Event-Based Variance-Constrained $\mathcal{H}_\infty$ Filtering for Stochastic Parameter Systems over Sensor Networks with Successive Missing Measurements

Licheng Wang, Zidong Wang, *Fellow*, *IEEE*, Qing-Long Han, *Senior Member*, *IEEE*, and Guoliang Wei

***Abstract*—This paper is concerned with the distributed $\mathcal{H}_\infty$ filtering problem for a class of discrete time-varying stochastic parameter systems with error variance constraints over a sensor network where the sensor outputs are subject to successive missing measurements. The phenomenon of the successive missing measurements for each sensor is modeled via a sequence of mutually independent random variables obeying the Bernoulli binary distribution law. To reduce the frequency of unnecessary data transmission and alleviate the communication burden, an event-triggered mechanism is introduced for the sensor node such that only some vitally important data is transmitted to its neighboring sensors when specific events occur. The objective of the problem addressed is to design a time-varying filter such that both the $\mathcal{H}_\infty$ requirements and the variance constraints are guaranteed over a given finite-horizon against the random parameter matrices, successive missing measurements and s-tochastic noises. By recurring to stochastic analysis techniques, sufficient conditions are established to ensure the existence of the time-varying filters whose gain matrices are then explicitly characterized in term of the solutions to a series of recursive matrix inequalities. A numerical simulation example is provided to illustrate the effectiveness of the developed event-triggered distributed filter design strategy.**

***Index Terms*—Distributed $\mathcal{H}_\infty$ filtering, error variance constraints, event-triggered mechanism, random parameter matrices, successive missing measurements, recursive matrix inequalities.**

I. Introduction

In the past few decades, the stochastic parameter systems have found applications in a variety of engineering domains such as digital control of chemical processes, radar control, missile track estimation and economic systems, see e.g. [3], [5], [7], [25], [37], [41]. On the other hand, the problem of filtering or state estimation for has long been a focus of research due to its engineering insights in many branches such as target tracking, orbit determination, image processing, fault diagnosis and biomedicine, see e.g. [2], [12], [42]. Many filter design approaches have been available in the literature such as [9], [10], [21], [23], [33], [35], [39], [40], among which the renowned Kalman filter and the $\mathcal{H}_\infty$ filter have proven to be most effective in dealing with Gaussian noises and energy-bounded noises, respectively. In systems with additive Gaussian noises, it is quite common that the performance requirements are described as the *upper bounds* on the filtering error variances, where the estimation error variance is no longer required to be the minimum as long as the engineering requirements are met [26]. In this case, the *variance-constrained* filters offer much design freedom that would facilitate the multi-objective design in order to reconcile between the performances of steady-state and transient behaviors, accuracy, robustness and disturbance rejection attenuation, etc.

Recent years have seen the widespread deployment of wireless sensor networks (WSNs) as a new generation of distributed embedded systems with a broad range of real-time applications [1], [13], [16], [29], [32], [36]. In the context of filtering or state estimation through a WSN, the measurement outputs are often collected through a network of smart sensing components installed in a spatial region of interest, where the individual sensor node can share the local information with its neighbors in the WSN. As one of the central issues in WSNs, the distributed filters aim to fuse the information not only from the individual sensor but also from its neighboring ones according to the given topology. So far, a number of distributed filtering algorithms have been proposed under various conditions on the target plant and the network topology, see e.g. [8], [27], [31], where most reported results have been concerned with time-invariant systems with fully available measurements. However, virtually almost all real-world systems are time-varying and, for in networked systems, the measurement signals may be missing during the network transmission resulting mainly from the limited bandwidth [19], [34], [44]. It is noted that the distributed filtering problem for time-varying systems with successive missing measurements has not received adequate attention yet despite its engineering importance.

For WSNs, the bandwidth of the wireless channels is typically limited and the capability of persistent power supply of each individual sensor is quite restrictive. As such, the scarcity of resources for WSNs has become a major concern and much attention has been devoted for the energy

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L. Wang and G. Wei are with Shanghai Key Lab of Modern Optical System, Department of Control Science and Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China. (Email: guoliang.wei1973@gmail.com)

Z. Wang is with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom. (Email: Zidong.Wang@brunel.ac.uk)

Q.-L. Han is with the School of Software and Electrical Engineering, Swinburne University of Technology, John Street, Hawthorn, Melbourne, VIC 3122, Australia.

saving purposes. It has been revealed that frequent data communications inevitably lead to a substantial proportion of energy consumption and, as compared to the traditional time-triggered communication protocols, the so-called event-triggered communication strategy would offer the possibility to avoid unnecessary waste of limited recourses [6], [11], [14], [18], [28], [45], [46]. The main idea of the event-triggered strategy is to transmit vitally important information only when certain event triggering condition is violated. Recently, a growing number of research results have been reported in the literature concerning event-triggered transmission schemes that have been applied in a variety of engineering systems, see e.g. [18], [30].

Summarizing the above discussions, a seemingly natural idea is to investigate the event-triggered distributed filter design problem for time-varying systems with mixed $\mathcal{H}_\infty$ and variance constraints subject to stochastic parameters and missing measurements. This appears to be a new yet challenging task because of the essential difficulties in 1) dealing with the asynchronous triggering of each individual sensor under a unified framework because each sensor is equipped with an event generator with separated triggering rate; 2) developing appropriate techniques to examine the impacts from the random parameter matrices onto the desired $\mathcal{H}_\infty$ performance requirement and the filtering error variance constraint; and 3) designing a set of easy-to-implement distributed filters that are insensitive to the randomly occurring successive missing measurements. We endeavor to handle the three identified difficulties in the present research.

In this paper, our research efforts are devoted to the problem of event-triggered distributed $\mathcal{H}_\infty$ filtering for a class of stochastic parameter systems with error variance constraints over a sensor network, where the underlying system is subject to successive missing measurements. *The main contributions of this paper are highlighted as follows: 1) the system model under consideration is quite general that covers time-varying parameters, random parameter perturbations and successive missing measurements, hence reflecting the reality more closely; 2) an event-triggering communication protocol is proposed to alleviate the network burden caused by the limited network bandwidth; 3) the mixed $\mathcal{H}_\infty$ performance index and error variance constraints are investigated, for the first time, for a class of time-varying systems; and 4) a novel filtering approach is developed in the form of recursive matrix inequalities that are suitable for online applications*.

The rest of this paper is organized as follows. In Section II, the target plant described by a discrete time-varying stochastic system with a network of N sensors is introduced and the problem under consideration is formulated. In Section III, the analysis and synthesis for the addressed event-based distributed filtering problem are investigated and a simulation example is given in IV to demonstrate the effectiveness of the main results. Finally, we conclude the paper in Section V.

Notation. The notations are quite standard. Throughout this paper, $\mathbb{Z}^+$, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the positive integer space, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. A^T represents the transpose of A. The notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). $\mathrm{diag}_N\{A_i\}$ stands for the block-diagonal matrix $\mathrm{diag}\{A_1,\ A_2\ ,\cdots, A_N\}$, and $\mathrm{vec}_N\{x_i\}$ denotes $[x_1 \quad x_2 \cdots x_N]$. I_n is the n-order identity matrix. $\mathbb{E}\{x\}$ stands for the expectation of stochastic variable x, and $\mathrm{Cov}\{x,\ y\}$ indicates the covariance of stochastic variables x and y. $\|x\|$ describes the Euclidean norm of a vector x, and $\lambda_{\max}(A)$ $(\lambda_{\min}(A))$ refers to the maximum eigenvalue (minimum eigenvalue) of matrix A.

II. Problem Formulation

For the WSN under consideration in this paper, the sensor nodes are distributed in space according to a fixed network topology represented by a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ of order N with the set of nodes (sensors) $\mathscr{V} = \{1, 2, \dots N\}$, set of edges $\mathscr{E} \subseteq \mathscr{V}\times\mathscr{V}$, and an adjacency matrix $\mathscr{A} = [a_{ij}]_{N\times N}$ with nonnegative adjacency elements a_{ij}. The edge $(i, j) \in \mathscr{E}$, if and only if, $a_{ij} > 0$, which represents that the ith node can receive the information from the jth node, otherwise, $a_{ij} = 0$. Furthermore, self-loops are not allowed here, i.e., $a_{ii} = 0$, for $i = 1, 2, \cdots , N$. The set of neighbors of node i is denoted by $\mathscr{N}_i = \{j \in \mathscr{V} |(i, j) \in \mathscr{E}\}$.

Consider the target plant described by the following discrete time-varying system:

$$\begin{cases} x_{k+1} = A_k x_k + B_k \omega_k, \\ \quad z_k = M_k x_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector of the target plant which is not directly available, z_k is the signal to be estimated, and $\omega_k \in \mathbb{R}^p$ denotes a zero mean Gaussian white noise sequence with covariance $S_k > 0$. $A_k \in \mathbb{R}^{n\times n}$ is the random parameter matrix to be defined later, and B_k, M_k are known time-varying matrices with appropriate dimensions.

The measurement output from the ith sensor is given by

$$y^i_k = \gamma^i_k (C^i_k x_k + D^i_k \omega_k) + (1 - \gamma^i_k) y^i_{k-1} \tag{2}$$

where $y^i_k \in \mathbb{R}^m$ is the measurement output of the ith sensor node. The random variable sequence γ^i_k characterizes the probability nature of the occurrence of successive missing measurements for each sensor and obeys the Bernoulli distribution with mathematical expectation $\bar{\alpha}^i$ and variance σ^2_i. $C^i_k \in \mathbb{R}^{m\times n}$ is a random parameter matrix and D^i_k is a known time-varying matrix with appropriate dimensions.

As in [7], the mutually independent random matrices A_k, C^i_k, which are also uncorrected with γ^i_k, have the following statistical properties:

$$\begin{aligned} &\mathbb{E}\{A_k\} = \bar{A}_k,\ \mathrm{Cov}\{a^k_{jr}, a^k_{ls}\} = T_{a^k_{jr} a^k_{ls}}, \\ &\mathbb{E}\{C^i_k\} = \bar{C}^i_k,\ \mathrm{Cov}\{c^{i,k}_{jr}, c^{i,k}_{ls}\} = T_{c^{i,k}_{jr} c^{i,k}_{ls}} \end{aligned} \tag{3}$$

where $T_{a^k_{jr} a^k_{ls}}$ and $T_{c^{i,k}_{jr} c^{i,k}_{ls}}$ are known scalars. a^k_{jr} and $c^{i,k}_{jr}$ are the (j, r)-th entries of matrices A_k and C^i_k, which can also be denoted as $[A_k]_{[j,r]}$ and $[C^i_k]_{[j,r]}$, respectively.

Remark 1: With the increasing complexity of real-time systems especially in process engineering, it is generally acknowledged that certain system parameters are subject to unavoidable perturbations that might result from changes in the interconnections of subsystems and modification of the operating point of a linearized model of a nonlinear system. In a networked environment, it is quite common that such parameter perturbations occur in a *random* manner due probably to random fluctuations of the network loads, random failures and repairs of the components as well as sudden environment changes, where the statistical properties of such random parameter perturbations could be acquired through statistical tests. This kind of systems is often referred to as the stochastic parameter systems as modeled in (1)-(2), where the probabilistic successive missing measurements are also taken into account. Both the stochastic parameters and the missing measurements, whose probability distribution laws could be obtained through statistical tests, are mainly caused by abrupt environmental changes in many engineering applications such as networked control systems, digital control of chemical processes as well as mobile robot localization systems.

For presentation convenience, we denote the estimation of x_k as $\hat{x}_k^i$ for sensor i. As discussed previously, for a sensor node i, the aim of the distributed filtering technique is to fuse the useful information not only from the local sensor i itself but also from its neighbors. In order to mitigate unnecessary data transmissions between the adjacent sensor nodes, an event-triggered communication mechanism is employed to determine whether the current estimated states need to be delivered to its neighbors or not. To this end, we define the event generator functions as follows:

$$\varphi(\hat{x}_k^i, \hat{x}_{k_i^t}^i, \theta_k^i) \triangleq (\hat{x}_k^i - \hat{x}_{k_i^t}^i)^T(\hat{x}_k^i - \hat{x}_{k_i^t}^i) - \theta_k^i \hat{x}_k^{iT}\hat{x}_k^i \le 0. \tag{4}$$

Here, k_i^t denotes the latest triggering instant for sensor i, the term $\hat{x}_k^i - \hat{x}_{k_i^t}^i$ is the difference of the ith sensor's estimation between the latest triggering instant and current sampling instant, and θ_k^i is a positive adjustable threshold. The event generators are triggered as long as the condition (4) is violated. Therefore, the sequence of event triggering instants $k_i^0 = 0 < k_i^1 < k_i^2 < \cdots < k_i^t < k_i^{t+1} < \cdots$ can be iteratively computed by

$$k_i^{t+1} = \min\{k \in \mathbb{N} | k > k_i^t, \varphi(\hat{x}_k^i, \hat{x}_{k_i^t}^j, \theta_k^i) > 0\}. \tag{5}$$

For system (1), the following distributed filter is adopted:

$$\begin{cases} \hat{x}_{k+1}^i = \bar{A}_k \hat{x}_k^i + K_k^i(y_k^i - \bar{\alpha}^i \bar{C}_k^i \hat{x}_k^i) \\ \qquad\qquad + \sum\limits_{j\in\mathcal{N}_i} h_{ij}(\hat{x}_{k_j^t}^j - \hat{x}_{k_i^t}^i), \\ \hat{z}_k^i = M_k \hat{x}_k^i \end{cases} \tag{6}$$

where $\hat{z}_k^i \in \mathbb{R}^m$ is the estimated output of the ith filter and K_k^i is the filter parameter to be determined.

Define $\rho_k^i \triangleq \hat{x}_k^i - \hat{x}_{k_i^t}^i$, $\tilde{C}_{\gamma_k}^i \triangleq \gamma_k^i C_k^i - \bar{\alpha}^i \bar{C}_k^i$ and the coupling configuration matrix $\mathscr{H} \triangleq [h_{ij}]_{N\times N}$ with $h_{ij} = a_{ij}$ (for $i \neq j$) and $h_{ii} = -\sum_{i=1, i\neq j}^N a_{ij}$. Let the filtering error and the output error be $e_k^i \triangleq x_k - \hat{x}_k^i$ and $\tilde{z}_k^i \triangleq z_k^i - \hat{z}_k^i$, respectively. Then, substituting (2) into (6) results in

$$\begin{cases} \hat{x}_{k+1}^i = \bar{A}_k \hat{x}_k^i + K_k^i \Big\{ \bar{\alpha}^i \bar{C}_k^i e_k^i + \tilde{C}_{\gamma_k}^i e_k^i + \tilde{C}_{\gamma_k}^i \hat{x}_k^i \\ \qquad + \bar{\alpha}^i D_k^i \omega_k + (\gamma_k^i - \bar{\alpha}^i) D_k^i \omega_k + (1-\bar{\alpha}^i) y_{k-1}^i \\ \qquad - (\gamma_k^i - \bar{\alpha}^i) y_{k-1}^i \Big\} + \sum\limits_{j=1}^N h_{ij} \hat{x}_k^j - \sum\limits_{j=1}^N h_{ij} \rho_k^j, \\ \hat{z}_k^i = M_k \hat{x}_k^i. \end{cases} \tag{7}$$

Letting $\tilde{A}_k \triangleq A_k - \bar{A}_k$, the error system can be easily obtained from (1) and (7) as follows:

$$\begin{cases} e_{k+1}^i = \bar{A}_k e_k^i + \tilde{A}_k e_k^i + \tilde{A}_k \hat{x}_k^i + B_k \omega_k \\ \qquad - K_k^i \Big\{ \bar{\alpha}^i \bar{C}_k^i e_k^i + \tilde{C}_{\gamma_k}^i e_k^i + \tilde{C}_{\gamma_k}^i \hat{x}_k^i + \bar{\alpha}^i D_k^i \omega_k \\ \qquad + (\gamma_k^i - \bar{\alpha}^i) D_k^i \omega_k + (1-\bar{\alpha}^i) y_{k-1}^i \\ \qquad - (\gamma_k^i - \bar{\alpha}^i) y_{k-1}^i \Big\} - \sum\limits_{j=1}^N h_{ij} \hat{x}_k^j + \sum\limits_{j=1}^N h_{ij} \rho_k^j, \\ \tilde{z}_k^i = M_k e_k^i. \end{cases} \tag{8}$$

For notation convenience, we set

$$\begin{aligned} e_k =& \mathrm{vec}_N^T\{e_k^{iT}\},\ \hat{x}_k = \mathrm{vec}_N^T\{\hat{x}_k^{iT}\},\ y_k = \mathrm{vec}_N^T\{y_k^{iT}\}, \\ z_k =& \mathrm{vec}_N^T\{z_k^{iT}\},\ \rho_k = \mathrm{vec}_N^T\{\rho_k^{iT}\},\ \mathcal{K}_k = \mathrm{diag}_N\{K_k^i\}, \\ \bar{\alpha} =& \mathrm{diag}_N\{\bar{\alpha}^i I_n\},\ \bar{\mathcal{A}}_k = I_N \otimes \bar{A}_k,\ \tilde{\mathcal{A}}_k = I_N \otimes \tilde{A}_k, \\ \mathcal{B}_k =& \mathbb{1} \otimes B_k,\ \mathcal{M}_k = I_N \otimes M_k,\ \mathcal{C}_k = \mathrm{diag}_N\{C_k^i\}, \\ \bar{\mathcal{C}}_k =& \mathrm{diag}_N\{\bar{C}_k^i\},\ \tilde{\mathcal{C}}_k = \mathrm{diag}_N\{\tilde{C}_k^i\} = \mathcal{C}_k - \bar{\mathcal{C}}_k, \\ \mathcal{D}_k =& \mathrm{vec}_N^T\{D_k^{iT}\},\ \theta_k = \mathrm{diag}_N\{\theta_k^i I_n\}, \\ \mathcal{N}_i =& \mathrm{diag}\{\underbrace{0\cdots 0}_{i-1}\ I_m\ \underbrace{0\cdots 0}_{N-i}\}, \\ e_{in} =& \mathrm{vec}_n^T\{\underbrace{0\cdots 0}_{i-1}\ 1\ \underbrace{0\cdots 0}_{n-i}\}, \\ \bar{e}_{in} =& \mathrm{vec}_N^T\{\underbrace{0\cdots 0}_{i-1}\ I_n\ \underbrace{0\cdots 0}_{N-i}\} \end{aligned} \tag{9}$$

where "$\mathbb{1}$" is a column vector with each element of one.

By using the matrix Kronecker product and considering $\tilde{C}_{\gamma_k}^i = (\gamma_k^i - \bar{\alpha}^i)\bar{C}_k^i + \gamma_k^i \tilde{C}_k^i$, we have the following filtering error system directly from (8)

$$\begin{cases} e_{k+1} = \mathscr{A}_k e_k + \hat{\mathscr{A}}_k \hat{x}_k + \mathscr{K}_k y_{k-1} + \mathscr{B}_k \omega_k + \bar{\mathscr{H}} \rho_k, \\ \tilde{z}_k = \mathcal{M}_k e_k \end{cases} \tag{10}$$

where

$$\begin{aligned} \mathscr{A}_k =& \bar{\mathcal{A}}_k + \tilde{\mathcal{A}}_k - \bar{\alpha}\mathcal{K}_k \bar{\mathcal{C}}_k - \sum_{i=1}^N (\gamma_k^i - \bar{\alpha}^i)\mathcal{K}_k \mathcal{N}_i \bar{\mathcal{C}}_k \\ &- \sum_{i=1}^N \gamma_k^i \mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k, \\ \hat{\mathscr{A}}_k =& \tilde{\mathcal{A}}_k - \sum_{i=1}^N (\gamma_k^i - \bar{\alpha}^i)\mathcal{K}_k \mathcal{N}_i \bar{\mathcal{C}}_k - \sum_{i=1}^N \gamma_k^i \mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k - \bar{\mathscr{H}}, \end{aligned}$$

$$\mathscr{B}_k = \mathcal{B}_k - \bar{\alpha}\mathcal{K}_k\mathcal{D}_k - \sum_{i=1}^{N}(\gamma_k^i - \bar{\alpha}^i)\mathcal{K}_k\mathcal{N}_i\mathcal{D}_k,$$
$$\mathscr{K}_k = \sum_{i=1}^{N}(\gamma_k^i - \bar{\alpha}^i)\mathcal{K}_k\mathcal{N}_i - (I_{Nn} - \bar{\alpha})\mathcal{K}_k, \tag{11}$$
$$\bar{\mathscr{H}} = \mathscr{H} \otimes I_n.$$

Remark 2: It is worth mentioning that a frequently used approach to deal with the filtering issues with missing measurements is to *augment* the system states, filter states and measurement outputs in a compact form, see e.g. [7]. Such an augmentation approach will inevitably lead to a high dimension, thereby imposing extra load of computation. Here, instead of state augmentation, we examine the error dynamics directly and, as will be shown later, the computation cost is reduced via a recursive algorithm.

To facilitate the further development, let us introduce a useful lemma as follows.

Lemma 1: [38] Let $Z_0(s)$, $Z_1(s), \cdots, Z_p(s)$ be quadratic functions of $s \in \mathbb{R}^n$ and $Z_i(s) = s^TQ_is$ $(i = 0, 1, \cdots, p)$ with $Q_i = Q_i^T$. If there exist positive scalars $\epsilon_1, \epsilon_2, \cdots, \epsilon_p > 0$ such that

$$Z_0(s) - \sum_{i=1}^{p}\epsilon_iZ_i(s) \leq 0, \tag{12}$$

then the implication

$$Z_1(s) \leq 0, Z_2(s) \leq 0, \cdots, Z_p(s) \leq 0 \Rightarrow Z_0(s) \leq 0$$

holds.

The filtering error covariance matrix governed by (10) is defined as

$$\bar{X}_k \triangleq \mathbb{E}\{e_ke_k^T\}. \tag{13}$$

In this paper, our purpose is to design a set of event-based distributed filters in form of (6) such that the filtering error system (10) satisfies the following two constraints simultaneously over a finite horizon $[0, L]$.

(R1) Given the disturbance attenuation level γ, matrix $W > 0$ and initial error e_0, the $\mathcal{H}_\infty$ performance constraint

$$\frac{1}{N}\sum_{k=0}^{L}\mathbb{E}\{\|\tilde{z}_k\|^2\} \leq \gamma^2\sum_{k=0}^{L}\mathbb{E}\left\{\|\omega_k\|^2\right\} + \gamma^2\mathbb{E}\left\{e_0^TWe_0\right\} \tag{14}$$

is satisfied in mean square subject to random parameter matrices and successive missing measurements.

(R2) At each sampling instant k, the error covariance of the filtering error system (10) satisfies

$$\bar{X}_k := \mathbb{E}\{e_ke_k^T\} \leq Q_k \qquad \forall k \in [0, L] \tag{15}$$

where Q_k is a sequence of positive definite matrices that are prespecified according to the engineering requirements.

III. Main Results

A. $\mathcal{H}_\infty$ Performance Analysis

Let us start with the $\mathcal{H}_\infty$ performance analysis for the filtering error system (10). By using the stochastic analysis techniques, a sufficient condition is presented in the following theorem under which the $\mathcal{H}_\infty$ performance index is satisfied.

Theorem 1: Consider the discrete-time stochastic parameter system (1). Let the distributed filter parameters $\{K_k^i\}_{0\leq k\leq L}$, the initial positive definite matrix $W > 0$ and a prescribed disturbance attenuation level γ be given. The filtering error system (10) achieves the $\mathcal{H}_\infty$ performance constraint (14) for all nonzero ω_k if there exist some families of positive scalars $\{\lambda_k\}_{0\leq k\leq L}$, $\{\delta_k^i\}_{0\leq k\leq L+1}$, $\{\varepsilon_k\}_{1\leq k\leq L+1}$, and positive matrices $\{\mathcal{P}_k\}_{1\leq k\leq L+1}$ satisfying the following recursive matrix inequalities:

$$\begin{cases} \Pi_k = \Pi_k^0 + \mathscr{L}_k^T\mathcal{P}_{k+1}\mathscr{L}_k + \mathscr{M}_k^T\mathcal{P}_{k+1}\mathscr{M}_k \\ \qquad + \sum_{i=1}^{N}(\mathscr{N}_k^{iT}\mathcal{P}_{k+1}\mathscr{N}_k^i + \mathscr{R}_k^{iT}\mathcal{P}_{k+1}\mathscr{R}_k^i) \leq 0, & \text{(16a)} \\ \sum_{i=1}^{N}\bar{\alpha}_i\mathcal{N}_i^T\mathcal{K}_k^T\mathcal{P}_{k+1}\mathcal{K}_k\mathcal{N}_i - \delta_{mk} \leq 0, & \text{(16b)} \\ \mathcal{P}_{k+1} - \varepsilon_{k+1}I_{Nn} \leq 0 & \text{(16c)} \end{cases}$$

for all $0 \leq k \leq L$ with initial condition

$$\mathbb{E}\{e_0^T\mathcal{P}_0e_0\} + \mu_0 + \nu_0 \leq \gamma^2\mathbb{E}\{e_0^TWe_0\} \tag{17}$$

where

$$\Pi_k^0 = \begin{bmatrix} \Pi_k^{01} & * & * & * & * \\ \Pi_k^{03} & \Pi_k^{02} & * & * & * \\ 0 & 0 & \tilde{\nu}_k & * & * \\ 0 & 0 & 0 & -\gamma^2I_p & * \\ 0 & 0 & 0 & 0 & -\lambda_kI_{Nn} \end{bmatrix},$$
$$\mathscr{L}_k = [0, 0, 0, \mathcal{B}_k - \bar{\alpha}\mathcal{K}_k\mathcal{D}_k, 0],$$
$$\mathscr{R}_k^i = [0, 0, 0, -\sigma_i\mathcal{K}_k\mathcal{N}_i\mathcal{D}_k, 0],$$
$$\mathscr{M}_k = [\bar{\mathcal{A}}_k - \bar{\alpha}\mathcal{K}_k\bar{\mathcal{C}}_k, -\bar{\mathscr{H}}\hat{x}_k, -(I_{Nn} - \bar{\alpha})\mathcal{K}_ky_{k-1}, 0, \bar{\mathscr{H}}],$$
$$\mathscr{N}_k^i = [-\sigma_i\mathcal{K}_k\mathcal{N}_i\bar{\mathcal{C}}_k, -\sigma_i\mathcal{K}_k\mathcal{N}_i\bar{\mathcal{C}}_k\hat{x}_k, \sigma_i\mathcal{K}_k\mathcal{N}_iy_{k-1}, 0, 0],$$
$$\Pi_k^{01} = \frac{1}{N}\mathcal{M}_k^T\mathcal{M}_k - \mathcal{P}_k + \varepsilon_{k+1}\Theta_{ak} + \delta_{nk}\Theta_{ck},$$
$$\delta_{nk} = \text{diag}_N\{\delta_k^iI_n\},\ \tilde{\mu}_k = \mu_{k+1} - \mu_k,\ \tilde{\nu}_k = \nu_{k+1} - \nu_k,$$
$$\Pi_k^{02} = \tilde{\mu}_k + \lambda_k\hat{x}_k^T\theta_k\hat{x}_k + \varepsilon_{k+1}\hat{x}_k^T\Theta_{ak}\hat{x}_k + \hat{x}_k^T\delta_{nk}\Theta_{ck}\hat{x}_k,$$
$$\Pi_k^{03} = \varepsilon_{k+1}\hat{x}_k^T\Theta_{ak}^T + \delta_{nk}\hat{x}_k^T\Theta_{ck}^T,\ \Theta_{ak} = I_N \otimes \hat{\Theta}_{ak},$$
$$\Theta_{ck} = \text{diag}_N\{\hat{\Theta}_{ck}^i\},\ \hat{\Theta}_{ak} = \sum_{i=1}^{n}\mathcal{T}_{a^{i,k}},$$
$$\hat{\Theta}_{ck}^i = \sum_{j=1}^{m}\mathcal{T}_{c^{j,k}}^i,\ \mu_k = \frac{\mu_0}{\sqrt{k+1}},\ \nu_k = \frac{\nu_0}{\sqrt{k+1}} \tag{18}$$

with μ_0 and ν_0 being given positive scalars. Here, $\mathcal{T}_{a^{i,k}} \in \mathbb{R}^{n\times n}$ and $\mathcal{T}_{c^{j,k}}^i \in \mathbb{R}^{n\times n}$ are two symmetric matrices with the (r, s)-th entries $T_{a_{ir}^ka_{is}^k}$ and $T_{c_{jr}^{i,k}c_{js}^{i,k}}$, respectively.

Proof: First, we define a positive real-value function $J_k = e_k^T\mathcal{P}_ke_k + \mu_k + \nu_k$. By noticing the uncorrelatedness between

γ_k^i, A_k, C_k^i and ω_k, we have from (10) that

$$\begin{aligned}
&\mathcal{J}_k \triangleq \mathbb{E}\{J_{k+1}-J_k\}\\
&=\mathbb{E}\{(\mathscr{A}_k e_k+\hat{\mathscr{A}}_k\hat{x}_k+\mathscr{K}_k y_{k-1}+\mathscr{B}_k\omega_k+\bar{\mathscr{H}}\rho_k)^T\mathcal{P}_{k+1}\\
&\quad\times(\mathscr{A}_k e_k+\hat{\mathscr{A}}_k\hat{x}_k+\mathscr{K}_k y_{k-1}+\mathscr{B}_k\omega_k+\bar{\mathscr{H}}\rho_k)\\
&\quad-e_k^T\mathcal{P}_k e_k+\tilde{\mu}_k+\tilde{\nu}_k\}\\
&=\mathbb{E}\{e_k^T\mathscr{A}_k^T\mathcal{P}_{k+1}\mathscr{A}_k e_k+\hat{x}_k^T\hat{\mathscr{A}}_k^T\mathcal{P}_{k+1}\hat{\mathscr{A}}_k\hat{x}_k+y_{k-1}^T\mathscr{K}_k^T\\
&\quad\times\mathcal{P}_{k+1}\mathscr{K}_k y_{k-1}+\omega_k^T\mathscr{B}_k^T\mathcal{P}_{k+1}\mathscr{B}_k\omega_k\\
&\quad+\rho_k^T\bar{\mathscr{H}}_k^T\mathcal{P}_{k+1}\bar{\mathscr{H}}_k\rho_k+2e_k^T\mathscr{A}_k^T\mathcal{P}_{k+1}\hat{\mathscr{A}}_k\hat{x}_k\\
&\quad+2e_k^T\mathscr{A}_k^T\mathcal{P}_{k+1}\mathscr{K}_k y_{k-1}+2e_k^T\mathscr{A}_k^T\mathcal{P}_{k+1}\bar{\mathscr{H}}_k\rho_k\\
&\quad+2\hat{x}_k^T\hat{\mathscr{A}}_k^T\mathcal{P}_{k+1}\mathscr{K}_k y_{k-1}+2\hat{x}_k^T\hat{\mathscr{A}}_k^T\mathcal{P}_{k+1}\bar{\mathscr{H}}_k\rho_k\\
&\quad+2y_{k-1}^T\mathscr{K}_k^T\mathcal{P}_{k+1}\bar{\mathscr{H}}_k\rho_k-e_k^T\mathcal{P}_k e_k+\tilde{\mu}_k+\tilde{\nu}_k\}\\
&=\mathbb{E}\{e_k^T[\bar{\mathcal{A}}_k^T\mathcal{P}_{k+1}\bar{\mathcal{A}}_k+(\bar{\alpha}\mathcal{K}_k\bar{\mathcal{C}}_k)^T\mathcal{P}_{k+1}(\bar{\alpha}\mathcal{K}_k\bar{\mathcal{C}}_k)\\
&\quad+\Phi_{1k}^T\mathcal{P}_{k+1}\Phi_{1k}-2\bar{\mathcal{A}}_k^T\mathcal{P}_{k+1}\bar{\alpha}\mathcal{K}_k\bar{\mathcal{C}}_k-\mathcal{P}_k]e_k\\
&\quad+\hat{x}_k^T[\Phi_{1k}^T\mathcal{P}_{k+1}\Phi_{1k}+\bar{\mathscr{H}}^T\mathcal{P}_{k+1}\bar{\mathscr{H}}]\hat{x}_k+y_{k-1}^T[\Phi_{2k}^T\mathcal{P}_{k+1}\\
&\quad\times\Phi_{2k}+((I_{Nn}-\bar{\alpha})\mathcal{K}_k)^T\mathcal{P}_{k+1}(I_{Nn}-\bar{\alpha})\mathcal{K}_k]y_{k-1}\\
&\quad+\omega_k^T[(\mathcal{B}_k-\bar{\alpha}\mathcal{K}_k\mathcal{D}_k)^T\mathcal{P}_{k+1}(\mathcal{B}_k-\bar{\alpha}\mathcal{K}_k\mathcal{D}_k)\\
&\quad+\mathcal{D}_k^T\Phi_{2k}^T\mathcal{P}_{k+1}\Phi_{2k}\mathcal{D}_k]\omega_k+\rho_k^T\bar{\mathscr{H}}^T\mathcal{P}_{k+1}\bar{\mathscr{H}}\rho_k\\
&\quad+2e_k^T[-(\bar{\mathcal{A}}_k-\bar{\alpha}\mathcal{K}_k\bar{\mathcal{C}}_k)^T\mathcal{P}_{k+1}\bar{\mathscr{H}}+\Phi_{1k}^T\mathcal{P}_{k+1}\Phi_{1k}]\hat{x}_k\\
&\quad+2e_k^T[-(\bar{\mathcal{A}}_k-\bar{\alpha}\mathcal{K}_k\bar{\mathcal{C}}_k)^T\mathcal{P}_{k+1}(I_{Nn}-\bar{\alpha})\mathcal{K}_k\\
&\quad-\Phi_{1k}^{1T}\mathcal{P}_{k+1}\Phi_{2k}]y_{k-1}+2e_k^T(\bar{\mathcal{A}}_k-\bar{\alpha}\mathcal{K}_k\bar{\mathcal{C}}_k)^T\mathcal{P}_{k+1}\bar{\mathscr{H}}\rho_k\\
&\quad+2\hat{x}_k^T[\bar{\mathscr{H}}^T\mathcal{P}_{k+1}(I_{Nn}-\bar{\alpha})\mathcal{K}_k-\Phi_{1k}^T\mathcal{P}_{k+1}\Phi_{2k}]y_{k-1}\\
&\quad-2\hat{x}_k^T\bar{\mathscr{H}}^T\mathcal{P}_{k+1}\bar{\mathscr{H}}\rho_k-2y_{k-1}^T\mathcal{K}_k^T(I_{Nn}-\bar{\alpha})^T\mathcal{P}_{k+1}\bar{\mathscr{H}}\rho_k\\
&\quad+(e_k+\hat{x}_k)^T\tilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\tilde{\mathcal{A}}_k(e_k+\hat{x}_k)+\tilde{\mu}_k+\tilde{\nu}_k\}
\end{aligned}\tag{19}$$

where $\Phi_{1k}=\Phi_{1k}^1+\Phi_{1k}^2$, $\Phi_{2k}=\sum_{i=1}^N(\gamma_k^i-\bar{\alpha}^i)\mathcal{K}_k\mathcal{N}_i$, $\Phi_{1k}^1=\sum_{i=1}^N(\gamma_k^i-\bar{\alpha}^i)\mathcal{K}_k\mathcal{N}_i\bar{\mathcal{C}}_k$, and $\Phi_{1k}^2=\sum_{i=1}^N\gamma_k^i\mathcal{K}_k\mathcal{N}_i\tilde{\mathcal{C}}_k$.

Next, by applying the property of matrix covariance, we obtain from (3) and (16c) that

$$\begin{aligned}
&\mathbb{E}\{(e_k+\hat{x}_k)^T\tilde{\mathcal{A}}_k^T\mathcal{P}_{k+1}\tilde{\mathcal{A}}_k(e_k+\hat{x}_k)\}\\
&\leq\varepsilon_{k+1}\mathbb{E}\{(e_k+\hat{x}_k)^T\tilde{\mathcal{A}}_k^T\tilde{\mathcal{A}}_k(e_k+\hat{x}_k)\}\\
&=\varepsilon_{k+1}(e_k+\hat{x}_k)^T\Theta_{ak}(e_k+\hat{x}_k).
\end{aligned}\tag{20}$$

On the other hand, it can infer from the definitions of $\Phi_1(k)$, $\Phi_2(k)$ and (16b) that

$$\begin{aligned}
&\mathbb{E}\{\Phi_{1k}^T\mathcal{P}_{k+1}\Phi_{1k}\}\\
&=\mathbb{E}\{\Phi_{1k}^{1T}\mathcal{P}_{k+1}\Phi_{1k}^1+\Phi_{1k}^{2T}\mathcal{P}_{k+1}\Phi_{1k}^2\}\\
&=\sum_{i=1}^N\sigma_i^2\bar{\mathcal{C}}_k^T\mathcal{N}_i^T\mathcal{K}_k^T\mathcal{P}_{k+1}\mathcal{K}_k\mathcal{N}_i\bar{\mathcal{C}}_k\\
&\quad+\mathbb{E}\Big\{\sum_{i=1}^N\bar{\alpha}_i\tilde{\mathcal{C}}_k^T\mathcal{N}_i^T\mathcal{K}_k^T\mathcal{P}_{k+1}\mathcal{K}_k\mathcal{N}_i\tilde{\mathcal{C}}_k\Big\}\\
&\leq\sum_{i=1}^N\sigma_i^2\bar{\mathcal{C}}_k^T\mathcal{N}_i^T\mathcal{K}_k^T\mathcal{P}_{k+1}\mathcal{K}_k\mathcal{N}_i\bar{\mathcal{C}}_k+\delta_{nk}\Theta_{ck},
\end{aligned}\tag{21}$$

$$\begin{aligned}
&\mathbb{E}\{\Phi_{1k}^T\mathcal{P}_{k+1}\Phi_{2k}\}=\mathbb{E}\{\Phi_{1k}^{1T}\mathcal{P}_{k+1}\Phi_{2k}\}\\
&=\sum_{i=1}^N\sigma_i^2\bar{\mathcal{C}}_k^T\mathcal{N}_i^T\mathcal{K}_k^T\mathcal{P}_{k+1}\mathcal{K}_k\mathcal{N}_i,
\end{aligned}\tag{22}$$

$$\mathbb{E}\{\Phi_{2k}^T\mathcal{P}_{k+1}\Phi_{2k}\}=\sum_{i=1}^N\sigma_i^2\mathcal{N}_i^T\mathcal{K}_k^T\mathcal{P}_{k+1}\mathcal{K}_k\mathcal{N}_i.\tag{23}$$

Consequently, based on (20)-(23), adding the zero term

$$\frac{1}{N}\mathbb{E}\{\|\tilde{z}_k\|^2\}-\gamma^2\mathbb{E}\{\|\omega_k\|^2\}-\Big(\frac{1}{N}\mathbb{E}\{\|\tilde{z}_k\|^2\}-\gamma^2\mathbb{E}\{\|\omega_k\|^2\}\Big)$$

to the right-hand side of (19) leads to

$$\mathcal{J}_k\leq\mathbb{E}\left\{\xi_k^T\bar{\Pi}_k\xi_k\right\}-\mathbb{E}\left\{\frac{1}{N}\|\tilde{z}_k\|^2-\gamma^2\|\omega_k\|^2\right\}\tag{24}$$

where

$$\begin{aligned}
\xi_k&=[e_k^T\ \ 1\ \ 1\ \ \omega_k^T\ \ \rho_k^T]^T,\ \bar{\Pi}_k=\bar{\Pi}_k^0+\bar{\Pi}_k^1,\\
\bar{\Pi}_k^0&=\Pi_k^0-\mathrm{diag}\{0,\lambda_k\hat{x}_k^T\theta_k\hat{x}_k,0,0,-\lambda_k I_{Nn}\},\\
\bar{\Pi}_k^1&=\mathscr{L}_k^T\mathcal{P}_{k+1}\mathscr{L}_k+\mathscr{M}_k^T\mathcal{P}_{k+1}\mathscr{M}_k\\
&\quad+\sum_{i=1}^N\mathscr{N}_k^{iT}\mathcal{P}_{k+1}\mathscr{N}_k^i+\sum_{i=1}^N\mathscr{R}_k^{iT}\mathcal{P}_{k+1}\mathscr{R}_k^i.
\end{aligned}\tag{25}$$

Meanwhile, by considering the event-triggering condition (4), one obtains that

$$\rho_k^T\rho_k-\hat{x}_k^T\theta_k\hat{x}_k\leq 0.\tag{26}$$

Thus, according to Lemma 1, it can be readily seen from (24) that

$$\begin{aligned}
\mathcal{J}_k&\leq\mathbb{E}\{\xi_k^T\bar{\Pi}_k\xi_k-\lambda_k(\rho_k^T\rho_k-\hat{x}_k^T\theta_k\hat{x}_k)\}\\
&\quad-\mathbb{E}\left\{\frac{1}{N}\|\tilde{z}_k\|^2-\gamma^2\|\omega_k\|^2\right\}\\
&=\mathbb{E}\{\xi_k^T\Pi_k\xi_k\}-\mathbb{E}\left\{\frac{1}{N}\|\tilde{z}_k\|^2-\gamma^2\|\omega_k\|^2\right\}.
\end{aligned}\tag{27}$$

It follows from (16a) and (27) that

$$\mathbb{E}\{J_{k+1}\}-\mathbb{E}\{J_k\}+\mathbb{E}\left\{\frac{1}{N}\|\tilde{z}_k\|^2-\gamma^2\|\omega_k\|^2\right\}\leq 0.\tag{28}$$

Subsequently, summing up (28) from 0 to L with respect to k yields

$$\frac{1}{N}\sum_{k=0}^L\mathbb{E}\{\|\tilde{z}_k\|^2\}\leq\sum_{k=0}^L\gamma^2\mathbb{E}\{\|\omega_k\|^2\}+\mathbb{E}\{e_0^T\mathcal{P}_0e_0\}+\mu_0+\nu_0.\tag{29}$$

The $\mathcal{H}_\infty$ performance is satisfied for (10) by substituting (17) into (29), and the proof of this theorem is now complete. ■

B. Variance Analysis

Having discussed the $\mathcal{H}_\infty$ performance analysis for the addressed filtering error system (10), we will now focus our attention on the variance analysis.

Theorem 2: Consider system (1) and let the filter parameter in (6) be given. We have $\bar{X}_k\leq Q_k$ for $1\leq k\leq L+1$, if there exists a family of positive definite matrices $\{Q_k\}_{1\leq k\leq L+1}$ satisfying the following matrix inequalities:

$$\Psi(Q_k)\leq Q_{k+1}\tag{30}$$

with initial condition $Q_0 = \bar{X}_0$. Here

$$\begin{aligned}\Psi(Q_k) = {} & 4\mathscr{O}_k + 4\mathscr{P}_k + 4\mathscr{S}_k + 4\mathscr{T}_k + 4\mathscr{Q}_k \\ & + 4\mathscr{U}_k + 4\mathscr{V}_k + 4\bar{\mathscr{H}}\hat{x}_k\hat{x}_k^T\bar{\mathscr{H}}^T \\ & + 4(I_{Nn} - \bar{\alpha})\mathcal{K}_k y_{k-1}y_{k-1}^T((I_{Nn} - \bar{\alpha}) \\ & \times \mathcal{K}_k)^T + 4\mathscr{R}_k + \mathscr{W}_k + 4\Xi_k\bar{\mathscr{H}}\bar{\mathscr{H}}^T\end{aligned} \tag{31}$$

where

$$\begin{aligned}\mathscr{O}_k &= (\bar{\mathcal{A}}_k - \bar{\alpha}\mathcal{K}_k\bar{\mathcal{C}}_k)Q_k(\bar{\mathcal{A}}_k - \bar{\alpha}\mathcal{K}_k\bar{\mathcal{C}}_k)^T, \\ \mathscr{P}_k &= \sum_{i=1}^{N}\sigma_i^2(\mathcal{K}_k\mathcal{N}_i\bar{\mathcal{C}}_k)Q_k(\mathcal{K}_k\mathcal{N}_i\bar{\mathcal{C}}_k)^T, \\ \mathscr{Q}_k &= \sum_{i=1}^{N}\sigma_i^2(\mathcal{K}_k\mathcal{N}_i\bar{\mathcal{C}}_k)\hat{x}_k\hat{x}_k^T(\mathcal{K}_k\mathcal{N}_i\bar{\mathcal{C}}_k)^T, \\ \mathscr{R}_k &= \sum_{i=1}^{N}\sigma_i^2(\mathcal{K}_k\mathcal{N}_i)y_{k-1}y_{k-1}^T(\mathcal{K}_k\mathcal{N}_i)^T, \\ \mathscr{S}_k &= \eta_k\mathcal{K}_k\vec{\Theta}_{ck}\mathcal{K}_k^T, \ \mathscr{V}_k = \ \beta_k\mathcal{K}_k\vec{\Theta}_{ck}\mathcal{K}_k^T, \\ \mathscr{W}_k &= (\mathcal{B}_k - \bar{\alpha}\mathcal{K}_k\mathcal{D}_k)S_k(\mathcal{B}_k - \bar{\alpha}\mathcal{K}_k\mathcal{D}_k)^T \\ &\quad + \sum_{i=1}^{N}\sigma_i^2(\mathcal{K}_k\mathcal{N}_i\mathcal{D}_k)S_k(\mathcal{K}_k\mathcal{N}_i\mathcal{D}_k)^T, \\ [\mathscr{T}_k]_{[r,s]} &= \mathscr{T}_k^{(r,s)} = \mathbb{E}\{\tilde{A}_kQ_k^{(r,s)}\tilde{A}_k^T\}, \\ [\mathscr{U}_k]_{[r,s]} &= \mathscr{U}_k^{(r,s)} = \mathbb{E}\{\tilde{A}_k(\hat{x}_k\hat{x}_k^T)^{(r,s)}\tilde{A}_k^T\}, \\ [\mathscr{T}_k^{(r,s)}]_{[l,m]} &= \sum_{i,j=1}^{n}T_{a_{li}^k\cdot a_{mj}^k}e_{in}^T\bar{e}_{rn}^TQ_k\bar{e}_{sn}e_{jn}, \\ [\mathscr{U}_k^{(r,s)}]_{[l,m]} &= \sum_{i,j=1}^{n}T_{a_{li}^k\cdot a_{mj}^k}e_{in}^T\bar{e}_{rn}^T\hat{x}_k\hat{x}_k^T\bar{e}_{sn}e_{jn}, \\ \Xi_k &= \hat{x}_k^T\theta_k\hat{x}_k, \ \eta_k = \lambda_{\max}\{Q_k\}, \\ \beta_k &= \lambda_{\max}\{\hat{x}_k\hat{x}_k^T\}, \ \vec{\Theta}_{ck} = \mathrm{diag}_N\{\bar{\alpha}^i\vec{\Theta}_{ck}^i\}\end{aligned}$$

and $Q_k^{(r,s)}$ $(r,s = 1,\cdots,N)$ is the (r,s)-th block element of matrix Q_k, $\vec{\Theta}_{ck}^i = \sum_{j=1}^{n}\vec{\mathcal{T}}_{c_j^{i,k}}$. $\vec{\mathcal{T}}_{c_j^{i,k}} \in \mathbb{R}^{m\times m}$ is a symmetric matrix with the (s,t)-th entry $T_{c_{sj}^{i,k}c_{tj}^{i,k}}$.

Proof: Noticing the filtering error system (10), the corresponding evolution of $\bar{X}_k$ is governed by

$$\begin{aligned}\bar{X}_{k+1} &= \mathbb{E}\{e_{k+1}e_{k+1}^T\} \\ &= \mathbb{E}\{(\mathscr{A}_ke_k + \hat{\mathscr{A}}_k\hat{x}_k + \mathscr{K}_ky_{k-1} + \mathscr{B}_k\omega_k + \bar{\mathscr{H}}\rho_k) \\ &\quad \times(\mathscr{A}_ke_k + \hat{\mathscr{A}}_k\hat{x}_k + \mathscr{K}_ky_{k-1} + \mathscr{B}_k\omega_k + \bar{\mathscr{H}}\rho_k)^T\} \\ &=\mathbb{E}\{\mathscr{A}_ke_ke_k^T\mathscr{A}_k^T + \hat{\mathscr{A}}_k\hat{x}_k\hat{x}_k^T\hat{\mathscr{A}}_k^T + \mathscr{K}_ky_{k-1}y_{k-1}^T\mathscr{K}_k^T \\ &\quad + \mathscr{B}_kS_k\mathscr{B}_k^T + \bar{\mathscr{H}}\rho_k\rho_k^T\bar{\mathscr{H}}^T + \mathscr{A}_ke_k\hat{x}_k^T\hat{\mathscr{A}}_k^T \\ &\quad + \hat{\mathscr{A}}_k\hat{x}_ke_k^T\mathscr{A}_k^T + \mathscr{A}_ke_ky_{k-1}^T\mathscr{K}_k^T + \mathscr{K}_ky_{k-1}e_k^T\mathscr{A}_k^T \\ &\quad + \mathscr{A}_ke_k\rho_k^T\bar{\mathscr{H}}^T + \bar{\mathscr{H}}\rho_ke_k^T\mathscr{A}_k^T + \hat{\mathscr{A}}_k\hat{x}_ky_{k-1}^T\mathscr{K}_k^T \\ &\quad + \mathscr{K}_ky_{k-1}\hat{x}_k^T\hat{\mathscr{A}}_k^T + \hat{\mathscr{A}}_k\hat{x}_k\rho_k^T\bar{\mathscr{H}}^T + \bar{\mathscr{H}}\rho_k\hat{x}_k^T\hat{\mathscr{A}}_k^T \\ &\quad + \mathscr{K}_ky_{k-1}\rho_k^T\bar{\mathscr{H}}^T + \bar{\mathscr{H}}\rho_ky_{k-1}^T\mathscr{K}_k^T\}.\end{aligned} \tag{32}$$

Then, substituting (11) into (32) and applying the property of conditional expectation as well as the elementary inequality, one has

$$\begin{aligned}\bar{X}_{k+1} &\le \mathbb{E}\{\mathscr{A}_ke_ke_k^T\mathscr{A}_k^T + \hat{\mathscr{A}}_k\hat{x}_k\hat{x}_k^T\hat{\mathscr{A}}_k^T + \mathscr{K}_ky_{k-1}y_{k-1}^T\mathscr{K}_k^T \\ &\quad + \mathscr{B}_kS_k\mathscr{B}_k^T + \bar{\mathscr{H}}\rho_k\rho_k^T\bar{\mathscr{H}}^T + \hat{\mathscr{A}}_k\hat{x}_k\hat{x}_k^T\hat{\mathscr{A}}_k^T \\ &\quad + \mathscr{A}_ke_ke_k^T\mathscr{A}_k^T + \mathscr{K}_ky_{k-1}y_{k-1}^T\mathscr{K}_k^T + \mathscr{A}_ke_ke_k^T\mathscr{A}_k^T \\ &\quad + \bar{\mathscr{H}}\rho_k\rho_k^T\bar{\mathscr{H}}^T + \mathscr{A}_ke_ke_k^T\mathscr{A}_k^T + \hat{\mathscr{A}}_k\hat{x}_k\hat{x}_k^T\mathscr{A}_k^T \\ &\quad + \mathscr{K}_ky_{k-1}y_{k-1}^T\mathscr{K}_k^T + \hat{\mathscr{A}}_k\hat{x}_k\hat{x}_k^T\hat{\mathscr{A}}_k^T + \bar{\mathscr{H}}\rho_k\rho_k^T\bar{\mathscr{H}}^T \\ &\quad + \mathscr{K}_ky_{k-1}y_{k-1}^T\mathscr{K}_k^T + \bar{\mathscr{H}}\rho_k\rho_k^T\bar{\mathscr{H}}^T\} \\ &= \mathbb{E}\{4\mathscr{A}_ke_ke_k^T\mathscr{A}_k^T + 4\hat{\mathscr{A}}_k\hat{x}_k\hat{x}_k^T\hat{\mathscr{A}}_k^T + 4\mathscr{K}_ky_{k-1}y_{k-1}^T \\ &\quad \times\mathscr{K}_k^T + \mathscr{B}_kS_k\mathscr{B}_k^T + 4\bar{\mathscr{H}}\rho_k\rho_k^T\bar{\mathscr{H}}^T\} \\ &= 4\tilde{\mathscr{O}}_k + 4\tilde{\mathscr{P}}_k + 4\mathbb{E}\{\bar{\mathscr{S}}_k\} + 4\mathbb{E}\{\bar{\mathscr{T}}_k\} + 4\mathscr{Q}_k \\ &\quad + 4\mathbb{E}\{\bar{\mathscr{U}}_k\} + 4\mathbb{E}\{\bar{\mathscr{V}}_k\} + 4\bar{\mathscr{H}}\hat{x}_k\hat{x}_k^T\bar{\mathscr{H}}^T + 4\mathscr{R}_k \\ &\quad + 4(I_{Nn} - \bar{\alpha})\mathcal{K}_ky_{k-1}y_{k-1}^T((I_{Nn} - \bar{\alpha})\mathcal{K}_k)^T \\ &\quad + \mathscr{W}_k + 4\bar{\mathscr{H}}\rho_k\rho_k^T\bar{\mathscr{H}}^T\end{aligned} \tag{33}$$

where

$$\begin{aligned}\tilde{\mathscr{O}}_k &= (\bar{\mathcal{A}}_k - \bar{\alpha}\mathcal{K}_k\bar{\mathcal{C}}_k)\bar{X}_k(\bar{\mathcal{A}}_k - \bar{\alpha}\mathcal{K}_k\bar{\mathcal{C}}_k)^T, \\ \tilde{\mathscr{P}}_k &= \sum_{i=1}^{N}\sigma_i^2(\mathcal{K}_k\mathcal{N}_i\bar{\mathcal{C}}_k)\bar{X}_k(\mathcal{K}_k\mathcal{N}_i\bar{\mathcal{C}}_k)^T, \\ \bar{\mathscr{S}}_k &= \sum_{i=1}^{N}\bar{\alpha}^i(\mathcal{K}_k\mathcal{N}_i\tilde{\mathcal{C}}_k)\bar{X}_k(\mathcal{K}_k\mathcal{N}_i\tilde{\mathcal{C}}_k)^T, \\ \bar{\mathscr{T}}_k &= \tilde{\mathcal{A}}_k\bar{X}_k\tilde{\mathcal{A}}_k^T, \ \bar{\mathscr{U}}_k = \tilde{\mathcal{A}}_k\hat{x}_k\hat{x}_k^T\tilde{\mathcal{A}}_k^T, \\ \bar{\mathscr{V}}_k &= \sum_{i=1}^{N}\bar{\alpha}^i(\mathcal{K}_k\mathcal{N}_i\tilde{\mathcal{C}}_k)\hat{x}_k\hat{x}_k^T(\mathcal{K}_k\mathcal{N}_i\tilde{\mathcal{C}}_k)^T.\end{aligned} \tag{34}$$

Furthermore, by some straightforward computations, it follows from the statistical properties of the random matrices in (3) that

$$\begin{aligned}&\mathbb{E}\{\bar{\mathscr{T}}_k\} = \tilde{\mathscr{T}}_k, \ [\tilde{\mathscr{T}}_k]_{[r,s]} = \tilde{\mathscr{T}}_k^{(r,s)} = \mathbb{E}\{\tilde{A}\bar{X}_k^{(r,s)}\tilde{A}^T\}, \\ &[\tilde{\mathscr{T}}_k^{(r,s)}]_{[l,m]} = \sum_{i,j=1}^{n}T_{a_{li}^k\cdot a_{mj}^k}e_{in}^T\bar{e}_{rn}^T\bar{X}_k\bar{e}_{sn}e_{jn}, \\ &\mathbb{E}\{\bar{\mathscr{U}}_k\} = \mathscr{U}_k, \ [\mathscr{U}_k]_{[r,s]} = \mathscr{U}_k^{(r,s)} = \mathbb{E}\{\tilde{A}(\hat{x}_k\hat{x}_k^T)^{(r,s)}\tilde{A}^T\}, \\ &[\mathscr{U}_k^{(r,s)}]_{[l,m]} = \sum_{i,j=1}^{n}T_{a_{li}^k\cdot a_{mj}^k}e_{in}^T\bar{e}_{rn}^T(\hat{x}_k\hat{x}_k^T)\bar{e}_{sn}e_{jn}.\end{aligned} \tag{35}$$

On the other hand, by using the properties of matrix operations, one can infer from the inequality (26) that the following inequality can always be guaranteed:

$$\rho_k\rho_k^T \le \Xi_kI_{Nn} \tag{36}$$

where $\Xi_k = \hat{x}_k^T\theta_k\hat{x}_k$.

Based on the above discussions, it can be found from (33)-(36) that

$$\begin{aligned}\bar{X}_{k+1} &\le 4\tilde{\mathscr{O}}_k + 4\tilde{\mathscr{P}}_k + 4\mathbb{E}\{\bar{\mathscr{S}}_k\} + 4\tilde{\mathscr{T}}_k + 4\mathscr{Q}_k \\ &\quad + 4\mathscr{U}_k + 4\mathbb{E}\{\bar{\mathscr{V}}_k\} + 4\bar{\mathscr{H}}\hat{x}_k\hat{x}_k^T\bar{\mathscr{H}}^T \\ &\quad + 4(I_{Nn} - \bar{\alpha})\mathcal{K}_ky_{k-1}y_{k-1}^T((I_{Nn} - \bar{\alpha})\mathcal{K}_k)^T \\ &\quad + 4\mathscr{R}_k + \mathscr{W}_k + 4\Xi_k\bar{\mathscr{H}}\bar{\mathscr{H}}^T \\ &= \bar{\Psi}(\bar{X}_k).\end{aligned} \tag{37}$$

Now we are ready to deal with the rest of the proof by induction. First, it is obvious that $\bar{X}_0 \leq Q_0$. Then, assuming $\bar{X}_k \leq Q_k$, it is not difficult to show that

$$
\begin{aligned}
\mathbb{E}\{\mathscr{S}_k\} &= \mathbb{E}\bigg\{ \sum_{i=1}^{N} \bar{\alpha}^i (\mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k) \bar{X}_k (\mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k)^T \bigg\} \\
&\leq \mathbb{E}\bigg\{ \sum_{i=1}^{N} \bar{\alpha}^i (\mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k) Q_k (\mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k)^T \bigg\} \\
&\leq \eta_k \mathbb{E}\bigg\{ \sum_{i=1}^{N} \bar{\alpha}^i (\mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k)(\mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k)^T \bigg\} \\
&= \mathscr{S}_k,
\end{aligned} \tag{38}
$$

$$
\begin{aligned}
\mathbb{E}\{\bar{\mathscr{V}}_k\} &= \mathbb{E}\bigg\{ \sum_{i=1}^{N} \bar{\alpha}^i (\mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k) \hat{x}_k \hat{x}_k^T (\mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k)^T \bigg\} \\
&\leq \beta_k \mathbb{E}\bigg\{ \sum_{i=1}^{N} \bar{\alpha}^i (\mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k)(\mathcal{K}_k \mathcal{N}_i \tilde{\mathcal{C}}_k)^T \bigg\} \\
&= \mathscr{V}_k.
\end{aligned} \tag{39}
$$

Accordingly, it can be readily concluded that

$$\bar{X}_{k+1} \leq \bar{\Psi}(\bar{X}_k) \leq \Psi(Q_k) \leq Q_{k+1},$$

which completes the proof. ∎

Remark 3: It should be pointed out that, from (31), we can see that $\bar{\mathscr{H}}\bar{\mathscr{H}}^T \geq 0$ in $\Psi(Q_k)$, and therefore the upper bound of error covariance increases as Ξ_k increases. In other words, the upper bound will increase as the threshold θ_k increases. In practical engineering, the event-triggering strategy reveals that the larger triggering threshold would lead to fewer data transmitted over the networks. Consequently, the threshold θ_k serves as an important factor on the tradeoff between the filtering performance and data transmission rate over the networks.

After establishing the analysis results for the addressed problem, we now proceed to present the design scheme of the finite-horizon distributed $\mathcal{H}_\infty$ filter for the discrete time-varying system (1).

For later presentation convenience, here Θ_{ak}^* and $\vec{\Theta}_{ck}^*$ are denoted, respectively, as the factorizations of Θ_{ak} and $\vec{\Theta}_{ck}$, i.e.

$$\Theta_{ak} = \Theta_{ak}^{*T} \Theta_{ak}^*, \quad \vec{\Theta}_{ck} = \vec{\Theta}_{ck}^{*T} \vec{\Theta}_{ck}^*.$$

Theorem 3: Let the disturbance attenuation level $\gamma > 0$, initial positive definite matrix $W = W^T > 0$ and a series of prespecified variance upper bounds $\{\Sigma_k\}_{0 \leq k \leq L+1}$ be given. The filtering error system (10) satisfies the $\mathcal{H}_\infty$ performance (R1) and the error variance constraint requirement (R2) simultaneously if there exist successions of positive definite matrices $\{\aleph_k\}_{1 \leq k \leq L+1}$, $\{Q_k\}_{1 \leq k \leq L+1}$, a set of matrices $\{\mathcal{K}_k\}_{0 \leq k \leq L} = \mathrm{diag}_N\{K_k^i\}_{0 \leq k \leq L}$ and some families of positive scalars $\{\lambda_k\}_{0 \leq k \leq L}$, $\{\bar{\varepsilon}_k = \varepsilon_k^{-1}\}_{1 \leq k \leq L+1}$ under the initial condition (17) and $Q_0 \leq \Sigma_0$, such that the following recursive linear matrix inequalities (LMIs):

$$
\begin{cases}
\tilde{\Pi}_k = \begin{bmatrix} \tilde{\Pi}_{11} & * & * \\ \tilde{\Pi}_{21} & \tilde{\Pi}_{22} & * \\ \tilde{\Pi}_{31} & 0 & \tilde{\Pi}_{33} \end{bmatrix} \leq 0, & \text{(40a)} \\
\hat{\Pi}_k = \begin{bmatrix} \hat{\Pi}_{11} & * & * \\ \hat{\Pi}_{21} & \hat{\Pi}_{22} & * \\ \hat{\Pi}_{31} & 0 & \hat{\Pi}_{33} \end{bmatrix} \leq 0, & \text{(40b)} \\
\begin{bmatrix} -\delta_k^i I_m & * \\ \sqrt{\bar{\alpha}_i} K_k^i & -\bar{e}_{in}^T \aleph_{k+1} \bar{e}_{in} \end{bmatrix} \leq 0, i = 1, \cdots, N, & \text{(40c)} \\
\bar{\varepsilon}_{k+1} I \leq \aleph_{k+1}, & \text{(40d)} \\
Q_{k+1} - \Sigma_{k+1} \leq 0 & \text{(40e)}
\end{cases}
$$

are satisfied for all $0 \leq k \leq L$. Moreover, the parameters are updated by

$$\mathcal{P}_{k+1} = \aleph_{k+1}^{-1} \tag{41}$$

where

$$
\begin{aligned}
\tilde{\Pi}_{11} &= \begin{bmatrix} \Delta_{11k}^1 & * \\ \Delta_{11k}^2 & \Delta_{11k}^3 \end{bmatrix}, \\
\Delta_{11k}^1 &= \mathrm{diag}\{\tilde{\Pi}_k^{01}, \tilde{\Pi}_k^{02}, \tilde{\nu}_k, -\gamma^2 I_p, -\lambda_k I_{Nn}\}, \\
\Delta_{11k}^2 &= [\mathscr{G}_k^T \ \mathscr{L}_k^T \ \mathscr{M}_k^T]^T, \\
\Delta_{11k}^3 &= -\mathrm{diag}\{\bar{\varepsilon}_{k+1} I_{Nn}, \aleph_{k+1}, \aleph_{k+1}\}, \\
\tilde{\Pi}_k^{01} &= \frac{1}{N} \mathcal{M}_k^T \mathcal{M}_k - \mathcal{P}_k, \ \tilde{\Pi}_k^{02} = \tilde{\mu}_k + \lambda_k \hat{x}_k^T \theta_k \hat{x}_k, \\
\mathscr{G}_k &= [\Theta_{ak}^*, \Theta_{ak}^* \hat{x}_k, 0, 0, 0], \ \tilde{\Pi}_{21} = [\tilde{\mathscr{N}}_k^T \ \tilde{\mathscr{R}}_k^T]^T, \\
\tilde{\mathscr{N}}_k &= [\bar{\mathscr{N}}_k \ 0], \ \bar{\mathscr{N}}_k = \mathrm{vec}_N^T\{\mathscr{N}_k^{iT}\}, \ \tilde{\mathscr{R}}_k = [\bar{\mathscr{R}}_k \ 0], \\
\bar{\mathscr{R}}_k &= \mathrm{vec}_N^T\{\mathscr{R}_k^{Ti}\}, \ \tilde{\Pi}_{22} = -\mathrm{diag}\{\bar{\aleph}_{k+1}, \tilde{\aleph}_{k+1}\}, \\
\tilde{\Pi}_{31} &= [\delta_{nk} \Theta_{ck}^*, \delta_{nk} \Theta_{ck}^* \hat{x}_k, 0, 0, 0], \ \tilde{\Pi}_{33} = -\delta_{nk}, \\
\bar{\aleph}_{k+1} &= I_N \otimes \aleph_{k+1}, \ \tilde{\aleph}_{k+1} = I_{Nnm^2} \otimes \aleph_{k+1}, \\
\hat{\Pi}_{11} &= \begin{bmatrix} \Upsilon_{11k}^1 & * \\ \Upsilon_{11k}^2 & \Upsilon_{11k}^3 \end{bmatrix}, \\
\Upsilon_{11k}^1 &= -Q_{k+1} + 4\mathscr{T}_k + 4\mathscr{U}_k, \ \Upsilon_{11k}^2 = [\Upsilon_{11k}^{21} \cdots \Upsilon_{11k}^{24}]^T, \\
\Upsilon_{11k}^3 &= -\frac{1}{4}\mathrm{diag}\{Q_k, 1, 1, I_{Nn}, 4S_k, 4I_N \otimes S_k\}, \\
\Upsilon_{11k}^{21} &= (\bar{\mathcal{A}}_k - \bar{\alpha}\mathcal{K}_k \bar{\mathcal{C}}_k) Q_k, \\
\Upsilon_{11k}^{22} &= [\bar{\mathscr{H}} \hat{x}_k, (I_{Nn} - \bar{\alpha}) \mathcal{K}_k y_{k-1}, \sqrt{\Xi_k} \mathscr{H}], \\
\Upsilon_{11k}^{23} &= \mathcal{B}_k - \bar{\alpha} \mathcal{K}_k \mathcal{D}_k, \ \Upsilon_{11k}^{24} = \mathrm{vec}_N\{\sigma_i \mathcal{K}_k \mathcal{N}_i \mathcal{D}_k S_k\}, \\
\hat{\Pi}_{21} &= [\Upsilon_{21k}^{1T} \ \Upsilon_{21k}^{2T} \ \Upsilon_{21k}^{3T}]^T, \ \Upsilon_{21k}^1 = [\bar{\Upsilon}_{21k}^1 \ \ 0], \\
\bar{\Upsilon}_{21k}^1 &= \mathrm{vec}_N^T\{\sigma_i \mathcal{K}_k \mathcal{N}_i \bar{\mathcal{C}}_k Q_k\}, \ \Upsilon_{21k}^2 = [\bar{\Upsilon}_{21k}^2 \ \ 0], \\
\bar{\Upsilon}_{21k}^2 &= \mathrm{vec}_N^T\{\sigma_i \mathcal{K}_k \mathcal{N}_i \bar{\mathcal{C}}_k \hat{x}_k\}, \ \Upsilon_{21k}^3 = [\bar{\Upsilon}_{21k}^3 \ \ 0], \\
\bar{\Upsilon}_{21k}^3 &= \mathrm{vec}_N^T\{\sigma_i \mathcal{K}_k \mathcal{N}_i y_{k-1}\}, \ \hat{\Pi}_{22} = -\frac{1}{4}\mathrm{diag}\{\mathcal{Q}_k, I_N, I_N\}, \\
\hat{\Pi}_{31} &= [\Upsilon_{31k}^{1T} \ \Upsilon_{31k}^{2T}]^T, \ \mathcal{Q}_k = I_N \otimes Q_k, \\
\Upsilon_{31k}^1 &= [\sqrt{\eta_k} \vec{\Theta}_{ck}^{*T} \mathcal{K}_k^T \ \ 0], \ \Upsilon_{31k}^2 = [\sqrt{\beta_k} \vec{\Theta}_{ck}^{*T} \mathcal{K}_k^T \ \ 0], \\
\hat{\Pi}_{33} &= -\frac{1}{4}\mathrm{diag}\{I_{Nm}, I_{Nm}\}.
\end{aligned} \tag{42}
$$

Proof: It follows from the Schur Complement Lemma that (16a) can be guaranteed if (40a) is satisfied. Moreover, we can see that inequality (40c) ensures that (16b) holds, and (40d) is equivalent to (16c). Also, it can be easily concluded from (40b) that (30) is true, and the rest of the proof follows directly from Theorem 1 and Theorem 2. To this end, the performance constraints (R1) and (R2) are achieved simultaneously and the proof of this theorem is complete. ■

Remark 4: In order to cope with the difficulty arising from the time-varying parameters for $\mathcal{H}_\infty$ filtering/control problems, some up-to-date techniques have recently been developed in the literature that include the differential/difference linear matrix inequality (DLMI) and recursive linear matrix inequality (RLMI) methods, see e.g. [10], [34]. In this paper, based on the RLMI approach, both the currently estimated states and previously obtained measurements have been exploited to obtain the filter parameters recursively, and therefore a better filtering performance is expected since more information is utilized.

Notice that the inequalities (40a)-(40c) in Theorem 3 are linear with respect to all unknown variables, which can be solved by the existing semi-definite programming method. In the following, according to Theorem 3, we summarize the event-based distributed $\mathcal{H}_\infty$ filtering algorithm as follows:

Algorithm 1: Event-Based Distributed Recursive Filter Design Algorithm.

Step 1. Given the $\mathcal{H}_\infty$ performance index γ and the positive definite matrix W. Let the initial values be generated randomly according to the uniform distribution over $[-0.2,\ 0.2]$. Select the initial values for matrices and scalars $\{\mathcal{P}_0,\ Q_0,\ \mu_0,\ \nu_0\}$ which satisfy the initial conditions (17) and $Q_0 \leq \Sigma_0$, then set $k = 0$.

Step 2. Obtain the values of matrices and scalars $\{\aleph_{k+1},\ Q_{k+1},\ \mu_{k+1},\ \nu_{k+1}\}$, the desired filter parameters K_k^i for the time step k by solving the LMIs (40a)-(40c).

Step 3. Set $k = k + 1$ and obtain $\mathcal{P}_{k+1}$ according to the parameter update formula (41).

Step 4. If $k = L$, then stop, else go to Step 2.

Remark 5: In Theorem 3, sufficient conditions for the existence and the derivation of the desired filters are provided, respectively. It is observed that all the system parameters, the information about the network topology and the statistic characteristics of the random sources (random parameters, random noises and random occurrence of the missing measurements) are reflected in the main results. The obtained time-varying filters are capable of, at each sampling time instant, guaranteeing prescribed variance upper bounds and also achieving prespecified $\mathcal{H}_\infty$ performance requirements. Furthermore, the proposed event-based distributed filter design algorithm is of a recursive form that would facilitate online applications.

IV. Numerical simulation

In this section, a numerical example is provided to illustrate the effectiveness of the developed distributed recursive filter design algorithm for the discrete time-varying stochastic systems with random parameters and successive missing measurements through sensor networks.

We consider the target plant as the model of (1), where ω_k is a zero-mean Gaussian white sequence with covariance $S_k = 1$, and the other corresponding system parameters are given as follows:

$$
\begin{aligned}
A_k &= \bar{A}_k + \tilde{A}_k \\
&= \begin{bmatrix} 0.25 + 0.02\sin(k) & 0.4 \\ 0.38 & 0.3 \end{bmatrix} + \zeta_k \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \\
B_k &= \begin{bmatrix} 0.05 \\ 0.05 \end{bmatrix},\ M_k = \begin{bmatrix} 0.4 \\ -0.6 \end{bmatrix}
\end{aligned}
$$

where ζ_k is a zero-mean scalar Gaussian white sequence with variance 1.

The topology of the sensor network is reflected by a graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ with the set of nodes $\mathscr{V} = \{1, 2, 3\}$, set of edges $\mathscr{E} = \{(1, 2), (2, 3), (3, 1)\}$, the adjacency elements with regard to the edges of the graph are $a_{ij} = 0.2$. The dynamics of each sensor node subject to successive missing measurements is constructed as (2) with the following parameters:

$$
\begin{aligned}
C_k^1 &= \bar{C}_k^1 + \tilde{C}_k^1 = [0.92\ 0.93] + \varsigma_k^1[0.1\ 0],\ D_k^1 = 0.2, \\
C_k^2 &= \bar{C}_k^2 + \tilde{C}_k^2 = [0.92\ 0.94] + \varsigma_k^2[0.1\ 0],\ D_k^2 = 0.2, \\
C_k^3 &= \bar{C}_k^3 + \tilde{C}_k^3 = [0.91\ 0.95] + \varsigma_k^3[0.1\ 0],\ D_k^3 = 0.2
\end{aligned}
$$

where $\varsigma_k^i (i = 1, 2, 3)$ are mutually independent zero mean Gaussian white sequences with unity variances.

In this simulation, the missing probabilities for the three sensor nodes at every sampling constant are taken, respectively, as 0.15, 0.1 and 0.15, and therefore $\bar{\alpha}^1 = 0.85$, $\bar{\alpha}^2 = 0.9$, $\bar{\alpha}^3 = 0.85$. The thresholds $\theta_k^i (i = 1, 2, 3)$ are all chosen as 0.1. The $\mathcal{H}_\infty$ performance index, positive matrix W and $\{\Sigma_k\}_{1 \leq k \leq L+1}$ are given as $\gamma = 0.5$, $\mathrm{diag}_6\{20\}$ and $\mathrm{diag}_6\{0.1\}$, respectively. The parameters' initial values are chosen as $P_0 = \mathrm{diag}_6\{0.3\}$ and $\mu_0 = \nu_0 = 0.5$ to satisfy conditions (17) and $Q_0 \leq \Sigma_0$. By implementing *Algorithm 1* and using Matlab (with the YALMIP 3.0), the LMIs in Theorem 3 can be solved recursively and some of the desired filter parameters are obtained as shown in Table I.

The corresponding simulation results are displayed in Figs. 1-5, where Figs. 1-2 show the trajectories for the actual states and their estimations, Figs. 3-4 depict the estimation error variances and their upper bounds for the first and second elements of the system state, respectively. Moreover, the estimation error $z_k^i\ (i = 1, 2, 3)$ is plotted in Fig. 5. From the above simulation results, we can clearly see that the designed filters have a satisfactory performance as expected and therefore the effectiveness of the proposed distributed filtering algorithm in this paper is well confirmed.

V. Conclusion

In this paper, we have dealt with the event-based distributed $\mathcal{H}_\infty$ filtering problem for the discrete time-varying system over a finite-horizon, where the stochastic noises, successive missing measurements and random parameter matrices have all been taken into account in order to better reflect the practical situations. The successive missing measurement phenomenon has been governed by a set of Bernoulli distributed white sequences. The event-triggered mechanism has been introduced in the process of filter analysis and design to ease the heavy burden on the communication channels. By utilizing the stochastic analysis techniques, some sufficient conditions

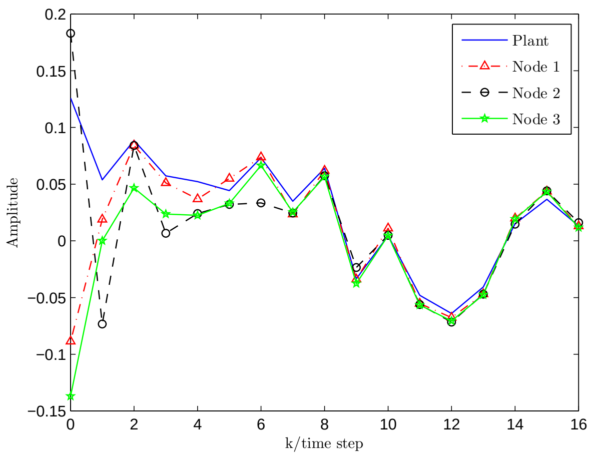

Fig. 1. The state trajectories of x_k^1 and $\hat{x}_k^{i,1}(i = 1, 2, 3)$.

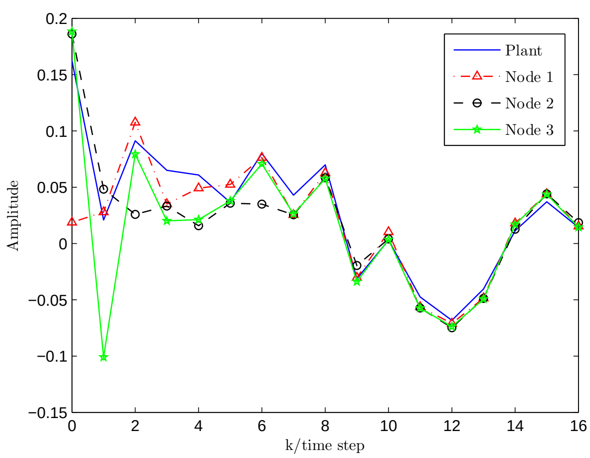

Fig. 2. The state trajectories of x_k^2 and $\hat{x}_k^{i,2}(i = 1, 2, 3)$.

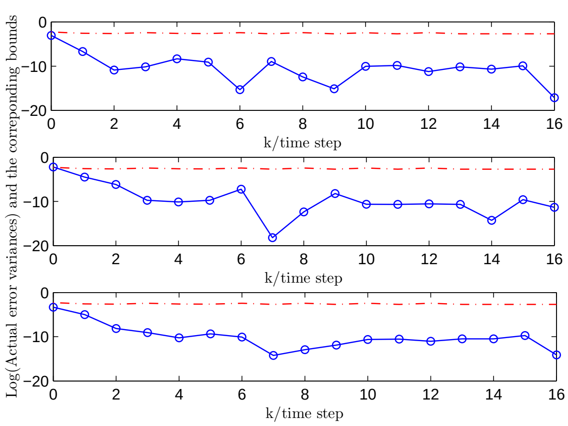

Fig. 3. The estimation error variances of x_k^1 and their upper bounds.

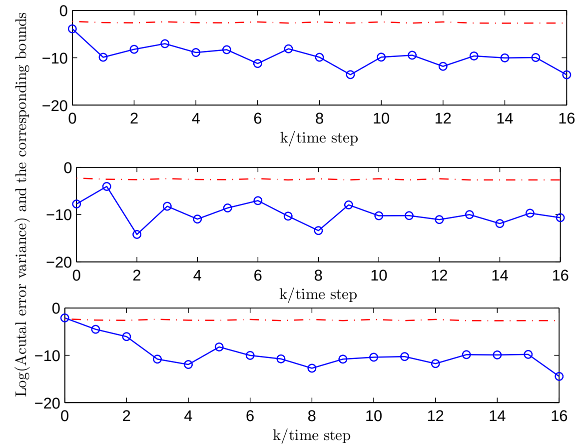

Fig. 4. The estimation error variances of x_k^2 and their upper bounds.

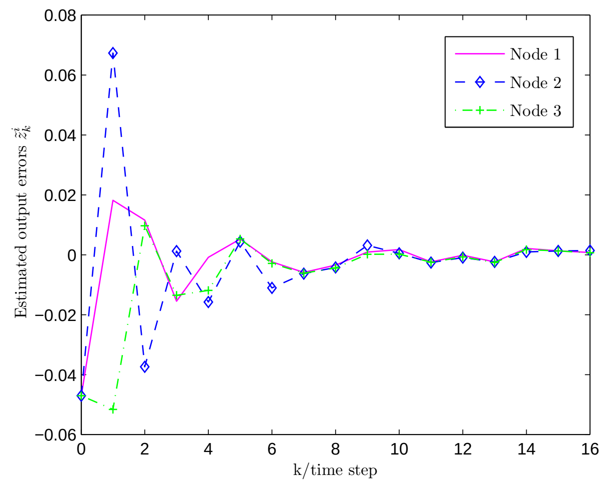

Fig. 5. Estimated output errors $\tilde{z}_k^i(i = 1,\ 2,\ 3)$.

have been obtained to guarantee both the $\mathcal{H}_\infty$ performance and variance constraint requirements. Moreover, by means of the feasibility of a series of RLMIs, the filter parameters have been explicitly expressed. A numerical simulation has been carried out to demonstrate the validity of the proposed filter design strategy. Further research topics would include the investigation on the distributed filtering problem over sensor networks subject to network-induced quantization effects [17], [20], [43] as well as the extension of our main results to the state estimation problems of neural networks [22], [24], Boolean networks [4] and genetic regulatory networks [15].

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TABLE I
FILTER PARAMETERS

k	0	1	2	3
K_k^1	$\begin{bmatrix} 0.3148 \\ 0.3266 \end{bmatrix}$	$\begin{bmatrix} 0.3268 \\ 0.3310 \end{bmatrix}$	$\begin{bmatrix} 0.2873 \\ 0.2883 \end{bmatrix}$	$\begin{bmatrix} 0.3130 \\ 0.3225 \end{bmatrix}$
K_k^2	$\begin{bmatrix} 0.3104 \\ 0.3209 \end{bmatrix}$	$\begin{bmatrix} 0.3259 \\ 0.3291 \end{bmatrix}$	$\begin{bmatrix} 0.2855 \\ 0.2858 \end{bmatrix}$	$\begin{bmatrix} 0.3130 \\ 0.3218 \end{bmatrix}$
K_k^3	$\begin{bmatrix} 0.3164 \\ 0.3262 \end{bmatrix}$	$\begin{bmatrix} 0.3237 \\ 0.3259 \end{bmatrix}$	$\begin{bmatrix} 0.2906 \\ 0.2899 \end{bmatrix}$	$\begin{bmatrix} 0.3149 \\ 0.3227 \end{bmatrix}$
k	4	5	6	$\cdots$
K_k^1	$\begin{bmatrix} 0.3124 \\ 0.3304 \end{bmatrix}$	$\begin{bmatrix} 0.2762 \\ 0.2920 \end{bmatrix}$	$\begin{bmatrix} 0.2667 \\ 0.2713 \end{bmatrix}$	$\cdots$
K_k^2	$\begin{bmatrix} 0.3127 \\ 0.3299 \end{bmatrix}$	$\begin{bmatrix} 0.2739 \\ 0.2888 \end{bmatrix}$	$\begin{bmatrix} 0.2747 \\ 0.2792 \end{bmatrix}$	$\cdots$
K_k^3	$\begin{bmatrix} 0.3143 \\ 0.3301 \end{bmatrix}$	$\begin{bmatrix} 0.2776 \\ 0.2912 \end{bmatrix}$	$\begin{bmatrix} 0.2707 \\ 0.2739 \end{bmatrix}$	$\cdots$

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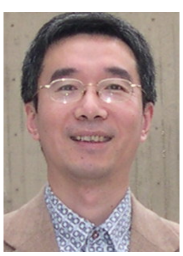
Licheng Wang received the B.Sc. degree in automation in 2011 from Weifang University, Weifang, China, and the M.Sc. degree in control science and engineering in 2014 from University of Shanghai for Science and Technology, Shanghai, China. He is now currently pursuing the Ph.D. degree in Control Science and Engineering at the University of Shanghai for Science and Technology, Shanghai, China. Since Nov. 2016, he has been a visiting PhD student in the Department of Electronic and Computer Engineering at Brunel University London in the UK. His research interests include nonlinear stochastic control and filtering, as well as complex networks and sensor networks.

Mr. Wang is a reviewer for some international journals.



Zidong Wang (SM'03-F'14) was born in Jiangsu, China, in 1966. He received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in electrical engineering in 1994, both from Nanjing University of Science and Technology, Nanjing, China.

He is currently Professor of Dynamical Systems and Computing in the Department of Computer Science, Brunel University London, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany and the UK. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published more than 200 papers in refereed international journals. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, William Mong Visiting Research Fellowship of Hong Kong.

Prof. Wang is a Fellow of the IEEE. He is serving or has served as an Associate Editor for 12 international journals, including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics - Systems. He is also a Fellow of the Royal Statistical Society and a member of program committee for many international conferences.

Qing-Long Han (M'09-SM'13) received the B.Sc. degree in Mathematics from Shandong Normal University, Jinan, China, in 1983, and the M.Sc. and Ph.D. degrees in Control Engineering and Electrical Engineering from East China University of Science and Technology, Shanghai, China, in 1992 and 1997, respectively.

From September 1997 to December 1998, he was a Post-doctoral Researcher Fellow with the Laboratoire d'Automatique et d'Informatique Industielle (now renamed as Laboratoire d'Informatique et d'Automatique pour les Systémes), Ecole Sup érieure d'Ing'enieurs de Poitiers (now renamed as Ecole Nationale Supérieure d'Ingé nieurs de Poitiers), Universit é de Poitiers, France. From January 1999 to August 2001, he was a Research Assistant Professor with the Department of Mechanical and Industrial Engineering at Southern Illinois University at Edwardsville, USA. From September 2001 to December 2014, he was Laureate Professor, Associate Dean (Research and Innovation) with the Higher Education Division, and Founding Director of the Centre for Intelligent and Networked Systems at Central Queensland University, Australia. From December 2014 to May 2016, he was Deputy Dean (Research) with the Griffith Sciences, and a Professor with the Griffith School of Engineering, Griffith University, Australia. In May 2016, he joined Swinburne University of Technology, Australia, where he is currently Pro Vice-Chancellor (Research Quality) and Distinguished Professor. In March 2010, he was appointed Chang Jiang (Yangtze River) Scholar Chair Professor by Ministry of Education, China.

Prof. Han is one of The World's Most Influential Scientific Minds: 2014-2016 and is a Highly Cited Researcher in Engineering according to Thomson Reuters. He is an Associate Editor of a number of international journals including IEEE Transactions on Industrial Electronics, IEEE Transactions on Industrial Informatics, IEEE Transactions on Cybernetics, and Information Sciences. His research interests include networked control systems, neural networks, time-delay systems, multi-agent systems and complex dynamical systems.

Guoliang Wei received the B.Sc. degree in mathematics from Henan Normal University, Xinxiang, China, in 1997 and the M.Sc. degree in applied mathematics and the Ph.D. degree in control engineering, both from Donghua University, Shanghai, China, in 2005 and 2008, respectively. He is currently a Professor with the Department of Control Science and Engineering, University of Shanghai for Science and Technology, Shanghai, China.

From March 2010 to May 2011, he was an Alexander von Humboldt Research Fellow in the Institute for Automatic Control and Complex Systems, University of Duisburg-Essen, Germany. From March 2009 to February 2010, he was a post doctoral research fellow in the Department of Information Systems and Computing, Brunel University, Uxbridge, UK, sponsored by the Leverhulme Trust of the UK From June to August 2007, he was a Research Assistant at the University of HongKong. From March to May 2008, he was a Research Assistant at the City University of Hong Kong.

His research interests include nonlinear systems, stochastic systems, and bioinformatics. He has published more than 20 papers in refereed international journals. He is a very active reviewer for many international journals.