On the root mean square error (RMSE) calculation for parameter estimation of photovoltaic models: A novel exact analytical solution based on Lambert *W* function

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Abstract

In the literature, one can find a lot of methods and techniques employed to estimate single diode solar photovoltaic (PV) cell parameters. The efficiency of these methods is usually tested by calculating the Root Mean Square Error (RMSE) between the measured and estimated values of the solar PV cell output current. In this work, first, the values of RMSE calculated using 69 different methods published in many journal papers for the well-known RTC France solar PV cell are presented and discussed. Second, a novel exact analytical solution for RMSE calculation based on the Lambert *W* function is proposed. The results obtained show that the RMSE values were not calculated correctly in most of the methods presented in the literature since the exact expression of the calculated cell output current was not used. Third, the precision of calculation of the methods used for analytical solving of Lambert *W* equation is presented and discussed. Fourth, the applicability of the proposed solution methodology in accordance with current-voltage characteristics measured in the laboratory for solar modules of Clean Energy Trainer Setup is checked. Identification of its unknown parameters is presented using three optimization techniques. Further, the proposed solution methodology is proven for Solarex MSX–60 PV module, and the most promising 5-parameter single diode parameters are estimated based on minimization of the precise RMSE values calculated. Finally, this work aimed to develop a good base for proper investigation and implementation of optimization algorithms to solve the parameter estimation problem of 5-parameter single diode PV equivalent circuits.

Keywords—Lambert W function; optimization; PV parameter estimation; root mean square error; RTC France solar cell; 5-parameter single diode model.

Abbreviations

ABC	Artificial bee colony
ABSO	Artificial bee swarm optimization
ABCDE	Artificial bee colony-differential evolution
A&I	Analytical and iterative based methods
BPFPA	Bee pollinator flower pollination algorithm
BMO	Bird mating optimizer
BBO	Biogeography–based optimization
BBO–M	Biogeography-based optimization algorithm with two mutation strategies
BC	Bézier curves
BHCS	Biogeography-based heterogeneous cuckoo search
CARO	Chaotic asexual reproduction optimization
COA	Chaotic optimization approach
CPMPSO	Classified perturbation mutation-based particle swarm optimization
CWOA	Chaotic whale optimization algorithm
CSO	Cat swarm optimization
DE	Differential evolution
DET	DE technique
EAs	Evolutionary algorithms
EHA–NMS	Eagle-based hybrid adaptive Nelder-Mead simplex algorithm
ER-WCA	Evaporation rate-based water cycle algorithm
FPSO	Flexible particle swarm optimization
FA	Firefly algorithm
FPA	Flower pollination algorithm
GGHS	Grouping-based global harmony search
GA	Genetic algorithm
GAMS	General algebraic modelling system
GOTLBO	Generalized oppositional teaching learning-based optimization
GWO	Grey wolf optimization
HFAPS	Hybrid firefly and pattern search algorithms
HS	Harmony search
HPEPD	High performing extraction procedure for the one-diode model
HISA	Hybridized interior search algorithm
HCLPSO	Chaotic heterogeneous comprehensive learning particle swarm optimizer
IADE	Improved adaptive DE
ICA	Imperialist competitive algorithm
ITLBO	Improved teaching-learning-based optimization
ISCE	Improved shuffled complex evolution
ILCOA	Improved Lozi map-based chaotic optimization algorithm
IGHS	Innovative global HS
IJAYA	Improved JAYA optimization algorithm
JAYA	Sanskrit word meaning victory or triumph
LI	Linear identification
LMSA	Levenberg–Marquardt algorithm combined with simulated annealing
MABC	Modified artificial bee colony algorithm
MADE	Memetic adaptive DE
MBA	Mine blast algorithm
MVO	Multi-verse optimization
MPCOA	Mutative-scale parallel chaos optimization algorithm
MPSO	Modified particle swarm optimization
MSSO	Modified simplified swarm optimization algorithm
NM	Newton method
NM–MPSO	Nelder–mead and modified particle swarm optimization
OBWOA	Opposition-based whale optimization algorithm
Rcr–IJADE	Rate crossover repairing improved adaptive DE
PCE	Population classification evolution
PS	Pattern search
PSO	Particle swarm optimization
PPSO	Parallel PSO
PGJAYA	Performance-guided JAYA algorithm
pSFS	Perturbed stochastic fractal search
PV	Photovoltaic

SA	Simulated annealing
SSE	Sum of squared error
STFT	Special trans function theory
TLBO	Teaching-learning-based optimization
TLABC	Teaching–learning-based artificial bee colony
TS	Taylor series
TVA-CPSO	Time-varying acceleration coefficients PSO
WCA	Water cycle algorithm
WDO	Wind-driven optimization

Nomenclature

G	Irradiance (W/m^2)
I–U	Current–voltage characteristics
I_i^{meas} and I_i^{calc}	Measured and estimated solar cell current at point *i*, respectively
I_{pv}	Photo-generated current
I_0	Reverse saturation current
K_{B}	Boltzmann constant
M	Positive integer
n	Ideality factor of the diode
N	The number of the measured points
Pr	Precision of calculation that reflects accuracy at higher values
P–U	Power–voltage characteristics
Pr_{STFT} and Pr_{Taylor}	Precision of calculation using STFT and TS, respectively
q	Electron's charge
RMSE	Root mean square error
R_P	Parallel resistance of the solar cell
R_S	Series resistance of the solar cell
T	Actual temperature in Kelvin
V_{th}	Thermal voltage
W	Solution of the Lambert equation
β	Real number in Lambert W equation

1. Introduction

Recently, significant scaling up renewable energy sources (RESs) capacity in modern power systems is experienced in response to several technical, economic, environmental, social, and political factors. The conventional fossil fuel generation sources are facing severe environmental problems such as the greenhouse gas emissions contributing to global warming, and techno-economic problems due to price fluctuations and fuel depletion across the world [1,2]. In this regard, the solar photovoltaic (PV) capacity is one of the primary drivers towards realizing emission-free power generation that can accelerate shifting power systems away from fossil fuel generation sources to renewable sources.

Solar PV modules considerably depend on the operating conditions such as solar irradiance, temperature, spectrum, and others, particularly for PV modules installed outdoors. These particular PV modules have different electrical characteristics from the reference characteristics given in the manufacturer datasheets. What makes the problem more difficult is the incomplete data and missing parameters in the data sheets provided by the manufacturers and vendors. This is why it is not simple to model the nonlinear electrical characteristics precisely with the missing data. Besides, the need for accurate solar PV modules becomes crucial, particularly with the fast-growing solar PV capacity across the world. The practical realization of any solar PV system requires an adequate solar PV cell model as well as accurately determined solar cell parameters to design reliable power systems with solar systems efficiently. Thus, it becomes essential to estimate such parameters for a complete and precise solar PV model that can closely match the experimental measures under different operation conditions [3,4].

The parameter estimation problem of solar PV cells represents a trendy scientific field for researches working with power and energy systems. In the literature, one can find a lot of methods and techniques employed to estimate single diode solar PV cell parameters [3–64], in which three primary classes for estimating the single diode solar PV cell parameters can be categorized into analytical, numerical (deterministic and metaheuristic), and hybrid methods [3]. The analytical methods use the *I–U* (current-voltage) and *P–U* (power-voltage) data curve and the other information provided in the datasheet to formulate the mathematical estimation optimization problem. They are easy to implement and imply less computational effort. However, these methods require simplifications or approximations of the expressions used, which has a significant impact on the solution's accuracy. Besides, the selected initial points from the *I–U* curve may influence the accuracy of the solution obtained [3]. On one hand, numerical methods include iterative techniques such as Newton-Raphson, Levenberg-

Marquardt, and linear identification [3,42,45] to provide accurate solutions; however, they suffer from locally optimal in non-convex optimization problems, in addition to the time consumption when obtaining global solutions because their performance is highly dependent on the initial values provided by the programmer. On the other hand, numerical methods include evolutionary algorithms (EAs) or metaheuristics. EAs are based on a trajectory or population of individuals that interact between exploration and exploitation phases to create a search path that can avoid local optima and achieve global/near-global solutions. EAs are classified into several categories as bio-inspired based algorithms, swarm intelligence-based algorithms, physics and chemistry-based algorithms, and others. Despite the fact that EAs have shown better-estimated parameters of PV equivalent circuits in terms of computational efficiency and precision of solutions compared to the analytical and deterministic methods, their performance is highly dependent on proper adjustment of the control parameters. The most popular EAs used for parameter estimation of PV equivalent circuits are the bio-inspired algorithms, which mimic ideas, processes, or biological behaviors that take place in nature. The main representatives of this group are MADE [12], ISCE [20], BPFPA [25], DET [37], FPA [44], DE [46], IADE [54], Rcr–IJADE [57], and GA [60]. Also, swarming-based EAs are modeled to mimic swarming behaviors of birds, cats, bees, fish, or others. CPMPSO [4], HCLPSO [6], FPSO [7], MPSO [18], FA [22], MSSO [24], CSO [31], MABC [35], TVA-CPSO [38], PPSO [39], ABC [48], ABSO [53,55], BMO [52], and PSO [61], are the main representatives of these swarming-based algorithms. Similarly, physics and chemistry-based algorithms that mimic physical, chemical ideas or concepts are used for parameter estimation of PV equivalent circuits. The main representatives of this group are ER-WCA [23], WDO [27], WCA [38], GGHS [56], HS [56], IGHS [56], and SA [59]. Another set is that inspired by the teaching and learning process such as GOTLBO [30], STLBO [47], and TLBO [47,51], or that inspired by chaotic behaviors such as ILCOA [11], COA [13], CWOA [28], CARO [41], and MPCOA [50]. In the optimization process, the objective function, in most cases, is to minimize the sum of the squared difference between the experimentally measured solar PV cell output current and the calculated one of a specified number of data points. The third category, hybrid methods, combines analytical and numerical optimization methods to achieve global solutions associated with high computational efficiency and convergence speed towards finding the global solution. However, hybrid methods such as HISA [5], BHCS [16], HFAPS [19], TLABC [21], NM–MPSO [32], EHA–NMS [40], and ABCDE [55] are high complexity level algorithms.

In many of the mentioned papers, comparisons of these algorithms were presented in terms of many criteria such as convergence characteristics, computational efficiency, complexity, time per iteration, and so forth [14]. Unfortunately, we can find a case in a research paper claiming that algorithm X gives better results in comparison with results obtained by using algorithms Y or Z. However, in another research paper, we can find results obtained by using algorithm Z are better than the results obtained with algorithm X without any comments on algorithm Y [46–48,55–58] even if the algorithms are compared after many runs (e.g. above 20). Besides, in a few recently published papers, we can also find different authors' discussion on the accuracy of the results obtained, and the root mean square error (RMSE) calculated with the measured and estimated values of the solar PV cell output current for the 5-parameter single diode model of solar cell [62–64]. It is clear that no method has been evidenced to be the most appropriate method in identifying the unknown parameters and no guarantee is observed for realizing global solutions of the different PV modules. This necessitates an assessment of the solutions provided by the different methods to solve the solar PV cell parameters extraction problem of the different PV modules.

In response to these discussions, in this work, first, the values of RMSE calculated using 69 different methods published in many journal papers for the well-known RTC France solar PV cell are presented and discussed. The RTC France solar cell is used in this work as it is one of the most popular solar cells across the world, and is commonly used in many published papers to test algorithms. Second, a novel exact analytical solution for RMSE calculation based on the Lambert *W* function is proposed. The results obtained show the shortcomings many of the reported works made with the determination of their RMSE calculations for the equivalent circuit modeling approach under focus. Third, the precision of two numerical approaches to numerically solve the Lambert *W* function is addressed and discussed with: (i) one based on Taylor series; and (ii) the other based on Special Trans Function Theory. Fourth, the applicability of the proposed solution methodology in accordance with current-voltage characteristics measured in the laboratory for solar modules of Clean Energy Trainer Setup is checked. Identification of its unknown parameters is presented using three optimization techniques: chaotic optimization algorithm (COA), evaporation rate water cycle algorithm (ER-WCA), and harmony search (HS). Further, the proposed solution methodology is applied for Solarex MSX–60 PV module, and the most promising 5 parameters are estimated based on the minimization of the precise RMSE values calculated. Finally, this work aimed to present a good base for proper investigation and implementation of optimization algorithms to solve the 5-parameter estimation problem of the single diode equivalent circuits of PVs.

The rest of the work is organized as follows: In Section 2, a short description of the 5-parameter single diode solar cell model is given. Also, the proposed RMSE expression based on Lambert *W* function is presented. Values obtained of the proposed RMSE expression and comparison of related works in the literature are presented and discussed in Section 3. Also, the precision of two numerical approaches to numerically solve the Lambert *W* function is addressed and discussed. In Section 4, the applicability of the proposed methodology is presented with the aid of experimental results and optimization methods. Lastly, the concluding remarks are drawn in Section 5.

2. Proposed RMSE calculation based on Lambert *W* function

Single diode solar cell model

The single diode solar cell model is a commonly used model for solar cell representation [4]. This model consists of one current source, one diode, and two resistances, namely a series resistance (R_S) and a parallel resistance (R_P), which represent the solar cell losses. The equivalent circuit of the single diode PV cell model is shown in Fig. 1.

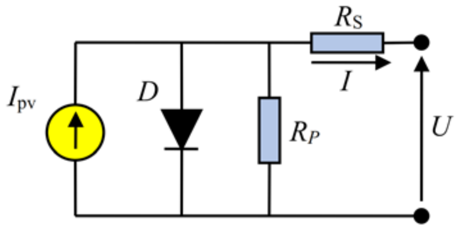


Fig. 1. Single diode model of the solar cell

In the mathematical sense, the *I*–*U* relationship of this model can be described, as follows:

$$I = I_{pv} - I_0\left(e^{\frac{U+IR_S}{n\times V_{th}}} - 1\right) - \frac{U+IR_S}{R_P} \qquad (1)$$

where I_{pv} represents the photo-generated current, I_0 is the reverse saturation current, n is the ideality factor of the diode, and $V_{th} = K_B T/q$ is the thermal voltage (K_B is Boltzmann constant, T is the temperature and q is the electron's charge). In addition to the single-diode model, two-diode and three-diode solar cell models can be found in the available literature [3]. However, in regard to two-diode and three-diode solar cell models, no exact analytical solution has been reached yet because of the high nonlinearity of the current expressions of these models.

Conventional RMSE calculation

In line with the literature [3-62], the following equation is usually expressed for the calculation of RMSE between measured and calculated output current of the solar PV cell.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(I_i^{meas} - I_i^{calc}\right)^2} \tag{2}$$

where *N* represents the number of the measured points. I_i^{meas} and I_i^{calc} represent the measured and estimated solar cell current at point *i*, respectively. Pseudo-substituting Eq. (1) into Eq. (2); at $U^{calc}=U^{meas}$, one can formulate the RMSE expression as follows:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(I_i^{meas} - \left(I_{pv} - I_0\left(e^{\frac{U_i^{meas}+I_i^{meas}\times R_S}{n\times V_{th}}} - 1\right) - \left(\frac{U_i^{meas} + I_i^{meas} R_S}{R_P}\right)\right)\right)^2} \tag{3}$$

However, with $I_{PV} - I_0\left(e^{\frac{U_i^{meas}+I_i^{meas}R_S}{n\times V_{th}}} - 1\right) - \left(\frac{U_i^{meas}+I_i^{meas}R_S}{R_P}\right)$ in the general case, the last part of this equation does not represent the calculated value of the solar cell output current, and therefore, Eq. (3) is not a correct expression of the RMSE measure. It should be noted that many reported works of the literature use this relation to estimate their RMSE.

Lambert W function

In mathematics, the Lambert *W* function is a set of functions, precisely the branches of the inverse relation of the function β given below.

$$f\left(x\right) = \beta = xe^x \tag{4}$$

where e^x is the exponential function, and *x* is any complex number. Many problems in engineering sciences can be described using this equation. Thus, one can get *x* as follows:

$$x = f^{-1}\left(xe^x\right) = W\left(\beta\right) \tag{5}$$

where *W* represents the solution of the Lambert equation [65–73]. A graphical representation of the function $\beta = xe^x$ is shown in Fig. 2.

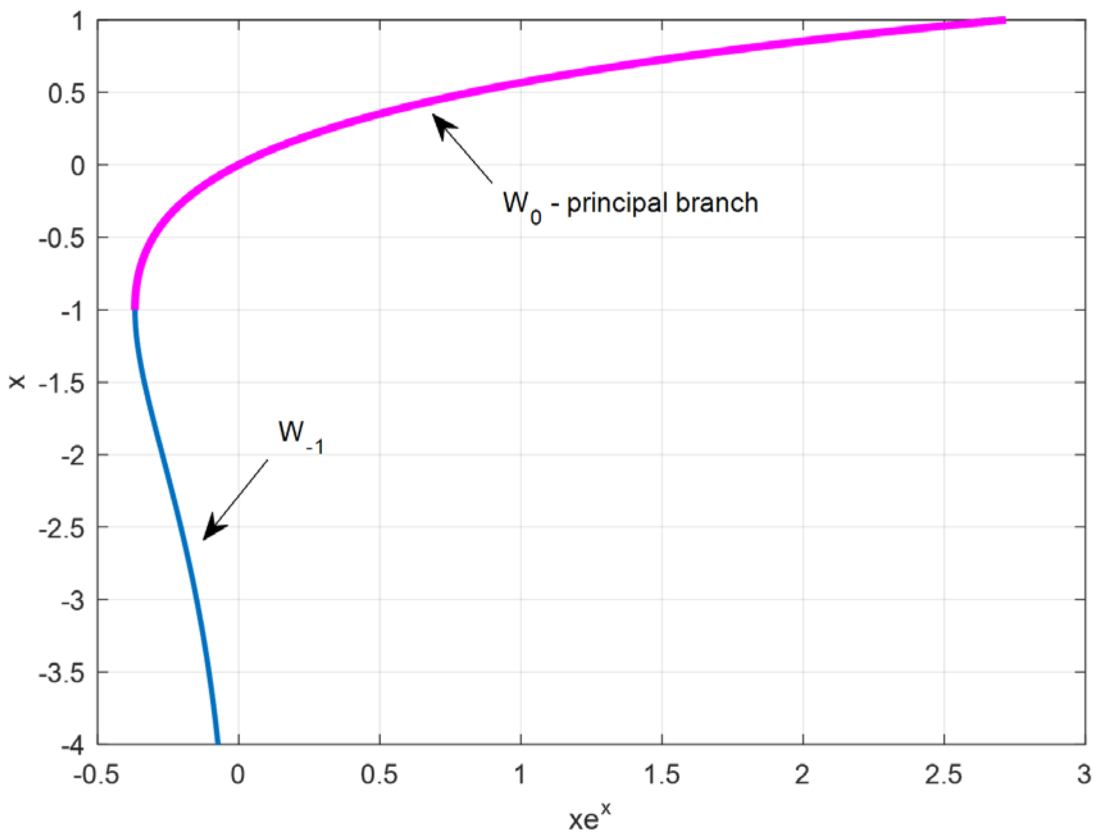


Fig. 2. Graphical representation of the function $\beta = xe^{x}$

It can be noticed from Fig. 2 that the illustration consists of two parts; the upper part (branch) with $W \geq -1$ representing the function W_0, which is called the principal branch, and the lower part (branch) with $W \leq -1$ representing the function W_{-1}.

It should be noted that for solving Lambert W functions, different techniques can be used such as the iterative methods [65] and program packages with corresponding solvers [66], in addition to analytical-based methods such Taylor series (TS) [67] and Special Trans Function Theory (STFT) [68–73].

The TS of W_0 around 0 is based on the usage of the Lagrange inversion formula as follows:

$$x = W(\beta) = \sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} \beta^{n} \tag{6}$$

In practice, Eq. (6) can be rewritten as follows:

$$x = W(\beta) = \sum_{n=1}^{M} \frac{(-n)^{n-1}}{n!} \beta^{n} \tag{7}$$

where M denotes a positive integer.

The Lambert W function can also be solved using STFT as follows:

$$x = \beta \frac{\sum_{n=0}^{M} \frac{\beta^n \left(M-n\right)^n}{n!}}{\sum_{n=0}^{M+1} \frac{\beta^n \left(M+1-n\right)^n}{n!}} \tag{8}$$

However, it should be noted that for the same value of positive integer *M*, the STFT gives the much more accurate results. It should be noted that the accuracy of using these methods is tested in many papers [66,68]. However, the general conclusion is that TS has good accuracy for small values of β, which is not the case for the higher values of β. However, STFT methods have a high level of accuracy for any value of β. To sum up, STFT is a more accurate method for analytical solving of the Lambert *W* equation. The reader can refer to [66,68,72] to find additional information and examples.

Proposed RMSE calculation

In order to calculate the estimated value of the solar cell output current, we need to solve Eq. (1). However, Eq. (1) is a highly nonlinear equation, *i.e.*, transcendental equation. Thus; it can be rearranged in the following form:

$$I = \frac{R_P\left(I_{pv}+I_0\right)-U}{R_S+R_P} - \frac{nV_{th}}{R_S} W(\beta) \tag{9}$$

So that

$$\beta = A \exp\left(\frac{R_P\left(R_S I_{pv}+R_S I_0+U\right)}{nV_{th}\left(R_S+R_P\right)}\right) \tag{10}$$

$$A = \frac{I_0 R_P R_S}{nV_{th}\left(R_S+R_P\right)} \tag{11}$$

In Eq. (9), *W* represents the solution of the Lambert *W* function. In practice, for one value of the solar cell voltage, we have a particular value of the solar cell current. Therefore, in the calculation process, if we take a particular value of the solar cell voltage (equals to the measured voltage value), we can calculate the solar cell current using Eq. (9). Thus, the calculation of the proposed RMSE expression between measured and estimated solar cell current can be realized in the following manner:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(I_i^{meas} - \left(\frac{R_P\left(I_{pv}+I_0\right)-U_i^{meas}}{R_S+R_P} - \frac{nV_{th}}{R_S} W\left(A\exp\left(\frac{R_P\left(R_S I_{pv}+R_S I_0+U_i^{meas}\right)}{nV_{th}\left(R_S+R_P\right)}\right)\right)\right)\right)^2} \tag{12}$$

Equation (12) represents the analytical solution of the RMSE expression calculated between the measured and estimated solar cell output current. Besides, the analytical solution of the proposed RMSE expression using TS is expressed in the following form:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(I_i^{meas} - \left(\frac{R_P\left(I_{pv}+I_0\right)-U_i^{meas}}{R_S+R_P} - \frac{nV_{th}}{R_S}\cdot\sum_{n=1}^{M}\frac{(-n)^{n-1}}{n!}\left(A\exp\left(\frac{R_P\left(R_S I_{pv}+R_S I_0+U_i^{meas}\right)}{nV_{th}\left(R_S+R_P\right)}\right)\right)^n\right)\right)^2} \quad (13)$$

Also, the analytical solution of the proposed RMSE expression using STFT is expressed as follows:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(I_i^{meas} - \left(\frac{R_P\left(I_{pv}+I_0\right)-U_i^{meas}}{R_S+R_P} - \frac{nV_{th}}{R_S}\cdot\left(A\exp\left(\frac{R_P\left(R_S I_{pv}+R_S I_0+U_i^{meas}\right)}{nV_{th}\left(R_S+R_P\right)}\right)\right)\frac{\sum_{n=0}^{M}\frac{\left(A\exp\left(\frac{R_P\left(R_S I_{pv}+R_S I_0+U_i^{meas}\right)}{nV_{th}\left(R_S+R_P\right)}\right)\right)^n (M-n)^n}{n!}}{\sum_{n=0}^{M+1}\frac{\left(A\exp\left(\frac{R_P\left(R_S I_{pv}+R_S I_0+U_i^{meas}\right)}{nV_{th}\left(R_S+R_P\right)}\right)\right)^n (M+1-n)^n}{n!}}\right)\right)^2} \quad (14)$$

As Lambert *W* function is very popular in science, many programming packages have implemented a solver for solving Lambert *W* function. For example, in Matlab, we have a function called *lambertw*. In Maple, it is simply called *W*, while in the *Mathematica* computer algebra framework, the function is implemented under the name *ProductLog*.

3. Numerical results and discussion

In Table 1, for the well-known RTC France solar PV cell, the values of RMSE calculated using 69 different methods reported in many journal papers are presented, which rely on the same experimental current-voltage characteristics. In the same table, the estimated values of the 5 parameters are given. Besides, the RMSE values calculated using Eq. (3) and the corrected RMSE values calculated using Eq. (12) are presented. It should be noted that the determination of the 5 parameters is not addressed by the current methodology; however, many optimization techniques rely on objective functions whose accuracy can be improved by the proposed RMSE calculation.

All the calculations were carried out into the Matlab program package, while the computer simulations were carried out on a PC with Intel(R) Core (TM) i3-7020U CPU @ 2.30 GHz and 4 GB RAM. Accuracy of the solution of the Lambert *W* function was tested by using the Lambert *W* equation embedded in Matlab, in which the accuracy was lower than 10^{-16} for all the points measured from the *I–U* characteristics. The central part of the Matlab code for

RMSE calculation based on the Lambert *W* function is given in Appendix 1. Also, a Mathematica code for solving the Lambert W equation is provided.

The procedure for RMSE calculation is summarized as follows:

For any measured point, we consider the voltage value and calculate the corresponding current using Eq. (9). We then apply Eq. (2) for all the points, thus giving Eq. (12). Eq. (12) is resolved in practice either by Eq. (13) or Eq. (14) numerical approaches (or both).

It can be seen from Table 1 that the differences between the calculated RMSE values and the RMSE values presented in the original papers are considerable. A graphical illustration of these differences, on a logarithmic scale, is shown in Fig. 3. We can find the same values of RMSE, as suggested in this work in just two papers [5,42] among 58. However, in [5,42], no comments were given about the exact analytical solution of RMSE calculation or the Lambert *W* function. Also, looking at the data given in Table 1, it can be noticed that many authors had used Eq. (3) for RMSE calculation, which is only a rough approximation.

Besides, some researchers use a very compact number value (0.0264V) or a semi-compact number value as (0.02638V) of V_{th} instead of the long number formatting value, which also has a significant impact on the calculated RMSE value. The impact of V_{th} on the calculated RMSE value is shown in Fig. 4.

Apart from the wrong approach for RMSE calculation, the reasons for results mismatching of the minimum RMSE can also be:

- Authors do not use all measured points for RMSE calculation (N=26),
- Complete dependence on heuristic optimization techniques to reach a better solution than the corresponding values reported in the literature. Each optimization technique may also have different complexity and/or parameters at different values from one paper to the other.
- Inaccurate solving of the Lambert *W* function.

However, with this analysis, we cannot conclude that algorithm X is the best, while algorithm Y is the worst. The cause lies in the fact that the authors should use the proposed straightforward RMSE equation during the solar cell parameters estimation process, which has a significant impact on the algorithm efficiency.

Table 1. Numerical results of the conventional and proposed RMSE for parameters estimation of the solar RTC France cell (Results with the minimum RMSE are given in bold)

#	Ref.	Authors, year	Method	I_{pv} (A)	I_0 (μA)	n	R_S (Ω)	R_P (Ω)	Original reported RMSE	RMSE calculated using Eq. (3)	Proposed RMSE calculated using Eq. (12)
1	[4]	Liang et al., 2020	CPMPSO	0.76077600	0.32302100	1.48118400	0.03637700	53.71852000	0.00098602	0.00098602	0.00077539
2	**[5]**	**Kler et al., 2019**	**HISA**	**0.76078797**	**0.31068459**	**1.47726778**	**0.03654695**	**52.88979426**	**0.00077301**	**0.00098911**	**0.00077301**
3	[6]	Dalia et al., 2019	HCLPSO	0.76079000	0.31062000	1.47710000	0.03654800	52.88500000	0.00077300	0.00112018	0.00083374
4	[7]	Ebrahimi et al., 2019	FPSO	0.76077552	0.32302000	1.48110817	0.03637000	53.71852000	0.00098602	0.00102203	0.00079112
5	[8]	Li et al., 2019	ITLBO	0.76080000	0.32300000	1.48120000	0.03640000	53.71850000	0.00098602	0.00099161	0.00077779
6	[9]	Chen et al., 2019	pSFS	0.76078000	0.32302000	1.48118000	0.03638000	53.71852000	0.00098602	0.00098608	0.00077542
7	[10]	Chen et al., 2019	ISCA	0.76077562	0.32301700	1.48118220	0.03637716	53.71821748	0.00098602	0.00098602	0.00077539
8	[11]	Pourmousa et al., 2019	ILCOA	0.76077500	0.32302100	1.48110800	0.03637700	53.71867900	0.00098602	0.00102229	0.00079167
9	[12]	Li et al., 2019	MADE	0.76080000	0.32300000	1.48120000	0.03640000	53.71850000	0.00098602	0.00099161	0.00077779
10	[13]	Calasan et al., 2019	COA	0.76077450	0.32300180	1.48117740	0.03637750	53.73000000	0.00098602	0.00098602	0.00077538
11	[14]	Yu et al., 2019	PGJAYA	0.76080000	0.32300000	1.48120000	0.03640000	53.71850000	0.00098602	0.00099161	0.00077779
12	[15]	Gnetchejo et al., 2019	GAMS	0.76077600	0.32302000	1.48118400	0.03637700	53.71852400	0.00098602	0.00098602	0.00077540
13	[16]	Chen et al., 2019	BHCS	0.76078000	0.32302000	1.48118000	0.03638000	53.71852000	0.00098602	0.00098608	0.00077542
14	[17]	Abd Elaziz et al., 2018	OBWOA	0.76077000	0.32320000	1.52080000	0.03630000	53.68360000	0.00009860	0.11416692	0.07674430
15	[18]	Manel et al., 2018	MPSO	0.76078700	0.31068300	1.47526200	0.03654600	52.88971000	0.00077301	0.00733027	0.00435991
16	[19]	Beigi et al., 2018	HFAPS	0.76077700	0.32262200	1.48106000	0.03638190	53.67840000	0.00098602	0.00098603	0.00077525
17	[20]	Gao et al., 2018	ISCE	0.76077553	0.32302083	1.48118360	0.03637709	53.71852771	0.00098602	0.00098602	0.00077539
18	[21]	Chen et al., 2018	TLABC	0.76078000	0.32302000	1.48118000	0.03638000	53.71636000	0.00098602	0.00098608	0.00077542
19	[22]	Louzazni et al., 2018	FA	0.76069712	0.43244110	1.45245666	0.03341059	53.40180803	0.00051382	0.28514264	0.14234113
20	[23]	Kler et al., 2017	ER-WCA	0.76077600	0.32269900	1.48108000	0.03638100	53.69100000	0.00098602	0.00098609	0.00077529
21	[24]	Lin et al., 2017	MSSO	0.76077700	0.32356400	1.48124400	0.03637000	53.74246500	0.00098607	0.00105990	0.00080916
22	[25]	Ram et al., 2017	BPFPA	0.76000000	0.31060000	1.47740000	0.03660000	57.71510000	0.00072700	0.00125359	0.00095551
23	[26]	Fathy et al., 2017	ICA	0.76030000	0.14650000	1.44210000	0.03890000	41.15770000	NG*	0.11581627	0.07502096
24	[27]	Derick et al., 2017	WDO	0.76080000	0.32230000	1.48080000	0.03676800	57.74614000	0.00008866	0.00115725	0.00089482
25	[28]	Oliva et al., 2017	CWOA	0.76077000	0.32390000	1.48120000	0.03636000	53.79870000	0.00098602	0.00134855	0.00094834
26	[29]	Yu et al., 2017	IJAYA	0.76080000	0.32280000	1.48110000	0.03640000	53.75950000	0.00098603	0.00098714	0.00077606
27	[30]	Chen et al., 2016	GOTLBO	0.76078000	0.33155200	1.48382000	0.03626500	54.11542600	0.00098744	0.00098744	0.00077979
28	[31]	Guo et al., 2016	CSO	0.76078000	0.32300000	1.48118000	0.03638000	53.71850000	0.00098602	0.00098612	0.00077544
29	[32]	Hamid et al., 2016	NM-MPSO	0.76078000	0.32306000	1.48120000	0.03638000	53.72220000	0.00098602	0.00098620	0.00077550

#	Ref.	Authors, year	Method	I_{pv} (A)	I_0 (μA)	n	R_S (Ω)	R_P (Ω)	Original reported RMSE	RMSE calculated using Eq. (3)	Proposed RMSE calculated using Eq. (12)
30	[33]	Zhang et al., 2016	PCE	0.76077600	0.32302100	1.48107400	0.03637700	53.71852500	0.00098602	0.00106059	0.00080924
31	[34]	Tong et al., 2016	TONG	0.76100000	0.36350000	1.49350000	0.03660000	62.57400000	NG	0.00238593	0.00150509
32	[35]	Jamadi et al., 2016	MABC	0.76077900	0.32132300	1.48138500	0.03638900	53.39999000	0.00098601	0.00276101	0.00172770
33	[36]	Ali et al., 2016	MVO	0.76160000	0.32094000	1.52520000	0.03650000	59.58840000	NG	0.12679796	0.08627778
34	[37]	Chellaswamy et al., 2016	DET	0.75100000	0.31500000	1.48700000	0.03600000	54.53200000	0.00093000	0.02448057	0.01584481
35	[38]	Jordehi, 2016	WCA	0.76090800	0.41355400	1.50438100	0.03536300	57.66948800	0.00094655	0.00760691	0.00464720
36			TLBO	0.76080900	0.31224400	1.47578000	0.03655100	52.84050000	0.00077487	0.00727229	0.00433487
37			GWO	0.76099600	0.24303880	1.45121900	0.03773200	45.11630900	0.00095145	0.00728451	0.00434963
38			TVA-CPSO	0.76078800	0.31068270	1.47525800	0.03654700	52.88964400	0.00077301	0.00734381	0.00436800
39	[39]	Ma et al., 2016	PPSO	0.76080000	0.32300000	1.48120000	0.03640000	53.71850000	NG	0.00099161	0.00077779
40	[40]	Chen et al., 2016	EHA-NMS	0.76080000	0.32302100	1.48118400	0.03637700	53.71852100	0.00098602	0.00098635	0.00077570
41	[41]	Yuan et al., 2015	CARO	0.76079000	0.31724000	1.48168000	0.03644000	53.08930000	0.00098665	0.00819692	0.00495024
42	[42]	Lim et al., 2015	LI	0.76094380	0.34565720	1.48799169	0.03614233	49.48220500	0.00105480	0.00134617	0.00105482
43	[43]	El-Fergany, 2015	MBA	0.76040000	0.23480000	1.48900000	0.03880000	44.61000000	NG	0.11672175	0.07620443
44	[44]	Alam et al., 2015	FPA	0.76079000	0.31067700	1.47707000	0.03654660	52.87710000	0.00077301	0.00121214	0.00087797
45	[45]	Dkhichi et al., 2014	LMSA	0.76078000	0.31849000	1.47976000	0.03643000	53.32644000	0.00098640	0.00098649	0.00077406
46	[46]	Niu et al., 2014	DE	0.76068000	0.35515000	1.49080000	0.03598000	56.55330000	0.00100000	0.00100348	0.00080173
47			BBO	0.76098000	0.86100000	1.58742000	0.03214000	78.85550000	0.00238000	0.00239295	0.00200212
48			BBO-M	0.76078000	0.31874000	1.47984000	0.03642000	53.36227000	0.00098634	0.00098656	0.00077423
49	[47]	Niu et al., 2014	STLBO	0.76078000	0.32302000	1.48114000	0.03638000	53.71870000	0.00098602	0.00099764	0.00078059
50			TLBO	0.76074000	0.32378000	1.48136000	0.03641000	54.40290000	0.00098845	0.00100164	0.00078421
51	[48]	Oliva et al., 2014	ABC	0.76080000	0.32510000	1.48170000	0.03640000	53.64330000	0.00098602	0.00109667	0.00083343
52	[49]	Laudani et al., 2014	HPEPD	0.76078840	0.31024820	1.47696410	0.03655304	52.85905600	0.00077301	0.00114867	0.00084728
53	[50]	Yuan et al., 2014	MPCOA	0.76073000	0.32655000	1.48168000	0.03635000	54.63280000	0.00094457	0.00231307	0.00146916
54	[51]	Patel et al., 2014	TLBO	0.76080000	0.32230000	1.48370000	0.03640000	53.76027000	NG	0.00969602	0.00585540
55	[52]	Askarzadeh et al., 2013	BMO	0.76077000	0.32479000	1.48173000	0.03636000	53.87160000	0.00098608	0.00098622	0.00077621
56	[53]	Askarzadeh et al., 2013	ABSO	0.76080000	0.30623000	1.47583000	0.03659000	52.29030000	0.00099124	0.00099125	0.00077368
57	[54]	Jiang et al., 2013	IADE	0.76070000	0.33613000	1.48520000	0.03621000	54.76430000	0.00098900	0.00099076	0.00078442
58	[55]	Hachana et al., 2013	ABSO	0.76080000	0.30623000	1.47986000	0.03659000	52.29030000	0.00098602	0.01416898	0.00855244
59			ABCDE	0.76077000	0.32302000	1.47986000	0.03637000	53.71850000	0.00098602	0.00485483	0.00292463
60			DE	0.76077000	0.32302000	1.48059000	0.03637000	53.71850000	0.00098602	0.00234235	0.00148100
61			MPSO	0.76077000	0.32302000	1.47086000	0.03637000	53.71850000	0.00098602	0.03902188	0.02247017

#	Ref.	Authors, year	Method	I_{pv} (A)	I_0 (μA)	n	R_S (Ω)	R_P (Ω)	Original reported RMSE	RMSE calculated using Eq. (3)	Proposed RMSE calculated using Eq. (12)
62	[56]	Askarzadeh et al., 2012	GGHS	0.76092000	0.32620000	1.48217000	0.03631000	53.06470000	0.00099097	0.00099089	0.00078146
63			HS	0.76070000	0.30495000	1.47538000	0.03663000	53.59460000	0.00099510	0.00099515	0.00077625
64			IGHS	0.76077000	0.34351000	1.48740000	0.03613000	53.28450000	0.00099306	0.00103345	0.00082116
65	[57]	Gong et al., 2013	Rcr-IJADE	0.76077600	0.32302100	1.48118400	0.03637700	53.71852600	0.00098602	0.00098602	0.00077539
66	[58]	AlHajri et al., 2012	PS	0.76170000	0.99800000	1.60000000	0.03130000	64.10256000	0.28630000	0.01493638	0.00981702
67	[59]	El-Naggar et al., 2012	SA	0.76200000	0.47980000	1.51720000	0.03450000	43.10345000	0.00170000	0.01899784	0.01165472
68	[60]	AlRashidi et al., 2011	GA	0.76190000	0.80870000	1.57510000	0.02990000	42.37288000	NG	0.01907803	0.01200884
69	[61]	Ye et al., 2009	PSO	0.76079800	0.32272100	1.48382000	0.03639400	53.79650000	NG	0.00965452	0.00583097

*NG: not given

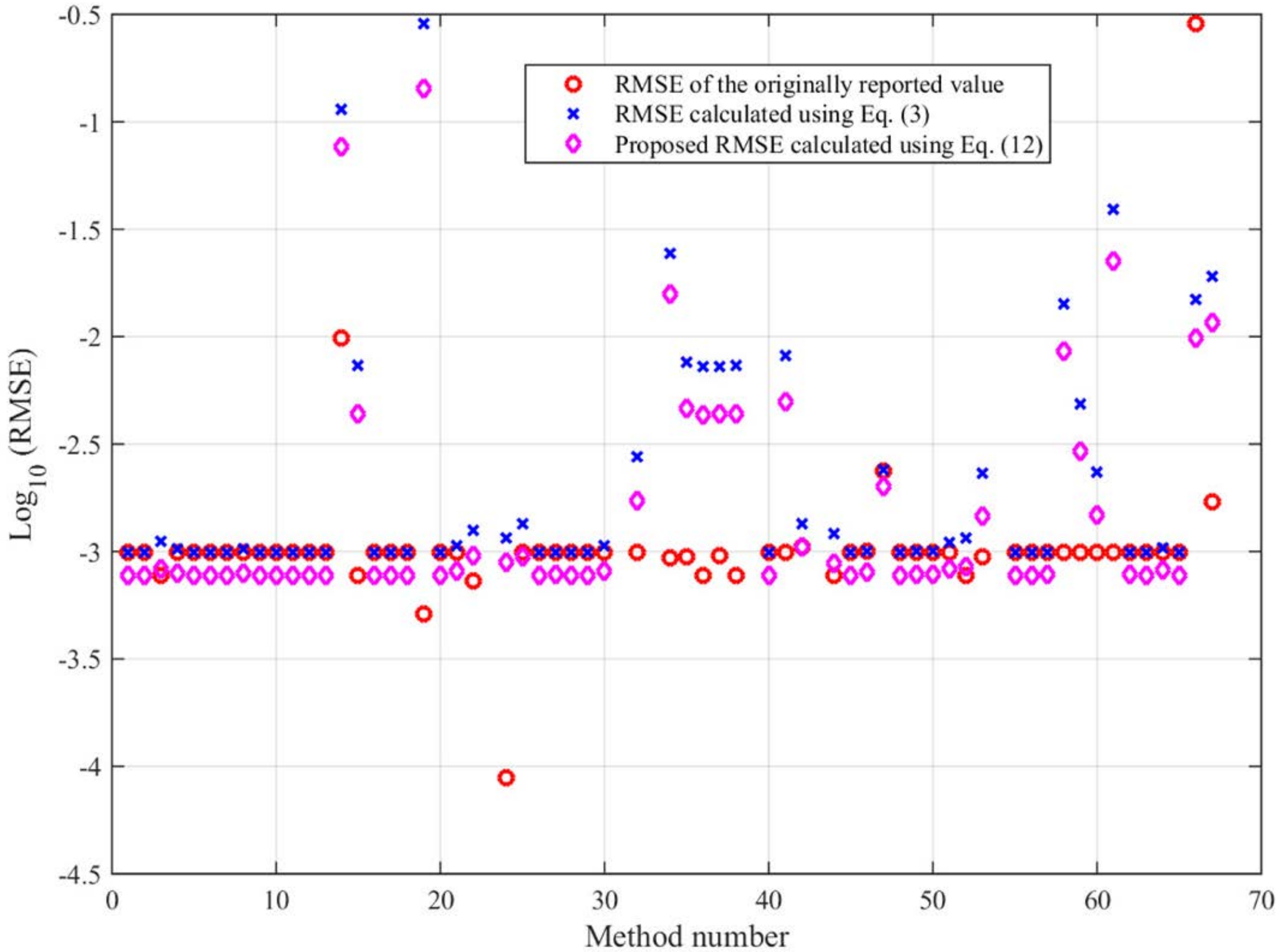


Fig. 3. The difference in a logarithmic scale between proposed RMSE values calculated using Eq. (12) and those calculated using different methods presented in the literature (Method number after Table 1)

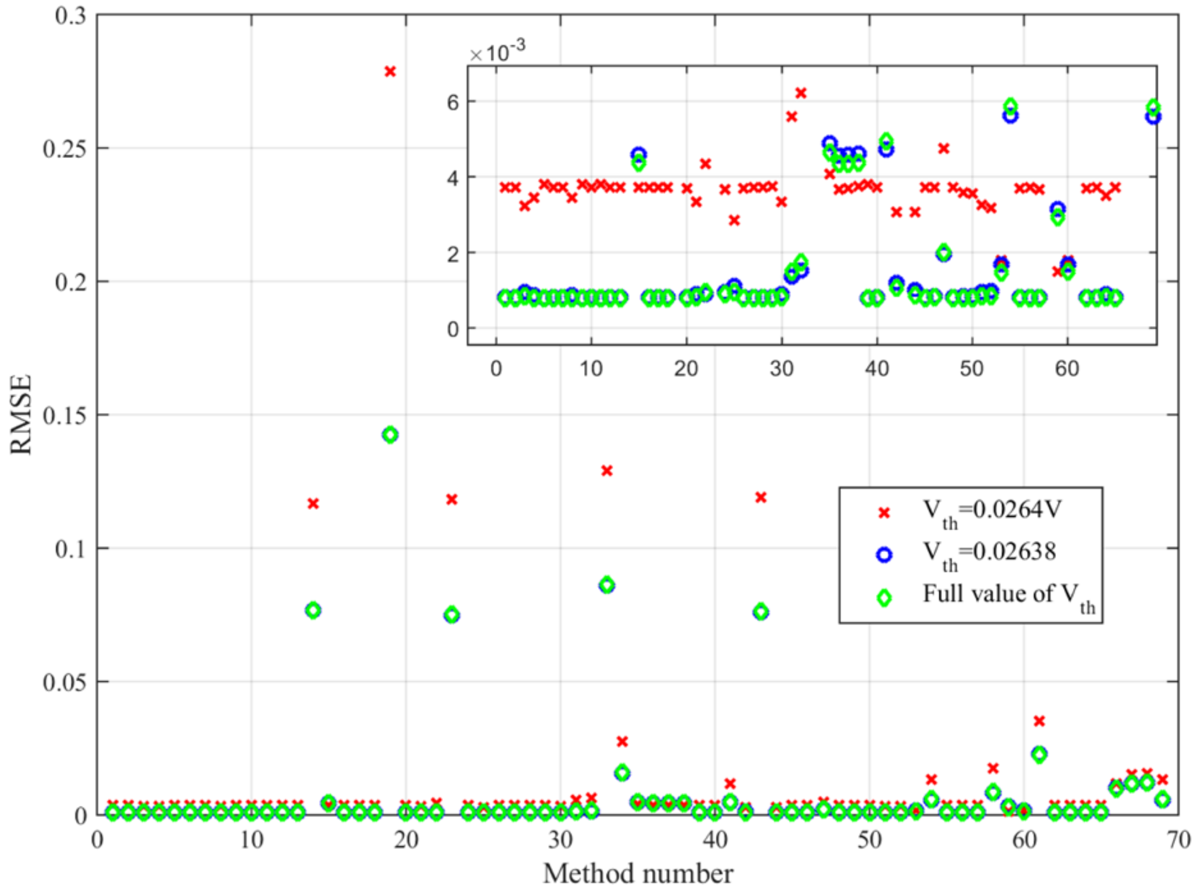


Fig. 4. Impact of V_{th} value on the RMSE value calculated using Eq. (12) (Method number after Table 1)

Based on the results shown in Table 1, we can see that the most accurate method for parameter estimation of single diode RTC France solar cell is the HISA method (method number 2) presented by *Kler* et al. in 2019 [5]. The results of this method are given in bold in Table 1. The *I*–*U* characteristic of the RTC France solar cells obtained for these parameters, as well as the measured characteristic, are shown in Fig. 5, in which the remarkable agreement between the measured and simulated characteristics is indicated.

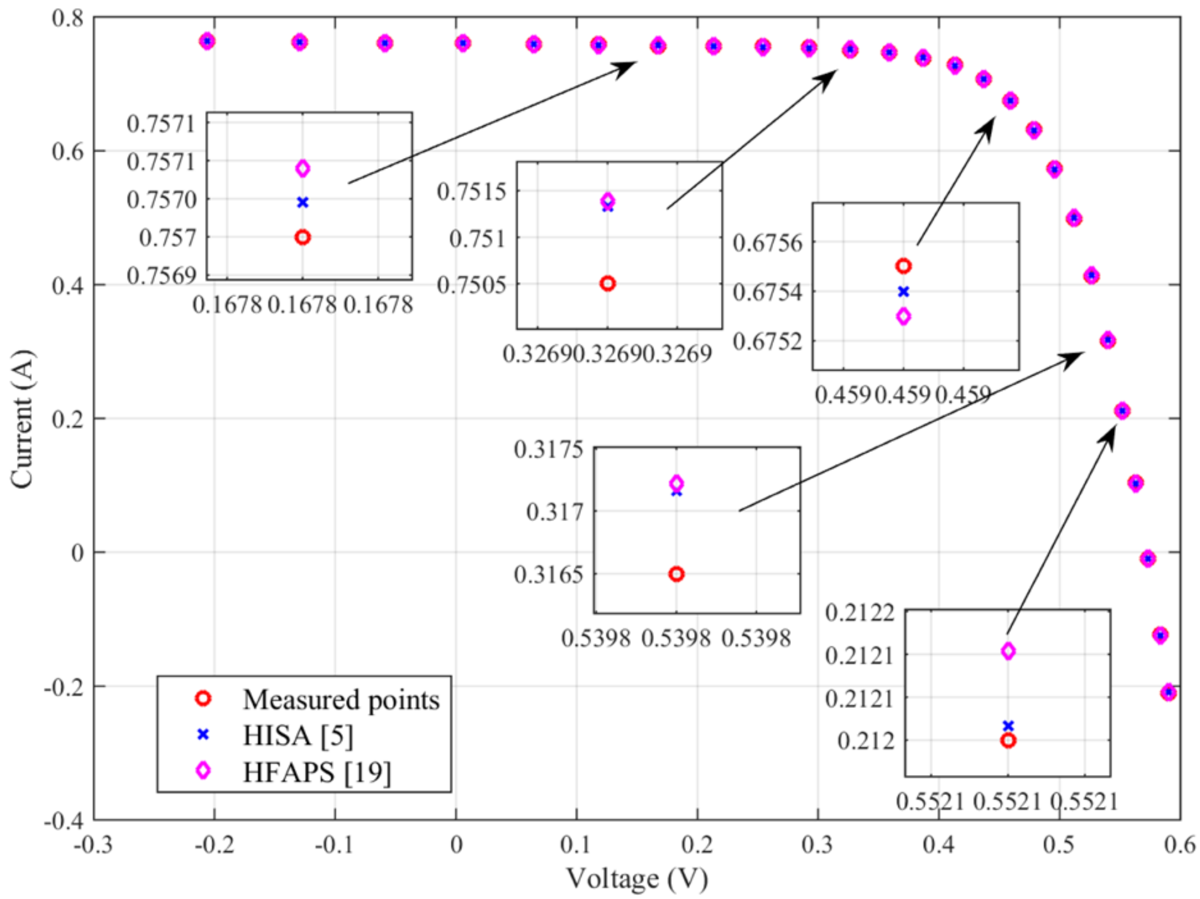


Fig. 5. Current-voltage characteristic of the RTC France solar cell

In order to show that the proposed method provides the best response, the characteristics of parameters determined using the HFAPS method (number 16) presented in [19], which also has good accuracy, is shown in the same figure. It is evident from the figure that a lower value of RMSE guarantees better matching between measured and estimated characteristics. The Matlab code for calculating *I–U* characteristics of the RTC solar cell parameters determined using the HFAPS method, method number 16 in [19], is presented in Appendix 2.

Furthermore, the relative difference between the current calculated using the proposed formula expressed in Eq. (9) and the conventional, erroneous, one expressed in Eq. (3), for the solar cell parameters presented, is shown in Fig. 6. Yet again, it is clear that there is a considerable difference in solar cell current values between the two formulas. Also, it evidences the importance of the proposed RMSE expression to develop a good base for proper investigation and implementation of optimization algorithms to solve the 5-parameter estimation problem of single diode PV equivalent circuits.

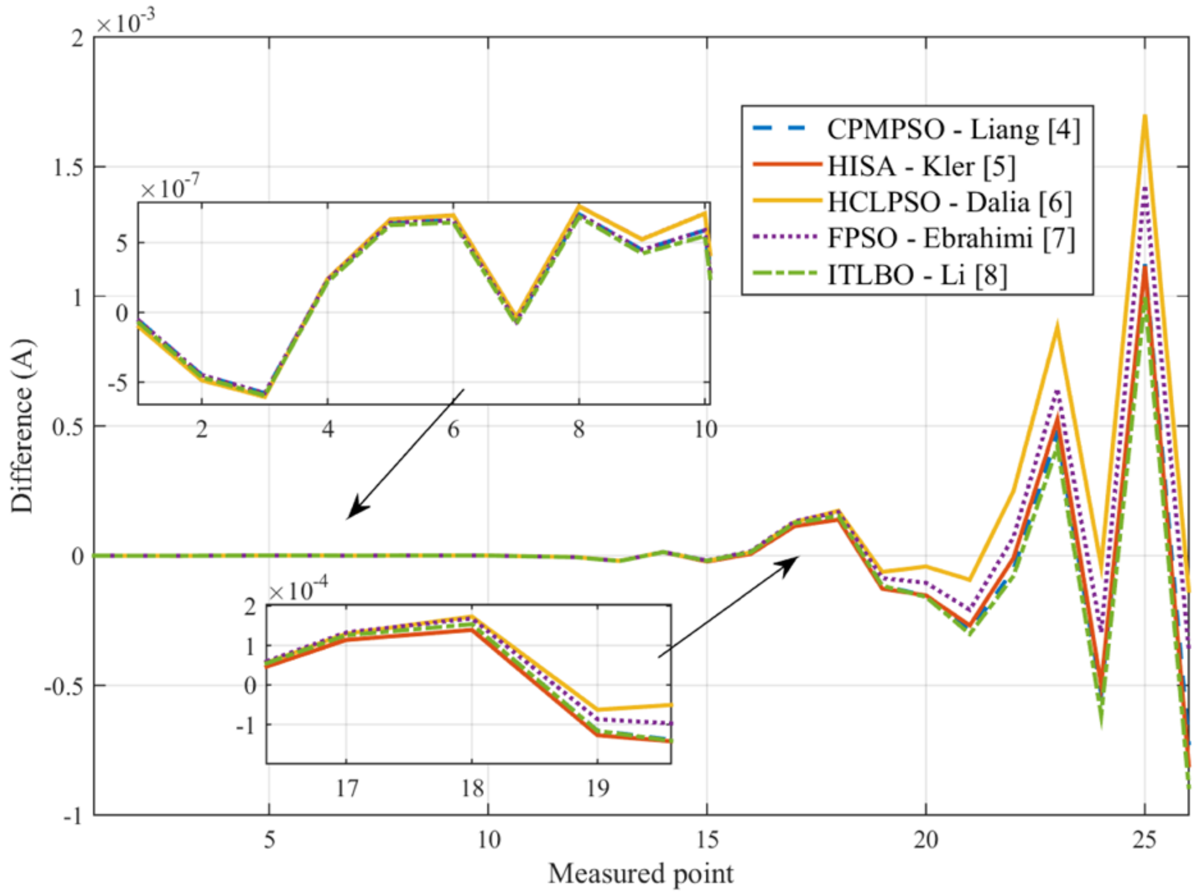


Fig. 6. The difference in current values calculated by Eq. (3) and last part of Eq. (9)

Precision of calculation of the methods used for analytical solving of the Lambert W equation

As mentioned before, for solving a Lambert *W* function, two methods, TS and STFT, are used. An investigation of the impact of β on the accuracy of the solution is presented. Fig. 7 shows a 3D illustration of β, measured points from the *I–U* characteristics of the RTC solar cell, and methods presented in Table 1. From the same figure, one can note that the minimal value of β is 1.46×10^{-9}, while the maximal value of β is 3.52. For that reason, some solutions to the Lambert *W* equation are obtained using TS and others are obtained using STFT. In this section, all calculations are performed in the environment of *Mathematica*.

The precision of calculation *Pr* that reflects accuracy, either based on TS or STFT (Pr_{STFT} and Pr_{Taylor}), is given as follows [72]:

$$Pr = \left|1 - log\left(\left|x - \beta\, exp\left(-x\right)\right|\right)\right| \tag{12}$$

Namely, for the high values of *Pr*, the accuracy of the solution is high. Table 2 presents Pr_{STFT} and Pr_{Taylor} for different values of β.

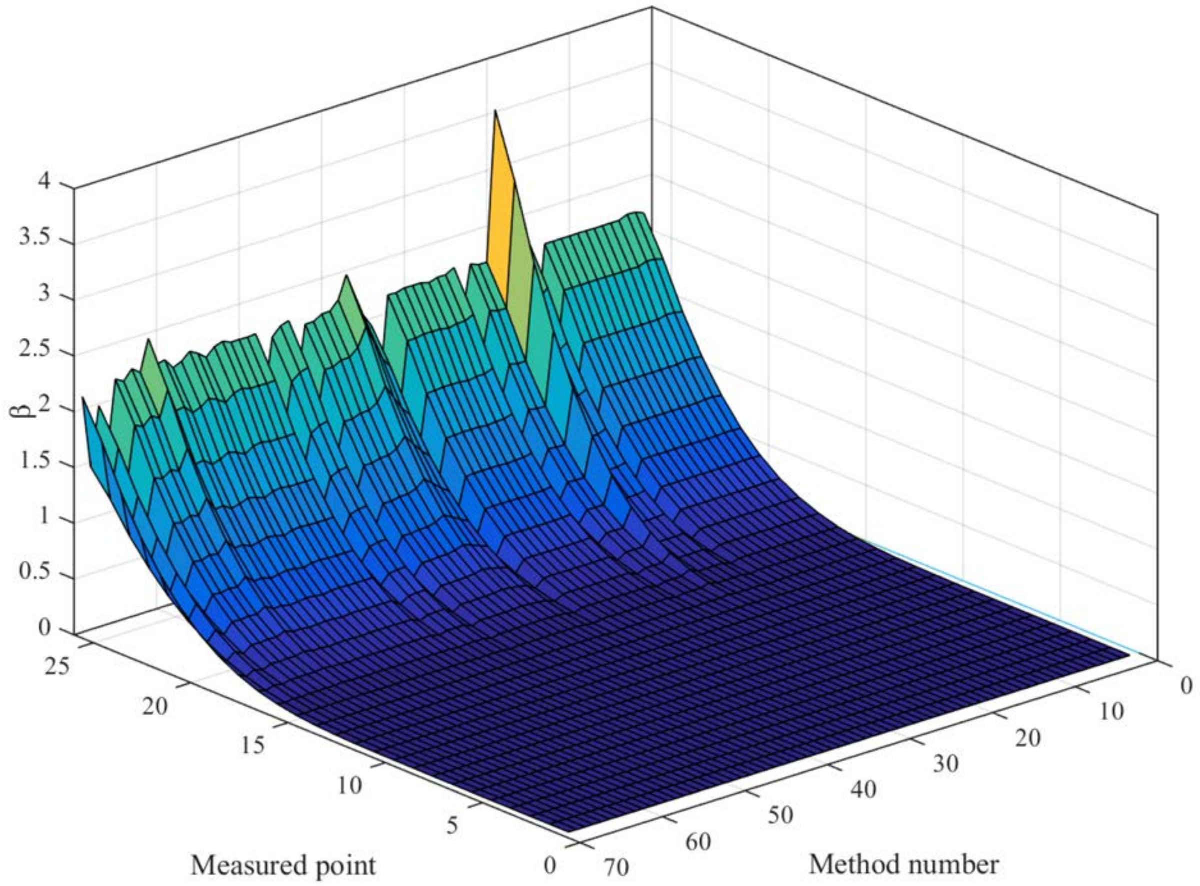


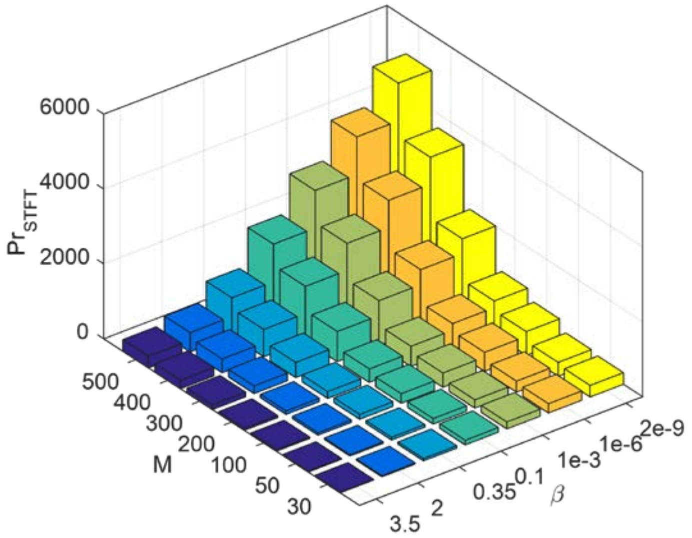
Fig. 7. 3D graph of β, measured points from the *I–U* characteristics of the RTC solar cell, and methods in Table 1

It should be noted that the peak region present in Fig. 7 is due to the parameters obtained using method 19 reported in Table 1 as the values of I_0 and n obtained using method 19 were quite different from the corresponding values obtained using the other methods.

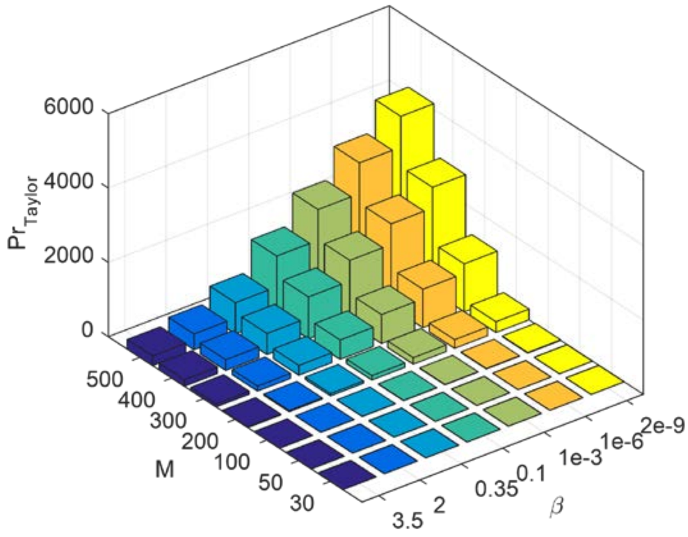
It can be seen that higher precision can be obtained if we use STFT for solving the Lambert *W* equation. Furthermore, for a higher value of β, TS cannot be used. Recalling Eqs. (7) and (8), it should be noted that for the same value of integer M, the STFT gives much more accurate results than TS. Also, the higher the value of M, the higher the accuracy is. The difference between the accuracy of Pr_{STFT} and Pr_{Taylor} for different values of β and M is illustrated in Fig. 8.

Table 2. Pr_{STFT} and Pr_{Taylor} for different values of M and β

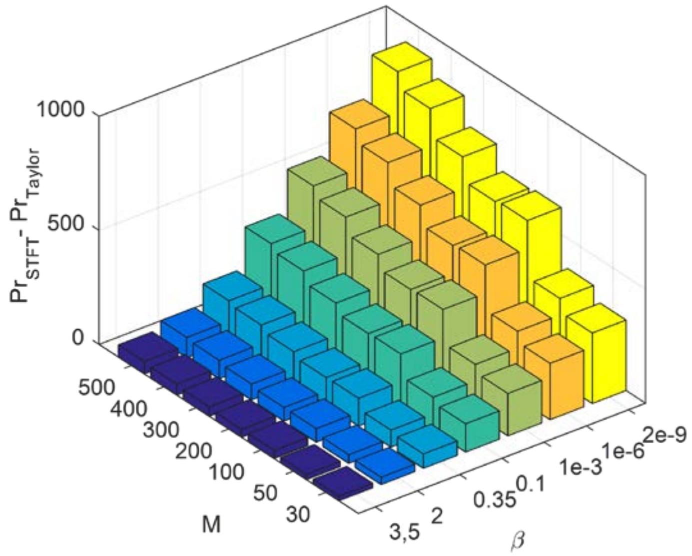
β	Solution, x	M	Pr_{STFT}	Pr_{Taylor}
2×10^{-9}	1.9999999960000000, 1199994253×10^{-9}	30	314	260
		50	515	426
		100	1019	840
		200	2025	1667
		300	3033	2493
		400	4039	3320
1×10^{-6}	9.9999900000149999, 7333338541×10^{-7}	30	225	177
		50	370	288
		100	732	567
		200	1455	1124
		300	2177	1681
		400	2901	2237
		500	3624	2794
1×10^{-3}	0.0009990014973385, 3088995782715	30	125	84
		50	205	135
		100	405	264
		200	804	521
		300	1203	778
		400	1604	1034
		500	2002	1291
0.1	0.0912765271608622, 6429989572142	30	57	22
		50	93	33
		100	182	62
		200	361	119
		300	540	176
		400	719	233
		500	897	289
0.35	0.2677773400403608, 4269261612680	30	40	5
		50	66	6
		100	129	7
		200	255	10
		300	381	12
		400	508	15
		500	634	17
2	0.8526055020137254, 9134647241469	30	23	0
		50	38	0
		100	75	0
		200	148	0
		300	221	0
		400	295	0
		500	368	0
3.5	1.1302893269741358, 2651855880912	30	20	0
		50	32	0
		100	63	0
		200	124	0
		300	186	0
		400	247	0
		500	309	0



(a)



(b)



(c)

Fig. 8. Pr_{STFT} and Pr_{Taylor} for different values of β and M: (a) Pr_{STFT}, (b) Pr_{Taylor}, and (c) $Pr_{STFT} - Pr_{Taylor}$

4. Application of the solution methodology

Experimental setup and parameters estimation of solar modules of clean energy trainer setup

In order to validate the solution methodology, the experimental results of a Clean Energy Trainer Setup module in the Laboratory of automatics at the University of Montenegro are presented and discussed, and the single diode parameters of the considered module are estimated. The experimental setup, shown in Fig. 9, consists of a computer with installed software for data acquisition and analysis, TES 1333R data logging solar power meter with high irradiance (*G*) resolution (0.1 W/m^2), lamp for sunlight simulation, USB data monitor for data acquisition purposes, and two modules of solar cells.

The *I–U* characteristic of one solar module at *G* equals 1330 W/m^{2,} and a temperature of 39 °C is measured. The 5 parameters to be determined are not directly addressed by the current methodology; however, the optimization techniques rely on objective functions whose accuracy can be improved by the proposed RMSE calculation. Then, for the measured *I–U* pairs, the 5-parameter single diode solar cell parameters are estimated using three optimization approaches, namely COA [13], ER-WCA [23], and HS [55,74]. The reader can refer to [13,23,55] for more details about these algorithms. The results obtained are presented in Table 3, in which all the values obtained using Eq. (12) are within their fitness function.

It can be seen from Table 3 that the results obtained using the optimization techniques are close to each other, in which all the values obtained using Eq. (12) are within their fitness function. However, the COA method gives the best accuracy in terms of RMSE. The simulated and measured *I–U* characteristics via the parameters obtained using the different optimization methods at G=1330 W/m^2 and T=39 °C are shown in Fig. 10.

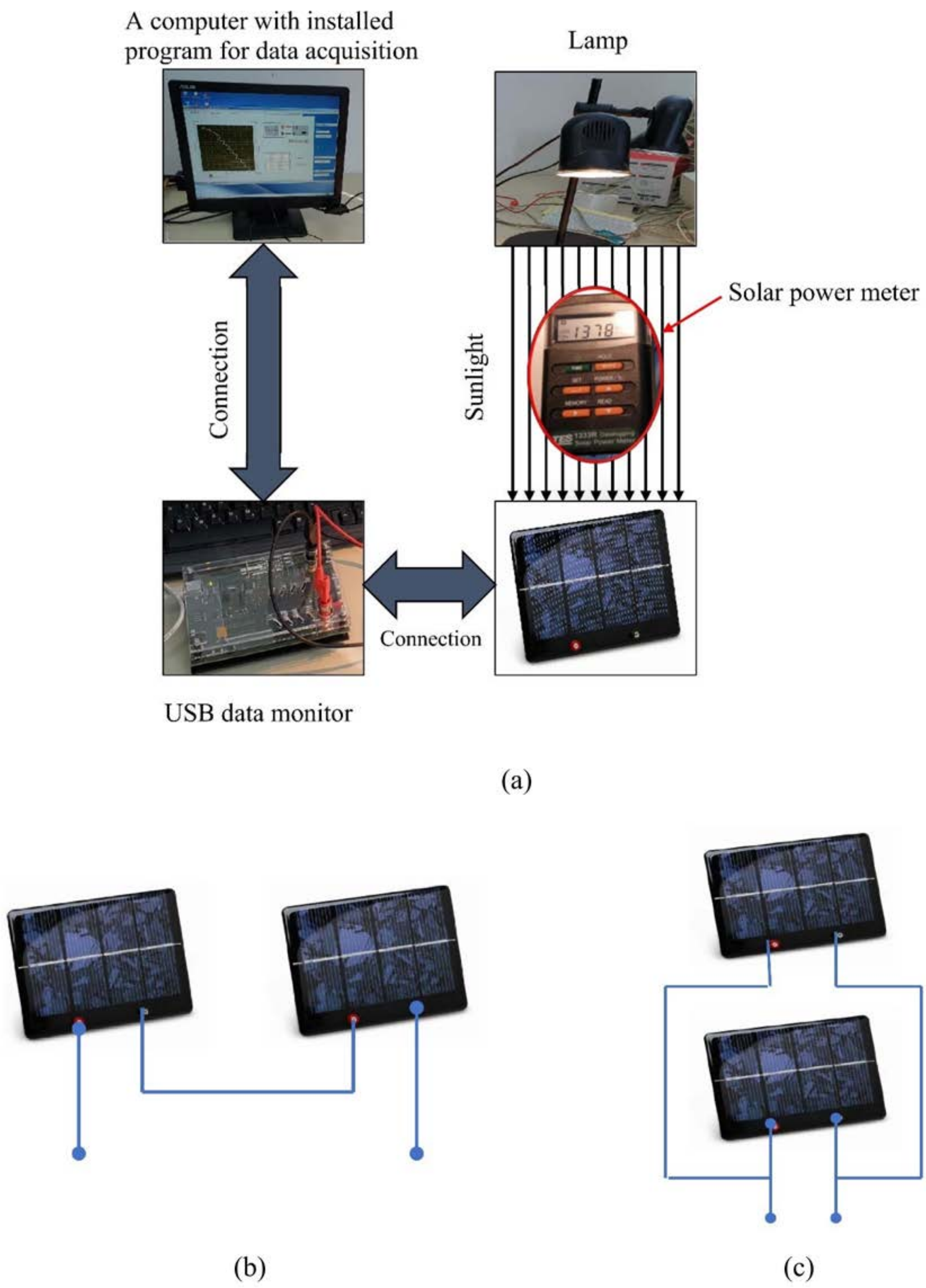


Fig. 9. Experimental setup: (a) Connection, (b) Series connection of solar modules, and (c) Parallel connection of solar modules

Table 3. Experimental results obtained from the tested solar module (Number of runs to achieve the optimum parameters for ER-WCA is 50, and 100 for COA and HS)

Parameters/Algorithm	COA	ER-WCA	HS
R_S (Ω)	0.114	0.121	0.114
R_P (Ω)	219.75	236.10	222.54
I_0 (A)	10.56×10^{-8}	9.13×10^{-8}	12.13×10^{-8}
I_{pv} (A)	0.2987	0.3012	0.3001
n	0.3441	0.3411	0.3456
RMSE calculated using Eq. (3)	0.00152416	0.00543925	0.01303987
Proposed RMSE calculated using Eq. (12)	0.00113727	0.00397174	0.00955046

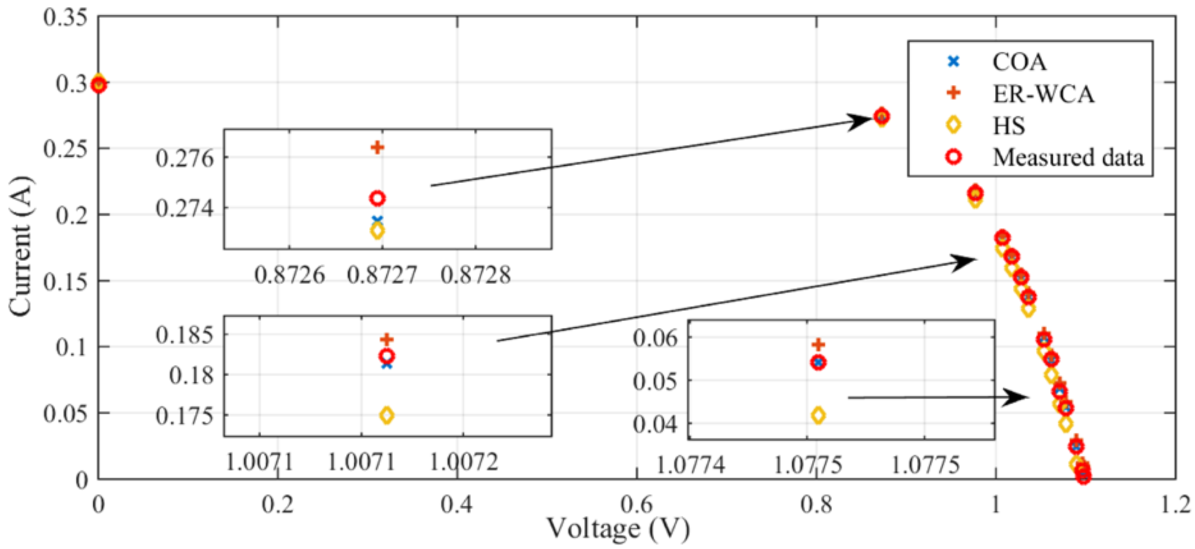
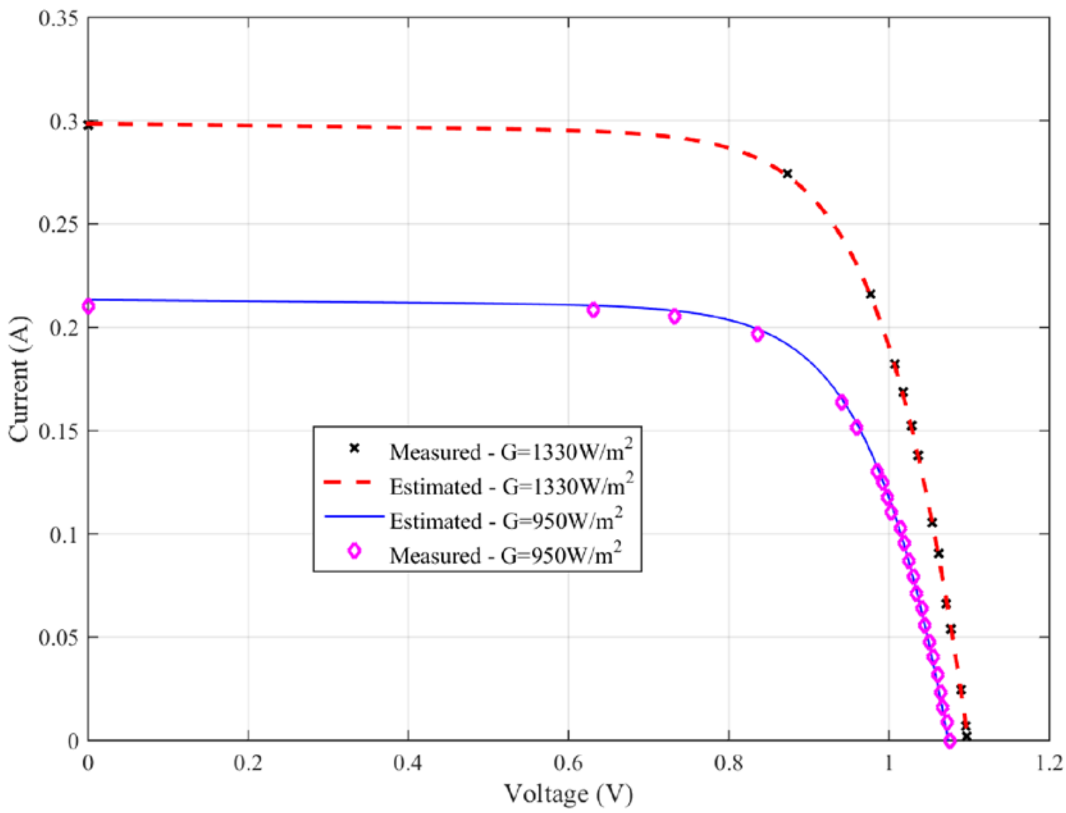


Fig. 10. The *I–U* characteristics of one solar module for parameters obtained using the three methods (G=1330 W/m^2 and T=39 °C)

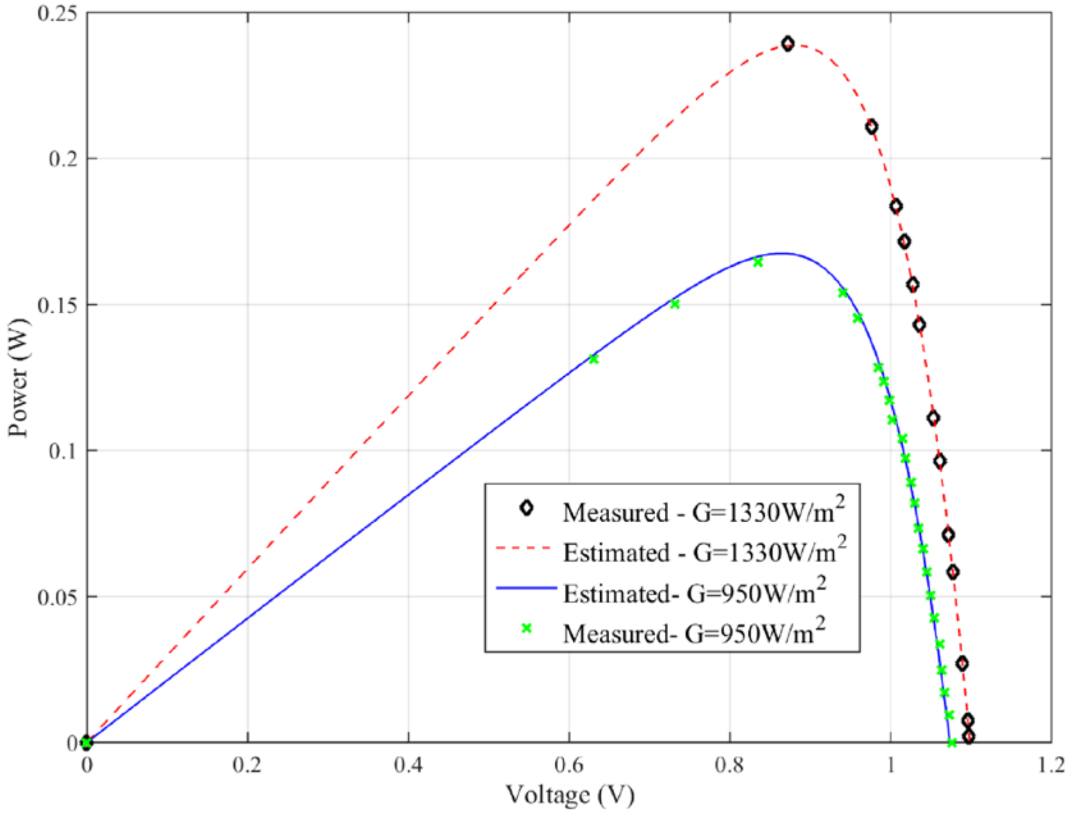
Besides, the *I–U* and *P–U* characteristics of one module at two different irradiance values at the same temperature (T = 39 °C), are presented in Fig. 11, respectively. In addition, the *I–U* and *P–U* characteristics for series and parallel connection of the modules are presented in Fig. 12. The agreement between both measured and estimated characteristics is remarkable for all presented figures. It should be noted that all the characteristics are presented for data obtained using the COA method while taking into consideration the change of parameters with irradiance and temperature [75].

Proposed RMSE expression and parameters estimation of Solarex MSX–60 PV solar module

In Table 4, for Solarex MSX–60 PV solar module, RMSE values calculated using the erroneous Eq. (3) as well as the RMSE values calculated using Eq. (12), are presented. Also, in the same table, the estimated values of the solar cell parameters using COA, ER-WCA, and HS, as well as estimated values of different methods in the literature are presented.

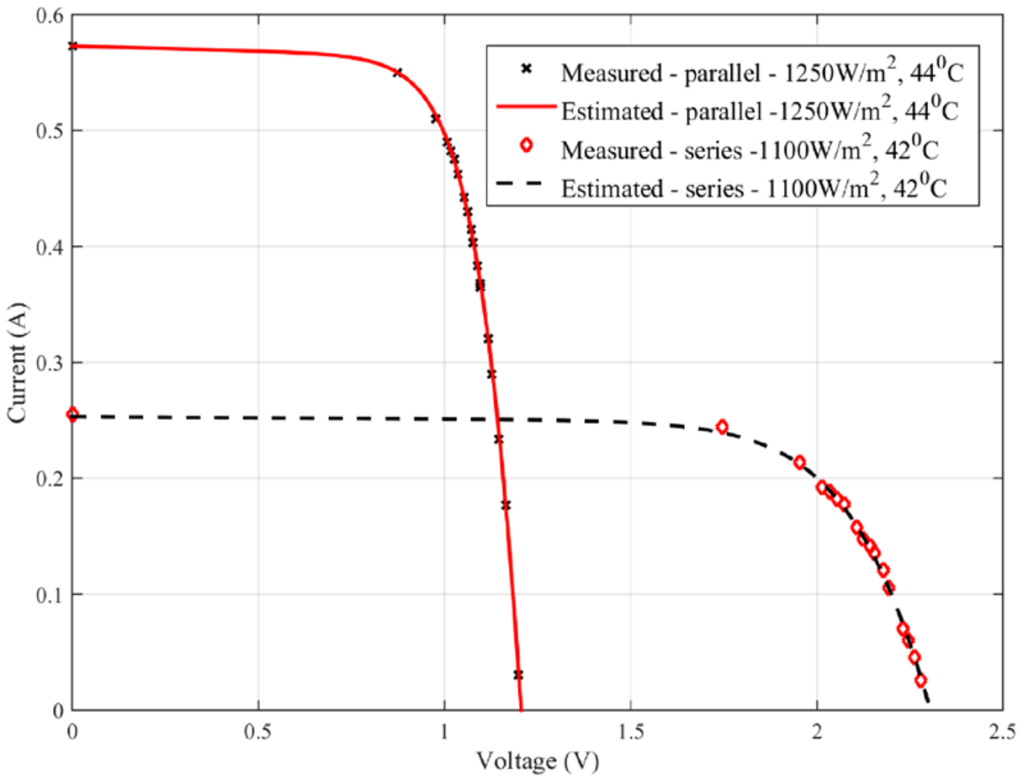


(a)

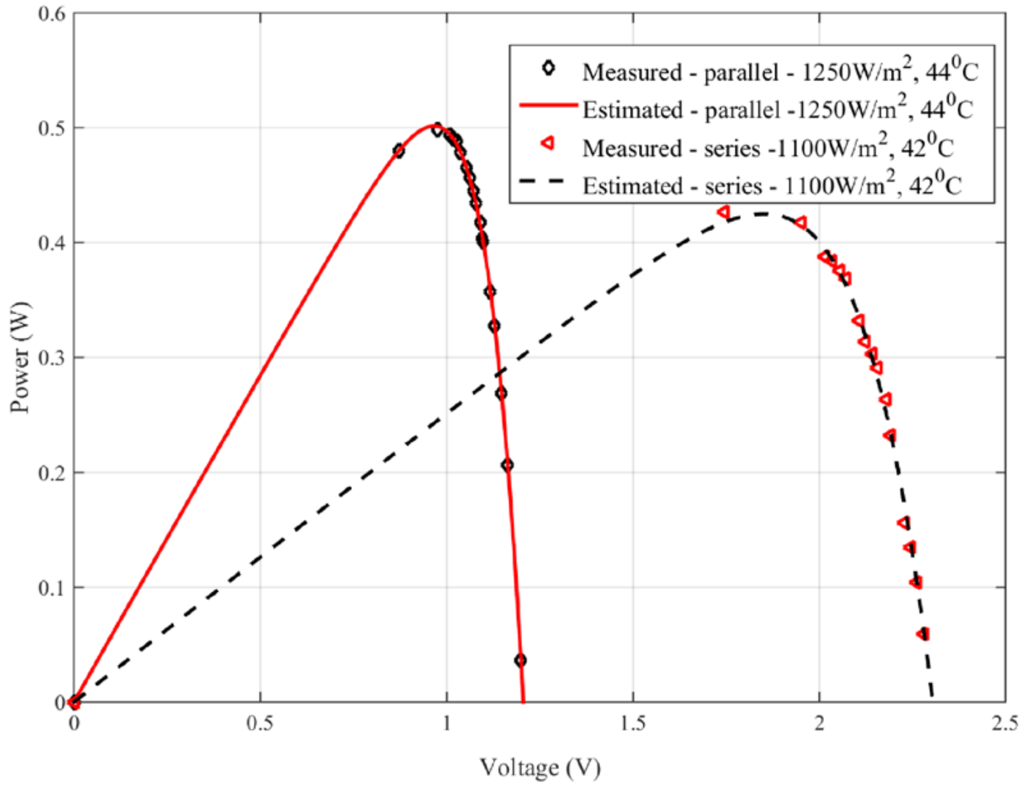


(b)

Fig. 11. The characteristics of one module at two different irradiance values at the same temperature (T = 39 °C): (a) I–U characteristic and (b) P–U characteristic



(a)



(b)

Fig. 12. The characteristics of series and parallel connections of modules: (a) *I–U* characteristic and (b) *P–U* characteristic

Table 4. Numerical results of the conventional and proposed RMSE for parameters estimation of Solarex MSX–60 PV solar module (Results with the minimum RMSE using the proposed approach are given in bold)

#	Ref.	Authors, year	Method	I_{pv} (A)	I_0 (A)	n	R_S (Ω)	R_P (Ω)	RMSE calculated using Eq. (3)	Proposed RMSE calculated using Eq. (12)
1	[76]	Villalva et al., 2009	A&I	3.8082	0.0000000012	1.0453	0.3160	146.081	0.03611840076252	0.02839662736179
2	[77]	Silva et al., 2016	A&I	3.7983	0.0000000679	1.2800	0.2510	582.728	0.02502525341693	0.01810661464843
3	[78]	Bana et al., 2018	NM	3.8084	0.0000000005	1.0003	0.3692	169.047	0.09613294311904	0.05563692889594
4	[79]	Szabo et al., 2018	BC	3.8080	0.0000000012	1.0450	0.3160	146.080	0.04202673823244	0.03072250565403
5	**Proposed**		**ER-WCA**	**3.8121**	**0.0000001399**	**1.3325**	**0.2235**	**914.689**	**0.01697330882512**	**0.01697330882512**
6			HS	3.8115	0.0000002265	1.3707	0.2129	1976.070	0.01775687114902	0.01775687114902
7			COA	3.8100	0.0000001783	1.3514	0.2184	2004.977	0.01705021261682	0.01705021261682

It can be seen from Table 4 that the differences between the proposed and conventional RMSE values are considerable. The *I–U* characteristics of the Solarex MSX-60 module are shown in Fig. 13, in which the agreement between both measured and estimated characteristics is remarkable for all the presented methods. However, it is noted that the results obtained using the ER-WCA method are the closest to the measured values. This is also validated by the proposed RMSE value calculated using Eq. (12) in Table 4, as the minimum RMSE value (0.011706768459604) is obtained using this method (method 5 given in bold in Table 4).

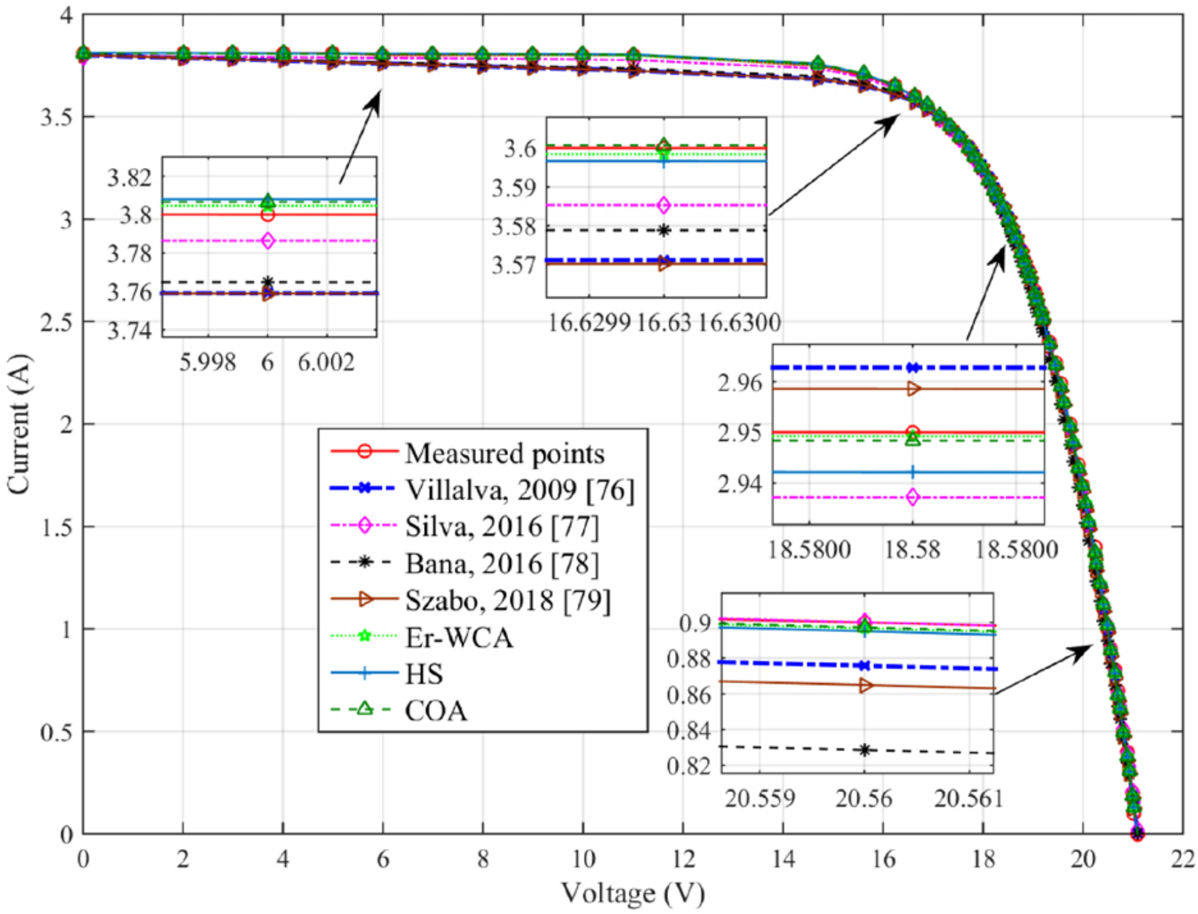


Fig. 13. The *I–U* characteristics of the Solarex MSX-60 module

Furthermore, the relative difference of the current in amperes calculated by Eq. (3) and last part of Eq. (9), for the parameters of Solarex MSX-60 module presented, is shown in Fig. 14. Yet again, it is clear that there is a considerable difference in solar cell current values between the two formulas, in which the difference in the solar cell current values shows the error in the conventional calculation methods since the exact expression of the calculated cell output current is not used. Other promising optimization techniques such as hybrid and improved algorithms [80–83] can be employed to address the problem using the proposed RMSE expression to get better solutions.

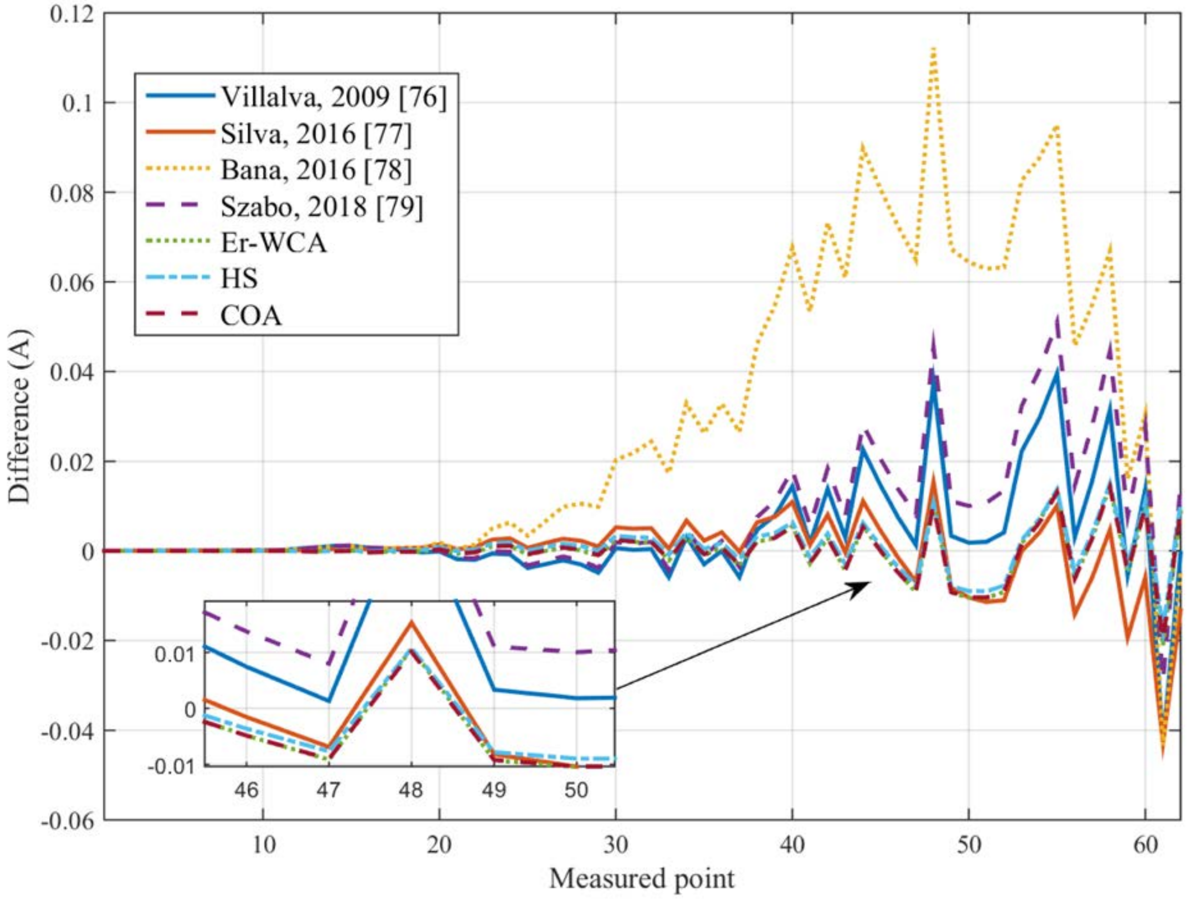


Fig. 14. The difference in current values calculated by Eq. (3) and last part of Eq. (9)

5. Conclusions

In the available literature, one can find a lot of methods and techniques employed to estimate single diode solar PV cell parameters. In this work, values of RMSE of RTC France solar PV cell and Solarex MSX-60 module are presented and discussed. We proposed an exact analytical solution for RMSE calculation based on the Lambert *W* function. The results obtained show that the RMSE values were not calculated correctly in most of the methods presented in the literature since the exact expression of the calculated cell output current was not used. The precision of two numerical approaches to numerically solve the Lambert *W* function is also addressed with: (i) one based on TS; and (ii) the other based on STFT. In addition, the impact of the thermal voltage on RMSE calculation is addressed. Further, the applicability of the proposed solution methodology is explored with the aid of the experimental results. It was found that the reasons for results mismatching of the minimum RMSE in the published papers are not using all the measured points for RMSE calculation, complete dependence on optimization techniques to reach a solution, approximation of the values of the operational factors and inaccurate solving of the Lambert *W* function. In regard to double and triple diode

PV equivalent circuits, no exact analytical solution has been reached yet because of the high nonlinearity of the current expressions of these models. Finally, this work aimed to develop a good base for proper investigation and implementation of optimization algorithms to solve the parameter estimation problem of 5-parameter single diode PV equivalent circuits.

APPENDIX 1

A1.1 Matlab code for solving Lambert *W* equation

The central part of the Matlab code for RMSE calculation based on the Lambert W function is given in Appendix 1 as follows [17,27]:

```
% RTC FRANCE CURRENTS
Iiz=[0.7640 0.7620 0.7605 0.7605 0.7600 0.7590 0.7570 0.7570 0.7555 0.7540 0.7505 0.7465
0.7385 0.7280 0.7065 0.6755 0.6320 0.5730 0.4990 0.4130 0.3165 0.2120 0.1035 -0.0100 -
0.1230 -0.2100];
% RTC FRANCE VOLTAGES
Uiz=[-0.2057 -0.1291 -0.0588 0.0057 0.0646 0.1185 0.1678 0.2132 0.2545 0.2924 0.3269
0.3585 0.3873 0.4137 0.4373 0.4590 0.4784 0.4960 0.5119 0.5265 0.5398 0.5521 0.5633
0.5736 0.5833 0.5900];
% Number of measured points
N=length(Uiz);
 % define Rp, Rs, Vth, a – Ideality factor, Io, Ipv, …
for t=1:N
        BETA=Io*10^-6*Rs*Rp/(a*Vth*(Rs+Rp))*exp((Rp*(Rs*Ipv+Rs*Io*10^-
6+Uiz(t)))/(a*Vth*(Rs+Rp)));
        CHECKING(t)=lambertw(BETA)- BETA *exp(-lambertw(BETA));
        CURRENTCALCULATED=(Rp*(Ipv+Io*10^-6)-Uiz(t))/(Rs+Rp)-
a*Vth*lambertw(BETA)/Rs;
        ERROR(t)=(CURRENTCALCULATED-Iiz(t))^2;
End
RMSE=sqrt(sum(ERROR)/N)
```

A1.2 Mathematica code for solving Lambert *W* equation

```
digitnumber=1000;
beta=35/10;

M=30;
Print["M= ", M];
```

$$F1=\sum_{x=0}^{M}(beta\text{^}x*(((M-x)\text{^}x)/x!));$$

$$F2=\sum_{x=0}^{M+1}(beta\text{^}x*(((M+1-x)\text{^}x)/x!));$$

```
SolutionTRANS = SetPrecision[beta*(F1/F2), digitnumber];
Print["SolutionTRANS= ", SolutionTRANS]
ErrorTRANS=SetPrecision[Abs[SolutionTRANS-beta*E^(-SolutionTRANS)], digitnumber];
Print["ErrorTRANS= ", ErrorTRANS]
```

$$solutionLAMBERT=\sum_{x=1}^{M}(beta\text{^}x*(((-x)\text{^}(x-1))/x!));$$

```
Print["solutionLAMBERT= ", SetPrecision[solutionLAMBERT, digitnumber]];
ErrorLAMBERT=SetPrecision[Abs[solutionLAMBERT-beta*E^(-solutionLAMBERT)],
digitnumber];
Print["ErrorLAMBERT= ", ErrorLAMBERT]
lambertMATH=N[ProductLog[beta],M]
Print["SOLUTIONlambertMATH= ",SetPrecision[lambertMATH, digitnumber]]
ErrorPRODUCT=Abs[lambertMATH-beta*E^(-lambertMATH)];
Print["ErrorPRODUCT= ",SetPrecision[ErrorPRODUCT, digitnumber]]
```

APPENDIX 2

A2. Matlab code for calculating *I–U* characteristics of the RTC solar cell

```
Ipv=0.76077700
Io=0.32262200
a=1.48106000
Rs=0.03638190
Rp=53.67840000
for t=1:N
    BETA=Io*10^-6*Rs*Rp/(a*Vth*(Rs+Rp))*exp((Rp*(Rs*Ipv+Rs*Io*10^-
6+Uiz(t)))/(a*Vth*(Rs+Rp)));
    CHECKING(t)=lambertw(BETA)- BETA *exp(-lambertw(BETA));
    Icalc(t)=(Rp*(Ipv+Io*10^-6)-Uiz(t))/(Rs+Rp)-a*Vth*lambertw(BETA)/Rs;
end
```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