

DYNAMIC PERFORMANCE OF SIMPLY-SUPPORTED RIGID-PLASTIC SQUARE PLATES SUBJECT TO LOCALISED BLAST LOADING

N. Mehreganian,

^a Department of Civil & Environmental Engineering, Skempton Building, South Kensington Campus, Imperial College
London, London SW7 2AZ, UK

A. S. Fallah ^{*},

^b Department of Mechanical, Aerospace and Civil Engineering, Brunel University London, Uxbridge UB8 3PH, UK

L.A. Louca

^a Department of Civil & Environmental Engineering, Skempton Building, South Kensington Campus, Imperial College
London, London SW7 2AZ, UK

ABSTRACT

This paper presents the theoretical solution to the response of a square plate undergoing plastic deformation due to a generic localised blast pulse. A representative localised blast load function was considered with spatial distribution of constant pressure over a central zone and exponentially decaying outside that zone. Considering an appropriate moment function and ignoring the membrane, transverse shear and rotary inertia effects, the static plastic collapse was initially found, whereby the analysis was extended to dynamic case by assuming a kinematically admissible, time dependent velocity profile. The analytical model, which was validated against the numerical results obtained through ABAQUS hydrocode, showed close correlation in terms of the permanent transverse deflection profile. In order to consider the effect of temporal pulse shape, the results were formulated for the rectangular as well as exponentially and linearly decaying pulses. For blast loads of high magnitude, the pressure load was replaced by an impulsive velocity. The calculations were simplified utilising the dimensionless form and the results were corroborated with the theoretical and experimental results from the literature. The model thus showed improvements in predicting the final deformation of square plates over the previous models of simplified loading function.

Keywords: Limit analysis, localised blast, impulsive loading, plastic hinge, plastic collapse pressure

* To whom correspondence should be addressed:

Email: as3@imperial.ac.uk (Arash S. Fallah)

Tel.: +44 (0) 2075945140

Fax: +44 (0) 2075941974

Notations

The following symbols are used in this paper:

Latin Upper Case

$A_i - G_i$	Loading parameters; [various]
\dot{D}	Internal Energy dissipation rate; [ML^2T^{-3}]
\dot{E}	External Work rate; [ML^2T^{-3}]
H	Plate characteristic thickness; [L]
I	Impulse; [MLT^{-1}]
L	Plate characteristic side length; [L]
M_0	Maximum moment of the plate per unit length; [MLT^{-2}]
T	End of motion time; [T]
V_0	Impulsive velocity; [LT^{-1}]
\dot{W}	Maximum transverse acceleration; [LT^{-2}]

Latin Lower Case

$f(z), g(z)$	Moment parameter function; [MLT^{-2}]
a	Exponent (loading parameter); [1]
b	Exponent (loading parameter); [L^{-1}]
p_1	Dynamic plastic collapse pressure; [$ML^{-1}T^{-2}$]
p_c	Static plastic collapse pressure; [$ML^{-1}T^{-2}$]
r_e	Loading constant central zone radius; [L]

\dot{w}	Transverse velocity; [LT^{-1}]
\ddot{w}	Transverse acceleration; [LT^{-2}]

Greek Lower case

α	Lower bound Static collapse coefficient; [1]
β	Upper bound Static collapse coefficient; [1]
ξ	Plastic hinge Generalised coordinate; [1]
ε_1	Impulse parameter; [L]
\dot{k}_x	Curvature rate in x direction; [T^{-1}]
\dot{k}_y	Curvature rate in y direction; [T^{-1}]
\dot{k}_{xy}	Warping curvature rate; [T^{-1}]
λ	Dimensionless kinetic energy; [1]
η	Dynamic overloading factor; [1]
μ	Areal density (ρH); [ML^{-2}]
τ	Duration of the pulse; [T]
ω_0	(r_e/L) ; [1]
σ_0	Static yield stress; [MLT^{-2}]

1 Introduction

2 With increasing risk of threats associated with blast loads in recent years, the dynamic performance of
 3 protective plated structures subject to short-term, high intensity pulse loads has been a topic of interest in the
 4 realm of civil, aeronautical, mechanical, defence and military engineering. This has become primarily significant
 5 for design purposes due to many major incidents which have occurred in the UK, in which explosions caused
 6 severe damage to structures (Chen et al., 2015). Attempts to mitigate the structural damage due to blast loads
 7 have, consequently, drawn the attention of engineers to plated metallic structures with the intention of protecting
 8 equipment, systems and people. One of the common forms into which plated structures cast is the quadrangular
 9 plates.

10 Most structural elements are made of steel or other alloys with high post-yield load carrying capacity, as well
 11 as enhanced energy-absorption. Since these elements can undergo large plastic deformations due to extreme
 12 dynamic loading, it is essential to investigate the blast response of such elements using theoretical methods
 13 suitable for inclusion of such effects. In theoretical calculations, the structure is often idealised to behave either

14 as an elastic-perfectly plastic or a rigid-perfectly plastic medium. The latter model- in which the elastic effects are
15 ignored for simplicity (and without great loss of accuracy)- is appropriate for the assessment of dynamic response
16 of blast-loaded structures, provided the loading is treated as a short duration pulse and the ratio of kinetic energy
17 to elastic strain energy is considerably high (Zheng et al., 2016).

18 In the previous analytical investigations (Florence, 1966; Jones, 1997; Jones et al., 1970; Li and Huang, 1989;
19 Nwankwo, Soleiman Fallah, Langdon, et al., 2013; Wierzbicki T and Florence AL, 1970), however, certain
20 limitations have been implemented, viz., loading has been characterised as a uniformly distributed pressure with
21 rectangular temporal pulse shape, which renders the results suitable for response to global blast loads.
22 Nevertheless, blast loads due to the proximal charges of short stand-off distance will lead to localised responses
23 with much higher local deformation and strains as well as the possibility of triggering potential tearing
24 mechanisms, a fact which necessitates the investigation of alternative loading functions. In the sequel, there is a
25 summary on different blast loads using associated characteristics for blast loading functions. The equations
26 proposed by researchers (Fallah et al., 2014; Jones, 1997; Karagiozova et al., 2010; Youngdahl, 1971) have been
27 used to lift restrictions on temporal and spatial distributions of pulse loads as well as providing a realistic yet
28 accurate approximation of localised blast load which proves feasible for such types of loading.

29 ***Localised Blast loads***

30 The blast loading is a high-pressure load arisen either from deflagration, (e.g. propagation of flame in a gas
31 explosion), or chemical detonation within a high explosive charge (e.g., Trinitrotoluene (TNT) explosion) (Schultz
32 et al., 1999). Due to the rather instantaneous chemical reaction within a charge, the latter is characterised with
33 much higher pressure and flame propagation velocity than the former. Furthermore, the duration of detonation is
34 generally much lower than that of deflagration and the rise time to maximum pressure for detonation is virtually
35 zero in contradistinction to the finite rise time for deflagration processes. Besides, depending on the proximity of
36 the blast source, the loading can be classified as global (e.g. far-field explosions) or localised (e.g. buried land
37 mines). While researchers have proposed a few definitions for the sake of classification of loading depending on
38 certain attributes (Jacob et al., 2007; Micallef et al., 2015), the concern has been regarding the structural response
39 rather than the definitions.

40 References (Gharababaei and Darvizeh, 2010) and (Jacob et al., 2007) used the ratio of stand-off distance to
41 plate radius to discern the localised load from global blast, while an empirical relationship by Hopkinson and
42 Cranz (DOD., 2008; Neuberger et al., 2007) describing a scaling parameter as ratio of stand-off-distance to the

43 cubic root of TNT equivalent charge weight can be used as an index to gauge the structural response type.
44 References (Chung Kim Yuen et al., 2012; Neuberger et al., 2007, 2009) incorporated the Hopkinson Crazz
45 scaling law to define a scaling model that correlates the response of circular plates to uniform blasts in various
46 load case scenarios. However, Micallef et al (Micallef et al., 2015) found the stand-off to charge diameter ratio a
47 more theoretically sound scaling factor to gauge the loading nature.

48 ***Static and Dynamic performance of the plate***

49 The first study on static collapse of plates was conducted by Hopkins and Prager (Hopkins and Prager, 1953)
50 who implemented the limit analysis theorems to determine the load carrying capacity of simply supported rigid-
51 perfectly plastic circular plates. The loading was subsequently considered with rectangular pulse shapes, the
52 dynamic effect of which was investigated by researchers (Xue and Hutchinson, 2004; Yuan and Tan, 2013)
53 establishing that the ratio of the blast duration to the total plate response time is pivotal in idealisation of the blast
54 with zero period, i.e. uniform momentum impulse. Nwankwo et al. examined the temporal and spatial distribution
55 of peel and shear stresses in beams' adhesive lap joints which were subject to a transverse, uniformly distributed
56 blast load. (Nwankwo, Soleiman Fallah and Louca, 2013). (Youngdahl, 1971) discussed the strong dependence
57 of the dynamic plastic deformation of plated structures on the pulse shape. An empirical relationship was proposed
58 to eliminate the pulse shape effect which was incorporated in the design and analysis of two-dimensional structural
59 members. He proposed a relationship which linked the maximum plastic deformation to square of effective
60 impulse multiplied by a function of effective pressure. This relationship was widely used for decades prior to
61 being theoretically established by researchers as the upper bound for the actual displacement while the response
62 time was found to be the lower bound on the actual response time (Li and Jones, 2005).

63 Rajendran and Lee (Rajendran and Lee, 2009) presented a detailed review of the pressure pulse from air and
64 underwater explosions. The study included a description of blast wave detonation as well as shock wave
65 propagation, various forms of pressure loads, and the plate wave interaction. Methods of calculating response to
66 such shocks were proposed by researchers (Balden and Nurick, 2006; Rajendran and Narasimhan, 2006;
67 Wierzbicki and Nurick, 1996). (Cox and Morland, 1959) obtained theoretical solutions for dynamic plastic
68 response of simply supported square plates as well as the response of n-sided polygonal plates subjected to uniform
69 dynamic load. Jones with co-authors presented an extensive series of theoretical and experimental research on
70 various structural elements subject to spatially uniform pressure loads of rectangular temporal pulse shape (Jones,
71 1968, 1971, 2013; Jones et al., 1970; Jones and Griffii, 1971; Li and Jones, 2000). In most cases, Jones' analytical

72 models for impulsive loading provided solutions in good agreement with experimental works, when the ratio of
73 kinetic energy to maximum strain energy stored elastically was more than ten (Zheng et al., 2016). (Komarov and
74 Nemirovskii, 1986) extended further the analyses of (Jones, 1997) to the dynamic case with travelling plastic
75 hinges to obtain the incipient plastic deformation in each of the two stages of motion.

76 For blasts with high magnitude impulses, large deformations are expected and for thin shells the membrane
77 forces dominate the structural behaviour. Yuan et al. proposed an analytical model to calculate the large
78 deformation of elastic-perfectly plastic beam systems. They investigated the influence of the catenary (membrane)
79 action in conjunction with its interaction with bending and shear actions in the plastic limits. Three distinct failure
80 deformations mode of beams subject to dynamic (non-impulsive) uniform loaded were investigated (Yuan et al.,
81 2016). The effect of membrane action was similarly studied by (Jones, 1997, 2012, 2014b) for blast loaded plates
82 and for plates struck by single and repeated masses, the latter being a phenomenon called pseudo-shakedown.
83 Chen and Yu (Chen and Yu, 2014) investigated the membrane effects on beams with inclusion of transient state
84 in the velocity profile. Other researchers (Yu and Chen, 1992) extended analysis of Komarov and Nemirovskii
85 (Komarov and Nemirovskii, 1986) to include the effect of catenary action on circular panels. It was shown,
86 theoretically, that the inclusion of transient phase-in which the velocity profile is time dependent- in analysis gave
87 better agreement with experimental results than analysis of (Jones, 1997). Jones later presented a mathematical
88 procedure for the strain rate sensitive ductile plate under impact and explosive loading. His equations to predict
89 the strain rate behaviour of plates were not only dependent on the material constants D and q (from Cowper
90 Symonds equation), but also on the flow stress and density of the plate material (Jones, 2014a).

91 Zheng et al. discussed the strain energy of a stiffened plate subject to uniform pressure load in terms of bending
92 strain energy, elastic-plastic membrane strain energy in plastic zones and strain energy of the stiffeners. The plate
93 deformation was represented by a cosine shape function to represent the global deformation of central zone and a
94 linear shape function used for simplified rigid-plastic model in the surroundings parts. The generalised
95 displacements were obtained numerically and used to determine the deflection time history (Zheng et al., 2016).

96 Nurick and co-authors (Cloete et al., 2006; Nurick et al., 2016; Nurick and Balden, 2010; Nurick and Martin,
97 1989; Nurick and Radford, 1997) conducted a series of experiments to determine the dynamic plastic deformation
98 of rectangular and circular plates and developed a range of dimensionless parameters, which are of practical
99 significance in the study of dynamic response of plates subject to localised and global blast loads. The numerical
100 and experimental studies on localised blast loading effects were further investigated by (Bonorchis and Nurick,
101 2007, 2009, 2010) and (Jacob et al., 2007) to account for the effects of boundary conditions and stand-off distance.

102 The previous studies on rectangular plates have focused on temporally rectangular pulse loads of a spatially
103 uniform pressure loading profile. The purpose of the study in this paper is to derive a theoretical model, which
104 determines the collapse load and dynamic plastic deformation of square plates subject to localised blast loads.
105 Similar work was done on circular plates, thick plates and membranes (Micallef et al., 2014) ,(Micallef et al.,
106 2012, 2016).

107 For this purpose, this paper is presented in seven sections: following the introduction, the initial assumptions
108 are discussed, the fundamental energy equilibrium equations and the characteristics of the localised blast load
109 function are presented. Then, in [Section 3](#), the solutions for the static collapse pressure load of a square plate are
110 obtained. This is followed by an investigation into the dynamic collapse phenomenon and displacement solutions
111 for each phase of motion. Subsequently, the dimensionless kinetic energy-displacement relation is obtained for
112 impulsive loading. In [Section 6](#), the proposed analytical model is validated against the numerical results obtained
113 through a Finite Element model set up in ABAQUS 6.14-4/Explicit® and the experimental studies extracted from
114 the literature in. Finally, [Section 7](#) presents the concluding remarks of the study.

115 **Statement of the problem**

116 The purpose of the current study is to extend, within the framework of limit analysis, the theoretical studies
117 mentioned in Section 1 to cases including localised blast loads through implementation of a modified loading
118 function(Fallah et al., 2014; Karagiozova et al., 2010), (Micallef et al., 2012). While the theoretical studies from
119 the literature have focused on the general solutions for dynamic plastic deformation of plates subject to impulsive
120 loads of uniform distribution, the objective of this work is to derive the general solutions for square plated
121 structures subject to any form of blast loading, i.e. localised or global.

122 Since the localised blasts affect only a small area of the structure severely, it is expected that boundary conditions
123 are not significant and full plate action may not be required (Micallef et al., 2012). Certain other assumptions and
124 the fundamental equilibrium equations are discussed hereunder.

125 **Assumptions**

126 For the simplicity of the analyses to be conducted, the following assumptions are made throughout the study:
127 1- In view of the Kirchhoff-Love plate theory, the quadrangular plate studied herein is assumed to be sufficiently
128 ‘thin’ such that the effects of transverse shear and rotatory inertia can be neglected but not thin to the extent
129 that in-plane actions can have a considerable effect on the plate response. Consequently, it is assumed that

130 bending action is predominant and its effect transcends those of the membrane, transverse shear or rotatory
131 inertia.

132 2- Furthermore, the blast pressure load is assumed to be exerted on the structural elements laterally, such that the
133 plate material particles follow straight trajectories normal to the un-deformed plate mid-surface. Consequently,
134 it is assumed that in-plane displacement components vanish from equilibrium equations (Wierzbicki and
135 Nurick, 1996). Moreover, the plate maintains its uniform thickness throughout the motion and through-
136 thickness dilatational waves are not considered.

137 **Material properties and Loading**

138 An initially flat, monolithic, ductile metallic square plate with side length of $2L$ and thickness of H , with
139 simply supported boundary conditions all along the periphery, is subjected to a representative axi-symmetric
140 localised blast load (Micallef et al., 2012; Nurick and Balden, 2010). Due to geometrical symmetry, only one
141 quarter of the plate is considered in the analyses.

142 In most works of the literature (Micallef et al., 2015), (Bonorchis and Nurick, 2009; Karagiozova et al., 2010;
143 Langdon et al., 2005), the localised blast load, which is a function of temporal and spatial variables, is assumed
144 multiplicative, i.e. $p(x, y, t) = f(x, y)q(t)$. The temporal part of the piecewise continuous loading function is
145 considered as a rectangular pulse shape given by Eqn. (1) and shown in Fig. 1, whereas the spatial form is assumed
146 to be constant over the central region and decay exponentially outside this zone, as given by Eqn. (2) and depicted
147 in Fig. 2. The influence of alternative pulse shapes will be discussed in Appendix A.

148

$$p_1(t) = \begin{cases} 1 & \text{for } 0 \leq t \leq \tau \\ 0 & \text{for } t \geq \tau \end{cases} \quad (1)$$

$$p_2(r) = \begin{cases} p_0 & 0 \leq r \leq r_e \\ ap_0e^{-br} & r_e \leq r \leq L \end{cases} \quad (2)$$

149

150 The constant ' a ' depends on the loading central diameter, $a = e^{br_e}$, and ' b ' can be found through regression
151 analysis on the pressure time histories obtained numerically or experimentally. The Loading constant central zone
152 parameter r_e is referred to as the loading radius hereinafter.

153 **Governing equations**

154 The rate of plastic energy dissipation for a rectangular plate with infinitesimal displacements is found from
155 Eqn. (3).

156

$$\dot{D} = \iint M_x \dot{k}_x + M_y \dot{k}_y + 2M_{xy} \dot{k}_{xy} dx dy \quad (3)$$

157 where $\dot{k}_x = -\partial^2 \dot{w} / \partial x^2$, $\dot{k}_y = -\partial^2 \dot{w} / \partial y^2$, $\dot{k}_{xy} = -\partial^2 \dot{w} / \partial x \partial y$ vectors in x, y and xy directions which are
158 perpendicular to the corresponding portion of the yield surface. In what follows we shall, as is customary, denote
159 differentiation with respect to time by placing a dot above a letter. The rate of external work on a finite plate area
160 A is:

161

$$\dot{E} = \int (p(x, y, t) - \mu \ddot{w}) \dot{w} dA \quad (4)$$

162

163 For a rectangular plate element, which undergoes infinitesimal displacement normal to the plane mid-surface,
164 when subject to lateral loads, the governing equations of motion are given by Eqn.'s (5)-(7):

165

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = \mu \ddot{w} - p(x, y, t) \quad (5)$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0 \quad (6)$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = 0 \quad (7)$$

166

167 where M_x , and M_y are bending moments per unit length (generalised stresses) in generalised coordinates and
168 M_{xy} is the twisting moment per unit length. In a similar fashion, Q_x and Q_y are the shear forces per unit length in
169 the Cartesian coordinate system, according to the normality requirements of plasticity (Jones, 1997).

170 **Yield Surface**

171 The principle bending moments according to (Timoshenko, S.; Woinosky-Kreiger, 1959) are given by Eqn.'s
172 (8)-(9). Provided the principle moments are arranged in this fashion, it can be assumed that the Johansen yield
173 criterion in two-dimensional moment space governs the plastic flow. The associated yield surface, together with

174 Tresca criterion, is shown in the Fig. 3 the condition satisfying it is given as $\text{Max}\{|M_1|, |M_2|\} \leq M_0$, where
 175 M_0 i.e. the maximum plastic moment per unit length is found by Eqn. (10).

176

$$M_1 = (M_x + M_y) / 2 - \frac{1}{2} [(M_x - M_y)^2 + 4M_{xy}^2]^{\frac{1}{2}} \quad (8)$$

$$M_2 = (M_x + M_y) / 2 + \frac{1}{2} [(M_x - M_y)^2 + 4M_{xy}^2]^{\frac{1}{2}} \quad (9)$$

$$M_0 = \frac{\sigma_0 H^2}{4} \quad (10)$$

177

178 Cox & Morland (Cox and Morland, 1959) investigated a particular theoretical solution to dynamic plastic
 179 deformation of square plates subject to uniformly distributed rectangular pressure pulse. It was found convenient
 180 to introduce an auxiliary dimensionless coordinate z , which is equated to $z = (x_1 + y_1)/\sqrt{2}L$ along the central
 181 axis. However, the Cartesian coordinates can be related to the polar coordinate r whereby the blast load is defined
 182 (Eqn. (2)), which lies on the equipotential surface with z for the range $0 < z < 1$, as it is assumed that the
 183 maximum loading range is on the inscribing circle to the plate, giving $p(x, y, t) = p(z, t)$. Fig. 4 and Eqn. (11)
 184 show the relationship between z and the in-plane Cartesian coordinates.

185

$$r = zL = \sqrt{x^2 + y^2}, \quad 0 < r < L \quad (11)$$

186

187 From the isotropy of stress-moment at the plate centre, $M_x = M_y = M_0$ and $M_{xy} = 0$, while along the diagonal
 188 plastic hinge lines-which constructs the collapse mechanism, $M_y = M_0$ when ($y = 0$ and $0 \leq x \leq \sqrt{2}L$) and
 189 $M_x = M_0$ when ($x = 0$ and $0 \leq y \leq \sqrt{2}L$). Rearranging the Eqn.'s (5)-(7) and eliminating the shear force
 190 reactions leads to:

191

$$\frac{\partial^2 M_x}{\partial x^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \mu \ddot{w} - p(x, y, t) \quad (12)$$

192

193 For brevity in analyses in the sequel, the solution to this Ordinary Differential Equation (ODE) is obtained by
 194 defining the generalised stresses (bending moments) and loads in terms of parameter z , as follows.

195 **Static collapse pressure**

196 **Lower bound calculations**

197 For the problem discussed earlier, the lower bound calculations can be conducted by noting that the distribution
198 of bending moment must satisfy the static equilibrium ($\mu\dot{w} = 0$) of the generalised stresses and must nowhere
199 violate the yield criterion. Using the boundary conditions, equation of motion (Eqn. (12)), and considering the
200 principle moments in Eqn.'s (8) and (9), we can assume that the generalised stresses are attributed to a moment
201 distribution function as represented in Eqn.'s (13)-(15):

202

$$M_x = M_0 + x^2 f(z) \quad (13)$$

$$M_y = M_0 + y^2 f(z) \quad (14)$$

$$M_{xy} = xyf(z) \quad (15)$$

203

204 These equations should satisfy the yield condition of Fig. 3 and Eqns. (8)-(9); viz., for any coordinates
205 $0 \leq x \leq \sqrt{2}L$ and $0 \leq y \leq \sqrt{2}L$:

206

$$M_1 = M_0 \quad (16a)$$

$$-M_0 \leq M_2 \leq M_0 \quad (16b)$$

207

208 Eqn. (16a) can be obtained by elementary calculations of (8) with the aid of Eqns. (13)-(15). Thus, the above
209 conditions determine the plastic flow in the regime AD of the yield criterion. The principle moment M_2 can be
210 evaluated in (17), when using Eqns. (11), (13)-(15) in (16b):

211

$$M_2 = M_0 + z^2 f(z) \quad (17)$$

212 The admissibility of Eqn. (17) will be verified in section 4.1.3. Combining Eqns. (13) - (15) into Eqn. (12) and
213 incorporating Eqn. (11) yields:

214

$$6f + 6z \frac{\partial f}{\partial z} + z^2 \frac{\partial^2 f}{\partial z^2} = -p_0, \quad 0 \leq z \leq r_e/L \quad (18)$$

$$6f + 6z \frac{\partial f}{\partial z} + z^2 \frac{\partial^2 f}{\partial z^2} = -ap_0 e^{-bLz} \quad r_e/L \leq z \leq 1 \quad (19)$$

215

216 which are valid for all coordinates across the plate provided the moments are arranged as per Eqns. (13)-(15).

217 Therefore, on integration, a piecewise general solution to differential Eqn. (18) is:

218

$$f(z) = \begin{cases} -\frac{p_0}{6} + \frac{A_1}{z^2} + \frac{B_1}{z^3} & 0 \leq z \leq r_e/L \\ \frac{-ap_0 e^{-bLz}(bLz + 2)}{(bLz)^3} + \frac{C_1}{z^2} + \frac{C_2}{z^3} & r_e/L \leq zL \leq L \end{cases} \quad (20a)$$

(20b)

219

220 The boundary conditions satisfying the bending moment distributions are given by $M_x = M_y = M_0$, $M_{xy} =$
 221 $Q_x = Q_y = 0$ at the plate centre, suggesting that the plastic flow in the centre of the plate to be governed by corner
 222 A of Johansen yield criteria. The arbitrary constants $A_1 - C_2$ are found by applying these boundary conditions,
 223 together with the continuity of moment and shear at $z = r_e/L$, and the boundary as follows.

224

$$\begin{cases} A_1 = B_1 = 0 & (21a) \\ C_1 = \frac{-p_0((br_e)^2 + 2br_e + 2)}{2(Lb)^2} & (21b) \\ C_2 = \frac{p_0((br_e)^3 + 3(br_e)^2 + 6br_e + 6)}{3(Lb)^3} & (21c) \end{cases}$$

225

226 The bending moment in an arbitrary section defined by normal n to the circle passing through the section is
 227 given by transformation formula (Eqn.(22)) according to (Braestrup, 2007; Timoshenko, S.;Woinosky-Kreiger,
 228 1959):

229

$$M_n = M_x \sin^2 \phi + M_y \cos^2 \phi + 2M_{xy} \sin \phi \cos \phi \quad (22)$$

230

231 It transpires that, at simply supported plate boundary ($z = 1$), $M_n = 0$ and $\phi = 45$. Evaluating Eqns. (20a-b)
 232 and (21a-c) in Eqn. (17) yields the lower bound for static collapse pressure as:

$$p_0 = p_c = \frac{M_0}{\alpha L^2} \quad (23)$$

233 where

$$\alpha = \left(\frac{3Lr_e^2 - 2r_e^3}{6L^3} \right) + \frac{(ae^{-Lb}(Lb + 2) + b^2r_e(L - r_e) + bL - 2br_e - 2)}{L^3b^3} \quad (24)$$

234 **Upper Bound Calculations**

235 The upper bound to the static collapse load of the square plate can be calculated by employing the principle
 236 of virtual velocities. In this manner, the rate of external work to the rate of internal energy dissipation through
 237 plastic work, i.e. $\dot{D} = \dot{E}$, Eqns. (3)-(4) require a kinematically admissible velocity field. It is assumed that the
 238 velocity profile is of a conical shape, which is the same as the transverse deflection profile throughout the entire
 239 static phase.

240

$$\dot{w} = \dot{W}(1 - z) \quad (25)$$

241

242 While it is physically reasonable to assume the conical shape in Fig. 5 for the velocity profile, it is also
 243 mathematically evident that for the case of $b \rightarrow 0$ (the case of uniform load) and $b \rightarrow \infty$ (i.e., the case of point
 244 load) the velocity shape function is of the form described in (25) and thus there is no reason as to why an alternative
 245 profile exists in the range of $0 < b < \infty$, i.e., the case studied hereunder. Thus, it is reasonable, as a first attempt,
 246 to assume the velocity field as this profile. With this assumption, the external work rate will furnish to the
 247 expression in (26):

248

$$\dot{E} = 2L^2\dot{W}_0 \left(\int_0^{r_e/L} p_u(1 - z)zdz + \int_{r_e/L}^1 p_u(1 - z)zae^{-bLz}dz \right) \quad (26)$$

249

250 Evaluating the integrals of (3), (26), the upper bound plastic collapse load is given as:

$$p_u = \frac{M_0}{\beta L^2} \quad (27)$$

251 and

$$\beta = \frac{ae^{-bL}(Lb + 2) + b^2r_e(L - r_e) + bL - 2br_e - 2}{(Lb)^3} + \frac{3Lr_e^2 - 2r_e^3}{6L^3} \quad (28)$$

252

253 It is evident from Eqns. (28) and (24) that, $\beta = \alpha$ thus the upper bound and lower bound are identical, hence
 254 $p_c = p_u = M_0/\beta L^2$ is the exact plastic collapse pressure. For the value of $b = 0$, $r_e = L$ the Eqn. (23) simplifies

255 to $p_c = 6M_0/L^2$ which corresponds to collapse load in the case of uniform pressure load. The variation of load
 256 parameter β as a function of centre region radius and parameter b is shown in Fig. 6. It is also evident from Fig.
 257 7 that principle moment distribution (ratio of M_2/M_0) is heavily dependent on the loading exponent b .

258 **Dynamic Analyses**

259 The dynamic analyses in this section are conducted by including the inertia term in equilibrium Eqns. (5)-(7).
 260 The kinematic relations in dynamic analyses are distinguished by two distinctive cases, as follows:

- 261 i. Case i: where $1 \leq \eta \leq \eta_{crit}$
 262 ii. Case ii: where $\eta \geq \eta_{crit}$, in which η is the dynamic overloading factor defined as $\eta = p_1/p_c$ and η_{crit}
 263 is defined in Eqn. (38).

264 It is pragmatic to introduce the following parameters

265

$$\ddot{\bar{w}} = M_0/\mu L^2, \quad \bar{\tau} = \eta\tau, \quad \bar{m} = M_2/M_0 \quad (29a-c)$$

266

267 **Case i- $1 \leq \eta \leq \eta_{crit}$**

268 During this case, we have assumed that the velocity profile is the same as the static case discussed in section
 269 30. However, the loading involves the temporal part of pulse shape in Eqn. (1). It is convenient to investigate the
 270 structural response in two distinctive phases, i.e., $0 \leq t \leq \tau$ and $\tau \leq t \leq T$, where τ is the duration of pulse load
 271 and T the end of motion time.

272 *First phase of motion $0 \leq t \leq \tau$*

273 During the first phase of motion, we maintain the temporal part to be of rectangular form (Eqn. (1)), while the
 274 spatial part follows Eqn. (2). Thus, the plastic flow of hinge lines is controlled by regime AB of the Johansen
 275 yield criterion. The ODE's in this phase are:

276

$$6g_1 + 6z \frac{\partial g_1}{\partial z} + z^2 \frac{\partial^2 g_1}{\partial z^2} = \mu \ddot{w} - p_1 \quad 0 \leq z \leq r_e/L \quad (30a)$$

$$6g_2 + 6z \frac{\partial g_2}{\partial u} + z^2 \frac{\partial^2 g_2}{\partial u^2} = \mu \ddot{w} - \alpha p_1 e^{-bLz} \quad r_e/L \leq z \leq 1 \quad (30b)$$

277

278 which have the same form as before except the static function $f(z)$ from Eqns. (18) and (19) is replaced with
 279 dynamic function $g(z)$. The general solutions to differential Eqns. (30a-b) are:

280

$$g_1(z) = \begin{cases} \frac{\mu\ddot{W} - p_1}{6} - \frac{\mu\ddot{W}z}{12} + \frac{A_1}{z^2} + \frac{B_1}{z^3} & 0 \leq z \leq r_e/L \quad (31a) \\ \frac{-ap_1 e^{-bLz}(bLz+2)}{(bLz)^3} + \frac{\mu\ddot{W}}{6} - \frac{\mu\ddot{W}z}{12} + \frac{D_1}{z^2} + \frac{E_1}{z^3} & r_e/L \leq z \leq 1 \quad (31b) \end{cases}$$

281

282 By employing the continuity of generalised stresses and shear forces at $z = 0$ and $z = r_e/L$, the integration
 283 constants are determined as follows:

$$\begin{cases} A_1 = B_1 = 0 & (32a) \\ D_1 = \frac{-p_1((br_e)^2 + 2br_e + 2)}{2(Lb)^2} & (32b) \\ E_1 = \frac{p_1((br_e)^3 + 3(br_e)^2 + 6br_e + 6)}{3L^3 b^3} & (32c) \end{cases}$$

284

285 An expression for \ddot{W}_1 is found as per Eqn. (33) by combining Eqns. (31a)-(32c), considering Eqns. (17), (23)
 286 and (29a) and invoking the boundary conditions at plate corners i.e. $\bar{m} = 0$:

287

$$\ddot{W}_1 = 12\ddot{\bar{w}}(\eta - 1) \quad (33)$$

288

289 Two time integrations of Eqn. (33) then yields the displacement field as $W_1(t) = (6\ddot{\bar{w}}(\eta - 1)t^2)$, when
 290 appreciating zero integration constants due to initial boundary conditions, i.e. $\dot{W}_1(0) = W_1(0) = 0$

291 *Second phase of motion $\tau \leq t \leq T$*

292 The second phase of motion is characterised by the fact that loading is absent and all motion is due to intrinsic
 293 inertia effects, the Eqn. (33) is applicable, although with $\eta = 0$ it becomes:

294

$$\ddot{W}_2 = -12\ddot{\bar{w}} \quad (34)$$

295

296 Again, the subsequent transverse displacement is achieved by two time integrations of Eqn. (34), while the
 297 constants of integration are determined by employing the continuity of the transverse velocity and displacement
 298 fields at $t = \tau$, presented in Eqn. (35).

299

$$W_2 = -6\ddot{\bar{w}}(t^2 - 2\bar{\tau}t + \eta\tau^2) \quad (35)$$

300

301 The second phase ceases at $t = \bar{\tau}$, which is when the transverse velocity \dot{W}_2 vanishes. Hence, the permanent
 302 displacement field at any point $0 \leq z \leq 1$ can be written as:

303

$$w_f = 6\ddot{\bar{w}}\bar{\tau}^2(\eta - 1)(1 - z)/\eta \quad (36)$$

304 *Static and kinematic admissibility*

305 It is essential to verify whether the mathematical treatment in 0 are statistically admissible, i.e. the Eqn. (16a)
 306 and inequality (16b) are not violated. Whilst Eqns. (13)-(15) satisfy $M_1 = M_0$, it is necessary to demonstrate that
 307 the shear forces at the centre vanish (i.e. $Q_x|_{z=0} = 0$ and $(\partial^2 M_x)/(\partial x^2) > 0$ (or $(\partial^2 M_y)/(\partial x^2) > 0$). While
 308 the former condition is clearly satisfied from Eqn. (31a), the latter condition, requires:

309

$$\frac{\partial^2 M_x}{\partial x^2} = \frac{\left(\left(6 \left(2L^4 z^3 - L^4 z^4 - \frac{5}{2} z^2 L^2 x^2 + 1/2 x^4 \right) \right) (\eta - 1) M_0 - p_1 L^6 z^3 \right)}{3L^6 z^3} > 0 \quad (37)$$

310 Which is obtained with the aid of variational parameters. Elementary calculation gives:

311

$$\eta \leq \left| \frac{12\beta}{12\beta - 1} \right| = \eta_{crit} \quad (38)$$

312

313 A similar procedure to establish the kinematic admissibility of the velocity profile is achieved by satisfying

314 Eqn. (16b), i.e. it is required to show that $-M_0 \leq M_2 = (M_x + M_y)/2 + 1/2[(M_x - M_y)^2 + 4M_{xy}^2]^{1/2} \leq M_0$.

315 This inequality simplifies to the following:

$$-2M_0 \leq (x^2 + y^2)g(z) \leq 0 \quad (39)$$

316 The right-hand side of the inequality-at the plate centre- requires that $(\mu\dot{W} - p_1)/6 - \mu\dot{W}z/12 \leq 0$, which,
 317 when using Eqns. (33), (23), will lead to an identical expression to Eqn. (38).

318 In the case of $r_e = L$ and $\beta = \frac{1}{6}$; the right-hand side of the inequality simplifies to the condition for the case
 319 of uniformly distributed load, i.e. $\eta \leq 2$.

320 In the second phase of motion, the Eqn. (16b) (or (37)) must still be satisfied, but with setting $\eta = 0$, which
 321 yields:

322

$$-2M_0 \leq \frac{-2 M_0(x^2 + y^2)}{L^2} \leq 0 \quad (40)$$

323

324 By considering the fact that $(x^2 + y^2)/L^2$ is universally positive (Eqn. (11)), Eqn. (40) is valid for all values
 325 of ω_0 and L at plate centre. A plot of bending moment distribution for various values of η is shown in Fig. 10. In
 326 this case, the loading parameter $b = 50$ and $\omega_0 = 0.125$, which are found by regression analyses on numerical
 327 results of registered pressure time history for an Armour steel model B4 found in (Mehreganian et al., 2018)

328 **Case ii: $\eta \geq \eta_{crit}$**

329 *First phase of motion*

330 For the blast loads with high magnitudes, $\eta > \eta_{crit}$ then $p_1 \gg p_c$ so the right-hand side of Eqn. (39) is no
 331 longer valid, since a yield violation occurs near the plate centre. This requires the velocity field profile to be
 332 modified. It is therefore assumed that the velocity profile is governed by three distinguishable phases, as the
 333 incipient plastic hinge forms in the central part of the plate (Fig. 8). It is also assumed that in this phase, the plastic
 334 flow in the plate centre is characterised by corner A of the yield condition in Fig. 3, which governs the central part
 335 of the plate for $0 \leq z \leq \xi_0$, whereas the remaining part of the plate $\xi_0 \leq z \leq 1$ is governed by the regime AB.
 336 Thus, the velocity profile will be of the form:

337

$$\dot{w} = \dot{W}_2 \quad 0 \leq z \leq \xi_0 \quad (41)$$

$$\dot{w} = \dot{W}_2 \frac{(1-z)}{(1-\xi_0)} \quad \xi_0 \leq z \leq 1 \quad (42)$$

338 where ξ_0 is time independent. It is also assumed that $r_e/L \leq \xi_0$. Thus, the moment function $g_1(z)$ is replaced
 339 by $g_2(z)$ in the following form:

340

$$g_2(z) = \begin{cases} \frac{\mu\ddot{W} - p_1}{6} + \frac{A_3}{z^2} + \frac{B_3}{z^3} & 0 \leq zL \leq r_e \quad (43a) \\ \frac{-ap_1 e^{-bLz}(bLz + 2)}{(bLz)^3} + \frac{\mu\ddot{W}}{6} + \frac{D_3}{z^2} + \frac{E_3}{z^3} & r_e \leq zL \leq \xi_0 L \quad (43b) \\ \frac{-ap_1 e^{-bLz}(bLz + 2)}{(bLz)^3} + \frac{\mu\ddot{W}}{6(1-\xi_0)} - \frac{\mu\ddot{W}z}{12(1-\xi_0)} + \frac{F_3}{z^2} + \frac{G_3}{z^3} & \xi_0 L \leq zL \leq L \quad (43c) \end{cases}$$

$$\begin{cases} A_3 = B_3 = 0 \\ D_3 = \frac{-p_1((br_e)^2 + 2br_e + 2)}{2(Lb)^2} \\ E_3 = \frac{p_1((br_e)^3 + 3(br_e)^2 + 6br_e + 6)}{3L^3 b^3} \end{cases} \quad (44)$$

341

342 The integration constants in Eqns. (43a-b) are obtained by applying the boundary conditions of $Q_x = 0$, $M_x =$
 343 $M_y = M_0$ at the midspan and owing to the continuity of shear force at $z = r_e/L$. It transpires that at the interval
 344 $0 \leq z \leq r_e/L$, $\mu\ddot{W}_2 = p_1$. Thus, using the kinematic conditions of $W_2 = \dot{W}_2 = 0$ at $t = 0$, the maximum
 345 displacement will become:

346

$$W_2 = \frac{\ddot{w}\eta t^2}{2\beta} \quad (45)$$

347

348 The expression for arbitrary moment function in the interval $\xi \leq z \leq 1$, is similar to Eqn. (31b), except the
 349 parameter \ddot{W} is now replaced with $\ddot{W}/(1-\xi_0)$ while the integration constants are F_3 and G_3 . The succeeding
 350 conditions of $Q_x = 0$ and $M = M_0$ at the interval $r_e/L < z < \xi_0$ would furnish these constants as:

351

$$\begin{cases} F_3 = \frac{p_1 \left(3a(-1 + \xi_0)(Lb\xi_0 + 1)e^{-Lb\xi_0} + L^2 b^2 \xi_0^2 \left(\xi_0 - \frac{3}{2} \right) \right)}{3L^2 b^2 (1 - \xi_0)} \\ G_3 = \frac{p_1 \left(4a(-1 + \xi_0) [(Lb\xi_0)^2 + 2Lb\xi_0 + 2] e^{-Lb\xi_0} + (Lb\xi_0)^3 \left(\xi_0 - \frac{4}{3} \right) \right)}{4L^3 b^3 (-1 + \xi_0)} \end{cases} \quad (46)$$

352

353 The expression (43c) should satisfy the at boundary condition of bending moment, i.e. $\bar{m} = 0$ at $z = 1$, which
 354 yields an expression of the incipient plastic hinge and the dynamic overloading factor in Eqn. (47).

$$\eta = \frac{12L^3 b^3 \beta}{\gamma} \quad (47)$$

355 where $\eta = p_1/p_c$, and the parameter γ is:

$$\gamma = 12a[b^2 L^2(1 - \xi_0)\xi_0 + (1 - 2\xi_0)bL - 2]e^{-Lb\xi_0} + 12a(Lb + 2)e^{-Lb} - 3(1 - \xi_0)^2(\xi_0 + 1/3)L^3 b^3 \quad (48)$$

356 The distribution of bending moment with various values of ξ_0 is presented in Fig. 11. For impulsive loading, $\eta \rightarrow$
 357 ∞ . thus $\gamma \rightarrow 0$ which occurs when the plastic hinges form at the supports, i.e. $\xi_0 \rightarrow 1$. The expression of ξ_0 in
 358 Eqn. (48) is highly nonlinear which can be solved with the aid of numerical methods.

359 *Second phase of motion* $\tau \leq t \leq T_1$

360 In this phase $p_1 = 0$ however, due to initial velocity at first phase, motion continues during the second phase.
 361 Concerning this, ξ_0 from Eqn. (42) is substituted by an active plastic hinge ξ , which moves inwards as
 362 demonstrated in Fig. 9. For the region $0 \leq z \leq \xi$, the equilibrium with $p_1 = 0$ predicts $\mu\ddot{W}_2 = 0$. Therefore, by
 363 implementing the continuity of the velocity parameter at $t = \tau$, the deformation and velocity are as in Eqn. (49),
 364 (50).

$$\dot{W}_2 = \ddot{w}\bar{\tau}/\beta \quad (49)$$

$$W_2 = \frac{\ddot{w}\bar{\tau}}{2\beta}(t - \tau) \quad (50)$$

365 For the second region, the differential Eqn.'s (30a) and (30b) will change to:

366

$$z^2 \frac{\partial^2 g_2}{\partial z^2} + 6z \frac{\partial g_2}{\partial z} + 6g_2 = \mu\ddot{w}_2 = \mu\dot{W}_2 \left(\frac{1-z}{1-\xi}\right) + \mu W_2 \dot{\xi} \left(\frac{1-z}{(1-\xi)^2}\right) \quad (51)$$

367 The general solution of which is:

$$g_2 = \frac{\mu\dot{W}_2(2-z)}{12(1-\xi)} + \frac{\mu W_2 \dot{\xi}(2-z)}{12(1-\xi)^2} + \frac{D_4}{z^2} + \frac{E_4}{z^3} \quad (52)$$

368 The integration constants are obtained by conditions of $Q_x = 0$ and $M_x = M_y = M_0$ at $0 \leq z \leq \xi$, the continuity
 369 of Q_x and M_x at $z = \xi$ gives:

370

$$\begin{cases} D_4 = \frac{p_1 \tau \dot{\xi} \xi^2 (2\xi - 3)}{6(\xi - 1)^2} \\ E_4 = \frac{-p_1 \tau \dot{\xi} \xi^3 (3\xi - 4)}{12(\xi - 1)^2} \end{cases} \quad (53)$$

371 An expression in terms of the travelling hinge ξ is found by employing the simply supported boundary condition
 372 (Eqn. (22)) at corners of the plate:

373

$$(3\xi^2 - 2\xi - 1)\dot{\xi} = \frac{12\beta}{\eta\tau} \quad (54)$$

374 An expression for the travelling hinge displacement is attained through time integration of $\dot{\xi}$ in (55), by ensuring
 375 that the plastic hinge at $t = \tau$ is stationary:

376

$$\xi^3 - \xi^2 - \xi - 1 = 12\beta t / \eta\tau + \bar{\xi} \quad (55)$$

377 where

$$\bar{\xi} = \xi_0^3 - \xi_0^2 - \xi_0 - 1 - \frac{12\beta}{\eta} \quad (56)$$

378

379 The integration constant $\bar{\xi}$ is determined by evaluating the continuities $\xi = \xi_0$ at $t = \tau$ in Eqn. (55).
 380 Subsequently, the second phase completes when $\xi = 0$, which occurs at time $T_1 = -(\bar{\xi} + 1)\eta\tau/12\beta$. The
 381 influence of dynamic load factor on the length of active plastic hinge (with $\tau = 50\mu s$) is presented in Fig. 12.
 382 Substituting this expression into Eqn. (50) gives the transverse displacement at the end of this phase as:

383

$$W_2 = -\frac{\ddot{w}\eta\tau^2(\eta\bar{\xi} + 6\beta + \eta)}{12\beta^2} \quad (57)$$

384 *Final phase of motion $T_1 \leq t \leq T$*

385 The final phase of the plate motion will essentially develop since the kinetic energy from the previous phase
 386 has to be somehow dissipated. The transverse velocity profile is the same as Fig. 5 as the plastic hinge closes ($\xi =$
 387 0). The incipient deformation is identical to the circumstance of infinitesimal blast loads, viz., the condition of
 388 inequality (38). Therefore, the solution to velocity and displacement fields at this phase is achieved by time
 389 integration of (34) and equating it with Eqns.(35), (49), (50), (57) at $t = T_1$. The transverse displacement at this
 390 phase is equivalent to:

$$W_3 = -\frac{p_1 \left[\tau^2 \eta^2 (\bar{\xi} + 1)^2 + 24\beta\eta\tau \left(t\bar{\xi} + \frac{1}{2}\tau \right) + 144\beta^2 t^2 \right]}{24\eta\beta\mu} \quad (58)$$

391 recalling the parameter $\bar{\xi}$ is defined as $\bar{\xi} = \xi_0^3 - \xi_0^2 - \xi_0 - 1 - 12\beta/\eta$. The plate rests when $\dot{W}_3 = 0$, which occurs
 392 at time T (Eqn. (59)).

$$T = -\frac{\tau \bar{\xi} \eta}{12\beta} \quad (59)$$

393 Thus, Eqn. (58) will simplify to Eqn. (60):

$$W_f = -\frac{\ddot{w} \eta \tau^2 \left[\left(\bar{\xi} + \frac{1}{2} \right) \eta + 6\beta \right]}{12\beta^2} \quad (60)$$

394 *Static and Kinematic Admissibility*

395 It is evident that the continuity requirements of generalised stresses i.e. $Q_x = Q_y = 0$, $M_x = M_y = M_0$ are
 396 satisfied at $z = 0$, $z = \xi_0$ and $z = \xi$ for the first, second, and third phases of motion. Furthermore, the Eqn. (16a)
 397 is satisfied throughout the entire motion, irrespective of the phase of motion. It can also be observed that the
 398 inequality (16b) is satisfied for $0 \leq z \leq \xi$ during the entire phases of motion.

399

400 *Impulsive Loading*

401 A blast load of rectangular pulse shape with very short duration ($\tau \rightarrow 0$) and very high amplitude ($\eta \rightarrow \infty$
 402 or $p_1 \gg p_c$) is known as impulsive loading. In the case of impulsive loading, the total change in momentum
 403 equals the total impulse imparted upon the system, hence the conservation of momentum implies that:

404

$$\int_0^{\frac{r_e}{L}} 8L^2 \tau p_1 z dz + \int_{\frac{r_e}{L}}^1 8L^2 \tau p_1 a e^{-bLz} z dz = \int_0^1 8L^2 \mu V_0 z dz \quad (61)$$

405 The solution to the Eqn. (61) yields:

$$V_0 = \frac{\varepsilon_1 \tau p_1}{L^2 \mu} \quad (62)$$

406 where $\varepsilon_1 = \frac{r_e^2 b^2 - 2e^{-Lb}(bL+1) + 2r_e b + 2}{b^2}$. Consequently, the Eqn. (60) can be recast in the form:

$$\frac{W_f}{H} = -\frac{\lambda}{12\eta} \left[\left(\bar{\xi} + \frac{1}{2} \right) \eta + 6\beta \right] \quad (63)$$

407 in which $\lambda = \frac{\mu V_0^2 L^2}{M_0 H} \left(\frac{L^4}{\varepsilon_1^2} \right)$ is the non-dimensional kinetic energy. For the case of $r_e = L$ and $\eta \rightarrow \infty$, $r_e \rightarrow L$, $\beta = \frac{1}{6}$

408 and $W_f/H \cong \lambda/8$, which conforms to results for the case of uniform pressure load. A plot of normalised

409 deflection vs. the dimensionless kinetic energy is shown in Fig. 15 for the case of $\omega_0 = 0.7$, $b = 50$ for various

410 values of load ratio. It can be observed that with increase of η the plot is only marginally different from impulsive
411 load, i.e. $\eta \rightarrow \infty$.

412 ***Fully Clamped Square Plate***

413 Although in practical applications the protective plate elements are designated with fully-clamped conditions,
414 the foregoing analysis for the simply supported plates can simply be extended to the case of fully-clamped plate:
415 the edge conditions of the moment, denoted by $\bar{m} = -1$, yields the static plastic collapse as $p_c = 2M_0/\beta L^2$; thus,
416 the foregoing results may be furnished for the fully-clamped plates by merely changing M_0 to $2M_0$ in the
417 parameter \bar{w} and associated expressions accordingly.

418 However, it should be noted that, in contradiction to the global blasts, the boundary effects are not significant
419 for the localised blast because such a blast impacts a small area of the plated structure (Micallef et al., 2014).
420 Furthermore, the difference of the permanent deformation of uniformly, impulsively loaded clamped and simply
421 supported plates is insignificant in Mode I (large inelastic deformation) (Yuen and Nurick, 2001).

422 **Numerical analyses**

423 ***Limitations of the study and Material model***

424 The analysis performed in Section 0 was predicated on the assumptions of Kirchhoff –Love plate theory,
425 which ignores the effects of transverse shear and rotatory inertia. Taking this limitation into account, it could be
426 assumed that for the range of $0.01 < H/L < 0.02$ and under infinitesimal loading conditions, the bending action
427 dominates the structural behaviour, such that the build-up of membrane action associated with the plastic collapse
428 can be disregarded.

429 Whilst considering the limitations of study, in this section, the analytical solutions are validated against
430 numerical simulations. The simulations are conducted in commercial Finite Element (FE) software ABAQUS
431 6.14-4/Explicit[®], which is capable of simulating the dynamic response for blast loading scenarios through analyses
432 of various degrees of complexity. For the sake of numerical analyses, the model parameters are as follows:

$$H = 4mm$$

$$L = 200mm$$

$$\sigma_0 = 330MPa$$

$$\mu = 30.4kg/m^2$$

433

434 These properties are typical of the protective plates made of mild steel, provided the strain rate sensitivity is
435 ignored. The loading parameters were selected consistent with constant values of $p_1 = 20MPa$, $b = 50$, $\tau =$
436 $30\mu s$ and the rectangular pulse shape considered in the analytical analyses of previous sections.

437 **Finite Element (FE) model**

438 A full 3-Dimensional FE model was set up in ABAQUS for rectangular plate of length $2L$, discretised with
439 S4(R) shell elements of double curvature with hourglass control and pinned along the periphery. These elements
440 incorporate reduced integration formulation to prevent shear locking. Shear locking is a major issue in
441 computational formulations of beams and plates leading to artificial over-stiffening. Due to symmetry, only a
442 quarter of the plate is modelled. The convergence of the displacements was satisfied with a mesh of element length
443 to thickness ratio of 1, giving a total of 2500. For each case of $\omega_0 = r_e/L$, a pressure matrix corresponding to
444 Cartesian coordinates was constructed by utilisation of Eqns. (2) and (11). This pressure matrix was mapped
445 directly onto the panel. For the case of $\omega_0=0.5$, a contour plot of transverse displacement and stress distributions
446 is shown in Fig. 13.

447 We can see from Fig. 17 that for the range of $H/L = 2\%$ the numerical results for displacement compare
448 favourably with the analytical results. The position of the stationery plastic hinge is obtained numerically for
449 various loading radii and plotted in Fig. 16. It is interesting to note that for most loading constant central zone
450 radii, the length of ξ_0 (which should satisfy $\eta\gamma - 12\beta^3 L^3 b^3 = 0$ from Eqn. (47)) is predicted at $\sim 0.89^{\text{th}}$ of the
451 plate length. For close-in blasts with small loading radii/plate length ratio, the length of the stationery plastic hinge
452 decreases accordingly. It turns out that in the case of $r_e/L = 0.1$, for example, a solution of ξ_0 in Eqn. (47). Exists
453 at the plate centre, in addition to the hinges formed near the supports. Numerical observations on the position of
454 first maximum equivalent plastic strain ($\bar{\epsilon}_p$) corroborate with this statement.

455 The theoretical solution for the impulsive loading from Eqn. (63) is validated in Fig. 14 against the
456 experimental results on ARMOX steel and mild steel MS4 specimens obtained from (Jacob et al., 2004; Langdon
457 et al., 2015), respectively. The tests from Jacob et al (Jacob et al., 2004) were those of Series I plates. The duration
458 time of $15\mu s$, typical of impulsive load cases, was chosen and the loading parameter $r_e = 25mm$ was taken as
459 the charge radius, whilst the loading decay parameter ' b ' - typically in the range of $50 \leq b \leq 100$ - was evaluated
460 by curve fitting methods on the registered pressure loads from the numerical results of (Mehreganian et al., 2018).
461 It is also important to note the deviation of data from the predicted curve when the dimensionless kinetic energy
462 increases ($\lambda > 200$), the limit beyond which the membrane action significantly affects the transverse deflections.

463 It should be noted that, for the case of ARMOX steel, the material is capable of higher energy absorption,
464 while the deformation is not significantly affected by strain rate sensitivity. However, the response can be affected
465 by the material elasticity which can pose a difficulty when implementing limit analysis. Regardless of this
466 limitation, the application of modified load function considerably enhances the prediction of plate response as
467 opposed to the previous models introduced by researchers (Jones, 1997), and (Komarov and Nemirovskii, 1986)
468 valid for uniform pressure.

469 **Concluding remarks**

470 This paper deals with a theoretical model devised to predict the transverse dynamic plastic displacement field
471 of a generic simply supported, monolithic, square plate subjected to localised blast loading. A piecewise
472 continuous load function formerly studied by (Karagiozova et al., 2010) and extended by (Micallef et al., 2015),
473 and (Micallef et al., 2012) was incorporated into the analyses, which is universal and adjustable, through alteration
474 of its parameters, to replicate various loading scenarios from proximal (localised) to distal (global) blast loads.

475 The plate was assumed thin, to enable making use of Kirchhoff-Love theory as opposed to more sophisticated
476 Mindlin-Reissner plate theory investigated by (Toolabi et al., 2018) using numerical methods. As such, transverse
477 shear and rotatory inertia effects can be ignored without loss of accuracy. The plate was, however, assumed thick
478 enough not to be rendered a membrane. The analyses were performed by means of limit analysis and the incipient
479 velocity profile was governed by the travelling plastic hinge in the three stages of analysis for high amplitude
480 loads (i.e. $p_1 \gg p_c$ or $\eta > \eta_{crit}$).

481 Close agreement was found when correlating the results of the theoretical model with the corresponding FE
482 model for different load distributions. The transverse deflection-impulsive load relation was validated for different
483 cases of load distributions. It was concluded that the analytical model yields satisfactory results for low impulse
484 where bending effect is dominant, while for larger impulses on the thin plate, the membrane effects become
485 significant. Moreover, the theoretical solutions for impulsive load give better estimate than the previous rigid-
486 plastic model proposed earlier by (Jones, 1997), or (Komarov and Nemirovskii, 1986) when validated against the
487 experimental data for close-in low impulse (kinetic energy) blast loads (i.e., $\lambda < 200$). The significance of the
488 study lies in the fact that its results are applicable to plates made of materials with little or no strain rate sensitivity,
489 such as aluminium and high strength armour steel. Structures made of armour steel undergo less deflection with
490 higher energy absorption capacity when compared to their Mild steel counterparts.

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494 **Declaration of Conflicting interest**

495 The Authors declare that there is no conflict of interest.

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638

639 **Appendix A.**

640 **A.1. Dynamic pressure pulse loading**

641 The analyses conducted in sections 0 were limited to the localised blast loads with rectangular temporal pulse
 642 shape. A blast peak pressure with dynamic pressure more than 10 times the static collapse can be idealised as
 643 rectangular pulse (Jones, 1997), (Rajendran and Lee, 2009), (Yuan et al., 2016). In real-life, the majority of near
 644 field explosions are non-impulsive, which may have linear or exponential pulse shapes. Since the pulse shape can
 645 significantly affects the structural response (Youngdahl, 1971), it is essential to understand the structural response
 646 under various pulse loads, which is the aim of this section. The linearly decaying pulse is defined as in Eqn. A. 1
 647 and Fig. A. 1.

648

$$p_2(t) = \begin{cases} \left(1 - \frac{t}{\tau}\right) & 0 \leq t \leq \tau \\ 0 & t \geq \tau \end{cases}, \quad \text{A. 1}$$

649 And a more general, exponentially decaying temporal variation of the pressure load is represented in the form
 650 of Fig. A.2 and Eqn. A. 2.

651

$$p_3(t) = \begin{cases} X e^{-\frac{Yt}{\tau}} & 0 \leq t \leq \tau \\ 0 & t \geq \tau \end{cases} \quad \text{A. 2}$$

652 which simplifies to Friedlander's equation (Rajendran and Lee, 2009) with a specific choice of parameters,
 653 i.e. $X = Y = 1$. Following the procedure outlined in Section 4, the final transverse deflection for the simplified
 654 exponential pulse shape is determined as in Eqn. A. 3:

655

$$W_f = \frac{\left(- \left(12 \left((12\beta + \eta) \ln \left(\frac{12\beta}{12\beta + \eta} \right) + \eta \right) \right) \beta \delta + \eta \left((\eta(\gamma_e + 1) + 24\beta)e - \eta(\gamma_e + 1) \right) \right) p_c e^{-2\tau^2}}{12\mu\beta} \quad \text{A. 3}$$

656 where $\gamma_e = \xi_0^3 - \xi_0^2 - \xi_0 - 1 + \frac{12\beta}{\eta(e^{-1}-1)}$ and δ is defined as:

$$\delta = e^{\frac{(\eta(\gamma_0+1)(1-e^{-1})+24\beta)}{12\beta}} \quad \text{A. 4}$$

657 The final deflection occurs at:

$$T_f = \tau \ln \left(\frac{12e\beta\tau + \eta(\xi_0 + \xi_0^2 - \xi_0^3)}{\tau(12\beta + \eta)} \right) \quad \text{A. 5}$$

658 For the linearly decaying pulse, the final deformation will be:

$$W_f = -\frac{\eta\ddot{w}\tau^2 \left(\left(\gamma_l + \frac{1}{2} \right) \eta + 8\beta \right)}{48\beta^2} \quad \text{A. 6}$$

659 where $\ddot{w} = p_c\beta/\mu$ also defined in (29a) and $\gamma_l = \xi_0^3 - \xi_0^2 - \xi_0 - 1 - 24\beta/\eta$. The permanent deformation
660 occurs at:

$$T_f = \frac{-\gamma_l\eta\tau}{24\beta} \quad \text{A. 7}$$

661 Comparing the Eqn. s' A. 6, A. 3 and (60), it is observed that, for non-impulsive dynamic loads the plastic
662 transverse deflection is highly dependent on the pulse shape. However, this effect can be practically eliminated
663 by incorporating Youngdahl's correlation parameters, namely total impulse, effective pressure and mean time
664 (t_m) (Jones, 1997), (Youngdahl, 1971) defined as follows:

665

$$I = \int_0^T P(t) dt \quad \text{A. 8}$$

$$t_m = \frac{1}{I} \int_0^T t.P(t) dt \quad \text{A. 9}$$

$$p_e = \frac{I}{2t_m} \quad \text{A. 10}$$

666 where I is the total impulse, $P(t) = p_1p(t)$ and p_e is the effective pressure, T is the time where plastic
667 deformation ceases and t_m is the centroid of the pulse. In the case of rectangular pressure pulse, for instance, $I =$
668 $p_1\tau$ and $p_e = p_1$ is used in lieu of dynamic pressure p_1 in Eqn. (60). Since Youngdahl's correlation parameters are
669 insensitive to small perturbations of the pulse, it transpires that the various pulse shapes virtually fall onto a single
670 curve, when using $\eta_e = p_e/p_c$. This obviates the need to record the pressure time histories and model the dynamic
671 loading for design purposes, as it is practically difficult to do so by laboratory tests.

672 Performing the analyses in Section 4 with effective pressure from Eqn.'s A. 8-A. 10, compiled with Eqn.'s A.
673 1-A. 2, a single plot can be obtained for the (effective) permanent displacement in terms of effective impulse and

674 pressure, as presented in Fig. A. 3. The influence of load parameters β and η on permanent displacement in such
675 case is presented in Fig. A.4.