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Long-term interest rates in Europe: A fractional cointegration analysis

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ABSTRACT

This paper uses fractional integration/cointegration techniques to examine the stochastic behaviour of long-term interest rates (on government securities with 10-year maturity) in 23 European countries as well as their long-run linkages on a pairwise basis over the period January 2001–February 2018. The results are mixed and sensitive to the (parametric and semi-parametric) estimation methods. Evidence is found for both unit roots and mean reversion in the series analysed. Various rates (especially in the case of smaller economies) appear to be fractionally cointegrated, but interestingly German, French and UK rates are not found to be linked to any other European rates.

1. Introduction

The aim of this paper is to examine the stochastic behaviour of long-term interest rates in Europe as well as their long-run linkages on a pairwise basis by using fractional integration and cointegration techniques respectively. It is well known that most interest rate series exhibit high persistence; however, the debate on the most appropriate statistical model for them is still open. Earlier studies were normally based on the I(0)/I(1) dichotomy. For instance, [Cox, Ingersoll, and Ross \(1985\)](#) characterized the short-term nominal interest rate as a stationary and mean-reverting I(0) process, whilst [Campbell and Shiller \(1987\)](#) concluded that it exhibits a unit root.

More general I(d)-type specifications have been adopted in subsequent studies. For instance, [Shea \(1991\)](#) found some evidence for the expectations hypothesis of the term structure estimating a long-memory model. [Backus and Zin \(1993\)](#) argued that the hyperbolic decline of the volatility of bond yields can be modelled using a fractionally integrated specification. [Lai \(1997\)](#) and [Phillips \(1998\)](#) and [Tsay \(2000\)](#) both found that a fractional integration framework is appropriate for US real interest rates and [Couchman, Gounder, and Su \(2006\)](#) presented similar evidence for sixteen countries. [Caporale and Gil-Alana \(2009\)](#) reported that in the case of the Federal Funds effective rate the fractional integration parameter is sensitive to the specification for the error term, whilst [Caporale and Gil-Alana \(2012, p. 4035\)](#) obtained evidence of long memory and fractional integration with cycles repeating every eight years. Finally, [Gil-Alana and Moreno \(2012\)](#) and [Abbritti, Gil-Alana, Lovcha, and Moreno \(2016\)](#) estimated a fractional integration model for the short-term interest rate and the term premium.

Concerning the euro area in particular, [Busch and Nautz \(2009\)](#) used a long memory framework and found that the persistence of the spread between market and policy rates has decreased, i.e. monetary policy can control interest rates more effectively. However, [Hassler and Nautz \(2008\)](#) and [Cassola \(2008, p. 982\)](#) estimated a fractional integration parameter around 0.25, which suggests less controllability (see also [Cassola & Morana, 2008](#), in the case of interest rates of one-week maturity).

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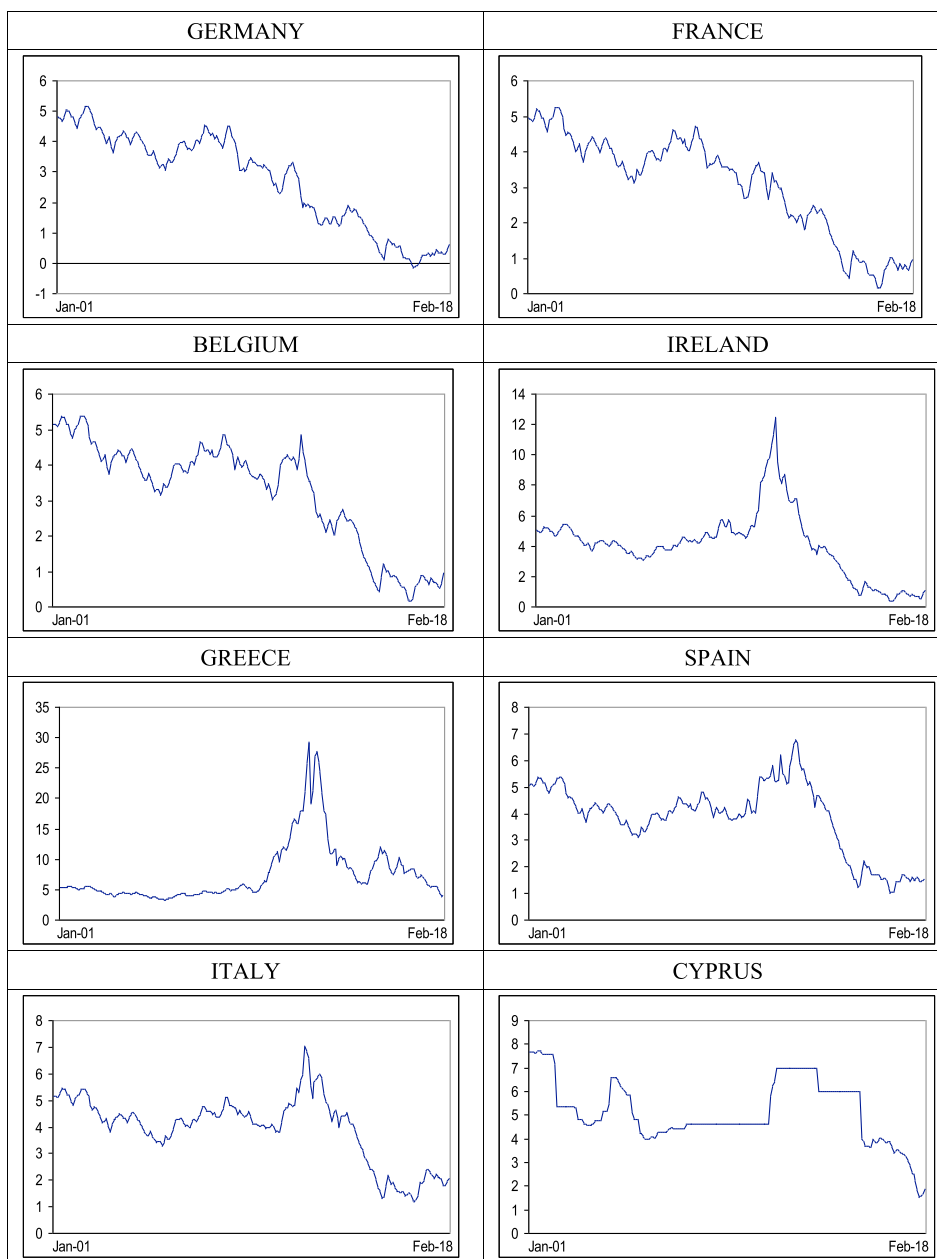


Fig. 1. Time series plots.

An interesting issue is whether interest rates are linked in the long run. According to the theory of interest rate parity, given perfect capital mobility, fixed exchange rates and perfect capital markets, interest rates will be equal across countries. However, the presence of market imperfections implies that interest rate differentials across countries will still be found. Recent papers on interest rate modelling include among others [Hacker, Karlsson, and Mansson \(2014\)](#), [Escobar, Ferrando, and Rubtson \(2016\)](#), [Garbers and Liu \(2018\)](#), and [Probst \(2019\)](#).

Interest rate linkages have often been investigated in the literature by carrying out cointegration analysis. For instance, unit root and cointegration tests were performed by [DeGennaro, Kunkel, and Lee \(1994\)](#) to examine the stochastic properties of the long-run relationships between interest rates on long-term government bonds issued by the US, Canada, Germany, UK and Japan. Regarding Europe specifically, [Karfakis and Moschos \(1990\)](#) and [Kirchgassner and Wolters \(1995\)](#) both investigated short-term nominal interest rate interdependencies between Germany and other EMS rates using Granger Causality and cointegration tests. [Wang, Jang, and Li \(2007\)](#) examined linkages among major Euro currency interest rates during 1994–2002 using a cointegrated VAR and found much stronger linkages when allowing for contemporaneous causality as well.

[Barkoulas, Baum, and Oguz \(1997\)](#) extended earlier studies on the system of long-term international interest rates by allowing for a fractionally integrated error correction term; in this case, even though mean reversion still occurs, the adjustment process towards the

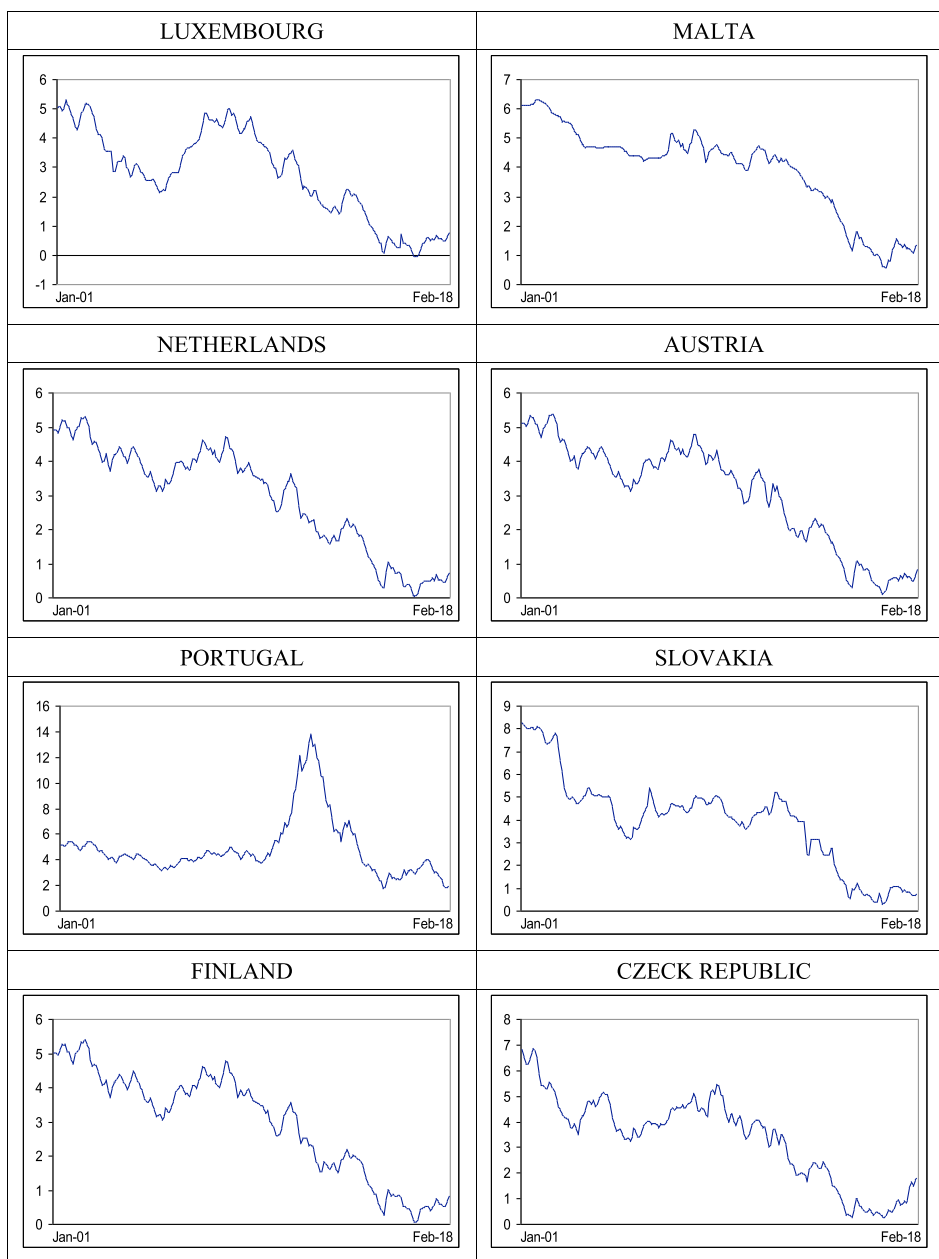


Fig. 1. (continued).

long-run equilibrium relationship can be very slow, i.e. shocks are allowed to have highly persistent effects, albeit disappearing in the long run. The motivation for this type of modelling approach is that the assumptions imposed by standard unit root and cointegration tests might be too restrictive; therefore it is important to consider the possibility of fractional orders of integration/cointegration with a slow rate of decay. Following their study, we also use a fractional integration/cointegration framework, but focus instead on long-term European rates. The layout of the paper is the following: Section 2 outlines the methodology; Section 3 presents the empirical results; Section 4 offers some concluding remarks.

2. Methodology

In this paper we use fractional integration and cointegration techniques widely employed for analyzing macroeconomic and financial time series (Gil-Alana, 2003; Gil-Alana & Hualde, 2009; Gil-Alana & Robinson, 1997). For our purposes, we define an I(0) process as a covariance stationary one with a spectral density function that is positive and finite at the zero frequency. This includes white noise as well as any type of stationary ARMA processes. Specifically, a process $\{x_t, t = 0, \pm 1, \dots\}$ is said to be I(d) if it can be represented as

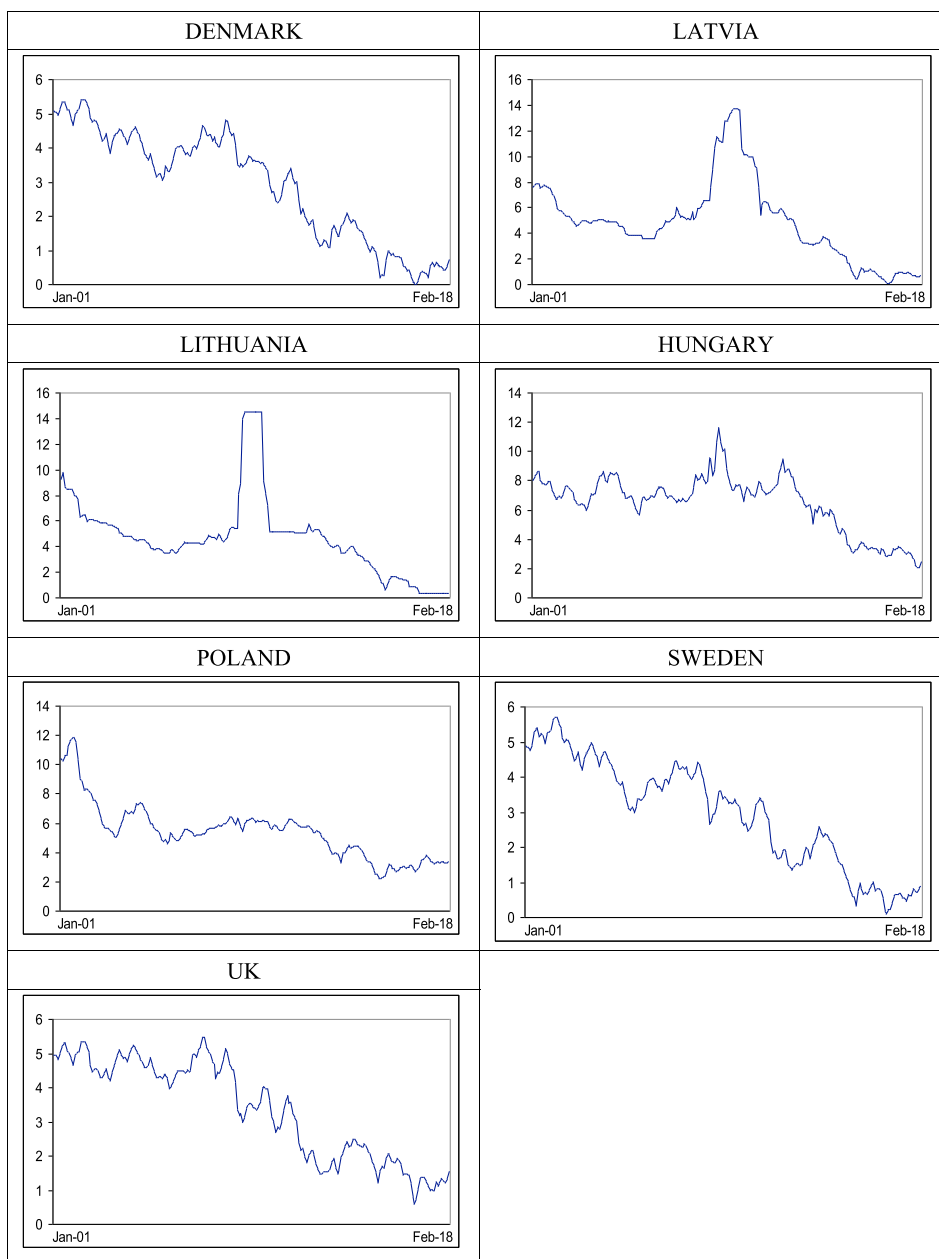


Fig. 1. (continued).

$$(1 - L)^d x_t = u_t, t = 1, 2, \dots \tag{1}$$

where L is the lag operator, d can be any real number, and u_t is assumed to be I (0).

We first consider parametric models where x_t from (1) can be the errors in a regression model of the form:

$$y_t = \beta_0 + \beta_1 t + x_t, t = 1, 2, \dots, \tag{2}$$

y_t being the observed time series (in our case the long-term interest rates), and β_0 and β_1 the coefficients of the intercept and the time trend. We consider both uncorrelated (white noise) and autocorrelated u_t under the I (0) assumption. In the latter case, we use the exponential spectral model of Bloomfield (1973), which is a non-parametric method for approximating ARMA processes with only a few parameters (Gil-Alana, 2004). Our approach is based on the Whittle function in the frequency domain (Dahlhaus, 1989), but we also use the Lagrange Multiplier (LM) test of Robinson (1994) for the null $d = d_0$ in (1) for any real value of d_0 . In addition, we apply a semi-parametric method that does not impose any functional form on the I (0) error term; this is based on a “local” Whittle approach

Table 1
Estimated values of d and 95% confidence bands with white noise errors.

	No regressors	An intercept	A linear trend
GERMANY	1.06 (0.97, 1.18)	1.19 (1.06, 1.36)	1.19 (1.06, 1.36)
FRANCE	1.04 (0.95, 1.17)	1.17 (1.03, 1.35)	1.17 (1.03, 1.35)
BELGIUM	1.06 (0.96, 1.18)	1.17 (1.05, 1.34)	1.17 (1.05, 1.34)
IRELAND	1.11 (1.02, 1.23)	1.18 (1.08, 1.31)	1.18 (1.08, 1.31)
GREECE	1.02 (0.93, 1.14)	1.03 (0.94, 1.15)	1.03 (0.94, 1.15)
SPAIN	1.03 (0.94, 1.15)	1.11 (1.01, 1.25)	1.11 (1.01, 1.25)
ITALY	1.04 (0.95, 1.16)	1.14 (1.02, 1.28)	1.14 (1.02, 1.28)
CYPRUS	1.02 (0.92, 1.15)	1.15 (1.06, 1.27)	1.15 (1.06, 1.27)
LUXEMBOURG	1.06 (0.97, 1.18)	1.27 (1.14, 1.43)	1.27 (1.14, 1.43)
MALTA	1.03 (0.94, 1.14)	1.22 (1.10, 1.38)	1.22 (1.10, 1.38)
NETHERLANDS	1.05 (0.96, 1.17)	1.18 (1.05, 1.35)	1.18 (1.05, 1.35)
AUSTRIA	1.05 (0.96, 1.17)	1.16 (1.03, 1.33)	1.16 (1.03, 1.33)
PORTUGAL	1.12 (1.04, 1.22)	1.20 (1.12, 1.29)	1.20 (1.12, 1.29)
SLOVAKIA	1.02 (0.93, 1.14)	1.31 (1.17, 1.50)	1.31 (1.17, 1.50)
FINLAND	1.05 (0.96, 1.17)	1.19 (1.05, 1.36)	1.19 (1.05, 1.36)
CZECH REP.	1.01 (0.93, 1.12)	1.21 (1.08, 1.40)	1.21 (1.08, 1.40)
DENMARK	1.05 (0.96, 1.17)	1.16 (1.03, 1.33)	1.16 (1.03, 1.33)
LATVIA	1.11 (1.02, 1.23)	1.25 (1.15, 1.38)	1.25 (1.15, 1.38)
LITHUANIA	1.11 (1.01, 1.23)	1.22 (1.12, 1.33)	1.22 (1.12, 1.33)
HUNGARY	1.05 (0.94, 1.19)	1.06 (0.94, 1.23)	1.06 (0.94, 1.23)
POLAND	1.02 (0.93, 1.14)	1.30 (1.18, 1.46)	1.30 (1.18, 1.46)
SWEDEN	1.05 (0.96, 1.16)	1.21 (1.08, 1.37)	1.21 (1.08, 1.37)
UK	1.07 (0.98, 1.20)	1.25 (1.10, 1.45)	1.25 (1.10, 1.45)

The estimated model is $y_t = \beta_0 + \beta_1 t + x_t; (1 - L)^d x_t = \varepsilon_t$. The values in bold refer to the significant cases according to the deterministic terms.

developed by Robinson (1995), Velasco (1999) and Abadir, Distaso, and Giraitis (2007) among others.

We then test for cointegration between long-term interest rates on a pairwise basis. A necessary condition for cointegration is that the two parent series display the same degree of integration. We use the statistic proposed by Robinson and Yajima (2002) to test homogeneity in the orders of integration of the two series denoted by d_x and d_y respectively, i.e. the null $H_0: d_x = d_y$. Finally we perform the Hausman test of Marinucci and Robinson (2001) for the null of no cointegration against the alternative of fractional cointegration.

3. Data and empirical results

The series analysed are the harmonized monthly long-term interest rates on government securities with 10-year maturity over the period January 2001–February 2018; these are denominated in euros for Germany, Austria, Belgium, Ireland, Greece, Spain, Italy, Cyprus, Luxembourg, Malta, Netherlands, Austria, Portugal, Slovakia, Finland, in Czech korunas for the Czech Republic, in Danish kroner for Denmark, in Latvian lats for Latvia, in Lithuanian litas for Lithuania, in Hungarian forint for Hungary, in Polish zlotys for Poland, in Swedish kronas for Sweden, and in pound sterling for the UK. See Fig. 1.

First we follow the parametric approach of Robinson (1994) with the Whittle estimates in the frequency domain (Dahlhaus, 1989). We consider the standard cases of i) no regressors, ii) an intercept, and iii) an intercept with a linear time trend. The results for the case of white noise disturbances are displayed in Table 1.

The most appropriate specification appears to be the one including an intercept only, since the time trend coefficients (not reported) are found to be statistically insignificant in all cases. The estimated values of d are always above 1, ranging from 1.03 (Greece) to 1.31 (Slovakia), and the unit root null hypothesis (i.e., $d = 1$) cannot be rejected only in the cases of Greece and Hungary where the 95% confidence bands include the value of 1. In all the other countries the value of d is significantly higher than 1. Thus, according to this specification, shocks are not mean reverting, with their effects persisting forever.

Table 2 focuses on the case of autocorrelated (Bloomfield) errors. It can be seen that the time trend is required in the majority of cases, the only exceptions being Cyprus, Greece, Ireland, Portugal, Latvia and Lithuania. The estimated values of d are now substantially smaller, and significantly higher than one only for Portugal (1.37) and Lithuania (1.30). The unit root null cannot be rejected in 17 cases (Germany, Belgium, Ireland, Greece, Spain, Italy, Cyprus, Luxembourg, Malta, Netherlands, Austria, Slovakia, Finland, Latvia, Poland, Sweden and the UK), while evidence of mean reversion (i.e., $d < 1$) is found in the cases of Hungary (0.71), France (0.72), Denmark (0.75), and the Czech Republic (0.76). Thus, for these four countries, mean reversion takes place and shocks have transitory effects that disappear in the long run, though very slowly. The lower values of d obtained with autocorrelated errors might reflect the competition between the fractional differencing and the Bloomfield parameters in capturing non-stationarity. The same has been found in other empirical applications (see, e.g. Gil-Alana & Robinson, 1997). Given the fact that the results are different depending on the specification of the error term, we also estimated d using semiparametric methods. Table 3 displays the results obtained from a “local” Whittle method (Robinson, 1995), using a selected number of bandwidth parameters.¹

We find evidence of mean reversion ($d < 1$) for Sweden, Denmark and UK for practically all bandwidth parameters, and also for Lithuania, Finland, Netherlands, France and Germany in a number of cases; the unit root null cannot be rejected for Belgium, Ireland, Greece, Spain, Italy, Cyprus, Luxembourg, Malta, Austria, Slovakia, the Czech Republic, Hungary and Poland; finally, there is evidence

¹ Very similar results were obtained when using the extension to this approach by Abadir et al. (2007).

Table 2
Estimated values of d and 95% confidence bands with autocorrelated errors.

	No regressors	An intercept	A linear trend
GERMANY	0.99 (0.85, 1.20)	0.84 (0.71, 1.06)	0.80 (0.63, 1.07)
FRANCE	0.98 (0.83, 1.17)	0.76 (0.67, 0.94)	0.72 (0.58, 0.92)
BELGIUM	0.98 (0.83, 1.18)	0.84 (0.73, 1.02)	0.83 (0.69, 1.02)
IRELAND	1.04 (0.89, 1.23)	1.03 (0.91, 1.20)	1.03 (0.90, 1.20)
GREECE	0.86 (0.72, 1.02)	0.87 (0.75, 1.05)	0.88 (0.75, 1.05)
SPAIN	0.95 (0.82, 1.14)	0.92 (0.81, 1.06)	0.91 (0.80, 1.06)
ITALY	0.98 (0.82, 1.17)	0.91 (0.77, 1.07)	0.90 (0.77, 1.07)
CYPRUS	0.91 (0.73, 1.18)	1.08 (0.93, 1.30)	1.08 (0.93, 1.30)
LUXEMBOURG	1.00 (0.85, 1.19)	0.97 (0.85, 1.17)	0.99 (0.84, 1.17)
MALTA	1.00 (0.87, 1.19)	0.97 (0.85, 1.17)	0.97 (0.82, 1.17)
NETHERLANDS	1.00 (0.85, 1.19)	0.83 (0.72, 1.03)	0.80 (0.65, 1.05)
AUSTRIA	0.99 (0.84, 1.19)	0.83 (0.73, 1.03)	0.81 (0.67, 1.04)
PORTUGAL	1.17 (1.00, 1.41)	1.37 (1.17, 1.59)	1.37 (1.17, 1.59)
SLOVAKIA	0.94 (0.81, 1.14)	0.92 (0.76, 1.12)	0.92 (0.79, 1.12)
FINLAND	0.88 (0.85, 1.20)	0.82 (0.71, 1.01)	0.80 (0.63, 1.01)
CZECH REP.	0.99 (0.81, 1.18)	0.76 (0.67, 0.92)	0.76 (0.62, 0.92)
DENMARK	0.88 (0.83, 1.19)	0.79 (0.68, 0.98)	0.75 (0.59, 0.98)
LATVIA	1.03 (0.89, 1.23)	1.08 (0.96, 1.26)	1.08 (0.96, 1.26)
LITHUANIA	1.03 (0.83, 1.27)	1.30 (1.07, 1.61)	1.30 (1.07, 1.60)
HUNGARY	0.85 (0.72, 1.03)	0.72 (0.62, 0.87)	0.71 (0.59, 0.86)
POLAND	1.02 (0.84, 1.24)	1.00 (0.82, 1.23)	1.00 (0.83, 1.23)
SWEDEN	1.01 (0.88, 1.21)	0.84 (0.69, 1.07)	0.78 (0.58, 1.07)
UK	0.95 (0.83, 1.17)	0.80 (0.66, 1.03)	0.75 (0.58, 1.03)

The estimated model is $y_t = \beta_0 + \beta_1 t + x_t$; $(1 - L)^d x_t = u_t$; u_t follows the exponential model of Bloomfield. The values in bold refer to the significant cases according to the deterministic terms.

Table 3
Estimated values of d with a semi-parametric “local” Whittle method.

	11	12	13	14	15	16
GERMANY	0.839	0.768	0.777	0.686*	0.726*	0.765*
FRANCE	0.828	0.809	0.849	0.703*	0.740*	0.774*
BELGIUM	0.845	0.894	0.943	0.855	0.910	0.941
IRELAND	1.034	1.065	1.124	1.130	1.152	1.198
GREECE	0.984	0.976	1.015	1.047	1.103	1.163
SPAIN	1.217	1.190	1.186	1.096	1.128	1.147
ITALY	1.096	1.176	1.202	1.128	1.194	1.172
CYPRUS	1.040	1.004	1.033	1.068	1.070	1.096
LUXEMBOURG	1.198	1.173	1.201	1.205	1.054	1.091
MALTA	1.098	0.978	1.025	0.992	1.008	1.028
NETHERLANDS	0.833	0.811	0.827	0.724*	0.763*	0.812
AUSTRIA	0.862	0.889	0.934	0.790	0.823	0.871
PORTUGAL	1.115	1.209	1.305**	1.324**	1.349**	1.388**
SLOVAKIA	0.939	1.008	0.976	0.973	1.029	1.051
FINLAND	0.843	0.847	0.866	0.760*	0.808	0.853
CZECH REP.	0.800	0.848	0.934	0.976	1.059	1.088
DENMARK	0.751*	0.751*	0.784	0.651*	0.693*	0.738*
LATVIA	1.294**	1.426**	1.446**	1.500**	1.500**	1.500**
LITHUANIA	0.653*	0.703*	0.750*	0.820	0.903	0.993
HUNGARY	0.767	0.816	0.773	0.826	0.853	0.881
POLAND	0.854	0.844	0.918	0.940	0.996	1.026
SWEDEN	0.645*	0.698*	0.770*	0.628*	0.690*	0.751*
UK	0.893	0.719*	0.712*	0.636*	0.656*	0.695*
95% Lower Intv.	0.752	0.762	0.771	0.780	0.787	0.794
95% Upper Intv.	1.247	1.237	1.228	1.219	1.212	1.205

The values are the estimated values of d using the semiparametric method of Robinson (1995).

The last two rows refer to the lower and upper bound for the I (1) case at the 95% level.

*: Evidence of mean reversion at the 95% level; **: Evidence of orders of integration above 1.

of $d > 1$ for Portugal and Latvia. When $d < 1$ mean reversion takes places and shocks have only transitory effects that disappear in the long run. This is generally the case for the countries belonging to Western Europe and Scandinavia (Belgium, France, Netherlands, Austria, Finland, Denmark, Sweden and the UK), as well as Poland.

Next, we carry out the cointegration analysis. First we test the homogeneity condition using the statistic proposed in Robinson and Yajima (2002). The results are displayed in Table 4; those in bold are the cases when the two series appear to have the same order of integration.

The same degree of integration is displayed by France, the Netherlands, Finland, Denmark, Sweden and the UK vis-à-vis Germany; Germany, the Netherlands, Finland, Denmark, Sweden and the UK vis-à-vis France; Austria, Lithuania and Hungary vis-à-vis Belgium; Spain, Italy and Cyprus vis-à-vis Ireland; Spain, Cyprus, Malta, Slovakia and Czech Republic vis-à-vis Greece; Ireland, Greece, Italy and Cyprus vis-à-vis Spain; Ireland, Spain, Cyprus and Luxembourg vis-à-vis Italy; Ireland, Greece, Spain, Italy and Malta vis-à-vis Cyprus; Italy and the Netherlands vis-à-vis Luxembourg; Greece, Cyprus, Czech Republic, Slovakia and Poland vis-à-vis Malta; Germany, France, Luxembourg, Austria, Finland and Denmark vis-à-vis the Netherlands; Belgium, Netherlands, Finland, Lithuania and Hungary vis-à-vis

Table 4
Homogeneity condition tests (Robinson and Yajima, 2001).

	FRA	BEL	IRE	GRE	SPA	ITA	CYP	LUX	MAL	NET	AUS
GER	-0.408	-4.056	-10.72	-8.664	-9.839	-10.61	-9.167	-12.45	-7.343	-0.911	-2.496
FRA	-	-3.648	-10.24	-8.256	-9.341	-10.20	-8.766	-12.04	-6.933	-0.503	-2.088
BEL	-	-	-6.599	-4.608	-5.783	-6.552	-5.111	-8.400	-3.287	3.144	1.599
IRE	-	-	-	1.991	0.816	0.048	1.448	-1.800	3.312	9.744	8.159
GRE	-	-	-	-	-1.176	-1.943	-0.504	-3.791	1.320	7.752	6.168
SPA	-	-	-	-	-	-0.768	0.671	-2.616	2.495	8.927	7.343
ITA	-	-	-	-	-	-	1.440	1.848	3.264	9.696	8.111
CYP	-	-	-	-	-	-	-	-3.288	1.824	8.256	6.671
LUX	-	-	-	-	-	-	-	-	5.112	1.154	9.960
MAL	-	-	-	-	-	-	-	-	-	6.432	4.847
NET	-	-	-	-	-	-	-	-	-	-	1.584
	POR	SVK	FIN	CZE	DNK	LTV	LTH	HUN	POL	SVD	U.K.
GER	-15.31	-6.888	-1.776	-69.60	0.839	-19.53	-2.952	-3.359	-6.096	1.391	1.200
FRA	-14.90	-6.479	-1.367	-6.559	1.248	-19.12	-2.807	-2.951	-5.681	1.799	1.608
BEL	-11.25	-2.831	2.280	-2.903	4.895	-15.47	0.840	0.696	-2.039	5.447	5.256
IRE	-4.656	3.768	8.880	3.695	11.49	-8.880	7.440	7.296	4.559	12.04	11.85
GRE	-6.648	1.776	6.888	1.704	9.504	-10.87	5.448	5.304	2.568	10.05	9.864
SPA	-5.472	2.951	8.063	2.879	10.68	-9.699	6.623	6.479	3.744	11.23	11.04
ITA	-4.703	3.720	8.832	3.648	11.44	-8.927	7.392	7.248	4.512	12.00	11.80
CYP	-6.144	2.279	7.391	2.208	10.00	-10.36	5.952	5.808	3.071	10.55	10.36
LUX	-2.856	5.568	10.68	5.496	13.30	-7.079	9.240	9.096	6.260	13.84	13.65
MAL	-7.968	0.456	5.567	0.384	8.184	-12.19	4.127	3.984	1.247	8.735	8.544
NET	-14.40	-5.976	-0.864	-6.048	1.752	-18.62	-2.304	-2.448	-5.184	2.304	2.112
AUS	-12.81	-4.391	0.720	-4.463	3.335	-17.04	-0.719	-0.863	-3.599	3.888	3.696
POR	-	8.424	13.53	8.352	16.15	-4.223	12.09	11.95	9.216	16.70	16.51
SVK	-	-	5.111	-0.072	7.727	-12.64	3.671	3.528	0.791	8.279	8.088
FIN	-	-	-	-5.184	2.616	-17.76	1.440	-1.583	-4.320	3.167	2.976
CZE	-	-	-	-	7.799	-12.57	3.744	3.600	0.864	8.352	8.160
DNK	-	-	-	-	-	-20.37	-4.059	-4.199	-6.935	0.552	0.360
LTV	-	-	-	-	-	-	16.32	16.17	13.44	20.92	20.73
LTH	-	-	-	-	-	-	-	-0.143	-2.880	4.607	4.466
HUN	-	-	-	-	-	-	-	-	-2.736	4.751	4.559
POL	-	-	-	-	-	-	-	-	-	7.487	7.296
SVD	-	-	-	-	-	-	-	-	-	-	-0.191

The values in bold indicate homogeneity in the degree of integration at the 5% level using the method of Robinson and Yajima (2002).

Table 5
Summary of the homogeneity test results.

GERMANY	France, Netherlands, Finland, Denmark, Sweden and U.K.
FRANCE	Germany, Netherlands, Finland, Denmark, Sweden and U.K.
BELGIUM	Austria, Lithuania and Hungary
IRELAND	Spain, Italy and Cyprus
GREECE	Spain, Cyprus, Malta, Slovakia and Czech Republic
SPAIN	Ireland, Greece, Italy and Cyprus
ITALY	Ireland, Spain, Cyprus and Luxembourg
CYPRUS	Ireland, Greece, Spain, Italy and Malta
LUXEMBOURG	Italy and Netherlands
MALTA	Greece, Cyprus, Czech Republic, Slovakia and Poland
NETHERLANDS	Germany, France, Luxembourg, Austria, Finland and Denmark
AUSTRIA	Belgium, Netherlands, Finland, Lithuania and Hungary
PORTUGAL	-
SLOVAKIA	Greece, Malta and Czech Republic and Poland
FINLAND	Germany, France, Belgium, Netherlands, Austria, Lithuania and Hungary
CZECH REPUB.	Greece, Malta, Slovakia and Poland
DENMARK	Germany, France, Netherlands, Austria, Sweden and U.K.
LATVIA	-
LITHUANIA	Belgica, Austria, Finland and Hungary
HUNGARY	Belgium, Austria, Finland and Lithuania
POLAND	Malta, Slovakia and Czech Republic
SWEDEN	Germany, France, Denmark and U.K.
U.K.	Germany, France, Denmark and Sweden

Austria; Greece, Malta, Czech Republic and Poland vis-à-vis Slovakia; Germany, France, Belgium, Netherlands, Austria, Lithuania and Hungary vis-à-vis Finland; Greece, Malta, Slovakia and Poland vis-à-vis Czech Republic; Germany, France, Netherlands, Austria, Sweden and the UK vis-à-vis Denmark; Belgium, Austria, Finland and Hungary vis-à-vis Lithuania; Belgium, Austria, Finland and Lithuania vis-à-vis Hungary; Malta, Slovakia and Czech Republic vis-à-vis Poland; Germany, France, Denmark and UK vis-à-vis Sweden, and finally, Germany, France, Denmark and Sweden vis-à-vis the UK (see Table 5). For the remaining vis-à-vis relationships we do not have evidence of equal orders of integration and therefore, we cannot go on further on a cointegration relationship.

Table 6 displays the cointegration results. German, French and UK interest rates are not cointegrated with any others, and the same holds for Luxembourg, Denmark, Latvia, Lithuania, Portugal and Sweden. Most cointegrating relationships are found between relatively small economies. Specifically, interest rates in Belgium are cointegrated with those in Hungary (0.855 and 0.826 are the integration

Table 6
Testing the null hypothesis of no cointegration against fractional cointegration.

Countries		H ₁₀	H ₂₀	d ₁	d ₂	d*
GERMANY	FRANCE	1.234	0.867	0.686	0.703	0.791
	NETHERLANDS	2.725	1.559	0.686	0.724	0.842
	FINLAND	0.027	0.533	0.686	0.760	0.691
	DENMARK	2.690	1.612	0.686	0.651	0.531
	SWEDEN	0.129	0.947	0.686	0.628	0.720
FRANCE	U.K.	3.123	5.273	0.686	0.636	0.852
	NETHERLANDS	0.121	0.326	0.703	0.724	0.670
	FINLAND	0.019	0.348	0.703	0.760	0.690
	DENMARK	1.981	3.833	0.703	0.651	0.836
	SWEDEN	1.330	3.791	0.703	0.628	0.812
BELGIUM	U.K.	10.693	15.834	0.703	0.636	1.012
	AUSTRIA	10.763	15.749	0.855	0.790	1.165
	LITHUANIA	3.430	2.195	0.855	0.820	0.680
	HUNGRIA	7.056	5.519	0.855	0.826	0.604
IRELAND	SPAIN	0.207	0.009	1.130	1.096	1.087
	ITALY	11.468	11.325	1.130	1.128	0.810
	CYPRUS	3.833	1.694	1.130	1.068	0.945
GREECE	SPAIN	1.188	2.587	1.047	1.096	0.944
	CYPRUS	4.799	5.822	1.047	1.068	0.840
	MALTA	0.054	0.664	1.047	0.992	1.069
	SLOVAKIA	0.018	0.847	1.047	0.973	1.060
SPAIN	CZECK REP.	0.049	0.280	1.047	0.976	1.026
	ITALY	87.721	94.179	1.096	1.128	0.211
	CYPRUS	12.871	10.832	1.096	1.068	0.757
ITALY	CYPRUS	6.668	3.791	1.128	1.068	0.884
	LUXEMBOURG	2.322	0.502	1.128	1.205	1.272
DDDD						
CYPRUS	MALTA	0.268	0.081	1.068	0.992	1.019
			46.0dsk			
LUXEMBOURG	NETHERLANDS	2.867	46.018	1.205	0.724	1.365
MALTA	CZECK REP.	14.274	13.023	0.992	0.976	0.635
	SLOVAKIA	13.563	12.122	0.992	0.973	0.644
	POLAND	0.036	0.548	0.992	0.940	1.010
NETHERLANDS	AUSTRIA	6.185	10.147	0.724	0.790	0.489
	FINLAND	13.023	15.918	0.724	0.760	0.383
	DENMARK	0.258	0.069	0.724	0.651	0.676
AUSTRIA	FINLAND	3.791	2.656	0.790	0.760	0.606
	LITHUANIA	1.951	2.939	0.790	0.820	0.60.28058
	HUNGARY	0.989	1.892	0.790	0.826	0.6968.104
SLOVAKIA	CZECK REP.	0.280	0.314	0.973	0.976	0.923
	POLAND	8.104	6.237	0.973	0.940	0.704
FINLAND	LITHUANIA	1.032	2.725	0.760	0.820	0.664
FINLAND	HUNGARY	0.502	1.981	0.760	0.826	0.693
CZECK REP.	POLAND	1.778	0.907	0.976	0.940	0.850
CZECK REP.						
DENMARK	SWEDEN	0.145	0.389	0.651	0.628	0.687
	U.K.	2.519	3.049	0.651	0.636	0.801
LITHUANIA	HUNGARY	2.975	3.198	0.820	0.826	0.657
SWEDEN	U.K.	0.533	0.416	0.628	0.636	0.697

In bold the cases with significant evidence of (fractional) cointegration at the 5% level using the method of [Marinucci and Robinson \(2001\)](#).

orders of the individual countries and 0.604 the cointegrating parameter). Irish interest rates are cointegrated with those in Italy (1.130 and 1.128 for the individual series, and 0.810 for the cointegration relation). Greek interest rates are cointegrated with those in Cyprus (1.047 and 1.068 are the integration orders and 0.840 the cointegrating parameter); Spanish interest rates are cointegrated with those in Italy (1.094 and 1.128; 0.211) and those in Malta (1.096 and 1.068; 0.757); the Italian ones with those in Cyprus (1.128 and 1.068; 0.884); also, the Maltese ones with those in the Czech Republic (0.992 and 0.970; 0.635) and Slovakia (0.992 and 0.972; 0.664); more cointegrating relationships are found between those in the Netherlands with the ones in Austria (0.724 and 0.790; 0.489) and Finland (0.724 and 0.700; 0.383) and, finally, those in Slovakia with the ones in Poland (0.973 and 0.940; 0.704).

The most notable cases when the cointegrating parameter is lower than the integration orders of the individual series and there is a stationary long-run equilibrium are those of Spain and Italy (the order of integration of the cointegrating residuals is 0.211), and the Netherlands vis-à-vis Finland (0.383) and Austria (0.489). In all the other cases the cointegrating parameter is found to be higher than 0.50, which implies mean-reverting but non-stationary behaviour of the cointegrating residuals.

In brief, there is no evidence of linkages between German, French and UK rates and other rates, whilst the interest rates of smaller economies appear to be linked. This might suggest that interest rates in the larger economies in Europe are more strongly linked to those of similarly sized economies outside Europe; it would be interesting to investigate in future work whether stronger linkages can indeed be found with the latter. No other clear patterns emerge; the specific features of the financial systems of individual countries should also be analysed by further research for a deeper understanding of the individual country results.

4. Conclusions

This paper has used fractional integration and cointegration techniques to examine the stochastic behaviour of long-term interest rates (on government securities with 10-year maturity) in 23 European countries as well as their long-run linkages on a pairwise basis over the period January 2001–February 2018. Our modelling approach allows for the possibility of a slow rate of decay of the effects of shocks (i.e. a slow mean-reversion process) instead of imposing the restrictive assumptions underlying the standard unit root and cointegration tests.

The results are mixed and sensitive to the (parametric and semiparametric) estimation methods. Evidence is found for both unit roots and mean reversion in the series analysed. Various rates appear to be fractionally cointegrated, especially in the case of smaller economies, which suggests that they are driven a common set of fundamentals exhibiting long memory, and implying bilateral long-run equilibrium bilateral relationships. Interestingly, however, German, French and UK rates are not found to be linked to any others, which might indicate that they are more responsive to external rates, an issue which is left for future work. The implications of these findings should be carefully taken into account by monetary authorities and market participants when designing their policies and investment strategies respectively.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.iref.2019.02.004>.

References

- Abadir, K. M., Distaso, W., & Giraitis, L. (2007). Nonstationarity-extended local Whittle estimation. *Journal of Econometrics*, *141*, 1353–1384.
- Abbritti, M., Gil-Alana, L. A., Lovcha, Y., & Moreno, A. (2016). Term structure persistence. *Journal of Financial Econometrics*, *14*(2), 331–352.
- Backus, D., & Zin, S. (1993). Long memory inflation uncertainty. Evidence from the term structure of interest rates. *Journal of Money, Credit, and Banking*, *25*, 681–700.
- Barkoulas, J. T., Baum, C. F., & Oguz, G. S. (1997). Fractional dynamics in a system of long term international interest rates. *International Journal of Finance*, *9*(2), 586–606.
- Bloomfield, P. (1973). An exponential model in the spectrum of a scalar time series. *Biometrika*, *60*, 217–226.
- Busch, U., & Nautz, D. (2009). *Controllability and persistence of money market rates along the yield curve: Evidence from the euro area* (Vol. 649). Goethe-Universität Frankfurt. SFB DP no.
- Campbell, J. Y., & Shiller, R. J. (1987). Cointegration and tests of present value models. *Journal of Political Economy*, *95*, 1062–1088.
- Caporale, G. M., & Gil-Alana, L. A. (2009). Persistence in US interest rates: Is it stable over time? *Quantitative and Qualitative Analysis in Social Sciences*, *3*(1), 63–77.
- Caporale, G. M., & Gil-Alana, L. A. (2012). *Persistence and cycles in the US federal Funds rate*. CESifo WP.
- Cassola, N. (2008). *Structural modeling of the spread between the Eonia swap rate and the minimum bid rate of the main refinancing operations of the eurosystem*. European Central Bank WP.
- Cassola, N., & Morana, C. (2008). Modeling short-term interest rate spreads in the euro money market. *International Journal of Central Banking*, *4*(4), 1–37.
- Couchman, J., Gounder, R., & Su, J. J. (2006). Long memory properties of real interest rates for 16 countries. *Applied Financial Economics Letters*, *2*, 25–30.
- Cox, J., Ingersoll, J., & Ross, S. (1985). A theory of term structure of interest rates. *Econometrica*, *53*, 385–408.
- Dahlhaus, R. (1989). Efficient parameter estimation for self-similar process. *Annals of Statistics*, *17*, 1749–1766.
- DeGennaro, R., Kunkel, R., & Lee, J. (1994). Modeling international long-term interest rates. *Financial Review*, *29*, 57–597.
- Escobar, M., Ferrando, S., & Rubtsov, A. (2016). Portfolio choice with stochastic interest rates and learning about stock returns predictability. *International Review of Economics & Finance*, *41*, 347–370.
- Garbers, C., & Liu, G. (2018). Macroprudential policy and foreign interest rate shocks. A comparison of loan-to-value and capital requirements. *International Review of Economics & Finance*, *58*, 683–698.
- Gil-Alana, L. A. (2003). Testing of fractional cointegration in macroeconomic time series. *Oxford Bulletin of Economics & Statistics*, *65*(4), 517–529.
- Gil-Alana, L. A. (2004). The use of the Bloomfield (1973) model as an approximation to ARMA processes in the context of fractional integration. *Mathematical and Computer Modelling*, *39*, 429–436.
- Gil-Alana, L. A., & Hualde, J. (2009). Fractional integration and cointegration. An overview with an empirical application. *The Palgrave Handbook of Applied Econometrics*, *2*, 434–472.
- Gil-Alana, L. A., & Moreno, A. (2012). Uncovering the US term premium. An alternative route. *Journal of Banking & Finance*, *36*(4), 1181–1193.
- Gil-Alana, L. A., & Robinson, P. M. (1997). Testing of unit roots and other nonstationary hypotheses in macroeconomic time series. *Journal of Econometrics*, *80*, 241–268.
- Hacker, R. S., Karlsson, H. K., & Mansson, K. (2014). An investigation of the causal relations between exchange rates and interest rate differentials using wavelets. *International Review of Economics & Finance*, *29*, 321–329.
- Hassler, U., & Nautz, D. (2008). On the persistence of the Eonia spread. *Economics Letters*, *10*(3), 184–187.
- Karfakis, J. C., & Moschos, D. M. (1990). Interest rate linkages within the European monetary system: A time series analysis. *Journal of Money, Credit, and Banking*, *22*, 388–394.
- Kirchgassner, G., & Wolters, J. (1995). Interest rate linkages in Europe before and after the introduction of the European monetary system. *Empirical Economics*, *20*, 435–454.
- Lai, K. S. (1997). Long term persistence in real interest rate. Some evidence of a fractional unit root. *International Journal of Finance & Economics*, *2*, 225–235.
- Marinucci, D., & Robinson, P. M. (2001). Semiparametric fractional cointegration analysis. *Journal of Econometrics*, *105*, 225–247.
- Phillips, P. C. B. (1998). *Econometric analysis of Fisher's equation*. Yale University. Cowles Foundation Discussion Paper 1180.
- Probst, J. (2019). Global real interest rate dynamics from the late 19th century to today. *International Review of Economics & Finance*, *59*, 522–547.
- Robinson, P. M. (1994). Efficient tests of nonstationary hypotheses. *Journal of the American Statistical Association*, *89*, 1420–1437.
- Robinson, P. M. (1995). Gaussian semiparametric estimation of long range dependence. *Annals of Statistics*, *23*, 1630–1661.
- Robinson, P. M., & Yajima, Y. (2002). Determination of cointegrating rank in fractional systems. *Journal of Econometrics*, *106*, 217–241.
- Shea, G. (1991). Uncertainty and implied variance bounds in long memory models of the interest rate term structure. *Empirical Economics*, *16*, 287–312.
- Tsay, W. J. (2000). The long memory story of the real interest rate. *Economics Letters*, *67*, 325–330.
- Velasco, C. (1999). Gaussian semiparametric estimation of nonstationary time series. *Journal of Time Series Analysis*, *20*, 87–127.
- Wang, Z., Jang, J., & Li, Q. (2007). Interest rate linkages in the Eurocurrency market: Contemporaneous and out-of-sample Granger causality tests. *Journal of International Money and Finance*, *26*, 86–103.