



XXIV Italian Group of Fracture Conference, 1-3 March 2017, Urbino, Italy

Analysis of failure in quasi-brittle materials by 3D multiplane cohesive zone models combining damage, friction and interlocking

Roberto Serpieri^a, Marco Albarella^a, Giulio Alfano^{b,*}, Elio Sacco^c

^a*Dipartimento di Ingegneria, Università degli Studi del Sannio, Piazza Roma, 21, Benevento, 82100, Italy*

^b*School of Engineering and Design, Brunel University, Uxbridge, UB8 3PH, UK*

^c*Dipartimento di Ingegneria Civile e Meccanica, Università di Cassino e del Lazio Meridionale, Via di Biasio n. 43, Cassino (FR), 03043, Italy*

Abstract

This paper presents the latest advances in the development of multiplane cohesive-zone models that are able to account for damage, friction and interlocking, including in particular their extension to a general three-dimensional (3D) case. Starting from the work proposed in a recent article by some of the authors, a simplified micromechanical formulation is used, whose main idea is to represent the asperities of the developing fracture surface in the form of a periodic arrangement of distinct inclined elementary planes, denominated Representative Multiplane Element (RME). The interaction between the two faces of each of these elementary planes is governed by the interface formulation proposed by Alfano and Sacco, which couples friction with damage but does not specifically account for the asperities of the fracture surface and the associated interlocking. A key feature of the model is that, for each elementary plane, it is possible to use a ‘base-line’ cohesive-zone model characterized by the same critical energy release rate in (local) modes I and II, because such value represents the ‘rupture’ energy needed to achieve decohesion, in absence of any frictional dissipation. Numerical results and their correlation with experimental data are presented to show how the model is able to capture the increase in total (measured) fracture energy on the RME with increasing mode I-to-mode II ratio thanks to the geometry of the elementary planes and their influence on the frictional dissipation. The model has then been further refined to account for the finite depth of the asperities of the fracture surface and for their wear as a result of frictional slip. The enhanced model has been validated against experimental results for problems involving monotonic and cyclic loading. Finally, the main strategy used to extend the model to a general 3D case is presented, and some of the key issues are discussed.

Copyright © 2017 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Peer-review under responsibility of the Scientific Committee of IGF Ex-Co.

Keywords: Interface Friction; Cohesive zone models; Interlocking; Fracture Energy.

* Corresponding author. Tel.: +44(0)1895 267062; fax: +44(0)1895 256392.

E-mail address: giulio.alfano@brunel.ac.uk

1. Introduction

A review of the literature reveals that dilation, occurring in quasi-brittle materials undergoing damage and fracture, is typically simulated through micromechanical models or, more often, phenomenological approaches. The latter are often easy to implement and integrate in finite element codes, but typically rely on fitting coefficients of difficult if not impossible physical interpretation, which often makes also their experimental determination difficult. Furthermore, phenomenological approaches can only be considered valid within the strict range of parameters in which the model has been validated against experimental results.

On the other hand, micromechanical approaches have the advantage of using a more rational derivation of the model based on the actual underlying physics. Accurate micro-mechanical modelling typically requires numerical approaches which can be integrated within a macroscopic analysis using computational homogenization methods; these can be based on nested or sequential techniques and in either case the computational cost is typically high and often excessive for real-life engineering computations, see, e.g., Geers et al. (2010). Alternatively, simplified micro-mechanical models can be used, which can be solved either analytically in closed form or with simple and fast numerical computations.

In this paper the 2D cohesive-zone models (CZMs) proposed by Serpieri and Alfano (2011), by Serpieri et al. (2015a) and by Serpieri et al. (2015b), which are capable of capturing progressive damage, crack initiation, propagation and their coupling with friction, interlocking and associated dilation, are reviewed and their extension to a general 3D model is discussed. The models by Serpieri and Alfano (2011), Serpieri et al. (2015a), and Serpieri et al. (2015b) are derived from a micromechanical analysis using a simplified approach, with the aim of capturing the interaction between de-cohesion, unilateral contact, friction and dilation. The micro-model is characterized by defining a Representative Multiplane Element (RME) with a schematic and simplified description of the geometry of the asperities, according to the scheme proposed by Serpieri and Alfano (2011) and, more recently, revisited by Serpieri et al. (2015a).

The finite depth of asperities is then accounted for in the more refined model proposed by Serpieri et al. (2015b), by enforcing equilibrium of the interfacing parts of the RME in the deformed configuration, so that the progressive reduction of the contact area for an increasing opening displacement is considered. Furthermore, wear of the asperities, leading to progressive flattening of the fracture surface at the micro-scale, and the associated reduction in the interlocking effect is also modelled by Serpieri et al. (2015b) by assuming that the angle of each inclined elementary plane is reduced as an exponential function of the energy dissipated by friction on that plane.

Issues arising when extending the proposed model to a general 3D case are also discussed in this paper. One aspect to consider is the question of whether the distinction between the so-called modes II and III, which is made in conventional fracture mechanics, is needed in this context. A second key point is how the geometry of the RME for a 3D case can be chosen, particularly when the actual response in the interface plane is expected to be isotropic.

2. Formulation of cohesive-zone models and their development

The main ideas behind the CZMs proposed in (Serpieri and Alfano, 2011), (Serpieri et al., 2015a) and (Serpieri et al., 2015b) and some key governing equations are recalled in this section. The reader is referred to the original articles for the complete set of governing equations and for the detailed implementation in the case of a 2D problem.

Within a body occupying a domain Ω , an interface Γ is pre-defined, where a crack can initiate and propagate. Accordingly, on Γ the displacement field is allowed to be discontinuous.

The key concept behind the models presented by Serpieri and Alfano (2011) and by Serpieri et al. (2015a) is to consider and link together two different length scales, which are assumed to be sufficiently separated, with the usual meaning that is given in computational homogenization (Geers et al., 2010). At the macro-scale, where a finite-element (FE) model is used, the interface is smooth, (actually flat in the numerical simulations considered here, see Figure 1(a)). This makes the interface easy to discretise in the FE mesh, whereby the only constraint to the element size is the requirement for a sufficiently refined mesh, which is typical of CZMs.

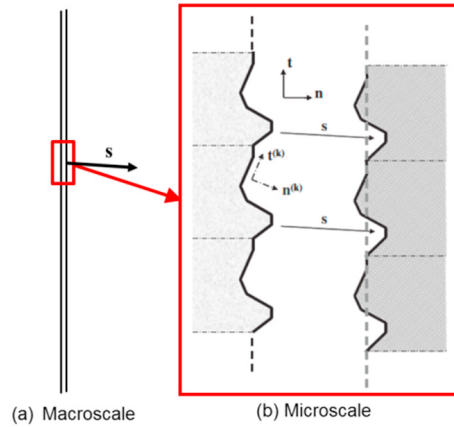


Fig. 1. CZM multiscale approach: (a) smooth macro-scale and (b) micro-scale analysis based on a repeated pattern of a finite-number of elementary planes.

At the micro-scale, instead, the actual geometry of the fracture surface is considered, but in a simplified way. To this end, a representative multiplane element made of a periodic pattern of N_p elementary planes is considered as a repeated unit (Figure 1(b)). While this approach implies the assumption of periodicity of the fracture surface geometry, it is reasonable to assume that it is a valid assumption in 2D, as long as the scales are sufficiently separated. The RME used by the authors for 2D problems is shown in Figure 2(a). For 3D models additional issues arise, which are discussed in a next section. A possible RME for 3D problems is shown in Figure 2(b).

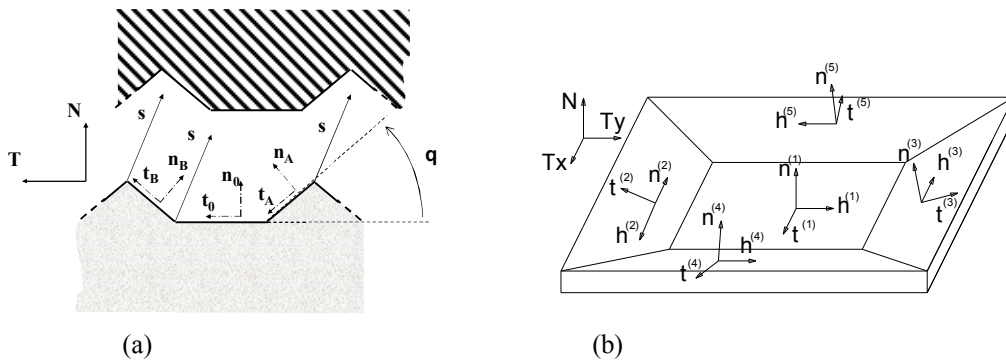


Fig. 2. RME for 2D and 3D formulations.

To link between the macro- and the micro-scale, at each integration point of interface elements used in a macro-scale displacement-based FE model, the associated relative-displacement-driven constitutive problem is resolved at the micro-scale. This is done by assuming that the two sides of the RME are subject to the assigned relative displacement s and that either side moves rigidly. In other words, the deformation of the asperities is neglected. On each elementary plane the local relative displacements are determined through suitable rotation matrices. Introducing the concept of undamaged and damaged parts of each infinitesimal area of the k^{th} elementary plane (Alfano and Sacco, 2006), and denoting by $\psi_u^{(k)}$ and $\psi_d^{(k)}$ the specific free energies on the undamaged and on the damaged parts, the specific free energy ψ at each integration point of the macro-scale model can be written in terms of a weighted sum of the contribution of each elementary plane:

$$\psi(\mathbf{s}, D_k, s_{fk}) = \sum_{k=1}^{N_p} \gamma_k \left[(1 - D_k) \psi_u^{(k)}(\mathbf{s}) + D_k \psi_d^{(k)}(\mathbf{s}, s_{fk}) \right] \tag{1}$$

where D_k denotes the damage variable on each elementary plane, ranging between 0 and 1 for no damage and full damage, s_{fk} denotes the frictional slip on each elementary plane (a scalar value in the 2D case and a two-component vector in the 3D model) and γ_k is a scalar weight providing the relative contribution of each elementary plane to the total structural response and is related to the ratio between the elementary plane area and the total area of the RME (Serpieri et al., 2015b).

Differentiating Equation (1) with respect to \mathbf{s} provides the macro-scale interface stress $\boldsymbol{\sigma}$ as a weighted sum of the stresses on each elementary plane:

$$\boldsymbol{\sigma} = \sum_{k=1}^{N_p} \gamma_k \left[(1 - D_k) \boldsymbol{\sigma}_u^k + D_k \boldsymbol{\sigma}_d^k \right] \tag{2}$$

Differentiating Equation (1) with respect to the damage variables provides energy variables Y_k which are work-conjugate to the damage parameters D_k . In general, different thermodynamically consistent methods can be used to associate Y_k to energy thresholds and, in this way, define damage evolution laws for D_k within the framework of thermodynamics with internal variables. Serpieri and Alfano (2011) use the 2D CZM formulation by Alfano and Sacco (2006) which is essentially based on a Coulomb friction model on the damaged part of the interface and a mixed-mode damage evolution law. The latter is based on the model by Alfano and Crisfield (2001) and considers different critical energy release rates, G_{cI} and G_{cII} , for local modes I and II on each elementary plane. Alfano and Crisfield (2001) show that this evolution law corresponds to the use of a non-associate evolution of damage and results in the bilinear relationships in pure modes I and II reported in Figure 3. These relationships are recovered with the models by Serpieri and Alfano (2011), Serpieri et al. (2015a) and Serpieri et al. (2015b) in the case of a flat RME (that is zero inclination angle of all elementary planes) and therefore will be denoted here as ‘base-line’ bilinear laws.

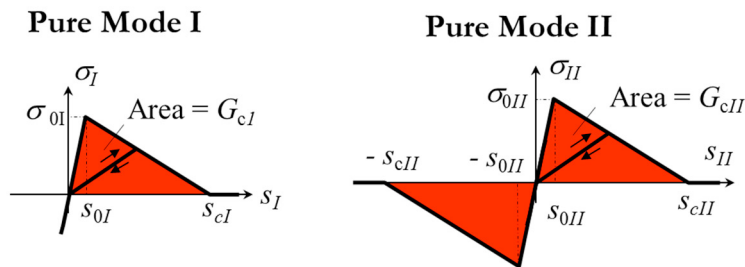


Fig. 3. Base-line bilinear laws in modes I and II [6].

More recently, Serpieri et al. (2015a) have shown a neater strategy to obtain thermodynamic consistency, for a mixed-mode CZM based on the use of an equivalent relative displacement in the case of mixed mode, which proceeds by assuming equal values of G_{cI} and G_{cII} . This results in an associate damage evolution law and is physically justified by the fact that, with the multiscale approach used in Serpieri and Alfano, (2011) Serpieri et al. (2015a), G_{cI} and G_{cII} do not include the contribution of friction to the total (measured) fracture energy. The increasing total fracture energy that is experimentally found for increasing mode II-to-mode I ratios is actually retrieved by the model because of the increasing contribution given by friction.

The finite depth of asperities is accounted for in the modified formulation presented by Serpieri et al. (2015b), by writing the governing equations at the micro-scale on the deformed configuration. In this way, the model captures the reduction in contact area between elementary planes for increased opening displacement, which leads to a reduction of the normal displacement in response to a slip test, until a maximum dilation value H_N is asymptotically attained (Figure 4). Furthermore, an additional feature of the model by Serpieri et al. (2015b) is the capability of modelling progressive wear of the asperities, which is simulated by reducing the angle θ_k of each inclined elementary plane as an exponentially decaying function of the energy W_k dissipated by friction on that elementary plane:

$$\theta_k = (\theta_{k0} - \theta_{kf}) e^{-W_k/W_0} + \theta_{kf}, \quad W_k(t) = \int_0^t \sigma_{kt} ds_{fk}, \quad (3)$$

where t denotes time, θ_{k0} and θ_{kf} are the initial (at time $t = 0$) and final (at time t) inclination angle of the elementary plane, while σ_{kt} denotes the (local) tangential stress on the elementary plane and W_0 is a characteristic decay exponent.

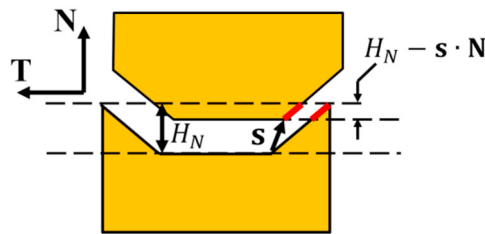


Fig. 4. CZM accounting for the finite depth of the fracture surface asperities [4].

3. Selected numerical results for 2D problems

3.1. DCB-UBM 2D simulation

The CZMs proposed in (Serpieri and Alfano, 2011), (Serpieri et al., 2015a) and (Serpieri et al., 2015b) allow decoupling the de-cohesion (rupture) energy, which can be assumed to be mode-mixity independent, from the frictional dissipation; in this way the interaction of the latter with the interlocking mechanisms induced by the inclined elementary plane allows retrieving the increase of the total (measured) fracture energy with increasing mode II-to-mode I ratio, which is typically observed in experiments. This was validated by Serpieri et al. (2015a) against the experimental results reported by Sorensen et al. (2006) for the delamination in a double cantilever beam, subject to uneven bending moments (DCB-UBM), made of E-glass-fibre-reinforced polyester matrix (Figure 5). Under different ratios of the applied moments M_1 and M_2 different mixed-mode ratios are obtained, ranging from pure mode I for $M_1 / M_2 = -1$ to pure mode II for $M_1 / M_2 = 1$. Figure 6 shows the numerical results reported by Serpieri et al. (2015a) expressed in terms of J-integral against the relative displacement at the initial crack tip compared with the experimental data reported by Sorensen et al. (2006). The CZM model used does not account for the finite-depth of the asperities and does not include modelling of asperities degradation. It was implemented in a user-subroutine of ABAQUS and used as constitutive model of 2D, linear, 2-node interface elements, whereas 4x230 4-noded, fully integrated, plane-strain elements were used for modelling the two arms of the specimen, also in accordance with the use of a 2D plane-strain model to compute the J integral in the paper by Sorensen et al. (2006). Following the original work by Sorensen et al. (2006), the behaviour of the bulk material was approximated with an isotropic material model with the Young's modulus and Poisson's ratio of 37 GPa and 0.3, respectively.

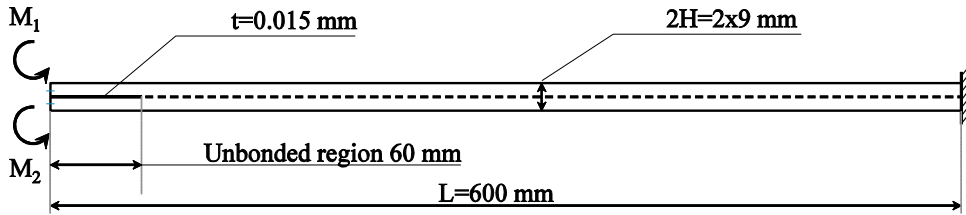


Fig. 5. geometry and loading of the DCB-UBM specimen [9] (the out of plane width is 30 mm).

The critical energy release rate $G_c = G_{c1} = G_{c2}$ represents the total fracture energy depurated of frictional dissipation, and therefore was calibrated based on the mode-I curve, together with the strengths $\sigma_{0I} = \sigma_{0II}$ in the base-line laws of Figure 3. The friction coefficient and the angle of the two inclined elementary planes were then calibrated based on the other curves with increasing mixed-mode ratio. The comparison of numerical and experimental results shows similar asymptotic values, but different slopes of the curves because in the experiment significant fibre bridging occurs, that is not captured by the CZM. However, the overall capability of capturing the mode-mixity dependence of the experimental results only as a result of the interaction of friction and interlocking is a remarkable feature of the proposed model.

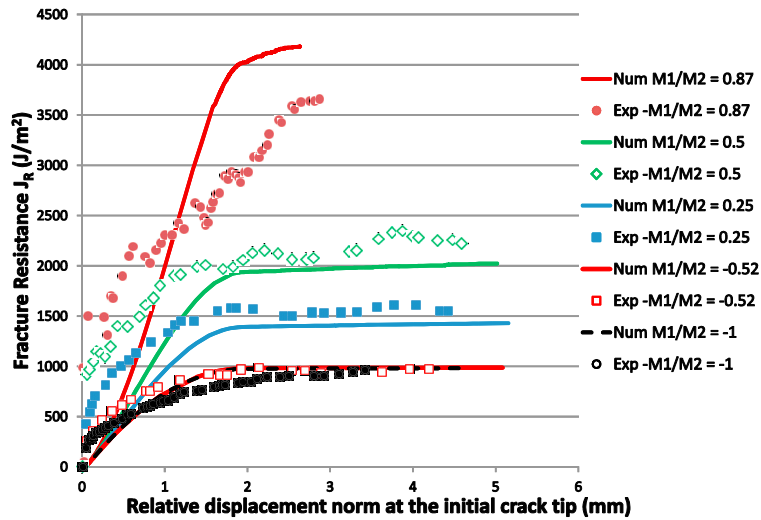


Fig. 6. Numerical (Serpieri et al., 2015a) against experimental (Sorensen et al., 2006) results for the DCB-UBM tests.

3.2. DCB-UBM 2D simulation

The capability of the further developed model proposed by Serpieri et al. (2015b) to capture the finite-depth of asperities and their degradation during cyclic loading was validated, through single-point numerical simulations, against the experimental results for granite joints tested under cyclic loading by Lee et al. (2001), with very good agreement shown in Figure 7. Such good correlation was obtained by assuming the friction coefficient measured in Lee et al. (2001), equal to 0.69, and with simple calibration of the other key parameters resulting in quite realistic model parameters, namely initial and final angles, θ_{k0} and θ_{kf} , equal to 30 and 9 degrees, respectively, an asperity depth of $H_N = 4$ mm and $W_0 = 6.666$ N/mm.

Further successful validation of the model was conducted by Serpieri et al. (2015b) against experimental results for pull-out tests of a steel bar from a concrete block, reported by Shima et al. (1987).

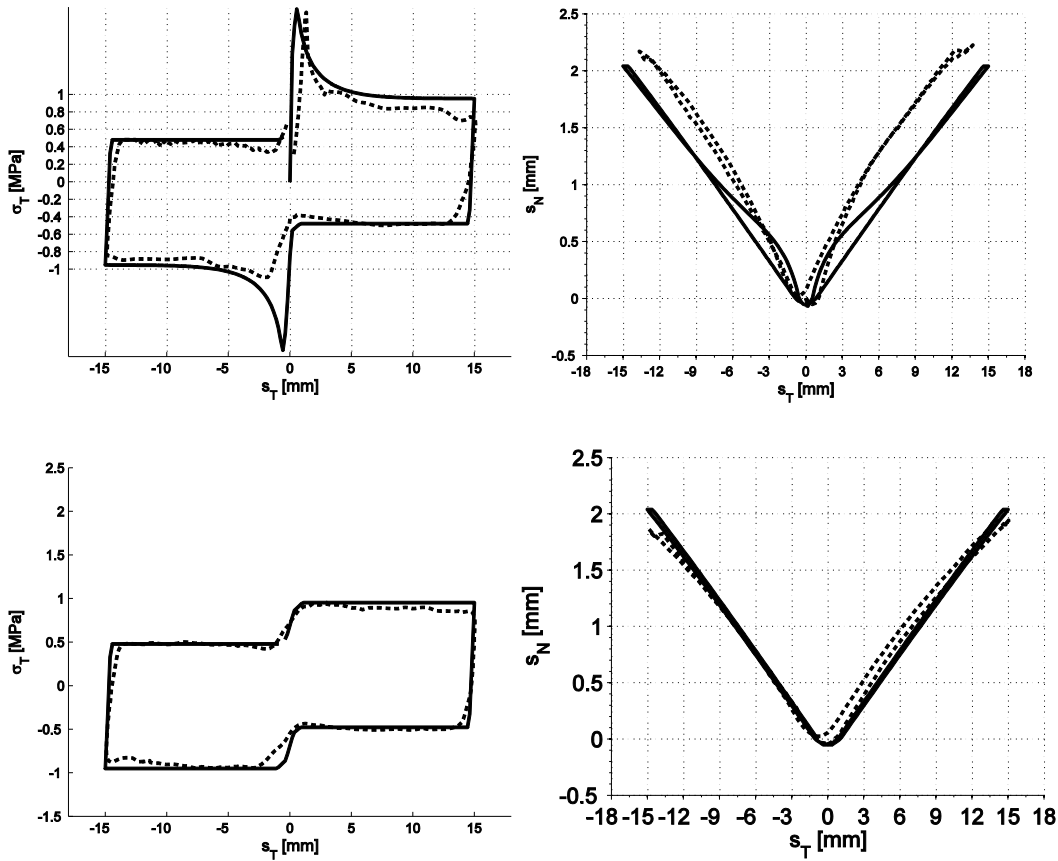


Fig. 7. Experimental (Lee et al., 2001) (dotted line) against numerical (Serpieri et al. 2015b) (solid line) results for the tests on granite joints subject to cyclic loading.

4. Extension to 3D problems

To generalize the modelling approach to a 3D case, one issue that can be raised is whether or not any distinction is needed between modes II and III, which is frequently introduced in fracture mechanics. In some cases, different responses can be argued due to the nature of the problem. For example, in the case of delamination of a laminated composites in which all plies are unidirectional and the crack front is orthogonal to the direction of fibres, one can argue that mode II involves sliding in the direction of the fibres while mode III results in sliding of two plies in a direction orthogonal to the fibres. This may induce a different influence of fibre bridging and may also result in different fracture surfaces. However, fibre bridging is a dissipative mechanism that could be introduced in the model separately, while the difference in the fracture surface between the cases of modes II and III can be captured using a suitable, — deliberately non-isotropic — 3D RME geometry. Therefore, the only distinction that will be made here is between tangential and opening relative displacements.

Furthermore, two situations can be distinguished: one in which the actual fracture surface has a well-defined geometry, such as in the case of ribbed steel bars used in reinforced concrete, and a second one in which the geometry of the fracture surface is not periodic and is statistically isotropic. This second case may be more challenging because of the problem of defining a 3D RME which is relatively simple, and therefore computationally not too expensive, but also complex enough to reproduce an isotropic response in the interface plane. This problem has been addressed in detail by Albarella et al. (2015). A key result that is presented here is that the 3D RME shown in Figure 2(b), despite being clearly non-isotropic, shows a response that is nearly isotropic in its plane. This can be

appreciated by considering the results shown in Figure 8(a), where the tangential stress, against a prescribed sliding relative displacement in different directions θ_L of the plane, is reported for the case of 5 MPa confining pressure. The result can be even better appreciated in Figure 8(b), where a polar plot of the fracture energy is shown for each direction of the (macro-scale) interface, in the case of no confining pressure.

For the detailed description of the 3D CZM model, the reader is referred to the paper by Albarella et al. (2015) and by Serpieri et al. (2017), where extensive numerical results and sensitive analyses for cases of monotonic and cyclic loading, using a wide range of RME geometries, are reported and critically discussed.

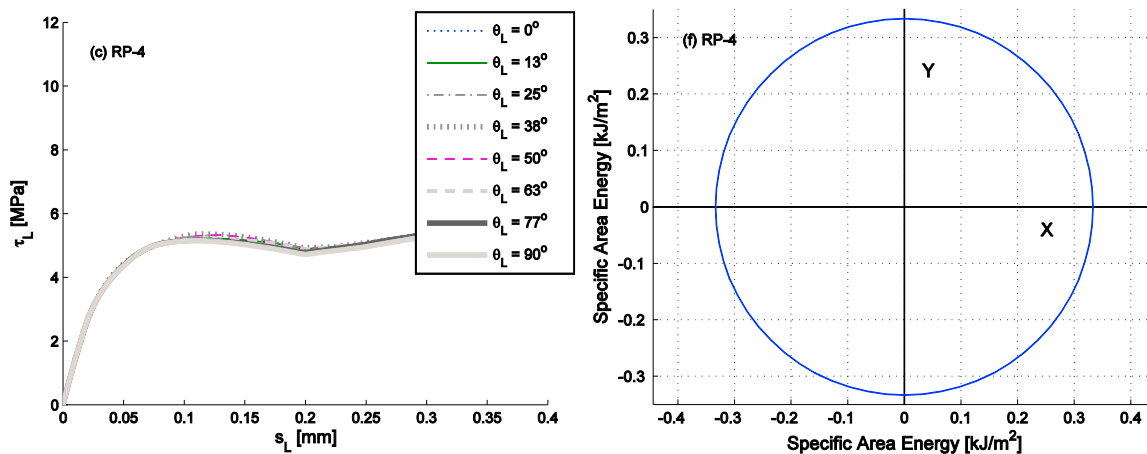


Fig. 8. Nearly isotropic response of the RME of Figure 2(b) for (a) 5MPa of confining pressure and (b) no confining pressure.

5. Conclusions

The cohesive-zone models (CZMs) proposed by Serpieri and Alfano (2011) has been theoretically revisited and analysed by Serpieri et al. (2015a) and further refined (Serpieri et al., 2015b) for 2D cases. These models, and their extension to a general 3D model addressed by Albarella et al. (2015), represent an original and effective method of capturing the complex interaction between damage, friction and interlocking, based on a simple yet physically well justified multi-scale approach. In its last version proposed by Serpieri et al. (2015b), the model also accounts for the finite depth of the fracture surface asperities and their wear and degradation, particularly in the case of cyclic loading.

The predicting capabilities of the model have been experimentally validated against a number of experimental tests of different nature, conducted and reported by different authors for problems involving structural interfaces made of different materials. While further validation is certainly useful, the effectiveness of the model is confirmed by the fact that the model input parameters have a clear physical meaning and their calibration performed for the aforementioned experimental validation always led to values which are well within the expected range. Furthermore, it is the authors' opinion that the clear physical meaning of the input parameters also facilitates the calibration itself and the interpretation of its results.

The capability of the model of capturing the mode-mixity dependence of the total (measured) fracture energy by using a single value of the 'decohesion' energy, in modes I and II, on each elementary plane of the geometry of the micro-scale, is a remarkable feature as it is based on the effective decoupling of the energy dissipated by friction and the amount of energy needed to achieve de-cohesion. This seems a significant step forward towards the possibility of characterizing mode-II, mode-III and mixed-mode fracture of materials and interfaces through parameters that can be considered as material properties, or, in other words, that are not dependent on the size and geometry of the structure or on the loading and boundary conditions.

Acknowledgements

This work was funded by Regione Campania through European funds (POR Campania FSE 2007–2013) for the Dottorato in azienda project (CUP F82I11001150002). The first author gratefully acknowledges the financial supports of the University of Sannio (FRA 2014). The third author gratefully acknowledges the financial supports of the University of Cassino and of the Consorzio RELUIS (Department of Civil Protection).

References

- Albarella, M., Serpieri, R., Alfano, G., Sacco, E., 2015. A 3D multiscale cohesive zone model for quasi-brittle materials accounting for friction, damage and interlocking. *Revue Européenne de Mécanique Numérique*, 24, 144–170. [10]
- Alfano, G., Crisfield, M.A., 2001. Finite element interface models for the delamination analysis of laminated composites: mechanical and computational issues. *International Journal for Numerical Methods in Engineering*, 50, 1701–1736.
- Alfano, G., Sacco, E., 2006. Combining interface damage and friction in a cohesive-zone model. *International Journal for Numerical Methods in Engineering*, 68, 542–582.
- Geers, M.G.D., Kouznetsova, V., Brekelmans, W.A.M., 2010. Multi-scale computational homogenization: trends and challenges. *Journal of Computational and Applied Mathematics*, 234, 2175–2182.
- Lee, H., Park, Y., Cho, T., You, K., 2001. Influence of asperity degradation on the mechanical behaviour of rough rock joints under cyclic shear loading. *International Journal of Rock Mechanics and Mining Sciences*, 38, 967–980.
- Serpieri, R., Alfano, G., 2011. Bond-slip analysis via a thermodynamically consistent interface model combining interlocking, damage and friction. *International Journal for Numerical Methods in Engineering*, 85, 164–186.
- Serpieri, R., Sacco, E., Alfano, G., 2015. A thermodynamically consistent derivation of a frictional-damage cohesive-zone model with different mode I and mode II fracture energies. *European Journal of Mechanics A/Solids*, 49, 13–25.
- Serpieri, R., Alfano, G., Sacco, E. 2015. A mixed-mode cohesive-zone model accounting for finite dilation and asperity degradation. *International Journal of Solids and Structure*, 67–68, 102–115.
- Serpieri, R., Albarella, M., Sacco, E. 2017. A 3D two-scale multiplane cohesive-zone model for mixed-mode fracture with finite dilation. *Computer Methods in Applied Mechanics and Engineering*, 313, 857–888.
- Shima, H., Chou, L., Okamura, H., 1987. Bond characteristics in post-yield range of deformed bars. *Concrete Library of JSCE*, 10, 113–124.
- Sorensen, B. F., Jorgensen, K., Jacobsen, T. K., 2006. DCB specimen loaded with uneven bending moments. *International Journal of Fracture*, 145, 159–172.