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Outlier-Resistant Observer-Based Control for A Class of Networked Systems under Encoding-Decoding Mechanism

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***Abstract*—This paper is concerned with a new outlier-resistant observer-based control problem for a class of networked systems (NSs) under the encoding-decoding communication mechanism (EDCM). In order to lighten the communication burden and enhance the data security, the EDCM is introduced in the observer-to-controller channel. In case of the measurement outliers, a specific saturation function is adopted in the observer structure to restrain the abnormal innovations so as to mitigate the side effects brought from the outliers. The aim of this paper is to design an observer-based controller such that, in the presence of measurement outliers, the closed-loop NS achieves the input-to-state stability (ISS) under the EDCM. By means of the uniform quantization technique, a criterion is first established in terms of the sizes of the encoding alphabet and the encoding period so as to ensure the detectability of the NS, and the requirement is also given on the capacity of the communication channel at each time instant. Then, with the help of ISS theory, the desired controller is obtained with its gain matrix parameterized by the solution to a certain inequality that can be solved via standard software packages. Finally, the effectiveness of the derived theoretical results is verified through three numerical simulation examples.**

***Index Terms*—Networked system, encoding-decoding communication mechanism, outlier-resistant observer, channel capacity, input-to-state stability.**

I. Introduction

Over the past several decades, networked systems (NSs) have received considerable research attention due to their prospective applications in various domains including, but are not limited to, unmanned aerial vehicles, remote diagnostics and troubleshooting, space and terrestrial exploration, and factory automation [4], [5], [11], [12], [14], [22], [25], [26], [29]–[31], [34], [36], [44]. In NSs, the information exchanges among actuators, sensors, controllers and estimators/filters are executed over networked media such as wireless or distributed networks [13]. Comparing to the conventional point-to-point communication, the network-based communication enjoys a number of distinctive merits. For example, the introduction of network renders the effectiveness in eliminating unnecessary wiring between system components, which indicates that the complexity and overhead of the corresponding system can be significantly diminished. Moreover, when some system components are required to be added/removed, NSs can be easily modified or upgraded without causing major changes in the system structure.

Despite the popularity of the NSs, the insertion of the communication network does give rise to some inevitable issues such as limited channel capacity and network security which, in turn, result in the so-called network-induced phenomena such as signal quantization [27], packet dropout [16], and communication delays [15]. These phenomena, if not adequately handled, could severely degrade the system performance and this has therefore triggered an ever-increasing research interest in both the analysis and synthesis problems for NSs in the past few decades. For example, many excellent results have been available on the estimation/control problems of NSs subject to network-induced phenomena, see e.g. [8], [32], [37]. From another perspective, to reduce the occurrence of the undesired network-induced phenomena, an active way is to employ certain communication protocols so as to regulate the data transmissions, and some widely deployed protocols include the event-triggering protocol [6], [7], [43], [50], the Round-Robin protocol [28], [35], [49], the try-once-discard protocol [21], [40] and the stochastic communication protocol [45], [48], to name just a few.

It is worth emphasizing that the main idea of the above-mentioned communication protocols is to alleviate the channel congestion by reducing the communication frequency and allowing for necessary network traffic only. Serving as yet another effective communication mechanism, the so-called encoding-decoding communication mechanism (EDCM) aims to send the symbolic data (rather than the original data) through communication channels to realize data compression. The employment of the EDCM not only reduces resource occupation but also enhances data security in the process of data transmission. Note that some pioneering EDCM-related work has been available in the literature, see e.g. [23], [24], [41], [42]. For instance, a sufficient condition has been presented in [23] to stabilize the linear time-invariant system by using the sampled encoded states/outputs. The EDCM has been further applied in [41] to research into the synchronization control problem for a kind of dynamical networks subject to packet dropouts, where a decoder-based controller has been constructed to guarantee the synchronization of the dynamical networks.

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In engineering practice, the measurement output may undergo abrupt yet large disturbances, which are referred to as *measurement outliers*, for a variety of reasons such as operation errors, sensor noises/failures, and unknown environmental disturbances. Up to now, the outlier detection approaches and the state-of-the-art outlier resistant/resilient control schemes have received particular research attention [17]. Different from the measurement noises [20], the measurement outliers may lead to abnormal magnitude changes which, if directly utilized in the observer/estimator implementation, would result in abnormal innovation and subsequently deteriorate the performance of the observer/estimator [46], [47]. As such, it makes practical sense to look into the outlier-resistant state estimation issue by removing/restraining the side-effects caused by measurement outliers, see [1]–[3], [10], [33], [38] for some initial results. In [1], a modified maximum likelihood estimator has been set up which is robust to the possible outliers. Moreover, in [10], a new Kalman filter has been put forward that is insensitive to measurement outliers. Nonetheless, when it comes to the observer-based control problems, the corresponding results have been very few and the main objective of this paper is therefore to propose an outlier-resistant observer to weaken the influence from the measurement outliers while maintaining the desired control performance.

Motivated by the above discussions, in this paper, we are concerned with the outlier-resistant observer-based control problem for a class of nonlinear NSs under EDCM. In doing so, three challenges we have to face are identified as follows: 1) how to establish a unified control-theoretic framework that takes the EDCM and measurement outliers into simultaneous consideration? 2) how to design an effective observer to mitigate the side-effects from the measurement outliers on the estimation performance? and 3) how to handle the decoding error (between the decoding system state and the actual system state) and use the incomplete decoding information to accomplish the desired control task?

To overcome the identified challenges, we make dedicated efforts in this paper with certain distinctive features outlined as follows: *1) a specific saturation function is introduced into the observer structure so as to attenuate the side-effects of the measurement outliers on the observation error; 2) a suitable EDCM is proposed for the addressed NSs to reduce communication resource occupation and also enhance data transmission security; 3) the combined effects from the encoding period as well as the size of encoding alphabets are analyzed in a quantitative way, and the requirement is given on the channel capacity is given; and 4) an easy-to-implement controller design algorithm is developed with explicitly characterized controller gain by using the ISS theory*.

The rest of this paper is structured as follows. In Section II, the problem to be addressed is presented and the basic idea of the EDCM is introduced. In Section III, the encoding-decoding-based detectability is analyzed and the desired outlier-resistant observer-based controller is designed. In Section IV, three simulation examples are provided and Section V concludes this paper.

Notation: The notation used here is fairly standard. I and 0 represent the identity matrix and zero matrix with proper dimensions, respectively. $\lambda_{\max}(A)(\lambda_{\min}(A))$ is the maximum (minimum) eigenvalue of a symmetric matrix A. For a matrix B, $B^{\perp}$ is an orthogonal basis of the null space for matrix B^T, i.e., $B^T B^{\perp}=0$. $\|y\|_2$ and $\|y\|_\infty$ represent the Euclidean norm and infinite norm of vector y, respectively. For a given real number x, $\lceil x \rceil$ denotes the least integer greater than or equal to x. In symmetric block matrices, "$*$" is used as an ellipsis for terms induced by symmetry. If $\gamma(\cdot):\mathbb{R}^+\mapsto\mathbb{R}^+$ is a continuous strictly increasing function with $\gamma(0)=0$, then $\gamma(\cdot)$ is called as a $\mathcal{K}$ class function. Further, if $\gamma(\cdot)\in\mathcal{K}$ with $\gamma(s)\to\infty$ as $s\to\infty$, we say that $\gamma(\cdot)$ is a $\mathcal{K}_\infty$ class function. A function $\Im(\cdot,\cdot):\mathbb{R}^+\times\mathbb{R}^+\mapsto\mathbb{R}^+$ is said to be of class $\mathcal{KL}$, if the mapping $\Im(s,k)$ is of class $\mathcal{K}$ for each fixed k, and is decreasing to 0 as $k\to\infty$ for each fixed s.

II. Problem Formulation and Preliminaries

Consider an NS shown in Fig. 1, where the data delivery between the observer and the controller is implemented via a network under a certain EDCM. The state estimates are encoded to certain codewords by a designed encoder, and then the received codewords are decoded by the corresponding decoder at the controller side. In the following, the physical plant, the EDCM, the observer and the controller will be presented in the state space.

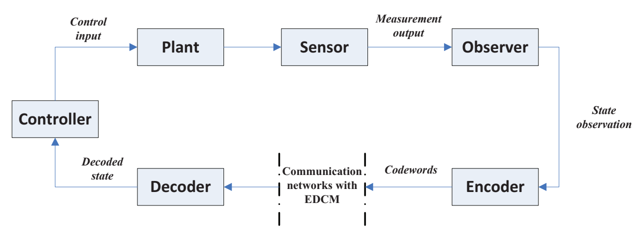

Fig. 1. Structure of the NS under EDCM.

A. The physical plant

In this paper, we consider the following class of nonlinear discrete-time systems:

$$\begin{cases} x_{k+1}=Ex_k+Df(x_k)+Bu_k \\ y_k=Nx_k \\ x_0=s_0 \end{cases} \tag{1}$$

where $x_k\in\mathbb{R}^{n_x}$ is the system state with the initial condition $x_0=s_0$ satisfying $\|s_0\|_2\le\epsilon_0$, and ϵ_0 is a known constant. $y_k\in\mathbb{R}^{n_y}$ denotes the measurement output. $u_k\in\mathbb{R}^{n_u}$ is the control input signal. $f(\cdot):\mathbb{R}^{n_x}\mapsto\mathbb{R}^{n_x}$ is a nonlinear vector-valued function. E, D, B and N are known real-valued matrices with compatible dimensions.

Assumption 1: The nonlinear vector-valued function $f(\cdot)$ satisfies $f(0)=0$ and

$$\|f(x_k+\delta_k)-f(x_k)\|_2\le\|U\delta_k\|_2 \tag{2}$$

where U is a known real matrix of appropriate dimensions and $\delta_k\in\mathbb{R}^{n_x}$ is a vector.

B. The outlier-resistant observer

The objective of this subsection is to design an observer on the basis of the available measurements. As discussed in the introduction, measurement outliers do exist which, if not properly dealt with, would deteriorate the observer performance or even destabilize the error dynamics. To resolve such a problem, a so-called *outlier-resistant* observer is constructed as follows:

$$\begin{cases} \hat{x}_{k+1} = E\hat{x}_k + Df(\hat{x}_k) + Bu_k + K_e\sigma\big(y_k - N\hat{x}_k\big) \\ \hat{x}_0 = 0 \end{cases} \quad (3)$$

where $\hat{x}_k \in \mathbb{R}^{n_x}$ is the state estimate and K_e is the observer gain to be designed. Here, the purposely introduced saturation function $\sigma(\cdot) : \mathbb{R}^{n_y} \mapsto \mathbb{R}^{n_y}$ is defined as follows:

$$\sigma(\varpi) = \begin{bmatrix} \sigma_1^T(\varpi_1) & \sigma_2^T(\varpi_2) & \cdots & \sigma_{n_y}^T(\varpi_{n_y}) \end{bmatrix}^T$$

with $\sigma_\iota(\varpi_\iota) \triangleq \mathrm{sign}(\varpi_\iota)\min\{\varpi_{\iota,\max}, |\varpi_\iota|\}$, $\iota = 1, 2, \ldots, n_y$, where $\varpi_{\iota,\max}$ is the ιth element of $\varpi_{\max}$ (i.e., the saturation level).

Denoting $e_k \triangleq x_k - \hat{x}_k$ and $\tilde{f}(e_k) \triangleq f(x_k) - f(\hat{x}_k)$, we obtain the corresponding error dynamics as follows:

$$e_{k+1} = Ee_k + D\tilde{f}(e_k) - K_e\sigma(Ne_k). \quad (4)$$

Remark 1: As a kind of contaminated measurements, the outliers are meant to deviate significantly from the normal values that have recently stirred quite a lot research attention. Measurement outliers can be caused by a variety of reasons such as sensor aging/failures, operational errors and environmental factors. In the context of state estimation, measurement outliers may lead to the calculation of *abnormal* innovation which would therefore have an adverse effect on the performance of observer. As such, we endeavor to propose an effective observer design scheme so as to prevent outliers from deteriorating the estimation accuracy.

Remark 2: The measurement outliers, if directly utilized in the innovation $y_k - N\hat{x}_k$, are likely to lead to abnormal deviation of the innovation from its usual pattern, thereby worsening estimation performance. Therefore, a specific saturation function $\sigma(\cdot)$ is introduced in the observer structure so as to constrain the innovation within a predefined range that can be determined *a priori* according to engineering practice. The proposed observer is referred to as an outlier-resistant one capable of attenuating the negative effects from the measurement outliers. It is worth mentioning that the proposed observer (3) will be degenerated into the traditional Luenberger-like one when the saturation level goes to infinity.

C. A general encoding-decoding procedure

In order to reduce the communication resource occupation and realize data transmission security, we are now in a position to introduce the EDCM in the observer-to-controller channel. To proceed further, let us present the general form of the encoding-decoding procedure for system (1).

The general form of the encoder is described by

$$\theta(ld) = \mathcal{J}\big(\hat{x}_{ld}\big) \quad (5)$$

and the decoder is described by

$$\breve{x}_{ld} = \mathcal{F}\big(\theta(ld)\big) \quad (6)$$

where $\theta(ld)$ $(l = 1, 2, \ldots)$ is the codeword generated at the encoding instant ld by the encoder, d is a constant representing the encoding period, $\breve{x}_{ld}$ is the decoded value of $\hat{x}_{ld}$ at time instant ld, and $\mathcal{J}(\cdot)$ and $\mathcal{F}(\cdot)$ are, respectively, encoder and decoder functions to be designed later.

D. Uniform quantization approach

As a critical part in the development of EDCM, the uniform quantization approach is briefly introduced as follows.

For given scale parameter $c > 0$ and integer q, we divide the hyperrectangle $\mathcal{B}_c = \{\zeta \in \mathbb{R}^{n_x} : \|\zeta\|_\infty \le c, i = 1, 2, \ldots, n_x\}$ into q^{n_x} hyperrectangles $\mathbb{I}^1_{\varepsilon_1}(c) \times \mathbb{I}^2_{\varepsilon_2}(c) \times \cdots \times \mathbb{I}^{n_x}_{\varepsilon_{n_x}}(c)$, where $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{n_x} \in \{1, 2, \ldots, q\}$ and

$$\begin{aligned} \mathbb{I}^i_1(c) &\triangleq \{\zeta_i | -c \le \zeta_i < -c + 2c/q\}, \\ \mathbb{I}^i_2(c) &\triangleq \{\zeta_i | -c + 2c/q \le \zeta_i < -c + 4c/q\}, \\ &\vdots \\ \mathbb{I}^i_q(c) &\triangleq \{\zeta_i | c - 2c/q \le \zeta_i < c\}. \end{aligned} \quad (7)$$

with ζ_i being the ith element of the vector ζ. The center of the hyperrectangle $\mathbb{I}^1_{\varepsilon_1}(c) \times \mathbb{I}^2_{\varepsilon_2}(c) \times \cdots \times \mathbb{I}^{n_x}_{\varepsilon_{n_x}}(c)$ is defined as

$$\hbar_c(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{n_x}) \triangleq \begin{bmatrix} -c + \frac{(2\varepsilon_1 - 1)c}{q} \\ -c + \frac{(2\varepsilon_2 - 1)c}{q} \\ \vdots \\ -c + \frac{(2\varepsilon_{n_x} - 1)c}{q} \end{bmatrix}. \quad (8)$$

Consequently, for any $\zeta \in \mathcal{B}_c$, there exist unique integers $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{n_x} \in \{1, 2, \ldots, q\}$ such that $\zeta \in \mathbb{I}^1_{\varepsilon_1}(c) \times \mathbb{I}^2_{\varepsilon_2}(c) \times \cdots \times \mathbb{I}^{n_x}_{\varepsilon_{n_x}}(c)$, which implies

$$\|\zeta - \hbar_c(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{n_x})\|_2 \le \frac{\sqrt{n_x}c}{q}.$$

For subsequent analysis, the following definitions are given.

Definition 1: [9] For a nonlinear function $\phi(\cdot) : \mathbb{R}^{n_y} \mapsto \mathbb{R}^{n_y}$ and real matrices W_1, $W_2 \in \mathbb{R}^{n_y \times n_y}$ where $W = W_2 - W_1$ is a positive definite matrix, if

$$\left(\phi(\varpi) - W_1\varpi\right)^T \left(\phi(\varpi) - W_2\varpi\right) \le 0, \ \forall \varpi \in \mathbb{R}^{n_y} \quad (9)$$

is true, then $\phi(\cdot)$ is said to satisfy a sector condition and belongs to the sector $[W_1, W_2]$.

Definition 2: [41] The nonlinear discrete-time system (1) is said to be detectable if there exist families of encoder-decoder pairs (5) and (6) with an encoding alphabet $\mathcal{H}$ of size $\mathcal{X}$ such that

$$\lim_{k \to \infty} \|x_k - \breve{x}_k\|_2 = 0 \quad (10)$$

holds for any solution of (1).

Definition 3: [18] Consider a nonlinear discrete-time system with the following form:

$$\rho_{k+1} = g(\rho_k, \nu_k) \quad (11)$$

where $\rho_k \in \mathbb{R}^n$, $\nu_k \in \mathbb{R}^p$, and $g(\cdot, \cdot) : \mathbb{R}^n \times \mathbb{R}^p \mapsto \mathbb{R}^n$ stand for the system state, exogenous input, and continuous nonlinear function with $g(0, 0) = 0$, respectively. For system (11), if there exist a $\mathcal{KL}$ function $\alpha(\cdot,\cdot)$ and a $\mathcal{K}$ class function $\beta(\cdot)$ such that the following condition

$$\|\rho_k\|_2 \leq \alpha(\|\rho_0\|_2, k) + \beta(\|\nu_k\|_\infty)$$

holds for $\forall k \geq 0$ and $\forall \rho_0 \in \mathbb{R}^n$ where $\|\nu_k\|_\infty \triangleq \sup_k\{\|\nu_k\|\}$, then the system (11) is said to be input-to-state stable.

This paper aims to lay emphasis on the control problem based on an outlier-resistant observer for the system (1) under the EDCM. The objective of this paper is to develop an efficient encoding-decoding procedure (5)-(6) and an outlier-resistant observer-based controller such that 1) the underlying system (1) is detectable; and 2) the closed-loop system is input-to-state stable.

III. Main Results

A. Detectability Analysis

In this subsection, we are ready to examine the detectability issue, that is, whether and how the encoded data could be recovered from the codewords with a prescribed accuracy constraint. To start with, we give the following lemmas that will be used in deriving our main results in the sequel.

Lemma 1: [9] There exist diagonal matrices M_1 and M_2 satisfying $0 \leq M_1 < I \leq M_2$ such that the saturation function $\sigma(Ne_k)$ in (4) can be divided into a linear part and a nonlinear part as

$$\sigma(Ne_k) = M_1 N e_k + \phi(Ne_k) \tag{12}$$

where $\phi(\cdot)$ is a nonlinear vector-valued function that satisfies the sector condition with $W_1 = 0$ and $W_2 = M$, where $M = M_2 - M_1$, i.e., $\phi(Ne_k)$ satisfies the following inequality:

$$\phi^T(Ne_k)\left[\phi(Ne_k) - MNe_k\right] \leq 0. \tag{13}$$

Lemma 2: Let the scalar $\varrho_1 > 0$ be given. If there exist a positive definite matrix $P > 0$ and a scalar $\mu_1 > 0$ such that the following linear matrix inequality (LMI)

$$\tilde{\Xi} = \begin{bmatrix} \tilde{\Xi}_{11} & * & * \\ 0 & -\mu_1 I & * \\ PE & PD & -P \end{bmatrix} < 0 \tag{14}$$

holds for any two solutions x_k^1 and x_k^2 of (1), then we have

$$\|x_{k+1}^1 - x_{k+1}^2\|_2 < \gamma_1 \|x_k^1 - x_k^2\|_2 \tag{15}$$

where

$$\tilde{\Xi}_{11} \triangleq -(1+\varrho_1)P + \mu_1 U^T U,$$
$$\gamma_1 \triangleq \sqrt{\left((1+\varrho_1)\lambda_{\max}(P)\right)/\lambda_{\min}(P)}.$$

Proof: Denoting $v_k \triangleq x_k^1 - x_k^2$, we easily obtain that $v_{k+1} = Ev_k + Df(x_k^1) - Df(x_k^2)$.

Consider the following Lyapunov function:

$$V_k = v_k^T P v_k$$

and it follows from $\triangle V_k = V_{k+1} - V_k$ that

$$\begin{aligned}\triangle V_k - \varrho_1 V_k = {} & [Ev_k + Df(x_k^1) - Df(x_k^2)]^T P \\ & \times [Ev_k + Df(x_k^1) - Df(x_k^2)] \\ & - (1+\varrho_1) v_k^T P v_k.\end{aligned} \tag{16}$$

In terms of Assumption 1, we have

$$\begin{aligned}&\triangle V_k - \varrho_1 V_k \\ &\leq v_k^T\left[E^T PE - (1+\varrho_1)P\right] v_k \\ &\quad + 2v_k^T E^T PD\left(f(x_k^1) - f(x_k^2)\right) \\ &\quad + \left(f(x_k^1) - f(x_k^2)\right)^T D^T PD\left(f(x_k^1) - f(x_k^2)\right) \\ &\quad - \mu_1\left(f(x_k^1) - f(x_k^2)\right)^T\left(f(x_k^1) - f(x_k^2)\right) \\ &\quad + \mu_1 v_k^T U^T U v_k \\ &= \eta_{1k}^T \Xi \eta_{1k}\end{aligned} \tag{17}$$

where

$$\eta_{1k} \triangleq \begin{bmatrix} v_k^T & f^T(x_k^1) - f^T(x_k^2) \end{bmatrix}^T,$$
$$\Xi \triangleq \begin{bmatrix} \Xi_{11} & * \\ D^T PE & D^T PD - \mu_1 I \end{bmatrix},$$
$$\Xi_{11} \triangleq E^T PE - (1+\varrho_1)P + \mu_1 U^T U.$$

Applying Schur Complement Lemma to (14), it is not difficult to see that $\Xi < 0$, which means $V_{k+1} < (1+\varrho_1)V_k$. Therefore, we have

$$\lambda_{\min}(P)\|v_{k+1}\|_2^2 < (1+\varrho_1)\lambda_{\max}(P)\|v_k\|_2^2.$$

and subsequently

$$\|v_{k+1}\|_2 < \gamma_1 \|v_k\|_2$$

which ends the proof. ■

Lemma 3: Let a positive scalar $\varrho_2 < 1$ be given and consider the error system (4). Assume that there exist a positive definite matrix $R > 0$, a matrix X and a positive scalar $\mu_2 >$ satisfying

$$\tilde{\Lambda} = \begin{bmatrix} \tilde{\Lambda}_{11} & * & * & * \\ 0 & -\mu_2 I & * & * \\ \mu_0 \frac{MN}{2} & 0 & -\mu_0 I & * \\ RE - XM_1N & RD & -X & -R \end{bmatrix} < 0 \tag{18}$$

where $\tilde{\Lambda}_{11} \triangleq -(1-\varrho_2)R + \mu_2 U^T U$. Let $d \in \mathbb{Z}^+$ be any positive integer such that $0 < \gamma_2 < 1$ where

$$\gamma_2 \triangleq \sqrt{\left((1-\varrho_2)^d \lambda_{\max}(R)\right)/\lambda_{\min}(R)}.$$

Then, we have

$$\|x_{k+d} - \hat{x}_{k+d}\|_2 < \gamma_2 \|x_k - \hat{x}_k\|_2. \tag{19}$$

In this case, the desired observer gain matrix is calculated by $K_e = R^{-1}X$.

Proof: For the error dynamics (4), we construct the following Lyapunov function

$$V_k = e_k^T R e_k$$

and then obtain

$$\begin{aligned}\triangle V_k + \varrho_2 V_k = {} & \left[Ee_k + D\tilde{f}(e_k) - K_e\sigma(Ne_k)\right]^T R \\ & \times \left[Ee_k + D\tilde{f}(e_k) - K_e\sigma(Ne_k)\right] \\ & - (1-\varrho_2) e_k^T R e_k.\end{aligned}$$

In light of (2) and (13), we further derive that

$$\triangle V_k + \varrho_2 V_k = \left[Ee_k + D\tilde{f}(e_k) - K_e\sigma(Ne_k)\right]^T R$$

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$$\begin{aligned}&\times\Big[Ee_k+D\tilde{f}(e_k)-K_e\sigma(Ne_k)\Big]\\&-(1-\varrho_2)e_k^TRe_k-\mu_2\tilde{f}^T(e_k)\tilde{f}(e_k)\\&+\mu_2e_k^TU^TUe_k-\mu_0\phi^T(Ne_k)\phi(Ne_k)\\&+\mu_0\phi^T(Ne_k)MNe_k\\=\;&\eta_{2k}^T\Lambda\eta_{2k}\end{aligned}\tag{20}$$

where

$$\eta_{2k}\triangleq\begin{bmatrix}e_k^T & \tilde{f}^T(e_k) & \phi^T(Ne_k)\end{bmatrix}^T,$$

$$\Lambda\triangleq\begin{bmatrix}\Lambda_{11} & * & *\\ \Lambda_{21} & D^TRD-\mu_2I & *\\ \Lambda_{31} & -K_e^TRD & K_e^TRK_e-\mu_0I\end{bmatrix},$$

$$\Lambda_{11}\triangleq(E-K_eM_1N)^TR(E-K_eM_1N)+\tilde{\Lambda}_{11},$$

$$\Lambda_{21}\triangleq D^TR(E-K_eM_1N),$$

$$\Lambda_{31}\triangleq-K_e^TR(E-K_eM_1N)+\mu_0\frac{MN}{2}.$$

Applying Schur Complement Lemma to (18) yields $\triangle V_k+\varrho_2V_k<0$, which implies

$$V_{k+d}<(1-\varrho_2)^dV_k\tag{21}$$

where d is any positive integer such that $0<\gamma_2<1$. Furthermore, we have

$$\lambda_{\min}(R)\|e_{k+d}\|_2^2<(1-\varrho_2)^d\lambda_{\max}(R)\|e_k\|_2^2$$

or

$$\|x_{k+d}-\hat{x}_{k+d}\|_2<\gamma_2\|x_k-\hat{x}_k\|_2,$$

which ends the proof. ■

In what follows, we shall present an encoding-decoding procedure and analyze the detectability of (1). Drawing on the ideas of EDCM in [42], we know that the decoded state $\breve{x}_k$ cannot be directly utilized to construct state observer (3) due to the constraint on the network bandwidth. Therefore, an auxiliary state $\bar{x}_k$ is introduced on the observer side which will be defined later. For the time being, the error vector at the encoding instant ld is denoted by $\zeta(ld)\triangleq\hat{x}_{ld}-\bar{x}_{ld}$.

In terms of Lemmas 2 and 3 for certain constants γ_1 and γ_2, the following encoding-decoding procedure is designed for (1).

Encoder: For $\zeta(ld)\triangleq\hat{x}_{ld}-\bar{x}_{ld}\in\mathbb{I}^1_{\varepsilon_1}(c(ld))\times\mathbb{I}^2_{\varepsilon_2}(c(ld))\times\cdots\times\mathbb{I}^{n_x}_{\varepsilon_{n_x}}(c(ld))\subset\mathcal{B}_{c(ld)}$, we have

$$\theta(ld)=[\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_{n_x}]\tag{22}$$

where $\bar{x}_{ld}$ is defined by

$$\begin{cases}\bar{x}_0=0,\\ \bar{x}_k=\breve{x}_k,\ k\neq ld,\\ \bar{x}_{ld}=E\breve{x}_{ld-1}+Df(\breve{x}_{ld-1})+Bu_{ld-1},\end{cases}$$

$$\begin{cases}\breve{x}_0=0,\\ \breve{x}_{k+1}=E\breve{x}_k+Df(\breve{x}_k)+Bu_k,\ k\neq ld-1,\\ \breve{x}_{ld}=\bar{x}_{ld}+\hbar_{c(ld)}(\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_{n_x}).\end{cases}\tag{23}$$

Decoder:

$$\begin{cases}\breve{x}_0=0,\\ \breve{x}_{k+1}=E\breve{x}_k+Df(\breve{x}_k)+Bu_k,\ k\neq ld-1,\\ \breve{x}_{ld}=\bar{x}_{ld}+\hbar_{c(ld)}(\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_{n_x}).\end{cases}\tag{24}$$

Remark 3: It is obvious from (24) that, for an encoding period $[ld,(l+1)d)$, a series of decoded state (denoted by $\breve{X}(ld)\triangleq\{\breve{x}(ld),\breve{x}(ld+1),\ldots,\breve{x}(l+1)d-1\}$) is generated, which means that the decoder can indeed produce the decoded states at both encoding instant ld and those non-encoding time instants $ld+1,\ldots,(l+1)d-1$. Here, the decoded state $\breve{X}(k),k\in(lh,(l+1)d)$ generated at non-encoding instants can be regarded as a "prediction" of the true states of the system.

The following notations are defined for presentation convenience.

$$c(d)\triangleq\gamma_2\epsilon_0+\gamma_1^d\epsilon_0,\tag{25}$$

$$c((l+1)d)\triangleq\gamma_2^l\epsilon_0(\gamma_1^d+\gamma_2)+\gamma_1^d\frac{\sqrt{n_x}c(ld)}{q}.\tag{26}$$

Lemma 4: The encoding-decoding procedure (22)-(24) satisfies the following constraint:

$$\|\hat{x}_{ld}-\bar{x}_{ld}\|_\infty\leq c(ld),\quad l=1,2,\ldots.\tag{27}$$

Proof: The proof is carried out by mathematical induction.

1) For $l=1$, by using the property of vector norm, we know that $\|\hat{x}_d-\bar{x}_d\|_2\leq\|\hat{x}_d-x_d\|_2+\|x_d-\bar{x}_d\|_2$, and it then follows from Lemmas 2 and 3 that

$$\begin{aligned}&\|\hat{x}_d-\bar{x}_d\|_2\\\leq\;&\|\hat{x}_d-x_d\|_2+\|x_d-\bar{x}_d\|_2\\\leq\;&\cdots\\\leq\;&\gamma_2\epsilon_0+\gamma_1^d\epsilon_0\end{aligned}\tag{28}$$

which guarantees

$$\|\hat{x}_{ld}-\bar{x}_{ld}\|_\infty\leq c(ld).$$

2) Assuming $\|\hat{x}_{hd}-\bar{x}_{hd}\|_\infty\leq c(hd)$ holds for all $h=2,3,\ldots,l$, one immediately has

$$\begin{aligned}&\|\hat{x}_{(l+1)d}-\bar{x}_{(l+1)d}\|_2\\\leq\;&\|\hat{x}_{(l+1)d}-x_{(l+1)d}\|_2+\|x_{(l+1)d}-\bar{x}_{(l+1)d}\|_2\\\leq\;&\cdots\\\leq\;&\gamma_2^{l+1}\epsilon_0+\gamma_1^d\|x_{ld}-\breve{x}_{ld}\|_2.\end{aligned}$$

According to the dynamics of $\breve{x}_{ld}$ in (24), it is further derived that

$$\begin{aligned}&\|x_{ld}-\breve{x}_{ld}\|_2\\=\;&\|x_{ld}-\bar{x}_{ld}-\hbar_{c(ld)}(\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_{n_x})\|_2\\\leq\;&\|\hat{x}_{ld}-\bar{x}_{ld}-\hbar_{c(ld)}(\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_{n_x})\|_2+\|x_{ld}-\hat{x}_{ld}\|_2\\\leq\;&\gamma_2^l\epsilon_0+\frac{\sqrt{n_x}c(ld)}{q}.\end{aligned}\tag{29}$$

Combining (28) and (29), we draw the conclusion that

$$\begin{aligned}&\|\hat{x}_{(l+1)d}-\bar{x}_{(l+1)d}\|_2\\\leq\;&\gamma_2^l\epsilon_0(\gamma_1^d+\gamma_2)+\gamma_1^d\frac{\sqrt{n_x}c(ld)}{q}.\end{aligned}\tag{30}$$

Moreover, by noting (28), (30) also implies

$$\|\hat{x}_{(l+1)d}-\bar{x}_{(l+1)d}\|_\infty\leq c((l+1)d).\tag{31}$$

Finally, from (28)-(31), it is straightforward to see that (27) is satisfied for $l \geq 1$, and the proof is now complete. ∎

In Lemma 4, it has been demonstrated that the decoding condition $\hat{x}_{ld} - \bar{x}_{ld} \in \mathcal{B}_{c(ld)}$ holds for all positive integers l. Subsequently, a sufficient condition for ensuring the detectability of (1) will be presented in the following theorem which imposes quantitative requirement on the network bandwidth.

Theorem 1: The nonlinear system (1) is detectable under the EDCM (22)-(24) if the following inequality

$$\frac{\gamma_1^d \sqrt{n_x}}{q} < 1 \tag{32}$$

holds where the parameters γ_1, d and q have been defined in Lemma 2, Lemma 3 and the uniform quantization scheme (7)-(8), respectively.

Proof: Considering the definition of $c(ld)$ and the fact that $0 < \gamma_2 < 1$, we see that $\lim_{l\to\infty} c(ld) = 0$. Then, it is inferred from (29) that

$$\lim_{l\to\infty} \|x_{ld} - \breve{x}_{ld}\|_2 = 0.$$

Moreover, noticing that x_k and $\breve{x}_k$ are actually the trajectories of (1) for those non-encoding instants $k \in (ld, (l+1)d)$, it follows from Lemma 2 that

$$\|x_k - \breve{x}_k\|_2^2 \leq \gamma_1^{k-ld} \|x_{ld} - \breve{x}_{ld}\|_2^2.$$

Consequently, we know that $\|x_k - \breve{x}_k\|_2$ is bounded at the non-encoding instants, which signifies that the nonlinear system (1) is detectable, namely, $\lim_{k\to\infty} \|x_k - \breve{x}_k\|_2 = 0$. The proof is now complete. ∎

In Theorem 1, a sufficient condition is provided for data reconfiguration which means that, if the condition $q > \gamma_1^d \sqrt{n_x}$ is satisfied, then the encoded data can be successfully restored to the true values. Therefore, for a certain q, the communication channel is required to be capable of transmitting $\lceil \log(n_x q + 1) + 1 \rceil$ bits of data at each time instant.

B. Decoder-based controller design

Lemma 5: [18] Assume that there exist an ISS-Lyapunov function $V(k, \rho_k) : [0, +\infty) \times \mathbb{R}^n \mapsto \mathbb{R}$, a $\mathcal{K}$ class function $\vartheta(\cdot)$ and three $\mathcal{K}_\infty$ class functions $\sigma_1(\cdot)$, $\sigma_2(\cdot)$ and $\sigma_3(\cdot)$ such that the following two inequalities

$$\sigma_1(\|\rho_k\|_2) \leq V(k, \|\rho_k\|_2) \leq \sigma_2(\|\rho_k\|_2)$$

and

$$V(k+1, \rho_{k+1}) - V(k, \rho_k) \leq -\sigma_3(\|\rho_k\|_2) + \vartheta(\|\nu_k\|_2)$$

hold for all $\rho_k \in \mathbb{R}^n$ and $\nu_k \in \mathbb{R}^p$. Then, the nonlinear discrete-time system (11) is input-to-state stable. Furthermore, the functions $\alpha(\cdot, \cdot)$ and $\beta(\cdot)$ in Definition 3 can be chosen as

$$\alpha(\cdot, k) = \sigma_1^{-1}(\psi^k \sigma_2(\cdot)), \quad 0 < \psi < 1$$

and

$$\beta(\cdot) = \sigma_1^{-1}\Big(\sigma_2\big(\sigma_3^{-1}(\vartheta(\cdot))\big)\Big)$$

where $\sigma_1^{-1}(\cdot)$ stands for the inverse function of the monotone function $\sigma_1(\cdot)$ and so does $\sigma_3^{-1}(\cdot)$.

According to the decoded state $\breve{x}_k$, the decoder-based controller is given as

$$u_k = K_c \breve{x}_k \tag{33}$$

where K_c is the controller gain to be determined.

Next, let $w_k \triangleq \breve{x}_k - x_k$ be the decoding error vector. In light of (33) and $\breve{x}_k = w_k + x_k$, the closed-loop system (1) is rewritten as

$$x_{k+1} = (E + BK_c)x_k + Df(x_k) + BK_c w_k. \tag{34}$$

On the basis of the detectability analysis, we know that the decoding error w_k is bounded. In this context, w_k can be regarded as a bounded input of (34) and, consequently, the ISS theory can be introduced to develop the encoding-decoding-based control scheme.

Theorem 2: Under the condition in Theorem 1, the decoder-based control system (34) is input-to-state stable if there exist positive definite matrices $Q > 0$ and $Z > 0$, matrices G_{11}, G_{12}, G_{22} and $\bar{K}_c$, and a positive scalar μ_3 satisfying

$$\tilde{\Pi} = \begin{bmatrix} -Q + \mu_3 U^T U & * & * & * \\ 0 & -\mu_3 I & * & * \\ 0 & 0 & -Z & * \\ G\Gamma E + \tilde{K}_c & G\Gamma D & \tilde{K}_c & \tilde{\Pi}_0 \end{bmatrix} < 0 \tag{35}$$

where

$$\tilde{\Pi}_0 \triangleq Q - G\Gamma - \Gamma^T G^T, \quad \Gamma \triangleq \begin{bmatrix} B(B^T B)^{-1} & (B^T)^\perp \end{bmatrix}^T,$$
$$G \triangleq \begin{bmatrix} G_{11} & G_{12} \\ 0 & G_{22} \end{bmatrix}, \quad \tilde{K}_c \triangleq \begin{bmatrix} \bar{K}_c \\ 0 \end{bmatrix}, \quad \bar{K}_c \triangleq G_{11} K_c.$$

Furthermore, the controller gain matrix is expressed as

$$K_c = G_{11}^{-1} \bar{K}_c.$$

Proof: Choosing the following ISS-Lyapunov function

$$V_k = x_k^T Q x_k,$$

we obtain

$$\begin{aligned} \triangle V_k &= V_{k+1} - V_k \\ &= [(E + BK_c)x_k + Df(x_k) + BK_c w_k]^T Q \\ &\quad \times [(E + BK_c)x_k + Df(x_k) + BK_c w_k] - x_k^T Q x_k \\ &\leq [(E + BK_c)x_k + Df(x_k) + BK_c w_k]^T Q \\ &\quad \times [(E + BK_c)x_k + Df(x_k) + BK_c w_k] \\ &\quad - x_k^T Q x_k - \mu_3 f^T(x_k) f(x_k) + \mu_3 x_k^T U^T U x_k \\ &= \eta_{3k}^T \Pi \eta_{3k} + w_k^T Z w_k \end{aligned}$$

where

$$\eta_{3k} \triangleq \begin{bmatrix} x_k^T & f^T(x_k) & w_k^T \end{bmatrix}^T,$$
$$\Pi \triangleq \begin{bmatrix} \Pi_{11} & * & * \\ \Pi_{21} & \Pi_{22} & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} \end{bmatrix},$$
$$\Pi_{11} \triangleq (E + BK_c)^T Q (E + BK_c) - Q + \mu_3 U^T U,$$
$$\Pi_{21} \triangleq D^T Q (E + BK_c), \quad \Pi_{22} \triangleq D^T Q D - \mu_3 I,$$
$$\Pi_{31} \triangleq (BK_c)^T Q (E + BK_c), \quad \Pi_{32} \triangleq (BK_c)^T Q D,$$
$$\Pi_{33} \triangleq (BK_c)^T Q (BK_c) - Z.$$

According to

$$\begin{aligned} &G\Gamma + \Gamma^T G^T - G\Gamma Q^{-1}\Gamma^T G^T - Q \\ = &-(G\Gamma - Q)Q^{-1}(G\Gamma - Q)^T \leq 0, \end{aligned}$$

one has

$$Q - G\Gamma - \Gamma^T G^T \geq -G\Gamma Q^{-1}\Gamma^T G^T. \tag{36}$$

In terms of Schur Complement Lemma and inequalities (35)-(36), we know that $\Pi < 0$. Consequently, it is clear that $V_{k+1} - V_k \leq -\lambda_{\min}(-\Pi)\|x_k\|_2^2 + \lambda_{\max}(Q)\|w_k\|_2^2$, and it is then inferred from Lemma 5 that the closed-loop system (34) is input-to-state stable by selecting

$$\begin{aligned} \vartheta(\|w_k\|_2) &= \lambda_{\max}(Z)\|w_k\|_2^2, \\ \sigma_1(\|x_k\|_2) &= \lambda_{\min}(Q)\|x_k\|_2^2, \\ \sigma_2(\|x_k\|_2) &= \lambda_{\max}(Q)\|x_k\|_2^2, \\ \sigma_3(\|x_k\|_2) &= \lambda_{\min}(-\Pi)\|x_k\|_2^2. \end{aligned}$$

Letting

$$\begin{aligned} \alpha(\|x_0\|_2, k) &= \sqrt{\frac{\psi^k \lambda_{\max}(Q)\|\epsilon_0\|_2^2}{\lambda_{\min}(Q)}}, \\ \beta(\|w_k\|_2) &= \sqrt{\frac{\lambda_{\max}(Q)\lambda_{\max}(Z)\|w_k\|_2^2}{\gamma\lambda_{\min}(Q)\lambda_{\min}(-\Pi)}} \end{aligned}$$

with $0 < \gamma < 1$, we have from Definition 3 that

$$\|x_k\|_2 \leq \alpha(\|x_0\|_2, k) + \beta(\|w_k\|_2), \tag{37}$$

which completes the proof. ■

Remark 4: It can be seen from (32) that there is a clear trade-off between the encoding period d, the number of the quantization intervals q and the error convergence indicator γ_1. It is often the case that the channel capacity is limited, and therefore adjusting d and q won't be a preferred option. Fortunately, the parameter γ_1 can be made as small as necessary to meet (32) as long as γ_1 lies within the interval $(0, 1)$ subject to (14).

Remark 5: In view of the analysis results presented in Theorem 1, the decoding error $\zeta(k)$ can be viewed as the exogenous bounded inputs. As such, in Theorem 2, an effective decoder-based control scheme has been designed such that the ISS of the system (34) can be guaranteed. In addition, in order to solve the controller gain design problem, the orthogonal decomposition is employed. We introduce a free matrix G with a unique structure and construct a matrix $\Gamma = \begin{bmatrix} B(B^TB)^{-1} & (B^T)^{\perp} \end{bmatrix}^T$ so as to cope with the coupling term QBK_c in Theorem 2.

Remark 6: Until now, the outlier-resistant observer-based control problem has been tackled for a class of NSs under the EDCM. The distinctive features with our main results are outlined as follows: 1) the negative effects from the measurement outliers are reduced by constructing a dedicated outlier-resistant observer; 2) the EDCM is employed for data transmission in order to reduce communication resource occupation and enhance the data security; 3) the interplay between the network bandwidth, the encoding accuracy and the error convergence is quantitatively analyzed; and 4) the existence condition of the decoder-based controller is parameterized by means of the solution to a certain matrix inequality.

IV. Numerical Example

In this section, to emphasize the effectiveness of the proposed outlier-resistant observer-based controller design scheme, we consider the following discrete-time nonlinear system (1) with parameters given by:

$$E = \begin{bmatrix} 1.05 & -1.0 \\ 0.05 & -0.8 \end{bmatrix}, \quad D = \begin{bmatrix} 0.2 & -0.6 \\ 0.2 & 0.4 \end{bmatrix},$$

$$B = \begin{bmatrix} -1.2 \\ -0.8 \end{bmatrix}, \quad N = \begin{bmatrix} 1.85 & -0.4 \end{bmatrix}.$$

The nonlinear function is selected as

$$f(x_k) = \begin{bmatrix} 0.05x_{1,k} - \tanh(0.05x_{1,k}) \\ 0.2x_{2,k} \end{bmatrix}.$$

It is readily seen that (2) is satisfied with $U = \mathrm{diag}\{0.1, 0.1\}$. The saturation function $\sigma(Ne_k)$ is described as follows:

$$\sigma(Ne_k) = \begin{cases} Ne_k, & \text{if } -e_{\max} \leq Ne_k \leq e_{\max}, \\ e_{\max}, & \text{if } \quad Ne_k \geq e_{\max}, \\ -e_{\max}, & \text{if } \quad Ne_k \leq -e_{\max} \end{cases}$$

where the saturation value is taken as $e_{\max} = 0.1$.

Example 1: The aim of this example is to verify the detectability of the nonlinear system (1). In this case, it is generally assumed that $u_k = 0$. Choosing $\varrho_1 = 0.8$, $\varrho_2 = 0.9$ and $d = 3$, we solve the matrix inequalities (15) and (18) to obtain $\gamma_1 = 3.4267$ and $\gamma_2 = 0.3423$. Moreover, the observer gain matrix K_e is calculated as $K_e = \begin{bmatrix} -1.4002 & 0.0917 \end{bmatrix}^T$. The simulation results are displayed in Figs. 2-5, where Fig. 2 characterizes the abnormal disturbances added to y_k, Figs. 3-4 plot the actual states of $x_{1,k}$ and $x_{2,k}$, their estimates $\hat{x}_{1,k}$ and $\hat{x}_{2,k}$, and their decoded values $\breve{x}_{1,k}$ and $\breve{x}_{2,k}$, and Fig. 5 depicts the corresponding decoding errors $w_{1,k}$ and $w_{2,k}$. It is easy to see from Fig. 5 that the errors between the actual states of the network and their decoded states asymptotically approach zero, which indicates that the desired detectability performance of the addressed NSs is well attained.

Example 2: The second example is given to test the validity of the designed decoder-based controller for the nonlinear system (1). The control input signal is taken as the form of (33). By solving (35) in Theorem 2, we obtain the desired controller parameter as $K_c = G_{11}^{-1}\bar{K}_c = \begin{bmatrix} 0.6286 & -0.8525 \end{bmatrix}$.

As stated in Theorem 2, the considered NS should achieve the ISS with the designed controller parameters given above. Such a theoretical result is confirmed by the simulation results presented in Figs. 6-8.

Example 3: In the third example, to further illustrate the superiority of our proposed outlier-resistant observer, a traditional observer is given as follows for the purpose of comparison:

$$\begin{cases} \hat{x}_{k+1} = E\hat{x}_k + Df(\hat{x}_k) + Bu_k + \tilde{K}_e(y_k - H\hat{x}_k) \\ \hat{x}_0 = 0 \end{cases} \tag{38}$$

where $\tilde{K}_e$ is the observer gain matrix.

Based on the traditional observer, the simulation results are exhibited in Figs. 9-11. It is obvious to see that the traditional observer is no longer effective with the appearance of measurement outliers, and this further shows the advantage of our proposed theoretical results.

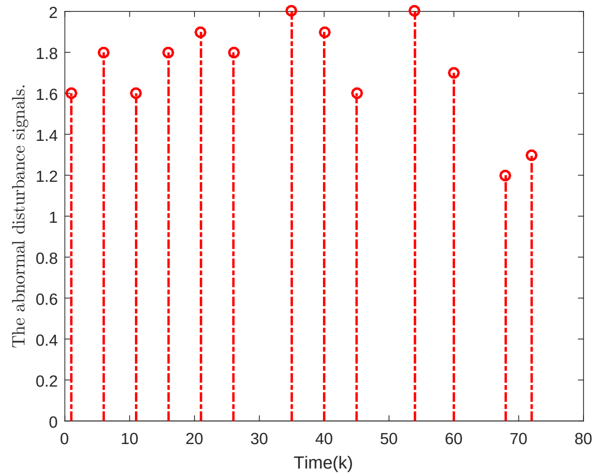

Fig. 2. The abnormal disturbance signals

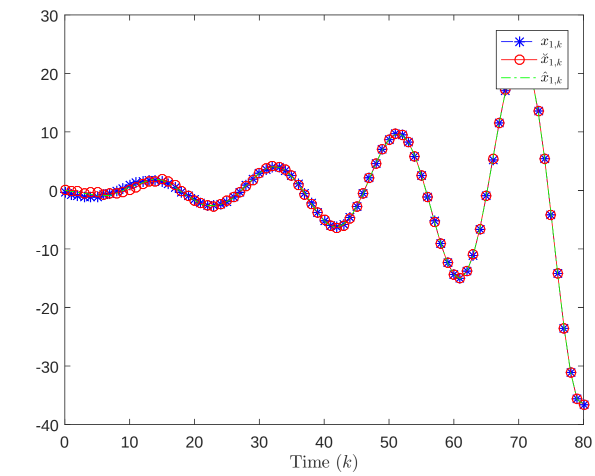

Fig. 3. $x_{1,k}$, its estimate $\hat{x}_{1,k}$ and its decoding value $\breve{x}_{1,k}$ with innovation constraint ($u_k = 0$)

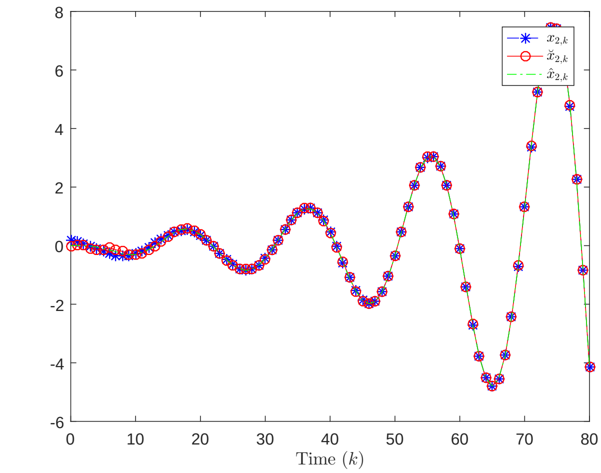

Fig. 4. $x_{2,k}$, its estimate $\hat{x}_{2,k}$ and its decoding value $\breve{x}_{2,k}$ with innovation constraint ($u_k = 0$)

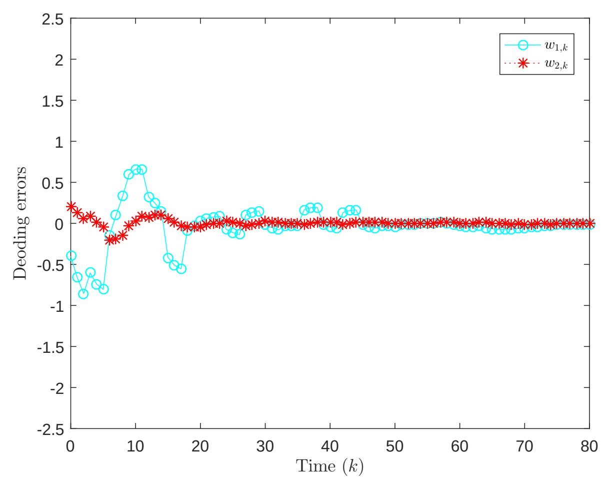

Fig. 5. Decoding errors $w_{1,k}$ and $w_{2,k}$ with innovation constraint ($u_k = 0$)

V. Conclusions

In this paper, considering the appearance of measurement outliers, the outlier-resistant observer-based control problem has been tackled for a class of NSs under the EDCM. A specific saturation function has been introduced in the state observer to mitigate the negative effects of measurement outliers on the error dynamics of the observation. With the help of the uniform quantization technique, the EDCM has been utilized in the observer-to-controller channel to realize the data compression and therefore reduce the communication resource occupation and enhance the data security. Based on the decoded data, an observer-based control scheme has been put forward. In terms of the solution to a certain matrix inequality constraint, a sufficient condition has been obtained such that the ISS of the closed-loop system can be guaranteed. Finally, three numerical examples have been conducted to verify the effectiveness and superiority of the proposed outlier-resistant observer-based control scheme. As a future research topic, the encoding-decoding-based estimation problem deserves further investigation especially for more complicated systems [11], [19], [39].

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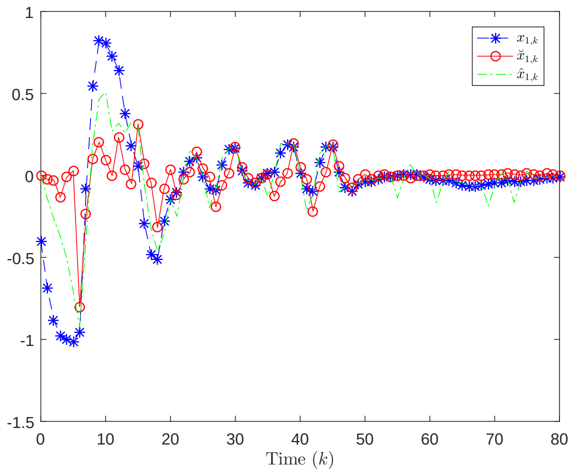

Fig. 6. $x_{1,k}$, its observed value $\hat{x}_{1,k}$ and its decoding value $\breve{x}_{1,k}$ with innovation constraint ($u_k \neq 0$)

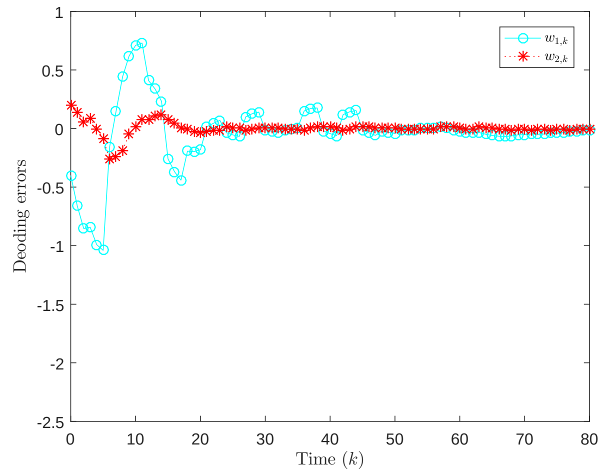

Fig. 8. Decoding errors $w_{1,k}$ and $w_{2,k}$ with innovation constraint ($u_k \neq 0$)

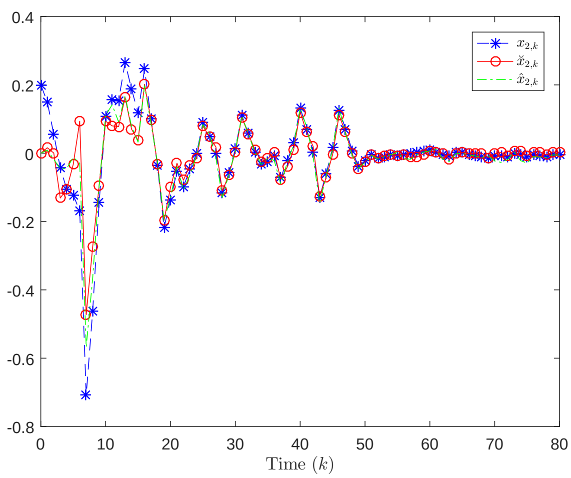

Fig. 7. $x_{2,k}$, its observed value $\hat{x}_{2,k}$ and its decoding value $\breve{x}_{2,k}$ with innovation constraint ($u_k \neq 0$)

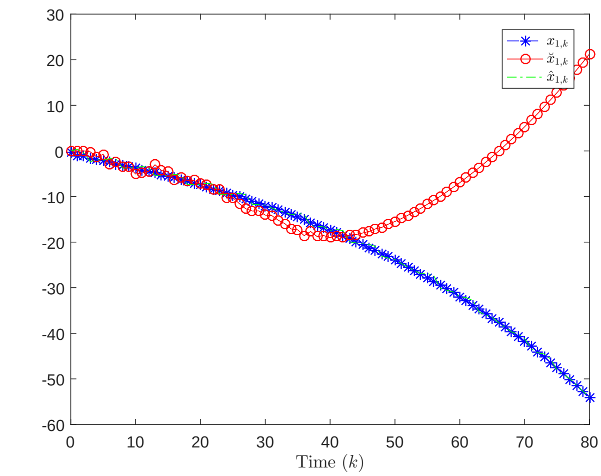

Fig. 9. $x_{1,k}$, its observed value $\hat{x}_{1,k}$ and its decoding value $\breve{x}_{1,k}$ without innovation constraint ($u_k \neq 0$)

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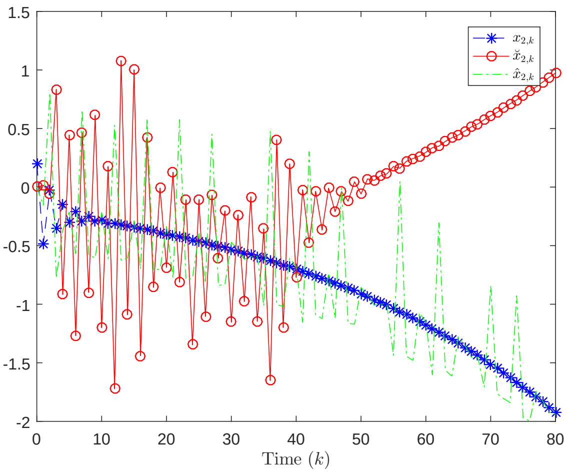

Fig. 10. $x_{2,k}$, its observed value $\hat{x}_{2,k}$ and its decoding value $\breve{x}_{2,k}$ without innovation constraint ($u_k \neq 0$)

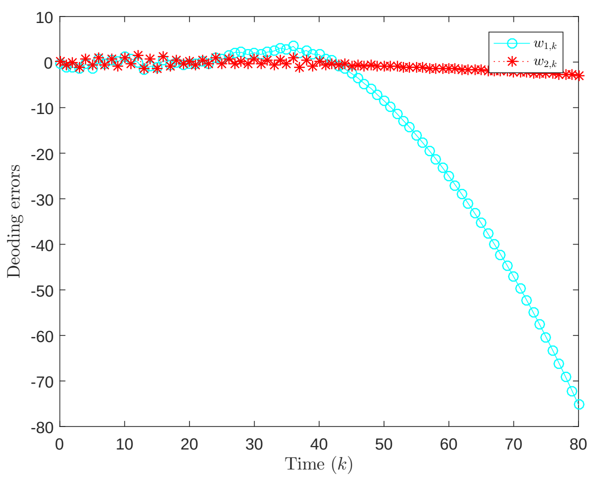

Fig. 11. Decoding errors $w_{1,k}$ and $w_{2,k}$ without innovation constraint ($u_k \neq 0$)

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