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Modelling of Mathematical Programs:
An Analysis of Strategy and An Outline
Description of a Computer
Assisted System

by

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0. Abstract

The salient components of the mathematical programming modeling activity are first analysed. Earlier generation systems such as program generators and procedural (modelling) languages are briefly discussed. A proposal for a computer assisted modelling scheme is then put forward. The proposed system contrasts with the earlier approaches in that no computer programming expertise is required on the part of the modeller. A mathematical programming model is usually constructed by progressive definition of dimensions, data tables, model variables, model constraints and the matrix coefficients which connect the last two entities. The philosophy and design of the experimental system supports this approach to model description. This aspect is illustrated by a few examples. The introduction of computer assistance in structuring of the data and the resulting model is novel and is in line with recent developments in friendly and flexible user interface.

1. Introduction

Linear Programming addresses itself to the question of making the most efficient use of scarce resources. The mathematics of Linear Programming, both formulation as well as computational algorithms have found extensive use over the last thirty years. All the major computer manufacturers such as IBM (MPSX) [1], CDC (APEX) [2], UNIVAC (FMPS) [3] or large software houses specializing in this area, namely SCICON(SCICONIC) [4] have developed Mathematical Programming systems for the solution of Linear and Integer programming problems. The user control aspect of these systems were all designed in the early to mid seventies and admit a number of criticisms which are set out below.

An engineer, a planner or an applied mathematician is a typical investigator wishing to use these systems.

- (i) In order to use these systems an investigator needs to learn a procedural language or a command driven language to communicate and control his application.
- (ii) An investigator further needs to possess some knowledge of computational methods for solving LP/IP, whereas most of his skills are vested in model definition and model analysis.
- (iii) Description of his model to these systems using a matrix generator program definitely calls for computer programming skill.

An experimental system is under development by the authors and Mr.M.Tamiz [5] and is called CAMPS: Computer Assisted Mathematical Programming (Modelling) System. In this report the philosophy and design of the computer assisted modelling (sub) system is put forward with a view to overcoming the criticism as outlined in (iii) above.

The contents of this report are organized as follows. The logical analysis of the modelling task, the derivation of the mathematical statement and a few illustrative examples of LP models are set out in section 2. The case for the computer assisted method is presented and argued in section 3. How the CAMPS system may be used to construct one of the models described in section 2, run optimize, prepare a report of the solution and finally document the model is discussed in section 4. The report is concluded with section 5 containing discussions and comments relating to the overall system.

2. Strategy and Tactics of LP Modelling

2.0 Introduction

Formulating linear and integer programming models for industrial (optimisation) problems requires an amount of experience and specialist skill. The method of analysing a physical problem is discussed in section 2.1. The logical sequence of steps which lead to a mathematical statement of the model are set out in section 2.2; these concepts are illustrated by an example. Having obtained a mathematical statement it is necessary to prepare the data for suitable processing by a computer based LP system. This aspect is discussed in section 2.3. Two further examples are considered in section 2.4 to explain these principles of modelling.

2.1 A Logical Analysis of the Problem

A modeller when he comes across an industrial (optimisation) problem does not necessarily find it well described in a summary form. It is more than likely he is presented with a description of the problem containing an amount of details which may be irrelevant for modelling purposes; further it may also contain a number of gaps. Hence the first task of the modeller is to consider only the modelling requirements and extract the quantitative relationships which are germane to that task. Having identified these items he produces a compact statement of the problem which contains only these pertinent details. The three examples which are presented in section 2 and the planning model considered later are first described in this summary form.

After identifying the key components of the model his next task is to discover the underlying structure in the model. This amounts to finding a way of categorizing the modelling information. The following is an illustrative list of typical categories that are found in practical problems.

- number of (decentralized) geographical locations
- number of planning periods
- number of different products
- number of grades of people
- number of age groups
- etc.,

This categorization helps him to decide to what details the quantitative information relating to the problems should be requested and incorporated in the model. It also indicates to what detail the answers are to be provided.

Model Variables

Once the categories are defined the model (decision) variables or the unknowns are broadly identified. An analysis of the decision variables may also suggest new categories at this stage. The point to note here is that the model variables are mostly detailed by categories. For the purpose of illustration a number of decision variables taken from different contexts are considered below.

- Production Planning: The quantity X_{pm} of a certain product p manufactured on a machine m .
- Distribution Planning: The quantity X_{prn} of a product p that is shipped from a source r to an outlet n .
- Inventory Scheduling: The quantity X_{pt} of a product p that is kept as closing stock at the end of period t .
- Project Analysis: Whether one should invest in project p at the beginning of time period t , or not invest in this project $Y_{pt} = 1$ or 0 may be represented by this zero-one variable Y_{pt} .

Model Constraints

The constraints connect the decision variables and express the physical restrictions of the problem. By and large these are also detailed by categories. A few examples of these are set out below.

- Material Balance Equation

$$XO_t + XP_t - XC_t - D_t, \quad t = 1, 2, \dots, T.$$

In this equation XO_t represents the opening inventory, XC_t represents the closing inventory, and XP_t the quantities to be produced. They are all decision variables pertaining to the time period t . D_t represents the customer demand for the product and is an input information.

- Capacity Restrictions

$$\sum_{p=1}^P X_{pm} \cdot t_{pm} \leq A_m, \quad m = 1, 2, \dots, M.$$

Here $p = 1, 2, \dots, P$ indicate the range of products which are manufactured on machines $m = 1, 2, \dots, M$. The rate of production is indicated by t_{pm} , that is, the time taken to produce one unit of product p on machine m . A_m indicates the number of hours the machine m is available. X_{pm} is the production variable and the constraints express the capacity of production for the machine m as limited by the number of hours of its availability.

- Blending Requirement

$$\sum_{c=1}^C X_{cp} \cdot b_{cr} \left\{ \begin{array}{l} \leq \\ \text{or} \\ = \\ \text{or} \\ \geq \end{array} \right\} Q_{pr} \quad \begin{array}{l} p = 1, \dots, P \\ r = 1, \dots, R \end{array}$$

In this case $c = 1, \dots, C$ are number of components which are used to blend $p = 1, \dots, P$ products. The components for instance could be different crudes and products could be different types of gasoline. $r = 1, \dots, R$ indicate quality requirements. Typical requirements are maximum vapour pressure, minimum volatility index etc. Thus b_{cr} , Q_{pr} are input information pertaining to linear blending rates and quality requirements respectively. X_{cp} is the decision variable indicating fractions (by volume or weight) of component c that are blended to derive product p . Thus

$$\sum_{c=1}^C X_{cp} = 1, \quad p = 1, \dots, p.$$

Note that in the discussion of the model variables and model constraints the subscripts p, m, n, c, r, t etc which have been introduced indicate categories taken from the context of the model. Thus identifying these amounts to setting out the basic structure of the model.

2.2 Derivation of a Mathematical Statement : An Example.

It follows from the preliminary analysis presented in the last section that in order to derive a mathematical statement of the model one has to formally define the matrix elements of the constraint relations. In order to do this it is necessary to define the subscripts and their ranges. Note that the matrix elements themselves may be derived out of tabular input information relating to the problem. These matrix elements may be considered to be model descriptors and are often referred to as "technology coefficients". The model (decision) variables in contrast are output information. Their values are obtained by solving the model. The sequence of steps leading to the derivation of a model thus naturally emerges and is set out below.

- Step 1 Define the subscripts and their ranges (sets and dimensions).
- Step 2 Define model variables, model constraints and the matrix coefficients in terms of these subscripts (step 1).
- Step 3 Specify the linear relationships in a row wise fashion which connect the items defined in step 2.

In its simplest and most standard form an LP model can be stated in the following way:

- Subscripts, Ranges:
 $i = 1, \dots, m, j = 1, \dots, n.$
- Variables, constraints, coefficients:
 $x : x_j, j = 1 \dots n, r : r, i = 1 \dots m,$
 $c : c_j, j = 1 \dots n, b : b_i, i = 1 \dots m,$
 $A : a_{ij} \quad i=1 \dots m, j=1 \dots n.$
- Linear objective function and constraints:

$$\text{Max} \quad \sum_{j=1}^n c_j x_j,$$

$$\text{Subject to} \quad r_i : \sum_{j=1}^n a_{ij} x_j = b_i, \quad i = 1, \dots, m$$

$$x_j \geq 0, \quad j=1, \dots, n.$$

However, in all real life applications the corresponding models possess more detailed structure than this standard form. As a result of such structure the A - matrix turns out to be highly sparse and b,c can also be sparse. In practice therefore formulating a model requires specifying only the nonzero coefficients of the A-matrix as used in stating the linear constraint relations.

In deriving the mathematical statement of an LP model and specially the linear constraint relations it is often convenient to prepare a material flow diagram for the problem. This enables the modeller to visualize and set out the balance relations, the capacity restrictions etc. The principles of LP modelling discussed so far are illustrated in the derivation of a production cum distribution model considered here and two further models described in section 2.4.

A Production cum Distribution Problem: An Example.

A clothing manufacturer has two factories Southail (FT1) and Leeds (FT2). In the Southail factory he can manufacture the products Shirts (P1), Denim Jeans (P3), and in Leeds he can manufacture Shirts (P1), Skirts (P2) and Denim Jeans (P3). The manufacturer ships these products direct to three main dealers in quantities of thousands. The dealers are Young Londoner (DL1), Beaute Paris (DL2) and Wiener Mode Anzug (DL3). The manufacturer knows his production costs, the transport costs and the monthly production capacity of his factories. The dealers send their requirements for the next month on the first day of each month. All the numerical data relating to the problem are set out in Table 2.1. The line diagram 2.1 illustrates the possible relationships between factories, products and dealers.

DEALERS REQUIREMENTS AND PRODUCTION CAPACITY IN UNITS OF THOUSANDS					
Product	Dealer Requirements			Factory Capacity	
	DL1	DL2	DL3	FT1	FT2
P1	50	10	30	36	54
P2	15	15	20	-	60
P3	20	60	30	85	45

PRODUCTION AND TRANSPORT COST IN POUND STERLING PER ITEM						
Factory	Production Costs			Dealer		
	P1	P2	P3	DL1	DL2	DL3
FT1	1.5	-	5.6	0.6	1.2	1.4
FT2	1.8	7.0	6.2	0.7	1.3	1.5

Table 2.1

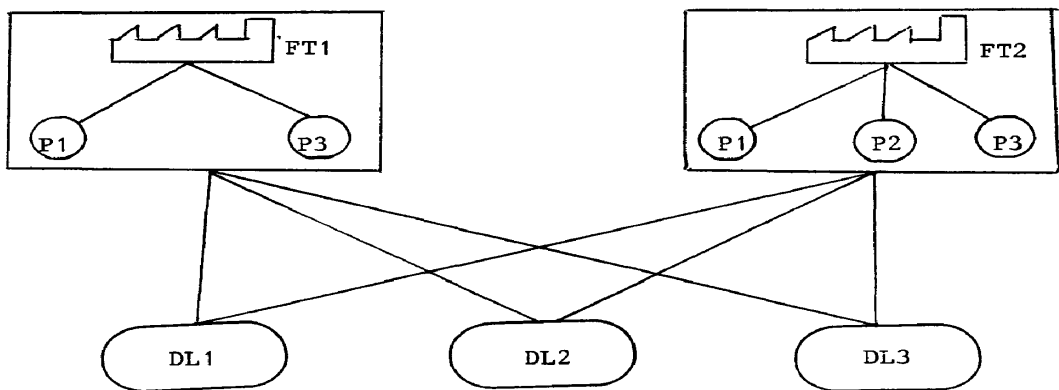


Diagram 2.1

The manufacturer at the beginning of each month, needs to formulate and solve a simple linear programming problem. A mathematical statement of this problem is set out below.

- Subscripts and Dimensions.

$i = 1, 2$ denotes the factories

$j = 1, 2, 3$ denotes the products

$k = 1, 2, 3$ denotes the dealers.

- Model Variables

x_{ijk} the quantity of product j manufactured in factory i and shipped to dealer k . However, for $i = 1$ (Southall) the product $j = 2$ skirts and its shipments are not defined.

That is $\left. \begin{array}{l} i = 1, \quad j = 1, 3, \\ i = 2, \quad j = 1, 2, 3 \end{array} \right\} k = 1, 2, 3$

- Model Coefficients (Descriptors)

p_{ij} the cost of producing one unit of product j at factory i ,

t_{ik} the cost of transporting one unit of each product from factory i to dealer k ,

c_{ijk} the derived cost of production as well as transport for given i, j, k which may be expressed as

$$c_{ijk} = p_{ij} + t_{ik}$$

a_{ij} the production capacity of the factory i for the product j ,

r_{jk} the requirement of the dealer k for the product j .

Linear Constraint Relations: A Mathematical Statement

Minimise

$$\text{cost} = \sum_{i=1}^2 \sum_{k=1}^3 \left(c_{i1k} x_{i1k} + c_{i3k} x_{i3k} \right) + \sum_{k=1}^3 c_{22k} x_{22k},$$

subject to the constraints:

capacity of production

$$\sum_{k=1}^3 x_{ijk} \leq a_{ij} \quad \left. \begin{array}{l} i = 1, \quad j = 1, 3 \\ i = 2, \quad j = 1, 2, 3 \end{array} \right\},$$

and satisfying dealer requirements

$$\left. \begin{array}{l} \sum_{i=1}^2 x_{ijk} = r_{jk}, \quad j = 1,3 \\ x_{22k} = r_{2k}, \end{array} \right\} k = 1,2,3$$

and $x_{ijk} \geq 0.$

2.3 LP User Formulation of the Model

The mathematical statement of the model set out in the last section is concise and convenient for communication and discussion by mathematicians and analysts. However, for the purpose of processing the model by a computer based LP system and deriving numerical solutions this form is abstract and not suitable.

Model information is usually presented to an industrial LP system in a compact form and it is appropriate to highlight a few features of LP input at this point.

- (i) All applicable LP models display a high degree of sparsity of the constraint matrix.
- (ii) Only the nonzero coefficients of the matrix are specified as input.
- (iii) Instead of a row index and a column index one uses a row name and a column name to specify a non-zero coefficient of the matrix.
- (iv) Feature (iii) requires that suitable name is given for the rows and columns of the model.

IBM's MPSX input format is industry's de facto standard for model specification: this format is described in [1] and also in the CAMPS manual [6].

To obtain the LP user formulation the following model variable and constraint names are first defined.

— Model Variable Name

FT1P1DL1 The amount of product P_i produced in the factory FT1 and shipped to the dealer DL1 etc.

PRODUCTION VARIABLES	FT1P1	FT1P2	FT1P3	FT1P4	FT1P5	FT1P6	FT2P1	FT2P2	FT2P3	FT2P4	FT2P5	FT2P6	FT2P7	RELATION	RHS VALUE	
COST	2.1	2.7	2.9	6.2	6.8	7.0	2.5	3.1	3.3	7.7	3.3	3.5	5.9	7.5	7.7	FREE
FACTORY CAP	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	30000
DEALER REQ	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	50000
REGP1DL1																15000
REGP2DL1																15000
REGP3DL1																20000
REGP1DL2																30000
REGP2DL2																30000
REGP3DL2																30000
REGP1DL3																30000
REGP2DL3																30000
REGP3DL3																30000

TABLEAU 2.1

MPS FORMAT LINE IMAGE INPJT.

NAME	EXPRDD			
ROWS				
N COSTROW				
L FT1P1CAP				
L FT1P3CAP				
L FT2P1CAP				
L FT2P3CAP				
L FT2P2CAP				
L REQP1DL1				
L REQP2DL1				
L REQP3DL1				
L REQP1DL2				
L REQP2DL2				
L REQP3DL2				
L REQP1DL3				
L REQP2DL3				
L REQP3DL3				
COLUMNS				
FT1P1DL1	FT1P1CAP	1.000000	REQP1DL1	1.000000
FT1P1DL1	COSTROW	2.100000	REQP1DL2	1.000000
FT1P1DL1	FT1P3CAP	1.000000	REQP1DL3	1.000000
FT1P1DL1	COSTROW	2.700000	REQP3DL1	1.000000
FT1P1DL1	FT1P3CAP	1.000000	REQP3DL2	1.000000
FT1P1DL1	COSTROW	2.900000	REQP3DL3	1.000000
FT1P3DL1	FT1P3CAP	1.000000	REQP1DL1	1.000000
FT1P3DL1	COSTROW	6.200000	REQP1DL2	1.000000
FT1P3DL1	FT1P3CAP	1.000000	REQP1DL3	1.000000
FT1P3DL1	COSTROW	6.800000	REQP2DL1	1.000000
FT1P3DL1	FT1P3CAP	1.000000	REQP2DL2	1.000000
FT1P3DL1	COSTROW	7.000000	REQP2DL3	1.000000
FT2P1DL1	FT2P1CAP	1.000000	REQP3DL1	1.000000
FT2P1DL1	COSTROW	2.500000	REQP3DL2	1.000000
FT2P1DL1	FT2P1CAP	1.000000	REQP3DL3	1.000000
FT2P1DL1	COSTROW	3.100000	REQP1DL1	1.000000
FT2P1DL1	FT2P1CAP	1.000000	REQP1DL2	1.000000
FT2P1DL1	COSTROW	3.300000	REQP1DL3	1.000000
FT2P1DL1	FT2P1CAP	1.000000	REQP2DL1	1.000000
FT2P1DL1	COSTROW	7.700000	REQP2DL2	1.000000
FT2P1DL1	FT2P1CAP	1.000000	REQP2DL3	1.000000
FT2P1DL1	COSTROW	8.300000	REQP3DL1	1.000000
FT2P1DL1	FT2P1CAP	1.000000	REQP3DL2	1.000000
FT2P1DL1	COSTROW	5.900000	REQP3DL3	1.000000
FT2P1DL1	FT2P1CAP	1.000000	REQP1DL1	1.000000
FT2P1DL1	COSTROW	7.500000	REQP1DL2	1.000000
FT2P1DL1	FT2P1CAP	1.000000	REQP1DL3	1.000000
FT2P1DL1	COSTROW	7.700000	REQP2DL1	1.000000
FT2P1DL1	FT2P1CAP	1.000000	REQP2DL2	1.000000
FT2P1DL1	COSTROW	7.700000	REQP2DL3	1.000000
FT2P3DL1	FT2P3CAP	1.000000	REQP3DL1	1.000000
FT2P3DL1	COSTROW	3.500000	REQP3DL2	1.000000
FT2P3DL1	FT2P3CAP	1.000000	REQP3DL3	1.000000
FT2P3DL1	COSTROW	5.900000	REQP1DL1	1.000000
FT2P3DL1	FT2P3CAP	1.000000	REQP1DL2	1.000000
FT2P3DL1	COSTROW	7.500000	REQP1DL3	1.000000
FT2P3DL1	FT2P3CAP	1.000000	REQP2DL1	1.000000
FT2P3DL1	COSTROW	7.700000	REQP2DL2	1.000000
FT2P3DL1	FT2P3CAP	1.000000	REQP2DL3	1.000000
FT2P3DL1	COSTROW	7.700000	REQP3DL1	1.000000
FT2P3DL1	FT2P3CAP	1.000000	REQP3DL2	1.000000
FT2P3DL1	COSTROW	7.700000	REQP3DL3	1.000000
RHS				
RHS VALUE	FT1P1CAP	35.000000	FT1P3CAP	35.000000
RHS VALUE	FT2P1CAP	54.000000	FT2P2CAP	60.000000
RHS VALUE	FT2P3CAP	45.000000	REQP1DL1	50.000000
RHS VALUE	REQP1DL2	10.000000	REQP1DL3	30.000000
RHS VALUE	REQP2DL1	15.000000	REQP2DL2	15.000000
RHS VALUE	REQP2DL3	20.000000	REQP3DL1	20.000000
RHS VALUE	REQP3DL2	60.000000	REQP3DL3	30.000000
ENDATA				

DISP AY 2.1

- Model Constraint Names

COSTROW	The objective row,
FT1PICAP	The capacity constraint corresponding to the product P1 produced in factory FT1,
REQP1DL1 DL1	The requirement of the product P1 by the dealer DL1 etc.

The sparse but complete constraint matrix in terms of these row and column names is set out in Tableau 2.1. The corresponding MPSX format input data file in line images is set out in Display 2.1.

2.4 Further Examples

Blending of Gasoline Products

An oil company in an off shore island maintains a reserve of five basic components Butane, Light Naptha, Heavy Naptha, Catalytic Naptha and Catalytic Reformate which are blended and replenished on a weekly basis to meet the demands for two grades of gasoline called GAS1 and GAS2. The availability, the linear blending coefficients and the costs for these components are tabulated in Table 2.2. The quality requirements and the volume demands for the two gasoline products are set out in Table 2.3. The oil company wishes to derive an LP model that must be solved on a weekly basis to find the optimal blending of the components.

Blending Componets

Componet	Availability, Thousand of Barrels	Reseach Octane number	Vapour pressure	Volatility Index	Code Name	Cost, cents Per gallon
Butane	3.5	120.0	60.0	105	BU	5.2
Light naphtha	2.0	84.5	18.0	30	LN	6.4
Heavy naphtha	4.0	73.0	4.0	12	HN	8.3
Catalytic naphtha	10.5	96.0	6.4	15	CN	10.2
Catalytic reformate	8.0	99.0	2.5	3	CR	11.0

Table 2.2

Gasoline Requirements

Needed volume, thousand of barrels	Minimum research octane number	Maximum vapour pressure	Minimum volatility index	Code name
10.0	95.0	11.0	18	GAS1
6.0	98.0	12.0	20	GAS2

Table 2.3

Diagram 2.2 shows how the two products connect the five components.

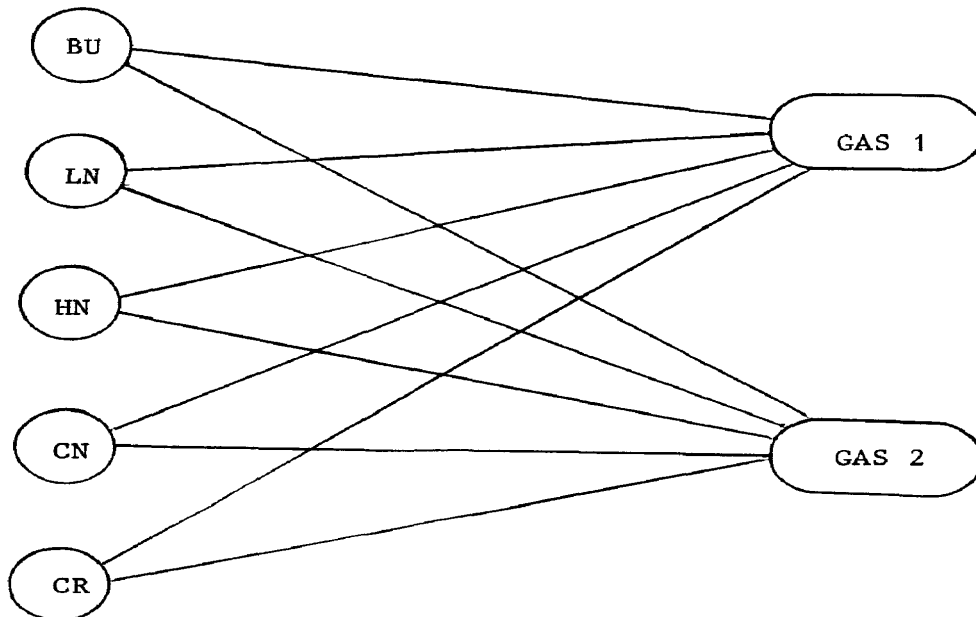


Diagram 2.2

- Subscripts and Dimensions

$i = 1, \dots, 5$ denotes the components,

$j = 1, 2, 3$ denotes the three quality indices: octane number
vapour pressure, volatility index,

$k = 1, 2$ denotes the two gasoline products.

- Model Variables

x_{ik} The amount of component i that is blended into the product k .

- Model Coefficients

a_i The amount of component i that is available for blending,

b_{ij} the linear blending coefficient for component i and quality
index j ,

c_i the cost of component i ,

r_{kj} the blending quality requirement for the product k against quality
index j ,

d_k the demand for the gasoline product k .

- Linear Constraint Relations: A Mathematical Statement.

$$\text{Minimise} \quad \sum_{i=1}^5 \sum_{k=1}^2 c_i x_{ik}$$

subject to

$$\text{Availability restriction} \quad \sum_{k=1}^2 x_{ik} \leq a_i, \quad i=1 \dots 5,$$

Demand balance

$$\sum_{i=1}^5 x_{ik} = d_k, \quad k=1, 2,$$

and

Blending requirements

$$\left. \begin{array}{l} \sum_{i=1}^5 x_{ik} b_{i1} \geq d_k \cdot r_{k1} \\ \sum_{i=1}^5 x_{ik} b_{i2} \leq d_k \cdot r_{k2} \\ \sum_{i=1}^5 x_{ik} b_{i3} \leq d_k \cdot r_{k3} \end{array} \right\} \begin{array}{l} j=1 \text{ Octane specification} \\ K=1, 2, j=\text{vapour pressure} \\ j=3 \text{ Volatility index} \end{array}$$

and $x_{ik} \geq 0 \quad i=1 \dots 5, k=1, 2.$

	B U G A S 1	L N G A S 1	H N G A S 1	C N G A S 1	C R G A S 1	B U G A S 2	L N G A S 2	H N G A S 2	C N G A S 2	C R G A S 2	RELATIONS	R H S V A L
COST	5.2	6.4	83	102	110	52	64	83	102	110	FREE	
<u>AVAILABILITY</u>												
AVAILBU	1					1					LE	35
AVAILLN		1					1				LE	20
AVAILAN			1					1			LE	40
AVAILCN				1					1		LE	105
AVAILCR					1					1	LE	80
<u>DEMANDS</u>												
DEMGAS1	1	1	1	1	1						EQ	100
DEMGAS2						1	1	1	1	1	EQ	60
<u>BLENDING</u>												
<u>REQUIREMENTS</u>												
BLOCTGS1	120	845	73	96	99						GE	9500
BLVAPGS1	60	18	4	64	25						LE	1100
BLVLTGS1	105	30	12	15	30						GE	1800
BLOCTGS2						120	845	73	96	99	GE	5880
BLVAPGS2						60	18	4	64	25	LE	720
BLVLTGS2						105	30	12	15	30	GE	1200

TABLEAU 2.2

LP User Formulation

- Model Variable Name

BUGAS1, LNGAS1... CRGAS2	The amount of Butane; used to produce GAS1... until amount of Catalytic Reformate used to produce GAS2.
-----------------------------	---

- Model Constraint Name

AVAILBU,... AVAILCR	The restrictions on availability for the five components.
DEMGAS1, DEMGAS2	The demand balance equations for the two Products ,
BLOCTGS1...BLVLTGS2	The six constraints for blending requirements ,

The matrix of the constraint relations is now set out in Tableau 2.2

A Multi Time Period Multi Mode Production Problem

A company manufactures three products P1, P2, P3 (NUTS, BOLTS, WASHERS) and has at its disposal three machines M1, M2, M3. The company can undertake normal and overtime production and needs to plan for two time periods, say WINTER and SUMMER. Any product left after the second time period has very little resale value. The necessary information concerning the operation of the company is set out in Tables 2.3, 2.4, 2.5.

It is necessary to find an LP formulation that maximizes the profit of the company's operation over the two periods.

- Subscripts and Dimensions

Let the four indices i, j, k, l be defined as

$i = 1, 2$	the index for the two time periods, Summer and Winter;
$j = 1, 2$	the index for the two modes of production, Normal, Overtime;
$k = 1, 2, 3$	the index for the three product types, P1, P2, P3;
$l = 1, 2, 3$	the index for the three machines, M1, M2, M3.

- Model Variables

x_{ijkl}	the quantity that is produced in the category i, j, k, l ,
y_{ik}	the quantity of product k stored in period i ,
z_{ik}	the quantity of product k sold in period i .

TABLE OF MACHINE HOURS (TABH)

		S U M M E R P E R I O D (H 1)							W I N T E R P E R I O D (H 2)								
		Normal (N) Working Hours			(O) Overtime			Total Hours (AV) Available		Normal (N) Working Hours			(O) Overtime			Total Hours (AV) Available	
		P1	P2	P3	P1	P2	P3	Normal w-Hrs	Overtime	P1	P2	P3	P1	P2	P3	Normal W-Hrs	Overtime
MACHINE 1	(M1)	4	5	6	3	4	5	100	80	5	6	7	4	5	5	110	90
MACHINE 2	(M2)	7	6	6	6	5	5	100	90	8	7	7	7	6	6	110	100
MACHINE 3	(M3)	3	-	-	2	-	-	40	30	4	-	-	3	-	-	50	40

P1 = MUTS P2 = BOLTS P3 = WASHERS

TABLE 2.3

TABLE OF PRODUCTION-COSTS (TABC)

	S U M M E R P E R I O D						W I N T E R P E R I O D					
	Normal Working Hours			Over time			Normal Working Hours			Over time		
	P1	P2	P3	P1	P2	P3	P1	P2	P3	P1	P2	P3
MACHINE 1	2	3	4	3	4	5	3	4	5	4	5	6
MACHINE 2	4	3	2	5	4	3	5	4	3	6	5	4
MACHINE 3	1	-	-	2	-	-	2	-	-	3	-	-

P1 = NUTS

P2 = BOLTS

P3=WASHERS

TABLE 2.4

TABLE OF ADDITIONAL-COSTS (TABC)

		S U M M E R P E R I O D			W I N T E R P E R I O D		
		MUTS	BOLTS	WASHERS	MUTS	BOLTS	WASHERS
SALEPRICE		10	10	9	11	11	10
MINIMUMDEMAND		25	30	30	30	25	25
STORAGE	CAPACITY	20	20	-			
	COST	1	1	1			
DATA	RESALEVALUE				2	2	1

TABLE 2.5

- Model Coefficients

The following information relating to the problem are available in the table TABH.

- $t_{ijk\ell}$ number of hours required to produce one unit of the product type k on the machine Z , in the time period i , using Normal or Overtime production j ;
- $a_{ijk\ell}$ machine availability in hours for the machine ℓ in period i and mode j .

In the table TABD,

- p_{ik} selling price } for the product type k in the time period i ;
 d_{ik} demand, }
- s_k storage cost for the product type k in one time period;
 h_k the corresponding storage capacity;
 r_k the final resale value at the end.

In the table TABC,

- $c_{ijk\ell}$ the production cost in the category i, j, k, ℓ .

- Linear Constraint Relation

The profit function of the problem may be expressed as

$$\begin{aligned} \text{profit} = & \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^3 \sum_{\ell=1}^3 (p_{ik} - c_{ijk\ell}) x_{ijk\ell} \\ & - \sum_{k=1}^3 s_k y_{1k} + \sum_{k=1}^3 (r_k - p_{2k}) y_{2k}. \end{aligned}$$

In an optimal plan Profit must be maximized subject to the constraints

- (i) machine availability,

$$\sum_{k=1}^3 t_{ijk\ell} \cdot x_{ijk\ell} \leq a_{ijk\ell}, \quad \text{for all } i, j, \ell;$$

- (ii) stock balance in the two periods,

$$\sum_{j=1}^2 \sum_{\ell=1}^3 x_{ijk\ell} - y_{1k} - z_{1k} = 0 \quad \text{for period 1, and all } k$$

and

$$\sum_{j=1}^2 \sum_{\ell=1}^3 x_{2jk\ell} - y_{1k} - y_{2k} = 0 \quad \text{for period 2, and all } k$$

(iii) minimum demand to be satisfied

$$z_{ik} \geq d_{ik}, \quad \text{for all } i, \text{ and } k;$$

(iv) upper bound on storage,

$$y_{1k} \leq h_k, \quad \text{for all } k;$$

(v) nonnegativity of the variables,

$$y_{ik} \geq 0 \quad \text{for all } i, k \quad \text{and} \quad x_{ijk\ell} \geq 0, \quad \text{for all } i, j, k, \ell.$$

LP User Formulation

- Model Variable Name

Production:

T1NP1M1...T20P3M3 The production variables $x_{1111} \dots x_{2233}$

Storage:

T1P1STR... The storage variables y_{11} etc.,

Amount meeting demand:

T1PID... the quantities which are allocated to satisfy demand z_{11} etc.,

- Model Constraint Name

PROFIT

Objective row.

T1MIAN...

Availability of machine 1, time period 1 and normal production,

T1P1ST

stock balance equation time period 1 product 1.

The other three constraints are satisfied by upper' bound and lower bound restrictions. The right hand side column is called RHS and the bound is called LIM and the full model is set out in Tableau 2.3.

3. Computer Assisted LP Modelling.

3.0 Earlier Generation Modelling Systems.

A number of matrix-generator report-writer (MGRW) systems are in use in industry, and for any organization that uses mathematical programming (MP) as a serious modelling tool such systems are of considerable value. Without exception such systems are used in conjunction with proven MP software: [7] contains a brief survey of such MGRW systems.

To understand the requirements of an MGRW system in which matrix-generator (MG) programs and report-writer (RW) programs are written it is necessary to identify the tasks performed by an MG or a RW. An MG or a RW may, of course, be written in a high-level computer language such as FORTRAN, ALGOL, PL1, or in an MGRW language. A typical modelling system using an MG, and OPTIMIZER, and a RW as shown in Diagram 3.1 may work in the

following way. The PROBLEM DATA is presented in the form of sets of tabulated information. These are read and processed by an MG program that produces a line image INPUT FILE. This usually contains,

- (a) the logical (i.e., slack and artificial) variables names or row names that are coded by suitable text expressions;
- (b) the structural variables names or column names that are similarly coded;
- (c) the coefficients of the problem matrix;
- (d) the RHS, BOUNDS, and RANGES information;
- (e) some information concerning the starting basis.

A RW is used primarily to extract only the pertinent information from the solution obtained by the OPTIMIZER, and it presents this information in a suitably tabulated format. The RW usually consults the PROBLEM DATA held as tables, and LINE IMAGE INPUT; it may also carry out some arithmetical operations on these solution values. An MGRW system therefore incorporates at least the following features:

- (i) input of PROBLEM DATA in tabular form;
- (ii) construction of row and column names by name expressions;
- (iii) using constants or arithmetic expressions to specify the matrix, RHS, BOUND, etc., coefficients by suitable row or column generator clause (procedure);
- (iv) accessing the solution file to obtain solution values, reduced costs, ranges, etc.;
- (v) format and print tabular information.

By and large these systems are either a procedural language compiler [8,9,10] or a program generator [11,12].

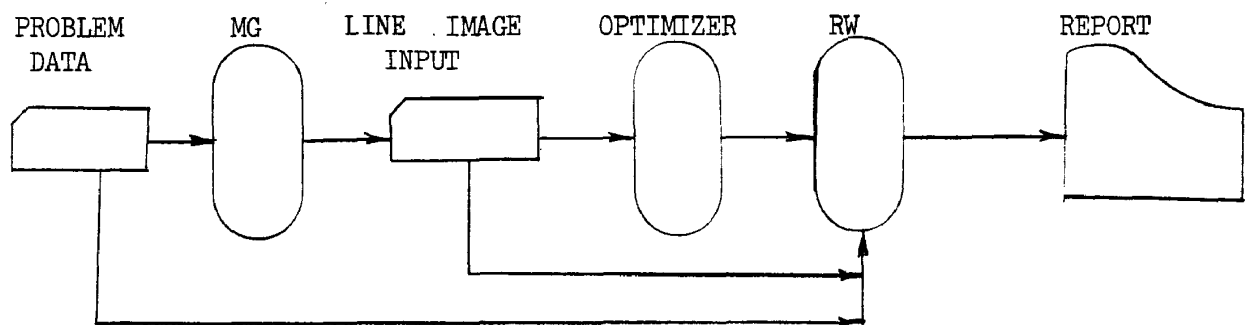


Diagram 3.1. Matrix generation, optimization, and report writing: a flow diagram.

In recent times a number of applications systems have avoided the use of a procedural language to communicate to a computer. The interactive program development and usage environment encourages this in contrast with the batch facilities of earlier computer operating systems. Some systems allow procedural language commands to be issued to the computer in the interactive mode. This latter approach does not provide the ideal interactive communication since the user of the system still needs to remember or refer to the syntax and semantics of a finite number of language commands.

3.1 Design Objectives of the system

In the construction of the computer assisted Mathematical Programming (Modelling) System: CAMPS we have followed the modern trends in system design and have aimed to meet the objectives set out below.

- (a) The (computer based) system should provide assistance to the modeller.
- (b) The modeller should be able to communicate his requirements via a set of menu options.
- (c) The modeller is considered to possess quantitative modelling skill and is not necessarily an expert in computer programming. He conceives and states mathematically his model and the system assists him in communicating this to the computer.

The progressive approach to model definition as described in section 2 of this report is strictly followed in CAMPS. A modelling system is of greatest assistance to the modeller when he is experimentally investigating and developing his model. The purpose of CAMPS is to provide support at this stage. The modeller by and large needs to maintain a mathematical documentation of the constructed model for his own reference and also to communicate with others. A mathematical statement of a completed model can also be obtained using CAMPS.

3.2 The set of Primary Options

CAMPS is fully described in the user document [6]. A set of five primary options showing the main functions are displayed in Table 3.1.

. . . CAMPS . . .			
Date :	5.11.83.		Time: 10.32
User :	CORMAC		Model: MTMPRD
MAIN OPTIONS			
1.	CREATE AND AMENDMODEL	...	CA
2.	GENERATE MODEL	...	GM
3.	RUN OPTIMISE	...	RO
4.	ANALYSIS AND REPORT	...	AR
5.	UTILITES	...	UT
6.	LOGOUT	...	LO

Table 3.1

The options 2 to 5 are connected with the task of experimentally investigating a model. The option 1 is used to formulate a model. A number of subfunctions which can be carried out using this modelling option are described in the next section.

3.3 The Set of Modelling Options.

Four out of five options (Table 3.2) which are provided under the heading of **CREATE AND AMEND MODEL** are based directly on the analysis of LP modelling strategy presented in section 2 of this report. It also follows from the analysis of section 2 that these options may be not entirely independent of each other. For instance using option 1 all the subscripts and their dimensions must be defined as the very first step. Only after this **DATA TABLES**, and **MODEL VARIABLES** can be defined independent of each other but consistent with the definitions under option 1. Under option 4 the model constraints (logical variables) are first defined and have to be consistent with the definitions under option 1. The linear constraint relationships are then stated and have to be consistent with definitions under options 1,2,3. Under option 5 test qualifiers for different items of data are introduced. These text qualifiers are subsequently put together in another part of the system to annotate the mathematical statement of the model (model documentation).

. . . CAMPS . . .			
Date:	5.11.83.	Time:	10. 32 .
User:	CORMAC	Model:	MTMPRD

CREATE AND AMEND MODEL	
1.	SUBSCRIPTS AND DIMENSIONS
2.	DATA TABLES
3.	MODEL VARIABLES
4.	MODEL CONSTRAINTS
5.	TEXT QUALIFIERS

Table 3.2

The hierarchical relationship of the command options and information flow through the system is illustrated in Diagram 3.1. It can be seen from the diagram that the system is designed to provide both modelling support and data handling support.

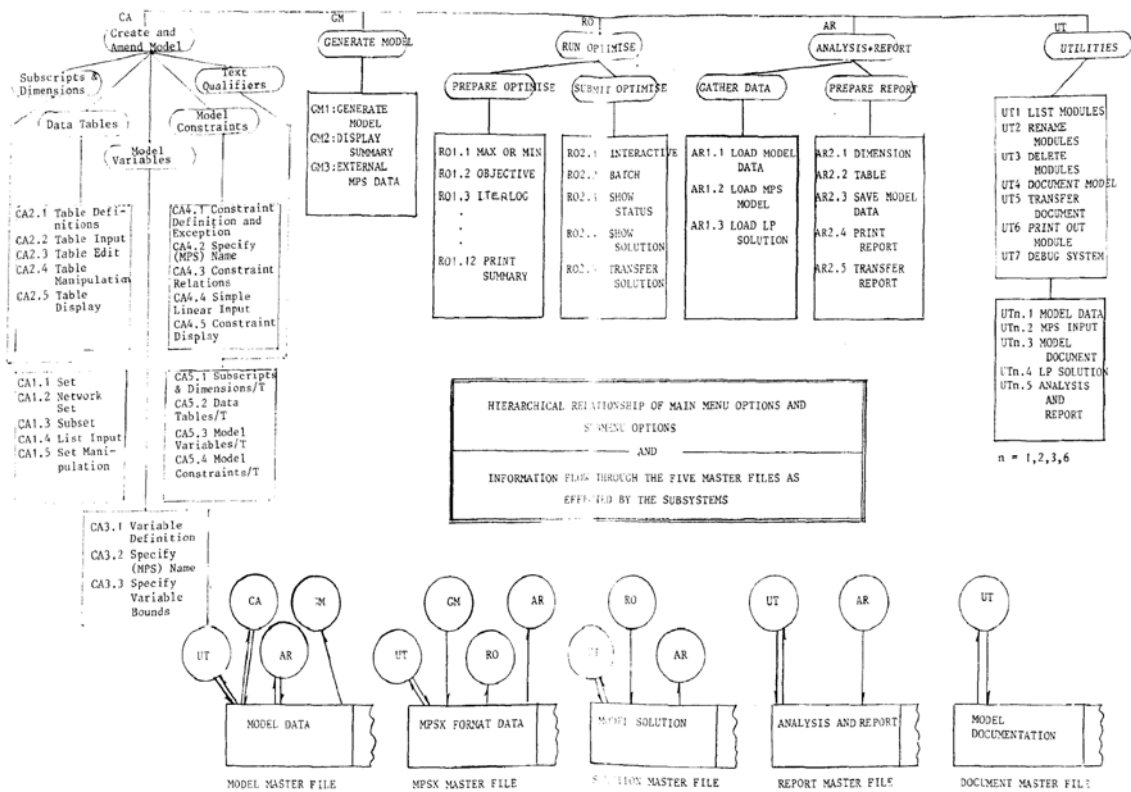


Diagram 3.1

4. Investigation of a Model using the System.

4.0 Introduction

In this section the use of CAMPS to create and investigate LP models is explained with an illustration. In section 4.1 the multitime period, multimode production problem of section 2 is considered to illustrate the modelling options of the CAMPS system. The optimisation controls, the method of reporting, and preparing model documentation are then explained in sections 4.2, 4.3, 4.4 respectively.

4.1 Model Construction

To construct a model the "**CREATE AND AMEND**" option (CA) of the **MAIN OPTIONS** menu (see Table 3.1) is first chosen. Alternatives available under this option such as **SUBSCRIPT** and **DIMENSIONS**,..., **TEXT QUALIFIERS**, are then chosen in their natural order to progressively construct the model.

- SUBSCRIPTS and DIMENSIONS

The full set of options under this command are set out in Table 4.1. For the given model only options 1 and 4 are used. Option 1 is used to declare the index sets (indices and their ranges).

. . . CAMPS . . .	
Date : 5.11.83.	Time : 10.32.
User : CORMAC	Model : MTMPRD

SUBSCRIPTS AND DIMENSIONS	
1.	SET
2.	NETWORK SET
3.	SUBSET
4.	LIST INPUT
5.	SET MANIPULATION

Table 4.1

Option 4 is used to create the text attribute set and the name attribute set corresponding to each entry is the index set as defined in option 1. The text and name elements are entered under the List Input facility.

- DATA TABLES

The tables **TABH**, **TABD**, **TABC** are defined using this command. The full set of options under this command are set out in Table 4.2. Under option 1 the dimensions of the tables and the type of its elements are specified.

. . . CAMPS . . .

Date :	5.11.83.	Time :	10. 32 .
User :	CORMAC	Model :	MTMPRD

DATA TABLE

1.	TABLE	DEFINTION
2.	TABLE	INPUT
3.	TABLE	EDIT
4.	TABLE	MANIPULATION
5.	TABLE	DISPLAY

Table 4.2

It is only necessary to introduce the name of a table (the dimensions are chosen using sets already defined) and its type. There are three possible types for a table; integer, real or text. Option 2 is used to enter data into the table or read it from some other data file held in the system. Option 5 is used to display and hence verify the contents of the tables set up in this way.

- **MODEL VARIABLES**

The three options under this command are set out in Table 4.3. The model variables x , y , z in terms of their appropriate subscripts

. . . CAMPS . . .

Date :	5.11.83.	Time :	10. 32 .
User :	CORMAC	Model :	MTMPRD

MODEL VARIABLES

1.	VARIABLE	DEFINTION
2.	SPECIFY	(MPS) NAME
3.	SPECIFY	VARIABLE BOUNDS

Table 4.3

$x(i, j, k, l), y(i, k), z(i, k)$ are first defined under option 1. To make the generated model variable names meaningful the **MPS** variable names such as T1NP1M1 etc. may be introduced under option 2. However, this is not obligatory and the system provides default names. Using option 3 the demand and storage bounds are specified.

- MODEL CONSTRAINTS

The four options under this command are shown in Table 4.4

...CAMPS...	
Date : 5. 11. 83	Time : 10.32
User : CORMAC	Model : MTMPRD

MODEL CONSTRAINTS	
1.	CONSTRAINT DEFINITION AND EXCEPTIONS
2.	SPECIFY (MPS) NAME
3.	CONSTRAINT RELATIONSHIP
4.	SIMPLE LINEAR INPUT
5.	CONSTRAINT DISPLAY

Table 4.4

Using option 1 the stock balance, the machine availability constraints and also the objective function are defined. These definitions amount to introducing the slack variables for the first set of constraints and the artificial variables for the second and one free row for the objective. Under option 2 the MPS names TIMIAN ..., TIPIST ..., PROFIT may be constructed for these which compare with the names used in the example. Again this step is not obligatory and default names are supplied by the system. Using option 3 the linear forms for these row groups are stated. Option 4 provides the facility of simple "equation mode" input. Using option 5 the constraints are displayed and verified.

- TEXT QUALIFIERS

This command leads to the set of options illustrated in Table 4.5. Under the four options the four groups of items

...CAMPS...	
Date : 5. 11. 83	Time : 10. 32 .
User : CORMAC	Model : MTMPRD

TEXT QUALIFIERS	
1.	SUBSCRIPTS AND DIMENSIONS/T
2.	DATA TABLES /T
3.	MODEL VARIABLES /T
4.	MODEL CONSTRAINTS /T

Table 4.5

DIMENSIONS,CONSTRAINTS which are defined earlier are annotated. Typical examples of these text annotations are

```

i ...      :      "number of time period(s) ."
j ...      :      "number of production mode(s) ."
           :
           :
           :
T(i, j, k, ℓ) :      "Table of production rate (hours)."  

A(i, j, ℓ)   :      "Table of machine availability (hours)."  

           :
           :
           :
X(i, j, k, ℓ) :      "The amount produced."  

y(i, k)      :      "The amount stored."  

           :
           :
           :
etc.        ...

```

These texts are stored together with the model data in the model master file. It can be used both by the Documentation subsystem and the Analysis and Report subsystem.

4.2 Optimisation Control

Using the Generate Model command at the main level the resulting model is produced in MPS format and summary information concerning the model can be examined. The Run Optimise command is then used to process the model. A small model can be processed interactively. Larger models are processed in the batch queue. Once the processing is complete a call back file is set up for the user to interrogate the solution. The optimisation controls such as choice of objective function, direction of optimisation (max or min). Maximum number of iterations, saving of the processed model, etc. are set up by a series of options.

4.3 Analysis and Report

Analysis of the solution and preparation of report usually amounts to preparing a few summary tables and printing these. In the given example for instance this could be production summary of storage used. From the point of view of data handling the user first needs to load the (input) Data Tables, solution to the model. He then defines and manipulates summary tables, assigns values to them, and finally prints these summary tables.

4.4 Model Documentation

The annotating texts which were introduced under the "Text Qualifier" option are used to construct the documentation of the model. The full documentation for the given example is set out below.

Title: Multi time Period Multimode Production Model.

Prepared by: C. Lucas Date: 5.11.83.

Subscripts and Dimensions.

$i = 1,2$ the number of time periods,
 $j = 1,2$ the number of production modes,
 $k = 1,\dots,3$ the number of products,
 $\ell = 1,\dots,3$ the number of machines.

Data Tables

$T(i, j, k, \ell) \dots$ the table of production rate (hours) for time period i , production mode j , product k , and machine ℓ ,
 $A(i, j, \ell) \dots$ the table of machine availability (hours) for time period i , production mode j and machine type ℓ ,
 $P(i, k) \dots$ the table of selling price (sterling) for time period i , and product k ,
 $D(i,k) \dots$ the table of demand for time period i and product k ,
 $S(k) \dots$ the table of storage cost for product k ,
 $H.(k) \dots$ the table of storage capacity for product k ,
 $R(k) \dots$ the table of resale value for product k ,
 $C(i,j,k, \ell) \dots$ the table of production cost for time period i , production mode j , product k and machine ℓ .

Model Variables

$X(i, j, k, \ell) \dots$ the amount produced in time period i , in production mode j , of product k , on machine ℓ ,
 $Y(i, k) \dots$ the amount of product k stored in time period i ,
 $Z(i, k) \dots$ the amount of product k sold in time period i ,

Model Constraints

Bounds

Minimum demand lower bounds Exception: None

$Z(i, k) \geq D(i, k)$ for all time periods i ,
and all products k .

Storage capacity upper bounds Exception: None

$Y(i, k) \leq H(k)$, all products k .

Objective and Linear Constraints

Objective

PROFIT:

$$\text{SUM...over } i \left[\text{SUM...over } j \left[\text{LSUM...over } k \left[\text{SUM...over } \ell \left[\left(p(i, k) - C(i, j, k, \ell) \right) * x(i, j, k, \ell) \right] \right] \right] \right]$$

Sales revenue less production costs

$$- \text{SUM...over } k [S(k) * Y(1, k)]$$

less storage cost

$$+ \text{SUM...over } k [(R(k) - P(2, k)) * Y(2, k)]$$

plus resale value less production cost

Machine Availability

Exceptions: None

MA(i, j, ℓ):

$$\text{SUM...over } k [T(i, j, k, \ell) * X(i, j, k, \ell)] \leq A(i, j, \ell)$$

all time periods i, all modes j, all machines ℓ.

Stock Balance 1

Exceptions: None

STBAL1(k):

$$\text{SUM over } j \left[\text{SUM...over } \ell [X(1, j, k, \ell) - Y(1, k) - Z(1, k)] \right] = 0$$

all products k.

Amount produced balances with storage and amount used to meet demand period 1

Stock Balance 2

Exceptions: None

STBAL 2(k):

$$\text{SUM...over } j \left[\text{SUM...over } \ell [X(2, j, k, \ell) + Y(1, k) - Y(2, k) - Z(2, k)] \right] = 0$$

all products k.

Amount produced balances with storage and amount used to meet demand period 2

The text qualifiers for dimensions are restated in defining the tables, model variables and model constraints. The bounds and other linear constraints which are expressed in groups may have exceptions in their specified ranges. Different expressions in linear forms are also annotated and these annotating texts are included within a pair of opening and closing hash marks #...# .

5. Discussion

The salient components of the mathematical programming models have been analysed. A number of earlier generation systems have been briefly considered. A computer assisted modelling scheme has been put forward which is derived out of the logical steps in the formulation of LP models. A clear and suitably annotated documentation of the model is an important requirement for the modeller. The system is designed to satisfy this requirement. A number of problems have been described and their mathematical formulations have been derived to illustrate the modelling principles and the design philosophy of the system. An example of applying the system to create, investigate and document a model is also set out to illustrate the LP modelling and data handling support provided by the system. A full user specification of the system can be found in [6].

6. Acknowledgements

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