Robust Fusion Filtering Over Multisensor Systems with Energy Harvesting Constraints ⋆

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Abstract

In this paper, a general theoretical framework is established for the robust fusion filtering problem of discrete time-varying stochastic multisensor systems under energy harvesting constraints. The energy harvesting technology is utilized to provide the needed energy for persistently maintaining the operation of the multisensor systems. The energy level at the energy harvester is characterized by a random variable obeying a certain probability distribution. For the communication between sensors and filters, we consider a scenario where the measurements received by sensors are broadcasted via networks and then obtained by filters according to a set of preassigned communication links. The aim of this paper is to design the fusion filter over a multisensor system with locally minimized variance of the estimation error. Specifically, the local filter is firstly designed such that, in the presence of energy harvesting constraints and parameter uncertainties, an upper bound on the filtering error covariance is guaranteed and subsequently minimized by appropriately choosing the filter parameters at each time instant. Then, all the local estimates obtained by local filters are fused by using the covariance intersection fusion strategy for fusion estimation purposes. Finally, an illustrative simulation is carried out to demonstrate the usefulness of the proposed fusion filtering scheme.

Key words: Covariance intersection fusion; Energy harvesting sensors; Fusion filtering; Robust estimation; Wireless communications.

1 Introduction

Along with the increasing popularity of the multisensor systems, the multi-sensor data fusion technology has received more and more attention due to its obvious advantages in improving the system reliability and robustness, enhancing the data credibility as well as increasing the information utilization efficiency [1, 6, 18, 22, 30, 34]. In fact, a large number of multi-sensor data fusion schemes have been developed with successful applications in many military and civilian fields such as inertial navigation, traffic control and marine surveillance and management. As a key issue in multi-sensor data fusion, the distributed fusion filtering/estimation problem has attracted considerable research interest and, in the past few years, a variety of distributed fusion filtering/estimation algorithms have been proposed, see. e.g [2, 5, 11, 20, 21, 23, 42–44]. For example, in [4, 36], the distributed Kalman fusion filtering schemes have been put forward over the multisensor systems and, in [15], the distributed particle fusion filtering approach has been proposed via the optimal fusion of Gaussian mixtures. In [41], an ellipsoidal fusion estimation method has been provided to deal with the bounded noises and, in [3], a distributed mixed H_2/H_∞ fusion estimator has been designed to handle the energy-bounded noises.

Parameter uncertainties are often encountered in practical engineering for various reasons such as modeling errors and inaccurate measurements, which would have a significant impact on the system performance. In the past few decades, there has been an enormous research effort directed towards the robust control/filtering problems with parameter uncertainties [10, 16, 19, 29, 38]. For example, in [24], the robust control problem has been investigated for systems with unknown norm-bounded parameter uncertainties while, in [8, 35, 39, 45], the robust filters have been analyzed/designed for uncertain systems. In particular, in [39], an online robust filtering algorithm has been proposed for uncertain discrete-time stochastic systems by using matrix decomposition approach, where the derived estimator minimizes an upper bound on the variance of the estimation error. Nevertheless, in the context of *fusion* filtering/estimation, the

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phenomena of parameter uncertainties have been largely overlooked and the corresponding results on the multi-sensor robust fusion filtering problems have been relatively few.

Energy supply is an indispensable component when monitoring and controlling a system simply because the information transmission/processing do require adequate energy consumption, and this is particularly true for multisensor systems where the information transmissions among the large number of sensor nodes demand large amounts of energy. As such, it becomes imperative to have effective energy replenishing schemes for energy collection/storage so as to maintain the normal operation of the overall network. Energy harvesting technology, which serves as an ideal solution for green energy supply, has recently attracted much attention from both academy and industry [7, 9, 26, 28] especially the control community. For example, in [13, 27], the power control problems of harvesting sensor have been investigated in the framework of remote state estimation. In [14, 25], several transmission schemes have been designed for state estimation and control with an energy harvesting sensor. In [31], the finite-horizon filter has been designed for nonlinear time-delayed systems with an energy harvesting sensor.

Apparently, it makes practical sense to consider the distributed fusion filtering problem over multisensor systems subject to energy harvesting constraints (EHCs), which looks to be particularly challenging due primarily to the fundamental difficulties brought from the introduction of the energy harvesting technology. For example, we are unavoidably confronted with the following issues: 1) how can we formulate the distributed fusion filtering problem with multiple sensors equipped with the energy harvesters? 2) how can we deal with the complex information communications within the multisensor systems with the EHCs? 3) how can we develop an effective distributed fusion filtering approach such that the desired fusion estimation error is achieved under the EHCs? It is, therefore, the primary motivation in this paper to provide satisfactory answers to the aforementioned three questions by designing a set of distributed fusion filters over multisensor systems with EHCs.

In the light of the discussions made above, in this paper, we are set to investigate the robust distributed fusion filtering problem for a class of discrete time-varying stochastic uncertain systems over multisensor systems with EHCs. *The main contributions of this paper are summarized as follows: 1) we make the first attempt to introduce the energy harvesting technology in the distributed filtering problem, and examine the effects from both EHCs and parameter uncertainties onto the multi-sensor-based fusion filtering performance; 2) the level of received energy at the energy harvester is characterized by a random variable obeying a certain probability distribution, and the parameter uncertainties are described by unknown but norm-bounded matrices; and 3) in the presence of EHCs and parameter uncertainties, the local filter is first designed to ensure the existence of an upper bound on the filtering error covariance that is subsequently minimized by choosing the filter parameters, and then all the local estimates obtained by local filters are fused by the covariance intersection (CI) fusion strategy.* Finally, an illustrative example is provided to verify the effectiveness of the proposed fusion filtering scheme.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ denotes the n dimensional Euclidean space. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). M^T represents the transpose of the matrix M. $\mathrm{tr}(A)$ means the trace of the matrix A. I denotes the identity matrix of compatible dimension and $\bar{I}$ means a square matrix of compatible dimension with all elements being 1. $\mathrm{diag}\{\dots\}$ stands for a block-diagonal matrix. $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $\mathrm{Prob}\{\cdot\}$ means the occurrence probability of the event "$\cdot$". Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

2 Problem Formulation

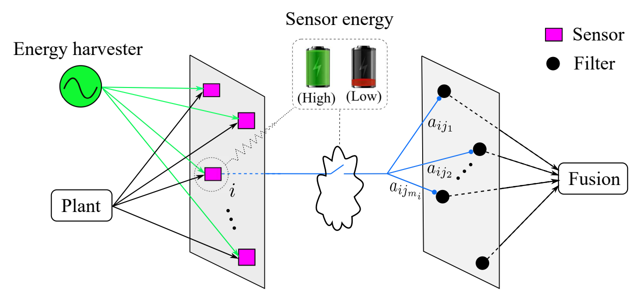

Fig. 1. Distributed fusion filtering structure.

A schematic diagram of the addressed distributed fusion filtering problem is given in Fig. 1, where the M sensors are responsible for sensing the measurements from the plant and broadcasting the measurements in the network. M filters, based on the received measurements, produce estimates separately for the plant state. Then, all the estimates produced by filters are fused in the fusion center. For the communications between sensors and filters, we consider the case where the measurement broadcasted by a sensor is received by filters according to a set of given communication links. The given communication links are represented by a scalar $a_{ij} \in \{0, 1\}$, where $a_{ij} = 1$ indicates that there is a communication channel between sensor i and filter j, and $a_{ij} = 0$ otherwise. Thus, the set of sensors from which filter i receives the measurements can be denoted by $\mathcal{M}_i \triangleq \{j \in \{1, 2, \cdots, M\} : a_{ji} = 1\}$.

The plant in Fig. 1 is described by the following class of discrete time-varying stochastic systems:

$$x_{k+1} = (A_k + \Delta A_k)x_k + E_k w_k \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the system state, $w_k \in \mathbb{R}^q$ is the process noise, A_k and E_k are known time-varying matrices with appropriate dimensions, and ΔA_k is an unknown matrix representing the parameter uncertainty. The initial value x_0 is a random variable with mean $\bar{x}_0$ and covariance X_0.

The parameter uncertainty matrix ΔA_k satisfies the following condition:

$$\Delta A_k = H_k O_k N_k \tag{2}$$

where O_k satisfies $O_kO_k^T \leq I$ and H_k and N_k are known time-varying matrices with appropriate dimensions.

For each sensor i ($i = 1, 2, \cdots, M$), the measurement models is expressed by

$$y_{i,k} = C_{i,k}x_k + v_{i,k} \tag{3}$$

where $y_{i,k} \in \mathbb{R}^p$ is the measurement sensed by sensor i, $v_{i,k} \in \mathbb{R}^p$ is the measurement noise, $C_{i,k}$ is a known time-varying matrix with appropriate dimensions.

The noises w_k and $v_{i,k}$ are represented by zero-mean white noises with covariances W and V_i, respectively. In this paper, the stochastic variables w_k, $v_{i,k}$ ($i = 1, 2, \cdots, M$), and x_0 are assumed to be mutually independent.

The measurement after being broadcasted by sensor i, denoted as $\bar{y}_{i,k}$, is expressed by

$$\bar{y}_{i,k} = \gamma_{i,k}y_{i,k} \tag{4}$$

where $\gamma_{i,k}$ is defined by

$$\gamma_{i,k} \triangleq \begin{cases} 1, \ z_{i,k} > 0, \\ 0, \ z_{i,k} = 0, \end{cases} \tag{5}$$

and $z_{i,k}$ is the energy level of sensor i at time instant k.

It is seen from (4) and (5) that the current energy status of sensor i determines whether or not the measurement sensed by sensor i is successfully broadcasted. In this paper, we consider the situation that each sensor broadcasts a measurement with 1 unit energy consumption if the energy level that the sensor stores is nonzero. Moreover, at each time instant, an extra energy is harvested from the environments and then stored in the sensor.

Let the set of energy levels of sensor i be denoted by $\{0, 1, 2 \cdots, S_i\}$ where S_i is the maximum number of energy units that sensor i can store. Then, the evolution of the amount of energy in the sensor i can be described by

$$z_{i,k+1} = \min\{z_{i,k} + h_{i,k} - \gamma_{i,k}, S_i\} \tag{6}$$

with the initial condition $z_{i,0} = \bar{z}_i \leq S_i$. Here, $h_{i,k}$ is the amount of energy harvested by sensor i at time instant k, which is an independently identically distributed random process with the following probability distribution

$$\text{Prob}(h_{i,k} = m) = p_m, \quad m = 0, 1, 2, \cdots, \tag{7}$$

where p_m satisfies $0 \leq p_m \leq 1$ and $\sum_{m=0}^{+\infty} p_m = 1$. Also, $h_{i,k}$ ($i = 1, 2, \cdots, M$) and other stochastic variables mentioned previously are mutually independent.

Remark 1 *It is seen from (5) and (6) that the random variable $\gamma_{i,k}$ is correlated with $\gamma_{i,l}$ ($l < k$), which makes the measurement model (4) distinguish from the traditional missing measurement model.*

Remark 2 *In the practical application, the probability distribution p_m can be obtained through the statistical methods on the sufficient energy data collected from the energy sources. In order to cater to more practical applications, the probability distribution (7) could be generalized to Prob$(h_{i,k} = m) = p_{i,m}$ and the corresponding fusion filtering method can be easily obtained by simply replacing p_m with $p_{i,m}$.*

For filter i, we adopt the following structure:

$$\begin{cases} \hat{x}_{i,k+1} = F_{i,k}\hat{x}_{i,k} + \sum_{j \in \mathcal{M}_i} G_k^{ij}\bar{y}_{j,k}, \\ \hat{x}_{i,0} = 0 \end{cases} \tag{8}$$

where $\hat{x}_{i,k} \in \mathbb{R}^n$ is the state estimate from filter i, $F_{i,k}$ and G_k^{ij} ($j \in \mathcal{M}_i$) are filter parameters to be determined. From the definition of the set $\mathcal{M}_i$, it can be seen that the effect of scalar a_{ij} is embodied in the set $\mathcal{M}_i$.

Denote by m_i the element number of the set $\mathcal{M}_i$ and reorder the elements in $\mathcal{M}_i$ as $\{j_1, j_2, \cdots, j_{m_i}\}$. Then, the information related to filter i can be organized as follows:

$$\begin{aligned} v_k^i &= \left[v_{j_1,k}^T \ v_{j_2,k}^T \ \cdots \ v_{j_{m_i},k}^T \right]^T, \\ G_k^i &= \left[G_k^{ij_1} \ G_k^{ij_2} \ \cdots \ G_k^{ij_{m_i}} \right], \\ C_k^i &= \left[C_{j_1,k}^T \ C_{j_2,k}^T \ \cdots \ C_{j_{m_i},k}^T \right]^T, \\ \Lambda_k^i &= \text{diag}\{\gamma_{j_1,k}I, \gamma_{j_2,k}I, \cdots, \gamma_{j_{m_i},k}I\}, \end{aligned}$$

and the filter i can be further expressed by

$$\hat{x}_{i,k+1} = F_{i,k}\hat{x}_{i,k} + G_k^i\Lambda_k^iC_k^ix_k + G_k^i\Lambda_k^iv_k^i. \tag{9}$$

Remark 3 *Note that the estimates derived from different filters according to (9) may contain the measurements from identical sensors. Due to the complicated networked communications and the randomness from the energy harvested, it is usually difficult to obtain the cross-covariance matrix of the estimation error for different estimates. In this case, we adopt the CI fusion strategy proposed in [12].*

Suppose that the estimation error covariance of the estimate $\hat{x}_{i,k}$ is bounded by a positive definite matrix $\bar{P}_{i,k}$. Then, the CI fusion criterion can be given by the following convex combination

$$\hat{x}_k^F = \bar{P}_k^F \sum_{i=1}^{M} \nu_{i,k}\bar{P}_{i,k}^{-1}\hat{x}_{i,k} \tag{10}$$

where

$$\bar{P}_k^F \triangleq \left(\sum_{i=1}^{M} \nu_{i,k}\bar{P}_{i,k}^{-1} \right)^{-1} \tag{11}$$

and $\nu_{i,k}$ ($i = 1, 2, \cdots, M$) are weighting coefficients satisfying $0 \leq \nu_{i,k} \leq 1$ and $\sum_{i=1}^{M} \nu_{i,k} = 1$.

We are now in a position to state the robust distributed filtering fusion problem as follows:

a) for each filter i ($i = 1, 2, \cdots, M$) of the structure (8), find an upper bound $\bar{P}_{i,k}$ for the estimation error covariance of the estimate $\hat{x}_{i,k}$, and then look for filter parameters $F_{i,k}$ and G_k^{ij} ($j \in \mathcal{M}_i$) such that the obtained upper bound $\bar{P}_{i,k}$ is minimized; and

b) fuse all the obtained estimates according to the CI fusion criterion (10) with (11). That is, based on the derived estimates $\hat{x}_{i,k}$ $(i = 1, 2, \cdots, M)$ and the locally minimized upper bound of its estimation error covariance $\bar{P}_{i,k}$ $(i = 1, 2, \cdots, M)$, determine the parameters $\nu_{i,k}$ $(i = 1, 2, \cdots, M)$ such that the trace of the fused covariance matrix $\bar{P}_k^F$ is minimized.

3 Main Results

3.1 The Computation of the Probability Distribution of Random Variable $\gamma_{i,k}$

It is known from (5) that the probability distribution of random variable $\gamma_{i,k}$ is closely related to that of the energy level $z_{i,k}$. Hence, let's start with the computation of the probability distribution of the energy level $z_{i,k}$.

Denote the probability distribution of the energy level $z_{i,k}$ by $\mathbf{p}_{i,k} \triangleq \begin{bmatrix} p_{i,k}^0 & p_{i,k}^1 & \cdots & p_{i,k}^{S_i} \end{bmatrix}^T$ where $p_{i,k}^m \triangleq \text{Prob}(z_{i,k} = m)$ for each $m = 0, 1, \cdots, S_i$. It is seen from (6) that the energy level $z_{i,k}$ is independent of $h_{i,k}$ and hence, for each $m = 0, 1, \cdots, S_i - 1$, one has

$$\begin{aligned} p_{i,k+1}^m =& \text{Prob}(z_{i,k+1} = m) \\ =& \text{Prob}(\min\{z_{i,k} + h_{i,k} - \gamma_{i,k}, S_i\} = m) \\ =& \text{Prob}(z_{i,k} + h_{i,k} - \gamma_{i,k} = m) \\ =& \text{Prob}(z_{i,k} = 0, h_{i,k} = m) \\ &+ \sum_{l=1}^{m+1} \text{Prob}(z_{i,k} = l, h_{i,k} = m + 1 - l) \\ =& \text{Prob}(z_{i,k} = 0)\text{Prob}(h_{i,k} = m) \\ &+ \sum_{l=1}^{m+1} \text{Prob}(z_{i,k} = l)\text{Prob}(h_{i,k} = m + 1 - l) \\ =& p_{i,k}^0 p_m + \sum_{l=1}^{m+1} p_{i,k}^l p_{m+1-l}. \end{aligned} \tag{12}$$

Then, the probability $p_{i,k+1}^{S_i}$ can be expressed by

$$\begin{aligned} p_{i,k+1}^{S_i} &= 1 - \sum_{m=1}^{S_i-1} p_{i,k+1}^m \\ &= 1 - p_{i,k}^0 \sum_{m=1}^{S_i-1} p_m - \sum_{m=1}^{S_i-1} \sum_{l=1}^{m+1} p_{i,k}^l p_{m+1-l}. \end{aligned} \tag{13}$$

By combining (12) and (13), a recursion for the probability distribution $\mathbf{p}_{i,k}$ is obtained as follows

$$\mathbf{p}_{i,k+1} = \mathbf{r}_i + \mathbf{M}_i \mathbf{p}_{i,k} \tag{14}$$

where

$$\mathbf{r}_i \triangleq [\underbrace{0 \cdots 0}_{S_i} \quad 1]^T,$$

$$\mathbf{M}_i \triangleq \begin{bmatrix} p_0 & p_0 & 0 & \cdots & 0 \\ p_1 & p_1 & p_0 & \cdots & 0 \\ p_2 & p_2 & p_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_{S_i-1} & p_{S_i-1} & p_{S_i-2} & \cdots & p_0 \\ -\sum_{m=0}^{S_i-1} p_m & -\sum_{m=0}^{S_i-1} p_m & -\sum_{m=0}^{S_i-2} p_m & \cdots & -p_0 \end{bmatrix}.$$

According to the recursion (14), a computation method for the probability distribution of $\gamma_{i,k}$ is provided in the following lemma.

Lemma 1 *The random variable $\gamma_{i,k}$ defined by (5) obeys the following probability distribution*

$$\textit{Prob}(\gamma_{i,k} = 1) = \bar{\gamma}_{i,k}, \quad \textit{Prob}(\gamma_{i,k} = 0) = 1 - \bar{\gamma}_{i,k} \tag{15}$$

where

$$\bar{\gamma}_{i,k} \triangleq 1 - \mathbf{a}_i \mathbf{p}_{i,k}, \quad \mathbf{a}_i \triangleq [1 \quad \underbrace{0 \cdots 0}_{S_i}], \tag{16}$$

and $\mathbf{p}_{i,k}$ is recursively obtained by (14).

Proof: From (5), one immediately has

$$\text{Prob}(\gamma_{i,k} = 0) = \text{Prob}(z_{i,k} = 0) = \mathbf{a}_i \mathbf{p}_{i,k},$$
$$\text{Prob}(\gamma_{i,k} = 1) = 1 - \text{Prob}(z_{i,k} = 0) = 1 - \mathbf{a}_i \mathbf{p}_{i,k}.$$

Then, the proof of this lemma follows readily. □

3.2 The Design of Filters

Setting $\tilde{x}_{i,k} \triangleq \begin{bmatrix} x_k^T & \hat{x}_{i,k}^T \end{bmatrix}^T$, we have from (1) and (9) that

$$\tilde{x}_{i,k+1} = (\tilde{A}_{i,k} + \tilde{H}_k O_k \tilde{N}_k)\tilde{x}_{i,k} + \tilde{w}_{i,k} + \tilde{C}_{i,k}^{\gamma} \tilde{x}_{i,k} \tag{17}$$

with the initial value $\tilde{x}_{i,k} = \begin{bmatrix} x_0^T & 0 \end{bmatrix}^T$, where

$$\begin{aligned} &\tilde{A}_{i,k} \triangleq \begin{bmatrix} A_k & 0 \\ G_k^i \bar{\Lambda}_k^i C_k^i & F_{i,k} \end{bmatrix}, \quad \tilde{H}_k \triangleq \begin{bmatrix} H_k \\ 0 \end{bmatrix}, \\ &\tilde{N}_k \triangleq \begin{bmatrix} N_k & 0 \end{bmatrix}, \quad \tilde{w}_{i,k} \triangleq \begin{bmatrix} E_k w_k \\ G_k^i \Lambda_k^i v_k^i \end{bmatrix}, \\ &\tilde{C}_{i,k}^{\gamma} \triangleq \begin{bmatrix} 0 & 0 \\ G_k^i (\Lambda_k^i - \bar{\Lambda}_k^i) C_k^i & 0 \end{bmatrix}, \\ &\bar{\Lambda}_k^i \triangleq \text{diag}\{\bar{\gamma}_{j_1,k} I, \bar{\gamma}_{j_2,k} I, \cdots, \bar{\gamma}_{j_{m_i},k} I\}. \end{aligned} \tag{18}$$

In order to derive the evolution of the covariance matrix $\tilde{X}_{i,k} \triangleq \mathbb{E}\{\tilde{x}_{i,k} \tilde{x}_{i,k}^T\}$, the following terms are calculated:

$$\begin{aligned} \tilde{W}_{i,k} &\triangleq \mathbb{E}\{\tilde{w}_{i,k} \tilde{w}_{i,k}^T\} \\ &= \text{diag}\left\{ E_k W E_k^T, G_k^i (\bar{\Lambda}_k^i \odot \tilde{V}^i) G_k^{iT} \right\}, \\ R_k^i(\tilde{X}_{i,k}) &\triangleq \mathbb{E}\{\tilde{C}_{i,k}^{\gamma} \tilde{x}_{i,k} \tilde{x}_{i,k}^T \tilde{C}_{i,k}^{\gamma T}\} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & G_k^i \big(\tilde{\Lambda}_k^i \odot (C_k^i Z_2 \tilde{X}_{i,k} Z_2^T C_k^{iT})\big) G_k^{iT} \end{bmatrix} \end{aligned} \tag{19}$$

where

$$Z_2 \triangleq \begin{bmatrix} I & 0 \end{bmatrix}, \quad \tilde{V}^i \triangleq \text{diag}\{V_{j_1}, V_{j_2}, \cdots, V_{j_{m_i}}\},$$
$$\tilde{\Lambda}_k^i \triangleq \text{diag}\{\bar{\gamma}_{j_1,k}(1-\bar{\gamma}_{j_1,k})\bar{I}, \bar{\gamma}_{j_2,k}(1-\bar{\gamma}_{j_2,k})\bar{I}, \cdots, \bar{\gamma}_{j_{m_i}}(1-\bar{\gamma}_{j_{m_i},k})\bar{I}\}.$$

Then, the difference equation that the covariance matrix $\tilde{X}_{i,k}$ satisfies is given by

$$\begin{aligned} &\tilde{X}_{i,k+1} \\ =&(\tilde{A}_{i,k} + \tilde{H}_k O_k \tilde{N}_k)\tilde{X}_{i,k}(\tilde{A}_{i,k} + \tilde{H}_k O_k \tilde{N}_k)^T + \tilde{W}_k \\ &+ R_k^i(\tilde{X}_{i,k}) + \mathbb{E}\{(\tilde{A}_{i,k} + \tilde{H}_k O_k \tilde{N}_k)\tilde{x}_{i,k}\tilde{x}_{i,k}^T \tilde{C}_{i,k}^{\gamma T}\} \\ &+ \mathbb{E}\{(\tilde{A}_{i,k} + \tilde{H}_k O_k \tilde{N}_k)\tilde{x}_{i,k}\tilde{x}_{i,k}^T \tilde{C}_{i,k}^{\gamma T}\}^T \end{aligned} \tag{20}$$

with initial value $\tilde{X}_{i,0} = \text{diag}\{X_0, 0\}$.
By using the elementary inequality $ab^T + ba^T \leq \varepsilon aa^T + \varepsilon^{-1}bb^T$ where a and b are vectors of compatible dimensions and ε is a positive scalar, it is immediately obtained that

$$\tilde{X}_{i,k+1} \leq \mathscr{F}_k(\tilde{X}_{i,k}) \tag{21}$$

where

$$\begin{aligned} &\mathscr{F}_k(\tilde{X}_{i,k}) \\ \triangleq&(1+\varepsilon_{i,k})(\tilde{A}_{i,k} + \tilde{H}_k O_k \tilde{N}_k)\tilde{X}_{i,k}(\tilde{A}_{i,k} + \tilde{H}_k O_k \tilde{N}_k)^T \\ &+ (1+\varepsilon_{i,k}^{-1})R_k^i(\tilde{X}_{i,k}) + \tilde{W}_k. \end{aligned} \tag{22}$$

The monotonicity of the matrix function $\mathscr{F}_k(\cdot)$ defined by (22) is discussed in the following lemma.

Lemma 2 *For any positive definite matrices X and Y as well as the matrix function $\mathscr{F}_k(\cdot)$ defined by (22), one has $\mathscr{F}_k(X) \leq \mathscr{F}_k(Y)$ if $X \leq Y$.*

Proof: According to (19), we have

$$\begin{aligned} &R_k^i(\tilde{X}_{i,k}) \\ =&\begin{bmatrix} 0 & 0 \\ 0 & G_k^i \mathbb{E}\left\{(\Lambda_k^i - \bar{\Lambda}_k^i)C_k^i x_k x_k^T C_k^{iT}(\Lambda_k^i - \bar{\Lambda}_k^i)\right\} G_k^{iT} \end{bmatrix}. \end{aligned}$$

Note that

$$\begin{aligned} &\mathbb{E}\left\{(\Lambda_k^i - \bar{\Lambda}_k^i)C_k^i x_k x_k^T C_k^{iT}(\Lambda_k^i - \bar{\Lambda}_k^i)\right\} \\ =&\mathbb{E}\left\{\mathbb{E}\left\{(\Lambda_k^i - \bar{\Lambda}_k^i)C_k^i x_k x_k^T C_k^{iT}(\Lambda_k^i - \bar{\Lambda}_k^i)|\Lambda_k^i\right\}\right\} \\ =&\mathbb{E}\left\{(\Lambda_k^i - \bar{\Lambda}_k^i)C_k^i \mathbb{E}\left\{x_k x_k^T|\Lambda_k^i\right\} C_k^{iT}(\Lambda_k^i - \bar{\Lambda}_k^i)\right\} \\ =&\mathbb{E}\left\{(\Lambda_k^i - \bar{\Lambda}_k^i)C_k^i Z_2 \tilde{X}_{i,k} Z_2^T C_k^{iT}(\Lambda_k^i - \bar{\Lambda}_k^i)\right\}. \end{aligned}$$

It is now easily seen that $\mathscr{F}_k(X)$ defined by (22) satisfies $\mathscr{F}_k(X) \leq \mathscr{F}_k(Y)$ if $X \leq Y$. The proof is complete. □

In the following theorem, an upper bound is provided for the covariance matrix $\tilde{X}_{i,k}$ given by (20).

Theorem 1 *For a given positive scalar ε, let the matrix $\bar{X}_{i,k}$ be a solution to the following Riccati-like difference equation*

$$\begin{aligned} &\bar{X}_{i,k+1} \\ =&(1+\varepsilon_{i,k})\tilde{A}_{i,k}(\bar{X}_{i,k} + \bar{X}_{i,k}\tilde{N}_k^T \tilde{N}_k \bar{X}_{i,k})\tilde{A}_{i,k}^T \\ &+ (1+\varepsilon_{i,k})\big(1 + \lambda_{\max}(\tilde{N}_k \bar{X}_{i,k}\tilde{N}_k^T)\big)\tilde{H}_k \tilde{H}_k^T + \tilde{W}_k \\ &+ (1+\varepsilon_{i,k}^{-1})R_k^i(\bar{X}_{i,k}) \end{aligned} \tag{23}$$

with the initial value $\bar{X}_{i,0} = \tilde{X}_{i,0}$. Then, the matrix $\bar{X}_{i,k}$ is an upper bound on the covariance matrix $\tilde{X}_{i,k}$ given by (20), i.e.,

$$\tilde{X}_{i,k} \leq \bar{X}_{i,k}$$

for all $k \geq 0$.

Proof: We prove this theorem by induction. Obviously, $\bar{X}_{i,0} = \tilde{X}_{i,0}$ is guaranteed by the initial condition. Assume that $\tilde{X}_{i,k} \leq \bar{X}_{i,k}$ holds and we need to show the inequality $\tilde{X}_{i,k+1} \leq \bar{X}_{i,k+1}$.

By using Lemma 2, we obtain from (21) that

$$\tilde{X}_{i,k+1} \leq \mathscr{F}_k(\tilde{X}_{i,k}) \leq \mathscr{F}_k(\bar{X}_{i,k}). \tag{24}$$

On the other hand, noting that

$$\begin{aligned} &(\tilde{A}_{i,k} + \tilde{H}_k O_k \tilde{N}_k)\bar{X}_{i,k}(\tilde{A}_{i,k} + \tilde{H}_k O_k \tilde{N}_k)^T \\ =&\tilde{A}_{i,k}\bar{X}_{i,k}\tilde{A}_{i,k}^T + \tilde{H}_k O_k \tilde{N}_k \bar{X}_{i,k}\tilde{N}_k^T O_k^T \tilde{H}_k^T \\ &+ \tilde{A}_{i,k}\bar{X}_{i,k}\tilde{N}_k^T O_k^T \tilde{H}_k^T + \tilde{H}_k O_k \tilde{N}_k \bar{X}_{i,k}\tilde{A}_{i,k}^T \\ =&\tilde{A}_{i,k}(\bar{X}_{i,k} + \bar{X}_{i,k}\tilde{N}_k^T O_k^T O_k \tilde{N}_k \bar{X}_{i,k})\tilde{A}_{i,k}^T \\ &+ \tilde{H}_k O_k \tilde{N}_k \bar{X}_{i,k}\tilde{N}_k^T O_k^T \tilde{H}_k^T + \tilde{H}_k \tilde{H}_k^T \\ &+ \tilde{A}_{i,k}\bar{X}_{i,k}\tilde{N}_k^T O_k^T \tilde{H}_k^T + \tilde{H}_k O_k \tilde{N}_k \bar{X}_{i,k}\tilde{A}_{i,k}^T \\ &- \tilde{A}_{i,k}\bar{X}_{i,k}\tilde{N}_k^T O_k^T O_k \tilde{N}_k \bar{X}_{i,k}\tilde{A}_{i,k}^T - \tilde{H}_k \tilde{H}_k^T \\ \leq&\tilde{A}_{i,k}(\bar{X}_{i,k} + \bar{X}_{i,k}\tilde{N}_k^T O_k^T O_k \tilde{N}_k \bar{X}_{i,k})\tilde{A}_{i,k}^T \\ &+ \lambda_{\max}(\tilde{N}_k \bar{X}_{i,k}\tilde{N}_k^T)\tilde{H}_k O_k O_k^T \tilde{H}_k^T + \tilde{H}_k \tilde{H}_k^T \\ \leq&\tilde{A}_{i,k}(\bar{X}_{i,k} + \bar{X}_{i,k}\tilde{N}_k^T \tilde{N}_k \bar{X}_{i,k})\tilde{A}_{i,k}^T \\ &+ \big(1 + \lambda_{\max}(\tilde{N}_k \bar{X}_{i,k}\tilde{N}_k^T)\big)\tilde{H}_k \tilde{H}_k^T, \end{aligned}$$

we have

$$\mathscr{F}_k(\bar{X}_{i,k}) \leq \bar{X}_{i,k+1} \tag{25}$$

where $\bar{X}_{i,k+1}$ is defined by (23).

It follows directly from (24) and (25) that $\tilde{X}_{i,k+1} \leq \bar{X}_{i,k+1}$, which completes the proof of this theorem. □

In Theorem 1, an upper bound is obtained for the covariance matrix $\tilde{X}_{i,k}$. Noting the relation $x_k - \hat{x}_{i,k} = Z_1\tilde{x}_{i,k}$ where $Z_1 = \begin{bmatrix} I & -I \end{bmatrix}$, we know easily that $\bar{P}_{i,k} \triangleq Z_1\bar{X}_{i,k}Z_1^T$ is an upper bound on the estimation error covariance matrix of the estimate $\hat{x}_{i,k}$. In what follows, the filter parameters $F_{i,k}$ and G_k^i shall be designed such that the obtained upper bound $\bar{P}_{i,k+1}$ is minimized.

Denote

$$\bar{X}_{i,k} \triangleq \begin{bmatrix} \bar{X}_{i,k}^1 & \bar{X}_{i,k}^{12} \\ \bar{X}_{i,k}^{12T} & \bar{X}_{i,k}^2 \end{bmatrix}$$

and select the filter parameters $F_{i,k}$ and G_k^i as follows:

$$\begin{aligned} F_{i,k} =&(A_k - G_k^i \bar{\Lambda}_k^i C_k^i)\big(I + \bar{P}_{i,k} N_k^T N_k \\ &\times (I + (\bar{X}_{i,k}^1 - \bar{P}_{i,k}) N_k^T N_k)^{-1}\big), \\ G_k^i =&(1+\varepsilon_{i,k}) A_k S_{i,k} C_k^{iT} \bar{\Lambda}_k^{iT} \Upsilon_{i,k}^{-1} \end{aligned} \tag{26}$$

where

$$\begin{aligned} S_{i,k} \triangleq& \bar{P}_{i,k} N_k^T (I + N_k(\bar{X}_{i,k}^1 - \bar{P}_{i,k}) N_k^T)^{-1} N_k \bar{P}_{i,k} \\ &+ \bar{P}_{i,k} \\ \Upsilon_{i,k} \triangleq& (1+\varepsilon_{i,k}) \bar{\Lambda}_k^i C_k^i S_{i,k} C_k^{iT} \bar{\Lambda}_k^{iT} + \bar{\Lambda}_k^i \odot \tilde{V}^i \\ &+ (1+\varepsilon_{i,k}^{-1})\big(\tilde{\Lambda}_k^i \odot (C_k^i \bar{X}_{i,k}^1 C_k^{iT})\big). \end{aligned} \tag{27}$$

Lemma 3 *Under the selection of the parameters $F_{i,k}$ and G_k^i as the form of (26), one has $\bar{X}_{i,k}^{12} = \bar{X}_{i,k}^2$ and $\bar{P}_{i,k} = \bar{X}_{i,k}^1 - \bar{X}_{i,k}^2$.*

Proof: See Appendix 6. □

By using Lemma 3, we have from (23) that

$$\begin{aligned} &\bar{X}_{i,k+1}^1 \\ =&(1+\varepsilon_{i,k}) A_k (\bar{X}_{i,k}^1 + \bar{X}_{i,k}^1 N_k^T N_k \bar{X}_{i,k}^1) A_k^T + E_k W E_k^T \\ &+ (1+\varepsilon_{i,k})\big(1 + \lambda_{\max}(N_k \bar{X}_{i,k}^1 N_k^T)\big) H_k H_k^T \end{aligned} \tag{28}$$

with initial value $\bar{X}_{i,0}^1 = X_0$, and

$$\begin{aligned} &\bar{P}_{i,k+1} \\ =&(1+\varepsilon_{i,k}) \begin{bmatrix} A_k - G_k^i \bar{\Lambda}_k^i C_k^i & -F_{i,k} \end{bmatrix} \\ &\times (\bar{X}_{i,k} + \bar{X}_{i,k} \tilde{N}_k^T \tilde{N}_k \bar{X}_{i,k}) \begin{bmatrix} A_k - G_k^i \bar{\Lambda}_k^i C_k^i & -F_{i,k} \end{bmatrix}^T \\ &+ (1+\varepsilon_{i,k}^{-1}) G_k^i \big(\tilde{\Lambda}_k^i \odot (C_k^i \bar{X}_{i,k}^1 C_k^{iT})\big) G_k^{iT} \\ &+ G_k^i (\bar{\Lambda}_k^i \odot \tilde{V}^i) G_k^{iT} + E_k W E_k^T \\ &+ (1+\varepsilon_{i,k})\big(1 + \lambda_{\max}(N_k \bar{X}_{i,k}^1 N_k^T)\big) H_k H_k^T \end{aligned} \tag{29}$$

with initial value $\bar{P}_{i,0} = X_0$.

In the following theorem, it is shown that the parameters $F_{i,k}$ and G_k^i defined in (26) minimize the upper bound $\bar{P}_{i,k+1}$.

Theorem 2 *The upper bound on the estimation error covariance matrix $\bar{P}_{i,k+1}$ given by (29) is minimized by the parameters $F_{i,k}$ and G_k^i defined in (26), and the minimized upper bound satisfies*

$$\begin{aligned} &\bar{P}_{i,k+1} \\ =&(1+\varepsilon_{i,k}) A_k S_{i,k} A_k^T + E_k W E_k^T \\ &- (1+\varepsilon_{i,k})^2 A_k S_{i,k} C_k^{iT} \bar{\Lambda}_k^{iT} \Upsilon_{i,k}^{-1} \bar{\Lambda}_k^i C_k^i S_{i,k} A_k^T \\ &+ (1+\varepsilon_{i,k})\big(1 + \lambda_{\max}(N_k \bar{X}_{i,k}^1 N_k^T)\big) H_k H_k^T \end{aligned} \tag{30}$$

with initial value $\bar{P}_{i,0} = X_0$, where $S_{i,k}$ and $\Upsilon_{i,k}$ are defined in (27).

Proof: Note that, in (29), only the first term is dependent on parameter $F_{i,k}$. By setting $\Psi_k^i \triangleq A_k - G_k^i \bar{\Lambda}_k^i C_k^i - F_{i,k}$ and using Lemma 3, the first term can be written as follows

$$\begin{aligned} &\begin{bmatrix} A_k - G_k^i \bar{\Lambda}_k^i C_k^i & -F_{i,k} \end{bmatrix} (\bar{X}_{i,k} + \bar{X}_{i,k} \tilde{N}_k^T \tilde{N}_k \bar{X}_{i,k}) \\ &\times \begin{bmatrix} A_k - G_k^i \bar{\Lambda}_k^i C_k^i & -F_{i,k} \end{bmatrix}^T \\ =&\Big((A_k - G_k^i \bar{\Lambda}_k^i C_k^i) Z_1 + \begin{bmatrix} 0 & \Psi_k^i \end{bmatrix}\Big)(\bar{X}_{i,k} + \bar{X}_{i,k} \tilde{N}_k^T \tilde{N}_k \bar{X}_{i,k}) \\ &\times \Big((A_k - G_k^i \bar{\Lambda}_k^i C_k^i) Z_1 + \begin{bmatrix} 0 & \Psi_k^i \end{bmatrix}\Big)^T \\ =&(A_k - G_k^i \bar{\Lambda}_k^i C_k^i) Z_1 (\bar{X}_{i,k} + \bar{X}_{i,k} \tilde{N}_k^T \tilde{N}_k \bar{X}_{i,k}) \\ &\times Z_1^T (A_k - G_k^i \bar{\Lambda}_k^i C_k^i)^T \\ &+ \begin{bmatrix} 0 & \Psi_k^i \end{bmatrix} (\bar{X}_{i,k} + \bar{X}_{i,k} \tilde{N}_k^T \tilde{N}_k \bar{X}_{i,k}) \begin{bmatrix} 0 & \Psi_k^i \end{bmatrix}^T \\ &+ \begin{bmatrix} 0 & \Psi_k^i \end{bmatrix} (\bar{X}_{i,k} + \bar{X}_{i,k} \tilde{N}_k^T \tilde{N}_k \bar{X}_{i,k}) Z_1^T (A_k - G_k^i \bar{\Lambda}_k^i C_k^i)^T \\ &+ (A_k - G_k^i \bar{\Lambda}_k^i C_k^i) Z_1 (\bar{X}_{i,k} + \bar{X}_{i,k} \tilde{N}_k^T \tilde{N}_k \bar{X}_{i,k}) \begin{bmatrix} 0 & \Psi_k^i \end{bmatrix}^T \\ =&(A_k - G_k^i \bar{\Lambda}_k^i C_k^i)(\bar{P}_{i,k} + \bar{P}_{i,k} N_k^T N_k \bar{P}_{i,k})(A_k - G_k^i \bar{\Lambda}_k^i C_k^i)^T \\ &+ \Psi_k^i (\bar{X}_{i,k}^2 + \bar{X}_{i,k}^2 N_k^T N_k \bar{X}_{i,k}^2) \Psi_k^{iT} \\ &+ \Psi_k^i \bar{X}_{i,k}^2 N_k^T N_k \bar{P}_{i,k} (A_k - G_k^i \bar{\Lambda}_k^i C_k^i)^T \\ &+ (A_k - G_k^i \bar{\Lambda}_k^i C_k^i) \bar{P}_{i,k} N_k^T N_k \bar{X}_{i,k}^2 \Psi_k^{iT} \\ =&(A_k - G_k^i \bar{\Lambda}_k^i C_k^i)(\bar{P}_{i,k} + \bar{P}_{i,k} N_k^T N_k \bar{P}_{i,k})(A_k - G_k^i \bar{\Lambda}_k^i C_k^i)^T \\ &+ (\Psi_k^i - \Psi_{o,k}^i)(\bar{X}_{i,k}^2 + \bar{X}_{i,k}^2 N_k^T N_k \bar{X}_{i,k}^2)(\Psi_k^i - \Psi_{o,k}^i)^T \\ &- (A_k - G_k^i \bar{\Lambda}_k^i C_k^i) \bar{P}_{i,k} N_k^T \big(I - (I + N_k \bar{X}_{i,k}^2 N_k^T)^{-1}\big) \\ &\times N_k \bar{P}_{i,k} (A_k - G_k^i \bar{\Lambda}_k^i C_k^i)^T \end{aligned}$$

where

$$\Psi_{o,k}^i \triangleq -(A_k - G_k^i \bar{\Lambda}_k^i C_k^i) \bar{P}_{i,k} N_k^T N_k (I + \bar{X}_{i,k}^2 N_k^T N_k)^{-1}.$$

Therefore, the upper bound matrix $\bar{P}_{i,k+1}$ defined by (29) is minimized by the parameter $F_{i,k}$ defined in (26) and the minimal $\bar{P}_{i,k+1}$ with respect to parameter $F_{i,k}$ is given as follows:

$$\begin{aligned} &\bar{P}_{i,k+1} \\ =&(1+\varepsilon_{i,k})(A_k - G_k^i \bar{\Lambda}_k^i C_k^i)(\bar{P}_{i,k} + \bar{P}_{i,k} N_k^T N_k \bar{P}_{i,k})(A_k \\ &- G_k^i \bar{\Lambda}_k^i C_k^i)^T - (1+\varepsilon_{i,k})(A_k - G_k^i \bar{\Lambda}_k^i C_k^i) \bar{P}_{i,k} N_k^T \\ &\times \big(I - (I + N_k \bar{X}_{i,k}^2 N_k^T)^{-1}\big) N_k \bar{P}_{i,k} (A_k - G_k^i \bar{\Lambda}_k^i C_k^i)^T \\ &+ (1+\varepsilon_{i,k}^{-1}) G_k^i \big(\tilde{\Lambda}_k^i \odot (C_k^i \bar{X}_{i,k}^1 C_k^{iT})\big) G_k^{iT} \\ &+ G_k^i (\bar{\Lambda}_k^i \odot \tilde{V}^i) G_k^{iT} + E_k W E_k^T \\ &+ (1+\varepsilon_{i,k})\big(1 + \lambda_{\max}(N_k \bar{X}_{i,k}^1 N_k^T)\big) H_k H_k^T \\ =&(1+\varepsilon_{i,k})(A_k - G_k^i \bar{\Lambda}_k^i C_k^i) S_{i,k} (A_k - G_k^i \bar{\Lambda}_k^i C_k^i)^T \\ &+ (1+\varepsilon_{i,k}^{-1}) G_k^i \big(\tilde{\Lambda}_k^i \odot (C_k^i \bar{X}_{i,k}^1 C_k^{iT})\big) G_k^{iT} \\ &+ G_k^i (\bar{\Lambda}_k^i \odot \tilde{V}^i) G_k^{iT} + E_k W E_k^T \\ &+ (1+\varepsilon_{i,k})\big(1 + \lambda_{\max}(N_k \bar{X}_{i,k}^1 N_k^T)\big) H_k H_k^T \\ =&(1+\varepsilon_{i,k}) G_k^i \bar{\Lambda}_k^i C_k^i S_{i,k} \bar{\Lambda}_k^{iT} G_k^{iT} + (1+\varepsilon_{i,k}) A_k S_{i,k} A_k^T \\ &- (1+\varepsilon_{i,k}) A_k S_{i,k} C_k^{iT} \bar{\Lambda}_k^{iT} G_k^{iT} \\ &- (1+\varepsilon_{i,k}) G_k^i \bar{\Lambda}_k^i C_k^i S_{i,k} A_k^T \end{aligned}$$

$$
\begin{aligned}
&+(1+\varepsilon_{i,k}^{-1})G_k^i\big(\tilde{\Lambda}_k^i\odot(C_k^i\bar{X}_{i,k}^1C_k^{iT})\big)G_k^{iT}\\
&+G_k^i(\bar{\Lambda}_k^i\odot\tilde{V}^i)G_k^{iT}+E_kWE_k^T\\
&+(1+\varepsilon_{i,k})\big(1+\lambda_{\max}(N_k\bar{X}_{i,k}^1N_k^T)\big)H_kH_k^T\\
=&(G_k^i-G_{o,k}^i)\Upsilon_{i,k}(G_k^i-G_{o,k}^i)^T\\
&-(1+\varepsilon_{i,k})^2A_kS_{i,k}C_k^{iT}\bar{\Lambda}_k^{iT}\Upsilon_{i,k}^{-1}\bar{\Lambda}_k^iC_k^iS_{i,k}A_k^T\\
&+(1+\varepsilon_{i,k})A_kS_{i,k}A_k^T+E_kWE_k^T\\
&+(1+\varepsilon_{i,k})\big(1+\lambda_{\max}(N_k\bar{X}_{i,k}^1N_k^T)\big)H_kH_k^T
\end{aligned}
$$

where

$$G_{o,k}^i\triangleq(1+\varepsilon_{i,k})A_kS_{i,k}C_k^{iT}\bar{\Lambda}_k^{iT}\Upsilon_{i,k}^{-1}.$$

It is now seen that the upper bound matrix $\bar{P}_{i,k+1}$ is further minimized by the parameter G_k^i given in (26), and the minimized $\bar{P}_{i,k+1}$ is given by (30). Therefore, the proof of this theorem is complete. □

3.3 The Fusion Scheme of State Estimates

For the obtained estimates $\hat{x}_{i,k}$ $(i=1,2,\cdots,M)$ and the locally minimized upper bounds $\bar{P}_{i,k}$ $(i=1,2,\cdots,M)$, we use the CI fusion criterion (10) with (11) to achieve the fusion estimation purpose, where the weighted parameters ν_i $(i=1,2,\cdots,M)$ are obtained by solving the following optimization problem:

$$
\begin{aligned}
&\min_{\{\nu_{1,k},\nu_{2,k},\cdots,\nu_{M,k}\}}\operatorname{tr}(\bar{P}_k^F)\\
&\text{subject to}\quad \textstyle\sum_{i=1}^M\nu_{i,k}=1,\\
&0\le\nu_{i,k}\le1,\;\; i=1,2,\cdots,M.
\end{aligned}\tag{31}
$$

Remark 4 *Note that the above optimization problem is nonlinear and, as pointed out in [40], it is difficult to derive the analytical solutions of the weighted parameters by directly solving the above nonlinear optimization problem. Fortunately, the corresponding numerical solutions can be obtained by utilizing the function "fmincon" in Matlab optimization toolbox. Also, the consistency of the CI fusion scheme can be demonstrated by following the similar lines in [37].*

Until now, the robust distributed fusion filtering problem has been solved, and the design algorithm of the fusion filtering scheme can be summarized as follows.

Remark 5 *In this paper, the distributed fusion filtering problem is dealt with over multisensor systems subject to EHCs. The distributed fusion filtering problem is first formulated with multiple sensors equipped with the energy harvesters. Then, the complex information communications are modelled within the multisensor systems with the EHCs. Afterwards, an effective distributed fusion filtering approach is developed to achieve the desired fusion estimation error under the EHCs. Note that, in the fusion filtering problem under consideration, in order to maintain the normal operation of the overall sensor network, an effective energy replenishing schemes, i.e., energy harvesting technology has been employed for energy collection/storage, which is different from the traditional ones in the existing literature [3, 4, 40, 44].*

Remark 6 *In comparison to the vast existing literature on the fusion estimation problems for multisensor systems, the main results developed in this paper exhibit the following three distinctive merits: 1) the energy harvesting mechanism is introduced into the framework of fusion filtering; 2) the parameter uncertainties are taken into account in the multi-sensor robust fusion filtering problems; and 3) a robust fusion filtering scheme is proposed in the simultaneous presence of EHCs and parameter uncertainties. In next section, the effectiveness of the proposed robust fusion filtering scheme will be verified through a numerical simulation example.*

Algorithm 1. Robust Distributed Fusion Filtering Algorithm

Step 1. Set initial conditions $\bar{X}_{i,0}^1=X_0$, $\bar{P}_{i,0}=X_0$, $\mathbf{p}_{i,0}=[\underbrace{0\cdots0}_{\bar{z}_i}\;1\;\underbrace{0\cdots0}_{S_i-\bar{z}_i}]^T$, $\hat{x}_{i,0}=0$ for all $i=\{1,2,\cdots,M\}$ and $k=0$;

Step 2. Compute the probability $\bar{\gamma}_{i,k}$ by (16), gain matrices $F_{i,k}$ and G_k^i by (26) with (27) for all $i=\{1,2,\cdots,M\}$, and derive the estimates $\hat{x}_{i,k+1}$ according to (8) for all $i=\{1,2,\cdots,M\}$;

Step 3. Obtain the weighted parameters $\nu_{i,k}$ $(i=1,2,\cdots,M)$ by solving the optimization problem (31) and derive the fused estimate $\hat{x}_k^F$ according to (10) with (11);

Step 4. Obtain the probability distribution $\mathbf{p}_{i,k+1}$, the upper bound matrices $\bar{X}_{i,k+1}^1$ and $\bar{P}_{i,k+1}$ according to (14), (28) and (30), respectively, and set $k=k+1$;

Step 5. If $k<N$, then go to *Step* 2., else go to *Step* 6.;

Step 6. Output the fused estimate $\hat{x}_k^F$;

Step 7. Stop.

4 An Illustrative Example

Consider a multisensor system with three sensor nodes. Let $a_{11}=a_{22}=a_{33}=a_{21}=1$, $a_{13}=a_{12}=a_{23}=a_{31}=a_{32}=0$ and the system parameters be given as follows:

$$
\begin{aligned}
&A_k=\begin{bmatrix}0.75&-0.66\sin(k)\\0.5&0.68\end{bmatrix},H_k=\begin{bmatrix}0.2&0\\-0.1\cos(k)&-0.2\end{bmatrix},\\
&O_k=\begin{bmatrix}0.1\sin(k)&0\\0&0.1\cos(k)\end{bmatrix},N_k=\begin{bmatrix}0.2&0.1\sin(2k)\\0&0.1\end{bmatrix},\\
&E_k=\begin{bmatrix}-0.2&0.1\end{bmatrix},C_{1,k}=\begin{bmatrix}0.6&0\end{bmatrix},\\
&C_{2,k}=\begin{bmatrix}0&2\end{bmatrix},C_{3,k}=\begin{bmatrix}1&0.5\end{bmatrix}.
\end{aligned}
$$

The covariances of noises are given as $W_k=0.05$, $V_{1,k}=0.82$, $V_{2,k}=0.2$ and $V_{3,k}=0.3$. The statistical properties of initial value are $\bar{x}_0=[0.3\;0.2]^T$ and $X_0=\operatorname{diag}\{0.3,0.4\}$.

Let the maximum number of energy units and the initial energy stored in sensors be $S_i=3$ and $\bar{z}_{i,0}=1$ $(i=1,2,3)$, respectively. Moreover, it is assumed that $p_0=0.4$, $p_1=0.2$, $p_2=0.1$ and $p_3=0.1$. Other parameters are chosen as $\varepsilon_{1,k}=0.1+(0.2\cos(k))^2$, $\varepsilon_{2,k}=0.05+(0.1\sin(k))^2$ and $\varepsilon_{3,k}=0.1+(0.2\sin(2k))^2$.

With the above parameters, the desired filter gains $F_{i,k}$ and G_k^i can be obtained by (26) and the minimized upper bound $\bar{P}_{i,k}$ can be recursively derived according to (30). Based on $\bar{P}_{i,k}$ and the CI-fusion method, $\bar{P}_k^F$ can be derived from (11). Then, by solving the optimization problem (31), the parameters $\nu_{i,k}$ $(i = 1, 2, 3, 4)$ at each time instant are obtained (see Table 1).

Table 1
The values of $\nu_{i,k}$ on each sampling instant

k	0	1	2	3	$\cdots$
ν_1	0.3333	0.3767	0.4917	0.4794	
ν_2	0.3333	0.0001	0.0002	0.2545	$\cdots$
ν_3	0.3334	0.6232	0.5018	0.2661	

In order to exhibit the fusion filtering performance, two experimental results are presented in Figs. 2-5. Fig. 2 shows the evolutions of the amounts of energy stored in each sensor on the first experiment and Fig. 3 plots the trajectory of real state x_k and its estimates in local filters and fusion center on this experiment. The corresponding results for the second experiment are shown in Figs. 4 and 5.

To compare the fusion filtering performance, we define the mean square errors (MSEs) as $\text{MSE}_i = \frac{\sum_{j=1}^{K}(x_k^j - \hat{x}_{i,k}^j)(x_k^j - \hat{x}_{i,k}^j)^T}{K}$ where, for $i \in \{1, 2, 3\}$, x_k^j and $\hat{x}_{i,k}^j$ are, respectively, the actual state and its estimates in local filters and fused estimate at the jth iteration of Algorithm 1.

Algorithm 1 has been implemented 100 times and the upper bound $\bar{P}_k$ and the MSEs are depicted in Figs. 6 and 7. Fig. 6 exhibits the first component in main diagonal of MSE_i and $\bar{P}_{i,k}$ $(i = 1, 2, 3)$, and the corresponding results for the second component are shown in Fig. 7. It can be seen from Figs. 6 and 7 that the MSE_i $(i = 1, 2, 3)$ stay below their upper bounds, which is consistent with the main results. Fig. 8 plots the traces of $\bar{P}_{i,k}$ and $\bar{P}_k^F$ from which it is seen that the trace of $\bar{P}_k^F$ is less than that of $\bar{P}_{i,k}$. Therefore, the designed fusion filtering approach performs very well.

Remark 7 *It is worth mentioning that the estimation accuracy is closely related to the given parameters* $\varepsilon_{i,k}$ *(*$i = 1, 2, 3$*). The optimal selection of these parameters could be obtained with the help of the intelligent optimization algorithms.*

5 Conclusions

In this paper, the robust distributed fusion filtering problem has been studied for a class of discrete time-varying stochastic uncertain systems over multisensor systems with EHCs. The energy level received by the energy harvester has been characterized by a random variable obeying a certain probability distribution and the parameter uncertainties have been assumed to be unknown yet norm-bounded matrices. The local filter has firstly been designed such that, for all possible parameter uncertainties and EHCs, an upper bound on the filtering error covariance is guaranteed and minimized by selecting the filter parameters. Then, all the local estimates obtained by local filters have been fused

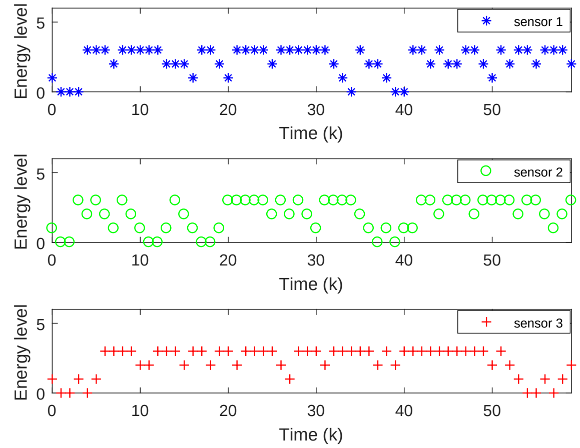

Fig. 2. Energy amounts in sensors on experiment 1.

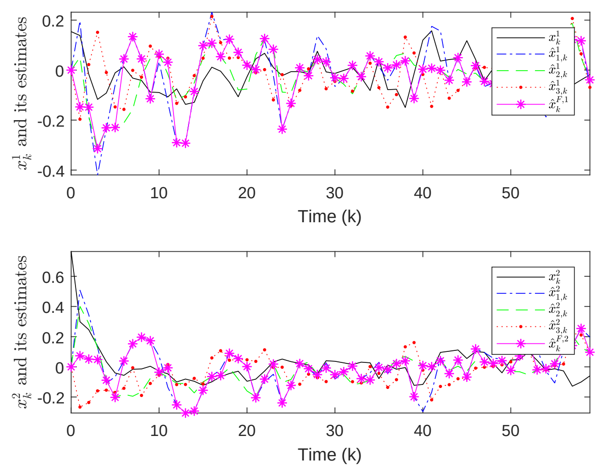

Fig. 3. State x_k and its estimates on experiment 1.

by CI fusion strategy. Finally, a numerically simulation example has been presented to demonstrate the effectiveness of the proposed fusion filtering scheme. It should be pointed out that, in this paper, the probability distribution of the amount of energy harvested is simply assumed to be exactly known. In the practical application, the probability distribution may contain uncertainties due to the unreliable measurement. Further research topics include the extension of the main results to 1) the set-membership filtering over multisensor systems subject to EHCs with uncertain probability distributions [17, 46]; 2) the moving horizon estimation problem over multisensor systems with EHCs [47, 48]; and 3) the state-saturated recursive filtering problem of networked systems subject to EHCs [32, 33].

6 Proof of Lemma 3

In order to prove Lemma 3, we employ the mathematical induction approach.

First, by setting the initial values as $\bar{X}_{i,0} = \text{diag}\{\bar{X}_0, 0\}$ and $\bar{P}_{i,0} = \bar{X}_0$, it is easily obtained that $\bar{P}_{i,0} = \bar{X}_{i,0}^1 -$

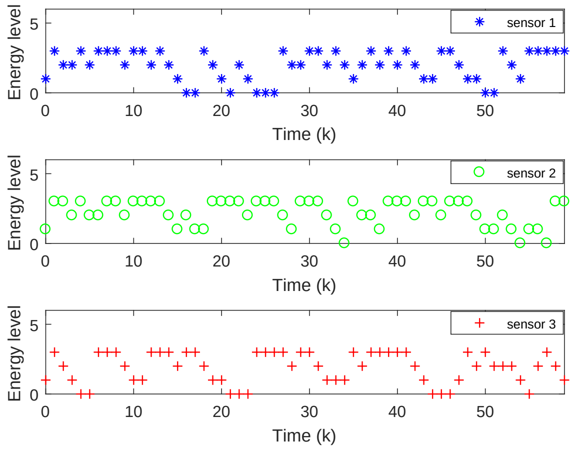

Fig. 4. Energy amounts in sensors on experiment 2.

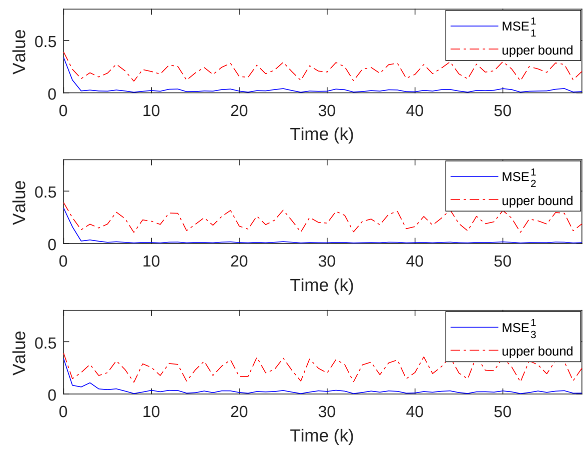

Fig. 6. MSEs and upper bounds for the 1st component.

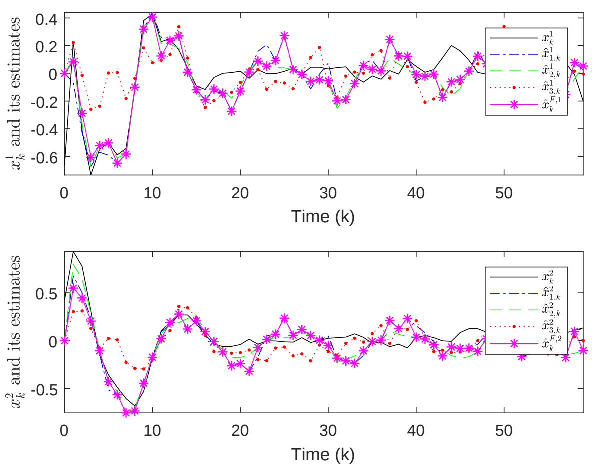

Fig. 5. State x_k and its estimates on experiment 2.

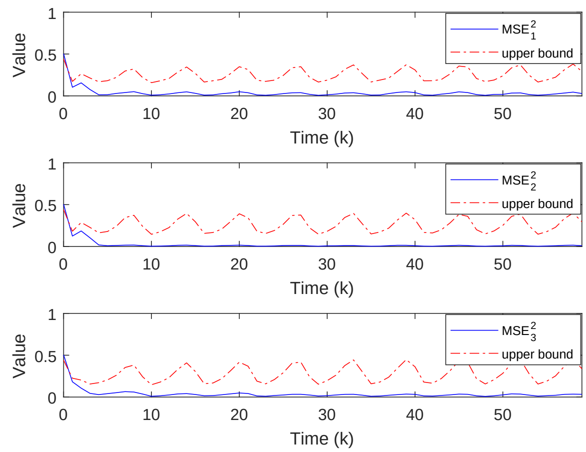

Fig. 7. MSEs and upper bounds for the 2nd component.

$\bar{X}^2_{i,0}$ and $\bar{X}^{12}_{i,0} = \bar{X}^2_{i,0}$. Then, assuming that $\bar{P}_{i,k} = \bar{X}^1_{i,k} - \bar{X}^2_{i,k}$ and $\bar{X}^{12}_{i,k} = \bar{X}^2_{i,k}$ hold at the sampling instant k, we are now in a position to show that $\bar{P}_{i,k+1} = \bar{X}^1_{i,k+1} - \bar{X}^2_{i,k+1}$ and $\bar{X}^{12}_{i,k+1} = \bar{X}^2_{i,k+1}$ are true at the sampling instant $k+1$.

It is obtained from (23) that

$$\begin{aligned}\bar{X}^{12}_{i,k+1} =&(1+\varepsilon_{i,k})A_k\Theta_{i,k,1}\bar{X}^2_{i,k}F^T_{i,k}\\ &+(1+\varepsilon_{i,k})A_k\Theta_{i,k,1}\bar{X}^1_{i,k}C^{iT}_k\bar{\Lambda}^{iT}_kG^{iT}_k,\\ \bar{X}^2_{i,k+1} =&(1+\varepsilon_{i,k})G^i_k\bar{\Lambda}^i_kC^i_k\Theta_{i,k,1}\bar{X}^1_{i,k}C^{iT}_k\bar{\Lambda}^{iT}_kG^{iT}_k\\ &+(1+\varepsilon_{i,k})F_{i,k}\Theta_{i,k,2}\bar{X}^2_{i,k}F^T_{i,k}\\ &+(1+\varepsilon_{i,k})G^i_k\bar{\Lambda}^i_kC^i_k\Theta_{i,k,1}\bar{X}^2_{i,k}F^T_{i,k}\\ &+(1+\varepsilon_{i,k})F_{i,k}\bar{X}^2_{i,k}\Theta^T_{i,k,1}C^{iT}_k\bar{\Lambda}^{iT}_kG^{iT}_k\\ &+(1+\varepsilon^{-1}_{i,k})G^i_k\big(\tilde{\Lambda}^i_k\odot(C^i_k\bar{X}^1_{i,k}C^{iT}_k)\big)G^{iT}_k\\ &+G^i_k(\bar{\Lambda}^i_k\odot\tilde{V}^i)G^{iT}_k\end{aligned} \quad (32)$$

where

$$\Theta_{i,k,1} = I + \bar{X}^1_{i,k}N^T_kN_k,\ \Theta_{i,k,2} = I + \bar{X}^2_{i,k}N^T_kN_k. \quad (33)$$

Note the fact that $\Theta_{i,k,1} = (I + \bar{P}_{i,k}N^T_kN_k\Theta^{-1}_{i,k,2})\Theta_{i,k,2}$, it is easily known from (26) that

$$F_{i,k} = (A_k - G^i_k\bar{\Lambda}^i_kC^i_k)\Theta_{i,k,1}\Theta^{-1}_{i,k,2}. \quad (34)$$

On the other hand, we have

$$\begin{aligned}&\Theta_{i,k,1}(\bar{X}^1_{i,k} - \bar{X}^2_{i,k}\Theta^{-T}_{i,k,2}\Theta^T_{i,k,1})\\ =&\Theta_{i,k,1}(\bar{X}^1_{i,k} - \Theta^{-1}_{i,k,2}\bar{X}^2_{i,k}\Theta^T_{i,k,1})\\ =&\Theta_{i,k,1}\Theta^{-1}_{i,k,2}(\Theta^{-1}_{i,k,2}\bar{X}^1_{i,k} - \bar{X}^2_{i,k}\Theta^T_{i,k,1})\\ =&\Theta_{i,k,1}\Theta^{-1}_{i,k,2}\bar{P}_{i,k} = S_{i,k}.\end{aligned} \quad (35)$$

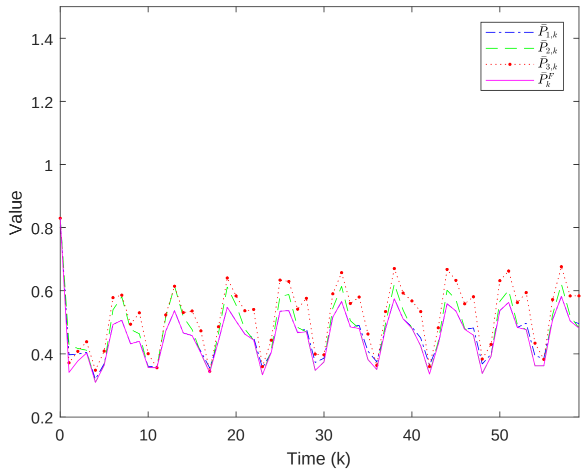

Fig. 8. Trace of the upper bounds.

Then, it follows from (32), (34) and (35) that

$$\begin{aligned}\bar{X}^{12}_{i,k+1} =&(1+\varepsilon_{i,k})A_k\Theta_{i,k,1}\Theta^{-T}_{i,k,2}\Theta^{T}_{i,k,1}A_k^T\\ &+(1+\varepsilon_{i,k})A_kS_{i,k}C_k^{iT}\bar{\Lambda}_k^{iT}G_k^{iT},\\ \bar{X}^{2}_{i,k+1} =&(1+\varepsilon_{i,k})A_k\Theta_{i,k,1}\Theta^{-T}_{i,k,2}\Theta^{T}_{i,k,1}A_k^T\\ &+(1+\varepsilon_{i,k})G_k^i\bar{\Lambda}_k^iC_k^iS_{i,k}C_k^{iT}\bar{\Lambda}_k^{iT}G_k^{iT}\\ &+(1+\varepsilon^{-1}_{i,k})G_k^i\big(\tilde{\Lambda}_k^i\odot(C_k^i\bar{X}^1_{i,k}C_k^{iT})\big)G_k^{iT}\\ &+G_k^i(\bar{\Lambda}_k^i\odot\tilde{V}^i)G_k^{iT}.\end{aligned}\tag{36}$$

Furthermore, from (26), (27) and (36), it is obtained that

$$\begin{aligned}&\bar{X}^2_{i,k+1}\\ =&(1+\varepsilon_{i,k})A_k\Theta_{i,k,1}\Theta^{-T}_{i,k,2}\Theta^{T}_{i,k,1}A_k^T\\ &+G_k^i\Big((1+\varepsilon_{i,k})\bar{\Lambda}_k^iC_k^iS_{i,k}C_k^{iT}\bar{\Lambda}_k^{iT}+\bar{\Lambda}_k^i\odot\tilde{V}^i\\ &+(1+\varepsilon^{-1}_{i,k})\tilde{\Lambda}_k^i\odot(C_k^i\bar{X}^1_{i,k}C_k^{iT})\Big)G_k^{iT}\\ =&(1+\varepsilon_{i,k})A_k\Theta_{i,k,1}\Theta^{-T}_{i,k,2}\Theta^{T}_{i,k,1}A_k^T\\ &+(1+\varepsilon_{i,k})A_kS_{i,k}C_k^{iT}\bar{\Lambda}_k^{iT}G_k^{iT}.\end{aligned}\tag{37}$$

By (36) and (37), it is readily obtained that $\bar{X}^2_{i,k+1}=\bar{X}^{12}_{i,k+1}$.

Utilizing (26), (28) and (36), we further obtain that

$$\begin{aligned}&\bar{X}^1_{i,k+1}-\bar{X}^2_{i,k+1}\\ =&(1+\varepsilon_{i,k})A_k(\bar{X}^1_{i,k}+\bar{X}^1_{i,k}N_k^TN_k\bar{X}^1_{i,k})A_k^T\\ &+(1+\varepsilon_{i,k})\big(1+\lambda_{\max}(N_k\bar{X}^1_{i,k}N_k^T)\big)H_kH_k^T\\ &-(1+\varepsilon_{i,k})A_k\Theta_{i,k,1}\Theta^{-T}_{i,k,2}\Theta^{T}_{i,k,1}A_k^T-G_k^i\Upsilon_{i,k}G_k^{iT}\\ &+E_kWE_k^T\\ =&(1+\varepsilon_{i,k})A_k\Theta_{i,k,1}(\bar{X}^1_{i,k}-\Theta^{-T}_{i,k,2}\Theta^{T}_{i,k,1})A_k^T\\ &+(1+\varepsilon_{i,k})\big(1+\lambda_{\max}(N_k\bar{X}^1_{i,k}N_k^T)\big)H_kH_k^T\\ &-(1+\varepsilon_{i,k})^2A_kS_{i,k}C_k^{iT}\bar{\Lambda}_k^{iT}\Upsilon^{-1}_{i,k}\bar{\Lambda}_k^iC_k^iS_{i,k}A_k^T\\ &+E_kWE_k^T.\end{aligned}\tag{38}$$

Then, the conclusion of $\bar{P}_{i,k+1}=\bar{X}^1_{i,k+1}-\bar{X}^2_{i,k+1}$ is drawn from (30), (35) and (38), and the proof of Lemma 3 is complete.

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