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Diffraction by an acoustically penetrable or an electromagnetically dielectric half plane II.

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Abstract

The present work gives a mathematical model for an acoustically penetrable or electromagnetically dielectric half plane. An approximate boundary condition is derived which depends on the thickness of, and the material constants which constitutes, the half plane. A solution is obtained, using the approximate boundary condition, for the problem of a line source field diffracted by a semi-infinite penetrable/dielectric half plane. The asymmetry of the approximate boundary condition results in a matrix Wiener-Hopf problem, which is solved explicitly.

1. Introduction

The present work arose in connection with noise reduction by means of barriers. Noise reduction by barriers is a common method of reducing noise pollution in heavily built up areas, Kurze[1]. Traffic noise from motorways, railways and airports, and other outdoor noises from heavy construction machinery or stationery installations, such as large transformers or plants, can be shielded by a barrier which intercepts the line-of-sight from source to receiver. Noise in open plan offices can also be reduced by means of barrier partitions. In most of the calculations with noise barriers, the field in the shadow region of the barrier is assumed to be solely due to diffraction at the edge. This assumption supposes that the barrier is perfectly rigid and therefore does not transmit sound. However, most practical barriers are made of wood or plastic and will consequently transmit some of the noise through the barrier. The object of the present work is to make some allowance for the transmitted field.

The present work also has applications in electromagnetism when considering diffraction by a dielectric half plane. Where appropriate the connection with electromagnetism will be outlined.

There have been a number of works* dealing with a penetrable barrier, including an earlier model of the authors, see Rawlins[2] where one can find an outline of the work carried out up to 1977 and a bibliography. Since that time the only other papers known to the author on this subject are by Anderson[3], Chakrabarti[4] and Volakis and Senior[5]. Chakrabarti's work was subsequently found to be in error, see Volakis and Senior[5]. These three authors use a boundary condition which makes the barrier almost transparent. The present work uses an alternative boundary condition which results in a matrix Wiener-Hopf problem. Matrix Wiener-Hopf problems are generally intractable. However, the present problem can be solved exactly. An interesting feature of the present solution is that the normal Weiner-Hopf arguments yield an unknown constant which must be determined from an analysis of the edge field behaviour. The edge field behaviour is also interesting in that it depends on the material constants of the half plane, and is more complex than the usual singular behaviour associated with a perfectly rigid or soft half plane in acoustics, or a perfectly conducting half plane in electromagnetics.

In section two the approximate boundary condition is derived. This is achieved by looking at the canonical problem of reflection and transmission of a plane wave incident upon a penetrable slab which is

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*(It is planned, in a future publication, to give numerical comparisons between the various mathematical models.)

assumed to be thin compared with the incident wavelength. A matching technique is used to obtain the approximate boundary condition from the canonical problem. In section three a scalar boundary value problem for the field diffracted by a penetrable barrier is formulated. The field being an acoustic potential function, or a component of a polarized electromagnetic wave. In section four the scalar boundary value problem is solved. In section five some asymptotic expressions for the far field in terms of sources and a diffracted field are given. An appendix consists of the calculation of the edge field behaviour which it is necessary to know in order to carry out the solution in section 4.

2. Approximate boundary condition

Consider the situation when an infinite slab occupies $-\infty < x < <\infty$, $-h < y < h$, where the y axis is normal to slab faces. When a plane wave $e^{-ik}(x\cos\theta_0+y\sin\theta_0)$-iwt * (*The factor e^{-iwt} will be dropped in the rest of the work) is incident upon an infinite penetrable medium of width 2h, which has a material propagation constant kn=k,, the field above and below the slab is given by (see Brekhovskikh[6] p.45, and Rawlins[2]),

$$u(x,y) = e^{-ik(x\cos\theta_o+y\sin\theta_o)} + Re^{-ik(x\cos\theta_o-y\sin\theta_o)}, \qquad y \geq h, \tag{1}$$

$$= Te^{-ik(x\cos\theta_o+y\sin\theta_o)}, \qquad y \leq -h, \tag{2}$$

where the reflection coefficient R is given by

$$R = \frac{(1-N^2)\sin 2k,h e^{-i2kh\sin\theta_o}}{(1+N^2)\sin 2k,h + 2iN\cos 2k,h}, \tag{3}$$

and the transmission coefficient T is given by

$$T = \frac{2iNe^{-i2kh\sin\theta_o}}{(1+N^2)\sin 2k,h + 2iN\cos 2k,h}, \tag{4}$$

where $k_1 = k(n^2 - \cos^2\theta_o)^{\frac{1}{2}}$.

For an acoustically penetrable slab $n = c/c_1$, $N = \kappa_1 p/kp_1\sin\theta_o)$ (where p, c and p_1, c_1 are the density and sonic velocity of the media |y|>h and |y|h respectively) and u represents the acoustic pressure. For a dielectric slab $n = [(\varepsilon_1 \mu_1)/(\varepsilon\mu)]^{\frac{1}{2}}$, $N = k, \varepsilon/(k\ \varepsilon_1 \sin\theta_1)$, (for $u=H_z$ magnetic vector parallel to the z axis), $N = k_1\mu/(k\mu, \sin\theta_0)$ (for $u=E_z$ electric vector parallel to z-axis) where u, ε and $\mu,_1$, ε, are the permeability and permitlivity of the media |y|>h and |y|<h respectively.

We shall now use the results (1) to (4) to obtain an approximate boundary condition for a penetrable slab whose width is small compared to the incident wave length, i.e. 2kh<<l. From the equations (1) and (2) we have

$$\sigma = \frac{u(x,h)}{u(x,-h)} = \frac{e^{-ikhsin\theta_0} + Re^{ikhsin\theta_0}}{Te^{ikhsin\theta_0}}$$

$$= \cos 2k_1 h + \sin 2k_1 h/(iN) \simeq 1 - 2ik_1 h/N + 0((kh)^2) \tag{5}$$

$$\tau = \frac{\frac{\partial u}{\partial y}(x,h)}{\frac{\partial u}{\partial y}(x,-h)} = \frac{-e^{-ikhsin\theta_0} + Re^{ikhsin\theta_0}}{-Te^{ikhsin\theta_0}}$$

$$= \cos 2k_1 h - iN\sin 2k_1 h \simeq 1 - i2k_1 Nh + 0((kh)^2). \tag{6}$$

Now assuming 2kh<<l then as far as the external field is concerned the slab is very thin and therefore can be modelled by the approximate boundary conditions

$$u(x,0^+) = \sigma u(x,0^-).$$
$$\frac{\partial u}{\partial y}(x,0+) = \tau \frac{\partial u}{\partial y}(x,0-). \tag{7}$$

$$\tau = 1-i2k_1\ Nh, \quad \sigma = 1-2ik_1\ h/N\ .$$

3. Formulation of the problem of line source field diffraction by a semi-infinite penetrable plane.

We consider the situation where a penetrable half plane occupies $x \le 0$, $y=0$. The line source is situated at (x_0,y_0), $y_0>0$. The problem is solved by finding a solution of the wave equation.

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial}{\partial y^2}+k^2\right]u(x,\ y)=\delta(x-x_0)\delta(y-y_0),\quad (|y|>h), \tag{8}$$

subject to the boundary conditions

$$\left.\begin{aligned} u(x,0^+) &= \sigma u(x,0^-), \\ \frac{\partial u}{\partial y}(x,\ o^+) &= \tau\frac{\partial u}{\partial y}(x,0^-), \end{aligned}\right\}\ x<0. \tag{9}$$

$\text{Im}\sigma \neq 0,\quad \text{Im}\tau \neq 0.$

$$u(x,0^+) = u(x,0^-),\ \frac{\partial u}{\partial y}(x,0^+) = \frac{\partial u}{\partial y}u(x,0^-),\quad x>0. \tag{10}$$

For a unique solution to the problem we also require the satisfaction of the radiation condition

$$\lim_{r\to\infty} r^{\frac{1}{2}}\left[\frac{\partial}{\partial r}-ik\right]u = 0, \tag{11}$$

and the edge condition

$$\begin{aligned} &\lim_{x\to 0} u(x,0) = 0(x^{\lambda}),\quad 0<\text{Re}\lambda\leq\frac{1}{2}, \\ &\lim_{r\to 0} r\ \text{grad}\ u = 0,\qquad \text{where } r = (x^2+y^2)^{\frac{1}{2}}. \end{aligned} \tag{12}$$

For the value of λ see appendix.

4. Solution of the boundary value problem

We shall assume, for analytical convenience, that $k=k_r+ik_i$, $k_r>0$, $k_i\geq 0$. At the end of the analysis we can set $k_i=0$.

Define $U(\alpha,y)$, where a is a complex variable by

$$U(\alpha,y)=\int_{-\infty}^{\infty}u(x,y)e^{i\alpha x}dx. \tag{13}$$

The radiation condition requires that the phase dependence of $u(x,y)$, as $|x|\to\infty$, behave like $e^{ik_i|x|}$. In view of this it can be seen that $U(\alpha,y)$ will exist for $-k_i < \text{Im}(\alpha) < k_i$. Then it follows from (8) that $U(\alpha,y)$ satisfies

$$\frac{d^2U}{dy^2}+k^2U=e^{i\alpha x_o}\delta(y-y_o),\ y_o>0 \tag{14}$$

where $k = (k^2 - \alpha^2)^{\frac{1}{2}}$ is defined to be that branch for which k = k when $\alpha = 0$. Then κ will always have a positive imaginary part in the region $|Im(\alpha)| < k_i$. A solution of (14) for α in the strip $|Im(\alpha)| < k_i$, which decays as $|y| \rightarrow \infty$, is given by

$$U(\alpha,y) = A(\alpha)\exp[iky] + \exp[i(\alpha x_0 + k \mid y-y_0 \mid)]/(2ik), \quad (y>0) \tag{15}$$

$$= B(\alpha)\exp[-iky], \qquad (y<0). \tag{16}$$

Let

$$\Phi_1^-(\alpha) = \int_{-\infty}^{0} \left[u(x,0^+) - u(x,0^-)\right] e^{i\alpha x} dx, \tag{17}$$

$$\Phi_2^-(\alpha) = \int_{-\infty}^{0} \left[\frac{\partial u}{\partial y}(x,0^+) - \frac{\partial u}{\partial y}(x,0^-)\right] e^{i\alpha x} dx, \tag{18}$$

$$\psi_1^+(\alpha) = \int_{-\infty}^{0} \left[u(x,0^+) - \sigma u(x,0^-)\right] e^{i\alpha x} dx, \tag{19}$$

$$\psi_2^+(\alpha) = \int_{-\infty}^{0} \left[\frac{\partial u}{\partial y}(x,0^+) - \sigma \frac{\partial u}{\partial y}(x,0^-)\right] e^{i\alpha x} dx, \tag{20}$$

Then $\Phi_{1,2}^-(\alpha)$ are analytic for $Im(\alpha) < k_i$, and $\psi_{1,2}^+(\alpha)$ are analytic for $Im(\alpha) > -k_i$. Throughout the rest of this work a superscript (or subscript) plus or minus sign attached to any function will mean that the function is analytic in $Im(\alpha) > -k_i$ or $Im(\alpha) < k_i$, respectively. Using the expressions (9),(10),(13),(15) and (16) in the expressions (17) to (20) gives

$$\Phi_1^-(\alpha) = A(\alpha) - B(\alpha) + \exp[i(\alpha x_0 + ky_0)]/(2ik), \tag{21}$$

$$\Phi_2^-(\alpha) = ik[A(\alpha) - B(\alpha)] + \exp[i(\alpha x_0 + ky_0)]/2, \tag{22}$$

$$\psi_1^+(\alpha) = A(\alpha) - \sigma B(\alpha)] + \exp[i(\alpha x_0 + ky_0)]/(2ik), \tag{23}$$

$$\psi_2^+(\alpha) = ik[A(\alpha) + \tau B(\alpha)] + \exp[i(\alpha x_0 + ky_0)]/2, \tag{24}$$

Eliminating A(α) and B(α) from (21) to (24) gives the matrix Wiener-Hopf equation

$$\psi_+(\alpha) = K(\alpha)\Phi_-(\alpha) + D(\alpha) \tag{25}$$

where

$$\psi_{+}(\alpha) = \begin{bmatrix} \psi_1^{+}(\alpha) \\ \psi_2^{+}(\alpha) \end{bmatrix}, \qquad \Phi_{-}(\alpha) = \begin{bmatrix} \Phi_1^{-}(\alpha) \\ \Phi_2^{-}(\alpha) \end{bmatrix}, \tag{26}$$

$$K(\alpha) = \frac{1}{2}\begin{bmatrix} (1+\sigma) & (1-\sigma)/(ik) \\ ik(1-\tau) & (1+\tau) \end{bmatrix}, \tag{27}$$

$$D(\alpha) = \frac{1}{2}\begin{bmatrix} (1-\sigma)\exp[i(\alpha x_0 + ky_0)]/(ik) \\ -(1-\tau)\exp\{i(\alpha x_0 + ky_0)\} \end{bmatrix}. \tag{28}$$

The matrix equation (25) constitutes a coupled system of Wiener-Hopf equations. The standard Wiener-Hopf technique can only be applied if the system (25) can be uncoupled into two separate Wiener-Hopf equations. This requires that the matrix function $K(\alpha)$ can be factorized. This is a nontrivial operation and it is not always obvious that one can in fact factorize the matrix. In the present problem we note that $K(\alpha)$ can be written as

$$K(\alpha) = CG(\alpha) \tag{29}$$

where

$$C = \frac{1}{2}\begin{bmatrix} 1+\sigma & 0 \\ 0 & 1+\tau \end{bmatrix}, \quad G(\alpha) = \begin{bmatrix} 1 & \left[\dfrac{1-\sigma}{1+\sigma}\right](ik)^{-1} \\ \left[\dfrac{1-\tau}{1+\tau}\right]ik & 1 \end{bmatrix} \tag{30}$$

The matrix $G(\alpha)$ given by (30) is of a special form which can be factorized immediately, (see Daniele[7] and Rawlins[8]), to give

$$G(\alpha) = G_{+}(\alpha)G_{-}(\alpha) \tag{31}$$

where

$$G_{\pm}(\alpha) = \sqrt{1-\varepsilon^2}\begin{bmatrix} \cosh\chi_{\pm} & \dfrac{\delta}{\gamma}\sinh\chi_{\pm} \\ -\dfrac{\gamma}{\delta}\sinh\chi_{\pm} & \cosh\chi_{\pm} \end{bmatrix}, \tag{32}$$

where

$$\begin{aligned} &\delta = [(1+\tau)(1-\sigma)/\{(1+\sigma)(1-\tau)\}]^{\frac{1}{2}}, \ \varepsilon = [(1-\tau)(1-\sigma)/\{(1+\tau)(1+\sigma)\}]^{\frac{1}{2}} \\ &\gamma = (\alpha^2 - k^2), \chi_{\pm}(\alpha) = \left[\frac{i}{2\pi}\right]\ell n[(1+\varepsilon)/(1-\varepsilon)]\ell n[(\gamma + (\pm\alpha - k)/(\gamma - ((\pm\alpha - k))] \end{aligned} \tag{33}$$

and the logarithms take values on the principle branch $\ell n(1) = 0$, $-\pi < \arg(\ ) \leq \pi$.

In order to be able to apply the usual Wiener-Hopf method we shall need some asymptotic growth estimates for the elements appearing in the matrices $G_{\pm}(a)$. It is not difficult to show that

$$\chi_{+}(\alpha) = i/(2\pi)\ell n[(1+\varepsilon)/(1-\varepsilon)]\ell n(2\alpha/k) + 0(\alpha^{-2}), \text{ as } |\alpha|\to\infty, \mathrm{Im}a\to -k_i \tag{34}$$

and hence

$$\cosh_{\chi+}(\alpha) = 0(\alpha^{\lambda}), \quad \sinh\chi_{+}(\alpha) = 0(\alpha^{\lambda}) \tag{35}$$

where

$$\mathrm{Re}\lambda = \frac{1}{2\pi}\left|\arg\left[\frac{1+\varepsilon}{1-\varepsilon}\right]\right|, \quad 0 < \mathrm{Re}\lambda \le \frac{1}{2}. \tag{36}$$

Similarly it can be shown that

$$\cosh\chi_{-}(\alpha) = 0(\alpha^{\lambda}), \quad \sinh\chi_{-}(\alpha) = 0(\alpha^{\lambda}), \text{ for } |\alpha|\to\infty, \ \mathrm{Im}a\to k_i. \tag{37}$$

By using results (29) and (31) in equation (25) we have

$$\begin{aligned}\psi_{+}(\alpha) &= CG(\alpha)\Phi_{-}(\alpha) + D(\alpha)\\ &= (CG_{+})G_{-}(\alpha)\,\Phi_{-}(\alpha) + D(\alpha),\end{aligned} \tag{38}$$

where

$$CG_{+}(\alpha) = \frac{\sqrt{1-\varepsilon^2}}{2}\begin{bmatrix} (1+\sigma)\cosh\chi_{+}(\alpha) & (1+\sigma)\frac{\delta}{\gamma}\cosh\chi_{+}(\alpha) \\ (1+\tau)\frac{\gamma}{\delta}\sinh\chi_{+}(\alpha) & (1+\tau)\cosh\chi_{+}(\alpha)\end{bmatrix},$$

is non singular since $\sigma \neq -1$ and $\tau \neq -1$. Thus CG+(ct) has an inverse and we can multiply across equation (38) by $(CG_{+}(a))'''$ to give

$$H^{+}(\alpha)\,\psi_{+}(\alpha) = J^{-}(\alpha)\,\Phi_{-}(\alpha) + \Delta(\alpha) \tag{39}$$

where

$$H_{+}(\alpha) = \frac{1}{(1+\tau)(1+\sigma)}\begin{bmatrix} (1+\tau)\cosh_{\chi+} & -(1+\sigma)\frac{\delta}{\gamma}\sinh_{\chi+} \\ -(1+\tau)\frac{\gamma}{\delta}\sinh_{\chi+} & (1+\sigma)\cosh_{\chi+}\end{bmatrix}, \tag{40}$$

$$J^{-}(\alpha) = \frac{1}{2}\left(1-\varepsilon^2\right)\begin{bmatrix} \cosh_{\chi-}(\alpha) & \frac{\delta}{\gamma}\sinh_{\chi-}(\alpha) \\ \frac{\gamma}{\delta}\sinh_{\chi-}(\alpha) & \cosh_{\chi-}(\alpha)\end{bmatrix}, \tag{41}$$

$$\Delta(\alpha) = \begin{bmatrix}\Delta_1(\alpha)\\ \Delta_2(\alpha)\end{bmatrix} = \frac{\exp[i(\alpha x_0 + ky_0)]}{2(1+\tau)(1+\sigma)}\begin{bmatrix} \frac{(1+\gamma)(1-\sigma)\cosh_{\chi+}(\alpha)}{ik} - \frac{(1+\sigma)(1-\tau)\delta\sinh_{\chi+}(\alpha)}{ik} \\ \frac{(1+\tau)(1-\sigma)}{\delta}\sinh_{\chi+}(\alpha) - (1+\sigma)(1-\tau)\cosh_{\chi+}(\alpha)\end{bmatrix} \tag{42}$$

We can now express (42), by means of the Cauchy integral theorem, see Noble[9], as

$$\Delta(\alpha) = \Delta_+(\alpha) + \Delta_-(\alpha), \tag{43}$$

where

$$\Delta_\pm(\alpha) = \begin{bmatrix} \Delta_1^\pm(\alpha) \\ \Delta_2^\pm(\alpha) \end{bmatrix},$$

$$\Delta_1^\pm(\alpha) = \pm \frac{1}{2\pi\, i} \int_{-\infty\mp iC}^{\infty\mp iC} \frac{\Delta_1(t)}{(t-\alpha)}\, dt, \Delta_2^\pm(\alpha) = \pm \frac{1}{2\pi\pi} \int_{-\infty\mp C}^{\infty\mp C} \frac{\Delta_2(t)dt}{t-\alpha}. \tag{44}$$

The representations (44) with the upper (lower) sign are valid when $\mathrm{Im}(\alpha)>-c(\mathrm{Im}(\alpha)<c)$ and define $\Delta^+_{1,2}(\alpha)(\Delta^-_{1,2}(\alpha))$ as analytic functions in

$\mathrm{Im}(\alpha)>-c(\mathrm{Im}(\alpha)<c)$. The exponential term in $\Delta_{1,2}(\alpha)$ ensures that the integrands of (44) are exponentially bounded as $t\to\pm\infty$ and therefore that the integrals exist. Standard asymptotics also show that

$$\Delta^\pm_{1,2}(\alpha) = 0(\alpha^{-1}) \text{ as } |\alpha| \to \infty \tag{45}$$

in their regions of regularity- We may now write (23) in terms of $\Delta_\pm(\alpha)$ as

$$H^+(\alpha)\psi_+(\alpha) - \Delta_+(\alpha) = J^-(\alpha)\Phi\_(\alpha) + \Delta\_(\alpha) \tag{46}$$

or written out in terms of elements of the matrices

$$\frac{\cosh\chi_+(\alpha)}{(1+\sigma)}\psi_1^+(\alpha) - \frac{\delta\sinh\chi_+(\alpha)}{(1+\tau)\gamma}\psi_1^+(\alpha) - \Delta_1^+(\alpha)$$

$$= \frac{1}{2}(1-\varepsilon^2)\left\{\cosh\chi_-(\alpha)\Phi_1^-(\alpha) + \frac{\delta}{\gamma}\sinh\chi_-(\alpha)\Phi_2^-(\alpha)\right\} + \Delta_1^-(\alpha). \tag{47}$$

$$-\frac{\gamma}{(1+\sigma)\delta}\sinh\chi_+(\alpha)\psi_1^+(\alpha) + \frac{\cosh\chi_+(\alpha)}{(1+\tau)}\psi_2^+(\alpha) - \Delta_2^+(\alpha)$$

$$= \frac{1}{2}(1-\varepsilon^2)\left\{\frac{\gamma}{\delta}\sinh\chi_-(\alpha)\Phi_1^-(\alpha) + \cosh\chi_-(\alpha)\Phi_2^-(\alpha)\right\} + \Delta_2^-(\alpha). \tag{48}$$

The edge condition (12) requires that the transformed functions must have the following asymptotic behaviour

$$\begin{aligned}&\Phi_1^-(\alpha) = 0(\alpha^{-\lambda-1}), \Phi_2^-(\alpha) = 0(\alpha^{-\lambda}), \text{ for } \mathrm{Im}(\alpha) < k_i, |\alpha| \to \infty;\\ &\psi_1^+(\alpha) = 0(\alpha^{-\lambda-1}), \psi_2^+(\alpha) = 0(\alpha^{-\lambda}), \text{ for } \mathrm{Im}(\alpha) > -k_i, |\alpha| \to \infty.\end{aligned} \tag{49}$$

By using the above asymptotic estimates (35),(37),(45) and (49) it can be shown that the left hand side of the equation (47) is regular, analytic and asymptotic to $0(\alpha^{-1})$ as $|\alpha|\to\infty$ in $\mathrm{Im}\alpha>-k_i$. Similarly the right hand side is regular, analytic and asymptotic to $o(\alpha^{-1})$ as $|\alpha|\to\infty$ in $\mathrm{Im}\alpha<k_i$. Hence by Liouville's theorem the analytic continuation of both sides in the entire complex plane is the constant zero. Hence

$$\frac{\cosh\chi_+(\alpha)}{(1+\sigma)}\psi_1^+(\alpha)-\frac{\delta\sinh\chi_+(\alpha)}{\gamma(1+\tau)}\psi_2^+(\alpha)-\Delta_1^+(\alpha)=0. \qquad (50)$$

Dealing with the equation (48) in a similar fashion it can be shown that the right and left hand side of this equation is asymptotic to $0(1)$ a constant in their respective regions of analycity. Hence by Liouville's theorem we have

$$-\frac{\gamma\sinh\chi_+(\alpha)}{(1+\sigma)\delta}\psi_1^+(\alpha)+\frac{\cosh\chi_+(\alpha)}{(1+\tau)}\psi_2^+(\alpha)-\Delta_2^+(\alpha)=a_0,$$ where a_0 is an unknown constant.

From (50) and (51) we have

$$\psi_1^+(\alpha)=(1+\sigma)\{\cosh\chi_+(\alpha)\Delta_1^+(\alpha)+\delta\sinh\chi_+(\alpha)(\Delta_2^+(\alpha)+a_0)/\delta\}, \qquad (52)$$

$$\psi_2^+(\alpha)=(1+\tau)\{\gamma\sinh\chi_+(\alpha)\Delta_1^+(\alpha)/\delta+\cosh\chi_+(\alpha)(\Delta_2^+(\alpha)+a_0\} \qquad (53)$$

From the equation (23) an (24) we have therefore

$$A(\alpha)=\frac{1}{(\sigma+\tau)}\Big[\{\tau(1+\sigma)\cosh\chi_+(\alpha)-\sigma(1+\tau)\sinh\chi_+(\alpha)/\delta\}\Delta_1^+(\alpha)$$
$$+\{\tau(1+\sigma)\delta\sinh\chi_+(\alpha)-\sigma(1+\tau)\cosh\chi_+(\alpha)\}(\Delta_2^+(\alpha)+a_0)/\gamma\Big]$$
$$+\frac{(\sigma-\tau)}{(\sigma+\tau)}\exp\big[i(\alpha x_0+ky_0)\big]/(2ik), \qquad (54)$$

$$B(\alpha)=\frac{-1}{(\sigma+\tau)}\Big[\{(1+\sigma)\cosh\chi_+(\alpha)+(1+\tau)\sinh\chi_+(\alpha)/\delta\}\Delta_1^+(\alpha)$$
$$+\{(1+\sigma)\delta\sinh\chi_+(\alpha)+(1+\tau)\cosh\chi_+(\alpha)\}(\Delta_2^+(\alpha)+a_0)/\gamma\Big]$$
$$+\exp\big[i(\alpha x_0+ky_0)\big]/((\sigma+\tau)ik). \qquad (55)$$

Hence we have solved the problem completely once we know the constant a_0. To determine this constant we analyse the edge field behaviour of the solution. We know from the appendix that the field near the edge behaves like $u(x,o) = 0(x^{\lambda})$ as $x \to 0^+$ where $\mathrm{Re}\lambda = \frac{1}{2\pi}\left|\arg\left[\frac{1+\varepsilon}{1-\varepsilon}\right]\right|$. Hence we know that the transformed quantities $A(\alpha)$ and $B(\alpha)$ should behave not greater than $0(\alpha^{-\lambda-1})$, as $|\alpha| \to \infty$ Letting $|\alpha| \to \infty$ in the expressions (54) and (55) give

$$A(\alpha) = \left[\frac{\delta\tau(1+\sigma) - \sigma(1+\tau)}{2(\sigma+\tau)}\right]\left[\frac{\Delta_1^+}{\delta} + a_0\right]\alpha^{\lambda-1} + 0\left[\alpha^{-\lambda-1}\right] \tag{56}$$

$$B(\alpha) = -\left[\frac{(1+\sigma)\delta + (1+\tau)}{2((\sigma+\tau)}\right]\left[\frac{\Delta_1^+}{\delta} + a_0\right]\alpha^{\lambda-1} + 0\left[\alpha^{-\lambda-1}\right] \tag{57}$$

If we exclude the trivial or non physical situations: $\sigma=\tau=0$; $\sigma=1$, and $\tau=-1$; $\sigma=1$ and $\tau=-1$; $\sigma=-1$, $\tau=1$; we must choose for the correct edge field behaviour

$$a_0 = -\tilde{\Delta}_1^+/\delta \; . \tag{58}$$

Hence the solution to the boundary value problem is given by

$$u(x, y) = \frac{1}{2\pi}\int_{-\infty+id}^{\infty+id}\left\{\left[\frac{\tau(\sigma+1)}{(\sigma+\tau)}\cosh\chi_+(\alpha) - \frac{\sigma(1+\tau)}{\delta(\sigma+\tau)}\sinh\chi_+(\alpha)\right]\Delta_1^+(\alpha)\right.$$

$$\left. + \left[\frac{\tau(1+\sigma)}{(\sigma+\tau)}\delta\sinh\chi_+(\alpha) - \frac{\sigma(1+\tau)}{(\sigma+\tau)}\cosh\chi_+(\alpha)\right]\frac{(\Delta_2^+(\alpha) - \tilde{\Delta}_1^+/\delta\delta}{\gamma}\right\}e^{-i\alpha x + iky}\,d\alpha$$

$$\frac{(\sigma-\tau)}{(\sigma+\tau)}\cdot\frac{1H_0^{(1)}}{4i}\left[k\{(x-x_0)^2 + (y+y_0)^2\}^{\frac{1}{2}}\right] + \frac{1}{4i}H_0^{(1)}\left[k\{(x-x_0)^2 + (y-y_0)^2\}^{\frac{1}{2}}\right],$$

$$y > 0. \tag{59}$$

$$= -\frac{1}{2\pi}\int_{-\infty+id}^{\infty+id}\left\{\left[\frac{(1+\sigma)}{(\sigma+\tau)}\cosh\chi_+(\alpha) + \frac{(1+\tau)}{(\sigma+\tau)\delta}\sinh\chi_+(\alpha)\right]\Delta_1^+(\alpha)\right.$$

$$\left. + \left[\frac{(1+\sigma)}{(\sigma+\tau)}\delta\sinh\chi_+(\alpha) + \frac{(1+\tau)}{(\sigma+\tau)}\cosh\chi_+(\alpha)\right]\frac{(\Delta_2^+(\alpha) - \tilde{\Delta}_1^+/\delta\delta}{\gamma}\right\}e^{-i\alpha x - iky}\,d\alpha\,,$$

$$+ \frac{2}{(\sigma+\tau)}\cdot\frac{1H_0^{(1)}}{4i}\left[k\left\{(x-x_0)^2 + (y-y_0)^2\right\}^{\frac{1}{2}}\right], \qquad y < 0. \tag{60}$$

The physical interpretation of the solution given by (59) and (60) is made more apparent by asymptotically evaluating the integrals for the receiver point (x,y) such that $k(x^2 + y^2)^{\frac{1}{2}} \to \infty$. This corresponds to the observer at (x,y) being in the far field. In practice if the line source at (x_0,y_0) and the receiver at (x,y) are more than two wavelengths from the edge (0,0) of the barrier then to a good approximation we can assume that we are in the far field, and the incident field in a plane wave.

5. Asymptotic expressions for the fax field

The asymptotic methods though straightforward are tedious. We shall merely give an outline of the calculations, more details of the techniques can be found in Noble[9]. Consider first $\Delta^{+}_{1,2}(\alpha)$ as given by (4.4); let k be real, then c=0 and the integration path along the real axis is indented below the point $t=\alpha$. Substitute $x_0 = r_0\cos\theta_0$, $y_0 = r_0\sin\theta_0$, $0<\theta_o<\pi$; $t= k\cos\xi$, $0 < \mathrm{Re}\xi < \pi$, then the integrand has a saddle point at $\xi = \theta_0$. The integration path is now deformed into the steepest descent-path $S(6_0)$ described by $\mathrm{Re}[\cos\{\xi-\theta_0)]=1$, $\mathrm{Imlcos}(\xi-\theta_0)] \geq 0$. In performing the deformation the pole at $k\cos\xi=a$ is intercepted if $\alpha < k\cos\theta_0$. The integral along $S(\theta_0)$ is asymptotically expanded as $kr_0\to\infty$ by means of the saddle point method. Thus it is found that

$$\Delta^{+}_{1,2}(\alpha) \sim \frac{A_{1,2}}{(k\cos\theta_\circ - \alpha)} + D_{1,2}(\alpha) H[k\cos\theta_\circ \alpha]\exp[i(\alpha x_\circ + ky_\circ)], \tag{61}$$

where

$$A_1 = -\frac{\{(1-\sigma)(1+\tau)\cosh\chi_+(k\cos\theta_\circ) - (1+\sigma)(1-\tau)\delta\sinh\chi_+(k\cos\theta_\circ)\}}{4\pi(1+\tau)(1+\sigma)}$$

$$-\left[\frac{2\pi}{kr_\circ}\right]^{\frac{1}{2}} \exp\left[i(kr_\circ - \pi/4)\right],$$

$$A_2 = \frac{\{(1+\tau)(1-\sigma)\sinh\chi_+(k\cos\theta_\circ) - \delta(1+\sigma)(1-\tau)\cosh\chi_+(k\cos\theta_\circ)\}}{4\pi i(1+\tau)(1+\sigma)\delta}$$

$$-k\sin\theta_\circ\left[\frac{2\pi}{kr_\circ}\right]^{\frac{1}{2}} \exp\left[i(kr_\circ - \pi/4)\right]$$

$$D_1(\alpha) = \frac{\{(1-\sigma)(1+\tau)\cosh\chi_+(\alpha) - (1+\sigma)(1-\tau)\delta\sinh\chi_+(\alpha)\}}{2ik(1+\tau)(1+\sigma)}$$

$$D_2(\alpha) = \frac{\{(1+\tau)(1-\sigma)\sinh\chi_+(\alpha) - \delta(1+\sigma)(1-\tau)\cosh\chi_+(\alpha)\}}{2(1+\tau)(1+\sigma)\delta}$$

and where H[x] = 1 for x >0, H[x] = 0 for x < 0 (Heaviside step function); the results are valid for $kr_0 \to \infty$, $-k < \alpha < k$; the term involving the functions $D_{1,2}$ arise from the residue contribution.

We can deal in a similar manner with $\tilde{\Delta}_1^+(\alpha)$, the only difference being that there is no pole contribution to worry about. Thus $\Delta_1^+ \sim \tilde{A}_1$ where

$$\overline{A}1 = -\frac{\{(1-\sigma)(1+\tau)\cosh\chi_+(k\cos\theta_0) - (1+\sigma)(1-\tau)\delta\sinh\chi_+(k\cos\theta_0)\}}{4\pi\pi(\ +\tau)(1+\sigma)}$$

$$-\left[\frac{2\pi}{kr_0}\right]^{\frac{1}{2}} \exp\left[i(kr_0 - \pi/4)\right] \tag{62}$$

The results (61) and (62) for $\Delta_{1,2}^+(\alpha)$ and $\tilde{\Delta}_1^+$ when inserted into (59) and (60) give

$$u(x,y) = u_d(x,y) + u_g(x,y) \tag{63}$$

where

$$u_a(x,y) = \frac{1}{2\pi}\int_{-\infty+id}^{\infty+id} \left\{\left[\frac{\tau(\sigma+1)}{(\sigma+\tau)}\cosh\chi_+(\alpha\alpha - \frac{\sigma(1+\tau)}{\delta(\sigma+\tau)}\sinh\chi_+(\alpha\alpha\right]\frac{A_1}{(k\cos\theta_0-\alpha)}\right.$$

$$\left. +\frac{1}{\gamma}\left[\frac{\tau(1+\sigma)}{(\sigma+\tau)}\delta\sinh\chi_+(\alpha\alpha - \frac{\sigma(1+\tau)}{(\sigma+\tau)}\cosh\chi_+(\alpha\alpha\right]\left(\frac{A_2}{(k\cos\theta_0-\alpha)} - \frac{\tilde{A}_1}{\delta}\right)\right\}$$

$$\exp[-i\alpha x + ik\mathrm{Y}]d\alpha, \quad \mathrm{Y} > 0, \tag{64}$$

$$= \frac{1}{2\pi}\int_{-\infty+id}^{\infty+id} \left\{\left[\frac{(1+\sigma)}{(\alpha+\tau)}\cosh\chi_+(\alpha\alpha + \frac{\sigma(1+\tau)}{(\sigma+\tau)\delta}\sinh\chi_+(\alpha\alpha\right]\frac{A_1}{(k\cos\theta_0-\alpha)}\right.$$

$$\left. +\frac{1}{\gamma}\left[\frac{(1+\sigma)}{(\sigma+\tau)}\delta\sinh\chi_+(\alpha\alpha + \frac{(1+\tau)}{(\sigma+\tau)}\cosh\chi_+(\alpha)\right]\left(\frac{A_2}{(k\cos\theta_0-\alpha)} - \frac{\tilde{A}_1}{\delta}\right)\right\}$$

$$-\exp[-i\alpha x - i]d\alpha, \quad \mathrm{Y} < 0; \tag{65}$$

and

$$u_g\ (X,Y) = \int_{-\infty+id}^{\infty+id} \left\{\left[\frac{\tau(\sigma+1)}{(\sigma+\tau)}\ \cosh\chi_+(\alpha) - \frac{\sigma(1+\tau)}{\delta(\sigma+\tau)}\sinh\chi_+\ (\alpha)\right] D_1\ (\alpha)\ H[k\cos\theta_0-\alpha]\right.$$

$$\left.+\left[\frac{\tau(1+\sigma)}{(\sigma+\tau)}\ \delta\sinh\chi_+(\alpha) - \frac{\sigma(1+\tau)}{(\sigma+\tau)}\cosh\chi_+\ (\alpha)\right]\frac{D_2(\alpha)}{\gamma}\ H[k\cos\theta_0-\alpha]\right\}$$

$$-\exp\left[-i\alpha(x-x_0) + ik\ (Y+Y_0)\right] d\alpha + \frac{1}{4i}\ H_0^{(1)}\left(k\left\{(x-x_0)^2 + (Y-Y_0)^2\right\}^{\frac{1}{2}}\right)$$

$$+\frac{(\sigma-\tau)}{4i(\sigma-\tau)} H_0^{(1)}\left(k\left\{(x-x_0)^2 + (Y+Y_0)^2\right\}^{\frac{1}{2}}\right), \qquad Y > 0,$$

$$= -\ \frac{1}{2\pi}\int_{-\infty+id}^{\infty+id} \left\{\left[\frac{(1+\sigma)}{(\sigma+\tau)}\ \cosh\chi_+(\alpha) + \frac{(1+\tau)}{(\sigma+\tau)\delta}\sinh\chi_+\ (\alpha)\right] D_1\ (\alpha)\ H[k\cos\theta_0-\alpha]\right.$$

$$\left.+\frac{1}{\gamma}\left[\frac{(1+\sigma)}{(\sigma+\tau)}\ \delta\ \sinh\chi_+(\alpha) + \frac{(1+\sigma)}{(\sigma+\tau)}\cosh\chi_+\ (\alpha)\right] D_2\ (\alpha)\ H[k\cos\theta_0-\alpha]\right\}$$

- exp -iα (x-x)+i<(Y₀ - Y) dα,

$$+\ 2/\ (\sigma+\tau)\ (1/4i)\ H_0^{(1)}\left(k\left\{(x-x_0)^2 + (Y-Y_0)^2\right\}^{\frac{1}{2}}\right), \qquad Y < 0. \tag{66}$$

The above expressions (66) can be considerably simplified to give

$$u_g(x,y) = -\frac{(\sigma-\tau)}{4\pi\pi i(-\tau)}\int_{-\infty+id}^{\infty+id} H[k\cos\theta_0-\alpha]\exp\left[-\ i\alpha(x-x_0) + ik(Y+Y_0)\right]\frac{d\alpha}{}$$

$$\frac{1}{4i}\ H_0^{(1)}\left(k\left\{(x-x_0)^2 + (Y-Y_0)^2\right\}^{\frac{1}{2}}\right) + \frac{(\sigma-\tau)}{(\sigma-\tau)}\frac{1}{4i}\ H_0^{(1)}\left(k\left\{(x-x_0)^2 + (Y-Y_0)^2\right\}^{\frac{1}{2}}\right),$$

$$Y > 0, \tag{67}$$

$$= -\frac{(2-(\sigma-\tau))}{4\pi(\sigma-\tau)}\int_{-\infty+id}^{\infty+id} H[k\cos\theta_0-\alpha]\exp\left[-\ i\alpha(x-x_0) + ik(Y+Y_0)\right]\frac{d\alpha}{ik}$$

$$+\frac{2}{(\sigma+\tau)}\frac{1}{4i}\ H_0^{(1)}\left(k\left\{(x-x_0)^2 + (Y-Y_0)^2\right\}^{\frac{1}{2}}\right), \qquad Y > 0. \tag{68}$$

The integrals in the expressions (64) and (65) can be asymptotically expanded for $kr\to\infty$ by the saddle point method following the usual steps: Substitute $x = r\cos\theta$, $Y=r\sin\theta$, $-\pi<\theta<\pi$; $a=k\cos\xi$, $0<\mathrm{Re}\,\xi<\pi$, then the integrand has a saddle point at $\xi=\pi-6$ and $\xi=\pi+\theta$, respectively; deform the path of integration into $S(\pi-6)$ and $S(\pi+6)$, respectively; apply the saddle point formula. This gives

$$u_d\ \ (r\cos\theta\ ,\ r\sin\theta) \sim \left[\frac{2\pi}{kr}\right]^{\frac{1}{2}} D\ (\theta\ ,\ \theta_0)\ e^{ikr}\ , \tag{69}$$

for $0<\theta$, $._2 <\pi$, $0<\theta_0<\pi$ we can rewrite the above expression for ug as

$$u_g\,(r\cos\theta\,,\,r\sin\theta)\ \sim \frac{1}{4i}\ H_0^{(1)}\ (KR_1)\ H[\theta-\theta_0+\pi]$$

$$+\frac{1}{4i}\,H_0^{(1)}\ (KR_2)\left(\frac{\sigma-\tau}{\sigma+\tau}\right)H\,[\theta+\theta_0-\pi]\ +\frac{1}{4i}\,.\,H_0^{(1)}\ (KR_1)\left(\frac{2}{\sigma+\tau}\right)\{1-H[\theta+\theta_0-\pi]\}\,,$$

$$-\pi<\pi<\pi,\quad kR_1\rightarrow\infty\quad kR_2\rightarrow\infty\quad . \qquad (72)$$

If the expressions (69) and (72) ae substituted into (63) we have finally the expression for the far field

$$u\,(r\cos\theta\,,\,r\sin\theta s\ =\frac{1}{4i}\,H^{(1)}\ (KR_1)\ H\,[\theta-\theta_0+\pi)$$

$$+\frac{1}{4i}\,H_0^{(1)}\ (KR_2)\left(\frac{\sigma-\tau}{\sigma+\tau}\right)H\,[\theta+\theta_0-\pi]\ +\frac{1}{4i}\,.\,H_0^{(1)}\ (KR_1)\left(\frac{2}{\sigma+\tau}\right)1-H[\theta+\theta_0]$$

$$kr\rightarrow\infty,\quad kr_0\rightarrow\infty,\quad \pi<\theta<\pi,\quad 0<\theta_\Theta<\pi,\quad \theta\neq\pm(\pi-\theta_\Theta+\pi)\,, \qquad (73)$$

where the diffraction coefficient $D(6,9_0)$ is given by (70) and (71) and the Hankel functions in (73) are understood to represent their asymptotic form.

The physical interpretation of the result (73) in conjunction with Fig 1. is now obvious. The first term represents the incident cylindrical wave due to a line source at (x_0/y_0). The second term is the wave reflected from the upper face of the half plane. This reflected wave appears to radiate from an image line source at $(X_0,-Y_0)$ the reflection coefficient being $(\sigma-\tau)/(\sigma-\tau)$. The third term represents a wave transmitted through the barrier. This wave appears to eminate from the line source at (X_0,Y_0); however its transmission coefficient is not unity, but $2/(\sigma-\tau)$. The first three terms represent the geometrical acoustic field and they will not exist everywhere. The regions where they are present are governed by the Heaviside step functions which multiply the Hankel functions. Physically these regions correspond to the shadow region behind the screen, and the insonified regions. On the boundary between these regions the arguments of the Heaviside step functions vanish. The last term of the expression (73) represents the diffracted field, which is a cylindrical wave which appears to radiate from the edge of the half plane, to all points of space.

Where

$$D(\theta,\theta_0) = \frac{1}{2\pi\pi + \tau)} \left\{\left[\tau(\sigma+1)\ \cosh\ \chi_+(-k\cos\theta c - \frac{\sigma}{\delta}(1+\tau)\ \sinh\chi_+(-k\cos\theta c\right]\right] \frac{A_1\,|\sin\theta\,|}{(\cos\theta_0 + \cos\theta o}$$

$$+ i\left[\tau(1+\sigma)\ \delta\sinh\chi_+\ (-k\cos\theta c - \sigma\ (1+\tau)\sinh\ \chi_+\ (k\cos\theta k\right]\left(\frac{A_2}{k(\cos\theta_0 + \cos\theta o} - \frac{\tilde{A}_1}{\delta}\right)\right\},$$

$$0 < \theta < \pi\ , \qquad (70)$$

$$= \frac{-1}{2\pi\pi(\ +\tau)} \left\{\left[(1+\sigma)\ \cosh\ \chi_+\ (-k\cos\theta c + \frac{(1+\tau)}{\delta}\ \sinh\chi_+\ (-k\cos\theta c\right] \frac{A_1\,|\sin\theta\,|}{k(\cos\theta_0 + \cos\theta o}\right.$$

$$\left.+ i\left[(1+\sigma)\ \delta\sinh\chi_+\ (-k\cos\theta c + (1+\tau)\ \cosh\chi_+ (k\cos\theta k\right] \left(\frac{A_2}{k(\cos\theta_0 + \cos\theta o} - \frac{\tilde{A}_1}{\delta}\right)\right\},$$

$$-\ \pi < \theta < 0\ . \qquad (71)$$

In a similar fashion the integrals appearing in the expressions (67) and (68) can be asymptotically evaluated by the saddle point method. In the integrand of the expression (67) let $x\text{-}x_0 = R_2 \cos\theta_2$, $Y\text{+}Y_0 = R_2 \sin\theta_2$, $0 < \theta_2 < \pi$, $\alpha=k\cos\xi$, $0<\mathrm{Re}\xi<\pi$; and in the expression (68) let $x\text{-}x_0=R_1$, $\cos\theta_1$, $Y\text{-}Y_0 = -R,\ \sin\theta,\ ,\ 0<\theta,<\pi$, $0=k\cos\xi$, $0<\mathrm{Re}\xi<\pi$, (see fig 1). The saddle point of (67) and (68) is then given by $\xi=\pi\text{-}\theta_2$ and $\xi=\pi\text{-}\theta_1$ respectively. Deforming the path of integration into $S(\pi\text{-}\theta_2)$ and $S(\pi\text{-}\theta_1)$, respectively, and applying the saddle point formula gives

$$u_g\ (r\cos\theta, r\sin\theta s \sim \frac{(\tau-\sigma)}{(\sigma-\tau)}\ \frac{1}{4i}.H_\circ^{(1)}\ (KR_2)\ \{H[\cos\theta_\circ + \cos\theta_\circ] - 1\}$$

$$+\frac{1}{4i}\ H_\circ^{(1)}\ (KR_1)\ , \qquad 0<\theta<\pi\ ,\ KR_1 \to \infty,\ KR_2 \to \infty;$$

$$\sim \frac{1}{4i}\ H_\circ^{(1)}\ (KR_1)\ H[\cos\theta_\circ + \cos\theta_1]$$

$$-\frac{2}{(\sigma+\tau)}\ \frac{1}{4i}.\ H_\circ^{(1)}\ (KR_1)\ \{H[\cos\theta_\circ + \cos\theta_1] - 1\},\ -\pi<\theta<0,\ KR_1 \to \infty\ ,\ KR_2 \to \infty\ ,$$

where we have used the asymptotic expression $H_\circ^{(1)}(z) \sim (2/\pi z^{\frac{1}{2}})\ \exp\ [i\ (z\text{-}\pi/4)]$

as$|z| \to\infty$. By using the fact that

$$H[\cos\theta_o + \cos\theta_o\ ] - 1 = -H[\theta+\theta_o\ -\pi]$$

$$H[\cos\theta_o + \cos\theta_1\ ] = H[\theta\text{-}\theta_o + \pi\ ]\ H\ [-\theta]$$

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Appendix

Here we derive the behaviour of the field near the edge of the half plane. We use the technique of Meixner [10] in assuming a series expansion in the low frequency situation kp→0, which satisfies Laplace's equation. Thus the problem can be posed thus :

Given

$$u(r,\ \theta) = C(\theta) + F(\theta)r^{\lambda} \tag{1}$$

Find the smallest value of Reλ such that

$$\nabla^2 u(r,\theta) = 0\ , \quad \nabla^2 \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r_2}\frac{\partial^2}{\partial\theta^2}\ , \tag{2}$$

$$u(r,\ \pi) = \sigma u\ (r, -\pi)\ ,\ \frac{\partial u}{\partial\theta}(r,\ \pi) = \tau\frac{\partial u}{\partial\theta}(r, -\pi)\ , \tag{3}$$

$$\operatorname{Im}\sigma \neq 0, \quad \operatorname{Im}\tau \neq 0\ .$$

$$\operatorname{Re}\lambda > 0 \quad \text{and} \quad \lim_{\rho\to 0} r\nabla u\,(r,\theta) = 0(1). \tag{4}$$

Substituting (1) into (2) gives on equating powers of p to zero

$$C''(\theta) = 0, \quad \Rightarrow C(\theta) = A\theta + B$$

$$F''\{\theta) + \lambda^2 F(\theta) = 0, \quad \Rightarrow \quad F(\theta) - C\cos\lambda\theta + D\sin\lambda\theta$$

Hence $\qquad u(r,6) = A\theta + B + (C\cos\lambda\theta + D\sin\lambda\theta)r^{\lambda}$ (5)

Substituting (5) into the boundary conditions (3) give

$$\begin{pmatrix} \pi(1+\sigma) & (1-\sigma) \\ (1-\tau) & 0 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} + \begin{pmatrix} (1-\sigma)\cos\lambda o & (1+\sigma)\sin\lambda\pi \\ -(1+\tau)\sin\lambda i & (1-\tau)\cos\lambda o \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} r^{\lambda} = 0\ .$$

This equation can only be satisfied by

$$(1-\sigma)(1-\tau) = 0 \quad \text{or} \quad A = B = 0 \tag{6}$$

and

$$\tan^2\lambda\pi = -\varepsilon^2 \quad \text{or} \quad C = D = 0. \tag{7}$$

Since Imσ $\neq$ 0, and Imτ $\neq$ 0, the only possible solution for (6) is the trivial case $A = B = 0$. For non trivial solutions to (7) we must have

$$\lambda = \pm \frac{1}{2\pi\pi} \ell n \left(\frac{1+\varepsilon}{1-\varepsilon}\right) .$$

$$\text{Re}\lambda = \pm \frac{1}{2\pi} \arg \left(\frac{1+\varepsilon}{1-\varepsilon}\right)$$

Hence

and since Reλ>0 we have

$$\text{Re}\lambda = \left| \frac{1}{2\pi} \arg \left(\frac{1+\varepsilon}{1-\varepsilon}\right) \right| \leq \frac{1}{2} .$$

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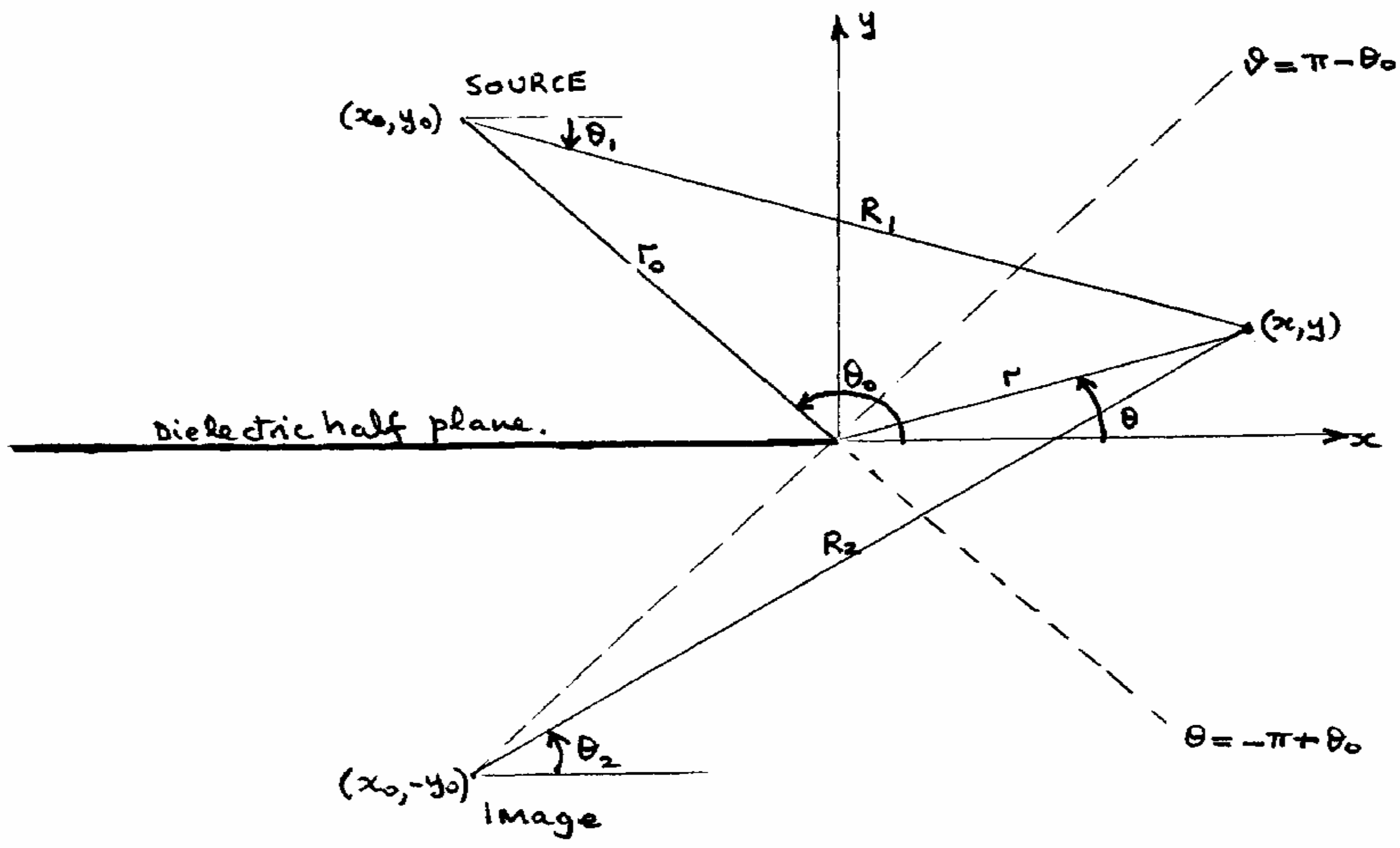

Figure1. Geometry of the diffraction problem