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Protocol-Based State Estimation for Delayed Markovian Jumping Neural Networks

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Abstract—**This paper is concerned with the state estimation problem for a class of Markovian jumping neural networks (MJNNs) with sensor nonlinearities, mode-dependent time delays and stochastic disturbances subject to the Round-Robin (RR) scheduling mechanism. The system parameters experience switches among finite modes according to a Markov chain. As an equal allocation scheme, the RR communication protocol is introduced for efficient usage of limited bandwidth and energy saving. The update matrix method is adopted to deal with the periodic time-delays resulting from the RR protocol. The objective of the addressed problem is to construct a state estimator for the MJNNs such that the dynamics of the estimation error is exponentially ultimately bounded in the mean square with a certain upper bound. Sufficient conditions are established for the existence of the desired state estimator by resorting to a combination of the Lyapunov stability theory and the stochastic analysis technique. Furthermore, the estimator gain matrices are characterized in terms of the solution to a convex optimization problem. Finally, a numerical simulation example is exploited to demonstrate the effectiveness of the proposed estimator design strategy.**

Index Terms—**Markovian jumping neural networks, exponentially ultimately bounded estimator, Round-Robin protocol, sensor nonlinearities, mode-dependent time delays.**

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I. Introduction

For decades, due to their extraordinary capabilities in parallel information processing, adaptable data processing and dynamical learning as well as imitation, the artificial neural networks (ANNs) have been extensively applied in a variety of subject areas such as brain science, cognitive science and computer science. The successful applications of ANNs are largely reliant on the dynamical behaviors (e.g. convergence and stability) of the ANNs and, accordingly, the dynamics analysis issues for ANNs have become a hot topic of research attracting an ever-increasing interest with many interesting results reported in the literature, see e.g., [7], [25], [32], [40], [50], [53]–[56] and the references therein. On the other hand, time delays inevitably occur in hardware implementation of the ANNs for a number of reasons including inherent restriction on physical devices during information transmission and limited processing speeds among the units of networks. It is well known that time delays, if not adequately handled, might lead to performance degradation of the underlying system or even undesirable behaviors such as oscillation or even instability. As such, dynamical behaviors of delayed ANNs have been thoroughly investigated in the past few years. For example, delay-dependent criteria have been established in [16], [36], [58] to estimate the neuron states of a class of delayed neural networks through the available output measurements. In [22], [46] and [21], [42], some attempts have been made on analyzing, respectively, the stability and synchronization issues of the time-delayed neural networks. In [29], the robust l_2-l_∞ state estimation problem has been discussed for uncertain Markovian jump neutral systems with distributed delays. Moreover, the dissipativity and passivity analysis issues with various types of delays have been considered in [15] and [27].

In reality, the structures and parameters of actual systems might be changeable because of internal component failures, system maintenances, sudden changes in external environment, coupled subsystems variations and modifications of the operating point of a linearized model for a nonlinear system. In many cases, such changes can be modeled by system switches between different structures and, among different switching systems, the so-called Markovian jumping systems have drawn considerable attention owing to their clear engineering insights [3], [4]. When it comes to recurrent neural networks (RNNs), it has been revealed in [43] that the switching between different RNN models can be governed by a Markovian chain. So far, the study on time-delay neural networks with Markovian jumping parameters has stirred much attention and fruitful results have been presented in the literature, see e.g. [5], [6], [17], [24], [34], [39]–[41], [47], [48]. Among others, in [34], the linear matrix inequality technique has been applied to ensure the existence of the required state estimator for the discrete-time neural networks subject to Markovian jumping parameters and mode-dependent delays. The mean-square asymptotic stability problem has been dealt with in [39] for Markovian jumping generalized neural networks with interval time-varying delays. Furthermore, for a class of Markovian-jumping-type delayed neural networks, a non-fragile state estimator has been designed in [17] in order to guarantee the stability of the overall estimation error dynamics. More recently, the result on decentralized event-triggered synchronization has been published in [40] for Markovian jumping neutral-type neural networks with mixed delays.

Over the past few decades, the state estimation issue has been serving as a central topic of research in signal processing and control engineering, and considerable attention has been devoted to this issue for different kinds of networked systems, see e.g. [8], [9], [11], [14], [23], [26], [28]. As for ANNs, it is of particular significance to access the information about neuron states so as to accomplish specific tasks such as approximation and optimization. Unfortunately, it is often the case in practice that only partial information of the neuron states is available through the network outputs due probably to the large scale of the network and the limited resource allocated to state observations. Accordingly, there appears to be a practical need to accurately estimate the neuron states via available but possibly noisy/imperfect output measurements, and the resulting state estimation problem has therefore received a great deal of research interest, see [13], [18], [30], [49], [57]. Moreover, in engineering practice, the sensors are often subject to nonlinear disturbances for many reasons such as harsh environments and channel noises, and the sensor nonlinearity has been extensively studied for both control and estimation problems. In the context of RNNs, an initial study has been carried out in [51] for the state estimation problem of delayed neural networks subject to randomly occurring sensor nonlinearity, and a Luenberger-type state estimator has been proposed in [48] for a class of coupled Markovian neural networks with the phenomenon of sensor nonlinearity.

Although much effort has been made on the state estimation problem for ANNs, little attention has been paid on the constrained communication issue between the network output and the possible remote estimator when implementing ANNs in a networked environment. Due to the large scale of ANNs, the amount of measurement outputs from a large number of sensors might bring much burden that surpasses the capacity of the transmission network with limited capacity, and it makes practical sense to mitigate the communication burden by resorting to certain communication protocols. In industry, a frequently used communication protocol is the so-called Round-Robin (RR) protocol that serves as a kind of equal resource allocation scheme. The RR protocol grants each sensor an equivalent right to access the data transmission service. From the methodological point of view, there are generally two approaches to handling the RR protocol, with one to transform the RR-induced effects into accumulated delays and another one to reflect such effects by periodic switches. For instance, in [59], the accumulated-delay method has been utilized to deal with the RR-protocol-based estimator design problem for a class of nonlinear dynamical networks. In [1], a switched controller structure has been adopted for the design of decentralized observer-based output-feedback controllers under the RR protocol. So far, the state estimation for ANNs under RR protocol has received some initial research attention in [35]. Different from the existing result, the Markovian jumping parameters and sensor nonlinearities have been taken into account in this paper and the methods proposed can ensure that the augmented system is exponentially ultimately bounded.

Motivated by the above discussion, the aim of this paper is to deal with the state estimation problem for a class of MJNNs under the RR protocol. We are interested in designing a state estimator such that, in the simultaneous presence of Markovian jumping parameters, sensor nonlinearities, mode-dependent time delays and stochastic disturbances, the estimation error dynamics is exponentially ultimately bounded in the mean square. An update matrix approach is proposed to tackle the complexities caused by the combinational use of the RR protocol and the zero-order holders (ZOHs). By virtue of the Lyapunov stability theory and the stochastic analysis technique, the exponential boundedness of the estimation error dynamics is investigated. Furthermore, the desired estimator parameters are acquired through a convex optimization problem that can be efficiently settled via the standard Matlab software. *The main contributions of this paper are highlighted as follows: 1) the state estimation problem is, for the first time, investigated for a class of Markovian jumping neural networks under the RR protocol; 2) a comprehensive system model is proposed to account for the phenomena of Markovian jumping parameters, sensor nonlinearities, mode-dependent time delays and stochastic disturbances; and 3) a combination of Lyapunov-Krasovskii functional, stochastic analysis technique and update matrix method is employed to establish the existence condition of the exponentially ultimately bounded estimator.*

Notation: Throughout this paper, for a matrix M, M^T and M^{-1} denote its transpose and inverse, respectively. $\mathbb{R}^n$ means the n dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. $\mathbb{Z}$ $(\mathbb{Z}^+, \mathbb{Z}^-)$ denote the set of all integers (nonnegative integers, negative integers). I and 0 denote the identity matrix and zero matrix, respectively. The notation $P > 0$ means that P is a real, symmetric and positive definite matrix. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$ represent, respectively, the expectation of a random variable x and the expectation of x conditional on y. $\|x\|$ stands for the Euclidean norm of a vector x. In symmetric block matrices, the shorthand $\mathrm{diag}\{A_1, A_2, \ldots, A_n\}$ represents a block diagonal matrix with diagonal blocks being the matrices $A_1, \ldots, A_n$, and the symbol $*$ denotes an ellipsis for terms induced by symmetry. If M is a symmetric matrix, $\lambda_{\text{max}}(M)$ and $\lambda_{\text{min}}(M)$ show the maximum eigenvalue and the minimum eigenvalue of M. $\mathrm{mod}(a, b)$ represents the unique nonnegative remainder on division of the integer a by the positive integer b. $\delta(a)$ is a binary function which equals to 1 for $a = 0$, and equals to 0 for $a \neq 0$. The symbol $\otimes$ denotes the Kronecker product. $\mathbf{1}_n = [1, 1, \ldots, 1]^T \in \mathbb{R}^n$. Matrices without explicitly stated dimensions are supposed to be compatible for matrix operations.

II. Problem Formulation and Preliminaries

In this paper, our effort is devoted to the design of a state estimator for a class of MJNNs through the available but possibly noisy/imperfect output measurements. As shown in Figure 1, the outputs of the MJNNs are transmitted to the estimator via a communication channel with limited bandwidth.

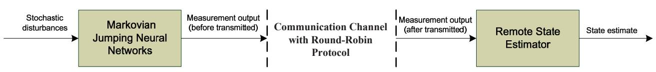

Fig. 1: State estimation under Round-Robin protocol.

A. Model Establishment

Consider a class of discrete-time MJNNs described by the following model:

$$\begin{cases} x(k+1) = \mathcal{C}(r(k))x(k) + \mathcal{B}(r(k))f(x(k)) \\ \qquad\qquad + \mathcal{A}(r(k))f(x(k-\tau(r(k)))) + \omega(k) \\ \quad y(k) = \mathcal{D}(r(k))x(k) + \mathcal{E}(r(k))g(x(k)) \end{cases} \tag{1}$$

with initial conditions

$$x(k) = \psi(k), \forall k \in \mathbb{Z}^-,$$

where $x(k) = [x_1(k) \quad \cdots \quad x_n(k)]^T \in \mathbb{R}^n$ is the neural state vector; $f(x(k)) = [f_1(x_1(k)) \quad \cdots \quad f_n(x_n(k))]^T : \mathbb{R}^n \to \mathbb{R}^n$ represents the nonlinear activation function; $\mathcal{C}(r(k)) = \mathrm{diag}\{\mathcal{C}_1(r(k)), \mathcal{C}_2(r(k)), \ldots, \mathcal{C}_n(r(k))\}$ is a diagonal matrix with $\mathcal{C}_\jmath(r(k))$ $(\jmath = 1, 2, \ldots, n)$ being positive scalars; $\mathcal{B}(r(k)) = [b_{ut}(r(k))]_{n\times n}$, $\mathcal{A}(r(k)) = [a_{ut}(r(k))]_{n\times n}$ are, respectively, the connection weight matrix and the delayed connection weight matrix; $\mathcal{D}(r(k))$ and $\mathcal{E}(r(k))$ are known constant matrices with compatible dimensions; $\tau(r(k))$ is the mode-dependent time delay and satisfy $\underline{\tau} \le \tau(r(k)) \le \bar{\tau}$ with $\underline{\tau}$ and $\bar{\tau}$ being two positive scalars; $y(k) = [y_1(k) \quad \cdots \quad y_m(k)]^T \in \mathbb{R}^m$ represents the measurement output; $g(x(k)) : \mathbb{R}^n \to \mathbb{R}^m$ denotes the sensor nonlinearity; $\omega(k)$ is a Gaussian white noise with

$$\begin{aligned} \mathbb{E}\{\omega(k)\} &= 0 \\ \mathbb{E}\{\omega^T(k)\omega(k)\} &= \upsilon^2 \\ \mathbb{E}\{\omega^T(i)\omega(j)\} &= 0 \ (i \neq j) \end{aligned}$$

where υ is a known positive scalar.

The Markov chain $r(k)$ $(k \ge 0)$ takes values in a finite state space $S = \{1, 2, \ldots, s\}$ with transition probability matrix $\Theta = [\theta_{ij}]_{s\times s}$ given by

$$\mathrm{Prob}\{r(k+1) = j | r(k) = i\} = \theta_{ij}, \ \forall i, j \in S$$

where $\theta_{ij} \ge 0$ $(i, j \in S)$ is the transition probability from i to j and $\sum_{j=1}^{s} \theta_{ij} = 1, \ \forall i \in S$.

For notational simplicity, in the sequel, for each possible $r(k) = i$ $(i \in S)$, a matrix $Z(r(k))$ will be denoted by Z_i. For example, $\mathcal{D}(r(k))$ is denoted by $\mathcal{D}_i$, $\bar{\mathcal{D}}(r(k))$ by $\bar{\mathcal{D}}_i$, etc.

Assumption 1: For the known constant matrices $\mathfrak{H}_1$, $\mathfrak{H}_2$, $\mathfrak{G}_1$ and $\mathfrak{G}_2$, the activation function $f(x(k))$ and the nonlinear function $g(x(k))$ satisfy the following sector-bounded conditions

$$[f(x(k)) - \mathfrak{H}_1 x(k)]^T [f(x(k)) - \mathfrak{H}_2 x(k)] \le 0, \tag{2}$$

$$[g(x(k)) - \mathfrak{G}_1 x(k)]^T [g(x(k)) - \mathfrak{G}_2 x(k)] \le 0. \tag{3}$$

Remark 1: Traditionally, the activation functions are required to satisfy the Lipschitz condition [19], [44]. As a more general description, the sector-bounded condition is capable of representing the activation functions in a more precise way. In fact, the sector-bounded condition has already been employed in dynamics analysis problems for RNNs, see e.g. [33], [52]. In particular, the assumption is less restricted than the Lipschitz condition, and a less conservative result has been achieved in [52].

B. Round-Robin Protocol Description

Consider the measurement outputs of a MJNN with m sensors labeled as $\{1, 2, \ldots, m\}$. Under the scheduling of the RR protocol, for sensors transmitting their data to the estimator via a shared communication channel, only one sensor is allowed to access the channel at each transmission instant k. A sensor $\zeta(k) \in \{1, 2, \ldots, m\}$ having the privilege to utilize the communication resource is determined as follows. Based on the scheduling rule of the RR protocol, the value of $\zeta(k)$ satisfies $\zeta(k+m) = \zeta(k)$ for all $k \in \mathbb{Z}^+$ and $\zeta(k) = k$ for $k \in \{1, 2, \ldots, m\}$. In other words, $\zeta(k)$ can be calculated as

$$\zeta(k) = \mathrm{mod}(k-1, m) + 1. \tag{4}$$

Let $\bar{y}_q(k)$ $(q \in \{1, 2, \ldots, m\})$ denote the received measurement from the qth sensor. With the help of the zero-order holders, the update of $\bar{y}_q(k)$ can be expressed by

$$\bar{y}_q(k) = \begin{cases} y_q(k), \ \text{if } \mathrm{mod}(k-q, m) = 0 \text{ and } k > 0 \\ \bar{y}_q(k-1), \ \text{otherwise} \end{cases} \tag{5}$$

which means that the received measurements in ZOHs will be updated periodically.

Letting $\Xi_{\zeta(k)} \triangleq \mathrm{diag}\{\delta(\zeta(k)-1), \ \delta(\zeta(k)-2), \ldots, \delta(\zeta(k)-m)\}$ be the update matrix, the actually received outputs can be described as follows:

$$\bar{y}(k) = \Xi_{\zeta(k)} y(k) + (I - \Xi_{\zeta(k)})\bar{y}(k-1) \tag{6}$$

where

$$\bar{y}(k) = [\bar{y}_1(k) \quad \bar{y}_2(k) \quad \cdots \quad \bar{y}_m(k)]^T. \tag{7}$$

Defining $\bar{x}(k) \triangleq [x^T(k) \quad \bar{y}^T(k-1)]^T$, system (1) with the RR protocol scheduling can be reformulated as

$$\begin{cases} \bar{x}(k+1) = \bar{\mathcal{C}}_{i,\zeta(k)}\bar{x}(k) + \bar{\mathcal{B}}_i\bar{f}(\bar{x}(k)) + \bar{\mathcal{A}}_i\bar{f}(\bar{x}(k-\tau_i)) \\ \qquad\qquad + \bar{\mathcal{E}}_{i,\zeta(k)}\bar{g}(\bar{x}(k)) + \bar{\omega}(k), \\ \bar{y}(k) = \bar{\mathcal{D}}_{i,\zeta(k)}\bar{x}(k) + \tilde{\mathcal{E}}_{i,\zeta(k)}\bar{g}(\bar{x}(k)) \end{cases} \tag{8}$$

where

$$\bar{\mathcal{C}}_{i,\zeta(k)} = \begin{bmatrix} \mathcal{C}_i & 0 \\ \Xi_{\zeta(k)}\mathcal{D}_i & I - \Xi_{\zeta(k)} \end{bmatrix}, \ \bar{\mathcal{B}}_i = \text{diag}\{\mathcal{B}_i, 0\},$$

$$\bar{\mathcal{A}}_i = \text{diag}\{\mathcal{A}_i, 0\}, \quad \bar{\mathcal{E}}_{i,\zeta(k)} = \begin{bmatrix} 0 & 0 \\ \Xi_{\zeta(k)}\mathcal{E}_i & 0 \end{bmatrix},$$

$$\bar{\mathcal{D}}_{i,\zeta(k)} = \begin{bmatrix} \Xi_{\zeta(k)}\mathcal{D}_i & I - \Xi_{\zeta(k)} \end{bmatrix},$$

$$\tilde{\mathcal{E}}_{i,\zeta(k)} = \begin{bmatrix} \Xi_{\zeta(k)}\mathcal{E}_i & 0 \end{bmatrix}, \quad \bar{\omega}(k) = \begin{bmatrix} \omega(k) \\ 0 \end{bmatrix},$$

$$\bar{f}(\bar{x}(k)) = \mathbf{1}_2 \otimes f(x(k)), \quad \bar{g}(\bar{x}(k)) = \mathbf{1}_2 \otimes g(x(k)).$$

Remark 2: In this paper, ZOHs have been employed to hold the value for the sensors which are not selected to transmit data. That is to say, although only one node has access to the limited communication resource at each transmission instant, the introduction of the ZOHs makes it possible to fully utilize the received measurements for the benefit of an accurate state estimation.

C. The State Estimator

The purpose of this paper is to design an effective estimator to estimate the neuron states based on the received output measurements $\bar{y}(k)$. As such, the state estimator for the dynamic system (8) is constructed as follows :

$$\begin{aligned} \hat{x}(k+1) = {} & \bar{\mathcal{C}}_{i,\zeta(k)}\hat{x}(k) + \bar{\mathcal{B}}_i\bar{f}(\hat{x}(k)) + \bar{\mathcal{A}}_i\bar{f}(\hat{x}(k-\tau_i)) \\ & + \bar{\mathcal{E}}_{i,\zeta(k)}\bar{g}(\hat{x}(k)) + \mathcal{K}_{i,\zeta(k)}\big(\bar{y}(k) \\ & - \bar{\mathcal{D}}_{i,\zeta(k)}\hat{x}(k) - \tilde{\mathcal{E}}_{i,\zeta(k)}\bar{g}(\hat{x}(k))\big) \end{aligned} \tag{9}$$

where $\hat{x}(k) \in \mathbb{R}^{m+n}$ is the estimate of $\bar{x}(k)$ and $\mathcal{K}_{i,\zeta(k)}$ is the gain matrices to be designed.

Denoting $e(k) \triangleq \bar{x}(k) - \hat{x}(k)$ as the estimation error and combing (8) with (9), one obtains the following estimation error dynamics:

$$\begin{aligned} e(k+1) = {} & \big(\bar{\mathcal{C}}_{i,\zeta(k)} - \mathcal{K}_{i,\zeta(k)}\bar{\mathcal{D}}_{i,\zeta(k)}\big)e(k) + \bar{\mathcal{B}}_i\big(\bar{f}(\bar{x}(k)) \\ & - \bar{f}(\hat{x}(k))\big) + \bar{\mathcal{A}}_i\big(\bar{f}(\bar{x}(k-\tau_i)) \\ & - \bar{f}(\hat{x}(k-\tau_i))\big) + \big(\bar{\mathcal{E}}_{i,\zeta(k)} - \mathcal{K}_{i,\zeta(k)}\tilde{\mathcal{E}}_{i,\zeta(k)}\big) \\ & \times \big(\bar{g}(\bar{x}(k)) - \bar{g}(\hat{x}(k))\big) + \bar{\omega}(k). \end{aligned} \tag{10}$$

For simplicity, we introduce the following notations:

$$\begin{aligned} \Im(k) &= \Big[\bar{x}^T(k) \quad e^T(k)\Big]^T, \\ \tilde{f}(\Im(k)) &= \Big[\bar{f}^T(\bar{x}(k)) \quad \bar{f}^T(\bar{x}(k)) - \bar{f}^T(\hat{x}(k))\Big]^T, \\ \tilde{g}(\Im(k)) &= \Big[\bar{g}^T(\bar{x}(k)) \quad \bar{g}^T(\bar{x}(k)) - \bar{g}^T(\hat{x}(k))\Big]^T. \end{aligned}$$

Then, taking system (8) and the estimation error (10) into consideration, an augmented system model is given as follows:

$$\begin{aligned} \Im(k+1) = {} & C_{i,\zeta(k)}\Im(k) + B_i\tilde{f}(\Im(k)) + A_i\tilde{f}(\Im(k-\tau_i)) \\ & + E_{i,\zeta(k)}\tilde{g}(\Im(k)) + W(k) \end{aligned} \tag{11}$$

where

$$\begin{aligned} C_{i,\zeta(k)} &= \text{diag}\{\bar{\mathcal{C}}_{i,\zeta(k)}, \bar{\mathcal{C}}_{i,\zeta(k)} - \mathcal{K}_{i,\zeta(k)}\bar{\mathcal{D}}_{i,\zeta(k)}\}, \\ B_i &= \text{diag}\{\bar{\mathcal{B}}_i, \bar{\mathcal{B}}_i\}, \quad A_i = \text{diag}\{\bar{\mathcal{A}}_i, \bar{\mathcal{A}}_i\}, \\ E_{i,\zeta(k)} &= \text{diag}\{\bar{\mathcal{E}}_{i,\zeta(k)}, \bar{\mathcal{E}}_{i,\zeta(k)} - \mathcal{K}_{i,\zeta(k)}\tilde{\mathcal{E}}_{i,\zeta(k)}\}, \\ W(k) &= \Big[\bar{\omega}^T(k) \quad \bar{\omega}^T(k)\Big]^T. \end{aligned}$$

Remark 3: Comparing with the existing results, such as [52], which has investigated the state estimation problem for a class of MJNNs with mixed time-delays, this paper has taken the constrained communication issue between the network output and the possible remote estimator into account. In order to alleviate the data transmission burden resulting from the limited capacity of the transmission network, the RR protocol is adopted to grant each sensor an equivalent right to access the data transmission service. The theoretical result can provide a basis of the practical applications.

Definition 1: [45] The augmented system (11) is said to be exponentially ultimately bounded in the mean square if there exist constants $\alpha \in [0,1), \beta > 0$ and $\varepsilon > 0$ such that

$$\mathbb{E}\{\|\Im(k)\|^2|\Im(0)\} \leq \alpha^k\beta + \varepsilon$$

where α is the decay rate and ε is an upper bound of $\mathbb{E}\{\|\Im(k)\|^2\}$. In this case, (9) is called an exponentially ultimately bounded estimator of the MJNNs (1).

The objective of this paper is twofold:

R1) Under the RR communication protocol, establish the existence condition of the exponentially ultimately bounded estimator (9);

R2) Derive the explicit expression of the estimator gain matrices $\mathcal{K}_{i,\zeta(k)}$ and provide a specific ultimate upper bound of the estimation error.

III. Main Results

Lemma 1 (Schur Complement [2]): Given constant matrices $\Re_1$, $\Re_2$, $\Re_3$ where $\Re_1 = \Re_1^T$ and $\Re_2 = \Re_2^T > 0$, then $\Re_1 + \Re_3^T\Re_2^{-1}\Re_3 < 0$ if and only if

$$\begin{bmatrix} \Re_1 & \Re_3^T \\ \Re_3 & -\Re_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Re_2 & \Re_3 \\ \Re_3^T & \Re_1 \end{bmatrix} < 0. \tag{12}$$

A. Exponentially Ultimate Boundedness

In this subsection, a sufficient condition is proposed to guarantee the ultimate boundedness of the dynamics of the estimation error (11) in the mean square.

Theorem 1: Under Assumption 1, let the estimator gain matrices $\mathcal{K}_{i,\zeta(k)}$ be given. For all $i \in S$, the augmented system (11) is exponentially ultimately bounded in the mean square if there exist positive definite matrices $\mathcal{P}_{1i,\zeta(k)} > 0$, $\mathcal{P}_{1i,\zeta(k+1)} > 0$ and $\mathcal{Q} > 0$, a scalar $0 < \kappa < 1$ as well as a set of positive constant scalars ρ_{1i}, ρ_{2i} and ρ_{3i} satisfying

$$\begin{bmatrix} \Gamma_{i,\zeta(k)} & * \\ \Sigma_{i,\zeta(k)} & -\bar{\mathcal{P}}^{-1}_{i,\zeta(k+1)} \end{bmatrix} < 0 \tag{13}$$

where

$$\Gamma_{i,\zeta(k)} = \begin{bmatrix} \Gamma_1 & * \\ 0 & \Gamma_2 \end{bmatrix}, \ \Gamma_1 = \begin{bmatrix} \Gamma_{11} & * \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix},$$

$$\Gamma_2 = \begin{bmatrix} \kappa\sigma\lambda_{\max}(Q) & * & * \\ 0 & \ddots & * \\ 0 & 0 & \kappa\sigma\lambda_{\max}(Q) \end{bmatrix},$$

$$\Gamma_{11} = \begin{bmatrix} \hat{\Gamma}_{i,\zeta(k)} & * & * \\ 0 & -\rho_{2i}\frac{I\otimes(\mathfrak{H}_1\mathfrak{H}_2+\mathfrak{H}_2^T\mathfrak{H}_1^T)}{2} & * \\ \rho_{1i}\frac{I\otimes(\mathfrak{H}_1+\mathfrak{H}_2)}{2} & 0 & \check{\Gamma}_i \end{bmatrix},$$

$$\Gamma_{21} = \begin{bmatrix} 0 & \rho_{2i}\frac{I\otimes(\mathfrak{H}_1+\mathfrak{H}_2)}{2} & 0 \\ \rho_{3i}\frac{I\otimes(\mathfrak{G}_1+\mathfrak{G}_2)}{2} & 0 & 0 \end{bmatrix},$$

$$\Gamma_{22} = \begin{bmatrix} -\mathcal{Q} - \rho_{2i}I & * \\ 0 & -\rho_{3i}I \end{bmatrix},$$

$$\hat{\Gamma}_{i,\zeta(k)} = -\mathcal{P}_{i,\zeta(k)} - \rho_{1i}\frac{I\otimes(\mathfrak{H}_1\mathfrak{H}_2+\mathfrak{H}_2^T\mathfrak{H}_1^T)}{2} - \rho_{3i}\frac{I\otimes(\mathfrak{G}_1\mathfrak{G}_2+\mathfrak{G}_2^T\mathfrak{G}_1^T)}{2} + \kappa\lambda_{\max}(\mathcal{P}_{i,\zeta(k)})I,$$

$$\check{\Gamma}_i = \sigma\mathcal{Q} - \rho_{1i}I, \ \mathcal{P}_{i,\zeta(k)} = \text{diag}\{\mathcal{P}_{1i,\zeta(k)}, \mathcal{P}_{1i,\zeta(k)}\},$$

$$\bar{\mathcal{P}}_{i,\zeta(k+1)} = \text{diag}\{\bar{\mathcal{P}}_{1i,\zeta(k+1)}, \ \bar{\mathcal{P}}_{1i,\zeta(k+1)}\},$$

$$\bar{\mathcal{P}}_{1i,\zeta(k+1)} = \sum_{j=1}^{s}\theta_{ij}\mathcal{P}_{1j,\zeta(k+1)},$$

$$\Sigma_{i,\zeta(k)} = \begin{bmatrix} C_{i,\zeta(k)} & 0 & B_i & A_i & E_{i,\zeta(k)} & \underbrace{0 \cdots 0}_{\bar{\tau}} \end{bmatrix},$$

$$\sigma = (1-\underline{\theta})(\bar{\tau}-\underline{\tau})+1, \quad \underline{\theta} = \min\{\theta_{ii}|i \in S\}.$$

Proof: Construct the following Lyapunov-Krasovskii functional candidate for system (11):

$$\mathcal{V}(\Im(k), i, \zeta(k)) = \mathcal{V}_1(\Im(k), i, \zeta(k)) + \mathcal{V}_2(\Im(k), i, \zeta(k)) \tag{14}$$

where

$$\mathcal{V}_1(\Im(k), i, \zeta(k)) = \Im^T(k)\mathcal{P}_{i,\zeta(k)}\Im(k),$$

$$\mathcal{V}_2(\Im(k), i, \zeta(k)) = \sum_{d=k-\tau_i}^{k-1}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d)) + (1-\underline{\theta}) \times \sum_{m=\underline{\tau}}^{\bar{\tau}-1}\sum_{d=k-m}^{k-1}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d)).$$

For $i, j \in S$, we have

$$\begin{aligned}
&\mathbb{E}\{\mathcal{V}_1(\Im(k+1), j, \zeta(k+1))|\Im(k), i, \zeta(k)\} \\
&\quad - \mathcal{V}_1(\Im(k), i, \zeta(k)) \\
&= \Big[C_{i,\zeta(k)}\Im(k) + B_i\tilde{f}(\Im(k)) + A_i\tilde{f}(\Im(k-\tau_i)) \\
&\quad + E_{i,\zeta(k)}\tilde{g}(\Im(k))\Big]^T\bar{\mathcal{P}}_{i,\zeta(k+1)}\Big[C_{i,\zeta(k)}\Im(k) \\
&\quad + B_i\tilde{f}(\Im(k)) + A_i\tilde{f}(\Im(k-\tau_i)) + E_{i,\zeta(k)}\tilde{g}(\Im(k))\Big] \\
&\quad - \Im^T(k)\mathcal{P}_{i,\zeta(k)}\Im(k) + \mathbb{E}\Big\{W^T(k)\bar{\mathcal{P}}_{i,\zeta(k+1)}W(k)\Big\}
\end{aligned} \tag{15}$$

and

$$\begin{aligned}
&\mathbb{E}\{\mathcal{V}_2(\Im(k+1),j,\zeta(k+1))|\Im(k),i,\zeta(k)\} \\
&\quad - \mathcal{V}_2(\Im(k),i,\zeta(k)) \\
=&\sum_{j=1}^{s}\theta_{ij}\sum_{d=k+1-\tau_j}^{k}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d)) \\
&+(1-\underline{\theta})\sum_{m=\underline{\tau}}^{\overline{\tau}-1}\sum_{d=k+1-m}^{k}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d)) \\
&-\sum_{d=k-\tau_i}^{k-1}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d)) \\
&-(1-\underline{\theta})\sum_{m=\underline{\tau}}^{\overline{\tau}-1}\sum_{d=k-m}^{k-1}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d)) \\
=&\theta_{ii}\Bigg(\sum_{d=k+1-\tau_i}^{k}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d)) \\
&-\sum_{d=k-\tau_i}^{k-1}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d))\Bigg) \\
&+(1-\underline{\theta})(\overline{\tau}-\underline{\tau})\tilde{f}^T(\Im(k))\mathcal{Q}\tilde{f}(\Im(k)) \\
&+\sum_{j=1,j\neq i}^{s}\theta_{ij}\Bigg(\sum_{d=k+1-\tau_j}^{k}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d)) \\
&-\sum_{d=k-\tau_i}^{k-1}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d))\Bigg) \\
&-(1-\underline{\theta})\sum_{d=k-\overline{\tau}+1}^{k-\underline{\tau}}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d)) \\
=&\sigma\tilde{f}^T(\Im(k))\mathcal{Q}\tilde{f}(\Im(k))-\tilde{f}^T(\Im(k-\tau_i))\mathcal{Q}\tilde{f}(\Im(k-\tau_i)) \\
&+\sum_{j=1,j\neq i}^{s}\theta_{ij}\Bigg(\sum_{d=k+1-\tau_j}^{k-1}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d)) \\
&-\sum_{d=k+1-\tau_i}^{k-1}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d))\Bigg) \\
&-(1-\underline{\theta})\sum_{d=k-\overline{\tau}+1}^{k-\underline{\tau}}\tilde{f}^T(\Im(d))\mathcal{Q}\tilde{f}(\Im(d)) \\
\le&\sigma\tilde{f}^T(\Im(k))\mathcal{Q}\tilde{f}(\Im(k))-\tilde{f}^T(\Im(k-\tau_i))\mathcal{Q}\tilde{f}(\Im(k-\tau_i)).
\end{aligned} \tag{16}$$

Next, it can be derived from (15)-(16) that

$$\begin{aligned}
&\mathbb{E}\{\mathcal{V}(\Im(k+1),j,\zeta(k+1))|\Im(k),i,\zeta(k)\} \\
&\quad-\mathcal{V}(\Im(k),i,\zeta(k)) \\
\le&\Big[C_{i,\zeta(k)}\Im(k)+B_i\tilde{f}(\Im(k))+A_i\tilde{f}(\Im(k-\tau_i)) \\
&+E_{i,\zeta(k)}\tilde{g}(\Im(k))\Big]^T\bar{\mathcal{P}}_{i,\zeta(k+1)}\Big[C_{i,\zeta(k)}\Im(k) \\
&+B_i\tilde{f}(\Im(k))+A_i\tilde{f}(\Im(k-\tau_i))+E_{i,\zeta(k)}\tilde{g}(\Im(k))\Big] \\
&-\Im^T(k)\mathcal{P}_{i,\zeta(k)}\Im(k)-\tilde{f}^T(\Im(k-\tau_i))\mathcal{Q}\tilde{f}(\Im(k-\tau_i)) \\
&+\sigma\tilde{f}^T(\Im(k))\mathcal{Q}\tilde{f}(\Im(k))+\mathbb{E}\Big\{W^T(k)\bar{\mathcal{P}}_{i,\zeta(k+1)}W(k)\Big\}.
\end{aligned} \tag{17}$$

In terms of the constraints (2) and (3), it is easy to verify that

$$\begin{aligned}
&\rho_{1i}\Big[\tilde{f}(\Im(k))-(I\otimes\mathfrak{H}_1)\Im(k)\Big]^T \\
&\times[\tilde{f}(\Im(k))-(I\otimes\mathfrak{H}_2)\Im(k)\Big]\le 0, \\
&\rho_{2i}\Big[\tilde{f}(\Im(k-\tau_i))-(I\otimes\mathfrak{H}_1)\Im(k-\tau_i)\Big]^T \\
&\times\Big[\tilde{f}(\Im(k-\tau_i))-(I\otimes\mathfrak{H}_2)\Im(k-\tau_i)\Big]\le 0, \\
&\rho_{3i}\Big[\tilde{g}(\Im(k))-(I\otimes\mathfrak{G}_1)\Im(k)\Big]^T \\
&\times\Big[\tilde{g}(\Im(k))-(I\otimes\mathfrak{G}_2)\Im(k)\Big]\le 0.
\end{aligned} \tag{18}$$

According to the definition of $\mathcal{V}(\Im(k),i,\zeta(k))$, the following inequality is true:

$$\begin{aligned}
&\kappa\left[\lambda_{\max}(\mathcal{P}_{i,\zeta(k)})\|\Im(k)\|^2+\sigma\lambda_{\max}(\mathcal{Q})\sum_{d=k-\overline{\tau}}^{k-1}\|\tilde{f}(\Im(d))\|^2\right] \\
&-\kappa\mathcal{V}(\Im(k),i,\zeta(k))\ge 0.
\end{aligned} \tag{19}$$

Then, it follows from (17)-(19) that

$$\begin{aligned}
&\mathbb{E}\{\mathcal{V}(\Im(k+1),j,\zeta(k+1))|\Im(k),i,\zeta(k)\} \\
&\quad-\mathcal{V}(\Im(k),i,\zeta(k)) \\
\le&\Big[C_{i,\zeta(k)}\Im(k)+B_i\tilde{f}(\Im(k))+A_i\tilde{f}(\Im(k-\tau_i)) \\
&+E_{i,\zeta(k)}\tilde{g}(\Im(k))\Big]^T\bar{\mathcal{P}}_{i,\zeta(k+1)}\Big[C_{i,\zeta(k)}\Im(k) \\
&+B_i\tilde{f}(\Im(k))+A_i\tilde{f}(\Im(k-\tau_i))+E_{i,\zeta(k)}\tilde{g}(\Im(k))\Big] \\
&-\Im^T(k)\mathcal{P}_{i,\zeta(k)}\Im(k)-\tilde{f}^T(\Im(k-\tau_i))\mathcal{Q}\tilde{f}(\Im(k-\tau_i)) \\
&+\sigma\tilde{f}^T(\Im(k))\mathcal{Q}\tilde{f}(\Im(k))+\mathbb{E}\Big\{W^T(k)\bar{\mathcal{P}}_{i,\zeta(k+1)}W(k)\Big\} \\
&-\Big(\rho_{1i}\tilde{f}^T(\Im(k))\tilde{f}(\Im(k))-\rho_{1i}\Im^T(k)(I\otimes(\mathfrak{H}_1+\mathfrak{H}_2))^T \\
&\times\tilde{f}(\Im(k))+\rho_{1i}\Im^T(k)\frac{I\otimes(\mathfrak{H}_1\mathfrak{H}_2+\mathfrak{H}_2^T\mathfrak{H}_1^T)}{2}\Im(k)\Big) \\
&-\Bigg(\rho_{2i}\tilde{f}^T(\Im(k-\tau_i))\tilde{f}(\Im(k-\tau_i))-\rho_{2i}\Im^T(k-\tau_i) \\
&\times(I\otimes(\mathfrak{H}_1+\mathfrak{H}_2))^T\tilde{f}(\Im(k-\tau_i))+\rho_{2i}\Im^T(k-\tau_i) \\
&\times\frac{I\otimes(\mathfrak{H}_1\mathfrak{H}_2+\mathfrak{H}_2^T\mathfrak{H}_1^T)}{2}\Im(k-\tau_i)\Bigg)-\Bigg(\rho_{3i}\tilde{g}^T(\Im(k))
\end{aligned}$$

$$\times \tilde{g}(\Im(k)) - \rho_{3i}\Im^T(k)(I \otimes (\mathfrak{G}_1 + \mathfrak{G}_2))^T\tilde{g}(\Im(k)) + \rho_{3i}\Im^T(k)\frac{I \otimes (\mathfrak{G}_1\mathfrak{G}_2 + \mathfrak{G}_2^T\mathfrak{G}_1^T)}{2}\Im(k)\Bigg) + \Bigg(\kappa\Big[\lambda_{\max}(\mathcal{P}_{i,\zeta(k)})\|\Im(k)\|^2 + \sigma\lambda_{\max}(\mathcal{Q}) \times \sum_{d=k-\bar{\tau}}^{k-1}\|\tilde{f}(\Im(d))\|^2\Big] - \kappa\mathcal{V}(\Im(k), i, \zeta(k))\Bigg)$$
$$=\Lambda^T(k,i)\tilde{\Gamma}_{i,\zeta(k)}\Lambda(k,i) + \mathbb{E}\Big\{W^T(k)\bar{\mathcal{P}}_{i,\zeta(k+1)}W(k)\Big\} - \kappa\mathcal{V}(\Im(k), i, \zeta(k)) \tag{20}$$

where

$$\tilde{\Gamma}_{i,\zeta(k)} = \Sigma^T_{i,\zeta(k)}\bar{\mathcal{P}}_{i,\zeta(k+1)}\Sigma_{i,\zeta(k)} + \Gamma_{i,\zeta(k)},$$
$$\Lambda(k,i) = \Big[\ \Im^T(k) \quad \Im^T(k-\tau_i) \quad \tilde{f}^T(\Im(k)) \quad \tilde{f}^T(\Im(k-\tau_i)) \quad \tilde{g}^T(\Im(k)) \quad \breve{f}^T(\Im(k))\ \Big]^T,$$
$$\breve{f}(\Im(k)) = \Big[\ \tilde{f}^T(\Im(k-\bar{\tau})) \quad \cdots \quad \tilde{f}^T(\Im(k-1))\ \Big]^T.$$

Moreover, it is straightforward to see that

$$\mathbb{E}\Big\{W^T(k)\bar{\mathcal{P}}_{i,\zeta(k+1)}W(k)\Big\} \le \lambda_{\max}(\bar{\mathcal{P}}_{i,\zeta(k+1)})\mathbb{E}\Big\{W^T(k)W(k)\Big\} = 2\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right).$$

By using the Schur Complement Lemma, it can be inferred from (13) that

$$\Sigma^T_{i,\zeta(k)}\bar{\mathcal{P}}_{i,\zeta(k+1)}\Sigma_{i,\zeta(k)} + \Gamma_{i,\zeta(k)} < 0$$

which implies

$$\mathbb{E}\{\mathcal{V}(\Im(k+1), j, \zeta(k+1))|\Im(k), i, \zeta(k)\} - \mathcal{V}(\Im(k), i, \zeta(k)) \le 2\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right) - \kappa\mathcal{V}(\Im(k), i, \zeta(k)).$$

For any scalar $\mu > 0$, one has

$$\mu^{k+1}\mathbb{E}\{\mathcal{V}(\Im(k+1), j, \zeta(k+1))|\Im(k), i, \zeta(k)\} - \mu^k\mathcal{V}(\Im(k), i, \zeta(k))$$
$$= \mu^{k+1}\Big(\mathbb{E}\{\mathcal{V}(\Im(k+1), j, \zeta(k+1))|\Im(k), i, \zeta(k)\} - \mathcal{V}(\Im(k), i, \zeta(k))\Big) + \mu^k(\mu-1)\mathcal{V}(\Im(k), i, \zeta(k))$$
$$\le \mu^{k+1}\Big(2\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right) - \kappa\mathcal{V}(\Im(k), i, \zeta(k))\Big) + \mu^k(\mu-1)\mathcal{V}(\Im(k), i, \zeta(k))$$
$$= \mu^k(\mu - \mu\kappa - 1)\mathcal{V}(\Im(k), i, \zeta(k)) + 2\mu^{k+1}\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right). \tag{21}$$

By applying the law of total expectation, it can be derived from (21) that

$$\mu^{k+1}\mathbb{E}\{\mathcal{V}(\Im(k+1), j, \zeta(k+1))\} - \mu^k\mathbb{E}\{\mathcal{V}(\Im(k), i, \zeta(k))\} \le \mu^k(\mu - \mu\kappa - 1)\mathbb{E}\{\mathcal{V}(\Im(k), i, \zeta(k))\} + 2\mu^{k+1}\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right). \tag{22}$$

Letting $\mu = \frac{1}{1-\kappa}$ and summing up both sides of the inequality (22) from 0 to $\iota - 1$ with respect to k, we have

$$\mu^\iota\mathbb{E}\{\mathcal{V}(\Im(\iota), r(\iota), \zeta(\iota))\} - \mathbb{E}\{\mathcal{V}(\Im(0), r(0), \zeta(0))\} \le \frac{2\mu(1-\mu^\iota)}{1-\mu}\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right),$$

which further indicates

$$\mathbb{E}\{\mathcal{V}(\Im(\iota), r(\iota), \zeta(\iota))\} \le \mu^{-\iota}\Big(\mathbb{E}\{\mathcal{V}(\Im(0), r(0), \zeta(0))\} + \frac{2\mu}{1-\mu}\upsilon^2\lambda_{\max}(\bar{\mathcal{P}}_{i,\zeta(k+1)})\Big) + \frac{2\mu}{\mu-1}\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right)$$
$$= (1-\kappa)^\iota\Big(\mathbb{E}\{\mathcal{V}(\Im(0), r(0), \zeta(0))\} - \frac{2}{\kappa}\upsilon^2\lambda_{\max}(\bar{\mathcal{P}}_{i,\zeta(k+1)})\Big) + \frac{2}{\kappa}\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right).$$

Noticing the fact that $\mathbb{E}\{\mathcal{V}(\Im(\iota), r(\iota), \zeta(\iota))\} \ge \lambda_{\min}\left(\mathcal{P}_{i,\zeta(k)}\right)\mathbb{E}\{\|\Im(\iota)\|^2\}$, we immediately arrive at

$$\mathbb{E}\{\|\Im(\iota)\|^2\} \le \frac{\Pi}{\lambda_{\min}\left(\mathcal{P}_{i,\zeta(k)}\right)} = \alpha^\iota\beta + \varepsilon \tag{23}$$

where

$$\Pi = (1-\kappa)^\iota\left(\mathbb{E}\{\mathcal{V}(\Im(0), r(0), \zeta(0))\} - \frac{2}{\kappa}\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right)\right) + \frac{2}{\kappa}\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right),$$
$$\alpha = (1-\kappa), \quad \varepsilon = \frac{\frac{2}{\kappa}\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right)}{\lambda_{\min}\left(\mathcal{P}_{i,\zeta(k)}\right)},$$
$$\beta = \frac{\mathbb{E}\{\mathcal{V}(\Im(0), r(0), \zeta(0))\} - \frac{2}{\kappa}\upsilon^2\lambda_{\max}\left(\bar{\mathcal{P}}_{i,\zeta(k+1)}\right)}{\lambda_{\min}\left(\mathcal{P}_{i,\zeta(k)}\right)}.$$

Therefore, the augmented system (11) is exponentially ultimately bounded in the mean square and the proof is now complete. ∎

B. Design of the State Estimator

After establishing the existence condition of the state estimator (9) in the previous subsection, we are now in a position to deal with the estimator design issue.

Theorem 2: For all $i \in S$, there exists an exponentially ultimately bounded estimator (9) for the MJNNs (1) if there exist positive definite matrices $\mathcal{P}_{1i,\zeta(k)} > 0$, $\mathcal{P}_{1i,\zeta(k+1)} > 0$, $\mathcal{Q} > 0$ and $\mathcal{X}_{i,\zeta(k)}$, scalars γ, ϱ and $0 < \kappa < 1$, a set of positive constant scalars ρ_{1i}, ρ_{2i} and ρ_{3i} such that the following linear matrix inequalities (LMIs) hold:

$$\begin{bmatrix} \Omega_{i,\zeta(k)} & * & * \\ \breve{\Sigma}_{i,\zeta(k)} & -\bar{\mathcal{P}}_{i,\zeta(k+1)} & * \\ \Upsilon & 0 & -I \end{bmatrix} < 0, \tag{24}$$

$$\mathcal{Q} < \varrho I, \tag{25}$$

$$\mathcal{P}_{i,\zeta(k)} < \gamma I \tag{26}$$

where

$$\Omega_{i,\zeta(k)} = \begin{bmatrix} \Omega_1 & * \\ 0 & \Omega_2 \end{bmatrix}, \ \Omega_1 = \begin{bmatrix} \Omega_{11} & * \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix},$$

$$\Omega_{11} = \begin{bmatrix} \hat{\Omega}_{i,\zeta(k)} & * & * \\ 0 & -\rho_{2i}\frac{I\otimes(\mathfrak{H}_1\mathfrak{H}_2+\mathfrak{H}_2^T\mathfrak{H}_1^T)}{2} & * \\ \rho_{1i}\frac{I\otimes(\mathfrak{H}_1+\mathfrak{H}_2)}{2} & 0 & \breve{\Gamma}_i \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} \varsigma I - I & * & * \\ 0 & \ddots & * \\ 0 & 0 & \varsigma I - I \end{bmatrix}, \ \delta = \kappa\gamma, \ \varsigma = \sigma\kappa\varrho,$$

$$\begin{aligned} \hat{\Omega}_{i,\zeta(k)} &= -\mathcal{P}_{i,\zeta(k)} - \rho_{1i}\frac{I\otimes(\mathfrak{H}_1\mathfrak{H}_2+\mathfrak{H}_2^T\mathfrak{H}_1^T)}{2} \\ &\quad - \rho_{3i}\frac{I\otimes(\mathfrak{G}_1\mathfrak{G}_2+\mathfrak{G}_2^T\mathfrak{G}_1^T)}{2} + \delta I, \\ \breve{\Sigma}_{i,\zeta(k)} &= \Big[\vec{C}_{i,\zeta(k)} \quad 0 \quad \bar{\mathcal{P}}_{i,\zeta(k+1)}B_i \quad \bar{\mathcal{P}}_{i,\zeta(k+1)}A_i \\ &\qquad \vec{E}_{i,\zeta(k)} \quad \underbrace{0 \cdots 0}_{\bar{\tau}} \Big], \\ \vec{C}_{i,\zeta(k)} &= \mathrm{diag}\{\bar{\mathcal{P}}_{1i,\zeta(k+1)}\bar{\mathcal{C}}_{i,\zeta(k)}, \bar{\mathcal{P}}_{1i,\zeta(k+1)}\bar{\mathcal{C}}_{i,\zeta(k)} \\ &\quad - \mathcal{X}_{i,\zeta(k)}\bar{\mathcal{D}}_{i,\zeta(k)}\}, \\ \vec{E}_{i,\zeta(k)} &= \mathrm{diag}\{\bar{\mathcal{P}}_{1i,\zeta(k+1)}\bar{\mathcal{E}}_{i,\zeta(k)}, \bar{\mathcal{P}}_{1i,\zeta(k+1)}\bar{\mathcal{E}}_{i,\zeta(k)} \\ &\quad - \mathcal{X}_{i,\zeta(k)}\tilde{\mathcal{E}}_{i,\zeta(k)}\}, \\ \Upsilon &= \Big[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \underbrace{I \cdots I}_{\bar{\tau}} \Big]. \end{aligned}$$

and the other relevant parameters are defined as those in Theorem 1. Moreover, the gain matrices of the estimator (9) are given as:

$$\mathcal{K}_{i,\zeta(k)} = \bar{\mathcal{P}}_{1i,\zeta(k+1)}^{-1}\mathcal{X}_{i,\zeta(k)}. \tag{27}$$

Proof: According to (19), (25) and (26), it can be inferred that

$$\begin{aligned} &\kappa\left[\gamma\|\Im(k)\|^2 + \varrho\sigma\sum_{d=k-\bar{\tau}}^{k-1}\|\tilde{f}(\Im(d))\|^2\right] \\ &- \kappa\mathcal{V}(\Im(k), i, \zeta(k)) \geq 0. \end{aligned} \tag{28}$$

Therefore, from Theorem 1, it can be easily derived that

$$\begin{bmatrix} \Omega_1 & * \\ \Omega_3 & \Omega_2' \end{bmatrix} < 0 \tag{29}$$

where

$$\Omega_2' = \begin{bmatrix} \varsigma I & * & * & * \\ 0 & \ddots & * & * \\ 0 & 0 & \varsigma I & * \\ 0 & 0 & 0 & -\bar{\mathcal{P}}_{i,\zeta(k+1)}^{-1} \end{bmatrix},$$

$$\Omega_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ C_{i,\zeta(k)} & 0 & B_i & A_i & E_{i,\zeta(k)} \end{bmatrix}.$$

Pre- and post-multiplying the inequality (29) by $\mathrm{diag}\{I, I, I, I, I, I, \ldots, I, \bar{\mathcal{P}}_{i,\zeta(k+1)}\}$, one has

$$\begin{bmatrix} \check{\Omega}_{i,\zeta(k)} & * \\ \bar{\mathcal{P}}_{i,\zeta(k+1)}\Sigma_{i,\zeta(k)} & -\bar{\mathcal{P}}_{i,\zeta(k+1)} \end{bmatrix} < 0 \tag{30}$$

where

$$\check{\Omega}_{i,\zeta(k)} = \begin{bmatrix} \Omega_1 & * \\ 0 & \Omega_2'' \end{bmatrix}, \ \Omega_2'' = \begin{bmatrix} \varsigma I & * & * \\ 0 & \ddots & * \\ 0 & 0 & \varsigma I \end{bmatrix}.$$

By Lemma 1, it can be deduced from (30) that

$$\begin{bmatrix} \Omega_{i,\zeta(k)} & * & * \\ \bar{\mathcal{P}}_{i,\zeta(k+1)}\Sigma_{i,\zeta(k)} & -\bar{\mathcal{P}}_{i,\zeta(k+1)} & * \\ \Upsilon & 0 & -I \end{bmatrix} < 0. \tag{31}$$

Letting $\mathcal{X}_{i,\zeta(k)} = \bar{\mathcal{P}}_{1i,\zeta(k+1)}\mathcal{K}_{i,\zeta(k)}$, it is easy to see that (31) is equivalent to (24) and the proof is now complete. ■

Remark 4: In comparison with the results in [17] without the RR protocol, a distinguished feature of our results is the specific form of the estimator gain matrices. More concretely, it is not difficult to see from (27) that the estimator parameter is not only determined by i (i.e. the mode at transmission instant k) but also by $\zeta(k+1)$ (i.e. the selected transmission node at transmission instant $k+1$), which means that the impact of the introduced RR protocol is completely reflected in the expression of the estimator parameter.

Remark 5: In Theorems 1 and 2, the existence condition for the exponentially ultimately bounded estimator and the gain matrices $\mathcal{K}_{i,\zeta(k)}$ have been given. It is observed that the main results involve all information including the periodic scheduling scheme (i.e. RR protocol), the Markovian jumping parameters, the mode-dependent time delays, the sensor nonlinearities and the stochastic disturbances. By using an equal allocation scheme, the designed state estimator is able to alleviate the data transmission congestion over finite communication resource.

IV. Numerical Example

In this section, we consider a two-neuron two-mode neural network (1) with the following system parameters:

$\tau(1) = 1,\ \tau(2) = 5,\ \mathcal{C}(1) = \text{diag}\{-0.64, -0.50\},$
$\mathcal{C}(2) = \text{diag}\{-0.64, -0.32\},\ \mathcal{B}(1) = \text{diag}\{0.30, 0.46\},$
$\mathcal{B}(2) = \text{diag}\{0.75, -0.34\},\ \mathcal{A}(1) = \text{diag}\{-0.58, 0.50\},$

$$\mathcal{A}(2) = \text{diag}\{-0.48, 0.68\},\ \mathcal{D}(1) = \begin{bmatrix} -4.00 & 0.30 \\ -0.32 & -0.20 \end{bmatrix},$$

$$\mathcal{D}(2) = \begin{bmatrix} -3.80 & -0.28 \\ -0.45 & -0.20, \end{bmatrix},\ \mathcal{E}(1) = \begin{bmatrix} 0.61 & 0.45 \\ 0.21 & 0.39 \end{bmatrix},$$

$$\mathcal{E}(2) = \begin{bmatrix} 0.60 & -0.20 \\ 0.54 & 0.40 \end{bmatrix}.$$

The constant matrices are given as follows:

$$\mathfrak{H}_1 = \begin{bmatrix} -0.63 & 0.54 \\ 0.51 & 0.58 \end{bmatrix},\ \mathfrak{H}_2 = \begin{bmatrix} -0.54 & 0.56 \\ 0.46 & 0.40 \end{bmatrix},$$

$$\mathfrak{G}_1 = \begin{bmatrix} -0.35 & 0.34 \\ -0.35 & 0.57 \end{bmatrix},\ \mathfrak{G}_2 = \begin{bmatrix} 0.40 & 0.43 \\ 0.32 & 0.48 \end{bmatrix}.$$

The activation functions and nonlinear function are chosen as

$$f(x(k)) = \begin{bmatrix} -0.5\sin(x_1(k)) + \sin(0.02x_2(k)) \\ 0.5\sin(x_2(k)) \end{bmatrix},$$

$$g(x(k)) = \begin{bmatrix} -0.32\sin(x_1(k)) + 0.25\sin(x_2(k)) \\ 0.43\sin(x_2(k)) \end{bmatrix}.$$

Suppose that $m = 2$, that is, there are two sensors to transmit the measurement outputs. The initial condition of system (1) is chosen as $\psi(0) = [0.34 \quad -0.65]^T$.

By solving a set of LMIs (24)-(26), we obtain the feasible solutions as follows (for space consideration, only parts of the solutions are listed here):

$$\kappa = 0.5,\quad \gamma = 420.95,\quad \varrho = 323.48,\quad \rho_1(1) = 437.14,$$
$$\rho_2(1) = 166.87,\quad \rho_3(1) = 230.96,\quad \rho_1(2) = 448.74,$$
$$\rho_2(2) = 160.14,\quad \rho_3(2) = 234.10.$$

The state estimator gains are obtained as shown in Table I. The simulation results are shown in Figs. 2-6. Figure 2 presents the evolution of Markovian chain. Under the Markovian chain depicted in Figure 2, the true states and their estimations are shown in Figs. 3-4. Figs. 5-6 represent the response of the estimation error. Moreover, from Table I, we can easily see that there are four groups of solutions derived. The simulation has verified that the proposed state estimation strategy is indeed effective.

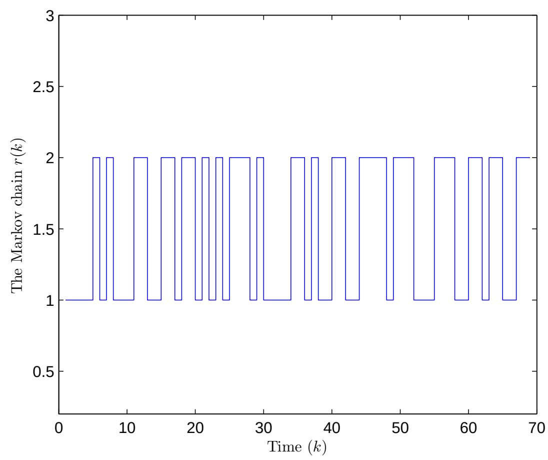

Fig. 2: Modes evolution.

V. Conclusions

In this paper, the state estimator has been designed for a class of MJNNs under the RR protocol with sensor nonlinearities, mode-dependent time delays and stochastic disturbances. For the purpose of easing the communication burden between the MJNNs and the state estimator, the RR protocol has been applied to allocate

TABLE I: Exponentially ultimately bounded state estimator parameters.

$\zeta(k), i$	$\mathcal{P}_{1i,\zeta(k)}$	$\mathcal{P}_{1i,\zeta(k+1)}$	$\mathcal{K}_{i,\zeta(k)}$
$\zeta(k)=1, i=1$	$\begin{bmatrix} 100.48 & 8.66 & 4.53 & 5.66 \\ 8.66 & 33.49 & 6.79 & 5.23 \\ 4.53 & 6.79 & 22.08 & 3.05 \\ 5.66 & 5.23 & 3.05 & 197.10 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 6.29 & 0 \\ 0 & 9.25 & 2.65 & 0 \\ 6.29 & 2.65 & 0 & -42.10 \\ 0 & 0 & -42.10 & -28.63 \end{bmatrix}$	$\begin{bmatrix} 0.10 & -0.20 \\ 0.10 & -0.21 \\ -0.31 & 0 \\ 0.01 & 0.39 \end{bmatrix}$
$\zeta(k)=1, i=2$	$\begin{bmatrix} 173.58 & -7.77 & 24.36 & 2.84 \\ -7.77 & 197.45 & 19.93 & 2.88 \\ 24.36 & 19.93 & 157.65 & -5.60 \\ 2.84 & 2.88 & -5.60 & 185.24 \end{bmatrix}$	$\begin{bmatrix} 22.02 & 34.92 & 14.33 & 15.25 \\ 34.92 & 99.75 & 41.27 & 55.58 \\ 14.33 & 41.27 & 746.65 & 22.13 \\ 15.25 & 55.58 & 22.13 & 83.94 \end{bmatrix}$	$\begin{bmatrix} -0.12 & -0.44 \\ -0.09 & -0.02 \\ 0.35 & 1.26 \\ -0.09 & -0.54 \end{bmatrix}$
$\zeta(k)=2, i=1$	$\begin{bmatrix} 173.14 & 13.30 & 4.69 & 4.32 \\ 13.30 & 39.70 & 12.56 & 10.87 \\ 4.69 & 12.56 & 41.94 & 3.51 \\ 4.32 & 10.87 & 3.51 & 175.42 \end{bmatrix}$	$\begin{bmatrix} 59.39 & 16.23 & 11.02 & 12.70 \\ 16.23 & 93.04 & 38.60 & 37.51 \\ 11.02 & 38.60 & 39.29 & 17.36 \\ 12.70 & 37.51 & 17.36 & 26.39 \end{bmatrix}$	$\begin{bmatrix} -0.04 & 0.01 \\ 0.08 & -0.22 \\ 0 & 0.51 \\ -0.11 & 0.01 \end{bmatrix}$
$\zeta(k)=2, i=2$	$\begin{bmatrix} 178.52 & -14.69 & -0.10 & 0.20 \\ -14.69 & 185.82 & -3.04 & -0.18 \\ -0.10 & -3.04 & 166.72 & -5.74 \\ 0.20 & -0.18 & -5.74 & 173.33 \end{bmatrix}$	$\begin{bmatrix} 27.98 & 469.43 & 0 & -84.05 \\ 469.43 & 0 & 0 & -24.56 \\ 0 & 0 & 0 & 0 \\ -84.05 & -24.56 & 0 & -42.22 \end{bmatrix}$	$\begin{bmatrix} -0.02 & 0 \\ 0 & 0 \\ 0.02 & 0 \\ 0 & 0 \end{bmatrix}$

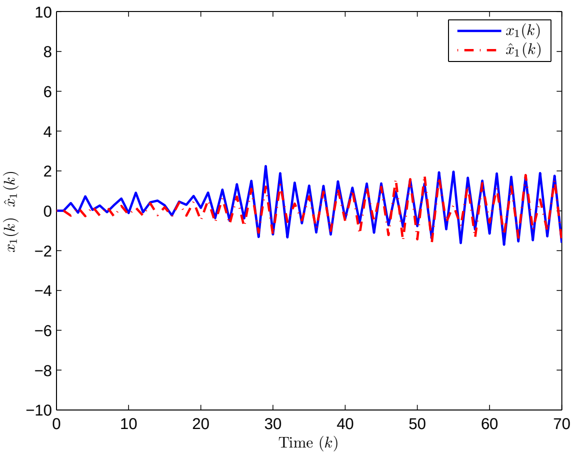

Fig. 3: $x_1(k)$ and its estimate $\hat{x}_1(k)$.

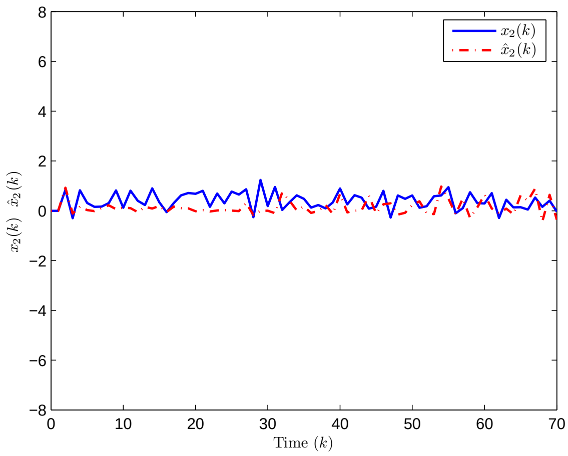

Fig. 4: $x_2(k)$ and its estimate $\hat{x}_2(k)$.

the limited communication resource. By means of the Lyapunov stability theory, stochastic analysis technique and the update matrix method, a sufficient condition has been presented for the exponential boundedness of the error dynamics. By solving a convex optimization problem via the standard Matlab software, the desired state estimator has been derived and a simulation example has been utilized to demonstrate the effectiveness of the proposed state estimation scheme. Finally, it is worth pointing out that it is possible to generalize our main results to other complex systems, such as the multi-agent systems with any communication protocol (e.g. Round-Robin protocol, Stochastic Communication protocol, Weighted Try-Once-Discard protocol, etc.), and some related results will be reported in future. Moreover, in order to reflect the reality more closely, other factors can be considered in the state estimation issues for MJNNs with time delays and sensor nonlinearities under RR protocol in the future. For example, it would be interesting to consider network attacks, incomplete measurements such as quantization and sensor saturation, estimator gain variations and event-triggered scheme

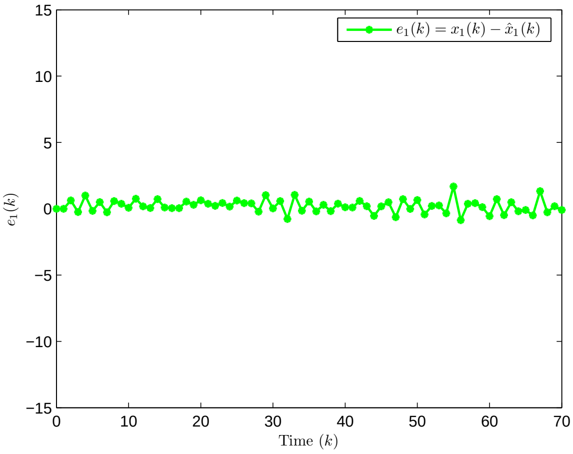

Fig. 5: Estimation error $e_1(k)$.

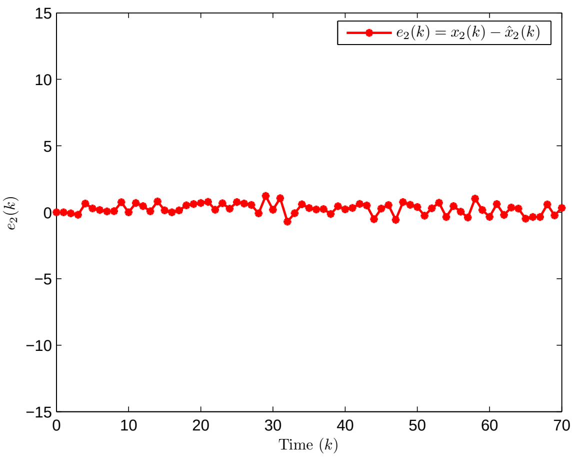

Fig. 6: Estimation error $e_2(k)$.

which have been investigated in [10], [12].

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