Inexact Copula-Based Stochastic Programming for Water Resources Management under Multiple Uncertainties

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Abstract: Extensive uncertainties exist in many resources and environmental management problems, which can be interrelated and thus amplify the complexity and nonlinearity of study systems. The interactions from dependent random variables pose significant impacts on the potential management strategies. In this study, an inexact copula-based stochastic programming (ICSP) method is developed to deal with interactive uncertainties with interval and stochastic characteristics as well as to address nonlinear dependence among multiple random variables. Specifically, the impacts of their interactions among random variables are revealed based on the concept of copula. ICSP can also reflect the risk of violating system constraints with linear and nonlinear dependences. The developed ICSP method is then applied to planning water-resources management problems; results (i.e. system benefit, economic penalty, water allocation, and flood diversion) under a variety of risk levels have been generated. Results are useful for generating desired strategies for water allocation and flood diversion under various individual and joint probabilities. Compared to the conventional joint-probabilistic chance-constrained programming (JCCP) approach, ICSP can

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better reveal multiple uncertainties and their interrelationships under nonlinear condition and generate more robust solutions.

Keywords：Copula, decision making, join probability, multiple uncertainties, planning, water resources

Introduction

Uncertainties exist in a variety of components in water resources systems, resulting in extensive complexities in problems of water management and flood prevention (Guang et al., 2017; Wan et al., 2017; Fletcher et al., 2017; Cheng et al., 2017; Jato-Espino et al., 2018; Booras et al., 2018). Moreover, such uncertainties may exhibit multiple formats (e.g., interval, fuzzy and/or random features) due to their inherent complexities and data unavailability. Correspondingly, extensive optimization approaches were proposed for tackling such complex uncertainties in water resources systems (Huang et al., 1996; Singh, 2012; Yager, 2014; Hu and Li, 2015; Kong et al., 2016; Ghassemi et al., 2017; Pastori et al, 2017; Garg and Joshi, 2017; Goharian et al., 2018).

Joint chance-constrained programming (JCCP) is an effective way for measuring the reliability of system constraints when multiple uncertain constraints are satisfied at a specific level (Parlar, 1985; Watanabe and Ellis, 1994). JCCP can not only reflect the reliability of satisfying system constraints, but also analyze the interactive effects among various system constraints. Previously, a number of JCCP methods were developed for water resources management. Li and Huang (2010) coupled inexact two-stage integer programming method with JCCP to reflect joint probabilities existing in water availabilities and storage capacities.

Zhuang et al. (2015) developed an inexact joint probabilistic programming (IJPP) approach, in which the conventional JCCP was improved to reflect the randomness in the left-hand-side of constraints. The previous JCCP methods handle problems in which the relations among random variables are linear so that the sum of individual probabilities satisfies the joint probability of constraints. Instead, nonlinear interactions exist among these random right-hand sides. However, such nonlinearities are hard to incorporate in JCCP because: (1) the traditional joint probability methods can only capture the dependence between some random variables with specific probability distribution (e.g. normal, lognormal or gamma) (Zhang and Singh, 2006, 2007; Zeng, 2016), and (2) nonlinearity among random variables leads to nonlinear constraints, resulting difficulties in the solution process of the optimization model under multiple uncertainties.

Stochastic mathematical programming (SMP) is effective for tackling uncertainties existing in decision making problems, which is an extension of mathematical programming whose coefficients are represented as chances or probabilities (Li et al., 2008). As an extension of the previous efforts, this study aims to develop an inexact copula-based stochastic programming (ICSP) method for water-resources planning under multiple uncertainties. As a function that links univariate distribution to a flexible multivariate distribution (instead of a multivariate normal distribution), copula can effectively capture system dependence (especially for nonlinear dependence) among multiple variables, as well as tail dependence of uncertain data (Zhao and Lin, 2011). The concept of copula will be introduced into the two-stage joint-probabilistic chance-constrained programming framework to simulate nonlinear dependence among random variables in the right-hand sides of constraints which can not be dealt with using the existing optimization methods. The ICSP method can also effectively handle uncertainties expressed as random variables and interval numbers when the constraints are nonlinear, and robustly manage the overall system risk. Based on the risk level, the interactive effects of multiple system constraints can be investigated through the copula function.

Modeling formulation

Two-Stage Joint-Probabilistic Chance-Constrained programming

In water resources planning problems, the storage capacities of multiple reservoirs may be satisfied with a given probability, representing the risk of violating the constraints under uncertainty. Joint-probabilistic chance-constrained programming (JCCP) method can be used to deal with the problems that interactions exist among random variables in the constraints. In real-world water-resources planning problems, uncertainties often exist in both parameters and system components (Fan et al., 2015). Some future changes may impact the optimization processes, which need to be taken into account (Asztalos and Kim, 2017; Kong et al., 2017). In general, two-stage stochastic programming (TSP) can be used for solving water-resources system planning problems under random uncertainty; interrelationships exist among random variables in the right-hand sides of constraints; however, TSP can not reflect the dependence of random variables (Gu et al., 2013). Joint-probabilistic chance-constrained programming (JCCP) that is able to deal with interactions among random variables in the constraints can be incorporated into the TSP framework, leading to a two-stage joint-probabilistic chance-constrained programming (TJCP) model as follows (Model A):

$$\text{Maximize:} \quad f = \sum_{j=1}^{n} c_j x_j - \sum_{j=1}^{n} d_j \left(\sum_{i=1}^{m} p_i y_{ij} \right) \tag{1}$$

Subject to:

$$\sum_{j=1}^{n} a_{rj} x_j \le b_r, \quad r = 1, 2, \ldots, R \tag{2}$$

$$\sum_{j=1}^{n} \left(a_{kj} x_j + a'_{kij} y_{ij} \right) \ge \tilde{\omega}_{ki}, \quad i = 1, 2, \ldots, m; \quad k = 1, 2, \ldots, K \tag{3}$$

$$\Pr\left\{ \sum_{j=1}^{n} \left(a_{1j} x_j + a'_{1ij} y_{ij} \right) \le b_i(t), \ldots, \sum_{j=1}^{n} \left(a_{sj} x_j + a'_{sij} y_{ij} \right) \le b_s(t), \ldots, \sum_{j=1}^{n} \left(a_{S'j} x_j + a'_{S'ij} y_{ij} \right) \le b_{S'}(t) \right\} \ge 1 - P \tag{4}$$

$$x_j \ge 0, \quad j = 1, 2, \ldots, n \tag{5}$$

$$y_{ij} \ge 0, \quad j = 1, 2, \ldots, n; \quad i = 1, 2, \ldots, m \tag{6}$$

where f stands for net system benefit; x_j denotes decision variable which can be decided before the realizations of random variables; y_{ij} indicates the recourse decision variable which can be determined after the random variables are known; c_j stands for coefficient of x_j; d_j stands for coefficient of y_{ij}; p_i stands for the probability of occurrence for scenario i; a_{rj}, b_r, a_{kj}, a'_{kij}, a_{sj} and a'_{sij} stand for parameters in the constraints; $\tilde{\omega}_{ki}$ and $b_s(t)$ stand for the random variables; $j = 1, 2, \ldots, n$ stands for index of decision variables, x_j; $i = 1, 2, \ldots, m$, $r = 1, 2, \ldots, R$, $k = 1, 2, \ldots, K$ and $s = 1, 2, \ldots, S'$ stand for indices of scenarios. The related joint constraints are satisfied at a level of probability $1 - P$, corresponding to a violation risk level of P. TJCP focuses on dealing with uncertainties expressed as random variables, and tackling joint probabilistic constraints with linear relationship among multiple random variables. The nonlinear dependence among random variables in the constraints is hard to be simulated, which could become more difficult under uncertainty.

Copula Function

A copula function is a joint-distribution function that links together univariate distribution functions to a flexible multivariate distribution (Karmakar and Simonovic, 2007; Komino and Blachowicz, 2015; Verhoest et al., 2015). If F is a d-dimensional distribution, and $X_1, X_2, \ldots, X_d$ are random variables, copula C can be expressed as follows (Joe and Xu, 1996):

$$F\left(x_1, \ldots, x_d\right) = C\left(F_1\left(x_1\right), F_2\left(x_2\right), \ldots, F_d\left(x_d\right)\right) \quad (7)$$

where $F_1, F_2, \ldots, F_d$ are univariate margin distributions; $x_1, x_2, \ldots, x_d$ are values of random variables $X_1, X_2, \ldots, X_d$. Compared with the classical multivariate joint probabilistic distribution modeling, the copula approach has an advantage that the marginal distributions and multivariate dependence can be constructed separately. It means that marginal and joint probability functions in copula can be chosen more flexible (Kong et al., 2015; Fan et al., 2016). Moreover, there is no assumption for the variables in copula functions to be independent or normal or have the same type of marginal distributions (Zhang and Singh, 2007).

The Archimedean copula is one of the widely applied copula classes nowadays as (i) it can be employed whether the correlation among variables is positive or negative; (ii) it can be constructed easily, and contains a variety of copulas; (iii) it can capture extensive dependence structures with different desirable properties (Genest and Mackay, 1986). For the above reasons, Gumbel-Hougaard, Clayton (Cook-Johnson) and Ali-Mikhail-Haq copulas that belong to the Archimedean copula family are adopted in this study. They are used for building the dependence among random variables in the right-hand sides of constraints.

Ali-Mikhail-Haq copula C_α^{AMH}:

$$C_\alpha^{AMh} = \frac{\prod_{j=1}^{d} u_j}{\left[1-\alpha\prod_{j=1}^{d}\left(1-u_j\right)\right]} \quad (8)$$

where $\alpha \in [-1,1)$ is the parameter of Ali-Mikhail-Haq copula. Clayton copula C_α^{CJ} :

$$C_\alpha^{CJ} = \left[\left(\sum_{j=1}^{d} u_j^{-\alpha}\right) - d + 1\right]^{-1/\alpha} \quad (9)$$

and $\alpha \in [-1,\infty) \setminus \{0\}$ is the parameter of Clayton copula. Gumbel-Hougaard copula C_α^{GH} :

$$C_\alpha^{GH} = \exp\left\{-\left[\sum_{j=1}^{d}\left(-\ln u_j\right)^{\alpha}\right]^{1/\alpha}\right\} \quad (10)$$

and $\alpha \in [1,\infty)$ is the parameter of Gumbel-Hougaard copula.

In this study, the method of inference functions for margins (IFM) is used for parameter estimation. The major advantage of IFM is that the parameters of marginal distributions and those of copula can be estimated separately, simplifying the calculations of estimation (Joe and Xu, (1996). The maximum likelihood (ML) estimation is to determine the parameter of copula. Rosenblatt transformation with Cramér von Mises statistic is applied to investigate the property of joint distribution generated by copula (The detail descriptions about estimation of copula parameter and goodness-of-fit statistics for copula are shown in Supplemental Information).

Inexact Copula-Based Stochastic Programming

The concept of copula is introduced into the TJCP framework for tackling joint and interactive uncertainties with nonlinear dependence existed among random variables. Besides, various uncertainties (e.g., interval values) may also exist in the objective function and/or the constraints. Interval-parameter programming (IPP) can deal with uncertainties represented as interval numbers. Consequently, IPP and copula are introduced into the TJCP framework to deal with uncertainties presented in terms of interval values and random variables as well as to handle joint probabilistic constraints with nonlinear relationship among random variables in the constraints. This leads to an inexact copula-based stochastic programming (ICSP) model as follows (Model B):

$$\text{Maximize:}\quad f^{\pm} = \sum_{j=1}^{n} c_j^{\pm} x_j^{\pm} - \sum_{j=1}^{n} d_j^{\pm}\left(\sum_{i=1}^{m} p_i y_{ij}^{\pm}\right) - \sum_{t=1}^{T} e_t^{\pm} z_t \tag{11}$$

Subject to:

$$\sum_{j=1}^{n} a_{rj}^{\pm} x_j^{\pm} \le b_r^{\pm},\quad r = 1,2,\ldots,R \tag{12}$$

$$\sum_{j=1}^{n}\left(a_{kj}^{\pm} x_j^{\pm} + a_{kij}'^{\pm} y_{ij}^{\pm}\right) \ge \tilde{\omega}_{ki}^{\pm},\quad i = 1,2,\ldots,m;\quad k = 1,2,\ldots,K \tag{13}$$

$$C\left(\sum_{j=1}^{n}\left(a_{sj}^{\pm} x_j^{\pm} + a_{sij}'^{\pm} y_{ij}^{\pm}\right) \le b_s(t),\quad s = 1,2,\ldots,S'\right) \ge 1 - P,\quad i = 1,2,\ldots,m \tag{14}$$

$$z_t = 0 \quad or \quad 1,\quad t = 1,2,\ldots,T \tag{15}$$

$$x_j^{\pm} \ge 0,\quad j = 1,2,\ldots,n \tag{16}$$

$$y_{ij}^{\pm} \ge 0,\quad j = 1,2,\ldots,n;\quad i = 1,2,\ldots,m \tag{17}$$

where $x_j^{\pm}$ and $y_{ij}^{\pm}$ are interval decision variables. $f^{\pm}$ is net system benefit with interval values; $c_j^{\pm}$, $d_j^{\pm}$, $e_t^{\pm}$ are interval parameters in the objective function; p_i is the probability of occurrence for scenario i; $a_{rj}^{\pm}$, $b_r^{\pm}$, $a_{kj}^{\pm}$, $a_{kij}'^{\pm}$, $a_{sj}^{\pm}$ and $a_{sij}'^{\pm}$ are parameters with interval values in the constraints; $\tilde{\omega}_{ki}$ and $b_s(t)$ are random variable; $C(\cdot)$ is the copula function; $1-P$ is a prescribed joint probability level that joint constraints are enforced to be satisfied; z_t is the binary variable; $j = 1,2,\ldots,n$ is index of decision variables; $i = 1,2,\ldots,m$, $r = 1,2,\ldots,R$, $k = 1,2,\ldots,K$ and $t = 1,2,\ldots,T$ are indices of scenarios. An interval number $x^{\pm} = \left[x^{-}, x^{+}\right] = \left\{t \in x \middle| x^{-} \le t \le x^{+}\right\}$ is an interval with known upper bound x^{+} and lower bound x^{-} but unknown distribution information (Li et al., 2008; Kong et al., 2015).

Solution Method for Solving ICSP

In ICSP, the joint probabilistic distribution of constraints is constructed by the copula function, thus the dependence (including nonlinear dependence) among random variables in the right-hand sides of the constraints can be reflected. However, considering copula relations leads to more complex nonlinear constraints, resulting difficulties in the solution process of the optimization model under uncertainty. Therefore, there is difficulty in solving model B because of constraint (14) associated with nonlinear and uncertain features.

For the conventional TJCP model, the nonlinear constraint (4) can be transformed to linear constraints by letting the random variable take a set of individual probabilistic constraints. The individual probabilities are determined through ensuring that the sum of these probabilities is equal to the joint probability. This is because the relationships among random variables in the constraints are linear in TJCP. For the ICSP model, the nonlinear constraint (14) can also be transformed to a linear constraint. As $a_{sj}\ (s=1,2,\ldots,S')$ are deterministic and $b_s\ (s=1,2,\ldots,S')$ are random, Equation (14) can be decomposed into:

$$\sum_{j=1}^{n}\left(a_{sj}^{\pm}x_j^{\pm}+a_{sij}'^{\pm}y_{ij}^{\pm}\right)\le b_s\left(t\right)^{\left(p_s^0\right)},\quad s=1,2,\ldots,S'\quad i=1,2,\ldots,m \tag{18}$$

$$C\Big(F\big(b_1(t)\big),\ldots,F\big(b_s(t)\big),\ldots,F\big(b_{S'}(t)\big)\Big)\ge 1-P \tag{19}$$

where $F\big(b_s(t)\big)=1-p_s^0$ denotes the cumulative distribution of random variable $b_s(t)$ under scenario s. Equation (18) is a set of individual probabilistic constraints. The individual probabilities of constraints can be determined by solving Equation (19) in the following way: if there are S' random variables, the violation probabilities of the former $S'-1$ constraints (i.e. $p_s^0, s=1,\ldots,S'-1$) are predefined and the last probability is obtained through solving the equation: $C\Big(F\big(b_1(t)\big)=1\text{-}p_1^0,\ldots,F\big(b_{S'-1}(t)\big)=1\text{-}p_{S'-1}^0,F\big(b_{S'}(t)\big)\Big)=1-P$. Because the copula function is a nondecreasing monotonous function, a single value for $F\big(b_{S'}(t)\big)$ can be obtained. Moreover, since the values of $p_s^0, s=1,\ldots,S'-1$ are predefined, a series of combinations of individual probabilities can be obtained for one overall constraint violation probability [i.e. P in Equation (19)]. Therefore, constraint (14) can be decomposed into constraint (18) which is a set of linear constraints with constraint (19) being solved. In other words, the joint probabilistic constraints associated with copula relationship can be transformed to linear by letting the random variable take a set of individual probabilistic constraints with the corresponding individual probabilities determined by copula.

After transforming the joint probabilistic constraints with copula relations to linear constraints, the ICSP model can be solved by two-step solution method (Fan and Huang, 2012). The main idea of the two-step solution method is to transform inexact model with interval numbers to two deterministic sub-models by analyzing interactions among parameters and variables in both objective function and constraints. The sub-models correspond to the upper-bound and lower-bound objective function values, respectively (See in Supplemental Information). Then solutions for the ICSP model can be obtained:

$$f_{opt}^{\pm}=\left[f_{opt}^{-},f_{opt}^{+}\right] \tag{20}$$

$$x_j^{\pm}=\left[x_j^{-},x_j^{+}\right],\quad j=1,2,\ldots,n_1 \tag{21}$$

$$y_j^{\pm}=\left[y_j^{-},y_j^{+}\right],\quad j=1,2,\ldots,n_2 \tag{22}$$

Combing the transformation from copula-based joint probabilistic constraints to linear constraints and the two-step method discussed above, the procedure of solution algorithm for the ICSP method is given below:

Step 1: Formulate an ICSP model.

Step 2: Acquire the parameters, including inexact parameters with interval values and probability distribution.

Step 3: Determine the best fit copula function which can effectively capture the dependence among random variables in the constraints. The copula function is determined in three steps. First, some copula functions are selected based on the several desirable properties. Second, the parameters of copula functions are estimated, and the corresponding joint distributions among random variables are obtained based on the marginal distributions. Then, the results generated by the chosen copula are compared using good-of-fit statistics to determine the best fit copula function.

Step 4: Transform the nonlinear constraints to linear constraints by the copula method. Thus a set of individual probability combinations based on the specific joint probability are obtained.

Step 5: Reformulate the ICSP model to two deterministic sub-models by the two-step method.

Step 6: Solve the sub-models through simplex method. The order of solving sub-models is based on the objective function. If the objective function is to be maximized, first solve the sub-model corresponding to f^{+} (Otherwise, first solve the sub-model corresponding to f^{-} instead). Then solve the other sub-model on the basis of the solutions above.

Step 7: The solution of entire ICSP model can be obtained based on the solutions of two sub-models shown in Equations (20)-(22).

Application

Overview of the Studied System

In this study, a synthetic water-resources system planning problem is applied to illustrate the applicability of the proposed ICSP approach. The studied system consists of a river, a tributary of the river, and two storage reservoirs (as shown in Fig. 1) (Li and Huang, 2010;

Gu et al., 2013). The main functions of these two reservoirs are flood control, and the auxiliary functions of these two reservoirs are irrigation, aquaculture, hydropower, industrial and municipal water supplies (Gu et al., 2013). Between the two reservoirs, there is a tributary that provide water downstream cities and countries. Water supply to downstream target area changes as the diversifications of water availabilities and storage capacities. It may be difficult for decision makers to determine water-allocation targets and schemes in the case of uncertain water inflows and demands (Li and Huang, 2009). In detail, if the value of promised water is regulated high (i.e. the decision makers promise too much water to be realized), shortage may be generated. Penalties may be resulted to the local economy in accordance with the exceeded expenses which should be paid for the alternative water instead of local water. If the value of promised water is regulated low (i.e. the decision makers promise too little water to water users) and high stream inflows may lead to a raised surplus, reservoirs may overflow during flooding events. Flood diversion must be taken to release the surplus water. Therefore, effective flood diversion during the high inflow season and reasonable allocation of water resources during the low inflow season are critical in this water-resources system planning problem.

Place Fig. 1 here

There are various uncertainties in many components of water resources systems. In detail, flows of river and tributary are uncertain and exhibit probabilistic characteristics; the total storage and dead storage capacities of reservoirs are affected by many impacts and could be characterized to be random variables; the promised amounts of water are uncertain resulted by the uncertain available water flow from rivers and reservoirs; the benefits, penalties and costs for flood diversion may be available as interval values. Moreover, outflow from reservoir 1 and inflow of tributary both flow downstream into Reservoir 2, leading to a joint probability problem (Li and Huang, 2010). Therefore, a water allocation model that can deal with such a complicated water-resources system planning problem is in demand.

ICSP for Water Resources Management

In order to control flood and prevent waterlogging, the study problem for water resources allocation based on ICSP can be designed as model C. The objective is to achieve a maximum net system benefit, which is made up of three parts. They are the benefit for satisfying the promised water, the penalty when the promised water is not delivered and the cost for flood diversion. Constraints (24) and (25) indicate the relationship between current storage levels and initial storage levels in reservoirs 1 and 2. Related to the water resources system in Fig. 2, the current storage of reservoir 1 is determined by initial storage of reservoir 1, inflow from river, release flow from reservoir 1 and water loss; the current storage of reservoir 2 is determined by initial storage of reservoir 2, outflow from reservoir 1, inflow of tributary, release flow from reservoir 2 and water loss. Constraints (26) to (28) indicate the water delivery to the water users and diversion if the condition of overflows during flooding seasons occurs. Constraints (29) and (30) indicate the storage water of reservoirs cannot be greater than the maximum amount of reservoirs, and cannot be lower than the minimum amount of reservoirs. Constraint (31) indicates the water-allocation target should fall in between the maximum amount of water demand and the water shortage. Constraint (32) indicates whether or not there is a need for flood diversion. Constraint (33) and (34) are non-negativity and technical constraints.

Maximize

$$f^{\pm} = B^{\pm} X^{\pm} - C^{\pm}\left(\sum_{k_1=1}^{K_1}\sum_{k_2=1}^{K_2} p_{k_1} p_{k_2} Y^{\pm}_{k_1k_2}\right) - \sum_{k_1=1}^{K_1}\sum_{k_2=1}^{K_2} p_{k_1} p_{k_2}\left(F^{\pm} + V^{\pm} U^{\pm}_{k_1k_2}\right) Z_{k_1k_2} \qquad (23)$$

Subject to:

(1) Relationship between current storage levels and initial storage levels in reservoirs 1 and 2

$$ST^{\pm}_{k_1} = ST^{0\pm}_1 + q^{\pm}_{k_1} - \left[\left(\left(ST^{0\pm}_1 + ST^{\pm}_{k_1}\right)/2\right)\cdot\left(C_{CL}+C_{SL}+C_{EL}\right)\right] - RE^{\pm}_{k_1}, \quad \forall k_1 \tag{24}$$

$$ST^{\pm}_{k_1k_2} = ST^{0\pm}_2 + \left(q^{\pm}_{k_2} + RE^{\pm}_{k_1}\right) - \left[\left(\left(ST^{0\pm}_2 + ST^{\pm}_{k_1k_2}\right)/2\right)\cdot\left(C_{CL}+C_{SL}+C_{EL}\right)\right] - RE^{\pm}_{k_1k_2}, \quad \forall k_1, k_2 \tag{25}$$

$$\left(X^{\pm} - Y^{\pm}_{k_1k_2}\right) \le RE^{\pm}_{k_1k_2} - U^{\pm}_{k_1k_2} Z_{k_1k_2}, \quad \forall k_1, k_2 \tag{26}$$

$$RE^{\pm}_{k_1k_2} - U^{\pm}_{k_1k_2} Z_{k_1k_2} \le X^{\pm}, \quad \forall k_1, k_2 \tag{27}$$

$$\left(X^{\pm} - Y^{\pm}_{k_1k_2}\right) \ge U^{\pm}_{k_1k_2} Z_{k_1k_2}, \quad \forall k_1, k_2 \tag{28}$$

(2) Water storage constraints

$$C\left(\Pr\left\{ST^{\pm}_{k_1} \le \tilde{T}R_1, \forall k_1\right\}, \Pr\left\{ST^{\pm}_{k_1k_2} \le \tilde{T}R_2, \forall k_1, k_2\right\}\right) \ge 1 - P \tag{29}$$

$$C\left(\Pr\left\{ST^{\pm}_{k_1} \ge \tilde{D}R_1, \forall k_1\right\}, \Pr\left\{ST^{\pm}_{k_1k_2} \ge \tilde{D}R_2, \forall k_1, k_2\right\}\right) \ge 1 - P' \tag{30}$$

(3) Water-allocation target constraints

$$Y^{\pm}_{k_1k_2} \le X^{\pm} \le X^{\pm}_{\max} \tag{31}$$

(4) 0-1 constraints

$$Z_{k_1k_2} = \begin{cases} 1, & \text{water diversion is undertaken} \\ 0, & \text{otherwise} \end{cases} \quad \forall k_1, k_2 \tag{32}$$

(5) Non-negativity and technical constraints

$$0 \le U^{\pm}_{k_1k_2} \le M^{\pm}_{k_1k_2\max} Z_{k_1k_2}, \quad \forall k_1, k_2 \tag{33}$$

$$Y^{\pm}_{k_1k_2} \ge 0, \quad \forall k_1, k_2 \tag{34}$$

where $f^{\pm}$ is net system benefit (\$); $B^{\pm}$ is net benefit per unit water allocated (\$/m^3); $X^{\pm}$ is water allocation target (m^3); $C^{\pm}$ is penalty of net benefit per unit of water not delivered (\$/m^3); $Y^{\pm}_{k_1k_2}$ is water shortage under scenarios k_1 and k_2 (m^3); p_{k_1} and p_{k_2} are probabilities of scenarios k_1 and k_2, respectively; $F^{\pm}$ is fixed charge for flooding diversion (\$); $V^{\pm}$ is floating charge for flooding diversion (\$/m^3); $U^{\pm}_{k_1k_2}$ is amount of flooding diversion under scenarios k_1 and k_2 (m^3); $Z_{k_1k_2}$ is used for identifying whether or not the flooding diversion is needed to be undertaken under scenarios k_1 and k_2 (binary variable); $ST_{k_1}^{\pm}$ is storage volume of water in reservoir 1 under scenario k_1 (m^3); $ST_{k_1k_2}^{\pm}$ is storage volume of water in reservoir 2 under scenarios k_1 and k_2 (m^3); $ST_1^{0\pm}$ and $ST_2^{0\pm}$ are initial storage volumes of water in reservoirs 1 and 2 (m^3), respectively; $q_{k_1}^{\pm}$ and $q_{k_2}^{\pm}$ are available water resources under scenarios k_1 and k_2 (m^3), respectively; C_{CL}, C_{SL} and C_{EL} are coefficients of canal loss, seepage loss and evaporation loss, respectively; $RE_{k_1}^{\pm}$ is release flow from reservoir 1 under scenario k_1 (m^3); $RE_{k_1k_2}^{\pm}$ is release flow from reservoir 2 under scenarios k_1 and k_2 (m^3); $\tilde{TR}_1$ and $\tilde{TR}_2$ are total storage capacities of reservoirs 1 and 2 (m^3), respectively; $\tilde{DR}_1$ and $\tilde{DR}_2$ are dead storage capacities of reservoirs 1 and 2 (m^3), respectively; $X_{\max}^{\pm}$ is maximum amount of water demand (m^3); $M_{k_1k_2\max}^{\pm}$ is maximum amount of flooding diversion under scenarios k_1 and k_2 (m^3); P and P' are joint probabilities of violating risk in the chance constraints of total and dead storage capacities, respectively; k_1 and k_2 are indices of scenarios for stream 1 and stream 2, respectively; K_1 and K_2 are total numbers of scenarios for stream 1 and stream 2, respectively.

Tables 1, 2 and 3 present the related parameter values including economic data, water resources availability and reservoir storages, which are based on literatures (Gu et al., 2013). Benefits would be produced for water users if the promised water targets are satisfied. However, expenditure will be paid when two cases happen: One case is that the water pre-allocation does not meet the requirement, and the other case is that flood diversion practices are required when the streamflow is too large. Table 2 is the streamflow data of streams. The corresponding probabilities can be identified as probability distributions of seasonal flows. It also indicated that varieties of uncertainties exist in the modeling parameters. The interval values of seasonal flows and the corresponding probabilities of occurrences are determined by scenario-based method for solving stochastic programming (Li and Huang, 2009). Both stream 1 and stream 2 have three streamflow levels including low, medium and high. For stream1, there are five scenarios including L1, M1, M2, M3 and H1. While for stream 2, there are ten scenarios (i.e. L1, L2, L3, M1, M2, M3, M4, H1, H2 and H3). Table 3 provides the storage capacities of reservoir 1 and reservoir 2. The total storage capacity and dead storage capacity are supposed to be normally distributed. The synthetic data is used in this study. The scenarios of storage and dead capacities of two reservoirs are generated based on the synthetic data under given marginal distributions. However, whether or not the dependence exists between data about two reservoirs is the premise of constructing the joint distribution, since percentage of synthetic data is determined by the given marginal distribution. Therefore, a rank-based coefficients of correlation method, Kendall’s tau, is applied to examine the dependence structures of storage capacities between two reservoirs. The dependences of total and dead storage capacities between two reservoirs are both strong with the value of Kendall’s tau 0.863 and 0.818, respectively. It demonstrates that there exists dependence between synthetic data of reservoirs 1 and 2. Then the joint distribution of storage capacities between two reservoirs can be modeled by copula based on the given distributions in Table 3.

Places Table 1 to 3 here

Moreover, the water resources system is associated with two reservoirs where interactive uncertainties exist in terms of storage capacities including total storage capacities and dead storage capacities. Therefore, a series of joint probabilities on storage capacities of two reservoirs would be considered. The joint probabilities can reflect the risk of violating the capacity constraints. For example, decreasing joint probability P will decrease the risk for violating the water storage constraints of reservoirs 1 and 2 and will finally affect the final decision on water-allocation and flood-diversion schemes. However, the dependence structure of water storage data is usually very complex, exhibiting nonlinear features. Consequently, the copula method would be employed in this study to quantify this complicated dependence between two reservoirs.

Result Analysis

In this study, the copula theory is used to capture the interactions among random variables in the right-hand sides of model constraints, as expressed by Equations (29) and (30). The Archimedean copulas are employed to quantify the dependence between random variables and the most appropriate one would be chosen based on the results of goodness-of-fit, as shown in Table 4. Where 'Maximum LL' represents the maximum value of log-likelihood (LL); 'Estimated parameter' represents the copula parameter α; and '*p*-value' represents the *p*-value of the Cramér von Mises statistic. The performances of joint distributions generated by different copulas are tested by Rosenblatt transformation with Cramér von Mises statistic, whose samples are derived from the specific marginal distribution of reservoir data given in Table 3. The *p*-values of goodness-of-fit statistics are obtained based on Monte Carlo simulation. Table 4 indicates that: (i) for total storage capacity $\tilde{T}R$ and dead storage capacity $\tilde{D}R$, the *p*-value of each copula stated in the study is much higher than 0.05, thus the null hypothesis is accepted, meaning that the copula function is a valid model to quantify the joint probability of $\tilde{T}R$ and $\tilde{D}R$; (ii) the Clayton copula reaches the maximum value of LL for total storage capacity $\tilde{T}R$, and the Gumbel-Hougaard copula generates the maximum value of LL for dead storage capacity $\tilde{D}R$; consequently, the Clayton copula is chosen to measure the dependence of total storage capacities of reservoirs 1 and 2 (i.e. $\tilde{T}R_1$ and $\tilde{T}R_2$), and the Gumbel-Hougaard copula is chosen to measure the dependence of dead storage capacities of reservoirs 1 and 2 (i.e. $\tilde{D}R_1$ and $\tilde{D}R_2$).

Place Table 4 here

The joint cumulative distribution function (CDF) for storage capacities of reservoirs 1 and 2 are presented in Fig. 2, wherein (a) shows the joint CDF for total storage capacities generated by the Clayton copula, and (b) shows the joint CDF for dead storage capacities generated by the Gumbel-Hougaard copula.

Places Fig. 2 here

Benefits, penalties and diversion expenditure of the water resources system are shown in Table 5, where P denotes the joint probability of violating risk in the chance constraints of total storage capacities (i.e. $\tilde{T}R_1$ and $\tilde{T}R_2$); different individual probabilistic combinations for joint probability P are expressed as p_1^0 and p_2^0; P' denotes the joint probability of violating risk in the chance constraints of dead storage capacities (i.e. $\tilde{D}R_1$ and $\tilde{D}R_2$), with different individual probabilistic combinations of $p_1'^0$ and $p_2'^0$. The joint probability $P = 0.05$ means the violation level for the total storage capacity of two reservoirs will less than 5%. The joint probability $P' = 0.05$ and $P' = 0.01$ mean that the violation levels for the dead storage capacity of two reservoirs will less than 5% and 1%, respectively. The value of joint probability is determined based on the significance level, which is usually set as 0.05 or 0.01. The results show that the system benefit and the corresponding system penalty vary with different joint and individual probabilities of reservoir-storage capacities constraints. p_1^0 and p_2^0 mean the individual probabilities of reservoir-total storage capacities, while $p_1'^0$ and $p_2'^0$ denote the individual probabilities of reservoir-dead storage capacities.

Place Table 5 here

Moreover, the system benefit increases and the corresponding system penalty decreases when joint probability of $\tilde{D}R_1$ and $\tilde{D}R_2$ decreases for a given joint probability of $\tilde{T}R_1$ and $\tilde{T}R_2$. Among all the combinations of probabilities, the combination 5# gains the maximum system benefit \$ [-5.207, 79.917] × 10^6. The minimum system penalty is \$ [73.028, 101.185] × 10^6 which corresponds to the combination 6#. The condition of system benefit less than zero means loss. In other words, if the system penalty is too large to exceed the possible benefit from the water pre-allocation, system benefit may go negative. In the study, fifty scenarios are generated for water shortage given a specific probabilistic combinations of total storage capacities (p_1^0 and p_2^0) and probabilistic combinations of dead storage capacities ($p_1'^0$ and $p_2'^0$). Fig. 3(a) gives the amount of water shortage under different scenarios to water user under combination 17#. The corresponding joint probabilities of total and dead storage capacities are $P = 0.05$ and $P' = 0.01$, respectively. The results show that the amount of water shortage decrease when the streamflow increase. The amounts of flood diversion under different scenarios under combination 2# are shown in Fig. 3(b). The corresponding joint probabilities of total and dead storage capacities are $P = 0.05$ and $P' = 0.05$, respectively. The results indicate that (i) there is no need for flood diversion under the streamflow levels L1, L2, L3, M1, M2 and M3 of stream 2; (ii) the amount of flood diversion increase when the streamflow increase. Therefore, schemes for water shortage and flood diversion should be designed under various streamflow levels and joint probabilities of storage capacities.

Place Fig. 3 here

Consequently, the preferences of decision makers under different probability level vary based on the trade-off among system benefit, penalty, flood diversion expenditure and risk of violating constraints. In detail, a higher joint probability of constraints leads to a higher system benefit but a higher risk of violation, vice versa.

Comparison of ICSP with JCCP

The study problem could also be solved through the conventional JCCP method through modeling joint probability of storage capacities of reservoirs with linear constraints (Li and Huang, 2010; Gu et al., 2013). In JCCP, the individual probabilistic combinations are set to the values adding up to the joint probability. For instance, the joint probability of total storage capacities is $P = 0.05$, the corresponding individual probabilistic combinations may be $p_1^0 = 0.01$ and $p_2^0 = 0.04$, $p_1^0 = 0.025$ and $p_2^0 = 0.025$, $p_1^0 = 0.04$ and $p_2^0 = 0.01$, and so on (Li and Huang, 2010). Fig. 4(a) shows the individual probabilistic combinations generated by ICSP and JCCP, respectively. Each probabilistic combination of constraints is enforced to be satisfied at the joint probability 0.05. It is indicated that for ICSP (1) the dependence of storage capacities between two reservoirs is nonlinear; (2) the dependence of total storage capacities between two reservoirs is different from that of dead storage capacities; (3) the individual probabilistic combinations under a specific joint probability would change as the distributions of $\tilde{TR}$ and $\tilde{DR}$ are varied. This is because the distributions of $\tilde{TR}$ and $\tilde{DR}$ which act as marginal distributions in the copula function are different from each other; and the copula function for estimating joint distribution of total storage capacities is different from that of dead storage capacities. In comparison for JCCP, the graph of combinations is straight line, indicating that the relationship of water storages between reservoir 1 and 2 is linear. Moreover, the curve of individual probabilistic combinations of $\tilde{TR}$ is coincident with that of $\tilde{DR}$ in JCCP. It means that the dependence structure of total storage capacities between two reservoirs is the same as that of dead storage capacities, which is not reasonable in reality. On the other hand, there is a tributary between two reservoirs. It leads to more complicated in the water-resources system planning problem, involving the relationship between two reservoirs. Copulas can exactly model the dependence structure among random variables, which have been widely used in hydrology and water resources research (Salvadori and Michele, 2004, Genest et al., 2009). From the above discussion, the nonlinear dependence modeled by the copulas in this study may be regarded as a more accurate representation of the reality. The individual probability combinations generated by ICSP and JCCP are employed to the water resources system, respectively. Fig.4(b) compares the system benefits from the two methods. $I_{\min}$ and $I_{\max}$ are the minimum and maximum system benefits generated by ICSP, respectively; $J_{\min}$ and $J_{\max}$ are the minimum and maximum system benefits generated by JCCP, respectively. The results show that the intervals of system benefit generated by ICSP are wider than those obtained by JCCP. Although the differences in ranges are very small, it still means that there are more decision alternatives feasible for the water resources system generated by ICSP than those generated by JCCP. This is because the relationship among random variables in constraints is simulated by linear method in JCCP, leading to a narrower feasible region than the actual interval solutions. The main limitation of JCCP is its estimation of joint distribution; it can only reflect linear dependence of storage capacities between two reservoirs. This also leads to that some potential decision alternatives that are still feasible for the water resources system may be neglected. In comparison, the ICSP method proposed in this study can directly incorporate copulas within its optimization framework, thus has advantages over the JCCP in reflecting dependence (including nonlinear dependence) of storage capacities between two reservoirs.

Place Fig. 4 here

Compared with JCCP, ICSP can effectively tackle water resources system planning problems in real life, where dependence among random variables in the joint probabilistic constraints may be nonlinear. The conventional JCCP is to generate optimal solutions subject to the joint probability estimated by linear approach, weakening the interactions among random variables. In additional, nonlinearity leads to more complex nonlinear constraints, resulting difficulties in the solution process of the optimization model under uncertainty. As an improvement of JCCP, the ICSP method can be applied to systems with complex relationship existing among random variables. The dependence (including nonlinear dependence) is effectively estimated by copulas, making the model more practical in reflecting the real-world system. However, when many random variables in the constraints exist, a large number of calculations will have to be conducted. This is because the estimation of parameters for multivariate copula function may be inconvenient. This may lead to difficulties in its application to large-scale problems. Therefore, development of a more advanced solution algorithm for further enhancing ICSP is in demand.

ICSP is applied to a synthetic water-resources system planning problem in this study. In general, real-world water-resources system planning problems are more complex than the synthetic case. However, the related parameter values including economic data, water resources availability and reservoir storages are based on information acquired from real-world studies in Gu et al (2013). Therefore, the synthetic case is sufficient to show substantive characteristics of real-world water-resources system planning problems. This study attempts to make a small improvement in theory and methods for dealing with interactions among random variables in the inexact optimization model. The proposed ICSP method can be further applied to real case studies that call for constructing joint probability in an optimization modeling framework in various planning systems.

Conclusion

In this study, an inexact copula-based stochastic programming (ICSP) method has been developed for tackling joint and interactive uncertainties with nonlinear dependence among random variables. The concept of copula is first introduced into the joint-probabilistic chance-constrained programming framework to reflect interactions among random variables. Archimedean copulas are applied to quantify the joint probabilities among dependent random variables in the right-hand sides of constraints. Although considering the copula relations leads to complex nonlinear constraints, these nonlinear constraints can be transformed to linear constraints by letting the random variable take a series of individual probabilistic constraints and determining the corresponding individual probabilities of constraints which are satisfied at the joint probability by copula. Then the optimization model under uncertainty can be solved by two-step solution method.

A case study of water-resources system planning has been provided to illustrate the applicability of ICSP. The dependence between total storage capacities is measured by the Clayton copula, and the dependence between dead storage capacities is measured by the Gumbel-Hougaard copula. Solutions under a set of joint probabilities and individual probabilities of reservoir-storage capacity constraints can be obtained by solving a series of deterministic sub-models. Results show that the system benefit and the corresponding system penalty vary with different joint and individual probabilities of constraints. Moreover, it is found that a higher joint probability of constraints leads to a higher system benefits but a higher risk of violating, vice versa. These could bring more useful information as well as enable managers to make better decisions on the system benefit, water allocation and flood diversion. The comparison between ICSP and JCCP shows that ICSP can better reflect nonlinear dependence between random variables and the generated individual probabilistic combinations would change as the copula function and marginal distributions of variables change. However, the interaction between variables generated by JCCP is linear and would not change along with the change of distributions of variables. Furthermore, the intervals of system benefit generated by ICSP are wider than those obtained by JCCP, which means that JCCP may neglect some feasible decision alternatives for the water resources system.

This study is the first attempt of introducing copula function into the inexact optimization modeling framework. It is found that the correlation between storage capacities of reservoirs is complex and nonlinear, which means that the joint probabilities and individual probabilities of reservoir-storage capacity constraints cannot be determined by a simple summation or multiplicative method. The development in the future should focus on the approximation of distributions of random variables using some reliable methods in the inexact optimization model and the variation of solution along with the continuous ranges of random variables.

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List of Table Captions

Table 1 Parameter related to water users and economic data

Parameters	Parameters meaning	Water user
$T^{\pm}$ ($10^6 m^3$)	Water allocation target	$[3800, 7800]$
$T^{\pm}_{max}$ ($10^6 m^3$)	Maximum allowable water allocation	15000
$NB^{\pm}$ ($/m^3$)	Net benefit when water demand is satisfied	$[0.018, 0.023]$
$PE^{\pm}$ ($/m^3$)	Penalty when water is not delivered	$[0.070, 0.084]$
$FC^{\pm}$ (10^6/m^3$)	Fixed expenditure for flood diversion	$[0.0015, 0.0022]$
$VC^{\pm}$ ($/m^3$)	Variable expenditure for flood diversion	$[0.020, 0.025]$

Table 2 Streamflow data under various probabilities of occurrences

Streamflow level	Probability and stream inflow of stream 1			Probability and stream inflow of stream 2		
		p_{k_1}	$q^{\pm}_{k_1}$ ($10^6 m^3$)		p_{k_2}	$q^{\pm}_{k_2}$ ($10^6 m^3$)
Low	L1	0.15	$[150, 550]$	L1	0.05	$[400, 850]$
				L2	0.05	$[1200, 1650]$
				L3	0.05	$[2000, 2450]$
Medium	M1	0.15	$[620, 770]$	M1	0.15	$[2950, 3400]$
	M2	0.20	$[850, 1000]$	M2	0.20	$[4250, 4700]$
				M3	0.20	$[6050, 6500]$
	M3	0.15	$[1080, 1230]$	M4	0.15	$[6850, 7300]$
High	H	0.15	$[1300, 1700]$	H1	0.05	$[7800, 8350]$
				H2	0.05	$[8600, 9050]$
				H3	0.05	$[9900, 10350]$

Table 3 Reservoir data

	Reservoir 1	Reservoir 2
Total storage capacity ($\tilde{TR}^{\pm}_1$ and $\tilde{TR}^{\pm}_2$ ($10^6 m^3$))	$N(283.50, 14.5^2)$	$N(498.5, 10.17^2)$
Dead storage ($\tilde{DR}^{\pm}_1$ and $\tilde{DR}^{\pm}_2$ ($10^6 m^3$))	$N(26.80, 0.53^2)$	$N(68.20, 0.60^2)$
Initial storage ($S^{0\pm}_1$ and $S^{0\pm}_2$ ($10^6 m^3$))	$[45.40, 51.20]$	$[88, 96.50]$

Table 4 Estimated parameter of copula and *p*-value of goodness-of-fit statistics

Copula	Joint distribution of $\widetilde{TR}$			Joint distribution of $\widetilde{DR}$		
	Maximum LL	Estimated parameters	*p*-value	Maximum LL	Estimated parameters	*p*-value
Clayton	**223.95**	**7.32**	0.8144	176.76	6.15	0.4725
Gumbel-Hougaard	164.50	3.48	0.9994	**198.27**	**5.48**	0.4915
Ali-Mikhail-Haq	113.05	0.90	0.8634	96.17	0.99	0.2485

Table 5 System benefits and penalties of the water resources system under different joint probabilities

Combinations	$\widetilde{TR}$			$\widetilde{DR}$			System benefit (10^6\$)	System penalty (10^6\$)
	Joint probability	Individual probability		Joint probability	Individual probability			
	P	p_1^0	p_2^0	P'	$p_1'^0$	$p_2'^0$		
1#	0.05	0.0510	0.0729	0.05	0.0510	0.2609	[-5.255, 79.876]	[73.028, 101.507]
2#					0.0710	0.0851	[-5.230, 79.892]	[73. 028, 101.504]
3#					0.2609	0.0510	[-5.660, 79.883]	[73. 028, 101.503]
4#				0.01	0.0110	0.0699	[-5.217, 79.908]	[73. 028, 101.189]
5#					0.0160	0.0249	**[-5.207, 79.917]**	[73. 028, 101.186]
6#					0.0699	0.0110	[-5.207, 79.913]	**[73. 028, 101.185]**
7#		0.0560	0.0570	0.05	0.0510	0.2609	[-5.432, 79.865]	[73. 028, 101.734]
8#					0.0710	0.0851	[-5.254, 79.881]	[73. 028, 101.513]
9#					0.2609	0.0510	[-5.259, 79.872]	[73. 028, 101.512]
10#				0.01	0.0110	0.0699	[-5.241, 79.897]	[73. 028, 101.198]
11#					0.0160	0.0249	[-5.230, 79.906]	[73. 028, 101.196]
12#					0.0699	0.0110	[-5.231, 79.902]	[73. 028, 101.194]
13#		0.0729	0.0510	0.05	0.0510	0.2609	[-5.438, 79.860]	[73. 028, 101.713]
14#					0.0710	0.0851	[-5.264, 79.876]	[73. 028, 101.517]
15#					0.2609	0.0510	[-5.270, 79.867]	[73. 028, 101.516]
16#				0.01	0.0110	0.0699	[-5.251, 79.892]	[73. 028, 101.202]
17#					0.0160	0.0249	[-5.237, 79.901]	[73. 028, 101.200]
18#					0.0699	0.0110	[-5.241, 79.898]	[73. 028, 101.199]

Figure 1 Click here to download Figure Fig. 1.pdf

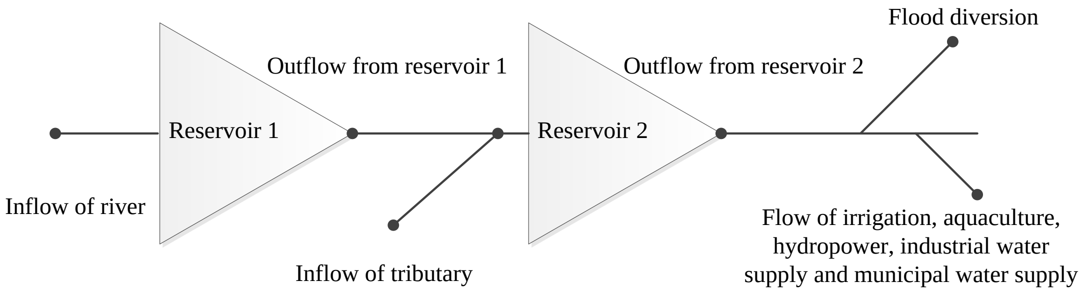
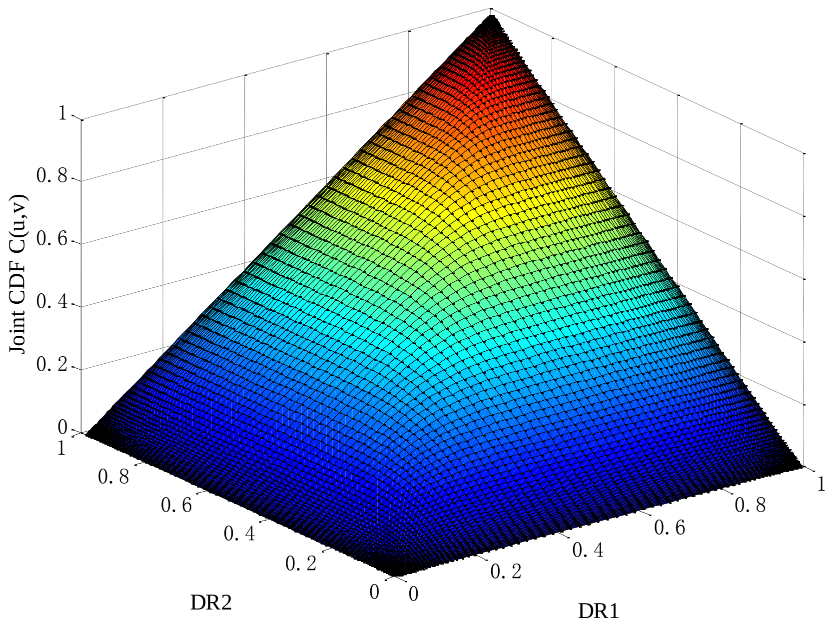
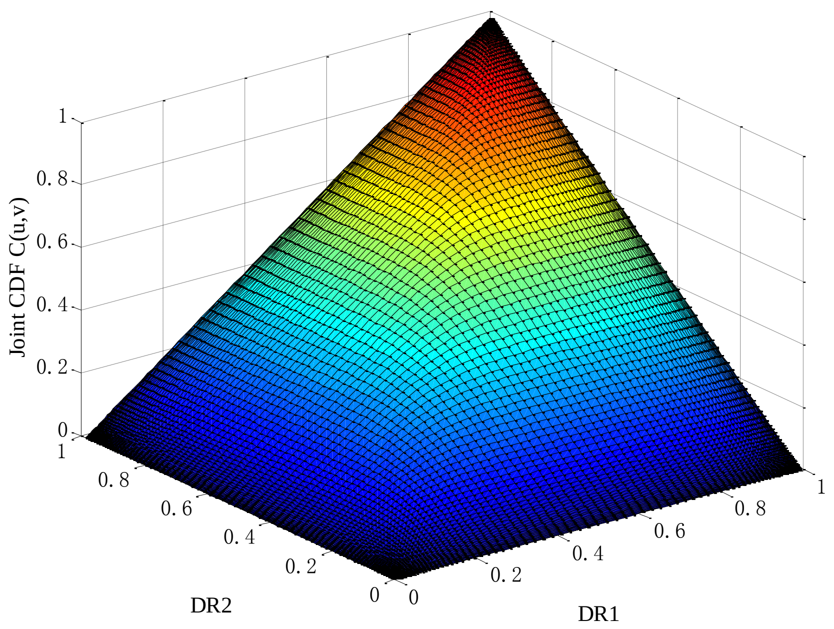


Fig. 1. Principle scheme of the water resources system

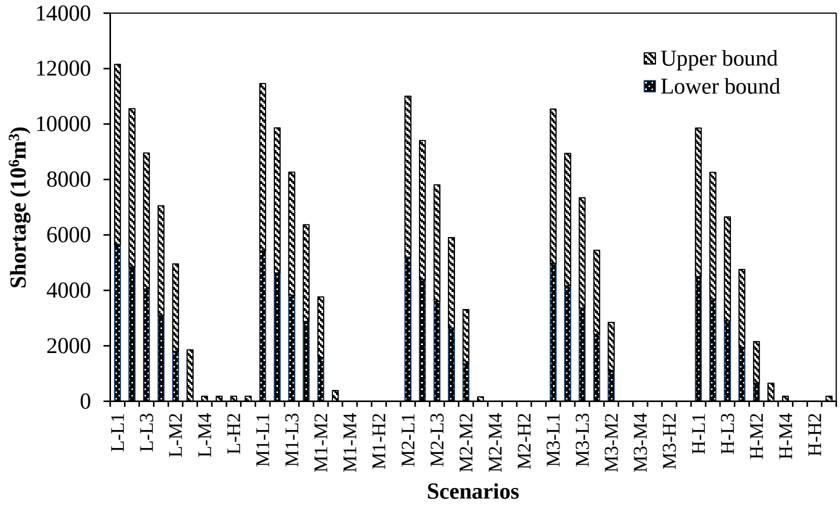


(a)

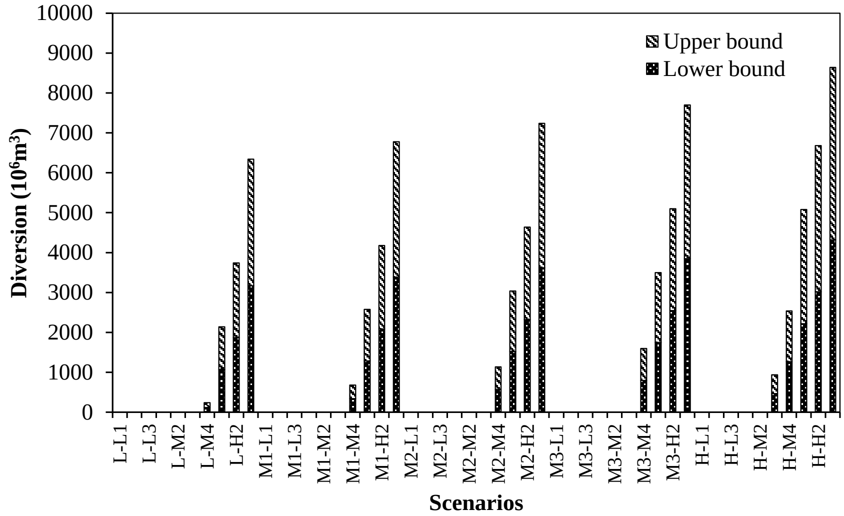


(b)

Fig. 2. Joint CDF for storage capacities, where (a) represents joint CDF for total storage capacities; (b) represents joint CDF for dead storage capacities



(a)



(b)

Fig. 3. Water shortage and flood diversion under different scenarios, where (a) represents water shortage under combination 17#; (b) represents flood diversion under combination 2#

Figure 4 Click here to download Figure Fig. 4.pdf

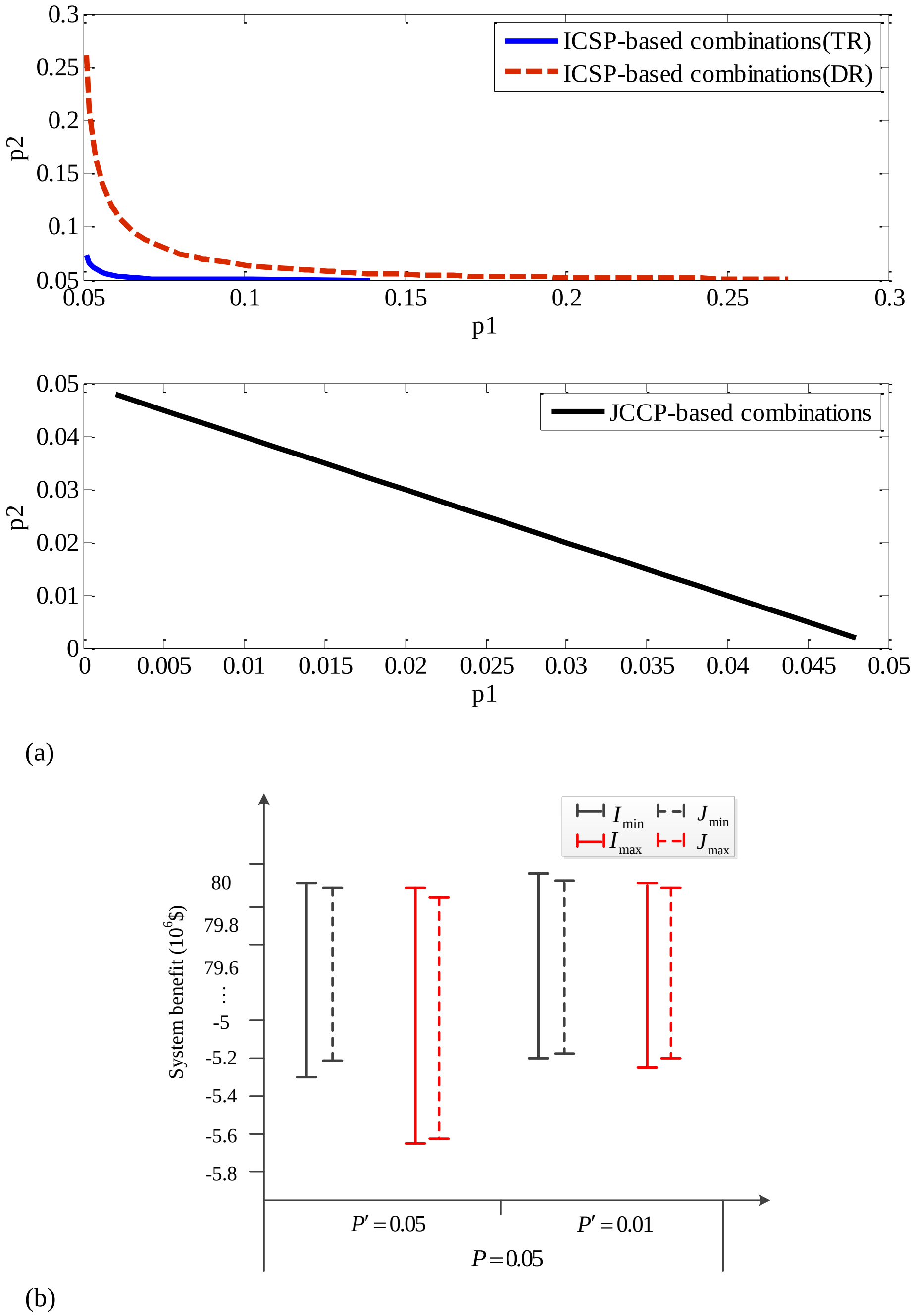


Fig. 4. Comparison between ICSP and JCCP, where (a) represents comparison of individual probabilistic combinations (joint probability = 0.05); (b) represents comparison of system benefit intervals