1988). Unlike their models where consumer heterogeneity is exogenous, in my setting price frame competition endogenizes consumer heterogeneity.

4 A wider message from these duopoly analyses is that an increase in prominence harms consumers if equilibrium pricing is frame-independent or if firms commit to price formats before competing in prices but it may benefit consumers if equilibrium pricing is frame-dependant. However, while all these papers explore strategic obfuscation in markets with confused consumers, they apply to different economic environments.

5 The results carry over unchanged if rational consumers are equally likely to choose either product when they are indifferent between them. But, my formulation fits better a default-bias interpretation, where a share σ of consumers are initially assigned to firm 1.
6 The firms' frame choices endogenize the share of confused consumers, but so long as $\alpha \in (0,1)$, Narasimhan's result applies because there are both confused and rational consumers.
7 (a) and (c) used the fact that $\alpha_0 = 0$, but the result still holds if $\alpha_0 \in (0, \alpha_H)$ (the reasoning in (b) applies).
8 Lemma 1 focuses on markets where $\sigma < 1$. However, the result carries over to the case where $\sigma = 1$ (with some modifications to the deviations in part (c) of the proof).
9 These results follow from Narasimhan (1988).

References

Armstrong, M., J. Vickers, and J. Zhou. 2009. "Prominence and Consumer Search." *The RAND Journal of Economics* 40 (2): 209–33.
Armstrong, M., and J. Zhou. 2011. "Paying for Prominence." *The Economic Journal* 121 (556): 368–95.
Carlin, B. 2009. "Strategic Price Complexity in Retail Financial Markets." *Journal of Financial Economics* 91 (3): 278–87.
Chioveanu, I. 2017. "Prominence, Complexity, and Pricing," MPRA Paper 81078.
Chioveanu, I., and J. Zhou. 2013. "Price Competition with Consumer Confusion." *Management Science* 59 (11): 2450–69.
Gu, Y., and T. Wenzel. 2014. "Strategic Obfuscation and Consumer Protection Policy." *The Journal of Industrial Economics* 62 (4): 632–60.
Narasimhan, C. 1988. "Competitive Promotional Strategies." *Journal of Business* 61 (4): 427–49.
Piccione, M., and R. Spiegler. 2012. "Price Competition under Limited Comparability." *The Quarterly Journal of Economics* 127 (1): 97–135.
Spiegler, R. 2011. *Bounded Rationality and Industrial Organization*. Oxford, UK: Oxford University Press.
Spiegler, R. 2016. "Choice Complexity and Market Competition." *Annual Review of Economics* 8: 1–25.
Varian, H. 1980. "A Model of Sales." *American Economic Review* 70 (4): 651–59.