H_∞ State Estimation for Coupled Stochastic Complex Networks with Periodical Communication Protocol and Intermittent Nonlinearity Switching

Yuqiang Luo, Zidong Wang, Yun Chen, and Xiaojian Yi

***Abstract*—In this paper, an H_∞ estimation approach is given for an array of coupled stochastic complex networks with intermittent nonlinearity switching. A set of binary random variables are adopted to characterize the intermittent switching behavior of the involved nonlinearities. To effectively alleviate data collisions and save energy, the Round-Robin protocol is utilized to curb network congestions in data communication. For the coupled stochastic complex networks, we design a protocol-based H_∞ estimator that not only resists stochastic disturbances, but also ensures the exponential mean square stability of the desired error system under a given disturbance attenuation level. With the help of the Lyapunov stability and convex optimization theories, sufficient conditions are provided for the expected estimator. Simulations are provided to illustrate the reasonability of our H_∞ approach.**

***Index Terms*—Stochastic complex networks, H_∞ performance, intermittent nonlinearity switching, Round-Robin protocol.**

I. Introduction

Inspired by real-world scenarios including brain networks and computer networks, the complex networks have attracted a persistence research attention recently owing to their potential applications in domains such as sociology, computer science, biology, physics and epidemiology, see e.g. [1], [14], [30], [41], [49], [51]. As two key network features, the dynamical behavior and network topology have long been the research hotspots within the field of complex networks, and associated achievements have been widely reported in [1], [3], [15], [40]. To be more specific, effective models and analytical tools including evolving networks and the random graphs have been presented in [1] to study the statistical mechanics of network dynamics and topology. In [40], the temporal correlations of complex networks coupled with delayed oscillators have been analyzed in a simple way that is conducive to the understanding of the relation between network topology and dynamics. In addition, unified analysis frameworks have been established in [9] and [5] to look into dynamical behaviors of chemical networks and social networks, respectively.

Apart from the network dynamics and topology, the network stability and synchronization have also played important roles in the analysis/synthesis of complex networks. For instances, the Hurwitz stability has been discussed in [11] for nonlinear complex dynamical networks where a special controller has been devised and implemented in a decentralized manner. With the help of a reference state and the Lyapunov stability theory, a set of conditions have been acquired in [22] to obtain the asymptotic stability of concerned complex networks. In [50], the global synchronization issue has been studied for coupled stochastic delayed complex networks susceptible to probabilistic nonlinearities. Additionally, synchronization control of complex networks has been considered in [23] in the presence of time-varying but bounded delays. On the other hand, state estimation for complex networks has been proven to be a particular important task since states of network nodes might have no access to end-users but the access to measurement outputs. Thus, state estimation problems of many kinds of complex networks have aroused a lot of attention, see [2], [8], [16], [20], [24], [31], [52] .

In practice, the implementation of complex networks often faces huge consumptions of resources (e.g. communication, processing and storage), giving rise to network congestions, unstable connections, high latency, and even link breaks [7], [47]. To reduce unnecessary resource consumption, it is significant to use communication protocols in data transmission [10], [19], [27], [43]. As such, the Round-Robin protocol (RRP), acting as a particularly attractive scheduling strategy in allocating network resources, has been adopted in this paper to regulate data transmission in the sensor-to-estimator channel. The RRP has the advantage of simple execution procedure where data transmission is implemented via a fixed circular order, see [13], [18], [21], [25], [34], [37], [38], [42], [44], [46].

It has been well recognized that nonlinearities are ubiquitous that occur in almost all sorts of networks in physical reality, and the investigation on nonlinearities has played a vital role in network synthesis/analysis. Basically, certain rigorous conditions (e.g. smoothness, Lipschitz continuity, differentiability, monotonicity and boundedness) are widely imposed on the nonlinearities to be tackled for the convenience of subsequent mathematical analysis, and thereby leading to considerable conservatism on the analysis results, [4], [35]. Intermittent

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behaviors of the nonlinearities may be unavoidable in complex networks due probably to stochastic component failures and repairs, sudden ambient noises and intermittent switchings among subsystems [12], [17]. In this regard, the intermittent nonlinearity switching would become inevitable and, if not properly settled, they might give rise to severe deterioration of network performance. Accordingly, filtering problems of complex networks (with such nonlinearities) have attracted a great many research attention. Nevertheless, little attention has been paid to filtering problems for coupled stochastic complex networks with the RRP and intermittent nonlinearity switching.

In this paper, a protocol-based H_∞ estimator is devised for coupled stochastic complex networks with the intermittent nonlinearity switching and RRP. The contributions are *1) the RRP is presented to manage data transmission in the sensor-to-estimator channel, which curbs communication frequency and saves communication resources; 2) a unified H_∞ estimation scheme is given to resolve complexities induced by the intermittent nonlinearity switching and RRP; and 3) a protocol-dependent index is built to achieve that the resulting estimation error system is exponentially mean-square stable (EMSS) under guaranteed H_∞ performance constraint.*

Notation $A > B$ (or $A \geq B$) implies $A - B$ is positive-definite (or semi-positive-definite). A^T represents the transpose of A. $\mathrm{diag}\{\dots\}$ is a block-diagonal matrix. $\delta(\cdot) \in \{0, 1\}$ is the Dirac delta function. $(\Omega, \mathscr{F}, \mathrm{Prob})$ stands for a complete probability space and Prob is the probability measure. β_s is the n-dimensional column vector with 1 on the sth row and 0 elsewhere. $\mathbb{Z}^+$ denotes the set of all non-negative integers.

II. Problem Formulation

Consider the following coupled stochastic complex network:

$$\begin{cases} x_i(k+1) = A_i x_i(k) + \varpi(k) B_i g\big(x_i(k)\big) + (1-\varpi(k)) \\ \qquad \times S_i h\big(x_i(k)\big) + \sum_{j=1}^{N} \lambda_{ij} \Gamma x_j(k) \\ \qquad + \rho_i\big(k, x_i(k)\big)\omega(k) + \vartheta_i(k), \\ y_i(k) = C_i x_i(k) + D_i \nu_i(k) \end{cases} \tag{1}$$

where

$$\begin{aligned} x_i(k) &\triangleq \left[x_{i1}(k),\ x_{i2}(k),\ \dots,\ x_{in}(k)\right]^T, \\ g\big(x_i(k)\big) &\triangleq \left[g_1\big(x_{i1}(k)\big),\ g_2\big(x_{i2}(k)\big),\ \dots,\ g_n\big(x_{in}(k)\big)\right]^T, \\ h\big(x_i(k)\big) &\triangleq \left[h_1\big(x_{i1}(k)\big),\ h_2\big(x_{i2}(k)\big),\ \dots,\ h_n\big(x_{in}(k)\big)\right]^T \end{aligned}$$

are the ith node's state vector and nonlinear vector-valued functions, respectively. $\rho_i\big(k, x_i(k)\big) : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is the diffusion coefficient satisfying $\rho_i(k, 0) \equiv 0$. $\omega(k)$ is the Wiener process defined on $(\Omega, \mathscr{F}, \mathrm{Prob})$ where

$$\begin{aligned} \mathbb{E}\{\omega(k)\} &= 0, \\ \mathbb{E}\{\omega(k)\omega(k)\} &= 1, \\ \mathbb{E}\{\omega(k)\omega(l)\} &= 0,\ k \neq l. \end{aligned}$$

$y_i(k) \in \mathbb{R}^m$ is the measurement output. $\nu_i(k)$ is the disturbance input which belongs to $l_2([0, +\infty); \mathbb{R}^{n_\nu})$. $B_i \in \mathbb{R}^{n\times n}$ and $S_i \in \mathbb{R}^{n\times n}$ are the connection weight matrices.

$$\vartheta_i(k) \triangleq [\vartheta_{i1}(k), \vartheta_{i2}(k), \dots \vartheta_{in}(k)]^T$$

is the energy-bounded external input which could be disturbance or control input. A_i, C_i and D_i are known matrices. $N \in \mathbb{R}$ is the total number of coupling nodes. $\Gamma \in \mathbb{R}^{n\times n}$ is the inner-coupling matrix [39] and $\lambda_{ij} \in \mathbb{R}$ $(i, j = 1, 2, \dots, N)$ are the outer-coupling strength indicating that if a connection exists between subnetworks i and j $(j \neq i)$, $\lambda_{ij} > 0$; otherwise, $\lambda_{ij} = 0$ $(j \neq i)$ [23], [45], [48]. To describe the intermittent switching behavior of the nonlinearities, the following random events are introduced:

$$\begin{cases} \text{Event 1: (1) experiences the nonlinear function } g(\cdot), \\ \text{Event 2: (1) experiences the nonlinear function } h(\cdot) \end{cases}$$

where $\varpi(k)$ is a random variable defined as $\varpi(k) = 1$ if Event 1 occurs and $\varpi(k) = 0$ if Event 2 occurs with

$$\begin{aligned} \mathrm{Prob}\{\varpi(k) = 1\} &= \varpi, \\ \mathrm{Prob}\{\varpi(k) = 0\} &= 1 - \varpi, \end{aligned}$$

and $\varpi \in \mathbb{R}$ is a constant.

Assumption 1: There exists a non-negative matrix ϱ such that

$$\rho_i^T\big(k, x_i(k)\big)\rho_i\big(k, x_i(k)\big) \leq x_i^T(k)\varrho x_i(k)$$

holds for all diffusion coefficient vectors $\rho_i\big(k, x_i(k)\big)$.

Assumption 2: [29] There exist constants r_j^-, r_j^+ $(j = 1, 2, \dots, n)$ such that

$$r_j^- \leq \frac{g_j(a(k)) - g_j(b(k))}{a(k) - b(k)} \leq r_j^+$$

holds for all $a(k), b(k) \in \mathbb{R}$ $(a(k) \neq b(k))$ with $g_j(0) = 0$.

Assumption 3: The sector-bounded condition

$$\begin{aligned} &[h\big(a(k)\big) - h\big(b(k)\big) - \Xi_1\big(a(k) - b(k)\big)]^T \\ &\times [h\big(a(k)\big) - h\big(b(k)\big) - \Xi_2\big(a(k) - b(k)\big)] \leq 0 \end{aligned} \tag{2}$$

holds for $\forall\ a(k), b(k) \in \mathbb{R}^n$ and $h(0) = 0$, where Ξ_1 and Ξ_2 are given constant matrices satisfying $\Xi_2 - \Xi_1 \geq 0$.

Assumption 4: The switching sequence $\varpi(k)$ is independent of $\varpi(l)$ $(\forall\ k \neq l)$ and $\omega(k)$ $(\forall\ k \in \mathbb{Z}^+)$.

Remark 1: Model (1) is quite general as it includes a set of well-known networks (e.g. neural networks) as its special cases. Moreover, the random variable $\varpi(k)$ is introduced to regulate the intermittent switching between nonlinearities $g(x_i(k))$ and $h(x_i(k))$. At each time k, either $g(x_i(k))$ or $h(x_i(k))$ is capable of exerting its influence on the network, thereby better reflecting the engineering practice.

To handle the network communication of large-scale data, the RRP is adopted to manage the transmission in the sensor-to-estimator channel, and the zero-order holder strategy is selected for the compensation of the measurement which is not received by remote estimators. Under the RRP scheduling mechanism, all the network nodes are prearranged in a predetermined order to gain the access token of the limited communication resource one by one. At each time, only one network node can access the channel for data transmission. Denote the update matrix as $\Psi_{\hbar(k)} \triangleq \mathrm{diag}\{\delta(\hbar(k) - 1), \delta(\hbar(k) -$

$2),\dots,\delta(\hbar(k)-m)\}$ and $\hbar(k)\triangleq \mathrm{mod}(k-1,m)+1$ as the node number that obtains the access token. Then, the measurement is eventually updated as [36]:

$$\begin{aligned}\vec{y}_i(k) &= \Psi_{\hbar(k)}y_i(k)+(I_m-\Psi_{\hbar(k)})\vec{y}_i(k-1)\\ &= C_i^{\hbar(k)}\phi_i(k)+\Psi_{\hbar(k)}D_i\nu_i(k)\end{aligned}\tag{3}$$

where

$$C_i^{\hbar(k)}\triangleq\begin{bmatrix}\Psi_{\hbar(k)}C_i & I_m-\Psi_{\hbar(k)}\end{bmatrix},\ \phi_i(k)\triangleq\begin{bmatrix}x_i(k)\\ \vec{y}_i(k-1)\end{bmatrix}.$$

Based on (3), (1) can be augmented as:

$$\begin{aligned}\phi_i(k+1)=&A_i^a(k)\phi_i(k)+B_i^a g\big(E_a\phi_i(k)\big)\\ &+S_i^a h\big(E_a\phi_i(k)\big)+\big(\varpi(k)-\varpi\big)\bigg(\frac{1}{\varpi}B_i^a\\ &\times g\big(E_a\phi_i(k)\big)-\frac{1}{1-\varpi}S_i^a h\big(E_a\phi_i(k)\big)\bigg)\\ &+\sum_{j=1}^{N}\lambda_{ij}\Gamma_a\phi_j(k)+F_a\rho_i\big(k,E_a\phi_i(k)\big)\\ &\times\omega(k)+D_i^a(k)\nu_i(k)+\Theta_i(k)\end{aligned}\tag{4}$$

where

$$A_i^a(k)\triangleq\begin{bmatrix}A_i & 0\\ \Psi_{\hbar(k)}C_i & I_m-\Psi_{\hbar(k)}\end{bmatrix},\qquad B_i^a\triangleq\varpi\begin{bmatrix}B_i\\ 0_{m\times n}\end{bmatrix},$$

$$S_i^a\triangleq(1-\varpi)\begin{bmatrix}S_i\\ 0_{m\times n}\end{bmatrix},\qquad E_a\triangleq\begin{bmatrix}I_n & 0_{n\times m}\end{bmatrix},$$

$$\Gamma_a\triangleq\begin{bmatrix}\Gamma & 0\\ 0_{m\times n} & 0_{m\times m}\end{bmatrix},\qquad F_a\triangleq\begin{bmatrix}I_n\\ 0_{m\times n}\end{bmatrix},$$

$$D_i^a(k)\triangleq\begin{bmatrix}0_{n\times n_\nu}\\ \Psi_{\hbar(k)}D_i\end{bmatrix},\qquad \Theta_i(k)\triangleq\begin{bmatrix}\vartheta_i(k)\\ 0_{m\times 1}\end{bmatrix}.$$

For the network model (1), construct the following improved full-order state estimator:

$$\begin{aligned}\hat{\phi}_i(k+1)=&A_i^a(k)\hat{\phi}_i(k)+B_i^a g\big(E_a\hat{\phi}_i(k)\big)\\ &+S_i^a h\big(E_a\phi_i(k)\big)+\Theta_i(k)\\ &+K_i\left[\vec{y}_i(k)-C_i^{\hbar(k)}\hat{\phi}_i(k)\right]\end{aligned}\tag{5}$$

where $\hat{\phi}_i(k)\triangleq\begin{bmatrix}\hat{x}_i^T(k) & \hat{q}_i^T(k-1)\end{bmatrix}^T$, and $\hat{x}_i(k)$ and $\hat{q}_i(k)$ are the estimator states. $K_i\in\mathbb{R}^{(n+m)\times m}$ is the to-be-designed parameter.

Remark 2: The update matrix $\Psi_{\hbar(k)}$ in (3) is modeled from the combined effects of the RRP and the ZOHs. With help of such a update matrix, the updated measurement can be established straightforward. From the scheduling mechanism of RRP, we know that $\Psi_{\hbar(k)}$ is time-varying but periodic. In the state estimator (5), only the measurement output of single node but are not the coupling information is used, which obviously results in a simple but effective structure. Although $\hat{x}_i(k)$ and $\hat{q}_i(k)$ in (5) are the estimator state variables. $\hat{x}_i(k)$ is clear in physics meaning of the estimation of $x_i(k)$. While $\hat{q}_i(k)$ is an auxiliary variable of the special estimator structure, which is introduced to facilitate analysis and design of the estimation system. In the algorithm development under RRP, it can be found that the resultant difficulties are conspicuous: 1) The measurement at the receiving end should be remodeled, see i.e. (3). 2) Structure of the estimator is of more specificity due to the measurement update model induced by the RRP and ZOHs. 3) Analysis and synthesis of the estimation system will be of more complexities.

Let $\Lambda\triangleq(\lambda_{ij})_{N\times N}$ be the linear outer-coupling configuration matrix. Consequently, the coupled complex network (4) can be reformulated as

$$\begin{aligned}\phi(k+1)=&(\mathcal{A}(k)+\Lambda\otimes\Gamma_a)\phi(k)\\ &+\mathcal{B}(k)G(\phi(k))+\mathcal{S}(k)H(\phi(k))\\ &+\big(\varpi(k)-\varpi\big)\bigg(\frac{1}{\varpi}\mathcal{B}(k)G(\phi(k))\\ &-\frac{1}{1-\varpi}\mathcal{S}(k)H(\phi(k))\bigg)+(I_N\otimes F_a)\\ &\times\rho(k)\omega(k)+D_1(k)\nu(k)+V(k)\end{aligned}\tag{6}$$

where

$$\begin{aligned}\phi(k)\triangleq&\begin{bmatrix}\phi_1^T(k), & \phi_2^T(k), & \dots, & \phi_N^T(k)\end{bmatrix}^T,\\ G(\phi(k))\triangleq&\big[g^T\big(E_a\phi_1(k)\big),\ g^T\big(E_a\phi_2(k)\big),\ \dots,\\ &g^T\big(E_a\phi_N(k)\big)\big]^T,\\ H(\phi(k))\triangleq&\big[h^T\big(E_a\phi_1(k)\big),\ h^T\big(E_a\phi_2(k)\big),\ \dots,\\ &h^T\big(E_a\phi_N(k)\big)\big]^T,\\ \rho(k)\triangleq&\big[\rho_1^T\big(k,E_a\phi_1(k)\big),\ \rho_2^T\big(k,E_a\phi_2(k)\big),\dots,\\ &\rho_N^T\big(k,E_a\phi_N(k)\big)\big]^T,\\ \mathcal{A}(k)\triangleq&\mathrm{diag}\big\{A_1^a(k),\ A_2^a(k),\dots,A_N^a(k)\big\},\\ \mathcal{B}(k)\triangleq&\mathrm{diag}\big\{B_1^a(k),\ B_2^a(k),\dots,B_N^a(k)\big\},\\ \mathcal{S}(k)\triangleq&\mathrm{diag}\big\{S_1^a(k),\ S_2^a(k),\dots,S_N^a(k)\big\},\\ \nu(k)\triangleq&\big[\nu_1^T(k),\ \nu_2^T(k),\dots,\nu_N^T(k)\big]^T,\\ D_1(k)\triangleq&\mathrm{diag}\big\{D_1^a(k),\ D_2^a(k),\ \dots,\ D_N^a(k)\big\},\\ V(k)\triangleq&\begin{bmatrix}\Theta_1^T(k), & \Theta_2^T(k), & \dots, & \Theta_N^T(k)\end{bmatrix}^T.\end{aligned}$$

Similarly, estimator (5) is rewritten as

$$\begin{aligned}\hat{\phi}(k+1)=&\mathcal{A}(k)\hat{\phi}(k)+\mathcal{B}(k)G(\hat{\phi}(k))+\mathcal{S}(k)H(\hat{\phi}(k))\\ &+V(k)+K(k)(\phi(k)-\hat{\phi}(k))+D_2(k)\nu(k)\end{aligned}\tag{7}$$

where

$$\begin{aligned}\hat{\phi}(k)\triangleq&\begin{bmatrix}\hat{\phi}_1^T(k), & \hat{\phi}_2^T(k), & \dots, & \hat{\phi}_N^T(k)\end{bmatrix}^T,\\ D_2(k)\triangleq&\mathrm{diag}\big\{K_1\Psi_{\hbar(k)}D_1,\ K_2\Psi_{\hbar(k)}D_2,\dots,K_N\Psi_{\hbar(k)}D_N\big\},\\ K(k)\triangleq&\mathrm{diag}\{K_1C_1^{\hbar(k)},\ K_2C_2^{\hbar(k)},\dots,\ K_NC_N^{\hbar(k)}\}.\end{aligned}$$

Defining $\tilde{\phi}(k)\triangleq\phi(k)-\hat{\phi}(k)$ and $e(k)\triangleq\begin{bmatrix}\phi^T(k), & \tilde{\phi}^T(k)\end{bmatrix}^T$, we have

$$\begin{aligned}e(k+1)=&\bar{A}(k)e(k)+\bar{B}\bar{G}(k)+\bar{S}\bar{H}(k)+\big(\varpi(k)-\varpi\big)\\ &\times\bigg(\frac{1}{\varpi}\widetilde{B}\bar{G}(k)-\frac{1}{1-\varpi}\widetilde{S}\bar{H}(k)\bigg)\\ &+\bar{F}\rho(k)\omega(k)+\bar{D}(k)\upsilon(k)\end{aligned}\tag{8}$$

where $\upsilon(k)\triangleq\begin{bmatrix}\nu^T(k) & V^T(k)\end{bmatrix}^T$,

$$\bar{A}(k)\triangleq\begin{bmatrix}\mathcal{A}(k)+\Lambda\otimes\Gamma_a & 0\\ \Lambda\otimes\Gamma_a & \mathcal{A}(k)-K(k)\end{bmatrix},$$

$$\bar{G}(k) \triangleq \begin{bmatrix} G(\phi(k)) \\ G(\phi(k)) - G(\hat{\phi}(k)) \end{bmatrix},$$

$$\bar{H}(k) \triangleq \begin{bmatrix} H(\phi(k)) \\ H(\phi(k)) - H(\hat{\phi}(k)) \end{bmatrix},$$

$$\bar{B} \triangleq \begin{bmatrix} \mathcal{B}(k) & 0 \\ 0 & \mathcal{B}(k) \end{bmatrix}, \bar{S} \triangleq \begin{bmatrix} \mathcal{S}(k) & 0 \\ 0 & \mathcal{S}(k) \end{bmatrix},$$

$$\widetilde{B} \triangleq \begin{bmatrix} \mathcal{B}(k) & 0_{N(n+m)\times Nn} \\ \mathcal{B}(k) & 0_{N(n+m)\times Nn} \end{bmatrix},$$

$$\widetilde{S} \triangleq \begin{bmatrix} \mathcal{S}(k) & 0_{N(n+m)\times Nn} \\ \mathcal{S}(k) & 0_{N(n+m)\times Nn} \end{bmatrix},$$

$$\bar{F} \triangleq \begin{bmatrix} I_N \otimes F_a \\ I_N \otimes F_a \end{bmatrix}, \bar{D}(k) \triangleq \begin{bmatrix} D_1(k) & I_{(n+m)N} \\ D_1(k) - D_2(k) & 0 \end{bmatrix}.$$

Definition 1: For all possible delays $\tau_\hbar$ $(-m+1 \le \tau_\hbar \le 0)$ induced by the RRP, the error dynamics (8) with $\upsilon(k) \equiv 0$ is EMSS if there exist constants $\alpha > 0$ and $\epsilon \in (0,1)$ such that

$$\mathbb{E}\left\{\|e(k)\|^2\right\} \le \alpha\epsilon^k \sup_{-m+1\le i\le 0} \mathbb{E}\left\{\|e(i)\|^2\right\}, \quad k \in \mathbb{Z}^+. \quad (9)$$

Our aim is to design estimator (5) such that the following design indexes are simultaneously achieved:

1) (8) is EMSS;
2) For given constant $\gamma > 0$ and zero initial condition, the disturbance attenuation constraint

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{e^T(k)e(k)\right\} \le \gamma^2 \sum_{k=0}^{\infty} \upsilon^T(k)\upsilon(k) \quad (10)$$

holds for any $\upsilon(k) \neq 0$.

Remark 3: Due to its remarkable advantages to alleviate communication burden, the RRP has been adopted extensively in network communications and therefore gain much research attention in recent years, see e.g. [13], [34], [38], [46]. It has been noticed that the state estimation problem of complex networks subject to the RRP has been studied in [53]. However, differentia between the literature and this paper are severalfold. By comparison, the peculiarities of this paper fall into but are not limited to the following points. Firstly, to better characterize the engineering practice, the intermittent switching between two kinds of nonlinearities $g(x_i(k))$ and $h(x_i(k))$ is considered. Secondly, a simple but effective structure of the estimator is constructed without the coupling information. Thirdly, the stability concern of the estimation error system is exponential mean-square stability but not the exponential ultimate boundedness in mean square. Fourthly, the H_∞ index indicating the disturbance attenuation level is discussed.

III. Main Results

Lemma 1: (Schur Complement) [6] Given matrices Σ_1, Σ_2 and Σ_3 where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, $\Sigma_1 - \Sigma_3^T\Sigma_2^{-1}\Sigma_3 > 0$ iff

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & \Sigma_2 \end{bmatrix} > 0 \quad \text{or} \quad \begin{bmatrix} \Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} > 0.$$

To carry out the exponential stability analysis for the error dynamics (8), we give the following notations for the presentation convenience:

$$\overline{R} \triangleq \mathrm{diag}\{r_1^- r_1^+, r_2^- r_2^+, \ldots, r_n^- r_n^+\},$$

$$\underline{R} \triangleq \mathrm{diag}\{-\frac{r_1^- + r_1^+}{2}, -\frac{r_2^- + r_2^+}{2}, \ldots, -\frac{r_n^- + r_n^+}{2}\},$$

$$Y_1 \triangleq \begin{bmatrix} I_n & 0_{n\times m} & 0 \\ 0 & 0 & I_n \end{bmatrix},$$

$$Y_2 \triangleq \begin{bmatrix} 0 & I_{(n+m)N} & 0 & 0 \\ 0_{nN\times(n+m)N} & 0 & 0_{nN\times nN} & I_{nN} \end{bmatrix},$$

$$Y_3 \triangleq \begin{bmatrix} I_{(n+m)N} & 0 & 0 & 0 \\ 0 & 0_{nN\times(n+m)N} & I_{nN} & 0_{nN\times nN} \end{bmatrix},$$

$$Y_4 \triangleq \big[I_{(n+m)N}\ 0_{(n+m)N\times(n+m)N}\ 0_{(n+m)N\times nN}\ 0_{(n+m)N\times nN}\big],$$

$$\overline{\varpi} \triangleq \varpi(1-\varpi),\ \widetilde{G}(k) \triangleq \begin{bmatrix} e(k) \\ \bar{G}(k) \end{bmatrix}, \quad \widetilde{H}(k) \triangleq \begin{bmatrix} e(k) \\ \bar{H}(k) \end{bmatrix},$$

$$\overline{\Xi}_2 \triangleq I_N \otimes (\Xi_2 E_a), \quad \overline{\Xi}_1 \triangleq I_N \otimes \left(E_a^T \Xi_1^T\right),$$

$$\overline{\Xi}_{12} \triangleq I_N \otimes \left(E_a^T \Xi_1^T \Xi_2 E_a\right),$$

$$\widetilde{\Xi}_1 \triangleq \begin{bmatrix} \overline{\Xi}_1 & 0 \\ 0 & \overline{\Xi}_1 \end{bmatrix}, \ \widetilde{\Xi}_2 \triangleq \begin{bmatrix} \overline{\Xi}_2 & 0 \\ 0 & \overline{\Xi}_2 \end{bmatrix}, \ \widetilde{\Xi}_{12} \triangleq \begin{bmatrix} \overline{\Xi}_{12} & 0 \\ 0 & \overline{\Xi}_{12} \end{bmatrix},$$

$$\widehat{\Xi}_{12} \triangleq \begin{bmatrix} \widetilde{\Xi}_{12} & * \\ -\dfrac{\widetilde{\Xi}_2 + \widetilde{\Xi}_1^T}{2} & I \end{bmatrix},$$

$$\widehat{A}(k) \triangleq \begin{bmatrix} \bar{A}(k) & \bar{B} \end{bmatrix}, \ \widehat{S}_1 \triangleq \begin{bmatrix} 0_{2N(n+m)\times 2N(n+m)} & \bar{S} \end{bmatrix},$$

$$\widehat{S}_2 \triangleq \begin{bmatrix} 0_{2N(n+m)\times 2N(n+m)} & \widetilde{S} \end{bmatrix},$$

$$\widehat{B} \triangleq \begin{bmatrix} 0_{2N(n+m)\times 2N(n+m)} & \widetilde{B} \end{bmatrix},$$

$$\widehat{Q}(\hbar(k)) \triangleq \begin{bmatrix} Q(\hbar(k)) & 0 \\ 0 & 0_{2nN\times 2nN} \end{bmatrix},$$

$$\beth \triangleq \begin{bmatrix} t_{ij} \end{bmatrix}_{(2n+m)N\times(2n+m)N},$$

$$t_{ij} \triangleq \begin{cases} I_{n+m} & \text{if } j = 2i-1 \text{ and } \ i \le N \\ I_n & \text{if } j = 2(i-N) \text{ and } \ N < i \le 2N \\ 0 & \text{else.} \end{cases}$$

Theorem 1: Let the parameters K_i $(i = 1, 2, \ldots, N)$ be given. Under Assumptions 1-4 if there exist matrices $Z_l = \mathrm{diag}\{Z_{l1}, Z_{l2}, \ldots, Z_{ln}\} > 0$ $(l = 1, 2)$ and $Q(\hbar(k)) > 0$ such that matrix inequalities

$$\Pi(k) = \begin{bmatrix} \Pi_{11}(k) & * & * \\ 0 & \Pi_{22}(k) & * \\ \Pi_{31}(k) & 0 & \Pi_{33}(k) \end{bmatrix} < 0 \quad (11)$$

hold, where

$$\widetilde{Z}_l \triangleq \begin{bmatrix} Z_l\overline{R} & Z_l\underline{R} \\ Z_l\underline{R} & Z_l \end{bmatrix},$$

$$\begin{aligned}
\Pi_{11}(k) \triangleq & \widehat{A}^T(k)Q(\hbar(k+1))\widehat{A}(k) - \widehat{Q}(\hbar(k)) \\
& + Y_4^T (I_N \otimes E_a)^T (I_N \otimes \varrho)(I_N \otimes E_a) Y_4 \\
& - \Big(Y_2^T \beth^T \Big(I_N \otimes Y_1^T \widetilde{Z}_1 Y_1\Big) \beth Y_2 + Y_3^T \beth^T \\
& \times \Big(I_N \otimes Y_1^T \ \widetilde{Z}_2 Y_1\Big) \beth Y_3\Big) \\
& + \frac{\overline{\varpi}}{\varpi^2} \widehat{B}^T Q(\hbar(k+1))\widehat{B},
\end{aligned}$$

$$\Pi_{22}(k) \triangleq \bar{F}^T Q(\hbar(k+1))\bar{F} - I,$$

$$\Pi_{31}(k) \triangleq \widehat{S}_1^T Q(\hbar(k+1))\widehat{A}(k) - \widehat{S}_2^T Q(\hbar(k+1))\widehat{B},$$

$$\Pi_{33}(k) \triangleq -\widehat{\Xi}_{12} + \widehat{S}_1^T Q(\hbar(k+1))\widehat{S}_1 + \frac{\overline{\varpi}}{(1-\varpi)^2}\widehat{S}_2^T Q(\hbar(k+1))\widehat{S}_2,$$

then the error dynamics (8) with intermittent nonlinearity switching is EMSS.

Proof: Define

$$\mathcal{V}(k) \triangleq e^T(k)Q(\hbar(k))e(k). \tag{12}$$

Consider an index $\mathcal{I}(k) = \mathbb{E}\{\mathcal{V}(k+1) - \mathcal{V}(k)|e(k)\}$, where $\upsilon(k) = 0$.

For any $s = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, N$, one knows from Assumptions 1–2 that

$$-\rho^T(k)\rho(k) + [(I_N \otimes E_a)\phi(k)]^T (I_N \otimes \varrho) \times [(I_N \otimes E_a)\phi(k)] \geq 0, \tag{13}$$

$$-\big[g_s\big(x_{is}(k)\big) - g_s\big(\hat{x}_{is}(k)\big) - r_s^+\big(x_{is}(k) - \hat{x}_{is}(k)\big)\big] \times \big[g_s\big(x_{is}(k)\big) - g_s\big(\hat{x}_{is}(k)\big) - r_s^-\big(x_{is}(k) - \hat{x}_{is}(k)\big)\big] \geq 0, \tag{14}$$

$$-\big[g_s\big(x_{is}(k)\big) - r_s^+ x_{is}(k)\big] \times \big[g_s\big(x_{is}(k)\big) - r_s^- x_{is}(k)\big] \geq 0 \tag{15}$$

hold. (14) and (15) can be, respectively, rewritten as

$$-\begin{bmatrix} x_{is}(k) - \hat{x}_{is}(k) \\ g_s\big(x_{is}(k)\big) - g_s\big(\hat{x}_{is}(k)\big)\end{bmatrix}^T \times \begin{bmatrix} r_s^- r_s^+ \beta_s\beta_s^T & -\frac{r_s^- + r_s^+}{2}\beta_s\beta_s^T \\ -\frac{r_s^- + r_s^+}{2}\beta_s\beta_s^T & \beta_s\beta_s^T \end{bmatrix} \times \begin{bmatrix} x_{is}(k) - \hat{x}_{is}(k)) \\ g_s\big(x_{is}(k)\big) - g_s\big(\hat{x}_{is}(k)\end{bmatrix} \geq 0, \tag{16}$$

$$-\begin{bmatrix} x_{is}(k) \\ g_s\big(x_{is}(k)\big)\end{bmatrix}^T \begin{bmatrix} r_s^- r_s^+ \beta_s\beta_s^T & -\frac{r_s^- + r_s^+}{2}\beta_s\beta_s^T \\ -\frac{r_s^- + r_s^+}{2}\beta_s\beta_s^T & \beta_s\beta_s^T \end{bmatrix} \times \begin{bmatrix} x_{is}(k) \\ g_s\big(x_{is}(k)\big)\end{bmatrix} \geq 0. \tag{17}$$

Pre- and post-multiplying (16) and (17) by Z_{ls} and summing up both sides of the resulting inequalities for $s = 1, 2, \ldots, n$ generate

$$\begin{cases} -\begin{bmatrix} x_i(k) - \hat{x}_i(k) \\ g\big(x_i(k)\big) - g\big(\hat{x}_i(k)\big)\end{bmatrix}^T \widetilde{Z}_1 \times \begin{bmatrix} x_i(k) - \hat{x}_i(k)) \\ g\big(x_i(k)\big) - g\big(\hat{x}_i(k)\end{bmatrix} \geq 0, \\ -\begin{bmatrix} x_i(k) \\ g\big(x_i(k)\big)\end{bmatrix}^T \widetilde{Z}_2 \begin{bmatrix} x_i(k) \\ g\big(x_i(k)\big)\end{bmatrix} \geq 0, \end{cases}$$

$$\Leftrightarrow \begin{cases} -\begin{bmatrix} x_i(k) - \hat{x}_i(k) \\ \vec{y}_i(k-1) - \hat{q}_i(k-1) \\ g\big(E_a\phi_i(k)\big) - g\big(E_a\hat{\phi}_i(k)\big)\end{bmatrix}^T Y_1^T\widetilde{Z}_1Y_1 \times \begin{bmatrix} x_i(k) - \hat{x}_i(k) \\ \vec{y}_i(k-1) - \hat{q}_i(k-1) \\ g\big(E_a\phi_i(k)\big) - g\big(E_a\hat{\phi}_i(k)\big)\end{bmatrix} \geq 0, \\ -\begin{bmatrix} x_i(k) \\ \vec{y}_i(k-1) \\ g\big(E_a\phi_i(k)\big)\end{bmatrix}^T Y_1^T\widetilde{Z}_2Y_1 \begin{bmatrix} x_i(k) \\ \vec{y}_i(k-1) \\ g\big(E_a\phi_i(k)\big)\end{bmatrix} \geq 0, \end{cases}$$

$$\Leftrightarrow \begin{cases} -\begin{bmatrix} \tilde{\phi}_i(k) \\ g\big(E_a\phi_i(k)\big) - g\big(E_a\hat{\phi}_i(k)\big)\end{bmatrix}^T Y_1^T\widetilde{Z}_1Y_1 \times \begin{bmatrix} \tilde{\phi}_i(k) \\ g\big(E_a\phi_i(k)\big) - g\big(E_a\hat{\phi}_i(k)\big)\end{bmatrix} \geq 0, \\ -\begin{bmatrix} \phi_i(k) \\ g\big(E_a\phi_i(k)\big)\end{bmatrix}^T Y_1^T\widetilde{Z}_2Y_1 \begin{bmatrix} \phi_i(k) \\ g\big(E_a\phi_i(k)\big)\end{bmatrix} \geq 0. \end{cases}$$

Consequently,

$$\begin{cases} -\begin{bmatrix} \tilde{\phi}(k) \\ G(\phi(k)) - G(\hat{\phi}(k))\end{bmatrix}^T \beth^T \Big(I_N \otimes (Y_1^T\widetilde{Z}_1Y_1)\Big) \times\beth \begin{bmatrix} \tilde{\phi}(k) \\ G(\phi(k)) - G(\hat{\phi}(k))\end{bmatrix} \geq 0, \\ -\begin{bmatrix} \phi(k) \\ G(\phi(k))\end{bmatrix}^T \beth^T \Big(I_N \otimes (Y_1^T\widetilde{Z}_2Y_1)\Big)\beth \begin{bmatrix} \phi(k) \\ G(\phi(k))\end{bmatrix} \geq 0, \end{cases}$$

which is equivalent to

$$\begin{cases} -\begin{bmatrix} e(k) \\ \bar{G}(k)\end{bmatrix}^T Y_2^T\beth^T \Big(I_N \otimes (Y_1^T\widetilde{Z}_1Y_1)\Big)\beth Y_2 \times \begin{bmatrix} e(k) \\ \bar{G}(k)\end{bmatrix} \geq 0, \\ -\begin{bmatrix} e(k) \\ \bar{G}(k)\end{bmatrix}^T Y_3^T\beth^T \Big(I_N \otimes (Y_1^T\widetilde{Z}_2Y_1)\Big)\beth Y_3 \times \begin{bmatrix} e(k) \\ \bar{G}(k)\end{bmatrix} \geq 0. \end{cases} \tag{18}$$

Moreover, Assumption 3 tells

$$\begin{cases} -[h\big(E_a\phi_i(k)\big) - \Xi_1E_a\phi_i(k)]^T \times [h\big(E_a\phi_i(k)\big) - \Xi_2E_a\phi_i(k)] \geq 0, \\ -[h\big(E_a\phi_i(k)\big) - h\big(E_a\hat{\phi}_i(k)\big) - \Xi_1\big(E_a\phi_i(k) - E_a\hat{\phi}_i(k)\big)]^T \times [h\big(E_a\phi_i(k)\big) - h\big(E_a\hat{\phi}_i(k)\big) - \Xi_2\big(E_a\phi_i(k) - E_a\hat{\phi}_i(k)\big)] \geq 0, \end{cases}$$

$$\Leftrightarrow \begin{cases} -[h\big(E_a\phi_i(k)\big) - \Xi_1E_a\phi_i(k)]^T \times [h\big(E_a\phi_i(k)\big) - \Xi_2E_a\phi_i(k)] \geq 0, \\ -[h\big(E_a\phi_i(k)\big) - h\big(E_a\hat{\phi}_i(k)\big) - \Xi_1E_a\tilde{\phi}_i(k)]^T \times [h\big(E_a\phi_i(k)\big) - h\big(E_a\hat{\phi}_i(k)\big) - \Xi_2E_a\tilde{\phi}_i(k)] \geq 0, \end{cases}$$

$$
\begin{aligned}
&\Leftrightarrow \begin{cases}
-h^T\big(E_a\phi_i(k)\big)h\big(E_a\phi_i(k)\big) + h^T\big(E_a\phi_i(k)\big) \\
\quad \times \Xi_2 E_a\phi_i(k) + \phi_i^T(k)E_a^T\Xi_1^T h\big(E_a\phi_i(k)\big) \\
\quad -\phi_i^T(k)E_a^T\Xi_1^T\Xi_2E_a\phi_i \geq 0, \\
-\Big(h\big(E_a\phi_i(k)\big) - h\big(E_a\hat{\phi}_i(k)\big)\Big)^T\Big(h\big(E_a\phi_i(k)\big) \\
\quad -h\big(E_a\hat{\phi}_i(k)\big)\Big) + \Big(h\big(E_a\phi_i(k)\big) - h\big(E_a\hat{\phi}_i(k)\big)\Big)^T \\
\quad \times \Xi_2E_a\tilde{\phi}_i(k) + \tilde{\phi}_i^T(k)E_a^T\Xi_1^T\Big(h\big(E_a\phi_i(k)\big) \\
\quad -h\big(E_a\hat{\phi}_i(k)\big)\Big) - \tilde{\phi}_i^T(k)E_a^T\Xi_1^T\Xi_2E_a\tilde{\phi}_i(k) \geq 0,
\end{cases} \\
&\Rightarrow \begin{cases}
-H^T(\phi(k))H(\phi(k)) + H^T(\phi(k))\overline{\Xi}_2\phi(k) \\
\quad +\phi^T(k)\overline{\Xi}_1H(\phi(k)) - \phi^T(k)\overline{\Xi}_{12}\phi(k) \geq 0, \\
-\Big(H(\phi(k)) - H(\hat{\phi}(k))\Big)^T\Big(H(\phi(k)) - H(\hat{\phi}(k))\Big) \\
\quad +\Big(H(\phi(k)) - H^T(\hat{\phi}(k))\Big)^T\overline{\Xi}_2\tilde{\phi}(k) \\
\quad +\tilde{\phi}^T(k)\overline{\Xi}_1\Big(H(\phi(k)) - H(\hat{\phi}(k))\Big) \\
\quad -\tilde{\phi}^T(k)\overline{\Xi}_{12}\tilde{\phi}(k) \geq 0,
\end{cases} \\
&\Rightarrow \begin{cases}
-\begin{bmatrix} H(\phi(k)) \\ H(\phi(k)) - H(\hat{\phi}(k)) \end{bmatrix}^T \begin{bmatrix} H(\phi(k)) \\ H(\phi(k)) - H(\hat{\phi}(k)) \end{bmatrix} \\
\quad + \begin{bmatrix} H(\phi(k)) \\ H(\phi(k)) - H(\hat{\phi}(k)) \end{bmatrix}^T \widetilde{\Xi}_2 \begin{bmatrix} \phi(k) \\ \tilde{\phi}(k) \end{bmatrix} \\
\quad + \begin{bmatrix} \phi(k) \\ \tilde{\phi}(k) \end{bmatrix}^T \widetilde{\Xi}_1 \begin{bmatrix} H(\phi(k)) \\ H(\phi(k)) - H(\hat{\phi}(k)) \end{bmatrix} \\
\quad - \begin{bmatrix} \phi(k) \\ \tilde{\phi}(k) \end{bmatrix}^T \widetilde{\Xi}_{12} \begin{bmatrix} \phi(k) \\ \tilde{\phi}(k) \end{bmatrix} \geq 0
\end{cases} \\
&\Leftrightarrow \begin{cases}
-\bar{H}^T(k)\bar{H}(k) + \bar{H}^T(k)\widetilde{\overline{\Xi}}_2e(k) + e^T(k)\widetilde{\overline{\Xi}}_1\bar{H}(k) \\
\quad -e^T(k)\widetilde{\overline{\Xi}}_{12}e(k) \geq 0
\end{cases} \\
&\Leftrightarrow -\widetilde{H}^T(k)\widehat{\Xi}_{12}\widetilde{H}(k) \geq 0. \qquad (19)
\end{aligned}
$$

In view of (8), (18) and (19), one easily knows

$$
\begin{aligned}
&\mathcal{I}(k) \\
= \;& \mathbb{E}\Bigg\{\bigg[\bar{A}(k)e(k) + \bar{B}\bar{G}(k) + \bar{S}\bar{H}(k) + \big(\varpi(k) - \varpi\big) \\
& \times\bigg(\frac{1}{\varpi}\widetilde{B}\bar{G}(k) - \frac{1}{1-\varpi}\widetilde{S}\bar{H}(k)\bigg) + \bar{F}\rho(k)\omega(k)\bigg]^T \\
& \times Q(\hbar(k+1))\bigg[\bar{A}(k)e(k) + \bar{B}\bar{G}(k) + \bar{S}\bar{H}(k) \\
& +\big(\varpi(k) - \varpi\big)\bigg(\frac{1}{\varpi}\widetilde{B}\bar{G}(k) - \frac{1}{1-\varpi}\widetilde{S}\bar{H}(k)\bigg) \\
& +\bar{F}\rho(k)\omega(k)\bigg] - e^T(k)Q(\hbar(k))e(k)\Bigg\}\Bigg|e(k)\Bigg\} \\
\leq \;& \mathbb{E}\Bigg\{\Big[\bar{A}(k)e(k) + \bar{B}\bar{G}(k) + \bar{S}\bar{H}(k)\Big]^TQ(\hbar(k+1)) \\
& \times\Big[\bar{A}(k)e(k) + \bar{B}\bar{G}(k) + \bar{S}\bar{H}(k)\Big] \\
& +\overline{\overline{\varpi}}\bigg(\frac{1}{\varpi}\widetilde{B}\bar{G}(k) - \frac{1}{1-\varpi}\widetilde{S}\bar{H}(k)\bigg)^TQ(\hbar(k+1)) \\
& \times\bigg(\frac{1}{\varpi}\widetilde{B}\bar{G}(k) - \frac{1}{1-\varpi}\widetilde{S}\bar{H}(k)\bigg) \\
& +\rho^T(k)\bar{F}^TQ(\hbar(k+1))\bar{F}\rho(k) - e^T(k)Q(\hbar(k))e(k) \\
& -\rho^T(k)\rho(k) + \left[(I_N\otimes E_a)\,\phi(k)\right]^T(I_N\otimes\varrho) \\
& \times\left[(I_N\otimes E_a)\,\phi(k)\right] \\
& -\widetilde{G}^T(k)\Big(Y_2^T\sqsupset^T\Big(I_N\otimes(Y_1^T\widetilde{Z}_1Y_1)\Big)\sqsupset Y_2 \\
& +Y_3^T\sqsupset^T\Big(I_N\otimes(Y_1^T\widetilde{Z}_2Y_1)\Big)\sqsupset Y_3\Big)\widetilde{G}(k) \\
& -\widetilde{H}^T(k)\widehat{\Xi}_{12}\widetilde{H}(k)\Bigg|e(k)\Bigg\} \\
= \;& \mathbb{E}\Bigg\{\big[\widehat{A}(k)\widetilde{G}(k) + \widehat{S}_1\widetilde{H}(k)\big]^TQ(\hbar(k+1)) \\
& \times\big[\widehat{A}(k)\widetilde{G}(k) + \widehat{S}_1\widetilde{H}(k)\big] \\
& +\overline{\overline{\varpi}}\bigg(\frac{1}{\varpi}\widehat{B}\widetilde{G}(k) - \frac{1}{1-\varpi}\widehat{S}_2\widetilde{H}(k)\bigg)^TQ(\hbar(k+1)) \\
& \times\bigg(\frac{1}{\varpi}\widehat{B}\widetilde{G}(k) - \frac{1}{1-\varpi}\widehat{S}_2\widetilde{H}(k)\bigg) \\
& +\rho^T(k)\bar{F}^TQ(\hbar(k+1))\bar{F}\rho(k) - \widetilde{G}^{T(k)}\widehat{Q}(\hbar(k))\widetilde{G}(k) \\
& -\rho^T(k)\rho(k) + \widetilde{G}^T(k)Y_4^T(I_N\otimes E_a)^T(I_N\otimes\varrho) \\
& \times(I_N\otimes E_a)Y_4\widetilde{G}(k) \\
& -\widetilde{G}^T(k)\bigg(Y_2^T\sqsupset^T\Big(I_N\otimes(Y_1^T\widetilde{Z}_1Y_1)\Big)\sqsupset Y_2 \\
& +Y_3^T\sqsupset^T\Big(I_N\otimes(Y_1^T\widetilde{Z}_2Y_1)\Big)\sqsupset Y_3\bigg)\widetilde{G}(k) \\
& -\widetilde{H}^T(k)\widehat{\Xi}_{12}\widetilde{H}(k)\Bigg|e(k)\Bigg\} \\
= \;& \mathbb{E}\Big\{\Big[\widetilde{G}^T(k) \quad \rho^T(k) \quad \widetilde{H}^T(k)\Big]\Pi(k) \\
& \times\Big[\widetilde{G}^T(k) \quad \rho^T(k) \quad \widetilde{H}^T(k)\Big]^T\Big|e(k)\Big\}.
\end{aligned}
$$

Accordingly, $\Pi(k) < 0$ in Theorem 1 refers to $\mathbb{E}\{\Delta\mathcal{V}(k)\} = \mathbb{E}\{\mathcal{V}(k+1) - \mathcal{V}(k)\} \leq \lambda_{\max}\big(\Pi(k)\big)\mathbb{E}\big\{\|e(k)\|^2\big\}$.

Letting k be sufficiently large, we know $\mathbb{E}\{\mu^k\mathcal{V}(k)\} = \mathbb{E}\Big\{\mathcal{V}(0) + \sum_{i=0}^{k-1}\mu^i\big(\mu\Delta\mathcal{V}(i) + (\mu-1)\mathcal{V}(i)\big)\Big\}$ holds for any scalar $\mu > 1$. In addition, it is not difficult to know that $\mathbb{E}\{\mathcal{V}(k)\} \leq \lambda_{\max}(Q(\hbar(k)))\mathbb{E}\big\{\|e(k)\|^2\big\}$. Consequently, one arrives at

$$
\begin{aligned}
&\mathbb{E}\{\mu^k\mathcal{V}(k)\} \\
\leq\,& \lambda_{\max}(Q(\hbar(0)))\mathbb{E}\big\{\|e(0)\|^2\big\} + \sum_{i=0}^{k-1}\Big(\mu\lambda_{\max}(\Pi(i)) \\
& + (\mu-1)\lambda_{\max}(Q(\hbar(i)))\Big)\mu^i\mathbb{E}\big\{\|e(i)\|^2\big\} \\
\leq\,& a(\mu)\sup_{-m+1\leq i\leq 0}\mathbb{E}\left\{\|e(i)\|^2\right\} + b(\mu)\sum_{j=0}^{k}\mu^j\mathbb{E}\big\{\|e(j)\|^2\big\}
\end{aligned}
$$

where $a(\mu) = \lambda_{\max}(Q(\hbar(0)))$ and $b(\mu) = \max_{0\leq i\leq m-1}\left[\mu\lambda_{\max}(\Pi(i)) + (\mu-1)\lambda_{\max}(Q(\hbar(i)))\right]$. Noticing $a(1) > 0$ and $b(1) < 0$, we conclude that there must be a scalar $z>1$ such that $a(z)>0$ and $b(z)<0$, which implies

$$\begin{aligned}\mathbb{E}\{\|e(k)\|^2\} &\leq \sum_{j=0}^{k}\mu^{j-k}\mathbb{E}\{\|e(j)\|^2\}\\ &\leq \mu^{-k}\frac{a(z)}{-b(z)}\Bigg(\sup_{-m+1\leq i\leq 0}\mathbb{E}\left\{\|e(i)\|^2\right\}\\ &\quad -\frac{1}{a(z)}\mathbb{E}\{z^kV(k)\}\Bigg)\\ &\leq \frac{a(z)}{-b(z)}\mu^{-k}\sup_{-m+1\leq i\leq 0}\mathbb{E}\left\{\|e(i)\|^2\right\}.\end{aligned}$$

Therefore, the error dynamics (8) is EMSS according to Definition 1. ■

In the following, we analyze the H_∞ performance of the error system (8).

Theorem 2: Let the parameters K_i $(i=1,2,\ldots,N)$ be given. Under Assumptions 1-4 and the zero initial condition, the error dynamics (8) satisfies the H_∞ constraint (10) for all $\upsilon(k)\neq 0$ if there exist matrices $Z_l=\mathrm{diag}\{Z_{l1},Z_{l2},\ldots,Z_{ln}\}>0$ $(l=1,2)$ and $Q(\hbar(k))>0$ such that

$$\mathcal{J}(k)=\begin{bmatrix}\mathcal{J}_{11}(k) & * & * & *\\ 0 & \Pi_{22}(k) & * & *\\ \Pi_{31}(k) & 0 & \Pi_{33}(k) & *\\ \mathcal{J}_{41}(k) & 0 & \mathcal{J}_{43}(k) & \mathcal{J}_{44}(k)\end{bmatrix}<0 \tag{20}$$

where

$$\begin{aligned}&\widehat{I}\triangleq\begin{bmatrix}I_{2N(n+m)} & 0\\ 0 & 0_{2nN\times 2nN}\end{bmatrix},\\ &\mathcal{J}_{11}(k)\triangleq\Pi_{11}(k)+\widehat{I},\quad \mathcal{J}_{41}(k)\triangleq\bar{D}^T(k)Q(\hbar(k+1))\widehat{A}(k),\\ &\mathcal{J}_{43}(k)\triangleq\bar{D}^T(k)Q(\hbar(k+1))\widehat{S}_1,\\ &\mathcal{J}_{44}(k)\triangleq\bar{D}^T(k)Q(\hbar(k+1))\bar{D}(k)-\gamma^2 I_{n_\nu+nN}.\end{aligned}$$

Proof: Firstly, define an index

$$J(k)\triangleq\sum_{k=0}^{\infty}\mathbb{E}\left\{e^T(k)e(k)-\gamma^2\upsilon^T(k)\upsilon(k)\right\}.$$

Based on $\upsilon(k)\neq 0$ and $\mathcal{V}(k)\geq 0$ $(\forall\, k\neq 0)$, we have

$$J(k)\leq\sum_{k=0}^{\infty}\mathbb{E}\left\{\mathcal{I}(k)+e^T(k)e(k)-\gamma^2\upsilon^T(k)\upsilon(k)\right\}.$$

The review of the above stability analysis leads to

$$\begin{aligned}J(k)\leq\sum_{k=0}^{\infty}\mathbb{E}\Bigg\{&\left[\widehat{A}(k)\widetilde{G}(k)+\widehat{S}_1\widetilde{H}(k)+\bar{D}(k)\upsilon(k)\right]^T\\ &\times Q(\hbar(k+1))\left[\widehat{A}(k)\widetilde{G}(k)+\widehat{S}_1\widetilde{H}(k)+\bar{D}(k)\upsilon(k)\right]\\ &+\varpi\left(\frac{1}{\varpi}\widehat{B}\widetilde{G}(k)-\frac{1}{1-\varpi}\widehat{S}_2\widetilde{H}(k)\right)^T Q(\hbar(k+1))\\ &\times\left(\frac{1}{\varpi}\widehat{B}\widetilde{G}(k)-\frac{1}{1-\varpi}\widehat{S}_2\widetilde{H}(k)\right)+\rho^T(k)\bar{F}^T\\ &\times Q(\hbar(k+1))\bar{F}\rho(k)-\widetilde{G}^T(k)\widehat{Q}(\hbar(k))\widetilde{G}(k)\\ &-\rho^T(k)\rho(k)+\widetilde{G}^T(k)Y_4^T\left(I_N\otimes E_a\right)^T\left(I_N\otimes\varrho\right)\\ &\times\left(I_N\otimes E_a\right)Y_4\widetilde{G}(k)\\ &-\widetilde{G}^T(k)\Big(Y_2^T\daleth^T\left(I_N\otimes Y_1^T\widetilde{Z}_1Y_1\right)\daleth Y_2\\ &+Y_3^T\daleth^T\left(I_N\otimes Y_1^T\widetilde{Z}_2Y_1\right)\daleth Y_3\Big)\widetilde{G}(k)-\widetilde{H}^T(k)\widehat{\Xi}_{12}\\ &\times\widetilde{H}(k)+\widetilde{G}^T(k)\widehat{I}\widetilde{G}(k)-\gamma^2\upsilon^T(k)\upsilon(k)\Big|e(k)\Bigg\}\\ =\mathbb{E}\Big\{&\left[\widetilde{G}^T(k)\quad\rho^T(k)\quad\widetilde{H}^T(k)\quad\upsilon^T(k)\right]\mathcal{J}(k)\\ &\times\left[\widetilde{G}^T(k)\quad\rho^T(k)\quad\widetilde{H}^T(k)\quad\upsilon^T(k)\right]^T\Big|e(k)\Big\}.\end{aligned}$$

Therefore, $\mathcal{J}(k)<0$ in Theorem 2 indicates $\sum_{k=0}^{\infty}\mathbb{E}\left\{e^T(k)e(k)\right\}\leq\gamma^2\sum_{k=0}^{\infty}\mathbb{E}\left\{\upsilon^T(k)\upsilon(k)\right\}$. ■

Remark 4: By constructing a simple yet practical Lyapunov function, the exponential stability and the H_∞ performance analysis are carried out in Theorems 1-2 via the same proof lines. It is apparent that conditions given by both theorems are access-token-dependent that contain the scheduling information of the adopted RRP, therefore greatly reducing the conservatism of our results.

Based on the above exponential stability and H_∞ performance analysis, we are now ready to provide an operable program of the solution to the estimation problem of the complex network (6). To gain an explicit expression of the parameter matrix for estimator (5), matrix $Q(\hbar(k))$ in Theorem 1 is supposed to have a special structure. Details of the design procedure are illustrated as follows.

Theorem 3: Under the RRP scheduling scheme, the exponential mean-square stability of the error dynamics (8) and the H_∞ performance constraint (10) can be simultaneously guaranteed for all $\upsilon(k)\neq 0$ if there exist positive-definite matrices $Z_l=\mathrm{diag}\{Z_{l1},Z_{l2},\ldots,Z_{ln}\}$ $(l=1,2)$, $Q_{2r-1}(j)\in\mathbb{R}^{n\times n}$ and $Q_{2r}(j)\in\mathbb{R}^{m\times m}$ $(r=1,2,\ldots,2N, j=1,2,\ldots,m)$, and the matrices $\mathscr{K}_i$ $(i=1,2,\ldots,N)$ such that

$$\mathscr{J}(j,t)<0 \tag{21}$$

where

$$\begin{aligned}&t=j+1 \text{ when } j\in\{1,\ 2,\ldots,m-1\},\ t=1 \text{ when } j=m,\\ &Q(j)\triangleq\mathrm{diag}\{Q_1(j),\ Q_2(j),\ldots,Q_{4N}(j)\},\\ &\mathscr{J}(j,t)\triangleq\begin{bmatrix}\mathscr{J}_{11}(j,t) & * & * & * & *\\ 0 & \Pi_{22}(t) & * & * & *\\ \mathscr{J}_{31}(t) & 0 & \mathscr{J}_{33}(t) & * & *\\ 0 & 0 & 0 & \mathscr{J}_{44} & *\\ \widehat{\mathscr{A}(j,t)} & 0 & Q(t)\widehat{S}_1 & Q(t)\bar{D}(j) & -Q(t)\end{bmatrix},\\ &\mathscr{J}_{11}(j,t)\triangleq-\widehat{Q}(j)+Y_4^T\left(I_N\otimes E_a\right)^T\left(I_N\otimes\varrho\right)\left(I_N\otimes E_a\right)\\ &\quad\times Y_4-\Big(Y_2^T\daleth^T\left(I_N\otimes(Y_1^T\widetilde{Z}_1Y_1)\right)\daleth Y_2\\ &\quad+Y_3^T\daleth^T\left(I_N\otimes(Y_1^T\widetilde{Z}_2Y_1)\right)\daleth Y_3\Big)\end{aligned}$$

$$
+\frac{\overline{\varpi}}{\varpi^2}\widehat{B}^TQ(t)\widehat{B},
$$

$$
\Pi_{22}(t)\triangleq \bar{F}^TQ(t)\bar{F}-I,\quad \mathscr{J}_{31}(t)\triangleq -\widehat{S}_2^TQ(t)\widehat{B},
$$

$$
\mathscr{J}_{33}(t)\triangleq -\widehat{\Xi}_{12}+\frac{\overline{\varpi}}{(1-\varpi)^2}\widehat{S}_2^TQ(t)\widehat{S}_2,
$$

$$
\mathscr{J}_{44}\triangleq -\gamma^2 I_{n_\nu+N(n+m)},\quad C_i^t\triangleq\begin{bmatrix}\Psi_tC_i & I_m-\Psi_t\end{bmatrix},
$$

$$
A_i^a(j)\triangleq\begin{bmatrix}A & 0\\ \Psi_jC_i & I_m-\Psi_j\end{bmatrix},
$$

$$
\begin{aligned}
\mathscr{A}_1(j,t)\triangleq{}&\mathrm{diag}\{Q_{2N+1}(t),\ Q_{4N}(t)\}\big(I_N\otimes A_i^a(j)\big)\\
&-\mathscr{K}(j,t),
\end{aligned}
$$

$$
\mathscr{A}(j,t)\triangleq\begin{bmatrix}\mathrm{diag}\{Q_1(t),\dots,Q_{2N}(t)\}\Big(I_N\otimes A_i^a(j)+\Lambda\otimes\Gamma_a\Big) & 0\\ \mathrm{diag}\{Q_{2N+1}(t),\ Q_{4N}(t)\}\big(\Lambda\otimes\Gamma_a\big) & \mathscr{A}_1(j,t)\end{bmatrix},
$$

$$
\mathscr{K}(j,t)\triangleq\mathrm{diag}\{\mathscr{K}_1(t)C_i^j,\ \mathscr{K}_2(t)C_i^j,\dots,\mathscr{K}_N(t)C_i^j\},
$$

$$
\widehat{\mathscr{A}}(j,t)\triangleq\begin{bmatrix}\mathscr{A}(j,t) & Q(t)\bar{B}\end{bmatrix},\quad D_i^a(j)\triangleq\begin{bmatrix}0_{n\times n_\nu}\\ \Psi_jD_i\end{bmatrix},
$$

$$
D_1(j)\triangleq\mathrm{diag}\big\{D_1^a(j),\ D_2^a(j),\ \dots,\ D_N^a(j)\big\},
$$

$$
D_2(j)\triangleq\mathrm{diag}\big\{K_i\Psi_jD_1,\ K_i\Psi_jD_2,\dots,K_i\Psi_jD_N\big\},
$$

$$
\bar{D}(j)\triangleq\begin{bmatrix}D_1(j) & I_{(n+m)N}\\ D_1(j)-D_2(j) & 0\end{bmatrix}.
$$

Furthermore, the estimator gain is calculated by

$$
K_i=\mathrm{diag}\{Q^{-1}_{2(N+i)-1}(t),\dots,Q^{-1}_{2(N+i)}(t)\}\mathscr{K}_i(t). \tag{22}
$$

Proof: Firstly, one knows from condition (22) that $\mathscr{K}_i=\mathrm{diag}\{Q_{2N+1}(t),\dots,Q_{4N}(t)\}K_i$. Secondly, it can be verified from the scheduling mechanism of RRP that

$$
\begin{cases}\hbar(k+1)=1+\hbar(k), & \text{if}\ \ \hbar(k)\in\{1,\ 2,\dots,m-1\}\\ \hbar(k+1)=1, & \text{if}\ \ \hbar(k)=m.\end{cases}
$$

Now, letting $j=\hbar(k)$ and $t=\hbar(k+1)$ and taking notice of $\mathscr{J}(j,t)<0$ in Theorem 3 result in

$$
\begin{bmatrix}
\bar{\mathcal{J}}_{11}(k) & * & * & * & *\\
0 & \Pi_{22}(k) & * & * & *\\
\bar{\mathcal{J}}_{31}(k) & 0 & \bar{\mathcal{J}}_{33}(k) & * & *\\
0 & 0 & 0 & \mathscr{J}_{44} & *\\
Q(\hbar(k+1))\widehat{A}(k) & 0 & Q(\hbar(k+1))\widehat{S}_1 & Q(\hbar(k+1))\bar{D}(k) & Q(\hbar(k+1))
\end{bmatrix}<0 \tag{23}
$$

where

$$
\begin{aligned}
\bar{\mathcal{J}}_{11}(k)\triangleq{}&-\widehat{Q}(\hbar(k))+\varrho\big(Y_4^T\left(I_N\otimes E_a\right)^T\left(I_N\otimes E_a\right)Y_4\big)\\
&-\Big(Y_2^T\beth^T\Big(I_N\otimes Y_1^T\widetilde{Z}_1Y_1\Big)\beth Y_2+Y_3^T\beth^T\big(I_N\\
&\otimes Y_1^T\widetilde{Z}_2Y_1\Big)\beth Y_3\Big)+\frac{\overline{\varpi}}{\varpi^2}\widehat{B}^TQ(\hbar(k+1))\widehat{B},
\end{aligned}
$$

$$
\bar{\mathcal{J}}_{31}(k)\triangleq-\widehat{S}_2^TQ(\hbar(k+1))\widehat{B},
$$

$$
\bar{\mathcal{J}}_{33}(k)\triangleq-\widehat{\Xi}_{12}+\frac{\overline{\varpi}}{(1-\varpi)^2}\widehat{S}_2^TQ(\hbar(k+1))\widehat{S}_2.
$$

Applying a congruence transformation

$$
\begin{aligned}
F={}&\mathrm{diag}\big\{I_{4N(n+m)},I_{N(n+m)},I_{4N(n+m)},\\
&I_{n_\nu+N(n+m)},Q^{-1}(\hbar(k+1))\big\}
\end{aligned}
$$

to (23) yields

$$
\begin{bmatrix}
\bar{\mathcal{J}}_{11}(k) & * & * & * & *\\
0 & \Pi_{22}(k) & * & * & *\\
\bar{\mathcal{J}}_{31}(k) & 0 & \bar{\mathcal{J}}_{33}(k) & * & *\\
0 & 0 & 0 & \bar{\mathcal{J}}_{44} & *\\
\widehat{A}(k) & 0 & \widehat{S}_1 & \bar{D}(k) & -Q^{-1}(\hbar(k+1))
\end{bmatrix}<0.
$$

It follows readily from Lemma 1 that $\mathcal{J}(k)<0$ in Theorem 2. Furthermore, $\Pi(k)<0$ in Theorem 1 is guaranteed by $\mathscr{J}(j,t)<0$ since $\Pi(k)$ is a principal submatrix of $\mathcal{J}(k)$. More specifically, the LMIs in Theorem 3 simultaneously ensure $\mathscr{J}(j,t)<0$ and the sufficient conditions in Theorems 1–2. ■

Remark 5: Until now, the estimation problem has been successfully solved for the concerned complex networks with intermittent nonlinearity switching. Actually, Theorem 3 provides a practical method to seek for the explicit solutions to the expected estimator, where the time-varying LMIs in Theorem 1 are replaced by a set of time-invariant LMIs. It should be noticed that matrix $Q(\hbar(k))$ in Theorem 1 is assumed to be of a block-diagonal form in Theorem 3 with the $Q_{2r-1}(j)\in\mathbb{R}^{n\times n}$ and $Q_{2r}(j)\in\mathbb{R}^{m\times m}$. Such an assumption is used to reduce the derivation difficulty embedded in the pursuit of parameter K_i that mainly comes from the augmentation operation in (4) and (8). By solving the LMIs $\mathscr{J}(j,t)<0$ in Theorem 1, the proposed H_∞ estimation algorithm can be easily executed.

Remark 6: In the past decade, state estimation tasks have been extensively studied for various stochastic complex networks with network-induced phenomena, and excellent results are available in literature. Compared to the existing literature, our main results exhibit the following distinctive novelties: 1) the introduced RRP is embedded to manage data transmission in the sensor-to-estimator communication with aim to reduce the transmission frequency of the data and get rid of the network-induced phenomena; 2) a new yet unified H_∞ estimation framework is established to handle the mathematical complexities resulting from the RRP and intermittent nonlinearity switching; and 3) a new protocol-dependent condition is derived to ensure the exponentially mean-square stability of the estimation error dynamics.

IV. Illustrative Example

Consider the stochastic nonlinear complex network (1) with three nodes. The parameters of the nodes are listed as follows:

$$
A_1=A_2=A_3=\begin{bmatrix}-0.62 & 0.42\\ 0.22 & -0.11\end{bmatrix},
$$

$$
B_1=B_2=B_3=\begin{bmatrix}0.52 & -0.32\\ 0.34 & -0.23\end{bmatrix},
$$

$$
S_1=S_2=S_3=\begin{bmatrix}0.32 & 0.32\\ 0.13 & -0.25\end{bmatrix},
$$

$$C_1 = C_2 = C_3 = \begin{bmatrix} -0.51 & -0.22 \\ -0.13 & -0.14 \end{bmatrix},$$
$$D_1 = D_2 = D_3 = \begin{bmatrix} -0.34 \\ 0.22 \end{bmatrix}, \ \Gamma = \begin{bmatrix} 0.47 & -0.23 \\ 0.12 & 0.13 \end{bmatrix}.$$

Set the linear outer-coupling configuration matrix as

$$\Lambda = \begin{bmatrix} 0.1 & -0.12 & 0.02 \\ -0.12 & 0.09 & 0.03 \\ 0.02 & 0.03 & -0.05 \end{bmatrix},$$

and the switching probability of the random variable $\varpi(k)$ as 0.64. Let the nonlinear vector-valued functions $g(\cdot)$ and $h(\cdot)$ be

$$\begin{aligned} g\left(\begin{bmatrix} a_1(k) \\ a_2(k) \end{bmatrix}\right) &= \tanh\left(\begin{bmatrix} 0.16 & 0 \\ 0 & -0.34 \end{bmatrix}\begin{bmatrix} a_1(k) \\ a_2(k) \end{bmatrix}\right) \\ &\quad -0.22\sin\left(\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} a_1(k) \\ a_2(k) \end{bmatrix}\right), \\ h\left(\begin{bmatrix} a_1(k) \\ a_2(k) \end{bmatrix}\right) &= -\tanh\left(\begin{bmatrix} 0.21 & 0.03 \\ 0 & 0.75 \end{bmatrix}\begin{bmatrix} a_1(k) \\ a_2(k) \end{bmatrix}\right) \\ &\quad + \begin{bmatrix} 0.54 & 0.25 \\ 0 & 0.96 \end{bmatrix}\begin{bmatrix} a_1(k) \\ a_2(k) \end{bmatrix}. \end{aligned}$$

Subsequently, it is verified that nonlinear functions $g(\cdot)$ and $h(\cdot)$ satisfy Assumption 2 and Assumption 3, respectively, with

$$\begin{aligned} r_1^- &= -0.22, & r_1^+ &= 0.38, \\ r_2^- &= -0.34, & r_2^+ &= 0.44, \\ \Xi_1 &= \begin{bmatrix} 0.33 & 0.22 \\ 0 & 0.21 \end{bmatrix}, & \Xi_2 &= \begin{bmatrix} 0.54 & 0.25 \\ 0 & 0.96 \end{bmatrix}. \end{aligned}$$

Choose the diffusion coefficient vector as

$$\begin{aligned} \rho_i\left(k, \begin{bmatrix} x_{i1}(k) \\ x_{i2}(k) \end{bmatrix}\right) &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\sin\left(\begin{bmatrix} x_{i1}(k) \\ x_{i2}(k) \end{bmatrix}\right) \\ &\quad + \begin{bmatrix} 0 & 0 \\ 0 & -0.3 \end{bmatrix}\begin{bmatrix} x_{i1}(k) \\ x_{i2}(k) \end{bmatrix}. \end{aligned}$$

Then, one knows that there exists a non-negative matrix $\varrho = \text{diag}\{1, \ 0.09\}$ such that Assumption 1 is valid.

Letting $m = 2$ and $\gamma = 1.09$, the solutions to the LMIs in Theorem 3 can be readily obtained via Matlab LMI toolbox, which are given as

$$Q_1(1) = \begin{bmatrix} 29.64 & 0.40 \\ 0.40 & 43.72 \end{bmatrix}, Q_1(2) = \begin{bmatrix} 29.53 & 0.29 \\ 0.29 & 43.64 \end{bmatrix},$$
$$Q_2(1) = \begin{bmatrix} 31.07 & -.0164 \\ -0.01 & 2.49 \end{bmatrix}, Q_2(2) = \begin{bmatrix} 1.19 & 0.08 \\ 0.08 & 33.75 \end{bmatrix},$$
$$Q_3(1) = \begin{bmatrix} 1.19 & 0.08 \\ 0.08 & 33.75 \end{bmatrix}, Q_3(2) = \begin{bmatrix} 35.49 & 0.27 \\ 0.27 & 35.31 \end{bmatrix},$$
$$Q_4(1) = \begin{bmatrix} 40.45 & 0.05 \\ 0.05 & 3.56 \end{bmatrix}, Q_4(2) = \begin{bmatrix} 1.70 & 0.06 \\ 0.06 & 37.24 \end{bmatrix},$$
$$Q_5(1) = \begin{bmatrix} 59.13 & 0.57 \\ 0.57 & 67.48 \end{bmatrix}, Q_5(2) = \begin{bmatrix} 68.45 & 0.59 \\ 0.59 & 59.11 \end{bmatrix},$$
$$Q_6(1) = \begin{bmatrix} 35.29 & 0.05 \\ 0.05 & 10.26 \end{bmatrix}, Q_6(2) = \begin{bmatrix} 6.62 & 0.05 \\ 0.05 & 35.29 \end{bmatrix},$$
$$Q_7(1) = \begin{bmatrix} 33.90 & 0.08 \\ 0.08 & 2.79 \end{bmatrix}, Q_7(2) = \begin{bmatrix} 34.87 & 0.54 \\ 0.54 & 38.74 \end{bmatrix},$$
$$Q_8(1) = \begin{bmatrix} 35.29 & 0.01 \\ 0.01 & 2.37 \end{bmatrix}, Q_8(2) = \begin{bmatrix} 1.16 & 0.01 \\ 0.01 & 35.29 \end{bmatrix},$$
$$Q_9(1) = \begin{bmatrix} 37.03 & -0.55 \\ -0.55 & 36.02 \end{bmatrix}, Q_9(2) = \begin{bmatrix} 37.38 & -0.38 \\ -0.38 & 35.98 \end{bmatrix},$$
$$Q_{10}(1) = \begin{bmatrix} 35.30 & 0.00 \\ 0.00 & 3.30 \end{bmatrix}, Q_{10}(2) = \begin{bmatrix} 1.58 & 0.00 \\ 0.00 & 35.31 \end{bmatrix},$$
$$Q_{11}(1) = \begin{bmatrix} 35.38 & 0.05 \\ 0.05 & 35.33 \end{bmatrix}, Q_{11}(2) = \begin{bmatrix} 35.36 & 0.07 \\ 0.07 & 35.35 \end{bmatrix},$$
$$Q_{12}(1) = \begin{bmatrix} 35.48 & 0.012 \\ 0.01 & 3.34 \end{bmatrix}, Q_{12}(2) = \begin{bmatrix} 1.60 & -0.00 \\ -0.00 & 35.38 \end{bmatrix}.$$
$$Z_1 = \begin{bmatrix} 479.70 & 0 \\ 0 & 265.56 \end{bmatrix}, Z_2 = \begin{bmatrix} 825.27 & 0 \\ 0 & 144.57 \end{bmatrix},$$

and the gain matrices are

$$K_1 = \begin{bmatrix} 0.0001 & -0.0001 \\ -0.0004 & -0.0030 \\ -0.0117 & -0.0003 \\ 0.0001 & -0.0028 \end{bmatrix}, K_2 = \begin{bmatrix} 0.0251 & 0.0027 \\ -0.0012 & 0.0065 \\ 52.6765 & -0.0342 \\ -0.0090 & -1.5537 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -0.0304 & -0.0006 \\ -0.0004 & -0.0001 \\ -0.0733 & 0.0006 \\ 0.0001 & 0.0186 \end{bmatrix}.$$

Assume the external and disturbance inputs to be $\vartheta_i(k) = e^{\frac{-k}{2}}\left[\sin(k) \ \ \cos(k)\right]^T$ and $\nu_i(k) = 0.02\cos(k)$ $(i = 1, 2, 3)$. It can be checked that

$$\phi(k) = \begin{bmatrix} x_{1,1}(k) \\ x_{1,2}(k) \\ \vec{y}_{1,1}(k-1) \\ \vec{y}_{1,2}(k-1) \\ x_{2,1}(k) \\ x_{2,2}(k) \\ \vec{y}_{2,1}(k-1) \\ \vec{y}_{2,2}(k-1) \\ x_{3,1}(k) \\ x_{3,2}(k) \\ \vec{y}_{3,1}(k-1) \\ \vec{y}_{3,2}(k-1) \end{bmatrix}, \qquad \hat{\phi}(k) = \begin{bmatrix} \hat{x}_{1,2}(k) \\ \hat{x}_{1,1}(k) \\ \hat{q}_{1,1}(k-1) \\ \hat{q}_{1,2}(k-1) \\ \hat{x}_{2,1}(k) \\ \hat{x}_{2,2}(k) \\ \hat{q}_{2,1}(k-1) \\ \hat{q}_{2,2}(k-1) \\ \hat{x}_{3,1}(k) \\ \hat{x}_{3,2}(k) \\ \hat{q}_{3,1}(k-1) \\ \hat{q}_{3,2}(k-1) \end{bmatrix}.$$

So one knows $x_{1,1}(k) - \hat{x}_{1,1}(k) = \tilde{\phi}_1(k)$, $x_{1,2}(k) - \hat{x}_{1,2}(k) = \tilde{\phi}_2(k)$, $x_{2,1}(k) - \hat{x}_{2,1}(k) = \tilde{\phi}_5(k)$, $x_{2,2}(k) - \hat{x}_{2,2}(k) = \tilde{\phi}_6(k)$, $x_{3,1}(k) - \hat{x}_{3,1}(k) = \tilde{\phi}_9(k)$ and $x_{3,2}(k) - \hat{x}_{3,2}(k) = \tilde{\phi}_{10}(k)$. The trajectories of the estimation error dynamics are shown in Fig. 1, from which one see that estimation errors converge quickly to zero.

V. Conclusions

The paper has concerned with the H_∞ estimation problem for coupled complex networks with intermittent nonlinearity switching. Apart from the boundedness, no other restrictions (e.g. Lipschitz, differentiability and continuity) have been imposed on the introduced nonlinearities and the switching of the nonlinearities has been regulated by a set of binary sequences. In large-scare complex networks, the combination use of RRP scheduling scheme and the zero-order holder strategy facilitates the communication efficiency between the network nodes and remote state estimators. The effectiveness

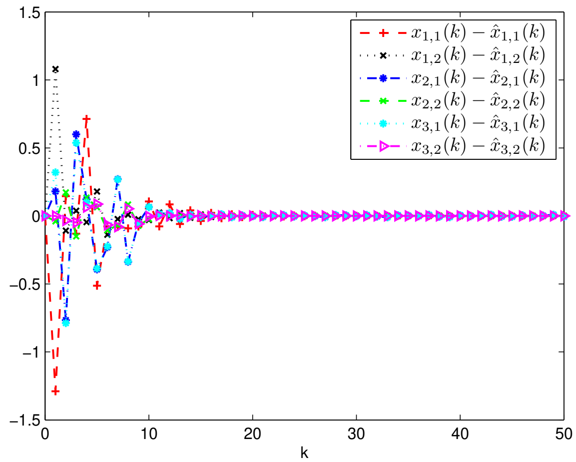

Fig. 1: Trajectory of error $x_i(k) - \hat{x}_i(k)$ $(i = 1, 2, 3)$

of our H_∞ approach has been verified by simulations. In the further research, we would extent the main results to more topics such as the moving horizon estimation problem [26], [55], the state estimation problem with dynamic quantization effects [28], [54], and the improvement of the state estimation performance by using some latest optimization algorithms [32], [33].

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