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# MODELLING MIXED-MODE RATE-DEPENDENT DELAMINATION IN LAYERED STRUCTURES USING GEOMETRICALLY NONLINEAR BEAM FINITE ELEMENTS

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## ABSTRACT

Delamination is one of the one most important problems for layered structures, which are widely used in industry (e.g. composite laminates) and also often present in nature (e.g. layered biological tissue). In this work delamination is studied using cohesive-zone models (CZMs) where a discontinuous displacement field and a non-linear traction-separation law on the considered interface are assumed. Authors of the present work have recently shown that beam elements can be used with very good accuracy to model delamination in layered structures both in geometrically linear and non-linear analysis. Beam elements also make use of a smaller number of degrees of freedom, with significant reduction in the overall computational burden. When the fracture process is significantly rate dependent, the traditional fracture-mechanics based approaches can only characterise the phenomenological dependence of the fracture energy on the crack speed. Instead, rate-dependent CZMs, recently developed by the authors, where the different dissipation mechanisms occurring during fracture are separated out, is less phenomenological and better linked to the underlying physics. Combining the highly efficient multi-layer beam model and the novel rate-dependent CZMs is the aim of the project on which the authors of this work are currently collaborating. This work gives a brief overview of authors' recent work which presents the background for developing a novel multi-layer beam finite element with rate-dependent mixed-mode delamination.

***Key Words:*** *multi-layer beam; delamination; finite element analysis; geometrically exact formulation*

## 1. Introduction

This work addresses the problem of how to efficiently model delamination or debonding in layered structures whose layers are efficiently modelled with beam elements and is divided in two main parts. In the first part, recent work by the first and the third author is presented. Their contribution is the development of a computationally efficient and numerically robust multi-layer beam model which accounts for mixed mode delamination. The model uses two- or three-node beam finite elements in conjunction with interface elements with embedded CZM as presented in [1]. In the second part, recent work of the second author on novel rate-dependent CZMs is presented. Building on the principles of thermodynamics, these CZMs are capable of capturing real behaviour of double cantilever beam (DCB) tests across a wide range of loading rates for mode I delamination. In the conclusions, an insight in the current collaboration between the authors is given.

## 2. Rate-independent geometrically exact multi-layer beam model

A multi-layer beam, composed of $n$ layers and $n-1$ interconnections is considered. Each layer $i$ is represented as a beam with a reference axis denoted by $X_{1,i}$ and since all layers are mutually parallel, $X_{1,i} = X_1$ for $i \in \langle 1, n \rangle$. The principal unknown functions of the problem are the displacements ($u_i(X_1)$ and $v_i(X_1)$) and the cross-sectional rotation $\theta_i(X_1)$ of each layer. Since the problem is highly non-linear in terms of the constitutive law of the interconnection and the kinematic equations of the layers, the solution is obtained numerically using the finite element method. Each layer is discretized in finite number ($N$) of nodes, which results in $3 \times n \times N$ degrees of freedom. Using beam finite elements instead of commonly used 2D plane-strain finite elements significantly reduces the total number of degrees-of-freedom (DOF). If 2-node beam finite elements are used instead of Q4 elements, the reduction in total number of DOF is 25%, while substituting Q8 elements with 3-node beam finite elements can reduce the total number of DOF up to 40%.

Governing equations are derived separately for layers and interconnections (see [6] and [5] for more detail). It is assumed that external loads can be applied only on layers, while the interconnection can produce only internal forces. Kinematic equations for the layers are geometrically exact and non-linear, based on the Reissner's beam theory, constitutive equations are linear elastic and equilibrium equations are derived from the principle of virtual work. The nodal vector of residual (internal - external) forces for layer $i$ and node $j$ (after the interpolation of the virtual quantities) is written in the following form

$$\boldsymbol{g}_{i,j}^{L} = \boldsymbol{q}_{i,j}^{INT,L} - \boldsymbol{q}_{i,j}^{EXT,L} = \int_{0}^{L} (\boldsymbol{D}_i \boldsymbol{P}_{i,j})^{\mathsf{T}} \boldsymbol{L}_i^{\mathsf{T}} \begin{Bmatrix} \boldsymbol{N}_i \\ M_i \end{Bmatrix} \mathrm{d}X_1 - \int_{0}^{L} \boldsymbol{P}_{i,j}^{\mathsf{T}} \begin{Bmatrix} \boldsymbol{f}_i \\ w_i \end{Bmatrix} \mathrm{d}X_1 - \boldsymbol{P}_{i,j}^{\mathsf{T}}(k) \begin{Bmatrix} \boldsymbol{F}_{i,k} \\ W_{i,k} \end{Bmatrix}, \tag{1}$$

where matrix $\boldsymbol{D}_i$ contains the relations between the virtual strains and virtual displacements (mostly derivatives), matrix $\boldsymbol{P}_{i,j}$ contains the interpolation functions with respect to the global vector of nodal unknowns, matrix $\boldsymbol{L}_i$ is basically the rotational matrix, $\boldsymbol{N}_i$ is the vector of internal (axial and shear) forces (stress-resultants) and $M_i$ is the internal bending moment. Members appearing with the negative sign originate from the virtual work of external forces; distributed external forces and bending moments are denoted by $\boldsymbol{f}_i$ and $w_i$, while concentrated forces and bending moments at the beam ends ($k = 0$ or $L$) are denoted by $\boldsymbol{F}_{i,k}$ and $W_{i,k}$.

The interconnection is basically attached to the edges of surrounding layers and the relative displacement at the interconnection can be easily defined using displacements, rotations and geometrical properties of surrounding layers. The constitutive law of the interconnection is based on the bi-linear CZM presented in [1] and the equilibrium equations are again derived from the principle of virtual work. The nodal vector of internal forces of the interconnection (after interpolating the virtual quantities) reads

$$\boldsymbol{q}_{\alpha,j}^{INT,I} = b_\alpha \int_{0}^{L} (\boldsymbol{Y}_\alpha \boldsymbol{R}_{\alpha,j})^{\mathsf{T}} \boldsymbol{\omega}_\alpha \mathrm{d}X_1, \tag{2}$$

where $b_\alpha$ is the constant width of the interconnection, matrix $\boldsymbol{Y}_\alpha$ contains relations between the virtual relative displacements of the interconnection and the virtual displacements and rotations of the surrounding layers, matrix $\boldsymbol{R}_\alpha$ contains the interpolation functions with respect to the global vector of nodal unknowns and $\boldsymbol{\omega}_\alpha$ is the vector of contact tractions at the interconnection. Finally, the nodal vector of internal, external and residual forces for all layers and all interconnections may be written as

$$\boldsymbol{q}_j^{INT} = \sum_{i=1}^{n} \left[ \boldsymbol{q}_{i,j}^{INT,L} + (1 - \delta_{in}) \boldsymbol{q}_{i,j}^{INT,I} \right], \quad \boldsymbol{q}_j^{EXT} = \sum_{i=1}^{n} \boldsymbol{q}_{i,j}^{EXT,L} \quad \text{and} \quad \boldsymbol{g}_j = \boldsymbol{q}_j^{INT} - \boldsymbol{q}_j^{EXT} = \boldsymbol{0}, \tag{3}$$

where $\delta_{in}$ is the Kronecker delta, which equals 1 when $i = n$ (otherwise is zero). By linearasing Eq. (3) nodal tangent stiffness matrix $\boldsymbol{K}_{j,k}$ can be obtained, where index $k$ refers to the node of interpolation of test functions. Global vector of residual forces (composed of global vector of internal and external forces) and the global tangent stiffness matrix can be assembled using the standard assembly procedures. During the simulation of delamination processes sharp oscillations around the exact solutions may occur unless the FE mesh is very dense. In [5], a novel modified arc-length method taking into account the damage of the system and capable of obtaining convergence for the most critical numerical tests is presented.

Although in many application the use of geometrically exact formulation will not produce results that significantly differ from those produced using the linear theory of small displacement and rotations, in [5] many examples where this differences cannot be neglected are reported. One of the best examples of that was found for the standard end-notch-specimen for mixed-mode delamination test (see Fig. 1). During the bending of the left-hand side of the upper layer, geometrically exact theory (unlike the linear one) allows the horizontal displacement ($u(0)$ in Fig. 1) which will eventually reduce the arm between the forces $F_1$ and $F_2$ and cause a drastic increases of force $F_1$ after the displacements exceed the limit where the geometrically non-linear effects cannot be neglected.

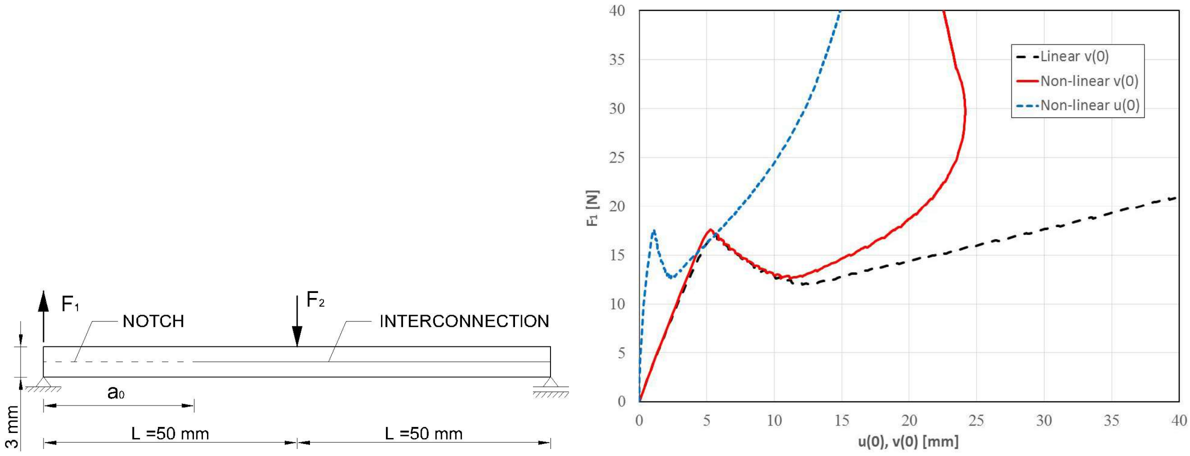


Figure 1: Differences between geometrically linear and non-linear formulations in mixed-mode delamination test

### 3. Rate-dependent cohesive zone model

Authors of the present work currently collaborate on a project aiming to incorporate rate-dependence in the above presented geometrically exact multi-layer beam model which accounts for mixed-mode delamination. The rheological representation of rate-dependent CZMs for mode I delamination presented by Musto and Alfano [3, 4] is given in Fig. 2. The model consists of an elastic and inelastic arm, where a single scalar $\alpha$ is used as an internal variable. $\alpha$ denotes the inelastic displacement jump in the viscous unit (Fig. 2(a) as presented in [3]) or visco-elastic unit (Fig. 2(b) as presented in [4]). Energy dissipated during the fracture process is the result of (i) rupture of elastic bonds (decohesion), which is controlled by the damage parameter $D$, and (ii) viscous flow, which is represented by the viscous or the elasto-viscous part $\alpha$ of the relative displacmenent at the interconnection. It is also assumed that decohesion is rate independent, while the rate-dependence is the result of the viscous dissipative mechanism introduced in the model. In the approach presented in [3] the interface response is that of an viscoelastic standard linear solid (SLS) model (see Fig. 2(a)), where the interface stress is the product of a Volterra convolution operator with exponential kernel, and a scaling factor $(1 - D)$ which accounts for interface damage. In [4], a fractional visco-elastic (FSLS) model for the undamaged response is considered. The dashpot in Fig. 2(a) is replaced by a new "Scott Blair element" in Fig. 2(b) in which the stress is proportional to the fractional derivative of order $\nu$ of the relative displacement of the interconnection ($\nu \in (0, 1)$). The fractional CZM in [4] led to capture with excellent agreement the experimental results for a DCB with rubber interconnection, across entire range of experimentally tested specimens, spanning almost 5 logarithmic decades.

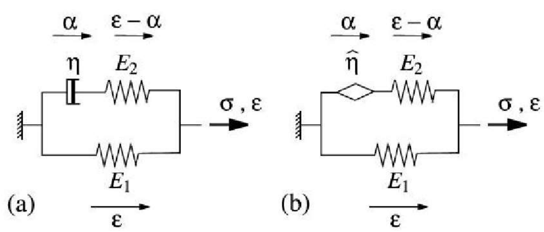


Figure 2: Rheological representation of the (a) classic (SLS) and (b) fractional Standard Linear Solid (FSLS) model

Both approaches [3, 4] assume that the damage evolution is driven by the energy stored in the elastic arm of either the SLS model or FSLS model. This results in a monotonically increasing total fracture energy $G_c$ with the prescribed relative-displacement speed $v$ resulting with a sigmoidal shape of $G_c - v$ curve. In [2], this assumption is reconsidered by taking into account two additional possibilities, namely, that damage is driven by (i) the energies in the two springs of the elastic and inelastic arms or (ii) the entire free energy including energy stored within the "Scott Blair element" in the FSLS model. Considering either of these assumptions, a non-monotonic $G_c - v$ relationship is obtained, with a bell rather than sigmoidal shape.

## 4. Conclusions

In this work, an overview of authors' recent work on which the current research is based, is presented. Combining multi-layered beam model with rate-dependent CZMs shall give a very efficient and robust model for capturing complex rate-dependent behaviour of structures undergoing mixed-mode delamination. The new model will be verified against experimental results for mode I delamination (DCB test) and computationally more expensive models which use plane-strain 2D finite elements with significantly higher number of degrees of freedom.

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