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Planning Water Resources Allocation under Multiple Uncertainties through A Generalized Fuzzy Two-Stage Stochastic Programming Method

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Abstract:
In this study, a generalized fuzzy two-stage stochastic programming (GFTSP) method is developed for planning water resources management systems under uncertainty. The developed GFTSP method can deal with uncertainties expressed as probability distributions, fuzzy sets, as well as fuzzy random variables. With the aid of a robust stepwise interactive algorithm, solutions for GFTSP can be generated by solving a set of deterministic submodels. Furthermore, the possibility information (expressed as fuzzy membership functions) can be reflected in the solutions for the objective function value and decision variables. The developed GFTSP is also applied to a water resources management and planning problem to demonstrate its applicability. Solutions of decision variables and objective function value are expressed as fuzzy membership functions, reflecting the fluctuating ranges of decision alternatives under different plausibilities. Moreover, comparison between solutions obtained through the membership functions derived from GFTSP and interval two-stage stochastic programming (ITSP) method suggests that the possibility information for the objective function value and decision variables is reasonable and robust. And thus the water alternatives can be directly derived from the obtained fuzzy membership functions when the preferred α value is predefined by decision makers.
Index Terms: Decision making, Fuzzy programming, dual-uncertainty, Planning, Water resources

1. Introduction

The availability of fresh water is a fundamental requirement for supporting socio-economic development, poverty reduction, and eco-environmental protection. Globally, demands on water are constantly increasing in terms of both sufficient quantity and satisfied quality, due to growing population, shrinking water availability, varying natural conditions, and deteriorating water quality. Conflicting issues of water resources allocation among competing municipal, industrial and agricultural interests are of increasing concern, forcing planners to contemplate comprehensive, complex and ambitious plans for water resources management systems [4], [35]. However, extensive uncertainties may exist in many system components and impact factors. For example, stream flow in a river is usually related to many meteorological and hydrological factors, and exhibits various uncertain features. Such uncertainties and their interactions can lead to additional complexities in planning efforts and affect consequent decision-making processes. Besides, these uncertainties may be further amplified by the multi-period, multi-layer, and multi-objective features of water systems. Therefore, it is desired that such uncertainties be reflected in efforts for identifying effective water resources management alternatives.

In response to the above concerns, innovative optimization techniques were developed for allocating and managing water in more efficient and environmental benign ways under uncertainty [12 - 19], [24], [39], [50]. Among the methods, two-stage stochastic programming (TSP), as a stochastic optimization method, was widely used for dealing with randomness in water management systems [20 - 22], [30 - 38], [54]. In TSP, a decision is firstly undertaken before values of random variables are known and, then, after the random events have taken place and their values are known, a second decision can be made in order to minimize “penalties” that may appear due to any infeasibility [33]. However, a potential limitation of the TSP is that it is extremely hard to solve a large-scale TSP model with all uncertain parameters being expressed as probability density functions (PDF), while non-PDF information cannot be incorporated within the TSP framework [37].

Fuzzy mathematical programming (FMP), as a branch of fuzzy set theory, was applied to tackle non-probabilistic uncertainties in water resources management [2-6] [23-24], [27-29], [40-45]. For example, Slowinski [49] proposed an interactive fuzzy multiobjective linear programming method and applied it to water supply planning. Chang et al. [3] proposed grey fuzzy multi-objective programming for optimal planning of a reservoir watershed. Bender and Simonovic [2] proposed a fuzzy compromise approach to water resources planning under imprecision uncertainty. Besides, generalized fuzzy linear programming (GFLP) [or fully fuzzy linear programming (FFLP)], as an extension of traditional FMP, exhibited great efficiency in dealing with various uncertainties through permitting uncertain information in the optimization process and resulting solutions [14-15]. Summarily, FMP or GFLP is effective in dealing with decision problems under fuzzy goal and constraints and handling ambiguous coefficients in the objective function and constraints; however, it has difficulties in tackling uncertainties expressed as probabilistic distributions in a non-fuzzy decision space [34].

Although amount of inexact programming methods (e.g. TSP, FMP) has been widely used to address various uncertainties in water resources management, further research is still required on tackling dual or multiple uncertainties stemming from interactions among uncertainties of many system components. In practical water resources management systems, extensive uncertainties exist in many impact factors and system components related to water availability, water demands, and economic coefficients. Some uncertainties can be quantified as probabilities while others may be characterized as fuzzy membership functions. For water management problems with fuzzy or probabilistic uncertainties, FMP, TSP and inexact two-stage fuzzy-stochastic programming methods can be used to tackle these uncertainties [35], [37], [48] [52-56]. However, due to various complexities in hydrological systems, some hydrological variables can hardly be quantified though a simple characterizing approach, such as probabilistic or fuzzy set theory. In other words, such variables may exhibit dual or multiple uncertainties. For example, the future stream flow is subject to numerous uncertainties in hydrological, hydrometeorological, and socio-economic factors, and may present dual or multiple uncertainties. For water systems with dual or multiple uncertainties, the FMP and TSP methods can hardly be applicable. Therefore, more effective approaches are desired to tackle such uncertainties.

Therefore, the objective of this study aims to develop a generalized fuzzy two-stage stochastic (GFTSP) programming method in response to the above challenges. In the GFTSP, techniques of generalized fuzzy linear programming (GFLP) and two-stage stochastic programming (TSP) will be integrated to deal with uncertainties expressed as fuzzy sets and fuzzy random variables (e.g., experiments whose outcomes are considered as fuzzy sets rather than deterministic values). A robust stepwise interactive algorithm (RSIA) will be proposed to solve the GFTSP problem and generate fuzzy solution. Comparison will be conducted between solutions obtained through RSIA and Monte Carlo method through a numerical example. A case study will then be provided for demonstrating how the developed method will support planning for water resources management. The results can help water managers identify desired alternatives for water management with maximized economic objectives.

This paper is organized as follows. In the next section, a generalized fuzzy two-stage stochastic programming (GFTSP) method and the related computational procedures will be introduced and investigated. A hypothetical case study in water resources management will be given in Section 3. Discussions will then be presented in reference to the results presented as membership functions from GFTSP and interval two-stage stochastic programming (ITSP) methods. Conclusions will then be offered in Section 5.

2. Methodology

2.1. Generalized fuzzy linear programming (GFLP) Method

The generalized fuzzy linear programming (GFLP) method was developed to deal with ambiguous coefficients expressed as fuzzy sets in the objective and constraints, and then generate fuzzy solutions. A GFLP model can be formulated as follows:

$$\text{Max } \tilde{f} = \tilde{c} \times \tilde{X} \quad \text{(1a)}$$

Subject to

$$\tilde{A} \times \tilde{X} \le \tilde{b} \quad \text{(1b)}$$

$$\tilde{X} \ge 0 \quad \text{(1c)}$$

where $\tilde{c} \in \{\tilde{R}\}^{1\times n}$, $\tilde{X} \in \{\tilde{R}\}^{n\times 1}$, $\tilde{b} \in \{\tilde{R}\}^{m\times 1}$, $\tilde{A} \in \{\tilde{R}\}^{m\times n}$, $\tilde{R}$ denote a set of fuzzy sets, and $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, ..., \tilde{c}_n)$, $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)^T$, $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, ..., \tilde{b}_n)^T$, $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$, $\forall i \in m,\ j \in n$.

The fuzzy parameters can show partial distributional information characterized as membership functions. For example, fuzzy parameter $\tilde{a}_{ij} = (a_{ij}^l, a_{ij}^m, a_{ij}^u)$ can be presented as a triangular fuzzy set with a_{ij}^l, a_{ij}^m, and a_{ij}^u being its lower bound, mid value, and upper bound, respectively. A fuzzy set (Ã) in X can be defined as a set of ordered pairs of Ã={x, $\mu_{Ã(x)}$| $x \in X$ }, where $\mu_{Ã(x)}$ is membership grade [34], [57]. The $\mu_{Ã(x)}$ value varies between 0 to 1, indicating the possibility that an element x belongs to Ã. The $\mu_{Ã(x)} = 1$ means that x definitely belongs to set A, while $\mu_{Ã(x)} = 0$ denotes that x does not belong to A. The closer $\mu_{Ã(x)}$ is to 1, the more likely x belongs to Ã; conversely, the closer $\mu_{Ã(x)}$ is to 0, the less likely x belongs to Ã [30], [34], [57].

To solve model (1), Fan et al. [14] proposed a stepwise interactive algorithm (SIA) based on the interactive algorithm for solving interval-parameter linear programming (ILP) problems as developed by Huang et al. [21]. However, the interactive algorithm would lead to violation of the best-case constraints when the decision point varies within the generated decision space [11]. Such a weakness would lead to potentially unacceptable solutions. Consequently, in this study, a robust stepwise interactive algorithm (RSIA) will be developed firstly to improve the previous SIA method. The RSIA will improve upon the SIA through incorporation of additional constraints into the solution procedures under each α-cut level. In the RSIA, the principles of fuzzy interval [1], [7-10], [26] will be employed to convert the GFLP problem to a set of ILP subproblems, and then the robust two-step method (RTSM) [11] will be used to solve these ILP subproblems. Compared with SIA, the RSIA can provide more robust solutions. Besides, the RSIA allows uncertain parameters to be transmitted into the optimization process, leading to simple intermediate models. Moreover, through discretization for the range of membership grade into a series of *α-cut* levels, the RSIA can help generate fuzzy interval solutions under each *α-cut* level; consequently, the membership function for each fuzzy variable can be approximated through statistical regression methods.

Since parameters in model (1) are available as fuzzy sets, they should be defuzzified before the model can be solved. A set of α-cut or α -level is one of the most important concepts introduced by Zadeh to establish a bridge between fuzzy set theory and traditional set theory. Each fuzzy set can be uniquely represented by a family of its α-cuts. As stated by Kreinovich [30], fuzzy data processing is computable for α-cuts but, in general, not computable for membership functions. Consequently, the fuzzy parameters and decision variables in model (1) should be defuzzified through the α-cut method instead of directly using their membership functions. Through the α-cut method, fuzzy parameters and decision variables in model (1) , as characterized by convex membership functions in a real number field (*R)*, will be converted into related fuzzy intervals.

For the range of membership grade (i.e. [0, 1]) for parameters $\tilde{c}_j$, $\tilde{x}_j$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ in model (1), we would discretize it into a finite number of *α-cut* levels. Thus, for any $\alpha \in [0, 1]$, the parameters of

$\tilde{c}_j$, $\tilde{x}_j$, $\tilde{a}_{ij}$, and $\tilde{b}_i$ can be denoted as $(\tilde{c}_j)_\alpha = [(c_j)^-_\alpha, (c_j)^+_\alpha]$, $\tilde{x}_j = [(x_j)^-_\alpha, (x_j)^+_\alpha]$, $\tilde{a}_{ij} = [(a_{ij})^-_\alpha, (a_{ij})^+_\alpha]$, and $\tilde{b}_i = [(b_i)^-_\alpha, (b_i)^+_\alpha]$.

With the property of α-cuts (i.e. if $\alpha_1 \geq \alpha_2$ then $a^-_{\alpha_1} \geq a^-_{\alpha_2}$, $a^+_{\alpha_1} \leq a^+_{\alpha_2}$), the examined α-cut levels will be applied to model (1) in sequence, in order to transfer the obtained fuzzy interval solutions (from the previous ILP submodel) to constraints in the forthcoming submodel. Therefore, before α-cut levels are used to defuzzify uncertain parameters, they would firstly be reordered in a sequence of $\alpha_{(1)}$, $\alpha_{(2)}$,…, $\alpha_{(q)}$, where $\alpha_{(1)} \geq \alpha_{(2)} \geq \ldots \geq \alpha_{(q)}$. We would appoint the maximum *α-cut* level ($\alpha_{(1)}$) as the first one to be examined. Then the related ILP submodel can be expressed as follows:

$$\text{Max } (f)^\pm_{\alpha_{(1)}} = \sum_{j=1}^{n} (c_j)^\pm_{\alpha_{(1)}} \times (x_j)^\pm_{\alpha_{(1)}} \tag{2a}$$

Subject to

$$\sum_{j=1}^{n} (a_{ij})^\pm_{\alpha_{(1)}} \times (x_j)^\pm_{\alpha_{(1)}} \leq (b_i)^\pm_{\alpha_{(1)}}, \text{ for } i = 1, 2, \ldots, m \tag{2b}$$

$$(x_j)^\pm_{\alpha_{(1)}} \geq 0 \text{ for } j = 1, 2, \ldots, m \tag{2c}$$

where $(c_j)^\pm_{\alpha_{(1)}}$, $(x_j)^\pm_{\alpha_{(1)}}$, $(a_{ij})^\pm_{\alpha_{(1)}}$, and $(b_i)^\pm_{\alpha_{(1)}}$ are fuzzy intervals under $\alpha_{(1)}$; $(c_j)^\pm_{\alpha_{(1)}} = [(c_j)^-_{\alpha_{(1)}}, (c_j)^+_{\alpha_{(1)}}]$; $(x_j)^\pm_{\alpha_{(1)}} = [(x_j)^-_{\alpha_{(1)}}, (x_j)^+_{\alpha_{(1)}}]$; $(a_{ij})^\pm_{\alpha_{(1)}} = [(a_{ij})^-_{\alpha_{(1)}}, (a_{ij})^+_{\alpha_{(1)}}]$; $(b_i)^\pm_{\alpha_{(1)}} = [(b_i)^-_{\alpha_{(1)}}, (b_i)^+_{\alpha_{(1)}}]$. Fuzzy intervals under other α-cut levels also have similar expressions. Furthermore, an interval number ($a^\pm$) can be defined as: $a^\pm = [a^-, a^+] = \{t \in a \mid a^- \leq t \leq a^+\}$.

Model (2) shows the formulation of interval-parameter linear programming (ILP) method with all parameters expressed as interval numbers. The ILP problem was developed through introducing the concept of interval analysis into a linear programming framework. If we suppose $(c_j)^\pm_{\alpha_{(1)}} \geq 0$ (for j = 1, 2, …, k) and $(c_j)^\pm_{\alpha_{(1)}} \leq 0$ (for $j = k + 1$, …, n), then according to the robust two-step method (RTSM) proposed by [11], model (1) can be transformed into two linear programming submodels. These two submodels with deterministic parameters would be solved to obtain interval solutions of model (2). In detail, the first submodel corresponding to $(f)^-_{\alpha_{(1)}}$ can be formulated as:

$$\text{Max } (f)^-_{\alpha_{(1)}} = \sum_{j=1}^{k} (c_j)^-_{\alpha_{(1)}} (x_j)^-_{\alpha_{(1)}} + \sum_{j=k+1}^{n} (c_j)^-_{\alpha_{(1)}} (x_j)^+_{\alpha_{(1)}} \tag{3a}$$

Subject to

$$\sum_{j=1}^{k} Sign((a_{ij})^\pm_{\alpha_{(1)}}) \left|(a_{ij})_{\alpha_{(1)}}\right|^+ (x_j)^-_{\alpha_{(1)}} + \sum_{j=k+1}^{n} Sign((a_{ij})^\pm_{\alpha_{(1)}}) \left|(a_{ij})_{\alpha_{(1)}}\right|^- (x_j)^+_{\alpha_{(1)}} \leq (b_i)^-_{\alpha_{(1)}}, \ \forall i \tag{3b}$$

$$(x_j)^\pm_{\alpha_{(1)}} \geq 0, \forall j \tag{3c}$$

Hence, solutions of $(x_{jopt})^-_{\alpha_{(1)}}$ (j = 1, 2, …, k) and $(x_{jopt})^+_{\alpha_{(1)}}$ (j = k+1, …, n) can be solved from submodel (3). Then the second submodel corresponding to $(f)^+_{\alpha_{(1)}}$ can be formulated based on the solutions from the first submodel, which can be expressed as follows:

$$\text{Max}\,(f)^+_{\alpha_{(1)}} = \sum_{j=1}^{k}(c_j)^+_{\alpha_{(1)}}(x_j)^+_{\alpha_{(1)}} + \sum_{j=k+1}^{n}(c_j)^+_{\alpha_{(1)}}(x_j)^-_{\alpha_{(1)}} \tag{4a}$$

Subject to

$$\sum_{j=1}^{k} Sign((a_{ij})^\pm_{\alpha_{(1)}})\left|(a_{ij})_{\alpha_{(1)}}\right|^-(x_j)^+_{\alpha_{(1)}} + \sum_{j=k+1}^{n} Sign((a_{ij})^\pm_{\alpha_{(1)}})\left|(a_{ij})_{\alpha_{(1)}}\right|^+(x_j)^-_{\alpha_{(1)}} \le (b_i)^+_{\alpha_{(1)}},\ \forall i \tag{4b}$$

$$\sum_{j=1}^{l_{i1}}(a_{ij})^-_{\alpha_{(1)}}(x_j)^+_{\alpha_{(1)}} + \sum_{j=l_{i1}+1}^{k}(a_{ij})^-_{\alpha_{(1)}}(x_{jopt})^-_{\alpha_{(1)}} + \sum_{j=k+1}^{l_{i2}}(a_{ij})^-_{\alpha_{(1)}}(x_j)^-_{\alpha_{(1)}} + \sum_{j=l_{i2}+1}^{n}(a_{ij})^-_{\alpha_{(1)}}(x_{jopt})^+_{\alpha_{(1)}} \le (b_i)^+_{\alpha_{(1)}} \tag{4c}$$

$$(x_j)^+_{\alpha_{(1)}} \ge (x_{jopt})^-_{\alpha_{(1)}},\ j = 1, 2,\ldots, k \tag{4d}$$

$$(x_j)^-_{\alpha_{(1)}} \le (x_{jopt})^+_{\alpha_{(1)}},\ j = k+1, k+2,\ldots n \tag{4e}$$

$$(x_j)^\pm_{\alpha_{(1)}} \ge 0,\ \forall j \tag{4f}$$

where $(a_{ij})^\pm_{\alpha_{(1)}} \ge 0$ (j = 1, 2, …, l_{i1}; j = l_{i2} + 1, …, n) and $(a_{ij})^\pm_{\alpha_{(1)}} \le 0$ (j = l_{i1} + 1, …, k, k + 1, …, l_{i2}); $(x_{jopt})^-_{\alpha_{(1)}}$ (j = 1, 2, …, k) and $(x_{jopt})^+_{\alpha_{(1)}}$ (j = k + 1, k + 2, …, n) are optimal solutions for model (3).

Models (3) and (4) are conventional linear programming model which can be solved through commercial software (e.g. Lingo and Matlab). In this study, we employed Lingo to solve them. Through solving model (4), solutions of $(x_{jopt})^+_{\alpha_{(1)}}$ (j = 1, 2,…, k) and $(x_{jopt})^-_{\alpha_{(1)}}$ (j = k + 1, k + 2, …, n) can be obtained. Therefore, the final solutions for the model (2) can be obtained as follows:

$$(x_{jopt})^\pm_{\alpha_{(1)}} = [(x_{jopt})^-_{\alpha_{(1)}}, (x_{jopt})^+_{\alpha_{(1)}}] \tag{5a}$$

$$(f_{opt})^\pm_{\alpha_{(1)}} = [(f_{opt})^-_{\alpha_{(1)}}, (f_{opt})^+_{\alpha_{(1)}}] \tag{5b}$$

After the ILP subproblem under $\alpha_{(1)}$ has been solved, the subproblem under $\alpha_{(2)}$ can be solved through incorporating the solutions under $\alpha_{(1)}$ as the constraints. These constraints are applied to ensure the final GFLP problem can generate feasible fuzzy membership functions for decision variables. Consequently, the ILP subproblems under other α-cut levels can be solved similarly. Therefore, considering $\alpha_{(2)}$ to $\alpha_{(q)}$ in sequence, a total of (q – 1) ILP submodels can be formulated as follows:

$$\text{Max}\ (f)^\pm_{\alpha_{(l)}} = \sum_{j=1}^{n}(c_j)^\pm_{\alpha_{(l)}} \times (x_j)^\pm_{\alpha_{(l)}} \tag{6a}$$

Subject to

$$\sum_{j=1}^{n}(a_{ij})^\pm_{\alpha_{(l)}} \times (x_j)^\pm_{\alpha_{(l)}} \le (b_i)^\pm_{\alpha_{(l)}},\ \text{for } i = 1, 2,\ldots, m \tag{6b}$$

$(x_j)^{\pm}_{\alpha(l)} \geq 0$ for $j = 1, 2, \ldots, m$ (6c)

$(x_j)^{\pm}_{\alpha(l)} \subseteq (x_{jopt})^{\pm}_{\alpha(l-1)}$ (6d)

where $\alpha_{(l)} \in \{\alpha_{(2)}\ldots, \alpha_{(q)}\}$ and $(x_{jopt})^{\pm}_{\alpha(l-1)}$ are the optimal solution obtained from the $(l\text{-}1)^{\text{th}}$ ILP model.

Solve model (6) in the sequence of $l = 2$ to q. Then, two submodels can be obtained based on RTSM to solve model (6) under each $\alpha_{(l)}$:

Submodel 1

$$\text{Max } (f)^{-}_{\alpha(l)} = \sum_{j=1}^{k}(c_j)^{-}_{\alpha(l)}(x_j)^{-}_{\alpha(l)} + \sum_{j=k+1}^{n}(c_j)^{-}_{\alpha(l)}(x_j)^{+}_{\alpha(l)} \tag{7a}$$

Subject to

$$\sum_{j=1}^{k} Sign((a_{ij})^{\pm}_{\alpha(l)})\left|(a_{ij})_{\alpha(l)}\right|^{+}(x_j)^{-}_{\alpha(l)} + \sum_{j=k+1}^{n} Sign((a_{ij})^{\pm}_{\alpha(l)})\left|(a_{ij})_{\alpha(l)}\right|^{-}(x_j)^{+}_{\alpha(l)} \leq (b_i)^{-}_{\alpha(l)},\ \forall i \tag{7b}$$

$$(x_j)^{-}_{\alpha(l)} \leq (x_{jopt})^{-}_{\alpha(l-1)},\ j = 1, 2, \ldots, k \tag{7c}$$

$$(x_j)^{+}_{\alpha(l)} \geq (x_{jopt})^{+}_{\alpha(l-1)},\ j = k+1, k+2, \ldots, n \tag{7d}$$

$$(x_j)^{\pm}_{\alpha(1)} \geq 0,\ \forall j \tag{7e}$$

Submodel 2

$$\text{Max} (f)^{+}_{\alpha(l)} = \sum_{j=1}^{k}(c_j)^{+}_{\alpha(l)}(x_j)^{+}_{\alpha(l)} + \sum_{j=k+1}^{n}(c_j)^{+}_{\alpha(l)}(x_j)^{-}_{\alpha(l)} \tag{8a}$$

Subject to

$$\sum_{j=1}^{k} Sign((a_{ij})^{\pm}_{\alpha(l)})\left|(a_{ij})_{\alpha(l)}\right|^{-}(x_j)^{+}_{\alpha(l)} + \sum_{j=k+1}^{n} Sign((a_{ij})^{\pm}_{\alpha(l)})\left|(a_{ij})_{\alpha(l)}\right|^{+}(x_j)^{-}_{\alpha(l)} \leq (b_i)^{+}_{\alpha(l)},\ \forall i \tag{8b}$$

$$\sum_{j=1}^{l_{i1}}(a_{ij})^{-}_{\alpha(l)}(x_j)^{+}_{\alpha(l)} + \sum_{j=l_{i1}+1}^{k}(a_{ij})^{-}_{\alpha(l)}(x_{jopt})^{-}_{\alpha(l)} + \sum_{j=k+1}^{l_{i2}}(a_{ij})^{-}_{\alpha(l)}(x_j)^{-}_{\alpha(l)} + \sum_{j=l_{i2}+1}^{n}(a_{ij})^{-}_{\alpha(l)}(x_{jopt})^{+}_{\alpha(l)} \leq (b_i)^{+}_{\alpha(l)} \tag{8c}$$

$$(x_j)^{+}_{\alpha(l)} \geq (x_{jopt})^{-}_{\alpha(l)},\ j = 1, 2, \ldots, k \tag{8d}$$

$$(x_j)^{-}_{\alpha(l)} \leq (x_{jopt})^{+}_{\alpha(l)},\ j = k+1, k+2, \ldots n \tag{8e}$$

$$(x_j)^{+}_{\alpha(l)} \geq (x_{jopt})^{+}_{\alpha(l-1)},\ j = 1, 2, \ldots, k \tag{8f}$$

$$(x_j)^{-}_{\alpha(l)} \leq (x_{jopt})^{-}_{\alpha(l-1)},\ j = k+1, k+2, \ldots, n \tag{8g}$$

$$(x_j)^{\pm}_{\alpha(1)} \geq 0,\ \forall j \tag{8h}$$

From submodels (7) and (8), we can obtain the final solutions for model (6):

$$(x_{jopt})^{\pm}_{\alpha(l)} = [(x_{jopt})^{-}_{\alpha(l)}, (x_{jopt})^{+}_{\alpha(l)}] \tag{9a}$$

$$(f_{opt})^{\pm}_{\alpha(l)} = [(f_{opt})^{-}_{\alpha(l)}, (f_{opt})^{+}_{\alpha(l)}] \tag{9b}$$

Based on formulas (2) to (9), we can obtain a series of fuzzy interval solutions for model (1) under different *α-cut* levels. Then we can approximately generate the membership function for every decision variable through statistical regression, based on the obtained fuzzy intervals. In this step, each GFLP model is considered as one experiment. The selected *α-cut* levels are its inputs (i.e., independent variables) and the lower and upper bounds of the decision variables and objective function are its outputs (i.e., dependent variables); *q α-cut* levels mean that the experiment will be conducted for *q* times, and *q* groups of solutions will be obtained. Finally, we can approximate membership functions for the decision variables through a regression method.

2.2. Generalized fuzzy two-stage stochastic programming (GFTSP) method

The developed generalized fuzzy linear programming method [i.e. model (1)] is effective in dealing with uncertainties (presented as fuzzy sets) that exist in coefficients of the objective function and constraints, and can generate solutions expressed as fuzzy sets. However, it can hardly tackle uncertainties expressed as random variables in a non-fuzzy decision space [23]; moreover, it is lack of linkage to economic consequences of violated policies as pre-regulated by authorities through taking recourse actions in order to correct any infeasibilities [35]. Two-stage stochastic programming (TSP) method is an effective method to deal with recourse problems where analysis of policy scenarios is desired and the related data are mostly random. In TSP, the initial action is called the first-stage decision, and the corrective one is named the second-stage decision. The first-stage decisions must be made before random events are known; subsequently, after random events have happened and their values are known, the second-stage decisions should be made so as to optimize the objective [39]. Generally, a two-stage stochastic linear programming model can be formulated as follows [31, 46-47]:

$$Max\ f = CX + E[Q(X, \omega)] \tag{10a}$$

Subject to

$$AX \leq B \tag{10b}$$

$$X \geq 0 \tag{10c}$$

where X is the first-stage decision vector made before the random variable are observed, and $Q(X, \omega)$ is the optimal value of the second-stage problem [47]:

$$Min\ q(Y, \omega) \tag{11a}$$

Subject to

$$W(\omega)y + T(\omega)x \leq h(\omega) \tag{11b}$$

$$y \geq 0 \tag{11c}$$

where y is the second-stage decision vector; $q(y, \omega)$ denotes the cost function in the second-stage problem; $\{ W(\omega), T(\omega), h(\omega) | \omega \in \Omega \}$ contains the data of the second-stage problem, and are the function of the random vector ω.

For given values of the first-stage variables (X), the second-stage problem can be decomposed into independent linear subproblems, with one subproblem for each realization of the uncertain parameters [31]. Assume that random vector ω has a finite number of possible realizations, $\Omega = \{\omega_1, \omega_1, \ldots, \omega_v\}$, with respective probabilities $p_1, p_2, \ldots, p_v$, the expectation of the second-stage optimization problem can be written as:

$$E[Q(X,\omega)] = \sum_{h=1}^{v} p_h Q(X,\omega_h) \tag{12}$$

Furthermore, the two-stage stochastic problem (10) can be formulated as one large linear programming problem:

$$Max\ f = C_{T_1} X + \sum_{h=1}^{v} p_h D_{T_2} Y \tag{13a}$$

Subject to

$$A_r X \le B_r \text{ , } r = 1, 2, \ldots, m_1 \tag{13b}$$

$$A_t X + A_t^{'} Y \le \omega_{th} \text{ , } t = 1, 2, \ldots, m_2;\ h = 1, 2, \ldots, v \tag{13c}$$

$$x_j \ge 0, x_j \in X \text{ , } j = 1, 2, \ldots, n_1 \tag{13d}$$

$$y_{jh} \ge 0, y_{jh} \in Y \text{ , } j = 1, 2, \ldots, n_2;\ h = 1, 2, \ldots, v \tag{13e}$$

where p_h is the probability of occurrence for scenario h, with $p_h > 0$ and $\sum_{h=1}^{v} p_h = 1$; C_{T_1} are coefficients of first-stage variables (X) in the objective function; D_{T_2} are coefficients of recourse variables (Y) in the objective function; A_r and A_t are coefficients of X in constraints r and t; $A_t^{'}$ are coefficients of Y in constraints t; ω_{th} is random variables of constraints t, which is associate with probability level p_h.

Obviously, model (13) can tackle uncertainties in the right-hand sides with probabilistic specifications for random variables. However, in many real-world problems, the quality of information on uncertainty is often not precise enough to be presented as PDFs; moreover, a large TSP model with all uncertain parameters being expressed as PDFs is extremely hard to be solved, even if their functions are available [22]. In fact, many parameters are estimated subjectively by experts due to data unavailability and, thus, are frequently expressed as fuzzy sets. Besides, some parameters may be highly uncertain and can hardly be presented by merely one type of presentation. For example, in water resources management problems, uncertainties may exist extensively, including left- and right-hand sides of the constraints and coefficients in the objective function. Some uncertainties may be characterized as random variables; at the same time, some random events cannot be estimated as deterministic values (and can only be quantified as fuzzy sets), leading to dual uncertainties presented in different formats in the uncertain system's components. Therefore, when multiple forms of uncertainties (e.g. fuzzy and stochastic parameters) exist in a model, one potential approach for handling them is to incorporate techniques of GFLP and TSP into one framework. This leads to a generalized fuzzy two-stage stochastic linear programming (GFTSP) method as follows:

$$Max\ \tilde{f} = \tilde{C}_{T_1} \tilde{X} + \sum_{h=1}^{v} p_h \tilde{D}_{T_2} \tilde{Y} \tag{14a}$$

Subject to

$$\tilde{A}_r \tilde{X} \le \tilde{B}_r \text{ , } r = 1, 2, \ldots, m_1 \tag{14b}$$

$$\tilde{A}_t \tilde{X} + \tilde{A}_t^{'} \tilde{Y} \geq \tilde{w}_{th}, \; t = 1, 2, \ldots, m_2; \; h = 1, 2, \ldots, v \tag{14c}$$

$$\tilde{x}_j \geq 0, \; \tilde{x}_j \in \tilde{X}, \; j = 1, 2, \ldots, n_1 \tag{14d}$$

$$\tilde{y}_{jh} \geq 0, \; \tilde{y}_{jh} \in \tilde{Y}, \; j = 1, 2, \ldots, n_2; \; h = 1, 2, \ldots, v \tag{14e}$$

where $\tilde{w}_{th}$ forms a set of discrete random variables, and the detailed values for these random events are quantified as fuzzy sets.

GFTSP can reflect uncertainties expressed as fuzzy sets, probability distributions and their combinations (fuzzy random variables); its left-hand sides (of constraints) contain fuzzy coefficients, and the right-hand ones present as fuzzy sets or fuzzy random variables. Moreover, GFTSP can also generate solutions presented as fuzzy sets, which can provide both fluctuating ranges and possibilistic distributions; these fuzzy solutions will be more effective in helping decision-makers analyze trade-offs between system benefit and process reliability through comparisons with solutions of ILP model.

A numerical example will be proposed to illustrate the solution process of the developed GFTSP model. Consider a farmer who has a total of 300 acres of land available for growing corn. The planting costs per acre are \$(200, 230, 260) (denoted as $\tilde{\mathrm{C}}$). Here the value of (200, 230, 260) is a triangular fuzzy number with the 200, 230 and 260 being the lower bound, mean value and upper bound, respectively. The farmer needs about (450 500, 550) (denoted as $\tilde{\mathrm{B}}$) tonne of corn for cattle feed which can be grown on the farm or bought from a wholesaler. The price of the corn (in tons) purchased from a wholesaler is \$(250, 275, 300) (expressed as $\tilde{\mathrm{D}}$) per tonne. The yield of the farmland is sensitive to, e.g. weather conditions. Consequently, three scenarios of weather conditions are considered (i.e. bad, average, good), with the probability of 0.2, 0.6, 0.2, (expressed as p_j, j = 1, 2, 3) respectively. The farmer knows that the yield on his land is about (2.0, 2.4, 2.8), (3.0, 3.4, 3.8) (4.0, 4.5, 5.0) (expressed as $\tilde{a}_j$, j = 1, 2, 3) tonne per acre for corn under bad, average, good scenario, respectively. Assume $\tilde{\mathrm{X}}$ to be the amount of areas planting corn and $\tilde{Y}_j$ to be the amount of the corn bought from a wholesaler under different scenarios, then a GFTSP model can be formulated as:

$$\text{Min } \tilde{f} = \tilde{C}\tilde{X} + \sum_{j=1}^{3} p_j \tilde{D}\tilde{Y}_j \tag{15a}$$

Subject to

$$\tilde{X} \leq 300 \tag{15b}$$

$$\tilde{a}_j \tilde{X} + \tilde{Y}_j \geq \tilde{B} \quad j = 1, 2, 3 \tag{15c}$$

$$\tilde{X}, \tilde{Y}_j \geq 0 \tag{15d}$$

The developed robust interactive algorithm (RSIA) will be applied to solve model (15). Based on RSIA, six *α-cut* (i.e. α = 0, 0.3, 0.5, 0.7, 0.9, 1) values are firstly selected to cut the GFTSP model, formulating six corresponding ITSP submodels; the robust two-step method will be employed to solve the ITSP submodel and generate fuzzy interval solution under each *α-cut* level; the membership functions of fuzzy variables and the objective function values will finally approximated through statistical regression methods. Table 1 presents the fuzzy interval solutions under different *α-cut* levels. Take the value of α = 0.5 as an example, the corresponding ITSP submodel under α = 0.5 would be expressed as:

$$\text{Min } (f)^{\pm}_{0.5} = (C)^{\pm}_{0.5}(X)^{\pm}_{0.5} + \sum_{j=1}^{3} p_j (D)^{\pm}_{0.5}(Y_j)^{\pm}_{0.5} \tag{16a}$$

Subject to

$$(X)^{\pm}_{0.5} \le 300 \tag{16b}$$

$$(a_j)^{\pm}_{0.5}(X)^{\pm}_{0.5} + (Y_j)^{\pm}_{0.5} \ge (B)^{\pm}_{0.5} \quad j = 1, 2, 3 \tag{16c}$$

$$(X)^{\pm}_{0.5} \supseteq (X_{opt})^{\pm}_{0.7}, (Y_j)^{\pm}_{0.5} \supseteq (Y_{jopt})^{\pm}_{0.7} \tag{16d}$$

where $(C)^{\pm}_{0.5}$, $(D_j)^{\pm}_{0.5}$, $(a_j)^{\pm}_{0.5}$ $(B)^{\pm}_{0.5}$ are the fuzzy interval under α = 0.5 and $(C)^{\pm}_{0.5}$= [200+(230-200)*0.5, 260-(260-230)*0.5] = [215, 245], $(D)^{\pm}_{0.5}$= [262.5, 287.5], $(a_1)^{\pm}_{0.5}$=[2.2, 2.6], $(a_2)^{\pm}_{0.5}$ = [3.2, 3.6], $(a_3)^{\pm}_{0.5}$= [4.25, 4.75], $(B)^{\pm}_{0.5}$=[475, 525]; $(X_{opt})^{\pm}_{0.7}$ and $(Y_{jopt})^{\pm}_{0.7}$ are the optimal solution under α = 0.5; $(X)^{\pm}_{0.5}$ and $(Y_j)^{\pm}_{0.5}$ are the optimal solutions to be solved. The robust two-step method proposed by Fan and Huang [11] will be applied to solve model (16), which will covert model (16) into two submodel as follows:

submodel 1

$$\text{Min } (f)^{+}_{0.5} = 245(X)^{+}_{0.5} + 0.2*287.5(Y_1)^{+}_{0.5} + 0.6*287.5(Y_2)^{+}_{0.5} + 0.2*287.5(Y_3)^{+}_{0.5} \tag{17a}$$

Subject to

$$(X)^{+}_{0.5} \le 300 \tag{17b}$$

$$2.2(X)^{+}_{0.5} + (Y_1)^{+}_{0.5} \ge 525 \tag{17c}$$

$$3.2(X)^{+}_{0.5} + (Y_2)^{+}_{0.5} \ge 525 \tag{17d}$$

$$4.25(X)^{+}_{0.5} + (Y_3)^{+}_{0.5} \ge 525 \tag{17e}$$

$$(X)^{\pm}_{0.5} \supseteq (X_{opt})^{\pm}_{0.7}, (Y_j)^{\pm}_{0.5} \supseteq (Y_{jopt})^{\pm}_{0.7} \tag{17f}$$

Submodel 2

$$Min\,(f)^{-}_{0.5} = 215(X)^{-}_{0.5} + 0.2*262.5(Y_1)^{-}_{0.5} + 0.6*262.5(Y_2)^{-}_{0.5} + 0.2*262.5(Y_3)^{-}_{0.5} \tag{18a}$$

Subject to

$$(X)^{-}_{0.5} \le 300 \tag{18b}$$

$$2.6(X)^{-}_{0.5} + (Y_1)^{-}_{0.5} \ge 475 \tag{18c}$$

$$Max\{(X)_{0.5}, (Y_1)_{0.5} \,|\, 2.2(X)_{0.5} + (Y_1)_{0.5}\} \ge 475 \tag{18d}$$

$$3.6(X)^{-}_{0.5} + (Y_2)^{-}_{0.5} \ge 475 \tag{18e}$$

$$Max\{(X)_{0.5}, (Y_1)_{0.5} \,|\, 3.2(X)_{0.5} + (Y_2)_{0.5}\} \ge 475 \tag{18f}$$

$$4.75(X)^{-}_{0.5} + (Y_3)^{-}_{0.5} \ge 475 \tag{18g}$$

$$Max\{(X)_{0.5},(Y_1)_{0.5} \mid 4.25(X)_{0.5}+(Y_3)_{0.5}\} \geq 475 \quad (18h)$$

$$(X)^{\pm}_{0.5} \supseteq (X_{opt})^{\pm}_{0.7}, (Y_j)^{\pm}_{0.5} \supseteq (Y_{jopt})^{\pm}_{0.7} \quad (18i)$$

where equations (18d), 18(f), (18h) are the equivalent expressions of equations (4c) [11]. From models (17) and (18), the fuzzy interval solutions of model (15) under $\alpha = 0.5$ can be obtained.

Table 1. The solutions of the numerical example under different α-cut levels

α	solutions
1	$(X)^{\pm}_1 = 166.7$, $(Y_1)^{\pm}_1 = 100$, $(Y_2)^{\pm}_1 = (Y_3)^{\pm}_1 = 0$ $(f)^{\pm}_1 = 4.38 \times 10^4$
0.9	$(X)^{\pm}_{0.9} = [163.9, 169.5]$, $(Y_1)^{\pm}_{0.9} = [95.1, 105.1]$, $(Y_2)^{\pm}_{0.9} = (Y_3)^{\pm}_{0.9} = 0$, $(f)^{\pm}_{0.9} = [4.24, 4.53] \times 10^4$
0.7	$(X)^{\pm}_{0.7} = [158.5, 175.2]$, $(Y_1)^{\pm}_{0.7} = [85.6, 115.6]$, $(Y_2)^{\pm}_{0.7} = (Y_3)^{\pm}_{0.7} = 0$, $(f)^{\pm}_{0.7} = [3.96, 4.84] \times 10^4$
0.5	$(X)^{\pm}_{0.5} = [153.2, 181.0]$, $(Y_1)^{\pm}_{0.5} = [76.6, 126.7]$, $(Y_2)^{\pm}_{0.5} = (Y_3)^{\pm}_{0.5} = 0$, $(f)^{\pm}_{0.5} = [3.70, 5.16] \times 10^4$
0.3	$(X)^{\pm}_{0.3} = [148.1, 187.1]$, $(Y_1)^{\pm}_{0.3} = [68.1, 138.4]$, $(Y_2)^{\pm}_{0.3} = (Y_3)^{\pm}_{0.3} = 0$, $(f)^{\pm}_{0.3} = [3.45, 5.51] \times 10^4$
0	$(X)^{\pm}_0 = [140.6, 196.4]$, $(Y_1)^{\pm}_0 = [56.3, 157.1]$, $(Y_2)^{\pm}_0 = (Y_3)^{\pm}_0 = 0$, $(f)^{\pm}_0 = [3.09, 6.05] \times 10^4$

Based on the fuzzy interval solutions under selected *α-cut* levels (as shown in Table 1), the membership functions of fuzzy variables can be generated through statistical regression methods. Fig. 1(a) shows the membership functions of $\tilde{X}$, $\tilde{Y}_1$ and $\tilde{f}$. It is indicated that the membership functions of decision variables can be well fitted based on the fuzzy intervals solutions under a series of *α-cut* levels. To further demonstrate the robustness and efficiency of the proposed method, we compare the solutions obtained through the fuzzy membership function with those obtained based on Monte Carlo method. In this comparison, 100 random *α-cut* levels are generated within a uniform distributed [0, 1] interval. Under each *α-cut* level, an ITSP submodel will be formulated and then generate associated interval solutions. Fig.1(b) shows the comparison between the values of $\tilde{X}$, $\tilde{Y}_1$ and $\tilde{f}$ obtained through the membership functions and those generated by Monte Carlo method. It suggests that, instead of solving the model again, the membership functions obtained through RSIA can be applied directly to generate related solutions of model (15) under any *α-cut* level.

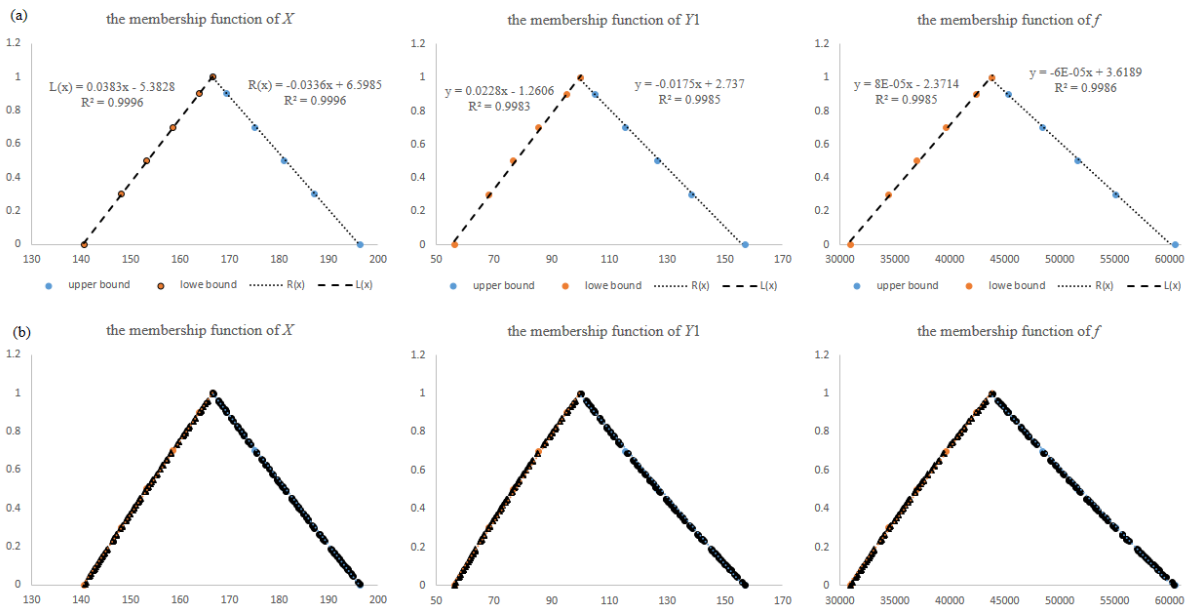

Figure 1. The membership functions and their comparison with Monte Carlo method

A case study of water resources management will be further proposed to illustrate the solution process of the GFTSP method and its applicability to practice problems. A general water resources management system involves several processes with various socio-economic and environmental implications. Extensive uncertainties exist in these processes. They can be sorted into two basic forms: uncertainties caused by inherent hydrologic variability and those due to fundamental lack of knowledge [48]. Fuzzy set theory was developed to capture judgmental belief, or uncertainty that is caused by the lack of knowledge or ambiguity [51]. In terms of the uncertainty derived from inherent variability, one example is hydrological prediction which is subject to numerous uncertainties, such as those generated in developing conceptual, mathematical and numerical models; specifically, some hydrological parameters are characterized by imprecise, vague, inconsistent, incomplete, or subjective information, which is insufficient for constructing reliable probability distributions and thus limits the application of conventional stochastic methods [16]. Therefore, complexities in hydrological systems can lead to difficulties in representing uncertainties associated with hydrological variability through a single characterizing approach based on either probability or fuzzy set theory. In other words, two types of uncertainty, which are fundamentally different from each other, may simultaneously exist in hydrological systems, leading to dual-uncertainties. Fuzzy random variable (FRV), which describes an experiment whose outcomes are considered as fuzzy sets rather than deterministic real values, is a potential alternative to deal with such dual-uncertainties [16-17].

Consider a water resources problem as follows: an authority is charged with delivering water to different sectors to meet demands for regional socio-economic development. The authority promises a range of allocation targets for each user in advance, which can help the users tailor their activities and investment plans. If the promised water is delivered, the net benefit to the local economy will be generated for each unit of water allocated. However, if the promised water is not delivered, then either the water must be obtained from higher priced alternatives or the demand must be curtailed by reduced production, resulting in a reduced net system benefit [4]. Furthermore, when the parameters related to water allocation targets such as economic data, are expressed as fuzzy sets; the hydrological data, such as flow distribution, are represented as fuzzy random variables, then the developed GFTSP method would be effective for dealing with various uncertainties to achieve a maximum benefit. The GFTSP model for water resources management can be formulated as follows:

$$\text{Max } \tilde{\tilde{f}} = \sum_{i=1}^{m}\sum_{k=1}^{T} \tilde{\tilde{NB}}_i \tilde{\tilde{T}}_{ik} - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{T} p_j \tilde{\tilde{C}}_i \tilde{\tilde{D}}_{ijk} \tag{19a}$$

Subject to

$$\tilde{\tilde{q}}_{jk} \geq \sum_{i=1}^{m} (\tilde{\tilde{T}}_{ik} - \tilde{\tilde{D}}_{ijk}) \quad \forall j, k \tag{19b}$$

$$\tilde{\tilde{T}}_{ik\max} \geq \tilde{\tilde{T}}_{ik} \geq \tilde{\tilde{D}}_{ijk} \geq 0 \quad \forall i, j, k \tag{19c}$$

where

Objective function:

$\tilde{\tilde{f}}$ = net system benefit ($/day);

Parameters:

$\tilde{\tilde{NB}}_i$ = net benefit to user i per unit of water allocated ($/unit);

$\tilde{\tilde{T}}_{ik}$ = fixed allocation target for water that is promised to user i (unit/day) in period k (the first-stage decision variable);

$\tilde{\tilde{T}}_{ik\max}$ = maximum allowable allocation amount for user i (unit/day) in period k;

$\tilde{\tilde{C}}_i$ = penalty rate for user i per unit of water not delivered ($\tilde{\tilde{C}}_i > \tilde{\tilde{NB}}_i$) ($/unit);

$\tilde{\tilde{q}}_{jk}$ = the amount of water availability under flow level j (m^3) in period k;

p_j = the probability of occurrence of flow level j;
i = the index of water user;
j = the index of flow level;
m = number of water users;
k = the total number of periods;
n = total number of flow levels.

Decision variables:

$\tilde{\tilde{D}}_{ijk}$ = the amount of shortage corresponding to water-allocation target $\tilde{\tilde{T}}_{ik}$ when the seasonal flow is $\tilde{\tilde{q}}_{jk}$ (m^3) with probability p_j (the second-stage decision variable).

In model (19), the objective (i.e., formula (19a)) is to maximize the net benefit of water supply to multiple users, which will cover both benefit of allocated water and penalty of water shortage. Constraint (19b) specifies that the total amount of water allocated to multiple users must not exceed the water availability. Formula (19c) defines technical constraints of non-negative variables and maximized allocation targets.

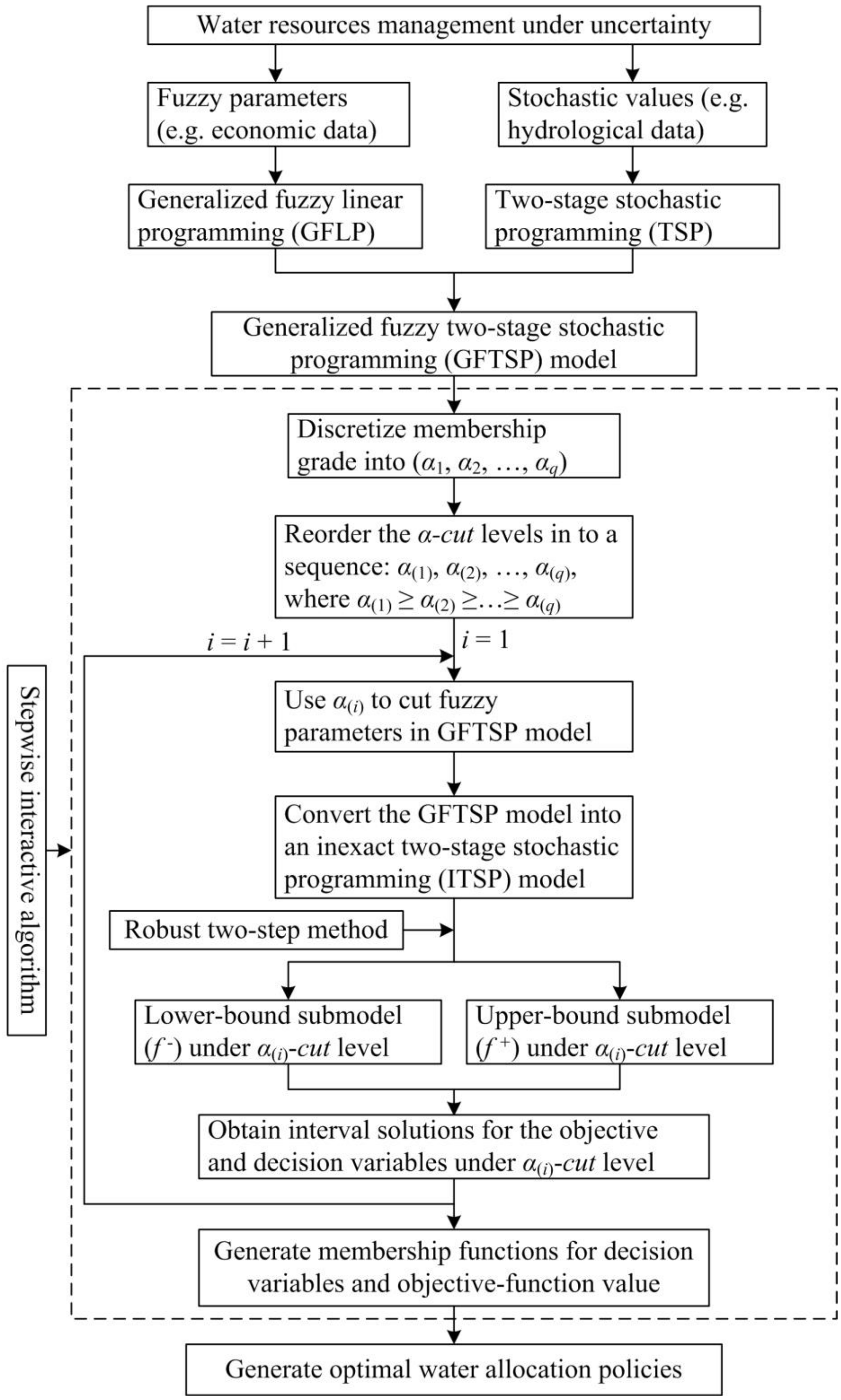

Figure 2. The schematic of the GFTSP model

According to the RSIA method, under each α-*cut* level, model (19) can be converted into an inexact two-stage stochastic programming (ITSP) problem as follows:

$$\text{Max}\,(f)_\alpha^\pm = \sum_{i=1}^{m}\sum_{k=1}^{T}(NB_i)_\alpha^\pm (T_{ik})_\alpha^\pm - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{T} p_j (C_i)_\alpha^\pm (D_{ijk})_\alpha^\pm \tag{20a}$$

Subject to

$$\sum_{i=1}^{m}((T_{ik})_\alpha^\pm - (D_{ijk})_\alpha^\pm) \le (q_{jk})_\alpha^\pm \quad \forall j,k \tag{20b}$$

$$(T_{ik\max})_\alpha^\pm \ge (T_{ik})_\alpha^\pm \ge (D_{ijk})_\alpha^\pm \ge 0 \quad \forall i,\ j,k \tag{20b}$$

In model (20), it is difficult to determine whether $(T_{ik})_\alpha^+$ or $(T_{ik})_\alpha^-$ will correspond to the upper-bound of the system net benefit (i.e. $(f)_\alpha^+$), because $(T_{ik})_\alpha^+$ corresponds to a higher benefit for water allocation but a higher risk of penalties if the promised water is not delivered, while $(T_{ik})_\alpha^-$ is associated with a lower benefit but a lower risk of penalties. Furthermore, if $(T_{ik})_\alpha^\pm$ are considered as uncertain inputs, the existing methods for solving inexact linear programming problems cannot be used directly [33]. Consequently, an optimized set of target values will be identified by having $(y_{ik})_\alpha$ in model (20) as decision variables. This optimized set will correspond to a maximized system benefit under uncertain water-allocation targets [22]. Accordingly, let us consider $(T_{ik})_\alpha^\pm = (T_{ik})_\alpha^- + \Delta(T_{ik})_\alpha (y_{ik})_\alpha$, where $\Delta(T_{ik})_\alpha = (T_{ik})_\alpha^+ - (T_{ik})_\alpha^-$ and $0 \le (y_{ik})_\alpha \le 1$; $(y_{ik})_\alpha$ are decision variables that are used for identifying an optimized set of allocation target values ($T_{ik}^\pm$) in order to support the related policy. Consequently, model (20) can be reformulated as follows:

$$\text{Max}\ (f)_\alpha^\pm = \sum_{i=1}^{m}\sum_{k=1}^{T}(NB_i)_\alpha^\pm[(T_{ik})_\alpha^- + \Delta(T_{ik})_\alpha (y_{ik})_\alpha] - \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{T} p_j (C_i)_\alpha^\pm (D_{ijk})_\alpha^\pm \tag{21a}$$

Subject to

$$\sum_{i=1}^{m}[(T_{ik})_\alpha^- + \Delta(T_{ik})_\alpha (y_{ik})_\alpha - (D_{ijk})_\alpha^\pm] \le (q_{jk})_\alpha^\pm \quad \forall j,k \tag{21b}$$

$$(T_{ik\max})_\alpha^\pm \ge (T_{ik})_\alpha^- + \Delta(T_{ik})_\alpha (y_{ik})_\alpha \ge (D_{ijk})_\alpha^\pm \ge 0 \quad \forall i,\ j,k \tag{21c}$$

$$0 \le (y_{ik})_\alpha \le 1 \quad \forall i,k \tag{21d}$$

Generally, the detailed solution process of RSIA for solving model (19) can be summarized as follows:

Step 1: Formulate model (19).
Step 2: Select a set of *α-cut* levels, $\alpha_1, \alpha_2, \ldots, \alpha_q$.
Step 3: Reorder these α values into a descending series (denoted as $\alpha_{(1)}, \alpha_{(2)}, \ldots, \alpha_{(q)}$, where $\alpha_{(1)} \ge \alpha_{(2)} \ge \ldots \ge \alpha_{(q)}$).
Step 4: Select $\alpha_{(1)}$ as the first *α-cut* level to cut model (19).
Step 5: Transform model (19) into an ITSP model presented as model (20) under $\alpha_{(1)}$.
Step 6: Convert model (20) into model (21) by introducing $(T_{ik})_\alpha^\pm = (T_{ik})_\alpha^- + \Delta(T_{ik})_\alpha (y_{ik})_\alpha$, where

$\Delta(T_{ik})_\alpha = (T_{ik})^+_\alpha - (T_{ik})^-_\alpha$ and $0 \le (y_{ik})_\alpha \le 1$.

Step 7: Solve model (21) through RTSM proposed by [11], in which model (21) will be transformed into two submodels corresponding to the lower and upper bounds of the objective function of model (21).

Step 8: Obtain the optimal solutions of the ITSP model under $\alpha_{(1)}$:

$(f_{opt})^\pm_{\alpha_{(1)}} = [(f_{opt})^-_{\alpha_{(1)}}, (f_{opt})^+_{\alpha_{(1)}}]$, $(D_{ijk\,opt})^\pm_{\alpha_{(1)}} = [(D_{ijk\,opt})^-_{\alpha_{(1)}}, (D_{ijk\,opt})^+_{\alpha_{(1)}}]$.

Step 9: Obtain the optimal water-allocation scheme under $\alpha_{(1)}$:

$(A_{ijk\,opt})^\pm_{\alpha_{(1)}} = (T_{ik\,opt})^\pm_{\alpha_{(1)}} - (D_{ijk\,opt})^\pm_{\alpha_{(1)}}$.

Step 10: Repeat Steps 4 to 9 in an order of $\alpha_{(2)}$, $\alpha_{(3)}$, …, $\alpha_{(q)}$, and obtain the resulting interval solutions as follows:

$(f_{opt})^\pm_{\alpha_{(l)}} = [(f_{opt})^-_{\alpha_{(l)}}, (f_{opt})^+_{\alpha_{(l)}}]$

$(D_{ijk\,opt})^\pm_{\alpha_{(l)}} = [(D_{ijk\,opt})^-_{\alpha_{(l)}}, (D_{ijk\,opt})^+_{\alpha_{(l)}}]$

$(A_{ijk\,opt})^\pm_{\alpha_{(l)}} = (T_{ik\,opt})^\pm_{\alpha_{(l)}} - (D_{ijk\,opt})^\pm_{\alpha_{(l)}}$

where l = 2, 3, …, q.

Step 11: Establish the membership functions for $\tilde{D}_{ijk\,opt}$, $\tilde{f}_{opt}$, as well as the relationship between T_{ik} and α [presented as $T_{ik}(\alpha)$] based on the solutions obtained in Steps 8 to 10.

Step 12: Stop.

Fig. 2 shows the schematic of the GFTSP model for water resources management. Obviously, the GFTSP model is an integration of generalized fuzzy linear programming (GFLP) and two-stage stochastic programming (TSP). Each method has a unique contribution in enhancing the capability of GFTSP in dealing with various uncertainties in water resources management. A stepwise interactive algorithm (RSIA) is proposed for solving the proposed GFTSP model, which can permit uncertainties to be directly communicated into the optimization process. Through RSIA, the developed GFTSP model will firstly be converted into several ITSP submodels, and then be further transformed into linear programming (LP) submodels. Consequently, the computational complexity of the GFTSP would be reasonable. For example, if *n α-cut* levels are identified in solving the GFTSP model, *n* ITSP submodels will be firstly generated. According to RTSM, each ITSP submodel can be further converted into two LP submodels; thus, the GFTSP model will finally result in 2*n* LP submodels with deterministic parameters.

3. Case Study

3.1. Overview of the study system

The following water resources management problem will be applied to demonstrate applicability of the developed GFTSP approach. An authority is charged with delivering water to different sectors to meet demands for regional socio-economic development. Three users would be involved in this studied case, including a municipality, an industry and an agricultural sector. The water users need to know the promised water supply firstly to planning their activities and investments. A net benefit will be generated for every unit water delivered; otherwise, penalties would be appear since water should be obtained from higher-priced alternatives or the demand must be curtailed by reducing production. Consequently, the problem under consideration is how to effectively allocate the limited water supply to multiple users in order to maximize the net benefit and minimize the associated penalties or negative consequences.

Table 2 shows the economic coefficients, including the net benefit to user *i* per m^3 of water allocated, as well as the loss to user *i* per m^3 of water not delivered. The data in Table 1 are assumed to be presented as triangular fuzzy numbers. Table 3 presents maximum allowable water allocation and the water allocation targets to different users. Table 4 shows the seasonal inflows under different probabilities, which are expressed as triangular fuzzy random variables. In this study, the triangular fuzzy numbers/fuzzy random variables are considered because (i) the triangular form is the simplest type of fuzzy numbers/fuzzy random variables; (ii) other types of fuzzy numbers can be expressed and estimated with this simple form of fuzzy number; (iii) a triangular fuzzy number can provide the most important information about a fuzzy number: lower and upper bounds of the number and its most possible value [43]. Obviously, there are two kinds of uncertainties in the proposed case: (i) fuzzy parameters in economic coefficients and water allocation targets, and (ii) fuzzy random variables in seasonal inflows. Both GFLP and TSP methods are not directly applicable for tackling such a problem due to the existence of dual-uncertainties. Consequently, the developed GFTSP method, which combines both capabilities of GFLP and TSP, is a potential approach for studying this case.

Table 2 Economic coefficients ($\$/m^3$)

	Users Municipal (i = 1)	Industrial (i = 2)	Agricultural (i = 3)
Net benefit when water demand is satisfied ($\widetilde{NB}_i$)	(100, 20, 20)	(50, 10, 10)	(30, 5, 5)
Reduction of net benefit when demand is not delivered ($\widetilde{C}_i$)	(250, 30, 30)	(100, 20, 20)	(60, 10, 10)

Table 3 water allocation targets ($\times 10^4$ m^3)

Season (k)	User Municipal (i = 1)	Industrial (i = 2)	Agricultural (i = 3)
Winter (k = 1)	(350, 30, 20)	(410, 60, 60)	(680, 50, 50)
Spring (k = 2)	(390, 40, 40)	(430, 50, 60)	(730, 60, 50)
Summer (k = 3)	(420, 50, 50)	(460, 70, 80)	(760, 60, 60)
Fall (k = 4)	(410, 40, 50)	(450, 70, 70)	(700, 50, 60)
Maximum allocation target ($\widetilde{T}_{ik\max}$)	900	900	900

Table 4 Seasonal water availability ($\times 10^4$ m^3) and associated probabilities

Season (k)	Flow level Low(j = 1)	Medium (j = 2)	High (j = 3)
Probability (p_j)	0.2	0.6	0.2
Water availability ($\widetilde{q}_{jk}$)			
Winter (k = 1)	(470, 50, 50)	(900, 100, 100)	(1200, 100, 100)
Spring (k = 2)	(490, 50, 50)	(1000, 100, 100)	(1300, 100, 100)

Summer (k = 3)	(530, 50, 50)	(1100, 100, 100)	(1400, 100, 100)
Fall (k = 4)	(480, 50, 50)	(950, 100, 100)	(1250, 100, 100)

3.2. Result Analysis

In this study, a GFTSP method is developed for supporting decision making in water resources management. A stepwise interactive algorithm (SIA) is proposed to solve the GFTSP model. Based on SIA, six *α-cut* levels (i.e. 0, 0.3, 0.5, 0.7, 0.85 and 1) are identified to defuzzify the fuzzy parameters. Under each *α-cut* level, the fuzzy parameters presented in Tables 2 and 3 would be converted into fuzzy intervals, and the seasonal water availability as shown in Table 4 would be converted into fuzzy random intervals. Consequently, the developed GFTSP model would be transformed into an inexact two-stage stochastic programming (ITSP) problem. The robust two-step method (RTSM) proposed by Fan and Huang [11] will then be employed to solve the ITSP submodels as derived from the GFTSP model. The feasibility and robustness of RTSM in solving the ITSP and ILP problems have been demonstrated by several cases in air/water quality management and energy system planning [8], [46], [48].

Table 5 shows the solutions of model (21) under different *α-cut* levels. They contain the water deficits of different users under different inflow levels [denoted as $(D_{ijk\,opt})^{\pm}_{\alpha(l)}$], as well as the values of additional variables [expressed as $(y_{ik})_{\alpha}$] for identifying desired water targets. The results indicate that the water allocation patterns would vary with temporal and spatial variations in water availability and economic conditions. Deficits would occur if the available flows do not meet user demands over the planning horizon. In case of insufficient water supply, the allotment to the agricultural sector would be first reduced, followed by the industrial sector's allocation. The municipal allocation should be of the highest priority since it brings the highest benefit when its water demand is satisfied; meanwhile, it is subject to the highest penalty if the promised water is not delivered. For example, under low inflow levels, the water demands of the municipal sector would almost be satisfied except for a small quantity of water shortage (i.e. [0, 20] × 10^4 m^3) in Summer; in comparison, the shortage of water in the agricultural sector would be as high as 680 × 10^4 m^3 over the planning horizon; the water deficit in the industrial sector would vary over the planning horizon due to variations in low water inflow over different periods. As the amount of inflow increases, the water shortage in the municipal sector would decrease first, followed by reduced water deficits in the industrial and agricultural sectors. As shown in Table 5, the promised water allocated to the municipal and industrial sectors would be satisfied as their respective inflow level is medium and high. In the case of the agriculture sector's water use, a water deficit would always exist even when the water inflow level is high, but the amount of deficit would decrease as water availability increases.

Table 5. Optimal solutions of $(D_{ijk})^{\pm}_{\alpha}$ and $(y_{ik})_{\alpha}$ obtained through model (21) under different *α-cut* levels

α-cut	1	0.85		0.7		0.5		0.3		0	
(i, j, k)	D_{ijk}	D_{ijk}	y_{ik}	D_{ijk}	y_{ik}	D_{ijk}	y_{ik}	D_{ijk}	y_{ik}	D_{ijk}	y_{ik}
(1, 1, 1)	0	0	1	0	1	0	1	0	1	0	0.94
(2, 1, 1)	290	[276.5, 291.5]	0	[263, 293]	0	[245, 295]	0	[227, 297]	0	[197, 297]	0
(3, 1, 1)	680	680	0.5	680	0.5	680	0.5	680	0.5	680	0.5
(1, 1, 2)	0	0	1	0	1	0	1	0	1	0	1

(2, 1, 2)	380	[362, 380]	0	[344, 380]	0	[320, 380]	0	[296, 380]	0	[270, 380]	0
(3, 1, 2)	680	680	0	680	0	680	0	680	0	680	0.091
(1, 1, 3)	20	[0.5, 20]	1	[0, 20]	1	[0, 20]	1	[0, 20]	1	[0, 20]	1
(2, 1, 3)	410	410	0	[391, 410]	0	[365, 410]	0	[339, 410]	0	[320, 410]	0.13
(3, 1, 3)	680	680	0	680	0	680	0	680	0	680	0
(1, 1, 4)	0	0	1	0	1	0	1	0	1	0	0.67
(2, 1, 4)	400	[382, 400]	0	[364, 400]	0	[345, 400]	0	[331, 400]	0	[301, 400]	0.15
(3, 1, 4)	680	680	0	680	0	680	0.091	680	0.195	680	0.273
(1, 2, 1)	0	0	1	0	1	0	1	0	1	0	0.94
(2, 2, 1)	0	0	0	0	0	0	0	0	0	0	0
(3, 2, 1)	540	[519, 549]	0.5	[498, 558]	0.5	[470, 570]	0.5	[442, 582]	0.5	[397, 597]	0.5
(1, 2, 2)	0	0	1	0	1	0	1	0	1	0	1
(2, 2, 2)	0	0	0	0	0	0	0	0	0	0	0
(3, 2, 2)	550	[524.5, 554.5]	0	[499, 559]	0	[465, 565]	0	[431, 571]	0	[390, 590]	0.091
(1, 2, 3)	0	0	1	0	1	0	1	0	1	0	1
(2, 2, 3)	0	0	0	0	0	0	0	0	0	0	0.13
(3, 2, 3)	540	[513, 543]	0	[486, 546]	0	[450, 550]	0	[414, 554]	0	[380, 580]	0
(1, 2, 4)	0	0	1	0	1	0	1	0	1	0	0.67
(2, 2, 4)	0	0	0	0	0	0	0	0	0	0	0.15
(3, 2, 4)	610	[584.5, 614.5]	0	[559, 619]	0	[530, 630]	0.091	[506, 646]	0.195	[461, 661]	0.273
(1, 3, 1)	0	0	1	0	1	0	1	0	1	0	0.94
(2, 3, 1)	0	0	0	0	0	0	0	0	0	0	0
(3, 3, 1)	240	[219, 249]	0.5	[198, 258]	0.5	[170, 270]	0.5	[142, 282]	0.5	[97, 297]	0.5
(1, 3, 2)	0	0	1	0	1	0	1	0	1	0	1
(2, 3, 2)	0	0	0	0	0	0	0	0	0	0	0
(3, 3, 2)	250	[224.5, 254.5]	0	[199, 259]	0	[165, 265]	0	[131, 271]	0	[90, 290]	0.091
(1, 3, 3)	0	0	1	0	1	0	1	0	1	0	1
(2, 3, 3)	0	0	0	0	0	0	0	0	0	0	0.13
(3, 3, 3)	240	[213, 243]	0	[186, 246]	0	[150, 250]	0	[114, 254]	0	[80, 280]	0
(1, 3, 4)	0	0	1	0	1	0	1	0	1	0	0.67
(2, 3, 4)	0	0	0	0	0	0	0	0	0	0	0.15
(3, 3, 4)	310	[284.5, 314.5]	0	[259, 319]	0	[230, 330]	0.091	[206, 346]	0.195	[161, 361]	0.273
$(f)^{\pm}_{\alpha}$	1.74	[1.59, 1.94]		[1.45, 2.13]		[1.24, 2.38]		[1.04, 2.62]		[0.72, 2.94]	

Notes: i = 1, 2, 3 indicates municipal, industrial and agricultural users, respectively; j = 1, 2, 3 means the low, medium, and high inflow levels, respectively; k = 1, 2, 3, 4 represents Winter, Spring, Summer, and Fall, respectively. $(D_{ijk})^{\pm}_{\alpha}$ indicates the water shortage under different α-*cut* levels ($\times 10^4$ m^3); $(f)^{\pm}_{\alpha}$ means the objective function value under different α-*cut* levels ($\times 10^9$)

The optimized water-allocation targets for the users could be obtained based on $(T_{ik})^{\pm}_{\alpha} = (T_{ik})^{-}_{\alpha} + \Delta(T_{ik})_{\alpha}(y_{ik})_{\alpha}$. The decision variable $(y_{ik})_{\alpha}$, where $0 \le (y_{ik})_{\alpha} \le 1$, is applied to identify the optimized water-allocation targets from the promised values. As shown in Table 5, $(y_{1k})_{\alpha} \ge 0.6$ for any k = 1, 2, 3, 4 and α = 0, 0.3, 0.5, 0.7, 0.85, 1, the municipal sector would usually obtain high allocation targets. This is because the water use in the municipal sector could lead to both high benefit (when their demands are satisfied) and high penalty (when the demands are not delivered); thus, the manager would acquire high system benefit through promising high water-allocation targets and reducing water deficits in the municipal sector. Conversely, the values of $(y_{2k})_{\alpha}$ and $(y_{3k})_{\alpha}$ are less than or equal to 0.5 for any k = 1, 2, 3, 4 and α = 0, 0.3, 0.5, 0.7, 0.85, 1, suggesting relatively low allocation targets to the industrial and agricultural sectors. This is due to the relatively low benefits generated by the two sectors. Generally, the identification of optimized water-allocation targets represents a compromise between the benefit of water delivered and the penalty of water not delivered. A higher target level would lead to a higher benefit but, at the same time, a higher risk of water shortage (and thus a higher penalty) when the water flow is low; however, a lower target level would result in a lower benefit as well as a lower risk of water shortage.

After the optimized allocation targets to different users are identified, the actual water allocation to different users can be obtained through $(A_{ijk\,opt})^{\pm}_{\alpha} = (T_{ik\,opt})^{\pm}_{\alpha} - (D_{ijk\,opt})^{\pm}_{\alpha}$. Table 6 presents the optimal water allocation target and actual allocation schemes under selected *α-cut* levels. It indicates that the water demand of the municipal sector would be satisfied first. For instance, the actual water-allocation amount of the municipal use is almost equal to its optimized water-allocation amount [i.e. $(A_{1jk\,opt})^{\pm}_{\alpha} = (T_{1k\,opt})_{\alpha}$], except some shortages as occurred in summer under low river inflow. In comparison, as shown in Table 6, the actual water allocation to the agricultural sector is always less than the optimized water-allocation targets over the planning horizon, regardless of the river inflow levels, indicating the existence of water shortage. As the water availability increases, the gap between the optimized allocation target and the actual allocation amount would decreases, showing a reduction of water shortage. In fact, the results in Table 6 are identical to those in Table 5. The values of $(A_{ijk\,opt})^{\pm}_{\alpha}$ and $(T_{ik\,opt})_{\alpha}$ indicate the actual allocation schemes given the decision variables $(D_{ijk\,opt})^{\pm}_{\alpha}$ and $(y_{ik})_{\alpha}$ in model (14).

Table 6. optimal water allocation targets $(A_{ijk\,opt})^{\pm}_{\alpha}$ and actual water allocation schemes $(T_{ik\,opt})_{\alpha}$ under different *α-cut* levels

α-cut / (i, j, k)	1		0.85		0.7		0.5		0.3		0	
	A_{ijk}	T_{ik}	A_{ijk}	T_{ik}	A_{ijk}	T_{ik}	A_{ijk}	T_{ik}	A_{ijk}	T_{ik}	A_{ijk}	T_{ik}
(1, 1, 1)	350	350	353	353	356	356	360	360	364	364	367	367
(2, 1, 1)	120	410	[109.5, 124.5]	401	[99, 129]	392	[85, 135]	380	[71, 141]	368	[53, 153]	350
(3, 1, 1)	0	680	0	680	0	680	0	680	0	680	0	680
(1, 1, 2)	390	390	396	396	402	402	410	410	418	418	430	430
(2, 1, 2)	50	430	[42.5, 60.5]	422.5	[35, 71]	415	[25, 85]	405	[15, 99]	395	[0, 110]	380
(3, 1, 2)	50	730	41	721	32	712	20	700	8	688	0	680
(1, 1, 3)	400	420	[407.5, 427]	427.5	[415, 435]	435	[425, 445]	445	[435, 455]	455	[450, 470]	470
(2, 1, 3)	50	460	39.5	449.5	[29, 48]	439	[15, 60]	425	[1, 72]	411	[0, 90]	410
(3, 1, 3)	80	760	71	751	62	742	50	730	38	718	20	700
(1, 1, 4)	410	410	417.5	417.5	425	425	435	435	445	445	430	430
(2, 1, 4)	50	450	[39.5, 57.5]	439.5	[29, 65]	429	[15, 70]	415	[0, 70]	401	[0, 100]	401
(3, 1, 4)	20	700	12.5	692.5	5	685	0	680	0	680	0	680
(1, 2, 1)	350	350	353	353	356	356	360	360	364	364	367	367
(2, 2, 1)	410	410	401	401	392	392	380	380	368	368	350	350
(3, 2, 1)	140	680	[131, 161]	680	[122, 182]	680	[110, 210]	680	[98, 238]	680	[83, 283]	680
(1, 2, 2)	390	390	396	396	402	402	410	410	418	418	430	430
(2, 2, 2)	430	430	422.5	422.5	415	415	405	405	395	395	380	380
(3, 2, 2)	180	730	[166.5, 196.5]	721	[153, 213]	712	[135, 235]	700	[117, 257]	688	[90, 290]	680
(1, 2, 3)	420	420	427.5	427.5	435	435	445	445	455	455	470	470
(2, 2, 3)	460	460	449.5	449.5	439	439	425	425	411	411	410	410
(3, 2, 3)	220	760	[208, 238]	751	[196, 256]	742	[180, 280]	730	[164, 304]	718	[120, 320]	700
(1, 2, 4)	410	410	417.5	417.5	425	425	435	435	445	445	430	430
(2, 2, 4)	450	450	439.5	439.5	429	429	415	415	401	401	401	401
(3, 2, 4)	90	700	[78, 108]	692.5	[66, 126]	685	[50, 150]	680	[34, 174]	680	[19, 219]	680
(1, 3, 1)	350	350	353	353	356	356	360	360	364	364	367	367
(2, 3, 1)	410	410	401	401	392	392	380	380	368	368	350	350
(3, 3, 1)	440	680	[431, 461]	680	[422, 482]	680	[410, 510]	680	[398, 538]	680	[383, 583]	680

(1, 3, 2)	390	390	396	396	402	402	410	410	418	418	430	430
(2, 3, 2)	430	430	422.5	422.5	415	415	405	405	395	395	380	380
(3, 3, 2)	480	730	[466.5, 496.5]	721	[453, 513]	712	[435, 535]	700	[417, 557]	688	[390, 590]	680
(1, 3, 3)	420	420	427.5	427.5	435	435	445	445	455	455	470	470
(2, 3, 3)	460	460	449.5	449.5	439	439	425	425	411	411	410	410
(3, 3, 3)	520	760	[508, 538]	751	[496, 556]	742	[480, 580]	730	[464, 604]	718	[420, 620]	700
(1, 3, 4)	410	410	417.5	417.5	425	425	435	435	445	445	430	430
(2, 3, 4)	450	450	439.5	439.5	429	429	415	415	401	401	401	401
(3, 3, 4)	390	700	[378, 408]	692.5	[366, 426]	685	[350, 450]	680	[334, 474]	680	[319, 519]	680

Notes: i = 1, 2, 3 indicates municipal, industrial and agricultural users, respectively; j = 1, 2, 3 means the low, medium, and high inflow levels, respectively; k = 1, 2, 3, 4 represents Winter, Spring, Summer, and Fall, respectively. $(T_{ik\ opt})_\alpha$ indicates the water allocation target under different *α-cut* levels (×10^4 m^3). $(A_{ijk\ opt})_\alpha^\pm$ means the actual water allocation scheme under different *α-cut* levels (×10^4 m^3).

Since parameters in model (19) are expressed as fuzzy sets, the fluctuating ranges of these inputs would vary under different plausibilities (*α-cut* levels), and thus result in variations in the generated solutions. For example, as shown in Table 5, under $\alpha = 0$ (the lowest plausibility degree), the value of $(D_{211})_0^\pm$ (i.e. amount of water shortage in the industrial sector in Winter under the scenario of low river inflow level) would be [197, 297] × 10^4 m^3; in comparison, under $\alpha = 1$ (highest plausibility degree), the value of $(D_{211})_1^\pm$ would be 290 × 10^4 m^3. As the value of *α-cut* level increases from 0 to 1, the lower bound of $(D_{211})_\alpha^\pm$ would increase (i.e. 227, 245, 263, 276.5 × 10^4 m^3 under α = 0.3, 0.5, 0.7, 0.85, respectively), while the upper bound of $(D_{211})_\alpha^\pm$ would decrease (i.e. 297, 295, 293, 291.5 × 10^4 m^3 under α = 0.3, 0.5, 0.7, 0.85, respectively). Fig. 3 shows the lower and upper bounds of water shortages under different *α-cut* levels. In this figure, each bar indicates the water shortage of a water user in different periods and inflow levels under different *α-cut* levels. For example, symbol D123 means water shortage for municipal sector (i.e. $i = 1$) in Summer (i.e. $k = 3$) under medium inflow level ($j = 2$); the axis with α values presents the *α-cut* levels that are selected to "cut" the model. They indicate that solutions of water allocation schemes for the three users would vary with the *α-cut* level. The lower bound would increase and the upper bound would decrease when the *α-cut* level is increased from 0 to 1. Such variations in water allocation would stem from the input fuzziness of model (11).

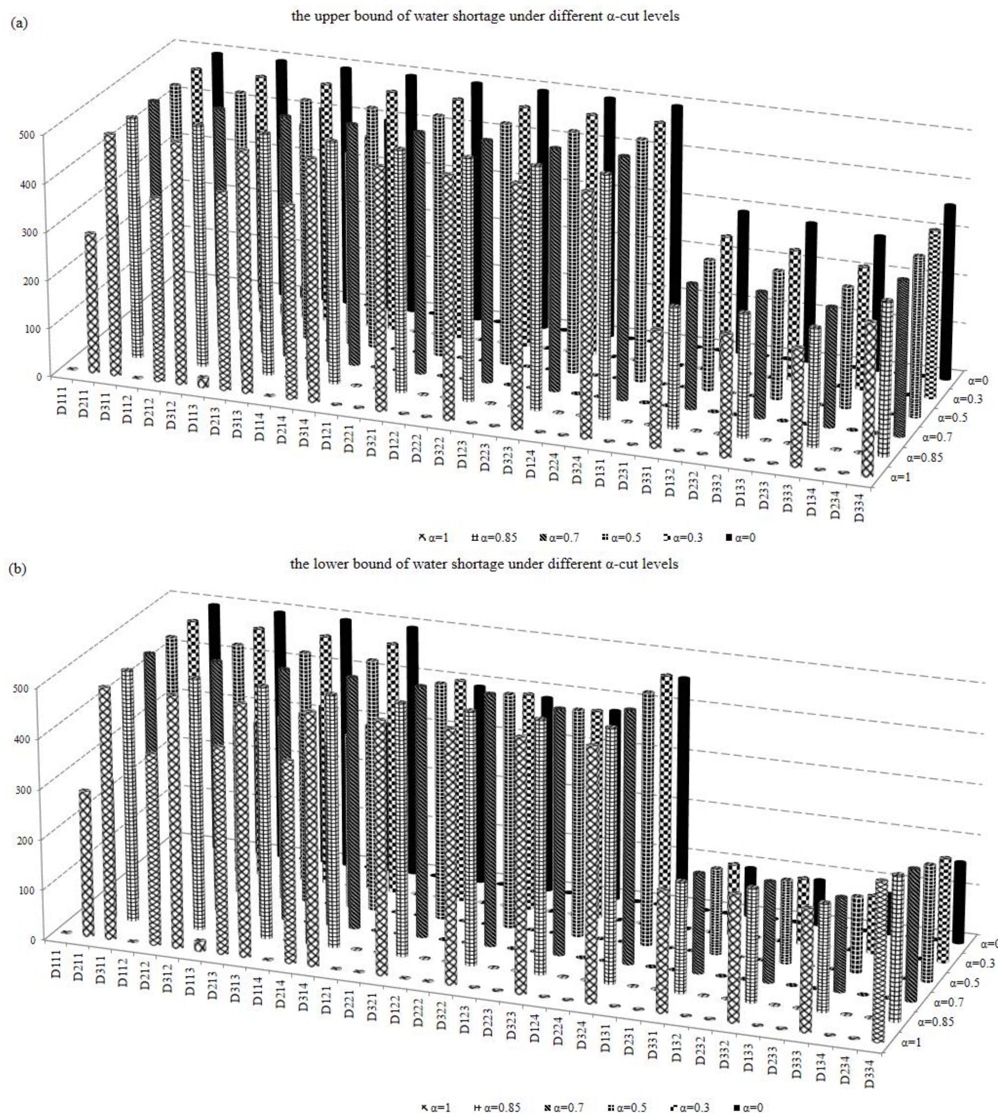

Figure 3. The lower and upper bounds of water shortages under different *α-cut* levels

4. Discussion

As shown in Table 5, a series of fuzzy interval solutions can be obtained through model (21) under different *α-cut* levels. Afterwards, the membership functions of the fuzzy decision variables in model (19) can be approximated through regression analysis based on the fuzzy interval solutions. Fig. 4 shows the obtained membership functions of the fuzzy variables (i.e. $\widetilde{D}_{ijk}$). It indicates that the membership functions of $\widetilde{D}_{ijk}$ can be well fitted through linear or polynomial regression methods based on the results of Table 5. Besides, Table 5 also provides the total system benefit [i.e., objective function of model (21)] under six α-cut levels. The results suggest that different plausibility degrees of uncertain inputs would lead to varied system benefits. The upper-bound of the objective function value corresponds to advantageous conditions, while the lower-bound one is associated with demanding conditions. Meanwhile, the membership function of the objective-function value can also be approximated based on the fuzzy interval solutions for the objective-function value in Table 5. Fig. 5 presents the obtained membership function, which can be well fitted through linear regression. From the membership functions of the decision variables (i.e. $\tilde{D}_{ijk}$) and objective-function value (i.e., $\tilde{f}$), the water shortage and the corresponding system benefit can be generated under any plausibility (*α-cut* level). For example, if the decision maker prefers to identify the water deficits in different sectors under an *α-cut* level of 0.6, the amount of water shortage in the industrial sector in period 2 under low inflow level would be [333.5, 380.0] × 10^4 m^3 [$(D_{212})^-_{0.6}$ = (2.1414 + 0.6)/0.0108 =333.5; $(D_{212})^+_{0.6}$ =380], and the corresponding system benefit would be \$[1.34, 2.24] × 10^9. The water shortages in other sectors under different inflow levels would be obtained in the same way.

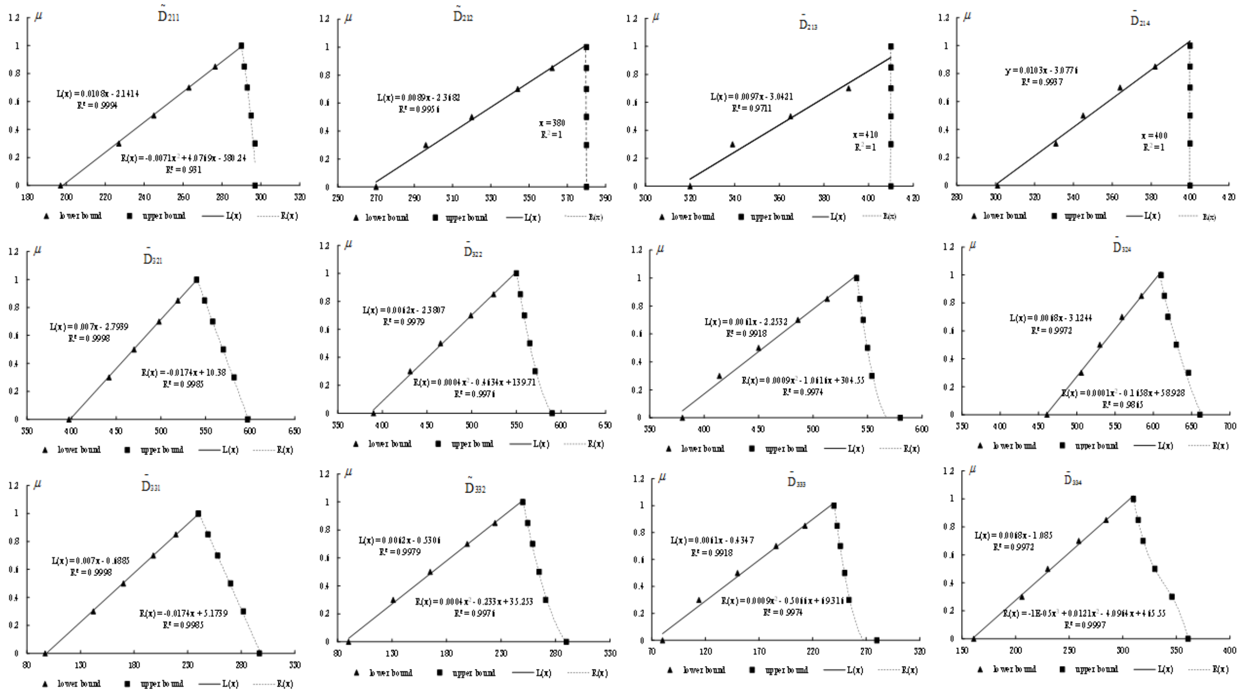

Figure 4. The membership functions of water shortages

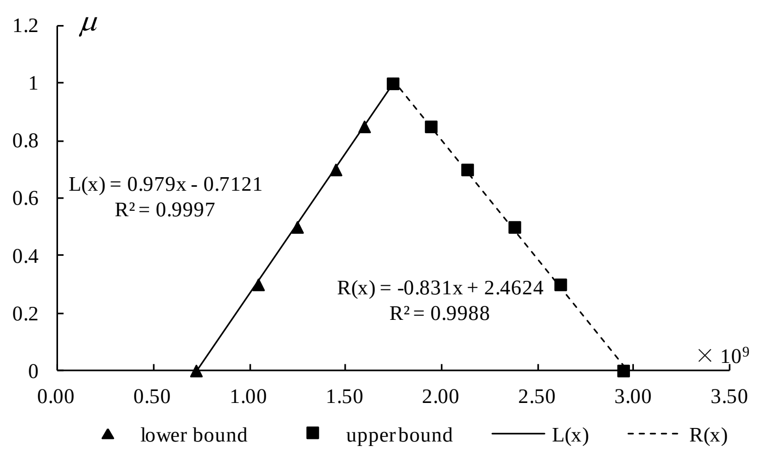

Figure 5. The membership function of the objective function

The solutions of decision variables $(y_{ik})_\alpha$ would vary under different α*-cut* levels, as presented in Table 5. The variation in the values of $(y_{ik})_\alpha$ would lead to fluctuation in the optimized water allocation targets (i.e. $(T_{ik\ opt})_\alpha$), as shown in Table 6. Consequently, regression functions can be established for the optimized water allocation targets and the α*-cut* levels. Fig. 6 shows the functions between the $(T_{ik\ opt})_\alpha$ and the α*-cut* level as obtained through linear or polynomial regression method. Through these regression functions, the optimized allocation targets to the three users in different periods can be directly generated, if the α*-cut* level is predefined by the decision maker. Afterwards, the actual water allocation schemes (i.e. $(A_{ijk\ opt})_\alpha^\pm$) can be obtained based on $(A_{ijk\ opt})_\alpha^\pm = (T_{ik\ opt})_\alpha^\pm - (D_{ijk\ opt})_\alpha^\pm$.

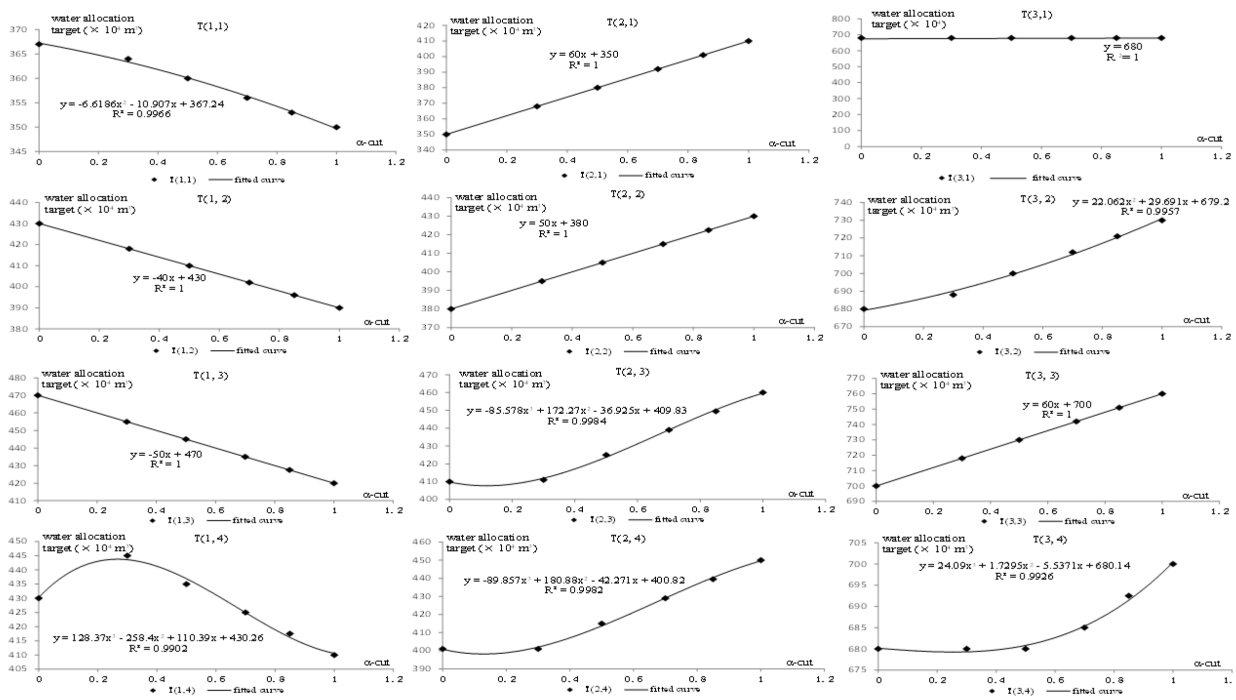

Figure 6. The curves fitted for water allocation target

To demonstrate the robustness of the developed GFTSP method, a comparison will be conducted between solutions obtained through the ITSP and the functions generated through the GFTSP, as presented in Figures 3 to 5. In this comparison, two values (i.e. 0.4, 0.8) are considered as the two α*-cut* levels for the membership functions of $\widetilde{D}_{ijk}$ and $\widetilde{f}$ (presented in Figs. 4 and 5, respectively), and the independent variable values for $T_{ik}(\alpha)$ (shown in Fig. 5). These lead to the associated solutions of $(D_{ijk\ opt})_\alpha^\pm$, $(f)_\alpha^\pm$ and $(A_{ijk\ opt})_\alpha^\pm$; correspondingly, these two α*-cut* levels are also applied to cut model (12), resulting in two ITSP models, which would be solved by the the robust two-step method (RTSM). Solutions obtained through the above two ways are presented in Table 7. They indicate, under these two α*-cut* levels, the allocation schemes obtained through the functions generated by the GFTSP are similar to those from the ITSP model. For example, the values of $(D_{211})^{\pm}_{0.4}$ and $(A_{211\,opt})^{\pm}_{0.4}$ obtained through the ITSP would be [236, 296] and [78, 138] × 10^4 m^3, while those from $\widetilde{D}_{ijk}$ and $T_{ik}(\alpha)$ would be [236, 296] and [78, 139] × 10^4 m^3. Furthermore, the system benefits acquired by these two ways are also close to each other. As shown in Table 7, under the scenario of $\alpha = 0.4$, the system benefit obtained through the ITSP and the $\widetilde{f}$ in Fig. 5 would be \$[1.18, 2.51] × 10^9, and \$[1.14, 2.48] × 10^9, respectively. The relative error (RE) of the lower and upper bounds of these two intervals would be -3.4% and -1.2%, respectively. Such REs under $\alpha = 0.8$ would be -1.3% and -0.5%, respectively. Therefore, the solutions obtained through the developed GFTSP are reasonable. The membership functions of $\widetilde{D}_{ijk}$ and $\widetilde{f}$, as well as the function of $T_{ik}(\alpha)$, can then be employed to generate desired water allocation schemes (instead of solving the optimization model again), as long as the preferred α value can be preregulated by decision makers.

5. Conclusions

In this study, a generalized fuzzy two-stage stochastic programming (GFTSP) method was developed through integrating methods of generalized fuzzy linear programming and two-stage stochastic programming into a general framework. The developed GFTSP method could deal with uncertainties expressed as fuzzy sets and fuzzy random variables. A robust stepwise interactive algorithm (RSIA) was proposed to solve the GFTSP model and generate solutions expressed as fuzzy sets. The proposed RSIA firstly defuzzified the inputs through the *α-cut* method, and then converted the GFTSP model into an inexact two-stage stochastic programming (ITSP) problem under each given *α-cut* level, which was then solved through the robust two-step method (RTSM) as developed by Fan and Huang [11]; finally, membership functions of the decision variables and the objective-function value were approximated through statistical regression.

The developed method was applied to a case of water resources management to illustrate its applicability. In this case, an authority is charged with delivering water to municipal, industrial and agricultural uses; the economical coefficients were assumed to be triangular fuzzy sets, while the seasonal inflow was estimated as triangular fuzzy random variables. The solutions of the decision variables, which were expressed as fuzzy sets with known membership functions, could provide the water shortages of different users in different periods under different water inflow levels, when the preferred plausibility (i.e. *α-cut* level) was predefined by decision makers. Correspondingly, the associated fluctuating interval of the objective-function value could be obtained through the membership functions of the objective-function values. These solutions were helpful for generating decision alternatives for water resources management under uncertainty.

To demonstrate the solution robustness of the developed GFTSP, a comparison was conducted between solutions obtained through the ITSP and the functions generated through the GFTSP. The results suggested that the allocation schemes based on the functions generated by the GFTSP were significantly similar to those acquired from the ITSP. The membership functions of $\widetilde{D}_{ijk}$ and $\widetilde{f}$, as well as those of $T_{ik}(\alpha)$, could be used for generating water allocation schemes (instead of solving the optimization model again), as long as the preferred α value can be predefined by decision makers.

The developed GFTSP could deal with various fuzzy sets and fuzzy random variables. However, it mainly focused on such uncertainties within a linear programming framework. It had difficulties in treating nonlinear constraints or objective function. Therefore, further improvements in the GFTSP are desired to enhance its capability in dealing with nonlinearity within the optimization framework.

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References

[1] Ammar, E.E., On solutions of fuzzy random multiobjective quadratic programming with applications in portfolio problem. *Inform. Sci.*, vol. 178, pp. 468 – 484, 2008.

[2] Bender M.J., Simonovic S.P., A fuzzy compromise approach to water resources planning under uncertainty. *Fuzzy Sets Syst.*, vol. 115, no.1, pp. 35-44, 2000.

[3] Chang NB, Wen CG, Chen YL, Yong YC, A grey fuzzy multiobjective programming approach for the optimal planning of a reservoir watershed. Part A: Theoretical development. *Water Res.* vol. 30, pp. 2329–2340, 1996.

[4] Chen C., Huang G.H., Li Y.P., Zhou Y., A robust risk analysis method for water resources allocation under uncertainty. *Stochastic Environmental Research and Risk Assessment*, vol. 27, pp. 713-723, 2013.

[5] Cheng GH, Huang GH, Li YP, Cao MF, Fan YR, Planning of municipal solid waste management systems under dual uncertainties: a hybrid interval stochastic programming approach. *Stochastic Environmental Research and Risk Assessment*, vol. 23, pp. 707-720, 2009.

[6] Chen S.-M., Chang Y.-C., Pan J.-S. Fuzzy Rules Interpolation for Sparse Fuzzy Rule-Based Systems Based on Interval Type-2 Gaussian Fuzzy Sets and Genetic Algorithms. *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 3, pp. 412-425, 2013.

[7] Dehghan, M., Hashemi, B., Ghatee, M., Computational methods for solving fully fuzzy linear systems. Applied Mathematics and Computation, vol. 179, pp. 328-343, 2006.

[8] Dehghan, M., Ghatee, M., Hashemi, B., Inverse of a fuzzy matrix of fuzzy numbers. International Journal of Computer Mathematics, vol. 86, no. 8, vol. 1433 – 1452, 2009.

[9] Du, P., Li, Y., and Huang, G., An Inexact Chance-Constrained Waste-Load Allocation Model for Water Quality Management of Xiangxihe River. *Journal of Environmental Engineering*, 10.1061/(ASCE)EE.1943-7870.0000724, 2013.

[10] Dubois, D. and Prade, H., *Fuzzy Sets and Systems: Theory and Applications*. London: Academic Press, 1980.

[11] Fan Y.R. and Huang G.H., A Robust Two-Step Method for Solving Interval Linear Programming Problems within an Environmental Management Context. *Journal of Environmental Informatics*, vol. 19, no. 1, pp. 1-9, 2012.

[12] Fan Y.R., Huang G.H., Guo P., Yang A.L., Inexact two-stage stochastic partial programming: application to water resources management under uncertainty, *Stochastic Environmental Research and Risk Assessment*, vol. 26, no.2, pp. 261-280, 2012a.

[13] Fan Y.R., Huang G.H., Li, Y.P., Robust interval linear programming for environmental decision making under uncertainty. *Engineering Optimization*, vol. 44, pp. 11, pp. 1321-1336, 2012b.

[14] Fan Y.R., Huang G.H., Veawab, A., A generalized fuzzy linear programming approach for environmental management problem under uncertainty. *Journal of the Air & Waste Management Association*, vol. 62, no.

1, pp. 72-86, 2012c.
[15] Fan Y.R., Huang G.H., Yang A.L., Generalized fuzzy linear programming for decision making under uncertainty: feasibility of fuzzy solutions and solving approach. *Information Sciences*, vol. 241, pp. 12-27, 2013.
[16] Faybishenko B., Fuzzy-probabilistic calculation of water-balance uncertainty. *Stochastic Environmental Research and Risk Assessment*, vol. 24, pp. 939-952, 2010.
[17] Fu G.T., Kapelan Z., Fuzzy probabilistic design of water distribution networks. *Water Resources Research*, vol. 47, W05538, 2011.
[18] Guo P, Huang GH, Two-stage fuzzy chance-constrained programming: application to water resources management under dual uncertainties. *Stochastic Environmental Research and Risk Assessment*, vol. 23, pp. 349-359, 2009.
[19] Guo P, Huang GH, He L, Zhu H, Interval-parameter Two-stage Stochastic Semi-infinite Programming: Application to Water Resources Management under Uncertainty. *Water Resources Management*, vol. 23, no. 8, pp. 1001-1023, 2009.
[20] Hu Q. Huang G.H. Liu Z.F. Fan Y.R. Li W. Inexact fuzzy two-stage programming for water resources management in an environment of fuzziness and randomness, *Stochastic Environmental Research and Risk Assessment*, vol. 26, no. 2, pp. 261-280, 2012.
[21] Huang G.H., Baetz B.W., Patry G.G., Grey fuzzy integer programming: An application to waste management planning under uncertainty. *European Journal of Operational Research*, vol. 83, pp. 594 – 620, 1995.
[22] Huang GH, Loucks DP, An inexact two-stage stochastic programming model for water resources management under uncertainty. *Civil Engineering and Environmental Systems*, vol. 17, pp. 95-118, 2000.
[23] Inuiguchi M., Ramík J., Possibilistic linear programming: a brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem. *Fuzzy Sets and Systems*, vol. 111, pp. 3-28, 2000.
[24] Juang, C.-F. , Juang, K.-J. Reduced Interval Type-2 Neural Fuzzy System Using Weighted Bound-Set Boundary Operation for Computation Speedup and Chip Implementation. *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 3, pp. 477-491, 2013.
[25] Jin L., Huang G.H., Fan Y.R., Nie X.H., Cheng G.H., A Hybrid Dynamic Dual Interval Programming for Irrigation Water Allocation under Uncertainty. *Water Resources Management*, vol. 26, no. 5, pp. 1183-1200, 2012.
[26] Kaufmann, A., Cupta, M., *Fuzzy Mathematical Models in Engineering and Many Science*, North Holland, Amsterdam, 1988.
[27] Kirnbauer M. C. and Baetz B.W., Allocating Urban Agricultural Reuse Strategies to Inventoried Vacant and Underutilized Land. *Journal of Environmental Informatics*, vol. 20, no. 1, pp. 1-11, 2012.
[28] Karmakar S., Mujumdar P.P., Grey fuzzy optimization model for water quality management of a river system. *Advances in Water Resources*, vol. 29, pp. 1088-1105, 2006.
[29] Karmakar S, Mujumdar P.P., A two-phase grey fuzzy optimization approach for water quality management of a river system. *Advances in Water Resources*, vol. 30, pp. 1218-1235, 2007.
[30] Kreinovich V., Membership Functions or α-Cuts? Algorithmic (Constructivist) Analysis Justifies an Interval Approach. *Mathematical Problems of Computer Science*, vol. 38, pp. 70-71, 2012.
[31] Li Y.P., Huang G.H., A recourse-based nonlinear programming model for stream water quality management. *Stochastic Environmental Research and Risk Assessment*, vol 26, pp. 207-223, 2012
[32] Li Y.P., Huang G.H., Huang Y.F., Zhou H.D., A multistage fuzzy-stochastic programming model for supporting sustainable water-resources allocation and management, *Environmental Modelling & Software*, vol. 24 no.7, pp. 786-797, 2009.
[33] Li Y.P., Huang G.H., Nie S.L., An interval-parameter multi-stage stochastic programming for water resources management under uncertainty. *Advances in Water Resources*, vol. 29, pp. 776-789, 2006.
[34] Li Y.P., Huang G.H., Interval-parameter Two-stage Stochastic Nonlinear Programming for Water Resources Management under Uncertainty, *Water Resources Management*, vol. 22, no. 6, pp. 681-698, 2008.
[35] Li Y.P., Huang G.H., Nie S.L., lanning water resources management systems using a fuzzy-boundary interval-stochastic programming method. *Advances in Water Resources*, vol. 33, pp. 1105-1117, 2010.

[36] Loucks DP, Stedinger JR, Haith DA, *Water Resource Systems Planning and Analysis*, Prentice-Hall, Englewood Cliffs, N.J., 1981.
[37] Loucks D.P. van Beek E., *Water Resources Systems Planning and Management: An Introduction to Methods*, Models and Applications. UNESCO Paris, 2005.
[38] Lu HW, Huang GH, Zeng GM, Maqsood I, He L, An Inexact Two-stage Fuzzy-stochastic Programming Model for Water Resources Management. *Water Resources Management*, vol. 22, no. 8, pp. 991-1016., 2009.
[39] Luo B, Maqsood I, Huang GH, Planning water resources systems with interval stochastic dynamic programming. *Water Resources Management, vol.* 21, no. 6, pp 997–1014, 2007.
[40] Mendel, J. M. On KM Algorithms for Solving Type-2 Fuzzy Set Problems. *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 3, pp. 426-446, 2013.
[41] Nourani V., Baghanam A.H., Gebremichael M., Investigating the Ability of Artificial Neural Network (ANN) Models to Estimate Missing Rain-gauge Data. *Journal of Environmental Informatics*, vol. 19, no. 1, pp. 38-50, 2012.
[42] Pagola M., Lopez-Molina C., Fernandez J, Barrenechea E, Bustince H., Interval Type-2 Fuzzy Sets Constructed From Several Membership Functions: Application to the Fuzzy Thresholding Algorithm, *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 2, pp. 230-244, 2013.
[43] Papageorgiou E.I., Iakovidis D.K., Intuitionistic Fuzzy Cognitive Maps, *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 2, pp. 342-354, 2013
[44] Recio B, Ibañez J, Rubio F, Criado JA, A decision support system for analyzing the impact of water restriction policies. *Decision Support Systems*, vol. 39, pp. 385–402, 2005.
[45] Sanz J.A., Fernandez A., Bustince H., Herrera F., IVTURS: A Linguistic Fuzzy Rule-Based Classification System Based On a New Interval-Valued Fuzzy Reasoning Method With Tuning and Rule Selection. *IEEE Trans. Fuzzy Syst.*, vol. 21, no. 3, pp. 399-411, 2013
[46] Shapiro A., Dentcheva D., Ruszczyński A., Lectures on Stochastic Programming: Modelling and Theory, SIAM (Society for Industrial and Applied Mathematics), Philadelhpia, PA, 2009.
[47] Shapiro A., Philpott A., A tutorial on stochastic programming. Technical Note, Georgia Institute of Technology, 2007, available at http://www2.isye.gatech.edu/people/faculty/Alex_Shapiro/TutorialSP.pdf
[48] Simonovic P., *Managing Water Resources: Methods and Tools for a Systems Approach*. UNESCO, Paris, 2009.
[49] Slowinski R, A multicriteria fuzzy linear programming method for water supply system development planning. *Fuzzy Sets and Systems*, vol. 19, pp. 217-237, 1986.
[50] Van Hop N., Solving fuzzy (stochastic) linear programming problems using superiority and inferiority measures. *Information Sciences*, vol. 177, pp. 1977-1991, 2007.
[51] Vucetic D., Simonovic, P., *Water Resources Decision Making under Uncertainty. Water Resources Research Report*, No. 73. Facility for Intelligent Decision Support (FIDS), the University of Western Ontario, London, Canada, 2011.
[52] Wang, J.-Q. ; Zhang, H.-Y. Multicriteria Decision-Making Approach Based on Atanassov's Intuitionistic Fuzzy Sets With Incomplete Certain Information on Weights. *IEEE Trans. Fuzzy Syst.*, vol. 21, no.3, pp. 510-515, 2013.
[53] Wang S., Huang G.H., Interactive Fuzzy Boundary Interval Programming for Air Quality Management Under Uncertainty. *Water, Air & Soil Pollution* 224:1574 DOI 10.1007/s11270-013-1574-5, 2013.
[54] Xia Meimei, Xu Zeshui, Liao Huchang, Preference Relations Based on Intuitionistic Multiplicative Information, *IEEE Trans. Fuzzy Syst.*, vol. 21, no.1, pp. 113-133, 2013.
[55] Xu Y., Huang G.H., Xu T.Y., Inexact Management Modeling for Urban Water Supply Systems, *Journal of Environmental Informatics*, vol. 20, no. 1, pp. 34-43, 2012.
[56] Zhu Y., Li Y.P., Huang G.H., Planning carbon emission trading for Beijing's electric power systems under dual uncertainties. Renewable and Sustainable Energy Reviews, vol. 23, pp. 113–128, 2013.
[57] Zimmermann H.J., *Fuzzy set theory and its application*. 3rd Edition. Kluwer Academic Publishers, 1995.