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Low Mortgage Rates and Securitization: A Distinct Perspective on the US Housing Boom*

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Abstract

In this paper, we analyse the impacts of low interest rates and lax underwriting standards on the US housing boom around the beginning of the new millennium. We suggest a time-varying mean of the log price-to-rent ratio (PtR) to capture the persistent changes in housing prices. We show that the increasing latent trend in the PtR was significantly affected by the increased securitization of residential mortgage loans and decreasing interest rates, with the former effect being about three times larger than the latter. In the absence of securitization, negative interest rates would have been needed to reproduce an equally large housing boom since 2003.

Keywords: Dynamic Gordon growth model; house price-to-rent ratio; particle filter; state-space model

JEL classification: C22; E44

I. Introduction

The US housing market's boom and bust around the turn of the twenty-first century has led to a chain reaction resulting in a global crisis. The role of credit supply in this housing boom has received ample attention. Among other factors, it has been pointed out that low interest rates (e.g. Himmelberg *et al.*, 2005; Leamer, 2007; Taylor, 2007), lax underwriting standards due to securitization practices (e.g. Mian and Sufi, 2009; Keys *et al.*, 2010), bad originator practices (Griffin and Maturana,

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2016), deregulation (Favara and Imbs, 2015), and risk-taking in lending by banks encouraged by low short-term interest rates (Maddaloni and Jose-Luis, 2011) have contributed to increased credit supply.

One of the most intense debates in this literature is the effect of the interest rates versus the effect of the underwriting standards (securitization). On one side, it is asserted that the low Federal Funds rate (Leamer, 2007; Taylor, 2007) and the (related) decline in mortgage rates (Himmelberg *et al.*, 2005) have contributed to the housing boom. Lower real interest rates imply lower financial costs of the mortgage loan, and potentially lower discount rates for future cash flows from owning houses (Poterba, 1984). These, in turn, lead to an increasing demand for housing and an acceleration of prices. On the other side, Bernanke (2010) argues that only a small proportion of the increase in house prices can be attributed to the stance of the monetary policy – supporting evidence can be found, for example, in Del Negro and Otrok (2007) and Glaeser *et al.* (2013) – and the deterioration in mortgage underwriting standards is likely a key explanation of the run-up of house prices. Using loan-level data, Keys *et al.* (2010) show that securitization practices led to significant relaxations of underwriting standards. Analysing data at the ZIP code level, Mian and Sufi (2009) confirm that the expansion in subprime mortgage credit from 2002 to 2005 was closely correlated with the increasing securitization of subprime mortgages. Through securitization, additional funding sources for mortgage loans have endogenized the credit supply (Shin, 2009).

In this paper, we contribute to the above-mentioned debate by looking at the effects of both interest rates and securitization simultaneously. We provide rare aggregate evidence on the relative impact of both factors on trends in house prices. The framework of the analysis follows the asset market approach using a dynamic variant of the Gordon growth model (e.g. Campbell *et al.*, 2009; Plazzi *et al.*, 2010) for the log house price-to-rent ratio (PtR). We adopt a generalized version of this approach, which allows the local mean of the PtR to be time-varying. This is consistent with the observed nonlinearity in house price dynamics. State and time dependence of house price dynamics have inspired time series studies applying structural breaks (e.g. Chien, 2010), regime-switching models (e.g. Hall *et al.*, 1997), and varying parameters (e.g. Guirguis *et al.*, 2005). Allowing a time-varying mean can be thought to generalize this line of modelling trends in house prices. Changes in the mean of the PtR could be due to persistent changes in expected return (risk) and economic fundamentals, such as productivity and income gains. For the increases in the US house prices within our considered period, however, the potential major contributors include irrational house price patterns, expansionary mortgage credit policies, and lax lending standards associated with securitization (Mian and Sufi, 2009).

Among these, our work focuses on the role of the measurable triggers (i.e., interest rate policies and securitization).

We estimate a latent state process reflecting the varying mean of the PtR in a nonlinear state-space model. A time-varying instead of a constant mean of the PtR is supported by log likelihood diagnostics. The identified trend is upward sloping from the early 1990s until 2007. The consideration of a continuously varying state enables us to investigate the particular relation between US housing markets and credit markets around the beginning of the twenty-first century in a stable manner. If the observed PtR were used instead, no stable relationship could be confirmed. We find that the observed PtR is integrated of order two in terms of stochastic trending, while other variables in the system are integrated of order one. Mixing variables with different integration orders likely invokes unstable relationships and spurious inference. By considering a trending mean of the PtR, we provide a new interpretation of the relationship between credit market conditions and housing markets – significant influences of credit market conditions on the housing market are likely to be relevant for long-run trends rather than short-run dynamics.

We overcome the difficulty of endogeneity among variables by using vector error correction models (VECMs), which allow endogenous effects from all involved variables directly. Our results show that both decreasing real mortgage rates and increasing securitization activities contributed significantly to observed increases in the mean of the PtR, which has adjusted to changes in mortgage rates and securitization activities significantly and with an increasing speed since 2002. The increase in the mean of the PtR has also positively contributed to securitization activities during the later 1990s and the early 2000s. Importantly, securitization played the most important role in describing the recent accelerations of the PtR. Respective impulse responses show that the impact of a standardized shock in securitization is about three times larger than the impact of a standardized shock in the real mortgage rates. A counterfactual analysis shows that in the absence of securitization activities, negative mortgage rates would have been needed to induce an equivalent housing boom since 2003. Without securitization activities, the nominal interest rate for a 30-year fixed conventional home mortgage would have had to have been as low as 4 percent in the early 1990s and decreased to about –6 percent in 2008 in order to obtain the same mean of the PtR. Our results strengthen the importance of regulatory and supervisory policies in the mortgage-backed securities market in stabilizing housing markets.

The time-varying mean of PtR also confirms that the US housing markets share similar features with the US stock markets. There is a large body of finance and macroeconomic literature documenting the persistent variations in the mean of the price to dividend ratio (e.g., Lettau *et al.*, 2008;

Herwartz *et al.*, 2016), and studying the causes and implications of these variations (e.g., Vissing-Jorgensen, 2002; Calvet *et al.*, 2004; McGrattan and Prescott, 2005; Lettau and Van Nieuwerburgh, 2008; Guvenen, 2009; Lustig and Van Nieuwerburgh, 2010).

In the next section, we illustrate the persistence in the PtR and its implications for the dynamic Gordon growth model. In Section III, we describe the adopted state-space model of the PtR in detail. We evaluate the model in Section IV and we compare it with a constant mean model. In Section V, we discuss results from subset VECMs and we conclude in Section VI. Detailed descriptions of the data, the applied particle filter algorithm, and approximation errors that result from the present-value approach are provided in the Appendix.

II. Empirical Observations

In this section, we document two empirical observations: the persistence in the PtR and its approximation error resulting from the dynamic Gordon growth model based on a constant mean. Both observations support a time-varying mean of the PtR. We use quarterly data of the Federal Housing Finance Agency (FHFA) housing price index and the rent of primary residences as a component of the consumer price index (CPI) for the time period of 1975–2009. To obtain the PtR illustrated in Figure 1, the housing price index is scaled so that the mean of the PtR is about 4.1581, as reported by Ayuso and Restoy (2006) for a similar time period. Focusing on the recent boom in house prices, the period of interest for this study is

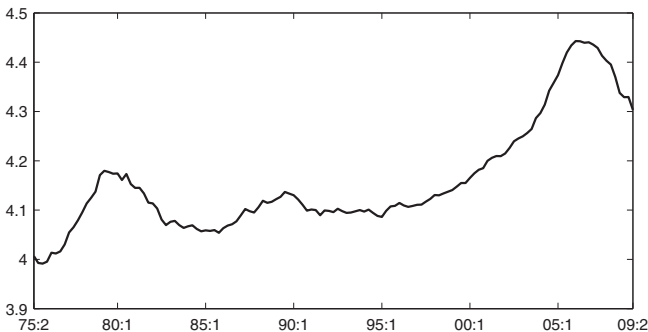


Fig. 1. Log housing PtR from 1975:Q2 to 2009:Q2 for the US (the total available sample period)

Table 1. *Unit root tests for the US PtR*

	ADF_t	PP_t	DF_{GLS}	KPSS
Test statistics	-2.36	-2.15	-1.45	0.96
Critical values at 10 percent	-2.58	-2.58	-1.62	0.35

Notes: A constant is included, and the Schwarz criterion lag length selection criterion is employed to obtain the above test statistics. ADF_t refers to the augmented Dickey–Fuller t -test. For PP_t , which is the t -test statistic considered in Phillips and Perron (1988), the spectral autoregressive estimator is used to calculate the long-run variance. DF_{GLS} refers to the modified Dickey–Fuller t -test proposed by Elliott *et al.* (1996). KPSS refers to the stationarity test proposed by Kwiatkowski *et al.* (1992). The time period is from 1975:2 to 2009:2, which is the total available period.

from the early 1990s to 2006. Data before 1990 are needed to prepare the estimation and data up to 2009 are used to assess model robustness.

A stylized fact of the housing market is the strong persistence in house prices. A change in house prices tends to be followed by a change in the same direction in the following year (Case and Shiller, 1989). Shocks have persistent effects on house prices over a long period. This observed serial dependence in changes of house prices might reflect inefficiencies in the housing market due to transaction costs, tax considerations, etc.

Our results confirm that the PtR is unlikely to be a stationary process. All unit root statistics do not obtain a rejection of the unit root hypothesis at the 10 percent significance level, as can be seen from Table 1. The same conclusion is obtained by the KPSS statistic (last column of Table 1), which circumvents the power weakness of unit root diagnostics under near integration. The null hypothesis of a stationary PtR is rejected. One might further argue that the lack of rejection of the unit root hypothesis can be due to the short time-span of the data. However, including data of the PtR until 2015, which displays a large fall of the PtR, the null hypothesis of a unit root is still not rejected.¹ The PtR shows characteristics of a non-stationary variable: shocks have persistent effects on the process.²

The dynamic Gordon growth model for the PtR (e.g. Campbell *et al.*, 2009) is based on the Campbell and Shiller (1989) present value model on the log stock price-to-dividend ratio in the finance literature, which decomposes the price-to-dividend ratio into the sum of discounted future dividend growth and expected future returns on the stock. In analogy, the PtR can be considered as the discounted sum of the expected growth rate of rents and required returns to housing. Results of this approach, however, should be interpreted with a caveat in mind – this approach relies on the assumption of the stationarity of the PtR, as shown in the following analysis.

¹Results are not shown due to space considerations.

²It is worth noting that while the PtR is persistent, it is a bounded process (i.e., the ratio of price-to-rent cannot fall below 0 and increase unlimedly). The bounded non-stationarity of the PtR can be confirmed by the tests suggested in Cavaliere and Xu (2014).

Let P_t and L_t denote the observed price and rental payment of housing at the end of period t . The realized log gross return at the end of period $t + 1$ is

$$\begin{aligned} r_{t+1} &\equiv \ln(P_{t+1} + L_{t+1}) - \ln(P_t) \\ &= -\eta_t + \ln[\exp(\eta_{t+1}) + 1] + \Delta l_{t+1}, \end{aligned} \tag{1}$$

where $\eta_t = \ln(P_t) - \ln(L_t)$ is the PtR and Δ is the difference operator such that, for example, $\Delta l_t = l_t - l_{t-1}$. The lowercase letters refer to the natural log of the corresponding uppercase letters. Equation (1) is nonlinear in terms of η_{t+1} . Using a linear approximation of equation (1) by means of a first-order Taylor expansion around a fixed point η , we obtain

$$r_{t+1} \approx \kappa - \eta_t + \rho\eta_{t+1} + \Delta l_{t+1}, \tag{2}$$

with $\rho \equiv 1/[1 + \exp(-\eta)]$ and $\kappa \equiv -\ln(\rho) - (1 - \rho)\ln(1/\rho - 1)$. Equation (2) can be thought as a formalization of the current PtR through the future PtR, returns, and rent growth. Notably, $\rho = P/(P + L)$ reflects the importance of the price relative to the sum of the price and the rent. The higher the price relative to the rent, the more weight is attached to the future PtR in the pricing equation.

In the empirical analysis, the observed sample mean is commonly used to approximate the fixed point η . This follows the idea that presuming stationarity of the PtR, the first-order Taylor expansion around the mean provides the best linear approximation on average. Iterating equation (2) forward, we obtain

$$\eta_t \approx \frac{\kappa}{1 - \rho} + \sum_{i=1}^{\infty} \rho^{i-1}(\Delta l_{t+i} - r_{t+i}) + \lim_{i \rightarrow \infty} \rho^i \eta_{t+i}. \tag{3}$$

Equation (3) provides a linear approximation of the current PtR η_t around its constant mean η . We evaluate the approximation error by comparing the PtR with the right-hand side of the equation (3), where the terminal value of η_T is set to the last observation from the sample.

We find that the approximation error with a constant mean shows a clear upward sloping and persistent trend, as can be seen in Figure 2. It appears stationary around a trend, but non-stationary around a constant mean. Common unit root tests confirm the non-stationarity. The two empirical features studied in this section highlight that the dynamic Gordon growth model with a constant mean might not be fully appropriate for a study of the dynamics of the persistent PtR.

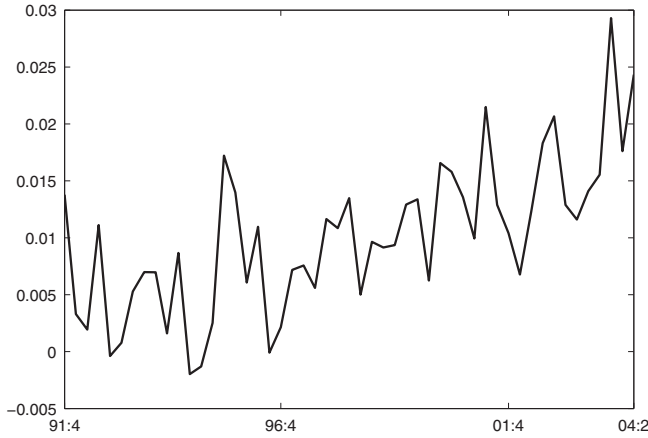


Fig. 2. Approximation error for the period of 1991:Q4 to 2004:Q2 from the present value model with the fixed steady state in equation (3)

Notes: The starting period 1991:Q4 is the same as the one for the final analysis of the effect of credit market conditions on housing markets. The ending period 2004:Q2 is chosen so that there are at least 20 observations available for the smallest forward looking time period.

III. State-Space Model

In this section, we propose a modified version of the present value model discussed in the previous section, and we formalize it within a state-space model. We assume that the local mean of η_t can be time-varying, and denote it by $\bar{\eta}_t$. A linear approximation for equation (1) can be obtained around $\bar{\eta}_t$ as

$$r_{t+1} \approx \kappa_t - \eta_t + \rho_t \eta_{t+1} + \Delta l_{t+1}, \tag{4}$$

with $\rho_t \equiv 1/[1 + \exp(-\bar{\eta}_t)]$ and $\kappa_t \equiv -\ln(\rho_t) - (1 - \rho_t) \ln(1/\rho_t - 1)$. This equation relates current η_t to future η_{t+1} , r_{t+1} , and Δl_{t+1} . A time-varying $\bar{\eta}_t$ corresponds to a time-varying ρ_t , which in turn implies a time-varying weight attached to the future cash flow. Allowing the fixed point used for the linear approximation to be time-varying reduces the approximation errors in comparison with those shown in Figure 2. It incorporates the evidence that the PtR is fluctuating around a trend rather than a constant mean. This approach can be compared with the approach of Herwartz *et al.* (2016), formalizing a time-varying mean of the stock price to dividend ratio. To obtain an explicit form of the iterated version of equation (4), which is comparable with equation (3), we make similar approximations as those in Lettau and Van Nieuwerburgh (2008):

$$E_t(\rho_{t+i}) \approx \rho_t, \quad E_t(\kappa_{t+i}) \approx \kappa_t, \quad E_t(\rho_{t+i} \eta_{t+i+1}) \approx E_t(\rho_{t+i}) E_t(\eta_{t+i+1}).$$

As shown in detail in the Appendix, the resulting errors from these three approximations are very small.

Taking the conditional expectation and iterating equation (4) forward, we find

$$\eta_t \approx \frac{\kappa_t}{1 - \rho_t} + \sum_{i=1}^{\infty} \rho_t^{i-1} E_t(\Delta l_{t+i} - r_{t+i}) + \lim_{i \rightarrow \infty} \rho_t^i E_t \eta_{t+i}. \quad (5)$$

This equation approximates the PtR by a deterministic term, discounted expected future rent growth rates and returns, and the discounted terminal value of the PtR. Compared with equation (3), the present value model in equation (5) allows for a time-varying deterministic term, which is a function of the local mean of the PtR. Because the mean of the PtR is time-varying, the future cash flows are also discounted at a time-varying rate ρ_t . Based on equation (5), the observation equation in the state-space model is formulated as

$$\eta_t = \frac{\kappa_t}{1 - \rho_t} + \sum_{i=1}^{\infty} \rho_t^{i-1} \tilde{E}_t(\Delta l_{t+i}^e - r_{t+i}^e) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2), \quad (6)$$

with $t = t_0, t_0 + 1, \dots, T$. The error term ε_t can capture rational bubbles ($\lim_{i \rightarrow \infty} \rho_t^i E_t \eta_{t+i}$) and other influences. Subtracting the risk-free rate r_t^f , $\Delta l_{t+i}^e = \Delta l_{t+i} - r_{t+i}^f$ is the excess rent growth rate, and $r_{t+i}^e = r_{t+i} - r_{t+i}^f$ is the excess return on housing. The operator \tilde{E}_t symbolizes objective expectations of a variable based on information available at the end of period t . Equation (6) decomposes the PtR into three components: a time-varying deterministic term, discounted objective expectations of future rent growth rates and returns, and an error term.

The state equation specifies the dynamics of the latent state process ρ_t , reflecting the varying mean of the PtR. The state process is bounded between 0 and 1 by construction, as it can be formulated as the ratio of the house price to the sum of the house price and the rent, and serves as the discount rate in the present value model in equation (5). Within these bounds, we can formulate the persistence by means of a non-stationary or a stationary autoregressive process. According to log likelihood diagnostics, a bounded non-stationary process is preferable to a bounded stationary process for our data, although the estimated latent states from both processes are similar. For space considerations, we concentrate on the formulation of ρ_t as a bounded random walk process (Cavaliere and Xu, 2014) for further analysis, i.e.

$$\rho_t = \rho_{t-1} + u_t, \quad \rho_{t_0} = \rho_0. \quad (7)$$

The disturbance term u_t is decomposed as $u_t = e_t + \underline{\xi}_t - \bar{\xi}_t$, where $e_t \sim N(0, \sigma_e^2)$ and $\underline{\xi}_t, \bar{\xi}_t$ are non-negative processes such that $\underline{\xi}_t > 0$ if and only if $\rho_{t-1} + e_t < 0$ and, similarly, $\bar{\xi}_t > 0$ if and only if $\rho_{t-1} + e_t > 1$.

The objective expectations \bar{E}_t for all future excess rent growth rates and excess returns are calculated as forecasts from two alternative vector autoregressive (VAR) models of order one.³ These comprise $y^{(1)} = (\eta_t, \Delta l_t^e, r_t^e)'$ and $y^{(2)} = (\eta_t, \Delta l_t^e, r_t^e, \pi_t)'$, where π_t is the smoothed inflation. The smoothed inflation is used such that short-term variations in the quarterly inflation are filtered out. Using VAR forecasts for objective expectations follows a long tradition proposed by Campbell and Shiller (1989).⁴ The reason for the inclusion of the smoothed inflation π_t in the VAR model of $y^{(2)}$ is the concern that inflation can have effects on the expected future rent growth rates and returns on housing, as considered in Brunnermeier and Julliard (2008). Importantly, including the PtR in the VAR provides unobservable market information about the future rents and returns. The reduced-form VAR is informative and, at the same time, general enough to be consistent with a present value relation with a gradually time-varying mean of the PtR.⁵

The nonlinear state-space model consisting of equations (6) and (7) is estimated by means of the so-called particle filter. Unlike the Kalman filter, the particle filter can cope with intrinsic nonlinearity of the state-space model. At time $t = t_0$, ρ_t is fixed to ρ_0 , which is later treated as a parameter and subjected to estimation. To allow for a dynamic pattern of ρ_t , the state equation formalizes that this process exhibits a bounded stochastic trend with innovation variance σ_u^2 . For given ρ_t , the in-sample determination of an implied model disturbance ε_t is straightforward. Its innovation variance is denoted by σ_ε^2 . Because ρ_t enters the observation equation in a highly nonlinear manner, the model in equations (6) and (7) cannot be implemented by means of linear conditional modelling. Consequently, it is not feasible to use the Kalman filter to evaluate the model's (log) density for given parameters. With known variance parameter

³Note that linear state-space models, such as the one in Van Binsbergen and Kojen (2010) including one state equation each for expected rent growth rates and expected returns, are not consistent with the observed persistence in the PtR. These models assume an exogenous fixed mean of the PtR.

⁴In related contexts, VAR-based predictions have also been used to approximate price expectations, for instance by Sbordone (2002) and Rudd and Whelan (2006). By means of a theoretical model on the generation of inflation expectations, Branch (2004) shows that economic agents more often use VAR forecasts for expectation formation in comparison with adaptive or naïve prediction rules.

⁵Note that we include a constant in the VAR model. The inclusion of a deterministic trend instead of a constant provides similar estimates for the latent process (ρ_t).

σ_{ε}^2 , however, the evaluation of the model's log density is straightforward for a given time path of $\rho_t, t = t_0, \dots, T$. As this process is not observable but explicitly formalized in equation (7), the particle filter allows a Monte Carlo based evaluation of the log-likelihood function for given parameters in $\theta = (\rho_0, \sigma_u^2, \sigma_{\varepsilon}^2)$.⁶

Moreover, as a particular rival model, we consider a degenerated state-space model with constant ρ , for which $\sigma_u^2 = 0$ is imposed. This model corresponds to the dynamic Gordon growth model with a constant mean of PtR. It will be of particular interest to evaluate the approximation losses in terms of the Gaussian log-likelihood when switching from the dynamic state-space model to its degenerate counterpart.

We employ the particle filter (Del Moral, 1996) as described with resampling in Cappé *et al.* (2007) for likelihood evaluation (Algorithm 3, using $\rho_t \sim N(\rho_{t-1}, \sigma_u^2)$ as the importance distribution). Model parameters in $\theta = (\rho_0, \sigma_u^2, \sigma_{\varepsilon}^2)$ are determined by means of a grid search. For those parameter combinations obtaining the maximum of the Gaussian log-likelihood, θ_{opt} , implied time paths $\hat{\rho}_t, t = t_0, \dots, T$, are determined by averaging over simulated particles. Noting the low dimension of θ , the number of particles is relatively small, $N = 2000$, but we perform the grid search multiple (i.e., 10) times to check if results are robust or suffer from prohibitive Monte Carlo errors. Details of the particle filter algorithm can be found in the Appendix.

IV. Model Evaluation

In this section, we evaluate the proposed state-space model using quarterly US data. The FHFA housing price index and the rent of primary residence as a component of the CPI are used to obtain $\eta_t, \Delta I_t$, and r_t . The 10-year Treasury Bill rate is adopted for r_t^f and smoothed inflation π_t is calculated from the CPI excluding shelter. Note that a long-term instead of short-term risk-free rate is considered to reflect the long-run holding time period of a home. Detailed descriptions of the data are provided in the Appendix. The first 30 observations are used to initiate the recursive VAR modelling and the provision of multi-step forecasts. Three conclusions can be drawn from our analysis.

First, the VAR model including inflation ($y^{(2)}$) has a better performance than the model without inflation ($y^{(1)}$). As can be seen from Table 2, the log-likelihood of the former (478.45) is about 35 percent higher than the

⁶Although we do not explicitly estimate correlation parameters that might be present in the error term, this does not restrict the estimated error term to be serially uncorrelated. Such a correlation could correspond to rational bubbles and other influences.

Table 2. *Parameter estimates and model evaluations*

VAR	Time-varying ρ				Constant ρ ($\sigma_e = 0$)		
	ρ_0	σ_ε	σ_e	log-likelihood	ρ_0	σ_ε	log-likelihood
$y^{(1)}$	0.984	8.27E-03	7.49E-05	354.33	0.985	1.31E-02	329.28
$y^{(2)}$	0.984	2.61E-03	1.29E-04	478.45	0.986	5.52E-03	416.26

Notes: This table documents the core parameter estimates (ρ_0 and standard deviations) and model diagnostics for the two dynamic specifications and their time-invariant counterparts. The time period is from 1982:3 to 2009:2. The first 30 observations from 1975:2 to 1982:2 are used to initiate recursive VAR forecasting to determine objective expectations \hat{E}_t in equation (6).

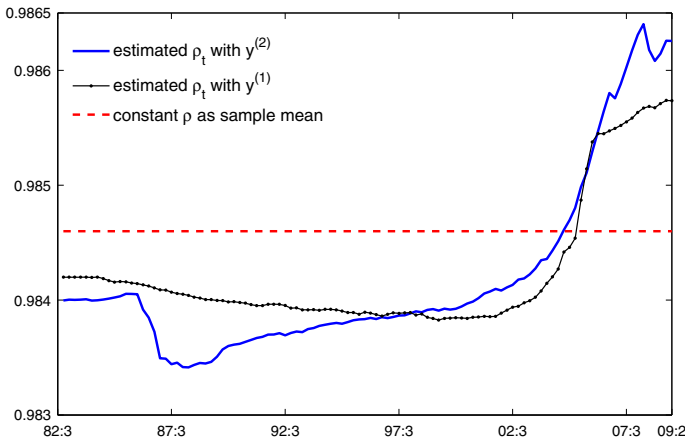


Fig. 3. Estimated ρ_t for the available time period

Notes: The first 30 observations from 1975:Q2 to 1982:Q2 are used to initiate recursive VAR forecasting for the estimation.

log-likelihood of the latter (354.33). This evidence supports the view that inflation influences the agents' expectation of rent growth rates and returns. For the further analysis in the next section, we consider the estimates based on the VAR model including inflation ($y^{(2)}$).

Secondly, the estimated time path of ρ_t is clearly time-varying. Figure 3 illustrates the estimated time path of ρ_t for the two alternative VAR models. Both paths of $\hat{\rho}_t$ are time-varying and different from the constant ρ (dashed line), which is the observed sample mean of $P_t/(P_t + D_t)$. Confirming the visual impression, it can be observed from Table 2 that, according to log-likelihood statistics, the model with the time-varying ρ_t is always strongly preferred over its constant parameter counterpart. When the VAR for $y^{(2)}$ is considered, the log-likelihood value of the time-varying ρ_t model (478.45) is about 15 percent higher than the one from the constant ρ model (416.26). Although one might question the validity of common likelihood (ratio)

comparisons of rival models in the present context, it is most unlikely that the reported log-likelihood improvement accords with repeated experiments under the null hypothesis of a constant ρ model.

Thirdly, the increase in the latent state ρ_t has a profound impact on the PtR. Equation (6) shows that there is an increase not only in the deterministic term but also in the sum of future discounted cash flows, *ceteris paribus*. The degree of increases in the PtR depends on the expectation of future rent growth rates and returns at a given time point. Consider 1982:Q3 for a simplified example. At this time point, the estimated state variable $\hat{\rho}_t$ is 0.984 and the observed risk-adjusted rent growth rate $\Delta l_t - r_t$ is around 0.006. Assuming a constant future risk-adjusted rent growth rate of 0.006, if the state variable increased to 0.986, the resulting PtR would increase by about 3 percent. This accounts for about 40 percent of the observed increase in the PtR from 1983:Q2 to 2006:Q4.

Apart from in-sample diagnosis, further out-of-sample (OOS) evidence is also in favour of trends governing ρ_t . To gauge the predictive content of the estimated time-varying state $\hat{\rho}_{t-1}$ for the PtR η_t during the housing boom, we consider an AR(2) process for the PtR as a baseline prediction model. This is the best performing model in OOS forecasting among a battery of considered models. We find that augmenting the AR(2) baseline model with the time-varying state as an additional explanatory variable improves the OOS forecasting performance further. The mean squared prediction error is reduced by about 15 percent compared with the baseline model.⁷

V. Cointegration Analysis

In this section, we investigate the relationship between the estimated latent state $\hat{\rho}_t$ from the housing market and easy credit market conditions.⁸ VECMs are applied due to the non-stationarity of the variables, their joint endogeneity, and the potential of common stochastic trends. Figure 4 illustrates the considered time series.

Preliminary Analysis

First, we discuss the data, and then we employ unit root tests and cointegrating rank tests for the latent state $\hat{\rho}_t$ for the PtR, the real mortgage rate rm_t , and the securitization ratio s_t .

⁷The recursive estimation and forecasting periods start at 1991:Q1 and 1997:Q1, respectively.

⁸Results are similar when using the estimates for the varying mean of the PtR ($\bar{\eta}_t$) backed out from $\hat{\rho}_t$.

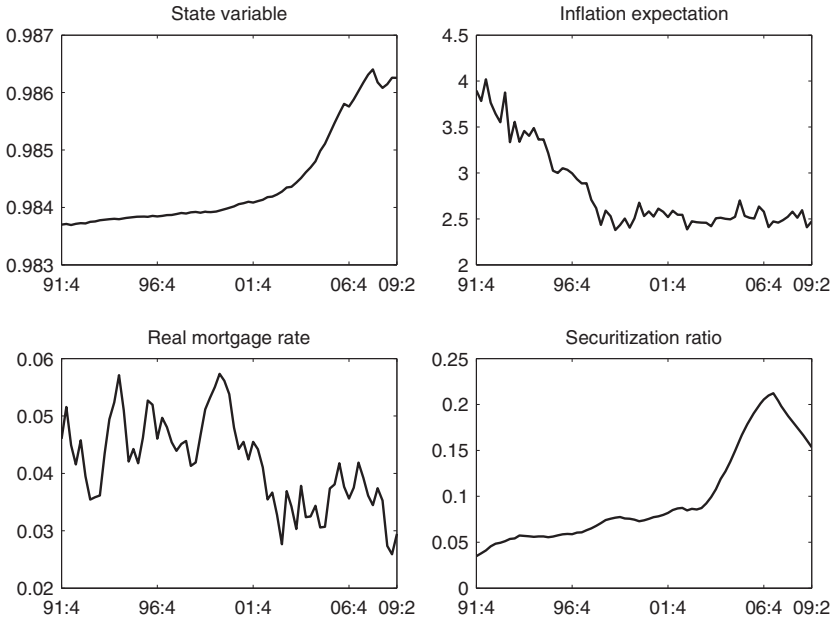


Fig. 4. The considered variables in VECMs

To obtain the real mortgage rate, we use nominal contract rates on the 30-year fixed-rate conventional home mortgage adjusted for inflation expectations. In the related body of literature, a long-term treasury bond rate rather than the mortgage rate has often been used to study the influence of interest rates on the housing market. The reason for this choice is to isolate endogenous fluctuations in market interest rates due to the housing market, because ordinary least-squares estimation cannot cope adequately with the endogeneity (e.g., Glaeser *et al.*, 2013). Because the VECM explicitly considers the effects of endogenous variables on each other, we use the mortgage rate to incorporate the potential dynamics in the data. With regards to inflation expectations, we draw the data from the Survey of Professional Forecasters at the Federal Reserve Bank of Philadelphia.⁹ These data are the means of forecasts for the annual average rates of CPI inflation over the next 10 years, which are available from 1991:Q4. As can be seen from Figure 4, inflation expectations have been very stable and have fluctuated around 2.5 percent since 1998.

⁹This survey is chosen instead of the Livingston and Michigan Survey of inflation expectations because it provides inflation expectations at the quarterly frequency over a long horizon.

Apart from the role of monetary policy, the lax underwriting standards of subprime mortgage loans seem to have contributed to the rapid expansion of mortgage supply and the subsequent crisis, as shown by analysis using data at the ZIP code or loan level (Mian and Sufi, 2009; Keys *et al.*, 2010). At an aggregate level, however, reliable direct measures of the underwriting standards are not publicly available. While Federal banking regulators and the Office of the Comptroller of the Currency conduct surveys to ask about banks' underwriting standards, these surveys do not include the non-bank financial sectors, which have been largely involved in underwriting subprime mortgages. As the increasing securitization practices have led to decreasing underwriting standards of subprime lenders (Keys *et al.*, 2010), we focus on securitization activities directly. For this purpose, we construct an aggregate measure of securitization practices based on data of private issuers (rather than the government sponsored enterprises), who were largely responsible for underwriting subprime mortgages. Specifically, we measure the securitization practices by means of the share of the outstanding home mortgages held by private issuers of asset backed securities, called the securitization ratio henceforth. Data are collected from the flow of funds accounts released by the Board of Governors of the Federal Reserve System.¹⁰

We find strong evidence to support the view that all considered variables are integrated of order one. Table 3 provides the unit root test statistics. The time periods used are the longest available periods for each variable. Results from alternative unit root tests are consistent with each other except for a few cases.

It is worth noticing that the observed PtR η_t is integrated of order two for the sample period of the cointegration analysis. The null hypothesis of a unit root cannot be rejected for $\Delta\eta_t$ (the test statistics are -1.23 , -1.50 , and -1.06 for ADF_t , PP_t , and DF_{GLS} , respectively) but can be rejected for the second difference $\Delta^2\eta_t$ (the test statistics are -10.70 , -11.12 , and -9.09 for ADF_t , PP_t , and DF_{GLS} , respectively). As an implication of distinct integration orders, a joint modelling of the observed PtR (integrated of order two) and the real mortgage rate and securitization (both integrated of order one) lacks econometric justification and results in unstable relationships and likely spurious inference. The instability of a cointegration model using the observed PtR can be confirmed by means of the so-called τ -statistic (Hansen and Johansen, 1999), which is based on the largest eigenvalue from the reduced-rank regression.¹¹ While the real mortgage rate and securitization cannot explain the dynamics of the observed PtR in a stable

¹⁰Data can be downloaded at <https://www.federalreserve.gov/datadownload/Choose.aspx?rel=Z1>. The securitization ratio is derived as FL673065105 over FL893065105.

¹¹Detailed results on this are available upon request.

Table 3. *Unit root test statistics*

	ADF_t	PP_t	DF_{GLS}
$\hat{\rho}_t$	1.29	1.56	1.09
$\Delta\hat{\rho}_t$	-5.22***	-5.22***	-5.16***
rm_t	-2.05	-2.05	-1.92*
Δrm_t	-6.93***	-8.41***	-2.30**
s_t	-2.05	-2.39	-1.47
Δs_t	-2.85*	-2.69*	-2.80***

Notes: ***, **, and * denote that test statistics are significant at the 1, 5, and 10 percent levels, respectively. The latent state for the PtR is denoted by $\hat{\rho}_t$, and rm_t is the real mortgage rate. The securitization ratio is represented by s_t . To provide an overview, we use the longest available periods for each variable. The sample periods for $\hat{\rho}_t$, rm_t , and s_t are 1982:Q3 to 2009:Q2, 1991:Q4 to 2009:Q2, and 1984:Q4 to 2009:Q2, respectively. See previous notes in Table 1 for detailed descriptions of the unit root tests.

Table 4. *Johansen trace tests for $(\hat{\rho}_t, rm_t, s_t)$*

Lagged differences	H_0	Test statistic	p -value
1	$r = 0$	41.12	0.01
	$r = 1$	18.11	0.10
	$r = 2$	2.37	0.70
2	$r = 0$	39.98	0.01
	$r = 1$	13.39	0.34
	$r = 2$	2.58	0.67
3	$r = 0$	67.70	0.00
	$r = 1$	21.87	0.03
	$r = 2$	2.19	0.74

Notes: Testing the cointegration rank for the latent state $\hat{\rho}_t$ for the PtR, the real mortgage rate rm_t , and the securitization ratio s_t . A constant is included. The sample period is from 1991:Q4 to 2009:Q2, which is the available common sample period for all variables.

and consistent manner, they can explain the time-varying mean of the PtR (as shown in the cointegration analysis below). Hence, the influences of credit market conditions on the housing market are likely important for long-run trends rather than short-run dynamics.

Given the non-stationarity of the time series, we continue with tests for the cointegrating rank. The overall evidence suggests that there is at least one cointegration relation. Table 4 reports the results from Johansen trace tests among $\hat{\rho}_t$, rm_t , and s_t for the common sample period. Because the Akaike information criterion suggests 3 as the lag order for the differences and the Schwarz criterion is minimized for lag order 1, lagged differences from 1 to 3 are considered.

As the next step, we adopt the so-called two-step (S2S) method to estimate the VECMs (Ahn and Reinsel, 1990). Brüggemann and Lütkepohl (2005) show that this estimator does not produce the outliers sometimes seen when following maximum likelihood estimation, particularly when

conditioning on small samples. Furthermore, to reduce the number of parameters and the estimation uncertainty, we apply a subset procedure. The cointegrating vector is estimated first. Then, linear restrictions on the parameters that characterize short-term dynamics are imposed. Explanatory variables with smallest absolute t -ratios are sequentially deleted until all t -ratios exceed 1.96 in absolute value. At each step, the entire system is estimated again and new t -ratios are updated within the reduced model. Estimation results are discussed in the following subsection.

Results

We find that real mortgage rates have a significantly negative effect on the long-term state of the PtR while the securitization ratio has a significantly positive effect (see Table 5). This evidence is consistent with theoretical considerations. Lower real mortgage rates reduce the financial costs of mortgage loans, and thereby stimulate the demand for houses. Moreover, larger proportions of the home mortgage funds from securitization activities can stimulate the credit supply as a result of agency problems along the securitization chain. Through the new financing model of mortgage funds, the cheap credit has led to the increases in the mean of the PtR. The results on the cointegration relation are robust. The same evidence can be found for the sample period from 1996 to 2006, which is characterized by the most intensive accelerations of house prices in relation to rents, and for an extended sample period from 1991 to 2009. Also, there is no significant autocorrelation in the residuals, as confirmed by Portmanteau statistics.

Moreover, we find evidence that the housing boom and the securitization activities have mutually influenced each other and the house prices have been strongly affected by the credit market conditions.

The latent state $\hat{\rho}_t$ for the PtR adjusts itself towards the equilibrium with the credit market condition (the cointegration relation); see the upper panel of Figure 5. The adjustment coefficient in VECMs measures the response of each variable to deviations from the equilibrium of the system (the cointegration relation). Recursive estimates of the adjustment coefficients (jointly with their respective 95 percent confidence intervals) for the state variable $\hat{\rho}_t$ differ significantly from zero. Its adjustment towards the cointegration relation is particularly significant since 2002, and reaches a level of about -0.12 at the end of the sample. It takes the mean of the PtR about two years to fully adjust to its equilibrium level with the real mortgage rates and securitization ratios.

The securitization ratio is also affected by the cointegration relation. In the late 1990s and early 2000s, there is mild evidence for significant adjustment towards the increasing mean of the PtR. If 90 percent confidence intervals are considered, this evidence becomes more obvious.

Table 5. *Cointegration parameters: $\hat{\rho}_t = \beta_1 rm_t + \beta_2 s_t$*

Time period	β_1	β_2	<i>p</i> -values for Portmanteau tests	
1996:Q1–2006:Q4	−0.0051 (−2.124)	0.013 (12.178)	lag order 2 0.798	lag order 4 0.996
1991:Q4–2009:Q2	−0.029 −4.396	0.018 (17.009)	lag order 4 0.184	lag order 8 0.147

Notes: A constant is included in the estimation. The S2S method is used to estimate the cointegration relation among the latent state $\hat{\rho}_t$ for the PtR, the real mortgage rate rm_t , and the securitization ratio s_t (*t*-statistics in parentheses). For period of 1996:Q1 to 2006:Q4, one lag is considered. For the time period of 1991:Q4 to 2009:Q2, the lag length of 3 is chosen. The lag length is chosen under consideration of diagnostics of residual autocorrelation.

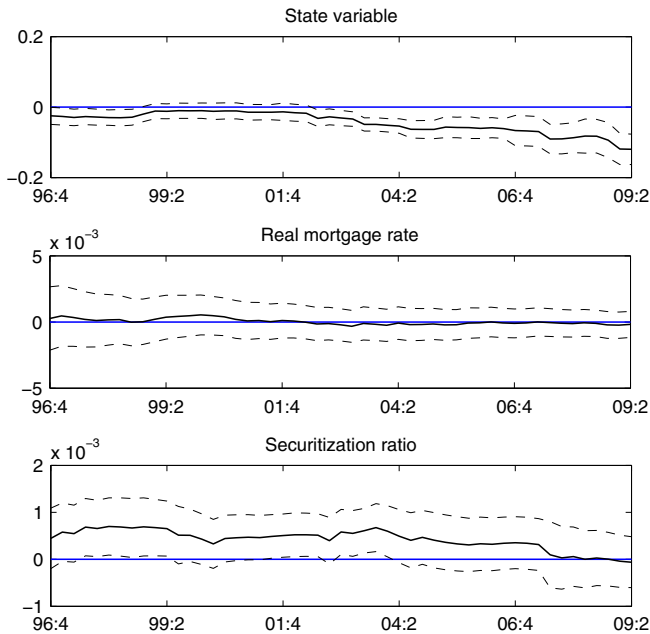


Fig. 5. Estimated adjustment coefficients from the VECMs

Notes: The recursive estimate obtained at 1996:Q4 using the sample from 1991:Q4 to 1996:Q4, and the one at 2009:Q2 using the sample from 1991:Q4 to 2009:Q2.

In contrast, real mortgage rates are not influenced by deviations from the cointegrating relation. Its adjustment coefficient is never significant over the entire recursion. The real mortgage rate is weakly exogenous towards its cointegration relation with the latent state in housing markets.¹²

¹²For an intuitive discussion of weak exogeneity as an indicator of long-run causality, the reader can consult Hall and Milne (1994).

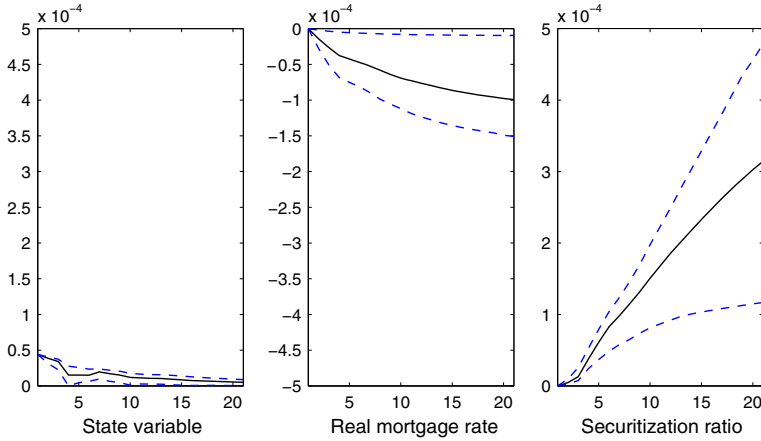


Fig. 6. Impulse responses of the latent state variable $\hat{\rho}_t$ for the PtR with respect to an innovation of the size of one standard deviation in the latent state, real mortgage rates, and the securitization ratio

Notes: The dashed lines are the 95 percent Efron (bootstrap) confidence intervals based on 299 bootstrap replications.

Furthermore, while the effect of a shock in the state variable itself decreases slowly over time, shocks in the real mortgage rates and the securitization have persistent effects on the state variable. Most strikingly, the securitization ratio has the highest impact on the long-term state of the PtR. This evidence is obtained from impulse response analysis, which provides a more comprehensive picture of the impact of a shock in credit markets on the latent state process in housing markets. In the impulse response analysis, the expected response of the state variable is traced out over the next five years given a one time innovation of the size of one standard deviation in the state variable, the real mortgage rate, and the securitization ratio. Figure 6 illustrates these impulse responses along with 95 percent bootstrap confidence intervals. After five years, the impact of a standardized shock in the securitization ratio on the state variable is about three times larger than the impact of a standardized shock in real mortgage rates, and 63 times larger than the impact of a standardized shock in the state variable itself. This evidence supports the view that the securitization of the residential mortgage loans has played the most important role in the recent increases of house prices relative to rents.

The results from the impulse response analysis are robust even when potential instantaneous correlations are taken into account. The impulse responses are obtained with only one shock in one variable at a time. If the shocks are instantaneously correlated, this analysis might only provide a partial picture. Table 6 provides results from tests for instantaneous

Table 6. *Wald tests for instantaneous causality*

H_0 : no instantaneous causality between	Test statistic	p -value
$\hat{\rho}_t$ and $(rm_t, s_t)'$	7.30	0.03
rm_t and $(\hat{\rho}_t, s_t)'$	5.53	0.06
s_t and $(\hat{\rho}_t, rm_t)'$	4.12	0.13

Notes: The test statistic is χ^2 distributed with two degrees of freedom. The latent state for the PtR is denoted by $\hat{\rho}_t$, and rm_t is the real mortgage rate. The securitization ratio is represented by s_t .

causality. Shocks in the securitization ratio do not instantaneously cause shocks in the state variable or the real mortgage rate. The correlations between estimated residuals from the securitization ratio and those from the state variable and real mortgage rates are 0.087 and 0.015, respectively. Similarly, shocks in the real mortgage rate are not instantaneously related to the shocks in the state variable and the securitization at the 5 percent significance level. Nevertheless, shocks in the state variable do instantaneously cause shocks in the remaining two variables. Therefore, it is likely that shocks in real mortgage rates and the state variable occur simultaneously. However, even when the instantaneous correlation between shocks in real mortgage rates and the state variable is taken into account by means of a structural VECM, the resulting impulse responses are similar to those in Figure 6. The reason for this is that not only the adjustment coefficient but also all short-run coefficients in the equation of the real mortgage rates are not significantly different from zero.

In addition, the effect of securitization can be highlighted by means of a counterfactual analysis. Based on the estimated cointegration relation, $\hat{\rho}_t = 0.983 - 0.029rm_t + 0.018s_t$, we address the following question. If there were no securitization activities, how much should the interest rate fall to result in the same increase in the long-run housing price? Conditional on the time series of $\hat{\rho}_t$ and imposing $s_t = 0, \forall t$, we can back out a counterfactual real mortgage rate in the absence of securitization activities. Our results suggest that negative financing costs for the mortgage loan would be needed to reproduce an equally strong housing boom since 2003, if easy credit market conditions were solely measured by means of the interest rate. The nominal rates on the 30-year fixed-rate conventional home mortgage varied around 8 percent in the early 1990s, and have decreased markedly to the region of 6 percent since 2002 (see the upper panel of Figure 7). Without securitization activities, however, these rates should already be as low as 4 percent in the early 1990s and decrease to about -6 percent in 2008 in order to lead to the same level of the mean of the PtR (see the lower panel of Figure 7).

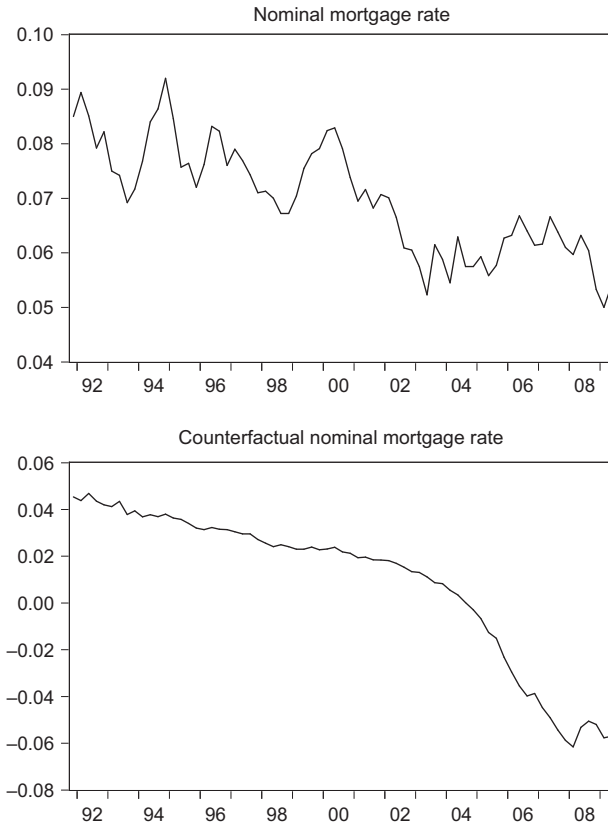


Fig. 7. Nominal mortgage rate with its counterfactual one if there were no securitization activities

VI. Conclusions

In this paper, we contribute to the debates about the effect of the interest rates and the effect of securitization on the recent boom in US housing prices, and we consider the effects of both factors simultaneously. We incorporate a time-varying mean of the log PtR in a dynamic Gordon growth model. The latent state in the adopted state-space model is estimated by means of particle filtering. We show that neglecting the time variation in the mean leads to a lower log-likelihood valuation. An increasing mean of the PtR from the early 1990s to 2007 is supported by the data.

We further analyse the endogenous relationship between the latent state for the PtR and credit market conditions. The results from VECMs confirm

the view that recent increases in the mean of the PtR have been significantly influenced by decreasing real mortgage rates and increasing securitization activities, especially since 2002. Moreover, increases in securitization activities have played an most important role in explaining the upward trend in the PtR. The effect of a standardized shock in securitization activities is about three times larger than the effect of a standardized shock in the real mortgage rates. Without securitization activities, negative nominal interest rates would have been needed to induce an equally strong housing boom since 2003.

For future research, two potential directions are worth considering. First, it would be of great interest to investigate the mean of the PtR at the level of regional or metropolitan areas. Second, to provide a full picture, we can consider a joint analysis of the effects of interest rates, securitization, and easy credit terms, such as the loan-to-value (LTV) ratio and the approval rate, on housing markets. The inclusion of easy credit terms could enrich the analysis, as agency problems associated with mortgage securitization contributed to easy credit terms. For both directions of research, the main obstacle is data availability. For the former direction of research, informative longitudinal (panel) data is needed. For the latter, Glaeser *et al.* (2013) show that the distribution of LTV ratios based on all mortgage debt did not change much over time, and the approval rate did not show any trend in increases. Any future analysis in this direction requires data allowing controls of different types of mortgages for the LTV ratio, and controls of characteristics of the marginal buyer for the approval rate.

Appendix

Data Description

We consider quarterly US data from the period 1975:Q1 to 2009:Q2 for the housing price index, the rent index, T-bill rates, and inflation. We use the FHFA (formerly the Office of Federal Housing Enterprise Oversight) housing price index, which provides the longest available quarterly time series of housing prices. The rent index is the rent of primary residence as a component of the consumer price index released by the United States Bureau of Labor Statistics (BLS). As the long-term risk-free rate, we use time series of the 10-Year Treasury Bill rate provided by the Board of Governors of the Federal Reserve System. The CPI excluding shelter from BLS is used to obtain time series of the smoothed inflation. Specifically, exponentially weighted moving averages of quarterly inflation are determined with a smoothing time period of 16 quarters.

The considered nominal mortgage rate is the contract rate on a 30-year fixed-rate conventional home mortgage. Data are provided by the Board of Governors of the Federal Reserve System. The real mortgage rate is obtained by deflating the nominal mortgage rate with inflation expectations as published in the Survey of Professional Forecasters at the Federal Reserve Bank of Philadelphia. It is the mean of forecasts for the annual average rate of CPI inflation over the next 10 years. Data are available for the period 1991:4 to 2009:2. To measure the securitization activities, we use the share of the home mortgage held by the private issuers of asset-backed securities. The related data are from the flow of funds accounts released by the Board of Governors of the Federal Reserve System. Notably, the data of total mortgage held by the issuers of asset backed securities are only available since 1984.

Approximations in the Present Value Model

Three approximations are adopted to derive the present value model in equation (5):

- (i) $E_t(\rho_{t+i}) \approx \rho_t$ for all $i \geq 1$;
- (ii) $E_t(\kappa_{t+i}) \approx \kappa_t$ for all $i \geq 1$;
- (iii) $E_t(\rho_{t+i}\eta_{t+i+1}) \approx E_t(\rho_{t+i})E_t(\eta_{t+i+1})$, with $i \geq 1$.

In this Appendix, we show that the resulting approximation errors are negligible.

Approximation (i) $E_t(\rho_{t+i}) \approx \rho_t$. First, note that the local mean of the PtR, $\bar{\eta}_t$, can be approximated by a martingale process. This is consistent with the empirical observation in Section II and the finance literature using a (bounded) martingale to approximate the steady-state log dividend to price ratio (Lettau and Van Nieuwerburgh, 2008). The martingale feature indicates that the local mean of the PtR is constant in expectation only, and can vary unexpectedly. It is also consistent with the observation by Case and Shiller, quoted for example in Shiller (2007), that times and places with high current home prices show high expectations of future home prices. Given that ρ_t is a concave function of $\bar{\eta}_t$, $\rho_t = 1/[1 + \exp(-\bar{\eta}_t)]$, and $\bar{\eta}_t$ is a martingale process, ρ_t is a supermartingale process, that is, $E_t(\rho_{t+i}) \leq \rho_t$, according to the Jensen inequality. However, the degree of the concavity in $\rho(\bar{\eta}_t)$ for the sensible range [4, 4.5] of $\bar{\eta}_t$ is very small. To evaluate the degree of the concaveness and its impact on the approximation error for $E_t(\rho_{t+i}) \approx \rho_t$, we compare the difference between

$b\rho(\bar{\eta}_1) + (1 - b)\rho(\bar{\eta}_2)$ and $\rho(b\bar{\eta}_1 + (1 - b)\bar{\eta}_2)$ for any $b \in [0, 1]$ and any $\bar{\eta}_1, \bar{\eta}_2 \in [4, 4.5]$. The maximal error is very small, about 0.00042.

Approximation (ii) $E_t(\kappa_{t+i}) \approx \kappa_t$. Given that κ_t is a concave function of ρ_t as $\kappa_t = -\ln(\rho_t) - (1 - \rho_t)\ln(1/\rho_t - 1)$, we find that $E_t(\kappa_{t+i}) \leq \kappa_t$ by the Jensen inequality. However, this concave function is approximately linear for the sensible range $[0.98, 0.99]$ of ρ_t for quarterly data. For any $b \in [0, 1]$ and any $\rho_1, \rho_2 \in [0.98, 0.99]$, the maximal difference between $b\kappa(\rho_1) + (1 - b)\kappa(\rho_2)$ and $\kappa(b\rho_1 + (1 - b)\rho_2)$ is about 0.00086. Thus, even though $\kappa(\rho_t)$ is a nonlinear function, it can be well approximated by means of a linear function such that the involved approximation error is almost negligible.

Approximation (iii) $E_t(\rho_{t+i}\eta_{t+i+1}) \approx E_t(\rho_{t+i})E_t(\eta_{t+i+1})$. The errors implied by this approximation are evaluated by Monte Carlo simulations. We generate ρ_t as defined in equation (7) with parameters as reported in the second row of Table 2. Given ρ_t , the local mean of the PtR can be obtained as $\bar{\eta}_t = -\ln(1/\rho_t - 1)$. Then, the PtR is generated as

$$\eta_t = \bar{\eta}_t + e_t + be_{t-1}, \quad b = 0.9754, \quad e_t \sim N(0, 0.0324). \quad (A1)$$

The deviation between the PtR η_t and the mean of the PtR $\bar{\eta}_t$ is formalized as an MA(1) process, which enables autocorrelations with high lags and, thus, persistence in the deviation $\eta_t - \bar{\eta}_t$. The MA parameter b and the variance of the error term are obtained from estimating the above equation by means of the available sample data. Conditional on time t , we follow two rival approaches to predict $\rho_{t+i}\eta_{t+i+1}$, $i = 1, 2, \dots, 120$. First, predictors are determined by means of high-order autoregressive models for $\phi_{t+i} \equiv \rho_{t+i}\eta_{t+i+1}$. Second, predictors of $\rho_{t+i}\eta_{t+i+1}$ are determined from the product of high-order autoregressive forecasts made separately for ρ_{t+i} and η_{t+i+1} . The adopted autoregression designs for both cases include 10 lags. From these two forecasts, we determine an absolute approximation error as $d_{t+i} = |\hat{\phi}_{t+i} - \hat{\rho}_{t+i}\hat{\eta}_{t+i+1}|$. To assess the magnitude of this approximation error, we consider relative approximation errors $\delta_{t+i} = d_{t+i}/\hat{\phi}_{t+i}$ that are determined for each forecast horizon i and time origin t and over a cross-section of simulated processes indicated by index r . The accuracy of approximation (iii) is assessed by means of

$$\bar{\delta}_i = \frac{1}{R(T - t_0 + 1)} \sum_{r=1}^R \sum_{t=t_0}^T \delta_{t+i}^{(r)}, \quad i = 1, 2, \dots, 120,$$

where $R = 1,000$ is the number of the MC replication, $T = 1,500$, and $t_0 = 500$ is the initial size of the estimation window for the high-order

autoregressive models. When t moves from t_0 to T , the estimation window expands accordingly. The simulation results show that the approximation error reaches about 0.0004 for the 120-step ahead forecasting. Thus, errors associated with approximation (iii) are also negligible. As a word of caution, it is fair to notice that the adopted simulation approach is, in particular, representative for the employed model specification. However, the magnitude of the mean approximation error remains small under (realistic) alternative parametrizations of the model, which incorporate the tight support of ρ and the high persistence of the log PtR. Results from detailed MC experiments are available from the authors upon request.

Particle Filter Algorithm

Step 1: Initialization ($t = 1$). Sample N particles $\tilde{\rho}_1^{(i)} \sim N(\rho_0, \sigma_\epsilon^2)$, $i = 1, \dots, N$, and determine importance weights

$$\tilde{w}_1^{(i)} = \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} \exp \left\{ -\frac{1}{2} \left[\frac{\tilde{\epsilon}_1^{(i)}}{\sigma_\epsilon} \right]^2 \right\}.$$

Normalized weights are obtained as

$$w_1^{(i)} = \frac{\tilde{w}_1^{(i)}}{\sum_i \tilde{w}_1^{(i)}}.$$

Step 2: Iteration ($t = 2, \dots, T$).

(a) Select N particles according to weights $w_{t-1}^{(i)}$. Set accordingly $\rho_{t-1}^{(i)} = \tilde{\rho}_{t-1}^{(i)}$ and $w_{t-1}^{(i)} = 1/N$ (resampling).

(b) For all particles draw

$$\tilde{\rho}_t^{(i)} \sim N(\rho_{t-1}^{(i)}, \sigma_\epsilon^2), \quad i = 1, \dots, N,$$

and determine raw weights

$$\tilde{w}_t^{(i)} = w_{t-1}^{(i)} \frac{1}{\sqrt{2\pi\sigma_\epsilon^2}} \exp \left\{ -\frac{1}{2} \left[\frac{\tilde{\epsilon}_t^{(i)}}{\sigma_\epsilon} \right]^2 \right\}.$$

(c) Normalize weights

$$w_t^{(i)} = \frac{\tilde{w}_t^{(i)}}{\sum_i \tilde{w}_t^{(i)}}.$$

(d) Go back to step (a).

By averaging over weighted draws, we obtain estimates of the contribution of ε_t to the Gaussian log-likelihood and, more interestingly, time-dependent estimates of ρ_t , that is,

$$\hat{\rho}_t = \frac{1}{N} \sum_{i=1}^N \rho_t^{(i)}, \quad t = 1, \dots, T.$$

With regard to the resampling step, we consider the so-called bootstrap particle filter proposed by Gordon *et al.* (1993).

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