This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI10.1109/TSMC.2019.2907649, IEEE Transactions on Systems, Man, and Cybernetics: Systems


# Partial-Nodes-Based Scalable $H_\infty$-Consensus Filtering with Censored Measurements over Sensor Networks

Fei Han, *Member, IEEE*, Zidong Wang, *Fellow, IEEE*, Hongli Dong*, *Senior Member, IEEE*, and Hongjian Liu

***Abstract*—This paper deals with the scalable distributed $H_\infty$-consensus filtering problem for a class of discrete time-varying systems subject to multiplicative noises and censored measurements over sensor networks. For the underlying sensor network, it is assumed that only the measurement outputs from partial sensor nodes are available. Also, the phenomenon of censored measurements is taken into account to reflect the limited capability in measuring. A new $H_\infty$-consensus performance index is put forward to evaluate the disturbance rejection level of the filters against the simultaneous presence of external disturbances, initial conditions as well as censoring effects. By utilizing the vector dissipativity theory and the recursive matrix inequality technique, sufficient conditions are established under which the prescribed $H_\infty$-consensus performance index is achieved. The parameters of the desired distributed filters are calculated via solving certain matrix inequalities, where such a calculation is conducted in a local sense so as to preserve the scalability of the filter design. Finally, a numerical simulation example is provided to demonstrate the validity and applicability of the proposed filtering strategy.**

***Index Terms*—Censored measurements, distributed filtering, $H_\infty$-consensus, multiplicative noises, partial-nodes-based measurements.**

## I. Introduction

During the past few years, distributed filtering problem for sensor networks (SNs) has gained tremendous research interest from the control and signal processing communities and found great applications in engineering practice such as intelligent transportation systems, people-centric networked systems, public health and environmental monitoring, navigation and tracking systems, etc., see e.g. [3], [6], [15], [18], [21], [22], [25], [30], [31], [37], [38]. The core idea of distributed filtering algorithm is to estimate the state information of the target plant in a collaborative manner by deploying a set of intelligent sensing facilities over the region of interest. Some commonly used distributed filtering strategies include distributed Kalman filtering methods, distributed set-membership filtering schemes, and distributed $H_\infty$ filtering approaches, see [5], [9], [29], [37], [41], [43] for some recent works. Among them, special attention has been focused on distributed $H_\infty$ filtering algorithms due primarily to their capacities of the disturbance rejection/attenuation against external disturbances, see e.g. [7], [8], [26], [28], [45]. Very recently, to improve the performance of the traditional distributed $H_\infty$ filters, the so-called distributed $H_\infty$-*consensus* filtering (DHCF) algorithm has been proposed with intention to account for the effects of the *consensus errors* related to neighboring nodes. Up to now, the DHCF issue has become an attractive research topic receiving considerable research attention, see e.g. [12], [13], [27], [39].

For a large-scale sensor network, the coupling complexities caused by mutual interactions among large quantities of sensor nodes constitute a major challenge for the analysis/synthesis issues on distributed filters. A commonly employed approach to coping with such a challenge is to augment the network states [5], [6], [21], [26], [38], [41], which implies that the global information of the network is available. In practice, however, each sensor can only access the local information from its neighbors and therefore the global information of the entire network is hardly accessible. As such, it is highly desirable to develop the so-called *scalable* filtering algorithm that only utilizes the neighbors' information of each sensor node in order to *locally* design the filter parameters. Clearly, such a *local* design scheme enjoys the flexibility/adaptability with respect to the scale/structure changes of the network. So far, some initial research effort has been made on the scalability issues for the distributed filter design, see e.g. [12], [13].

As a kind of typical phenomena encountered in practical engineering, the measurement censoring caused mainly by the inevitable data truncation has recently aroused some research attention with applications ranging from biological and economic systems, computer vision problems, to distributed detection scenarios. Various models have been utilized in the literature to characterize the censored measurements, among which the Tobit model (proposed by economist James Tobin in the middle of last century) has been recognized to be the most popular one. In recent years, the Tobit model has

This work was supported in part by the National Natural Science Foundation of China under Grants 61873148 and 61873058, the China Postdoctoral Science Foundation under Grant 2017M621242, the PetroChina Innovation Foundation under Grant 2018D-5007-0302, the Fundamental Research Fund for Undergraduate Universities affiliated to Heilongjiang Province under Grant 2018QNL-05, the Natural Science Foundation of Heilongjiang Province of China under Grant F2018005 and the Alexander von Humboldt Foundation of Germany. (*Corresponding author: Hongli Dong*).

F. Han and H. Dong are with the Institute of Complex Systems and Advanced Control, Northeast Petroleum University, Daqing 163318, China. They are also with the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Northeast Petroleum University, Daqing 163318, China. (emails: tomcumt@126.com; shiningdhl@vip.126.com).

Z. Wang is with the Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom. (email: Zidong.Wang@brunel.ac.uk).

H. Liu is with the School of Mathematics, Anhui Polytechnic University, Wuhu 241000, China. He is also with Northeast Petroleum University, Daqing 163318, China. (e-mail: hjliu2006@ahpu.edu.cn).

attracted some initial research interest in the area of signal processing, and special effort has been directed towards the Tobit Kalman filtering problems, see e.g. [1], [2], [10], [11], [16]. It is worth mentioning that most results concerning Tobit Kalman filtering have been based on the assumption that the probability density function of the system noises obeys the Gaussian distribution. Apparently, these existing results are not applicable to systems suffering from non-Gaussian noises. Very recently, a set-membership filtering approach has been proposed in [17] for a class of time-varying nonlinear systems with censored measurements where the noises reside within certain ellipsoids. Nevertheless, when it comes to the DHCF problem subject to censoring effects, the corresponding results have been very few due to the lack of appropriate techniques capable of tackling the scalability issue in a large-scale network environment.

In the practical applications of SNs, it is essentially difficult to guarantee that system measurements are available from *all* sensor nodes due to a variety of physical restrictions. For example, certain sensor nodes only have the transmission capability (i.e. without measuring capability) because of the limited resource that prevents the information collection from all nodes. The unavailability of measurement outputs could also due to the sensor failures in certain severe circumstances, e.g., colliery, nuclear plant, and military battlefield. Therefore, in reality, instead of requiring the measurement information from all the sensor nodes, it is more reasonable to assume that the measurement outputs utilized for the state estimation/filtering tasks are only available from a *fraction* of sensor nodes, which gives rise to the so-called partial-nodes-based (PNB) distributed filtering problem. To be more specific, the main idea of the proposed PNB distributed filtering problem is to make use of the measurement information from *partial* sensor nodes to achieve the desired estimation performance. By now, some pioneering work has been reported on the PNB state estimation topics for complex networks, see e.g. [19], [20]. Nevertheless, the PNB distributed consensus filtering issue has not been fully investigated yet and this motivates us to launch a study on such a problem of clear engineering insights.

According to above discussions, it can be concluded that, despite its practical significance, the PNB scalable DHCF problem has received very little attention for time-varying systems with censored measurements. This is due mainly to the following identified challenges: 1) how to design the distributed filters based on the measurement outputs for partial sensor nodes? 2) how to examine the impact of the censored measurements by an appropriate filtering performance index? and 3) how to achieve and verify the scalability of the distributed filtering scheme? The purpose of this paper is, therefore, to handle the aforementioned challenges and provide a feasible trade-off between the availability (of the sensor measurements), the robustness (against the censoring measurements), the scalability (of the computational complexity) and the accuracy (by means of the $H_\infty$-consensus). The main contributions of this paper can be summarized below: *1) according to our literature review, the censored measurement phenomenon is, for the first time, investigated for the distributed filtering problem within a local design framework; 2) compared with [19], [20], a new partial-nodes-based distributed filtering strategy is proposed in terms of the measurement outputs from a fraction of sensor nodes, where the distributed filtering schemes are designed, respectively, for nodes with and without measurements, thereby better reflecting the engineering practice; and 3) in light of the existing results concerning the DHCFs and the censored measurements, a new $H_\infty$ performance index is established to evaluate the impact from the censored measurements on the filtering performance.*

The remainder of this paper is organized as follows. In Section II, the target plant described by a discrete time-varying system is introduced and the distributed filtering problem to be addressed is given. In Section III, sufficient conditions are derived to guarantee the desired filtering performance index and the filter parameters are calculated by using the local performance analysis method. An illustrative example is presented in Section IV to demonstrate the effectiveness of the proposed filtering strategy. Finally, conclusions are drawn in Section V.

**Notation:** Let $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ be the sets of the $n$-dimensional vectors and $n \times m$ real matrices, respectively. For column vectors $x = [x_1, x_2, \ldots, x_n]^T$ and $y = [y_1, y_2, \ldots, y_n]^T$, $x \gg y$ (respectively, $x \ll y$) represents that $x_i > y_i$ ($x_i < y_i$), $\forall\ i = 1, 2, \ldots, n$. Denote by $\mathbf{1}$ a column vector of appropriate dimension with all elements being 1. A square matrix $U = [U_{ij}]$ is called column substochastic if $\mathbf{1}^T U \ll \mathbf{1}^T$ and $U_{ij} \geq 0$. $l_2[0, n-1]$ means the set of summable vectors over $[0, n-1]$. For a vector $w_k \in l_2[0, n-1]$ and a matrix $Q_k$ with compatible dimensions, $\|w_k\|^2_{Q_k} = w_k^T Q_k w_k$. Given two matrices $X$ and $Y$, $X^T Y(\bullet)$ means $X^T Y X$. A block diagonal matrix is denoted as diag$\{\ldots\}$, moreover, $\text{diag}_r\{A\} = \text{diag}\{\underbrace{A, \ldots, A}_{r}\}$.

## II. Problem Formulation

Consider an SN with $N$ nodes scattered in certain area of interest. The topology of the SN is characterized by a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathscr{A})$, where $\mathcal{V} = \{i \mid i = 1, 2, \ldots, N\}$ is the node set, $\mathcal{E} = \{(i, j) \mid (i, j) \in \mathcal{V} \times \mathcal{V}\}$ is the edge set, and $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix. For different nodes $i$ and $j$, if $(i, j) \in \mathcal{E}$, which implies that $j$ can transmit messages to $i$, then $a_{ij} = 1$,; otherwise $a_{ij} = 0$. The in-degree and the out-degree of node $i$ are, respectively, defined as $p_i \triangleq \sum_{j=1}^{N} a_{ij}$ and $q_i \triangleq \sum_{j=1}^{N} a_{ji}$. The set of neighbors of node $i$ with in-degree $p_i$ is denoted by $\mathcal{N}_i = \{j_{i1}, j_{i2}, \ldots, j_{ip_i}\}$.

Consider the discrete time-varying stochastic plant over the finite time-horizon $\mathcal{H} \triangleq \{0, 1, \ldots, n-1\}$:

$$x_{k+1} = (A_{0,k} + \sum_{t=1}^{r} \phi_{t,k} A_{t,k}) x_k + B_k w_k \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the system state, $w_k \in \mathbb{R}^{n_w}$ is the external disturbance belonging to $l_2[0, n-1]$ and $\phi_{t,k}$ ($t = 1, 2, \ldots, r$) $\in \mathbb{R}$ are mutually independent multiplicative noises with zero means and unity variances. $A_{0,k}$, $A_{t,k}$ and $B_k$ are known time-varying matrices with compatible dimensions.

Without loss of generality, for the sensor network under consideration, it is assumed that the measurement outputs of

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI10.1109/TSMC.2019.2907649, IEEE Transactions on Systems, Man, and Cybernetics: Systems


the first $l_0$ ($l_0 < N$) nodes are available with the following form:

$$y^*_{i,k} = C_{i,k}x_k + D_{i,k}\xi_{i,k}, \quad \forall i = 1, 2, \ldots, l_0 \tag{2}$$

where $y^*_{i,k} \triangleq [y^*_{i,k}(1) \quad y^*_{i,k}(2) \quad \ldots \quad y^*_{i,k}(n_y)]^T \in \mathbb{R}^{n_y}$ is the measurement output of node $i$ and $\xi_{i,k} \in \mathbb{R}^{n_\vartheta}$ is the external disturbance belonging to $l_2[0, n-1]$. $C_{i,k}$ and $D_{i,k}$ are known time-varying matrices with compatible dimensions.

As stated in [1], [2], [17], in practical engineering, the sensors often suffer from the measurement censoring, which can be formulated by the following well-known Tobit type $I$ model:

$$y_{i,k}(t) = \begin{cases} y^*_{i,k}(t), & \text{if } y^*_{i,k}(t) > \Gamma_i(t); \\ \Gamma_i(t), & \text{otherwise}, \end{cases} \tag{3}$$

where $\Gamma_i(t) \in \mathbb{R}$ is a known threshold.

For convenience, we introduce the following indicative variable $p_{i,k}(t)$:

$$p_{i,k}(t) = \begin{cases} 1, & \text{if } y^*_{i,k}(t) > \Gamma_i(t); \\ 0, & \text{otherwise}. \end{cases} \tag{4}$$

Then, according to (3)-(4), the actual measurement output $y_{i,k}(t)$ is described as follows:

$$y_{i,k}(t) = p_{i,k}(t)y^*_{i,k}(t) + (1 - p_{i,k}(t))\Gamma_i(t). \tag{5}$$

Denoting

$$\begin{aligned} \Gamma_i &\triangleq [\Gamma_i(1) \quad \Gamma_i(2) \quad \cdots \quad \Gamma_i(n_y)]^T, \\ y_{i,k} &\triangleq [y_{i,k}(1) \quad y_{i,k}(2) \quad \cdots \quad y_{i,k}(n_y)]^T, \\ p_{i,k} &\triangleq \text{diag}\{p_{i,k}(1), p_{i,k}(2), \cdots, p_{i,k}(n_y)\}, \end{aligned}$$

one has

$$y_{i,k} = p_{i,k}y^*_{i,k} + (I - p_{i,k})\Gamma_i. \tag{6}$$

Noticing that the measurement outputs are only available from partial nodes, the following PNB distributed filters are constructed:

$$\begin{aligned} \hat{x}_{i,k+1} &= A_{0,k}\hat{x}_{i,k} + L_{i,k}(y_{i,k} - C_{i,k}\hat{x}_{i,k}) \\ &\quad + \sum_{j\in\mathcal{N}_i} K_{ij,k}(\hat{x}_{j,k} - \hat{x}_{i,k}), \\ &\quad i = 1, 2, \ldots, l_0, \end{aligned} \tag{7}$$

$$\begin{aligned} \hat{x}_{i,k+1} &= A_{0,k}\hat{x}_{i,k} + \sum_{j\in\mathcal{N}_i} K_{ij,k}(\hat{x}_{j,k} - \hat{x}_{i,k}), \\ &\quad i = l_0 + 1, l_0 + 2, \ldots, N, \end{aligned} \tag{8}$$

where $\hat{x}_{i,k} \in \mathbb{R}^{n_x}$ is the estimate of $x_k$ by node $i$, and $L_{i,k}$ and $K_{ij,k}$ are the filter parameters to be calculated later.

Setting $e_{i,k} \triangleq x_k - \hat{x}_{i,k}$, the filtering error dynamics of node $i$ is obtained from (1)-(8) as follows:

$$\begin{aligned} e_{i,k+1} =& (A_{0,k} - L_{i,k}C_{i,k} - \sum_{j\in\mathcal{N}_i} K_{ij,k})e_{i,k} + \sum_{t=1}^{r} \phi_{t,k}A_{t,k}x_k \\ &+ B_k w_k - L_{i,k}p_{i,k}D_{i,k}\xi_{i,k} + \sum_{j\in\mathcal{N}_i} K_{ij,k}e_{j,k} \\ &- L_{i,k}(I - p_{i,k})(\Gamma_i - C_{i,k}x_k), \\ &i = 1, 2, \ldots, l_0, \\ e_{i,k+1} =& (A_{0,k} - \sum_{j\in\mathcal{N}_i} K_{ij,k})e_{i,k} + \sum_{t=1}^{r} \phi_{t,k}A_{t,k}x_k \\ &+ B_k w_k + \sum_{j\in\mathcal{N}_i} K_{ij,k}e_{j,k}, \\ &i = l_0 + 1, l_0 + 2, \ldots, N. \end{aligned}$$

Denoting $\eta_{i,k} \triangleq [x^T_k \quad e^T_{i,k}]^T$, $\nu_{i,k} \triangleq [w^T_k \quad \xi^T_{i,k}]^T$, $\varpi_i \triangleq [0 \quad \Gamma^T_i]^T$, $z_{i,k} \triangleq \mathcal{F}\eta_{i,k}$ and $\mathcal{F} \triangleq [0 \quad I]$, the filtering error system is obtained with the following compact form:

$$\begin{cases} \eta_{i,k+1} = (\mathcal{A}_{0,k} + \mathcal{C}_{i,k} - \sum_{j\in\mathcal{N}_i} \mathcal{K}_{ij,k})\eta_{i,k} \\ \qquad + (\mathcal{B}_k + \mathcal{D}_{i,k})\nu_{i,k} + \sum_{j\in\mathcal{N}_i} \mathcal{K}_{ij,k}\eta_{j,k}, \\ \qquad - \mathcal{E}_{i,k}\varpi_i + \sum_{t=1}^{r} \phi_{t,k}\mathcal{A}_{t,k}\eta_{i,k} \\ \qquad i = 1, 2, \ldots, l_0, \\ \eta_{i,k+1} = (\mathcal{A}_{0,k} - \sum_{j\in\mathcal{N}_i} \mathcal{K}_{ij,k})\eta_{i,k} + \mathcal{B}_k\nu_{i,k} \\ \qquad + \sum_{t=1}^{r} \phi_{t,k}\mathcal{A}_{t,k}\eta_{i,k} + \sum_{j\in\mathcal{N}_i} \mathcal{K}_{ij,k}\eta_{j,k}, \\ \qquad i = l_0 + 1, l_0 + 2, \ldots, N, \\ z_{i,k} = \mathcal{F}\eta_{i,k}, \quad i = 1, 2, \ldots, N \end{cases} \tag{9}$$

where

$$\begin{aligned} &\mathcal{A}_{0,k} = \text{diag}\{A_{0,k}, A_{0,k}\}, \quad \mathcal{D}_{i,k} = \text{diag}\{0, -L_{i,k}p_{i,k}D_{i,k}\}, \\ &\mathcal{E}_{i,k} = \text{diag}\{0, L_{i,k}(1 - p_{i,k})\}, \quad \mathcal{K}_{ij,k} = \text{diag}\{0, K_{ij,k}\}, \\ &\mathcal{A}_{t,k} = \begin{bmatrix} A_{t,k} & 0 \\ A_{t,k} & 0 \end{bmatrix}, \quad \mathcal{B}_k = \begin{bmatrix} B_k & 0 \\ B_k & 0 \end{bmatrix}, \\ &\mathcal{C}_{i,k} = \begin{bmatrix} 0 & 0 \\ L_{i,k}(1 - p_{i,k})C_{i,k} & -L_{i,k}C_{i,k} \end{bmatrix}. \end{aligned}$$

In what follows, we are to examine the impact from the censored measurements on the system performance. Before proceeding, the following notations are introduced:

$$\begin{aligned} \nu_k &\triangleq [\nu^T_{1,k} \quad \ldots \quad \nu^T_{N,k}]^T, \\ z_k &\triangleq [z^T_{1,k} \quad \ldots \quad z^T_{N,k}]^T, \\ \bar{\mathbf{W}}_{i,k} &\triangleq [W_{i,k}(1) \quad W_{i,k}(2) \quad \ldots \quad W_{i,k}(n_y)], \\ \mathbf{exp}(\varpi_i) &\triangleq [\exp(\Gamma_i(1)) \quad \exp(\Gamma_i(2)) \quad \ldots \quad \exp(\Gamma_i(n_y))]^T, \end{aligned}$$

where $W_{i,k}(t) > 0$ for $t = 1, 2, \ldots, n_y$.

*Definition 1:* Let the disturbance attenuation level $\gamma > 0$, the weighted matrices $U_{i1} > 0$, $U_{i2} > 0$, $R_{i,k} > 0$, $Q_{i,k} > 0$, $T_{i1,k} > 0$, $T_{i2,k} > 0$ and $\bar{\mathbf{W}}_{i,k}$ be given. The filtering error system (9) satisfies the $H_\infty$-consensus performance constraint over the finite horizon $\mathcal{H}$ if the following inequality holds

$$\begin{aligned} \mathbb{E}\left\{\sum_{k=0}^{n-1} \Phi_{\mathcal{G}}(z_k)\right\} \le& \gamma^2 \sum_{i=1}^{l_0} \sum_{k=0}^{n-1} \bar{\mathbf{W}}_{i,k}\mathbf{exp}(\varpi_i) \\ &+ \gamma^2 \sum_{i=1}^{N} \left(\eta^T_{i,0}\mathcal{U}_i\eta_{i,0} + \sum_{k=0}^{n-1} \|\nu_{i,k}\|^2_{\mathcal{T}_{i,k}}\right), \end{aligned} \tag{10}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI10.1109/TSMC.2019.2907649, IEEE Transactions on Systems, Man, and Cybernetics: Systems


where

$$\Phi_{\mathcal{G}}(z_k) \triangleq \sum_{i=1}^{N} \left( \sum_{j\in\mathcal{N}_i} \|z_{j,k} - z_{i,k}\|^2_{R_{i,k}} + \|z_{i,k}\|^2_{Q_{i,k}} \right),$$
$$\mathcal{U}_i = \text{diag}\{U_{i1}, U_{i2}\}, \quad \mathcal{T}_{i,k} = \text{diag}\{T_{i1,k}, T_{i2,k}\}.$$

*Remark 1:* In comparison with the traditional $H_\infty$-consensus performance index, the effect of the measurement censoring is reflected in inequality (10) by introducing the second term on its right-hand side. It is noted that such a term is characterized by a weighted sum with respect to an exponential function of the censoring threshold over the time horizon $\mathcal{H}$, which would increase as the censoring threshold grows. This implies that the $H_\infty$-consensus performance would become worse as the censoring phenomena becomes more severe, which is in agreement with the engineering practice. As such, the proposed new performance index (10) accounts for the influence of the measurement censoring very well.

This paper aims to find the filter parameters $L_{i,k}$ and $K_{ij,k}$ respectively for node $i$ to guarantee that the dynamics of the system (9) satisfies the desirable $H_\infty$-consensus performance index (10).

## III. Main results

In this section, by resorting to the vector dissipativity theory, the analysis and design problems of the distributed $H_\infty$-consensus filter (7) will be discussed. To facilitate the subsequent development, we give the following notations

$$\begin{aligned}
\varpi &\triangleq [\varpi_1^T \quad \varpi_2^T \quad \cdots \quad \varpi_{l_0}^T]^T,\\
S_i(z_k, \nu_{i,k}, \varpi_i) &\triangleq \gamma^2(\|\nu_{i,k}\|^2_{\mathcal{T}_{i,k}} + \bar{\mathbf{W}}_{i,k}\mathbf{exp}(\varpi_i))\\
&\quad - \sum_{j\in\mathcal{N}_i} \|z_{j,k} - z_{i,k}\|^2_{R_{i,k}} - \|z_{i,k}\|^2_{Q_{i,k}},\\
&\quad i = 1, 2, \ldots, l_0, \qquad (11)\\
S_i(z_k, \nu_{i,k}) &\triangleq \gamma^2\|\nu_{i,k}\|^2_{\mathcal{T}_{i,k}} - \|z_{i,k}\|^2_{Q_{i,k}}\\
&\quad - \sum_{j\in\mathcal{N}_i} \|z_{j,k} - z_{i,k}\|^2_{R_{i,k}},\\
&\quad i = l_0 + 1, l_0 + 2, \ldots, N.
\end{aligned}$$

*Definition 2:* The filtering error dynamics (9) is strictly stochastic vector-dissipative over the finite horizon $\mathcal{H}$ regarding the vector-valued supply rate function

$$\begin{aligned}
\mathbf{S}(z_k, \nu_k, \varpi) \triangleq &[S_1(z_k, \nu_{1,k}, \varpi_1), \ldots, S_{l_0}(z_k, \nu_{l_0,k}, \varpi_{l_0}),\\
&S_{l_0+1}(z_k, \nu_{l_0+1,k}) \ldots, S_N(z_k, \nu_{N,k})]^T,
\end{aligned}$$

if there exist a vector-valued storage function $\mathbf{V}(\eta_k) \triangleq [V_1(\eta_{1,k}), \ldots, V_N(\eta_{N,k})]^T$ (with $\mathbf{V}(0) = 0$ and $V_i(\eta_{i,k}) \geq 0$, $i = 1, 2, \ldots N$) and a dissipation matrix ( [12], [13]) sequence $U_k \in \mathbb{R}^{N\times N}$ such that the following inequality holds for any $k \in \mathcal{H}$:

$$\mathbb{E}\{\mathbf{V}(\eta_{k+1})\} \ll U_k\mathbb{E}\{\mathbf{V}(\eta_k)\} + \mathbb{E}\{\mathbf{S}(z_k, \nu_k, \varpi)\}. \qquad (12)$$

In the following, sufficient conditions are provided to guarantee that the filtering error dynamics (9) is strictly stochastic vector-dissipative over the finite horizon $\mathcal{H}$. In order to construct a desirable dissipation matrix $U_k$, we first define an interval-valued function $\mathscr{I}_{q_i}$ concerning out-degree $q_i$ as follows:

$$\mathscr{I}_{q_i} = \begin{cases} (0, \frac{1+q_i}{2q_i}), & \text{if } q_i \neq 0;\\ (0, 1], & \text{if } q_i = 0. \end{cases} \qquad (13)$$

*Theorem 1:* For given the real number $\gamma > 0$, the constant sequence $\alpha_{i,k} \in \mathscr{I}_{q_i}$, the matrices $R_{i,k} > 0$, $Q_{i,k} > 0$, $\mathcal{T}_{i,k} > 0$, $\bar{\mathbf{W}}_{i,k}$, and the filter parameters $L_{i,k}$ and $K_{ij,k}$, the filtering error dynamics (9) is strictly stochastic vector-dissipative over the finite horizon $\mathcal{H}$ regarding the vector-valued supply rate function $\mathbf{S}(z_k, \nu_k, \varpi)$ and also satisfies the $H_\infty$-consensus performance constraint (10), if there exists a vector-valued storage function $\mathbf{V}(\eta_k)$ (whose element is $V_i(\eta_{i,k}) = \eta_{i,k}^T\mathcal{P}_{i,k}\eta_{i,k}$, where $\{\mathcal{P}_{i,k}\}_{k\in\mathcal{H}\cup\{n\}}$ is a sequence of positive definite matrices with the initial condition $\mathcal{P}_{i,0} \leq \gamma^2 U_i$), such that the following conditions hold for $k \in \mathcal{H}, \forall i = 1, 2, \ldots, N$:

$$\begin{cases}
\Xi_{i,k} \triangleq \begin{bmatrix} \Xi_{i,k}^{11} & \Xi_{i,k}^{12} & \Xi_{i,k}^{13}\\ * & \Xi_{i,k}^{22} & \Xi_{i,k}^{23}\\ * & * & \Xi_{i,k}^{33} \end{bmatrix} < 0, & \text{(14a)}\\
\Psi_{i,k} \triangleq -\frac{1}{\gamma^2}(\mathcal{B}_k + \mathcal{D}_{i,k})\mathcal{T}_{i,k}^{-1}(\mathcal{B}_k + \mathcal{D}_{i,k})^T & \\
\qquad + \mathcal{P}_{i,k+1}^{-1} > 0, & \text{(14b)}\\
\forall i = 1, 2, \ldots, l_0, &
\end{cases}$$

and

$$\begin{cases}
\Upsilon_{i,k} \triangleq \begin{bmatrix} \Upsilon_{i,k}^{11} & \Upsilon_{i,k}^{12}\\ * & \Upsilon_{i,k}^{22} \end{bmatrix} < 0, & \text{(15a)}\\
\bar{\Psi}_{i,k} \triangleq \mathcal{P}_{i,k+1}^{-1} - \frac{1}{\gamma^2}\mathcal{B}_k\mathcal{T}_{i,k}^{-1}\mathcal{B}_k^T > 0, & \text{(15b)}\\
\forall i = l_0 + 1, l_0 + 2, \ldots, N, &
\end{cases}$$

where

$$\begin{aligned}
\Xi_{i,k}^{11} =& \sum_{t=1}^{r} \mathcal{A}_{t,k}^T\mathcal{P}_{i,k+1}\mathcal{A}_{t,k} + \mathcal{F}^T(Q_{i,k} + R_{i,k})\mathcal{F} - \rho_{i,k}\mathcal{P}_{i,k}\\
&+ (\mathcal{A}_{0,k} + \mathcal{C}_{i,k} - \sum_{j\in\mathcal{N}_i} \mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}(\bullet),\\
\Xi_{i,k}^{12} =& \Big[(\mathcal{A}_{0,k} + \mathcal{C}_{i,k} - \sum_{j\in\mathcal{N}_i} \mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}\mathcal{K}_{ij_1,k} - \mathcal{F}^TR_{i,k}\mathcal{F} \ \cdots\\
&(\mathcal{A}_{0,k} + \mathcal{C}_{i,k} - \sum_{j\in\mathcal{N}_i} \mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}\mathcal{K}_{ij_{p_i},k} - \mathcal{F}^TR_{i,k}\mathcal{F}\Big],\\
\Xi_{i,k}^{22} =& [\mathcal{K}_{ij_1,k} \ \cdots \ \mathcal{K}_{ij_{p_i},k}]^T\Psi_{i,k}^{-1}[\mathcal{K}_{ij_1,k} \ \cdots \ \mathcal{K}_{ij_{p_i},k}]\\
&- \text{diag}\Big\{\frac{\alpha_{j_{i1},k}}{1 + q_{j_{i1}}}\mathcal{P}_{j_{i1},k} - \mathcal{F}^TR_{i,k}\mathcal{F}, \ldots,\\
&\frac{\alpha_{j_{ip_i},k}}{1 + q_{ij_{p_i}}}\mathcal{P}_{ij_{p_i},k} - \mathcal{F}^TR_{i,k}\mathcal{F}\Big\},\\
\Xi_{i,k}^{13} =& -(\mathcal{A}_{0,k} + \mathcal{C}_{i,k} - \sum_{j\in\mathcal{N}_i} \mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}\mathcal{E}_{i,k}\varpi_i,\\
\Xi_{i,k}^{23} =& -\left[(\mathcal{K}_{ij_1,k}\Psi_{i,k}^{-1}\mathcal{E}_{i,k}\varpi_i)^T \ \cdots \ (\mathcal{K}_{ij_{p_i},k}\Psi_{i,k}^{-1}\mathcal{E}_{i,k}\varpi_i)^T\right]^T,\\
\Xi_{i,k}^{33} =& \varpi_i^T\mathcal{E}_{i,k}^T\Psi_{i,k}^{-1}\mathcal{E}_{i,k}\varpi_i - \gamma^2\bar{\mathbf{W}}_{i,k}\mathbf{exp}(\varpi_i),\\
\Upsilon_{i,k}^{11} =& \sum_{t=1}^{r} \mathcal{A}_{t,k}^T\mathcal{P}_{i,k+1}\mathcal{A}_{t,k} + \mathcal{F}^T(Q_{i,k} + R_{i,k})\mathcal{F} - \rho_{i,k}\mathcal{P}_{i,k}
\end{aligned}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI10.1109/TSMC.2019.2907649, IEEE Transactions on Systems, Man, and Cybernetics: Systems


$$
\begin{aligned}
&+(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\bar{\Psi}_{i,k}^{-1}(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}),\\
\Upsilon_{i,k}^{12}=&\Big[(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\bar{\Psi}_{i,k}^{-1}\mathcal{K}_{ij_1,k}-\mathcal{F}^TR_{i,k}\mathcal{F}\\
&\cdots\ (\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\bar{\Psi}_{i,k}^{-1}\mathcal{K}_{ij_{p_i},k}-\mathcal{F}^TR_{i,k}\mathcal{F}\Big],\\
\Upsilon_{i,k}^{22}=&[\mathcal{K}_{ij_1,k}\ \cdots\ \mathcal{K}_{ij_{p_i},k}]^T\bar{\Psi}_{i,k}^{-1}[\mathcal{K}_{ij_1,k}\ \cdots\ \mathcal{K}_{ij_{p_i},k}]\\
&-\mathrm{diag}\Big\{\frac{\alpha_{j_{i1},k}}{1+q_{j_{i1}}}\mathcal{P}_{j_{i1},k}-\mathcal{F}^TR_{i,k}\mathcal{F},\ldots,\\
&\frac{\alpha_{j_{ip_i},k}}{1+q_{j_{ip_i}}}\mathcal{P}_{j_{ip_i},k}-\mathcal{F}^TR_{i,k}\mathcal{F}\Big\},\\
\rho_{i,k}=&\frac{1+q_i(1-\alpha_{i,k})}{1+q_i}.
\end{aligned}
$$

*Proof:* For presentation clarity, the proof is divided into the following four steps.

1) *Proof of the stochastic vector-dissipativity for the nodes $i$ $(i=1,2,\ldots,l_0)$ over the finite-horizon $\mathcal{H}$.*

First, for node $i$ $(i=1,2,\ldots,l_0)$, it follows that

$$
\begin{aligned}
&\mathbb{E}\{V_i(\eta_{i,k+1})|\eta_{i,k}\}\\
=&\mathbb{E}\{\eta_{i,k+1}^T\mathcal{P}_{i,k+1}\eta_{i,k+1}|\eta_{i,k}\}\\
=&\mathbb{E}\Big\{\Big((\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})\eta_{i,k}+\sum_{t=1}^{r}\phi_{t,k}\mathcal{A}_{t,k}\eta_{i,k}\\
&+(\mathcal{B}_k+\mathcal{D}_{i,k})\nu_{i,k}+\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}-\mathcal{E}_{i,k}\varpi_i\Big)^T\mathcal{P}_{i,k+1}\\
&\times\Big((\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})\eta_{i,k}+\sum_{t=1}^{r}\phi_{t,k}\mathcal{A}_{t,k}\eta_{i,k}\\
&+(\mathcal{B}_k+\mathcal{D}_{i,k})\nu_{i,k}+\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}-\mathcal{E}_{i,k}\varpi_i\Big)\Big|\eta_{i,k}\Big\}\\
=&\eta_{i,k}^T\Big(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\Big)^T\mathcal{P}_{i,k+1}\Big(\bullet\Big)\eta_{i,k}\\
&+\nu_{i,k}^T(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}(\mathcal{B}_k+\mathcal{D}_{i,k})\nu_{i,k}\\
&+\Big(\sum_{j\in\mathcal{N}_i}\eta_{j,k}^T\mathcal{K}_{ij,k}^T\Big)\mathcal{P}_{i,k+1}\Big(\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\Big)\\
&+2\eta_{i,k}^T(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\mathcal{P}_{i,k+1}(\mathcal{B}_k+\mathcal{D}_{i,k})\nu_{i,k}\\
&+2\eta_{i,k}^T(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
&-2\eta_{i,k}^T(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\mathcal{P}_{i,k+1}\mathcal{E}_{i,k}\varpi_i\\
&+2\nu_{i,k}^T(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
&-2\nu_{i,k}^T(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}\mathcal{E}_{i,k}\varpi_i\\
&-2\varpi_i^T\mathcal{E}_{i,k}^T\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
&+\eta_{i,k}^T\sum_{t=1}^{r}\mathcal{A}_{t,k}^T\mathcal{P}_{i,k+1}\mathcal{A}_{t,k}\eta_{i,k}\\
&+\varpi_i^T\mathcal{E}_{i,k}^T\mathcal{P}_{i,k+1}\mathcal{E}_{i,k}\varpi_i\\
&+\gamma^2\nu_{i,k}^T\mathcal{T}_{i,k}\nu_{i,k}-\gamma^2\nu_{i,k}^T\mathcal{T}_{i,k}\nu_{i,k}. \qquad (16)
\end{aligned}
$$

Next, it is readily obtained from (14c) that $\Psi_{i,k}>0$. Then, by applying the Schur Complement Lemma to (15b), one has $\Phi_{i,k}^{-1}\triangleq\gamma^2\mathcal{T}_{i,k}-(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}(\mathcal{B}_k+\mathcal{D}_{i,k})>0$. On the other hand, it is easy to see that the following inequality is true:

$$
\begin{aligned}
&\Big(\nu_{i,k}-\Phi_{i,k}(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})\eta_{i,k}\\
&-\Phi_{i,k}(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}+\Phi_{i,k}(\mathcal{B}_k+\mathcal{D}_{i,k})^T\\
&\times\mathcal{P}_{i,k+1}\mathcal{E}_{i,k}\varpi_i\Big)^T\Phi_{i,k}^{-1}\Big(\bullet\Big)\geq 0,
\end{aligned}
$$

and therefore

$$
\begin{aligned}
&\nu_{i,k}^T((\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}(\mathcal{B}_k+\mathcal{D}_{i,k})-\gamma^2\mathcal{T}_{i,k})\nu_{i,k}\\
&+2\eta_{i,k}^T(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\mathcal{P}_{i,k+1}(\mathcal{B}_k+\mathcal{D}_{i,k})\nu_{i,k}\\
&+2\nu_{i,k}^T(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
&-2\nu_{i,k}^T(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}\mathcal{E}_{i,k}\varpi_i\\
\leq&\eta_{i,k}^T(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\mathcal{P}_{i,k+1}(\mathcal{B}_k+\mathcal{D}_{i,k})\\
&\times\Phi_{i,k}(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})\eta_{i,k}\\
&+\sum_{j\in\mathcal{N}_i}\eta_{j,k}^T\mathcal{K}_{ij,k}^T\mathcal{P}_{i,k+1}(\mathcal{B}_k+\mathcal{D}_{i,k})\Phi_{i,k}(\mathcal{B}_k+\mathcal{D}_{i,k})^T\\
&\times\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}+2\eta_{i,k}^T(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\\
&\times\mathcal{P}_{i,k+1}(\mathcal{B}_k+\mathcal{D}_{i,k})\Phi_{i,k}(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
&+\varpi_i^T\mathcal{E}_{i,k}^T\mathcal{P}_{i,k+1}(\mathcal{B}_k+\mathcal{D}_{i,k})\Phi_{i,k}(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}\mathcal{E}_{i,k}\varpi_i\\
&-2\eta_{i,k}^T(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\mathcal{P}_{i,k+1}(\mathcal{B}_k+\mathcal{D}_{i,k})\\
&\times\Phi_{i,k}(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}\mathcal{E}_{i,k}\varpi_i-2\sum_{j\in\mathcal{N}_i}\eta_{j,k}^T\mathcal{K}_{ij,k}^T\mathcal{P}_{i,k+1}\\
&\times(\mathcal{B}_k+\mathcal{D}_{i,k})\Phi_{i,k}(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}\mathcal{E}_{i,k}\varpi_i. \qquad (17)
\end{aligned}
$$

Moreover, with the help of the matrix inverse formula, we derive that

$$
\begin{aligned}
&\mathcal{P}_{i,k+1}(\mathcal{B}_k+\mathcal{D}_{i,k})\Phi_{i,k}(\mathcal{B}_k+\mathcal{D}_{i,k})^T\mathcal{P}_{i,k+1}+\mathcal{P}_{i,k+1}\\
=&(\mathcal{P}_{i,k+1}^{-1}-\frac{1}{\gamma^2}(\mathcal{B}_k+\mathcal{D}_{i,k})\mathcal{T}_{i,k}^{-1}(\mathcal{B}_k+\mathcal{D}_{i,k})^T)^{-1}\\
\triangleq&\Psi_{i,k}^{-1}. \qquad (18)
\end{aligned}
$$

In combination of (16)-(18), one finds

$$
\begin{aligned}
&\mathbb{E}\{V_i(\eta_{i,k+1})|\eta_{i,k}\}\\
\leq&\eta_{i,k}^T(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}(\bullet)\eta_{i,k}\\
&+2\eta_{i,k}^T(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
&-2\eta_{i,k}^T(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}\mathcal{E}_{i,k}\varpi_i
\end{aligned}
$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI10.1109/TSMC.2019.2907649, IEEE Transactions on Systems, Man, and Cybernetics: Systems


$$
\begin{aligned}
&-2\sum_{j\in\mathcal{N}_i}\eta_{j,k}^T\mathcal{K}_{ij,k}^T\Psi_{i,k}^{-1}\mathcal{E}_{i,k}\varpi_i\\
&+\sum_{j\in\mathcal{N}_i}\eta_{j,k}^T\mathcal{K}_{ij,k}^T\Psi_{i,k}^{-1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
&+\eta_{i,k}^T\sum_{t=1}^{r}\mathcal{A}_{t,k}^T\mathcal{P}_{i,k+1}\mathcal{A}_{t,k}\eta_{i,k}\\
&+\varpi_i^T\mathcal{E}_{i,k}^T\Psi_{i,k}^{-1}\mathcal{E}_{i,k}\varpi_i+\gamma^2\nu_{i,k}^T\mathcal{T}_{i,k}\nu_{i,k}\\
&\triangleq\chi_{i,k}^T\Pi_{i,k}\chi_{i,k}+\gamma^2\nu_{i,k}^T\mathcal{T}_{i,k}\nu_{i,k} \qquad (19)
\end{aligned}
$$

where $\chi_{i,k}=[\eta_{i,k}^T\ \eta_{\mathcal{N}_i,k}^T\ 1]^T$, $\eta_{\mathcal{N}_i,k}=[\eta_{j_1,k}^T\ \cdots\ \eta_{j_{p_i},k}^T]^T$ and

$$
\Pi_{i,k}=\begin{bmatrix}\Pi_{i,k}^{11} & \Pi_{i,k}^{12} & \Pi_{i,k}^{13}\\ * & \Pi_{i,k}^{22} & \Pi_{i,k}^{23}\\ * & * & \Pi_{i,k}^{33}\end{bmatrix}
$$

with

$$
\begin{aligned}
\Pi_{i,k}^{11}=&(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}\\
&-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})+\sum_{t=1}^{r}\mathcal{A}_{t,k}^T\mathcal{P}_{i,k+1}\mathcal{A}_{t,k},\\
\Pi_{i,k}^{12}=&\Big[(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}\mathcal{K}_{ij_1,k}\ \cdots\\
&(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}\mathcal{K}_{ij_{p_i},k}\Big],\\
\Pi_{i,k}^{13}=&-(\mathcal{A}_{0,k}+\mathcal{C}_{i,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}\mathcal{E}_{i,k}\varpi_i,\\
\Pi_{i,k}^{22}=&\big[\mathcal{K}_{ij_1,k}\ \ldots\ \mathcal{K}_{ij_{p_i},k}\big]^T\Psi_{i,k}^{-1}\big[\mathcal{K}_{ij_1,k}\ \ldots\ \mathcal{K}_{ij_{p_i},k}\big],\\
\Pi_{i,k}^{23}=&-\big[(\mathcal{K}_{ij_1,k}\Psi_{i,k}^{-1}\mathcal{E}_{i,k}\varpi_i)^T\ \ldots\ (\mathcal{K}_{ij_{p_i},k}\Psi_{i,k}^{-1}\mathcal{E}_{i,k}\varpi_i)^T\big]^T,\\
\Pi_{i,k}^{33}=&\ \varpi_i^T\mathcal{E}_{i,k}^T\Psi_{i,k}^{-1}\mathcal{E}_{i,k}\varpi_i.
\end{aligned}
$$

By some straightforward algebraic manipulations, we can see that the following inequality is true:

$$
\begin{aligned}
&\chi_{i,k}^T\Pi_{i,k}\chi_{i,k}+\sum_{j\in\mathcal{N}_i}\|z_{j,k}-z_{i,k}\|_{R_{i,k}}^2-\rho_{i,k}\eta_{i,k}^T\mathcal{P}_{i,k}\eta_{i,k}\\
&-\sum_{j\in\mathcal{N}_i}\frac{\alpha_{j,k}}{1+q_j}\eta_{j,k}^T\mathcal{P}_{j,k}\eta_{j,k}+\|z_{i,k}\|_{Q_{i,k}}^2-\gamma^2\bar{\mathbf{W}}_{i,k}\mathbf{exp}(\varpi_i)\\
=&\chi_{i,k}^T\Xi_{i,k}\chi_{i,k}<0, \qquad (20)
\end{aligned}
$$

which implies

$$
\begin{aligned}
\chi_{i,k}^T\Pi_{i,k}\chi_{i,k}<&-\sum_{j\in\mathcal{N}_i}\|z_{j,k}-z_{i,k}\|_{R_{i,k}}^2-\|z_{i,k}\|_{Q_{i,k}}^2\\
&+\sum_{j\in\mathcal{N}_i}\frac{\alpha_{j,k}}{1+q_j}\eta_{j,k}^T\mathcal{P}_{j,k}\eta_{j,k}+\rho_{i,k}\eta_{i,k}^T\mathcal{P}_{i,k}\eta_{i,k}\\
&+\gamma^2\bar{\mathbf{W}}_{i,k}\mathbf{exp}(\varpi_i).
\end{aligned}
$$

Recalling the expression of $S_i(z_k,\nu_{i,k},\varpi_i)$ and (14a), one has

$$
\begin{aligned}
&\mathbb{E}\{V_i(\eta_{i,k+1})|\eta_{i,k}\}\\
\le&\chi_{i,k}^T\Pi_{i,k}\chi_{i,k}+\gamma^2\nu_{i,k}^T\mathcal{T}_{i,k}\nu_{i,k}\\
<&\gamma^2\nu_{i,k}^T\mathcal{T}_{i,k}\nu_{i,k}+\gamma^2\bar{\mathbf{W}}_{i,k}\mathbf{exp}(\varpi_i)-\sum_{j\in\mathcal{N}_i}\|z_{j,k}-z_{i,k}\|_{R_{i,k}}^2\\
&-\|z_{i,k}\|_{Q_{i,k}}^2+\sum_{j\in\mathcal{N}_i}\frac{\alpha_{j,k}}{1+q_j}\eta_{j,k}^T\mathcal{P}_{j,k}\eta_{j,k}+\rho_{i,k}\eta_{i,k}^T\mathcal{P}_{i,k}\eta_{i,k}\\
=&[\frac{a_{i1}\alpha_{1,k}}{1+q_1},\ldots,\rho_{i,k},\ldots,\frac{a_{iN}\alpha_{N,k}}{1+q_N}]\mathbf{V}(\eta_k)+S_i(z_k,\nu_{i,k},\varpi_i)\\
=&[U_k\mathbf{V}(\eta_k)]_i+S_i(z_k,\nu_{i,k},\varpi_i), \qquad (21)
\end{aligned}
$$

where

$$
U_k=\begin{bmatrix}
\rho_{1,k} & \cdots & \frac{a_{1i}\alpha_{i,k}}{1+q_i} & \cdots & \frac{a_{1N}\alpha_{N,k}}{1+q_N}\\
\vdots & \ddots & \vdots & \cdots & \vdots\\
\frac{a_{i1}\alpha_{1,k}}{1+q_1} & \cdots & \rho_{i,k} & \cdots & \frac{a_{iN}\alpha_{N,k}}{1+q_N}\\
\vdots & \cdots & \vdots & \ddots & \vdots\\
\frac{a_{N1}\alpha_{1,k}}{1+q_1} & \cdots & \frac{a_{Ni}\alpha_{i,k}}{1+q_i} & \cdots & \rho_{N,k}
\end{bmatrix}. \qquad (22)
$$

Since $\alpha_{i,k}\in\mathscr{I}_{q_i}$, it is inferred that $U_k$ is nonsingular. Notice $\mathbf{1}^TU_k=\mathbf{1}^T$ and, therefore, $U_k$ is the dissipation matrix. Moreover, in terms of the property of the conditional expectation, the following inequality is acquired:

$$
\begin{aligned}
&\mathbb{E}\{V_i(\eta_{i,k+1})\}\\
<&\mathbb{E}\{[U_k\mathbf{V}(\eta_k)]_i\}+\mathbb{E}\{S_i(z_k,\nu_{i,k},\varpi_i)\}, \qquad (23)\\
&i=1,2,\ldots,l_0.
\end{aligned}
$$

2) *Proof of the stochastic vector-dissipativity for the nodes* $i$ $(i=l_0+1,l_0+2,\ldots,N)$ *over the finite horizon* $\mathcal{H}$.

For the node $i$ $(i=l_0+1,\ldots,N)$, it follows immediately from the system dynamics (9) that

$$
\begin{aligned}
&\mathbb{E}\{V_i(\eta_{i,k+1})|\eta_{i,k}\}\\
=&\mathbb{E}\{\eta_{i,k+1}^T\mathcal{P}_{i,k+1}\eta_{i,k+1}|\eta_{i,k}\}\\
=&\mathbb{E}\Big\{\Big((\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})\eta_{i,k}+\sum_{t=1}^{r}\phi_{t,k}\mathcal{A}_{t,k}\eta_{i,k}+\mathcal{B}_k\nu_{i,k}\\
&+\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\Big)^T\mathcal{P}_{i,k+1}\Big(\bullet\Big)\Big|\eta_{i,k}\Big\}\\
=&\eta_{i,k}^T\Big(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\Big)^T\mathcal{P}_{i,k+1}\Big(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\Big)\eta_{i,k}\\
&+\eta_{i,k}^T\sum_{t=1}^{r}\mathcal{A}_{t,k}^T\mathcal{P}_{i,k+1}\mathcal{A}_{t,k}\eta_{i,k}\\
&+\nu_{i,k}^T\mathcal{B}_k^T\mathcal{P}_{i,k+1}\mathcal{B}_k\nu_{i,k}\\
&+\Big(\sum_{j\in\mathcal{N}_i}\eta_{j,k}^T\mathcal{K}_{ij,k}^T\Big)\mathcal{P}_{i,k+1}\Big(\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\Big)\\
&+2\eta_{i,k}^T\Big(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\Big)^T\mathcal{P}_{i,k+1}\mathcal{B}_k\nu_{i,k}\\
&+2\eta_{i,k}^T\Big(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\Big)^T\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
&+2\nu_{i,k}^T\mathcal{B}_k^T\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
&+\gamma^2\nu_{i,k}^T\mathcal{T}_{i,k}\nu_{i,k}-\gamma^2\nu_{i,k}^T\mathcal{T}_{i,k}\nu_{i,k}. \qquad (24)
\end{aligned}
$$

It is obvious that (15b) leads to $\bar{\Psi}_{i,k}>0$. Then, by applying the Schur Complement Lemma again, one obtains $\bar{\Phi}_{i,k}^{-1}=\gamma^2\mathcal{T}_{i,k}-\mathcal{B}_k^T\mathcal{P}_{i,k+1}\mathcal{B}_k>0$. On the basis of the following inequality

$$
\Big(\nu_{i,k}-\bar{\Phi}_{i,k}\mathcal{B}_k^T\mathcal{P}_{i,k+1}(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})\eta_{i,k}-\bar{\Phi}_{i,k}\mathcal{B}_k^T
$$

$$\times \mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\Big)^T\bar{\Phi}_{i,k}^{-1}\Big(\bullet\Big)\geq 0,$$

we have

$$\begin{aligned}
&\nu_{i,k}^T(\mathcal{B}_k^T\mathcal{P}_{i,k+1}\mathcal{B}_k-\gamma^2\mathcal{T}_{i,k})\nu_{i,k}\\
&+2\eta_{i,k}^T(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\mathcal{P}_{i,k+1}\mathcal{B}_k\nu_{i,k}\\
&+2\nu_{i,k}^T\mathcal{B}_k^T\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
<&\eta_{i,k}^T(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\mathcal{P}_{i,k+1}\mathcal{B}_k\bar{\Phi}_{i,k}\mathcal{B}_k^T\mathcal{P}_{i,k+1}(\mathcal{A}_{0,k}\\
&-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})\eta_{i,k}+2\eta_{i,k}^T(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\mathcal{P}_{i,k+1}\\
&\times\mathcal{B}_k\bar{\Phi}_{i,k}\mathcal{B}_k^T\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}+\sum_{j\in\mathcal{N}_i}\eta_{j,k}^T\mathcal{K}_{ij,k}^T\\
&\times\mathcal{P}_{i,k+1}\mathcal{B}_k\bar{\Phi}_{i,k}\mathcal{B}_k^T\mathcal{P}_{i,k+1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}. \qquad (25)
\end{aligned}$$

According to the matrix inverse formula, one derives

$$\begin{aligned}
&\mathcal{P}_{i,k+1}\mathcal{B}_k\bar{\Phi}_{i,k}\mathcal{B}_k^T\mathcal{P}_{i,k+1}+\mathcal{P}_{i,k+1}\\
=&(\mathcal{P}_{i,k+1}^{-1}-\frac{1}{\gamma^2}\mathcal{B}_k\mathcal{T}_{i,k}^{-1}\mathcal{B}_k^T)^{-1}\\
\triangleq&\bar{\Psi}_{i,k}^{-1}. \qquad (26)
\end{aligned}$$

Then, it is not difficult to verify from (24)-(26) that

$$\begin{aligned}
&\mathbb{E}\{V_i(\eta_{i,k+1})|\eta_{i,k}\}\\
\leq&\sum_{j\in\mathcal{N}_i}\eta_{j,k}^T\mathcal{K}_{ij,k}^T\Psi_{i,k}^{-1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
&+\eta_{i,k}^T\sum_{t=1}^{r}\mathcal{A}_{t,k}^T\mathcal{P}_{i,k+1}\mathcal{A}_{t,k}\eta_{i,k}+\gamma^2\nu_{i,k}^T\mathcal{T}_{i,k}\nu_{i,k}\\
&+2\eta_{i,k}^T(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\bar{\Psi}_{i,k}^{-1}\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k}\eta_{j,k}\\
&+\eta_{i,k}^T(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\bar{\Psi}_{i,k}^{-1}(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})\eta_{i,k}\\
\triangleq&\bar{\chi}_{i,k}^T\bar{\Pi}_{i,k}\bar{\chi}_{i,k}+\gamma^2\nu_{i,k}^T\mathcal{T}_{i,k}\nu_{i,k}, \qquad (27)
\end{aligned}$$

where

$$\begin{aligned}
\bar{\chi}_{i,k}=&[\eta_{i,k}^T\ \eta_{\mathcal{N}_i,k}^T]^T,\quad \eta_{\mathcal{N}_i,k}=[\eta_{j_1,k}^T\ \cdots\ \eta_{j_{p_i},k}^T]^T,\\
\bar{\Pi}_{i,k}=&\begin{bmatrix}\bar{\Pi}_{i,k}^{11} & \bar{\Pi}_{i,k}^{12}\\ * & \bar{\Pi}_{i,k}^{22}\end{bmatrix},\\
\bar{\Pi}_{i,k}^{11}=&(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\bar{\Psi}_{i,k}^{-1}(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})\\
&+\sum_{t=1}^{r}\mathcal{A}_{t,k}^T\mathcal{P}_{i,k+1}\mathcal{A}_{t,k},\\
\bar{\Pi}_{i,k}^{12}=&\Big[(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\bar{\Psi}_{i,k}^{-1}\mathcal{K}_{ij_1,k}\ \ldots\\
&(\mathcal{A}_{0,k}-\sum_{j\in\mathcal{N}_i}\mathcal{K}_{ij,k})^T\Psi_{i,k}^{-1}\mathcal{K}_{ij_{p_i},k}\Big],\\
\bar{\Pi}_{i,k}^{22}=&[\mathcal{K}_{ij_1,k}\ \ldots\ \mathcal{K}_{ij_{p_i},k}]^T\bar{\Psi}_{i,k}^{-1}[\mathcal{K}_{ij_1,k}\ \ldots\ \mathcal{K}_{ij_{p_i},k}].
\end{aligned}$$

Moreover, (15a) implies

$$\begin{aligned}
&\bar{\chi}_{i,k}^T\bar{\Pi}_{i,k}\bar{\chi}_{i,k}+\sum_{j\in\mathcal{N}_i}\|z_{j,k}-z_{i,k}\|_{R_{i,k}}^2+\|z_{i,k}\|_{Q_{i,k}}^2\\
&-\sum_{j\in\mathcal{N}_i}\frac{\alpha_{j,k}}{1+q_j}\eta_{j,k}^T\mathcal{P}_{j,k}\eta_{j,k}-\rho_{i,k}\eta_{i,k}^T\mathcal{P}_{i,k}\eta_{i,k}\\
=&\zeta_{i,k}^T\Upsilon_{i,k}\zeta_{i,k}<0, \qquad (28)
\end{aligned}$$

namely,

$$\begin{aligned}
\bar{\chi}_{i,k}^T\bar{\Pi}_{i,k}\bar{\chi}_{i,k}<&-\sum_{j\in\mathcal{N}_i}\|z_{j,k}-z_{i,k}\|_{R_{i,k}}^2-\|z_{i,k}\|_{Q_{i,k}}^2\\
&+\sum_{j\in\mathcal{N}_i}\frac{\alpha_{j,k}}{1+q_j}\eta_{j,k}^T\mathcal{P}_{j,k}\eta_{j,k}+\rho_{i,k}\eta_{i,k}^T\mathcal{P}_{i,k}\eta_{i,k}.
\end{aligned}$$

Noticing $S_i(z_k,\nu_{i,k})$ and (14a), we have the following conclusion:

$$\begin{aligned}
&\mathbb{E}\{V_i(\eta_{i,k+1})|\eta_{i,k}\}\\
\leq&\bar{\chi}_{i,k}^T\bar{\Pi}_{i,k}\bar{\chi}_{i,k}\\
<&\gamma^2\nu_{i,k}^T\mathcal{T}_{i,k}\nu_{i,k}-\sum_{j\in\mathcal{N}_i}\|z_{j,k}-z_{i,k}\|_{R_{i,k}}^2-\|z_{i,k}\|_{Q_{i,k}}^2\\
&+\sum_{j\in\mathcal{N}_i}\frac{\alpha_{j,k}}{1+q_j}\eta_{j,k}^T\mathcal{P}_{j,k}\eta_{j,k}+\rho_{i,k}\eta_{i,k}^T\mathcal{P}_{i,k}\eta_{i,k}\\
=&\Big[\frac{a_{i1}\alpha_{1,k}}{1+q_1},\ldots,\rho_{i,k},\ldots,\frac{a_{iN}\alpha_{N,k}}{1+q_N}\Big]\mathbf{V}(\eta_k)+S_i(z_k,\nu_{i,k})\\
=&[U_k\mathbf{V}(\eta_k)]_i+S_i(z_k,\nu_{i,k}). \qquad (29)
\end{aligned}$$

By applying the property of the conditional expectation to (29), we further obtain the following relationship:

$$\begin{aligned}
\mathbb{E}\{V_i(\eta_{i,k+1})\}<\mathbb{E}\{[U_k\mathbf{V}(\eta_k)]_i\}+\mathbb{E}\{S_i(z_k,\nu_{i,k})\},\\
i=l_0+1,\ldots,N. \qquad (30)
\end{aligned}$$

3) *Proof of the stochastic vector-dissipativity for the nodes* $i=1,2,\ldots,l_0,l_0+1,\ldots,N$.

Based on the above results, it is observed from (23) and (30) that the following vector inequality is satisfied:

$$\mathbb{E}\{\mathbf{V}(\eta_{k+1})\}\ll U_k\mathbb{E}\{\mathbf{V}(\eta_k)\}+\mathbb{E}\{\mathbf{S}(z_k,\nu_k,\varpi)\}. \qquad (31)$$

Consequently, it is readily seen from Definition 2 that the dynamics (9) is stochastic vector-dissipative over the finite horizon $\mathcal{H}$.

4) *Proof of the* $H_\infty$*-consensus performance index.*

In this step, we aim to prove the guaranteed $H_\infty$-consensus performance of the distributed filtering scheme. First, left-multiplying $\mathbf{1}^T$ on both sides of (31) yields

$$\mathbb{E}\{\mathbf{1}^T\mathbf{V}(\eta_{k+1})\}<\mathbb{E}\{\mathbf{1}^T\mathbf{S}(z_k,\nu_k,\varpi)\}+\mathbb{E}\{\mathbf{1}^TU_k\mathbf{V}(\eta_k)\}. \qquad (32)$$

Denoting $v(\eta_k)\triangleq\mathbf{1}^T\mathbf{V}(\eta_k)$, (32) is further reformulated as follows:

$$\begin{aligned}
\mathbb{E}\{v(\eta_{k+1})\}<&\sum_{i=1}^{l_0}\mathbb{E}\{S_i(z_k,\nu_{i,k},\varpi_i)\}\\
&+\sum_{i=l_0+1}^{N}\mathbb{E}\{S_i(z_k,\nu_{i,k})\}+\mathbb{E}\{v(\eta_k)\}. \qquad (33)
\end{aligned}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI10.1109/TSMC.2019.2907649, IEEE Transactions on Systems, Man, and Cybernetics: Systems


Keeping the notation of $S_i(z_k, \nu_{i,k}, \varpi_i)(i = 1, 2, \dots, l_0)$ and $S_i(z_k, \nu_{i,k})(i = l_0 + 1, \dots, N)$ in mind, we arrive at

$$\begin{aligned}&\mathbb{E}\{v(\eta_{k+1})\} - \mathbb{E}\{v(\eta_k)\} + \Phi_{\mathcal{G}}(z_k)\\ &< \sum_{i=1}^{N} \gamma^2 \nu_{i,k}^T \mathcal{T}_{i,k} \nu_{i,k} + \sum_{i=1}^{l_0} \gamma^2 \bar{\mathbf{W}}_{i,k} \mathbf{exp}(\varpi_i).\end{aligned} \tag{34}$$

Summing up both sides of (34) from 0 to $n-1$ with respect to $k$ yields

$$\begin{aligned}&\mathbb{E}\{v(\eta_n)\} - \mathbb{E}\{v(\eta_0)\} + \sum_{k=0}^{n-1} \Phi_{\mathcal{G}}(z_k)\\ &<\gamma^2 \sum_{k=0}^{n-1}\sum_{i=1}^{N} \nu_{i,k}^T \mathcal{T}_{i,k} \nu_{i,k} + \sum_{k=0}^{n-1}\sum_{i=1}^{l_0} \gamma^2 \bar{\mathbf{W}}_{i,k} \mathbf{exp}(\varpi_i).\end{aligned} \tag{35}$$

Taking the fact $\mathbb{E}\{v(\eta_n)\} > 0$ and the initial condition $\mathcal{P}_{i,0} \leq \gamma^2 U_i$ into account, the $H_\infty$-consensus performance index (10) is satisfied. Thus, the proof of this theorem is now complete. ■

*Theorem 2:* For given the real number $\gamma > 0$, the constant sequence $\alpha_{i,k} \in \mathscr{I}_{q_i}$, and the matrices $R_{i,k}, Q_{i,k}, \mathcal{T}_{i,k}$ and $\bar{\mathbf{W}}_{i,k}$ be given. The filtering error system (9) achieves the $H_\infty$-consensus performance index over the finite horizon $\mathcal{H}$, if there exist a sequence of matrices $\{\mathcal{P}_{i,k}\}_{k\in\mathcal{H}\cup\{n\}} \triangleq \mathrm{diag}\{P_{i,k}^1, P_{i,k}^2\}$ ($P_{i,k}^1 > 0$ and $P_{i,k}^2 > 0$), and matrices $E_{i,k}$ and $F_{ij,k}$ satisfying the initial condition $\mathcal{P}_{i,0} \leq \gamma^2 U_i$ ($\forall i = 1, 2, \dots, N$), such that the following recursive linear matrix inequalities are satisfied for all $k \in \mathcal{H}$:

$$\begin{cases}\Omega_{i,k} \triangleq \begin{bmatrix} \Omega_{i,k}^{11} & \Omega_{i,k}^{12} & 0 & \Omega_{i,k}^{14} & 0 & \Omega_{i,k}^{16}\\ * & \Omega_{i,k}^{22} & 0 & \Omega_{i,k}^{24} & 0 & 0\\ * & * & \Omega_{i,k}^{33} & \Omega_{i,k}^{34} & 0 & 0\\ * & * & * & \Omega_{i,k}^{44} & \Omega_{i,k}^{45} & 0\\ * & * & * & * & \Omega_{i,k}^{55} & 0\\ * & * & * & * & * & \Omega_{i,k}^{66}\end{bmatrix} < 0, & \text{(36a)}\\ \Phi_{i,k}^{-1} \triangleq -(\mathcal{B}_k + \mathcal{D}_{i,k})^T \mathcal{P}_{i,k+1} (\mathcal{B}_k + \mathcal{D}_{i,k}) + \gamma^2 \mathcal{T}_{i,k} > 0, & \text{(36b)}\\ \forall i = 1, 2, \dots, l_0, \end{cases}$$

and

$$\begin{cases}\Delta_{i,k} \triangleq \begin{bmatrix} \Delta_{i,k}^{11} & \Delta_{i,k}^{12} & \Delta_{i,k}^{13} & 0 & \Delta_{i,k}^{15}\\ * & \Delta_{i,k}^{22} & \Delta_{i,k}^{23} & 0 & 0\\ * & * & \Delta_{i,k}^{33} & \Delta_{i,k}^{34} & 0\\ * & * & * & \Delta_{i,k}^{44} & 0\\ * & * & * & * & \Delta_{i,k}^{55}\end{bmatrix} < 0, & \text{(37a)}\\ \bar{\Phi}_{i,k}^{-1} \triangleq \gamma^2 \mathcal{T}_{i,k} - \mathcal{B}_k^T \mathcal{P}_{i,k+1} \mathcal{B}_k > 0, & \text{(37b)}\\ \forall i = l_0 + 1, l_0 + 2, \dots, N, \end{cases}$$

where

$$\begin{aligned}
\Omega_{i,k}^{11} =& \mathcal{F}^T Q_{i,k} \mathcal{F} + \mathcal{F}^T R_{i,k} \mathcal{F} - \rho_{i,k} \mathcal{P}_{i,k},\\
\Omega_{i,k}^{12} =& -\big[\underbrace{\mathcal{F}^T R_{i,k} \mathcal{F} \ \dots \ \mathcal{F}^T R_{i,k} \mathcal{F}}_{p_i}\big],\\
\Omega_{i,k}^{22} =& -\mathrm{diag}\Big\{\frac{\alpha_{j_{i1},k}}{1 + q_{j_{i1}}} \mathcal{P}_{j_{i1},k} - \mathcal{F}^T R_{i,k} \mathcal{F}, \dots,\\
&\frac{\alpha_{j_{ip_i},k}}{1 + q_{j_{ip_i}}} \mathcal{P}_{j_{ip_i},k} - \mathcal{F}^T R_{i,k} \mathcal{F}\Big\},\\
\Omega_{i,k}^{33} =& -\gamma^2 \bar{\mathbf{W}}_{i,k} \mathbf{exp}(\varpi_i),\\
\Omega_{i,k}^{14} =& \Big(\bar{\mathcal{A}}_{0,k} + \bar{\mathcal{C}}_{i,k} - \sum_{j\in\mathcal{N}_i} \bar{\mathcal{K}}_{ij,k}\Big)^T, \quad \Omega_{i,k}^{34} = \bar{\mathcal{E}}_{i,k}^T,\\
\Omega_{i,k}^{24} =& [\bar{\mathcal{K}}_{ij_1,k} \ \dots \ \bar{\mathcal{K}}_{ij_{p_i},k}]^T, \quad \bar{\mathcal{K}}_{ij,k} = \mathrm{diag}\{0, F_{ij,k}\},\\
\Omega_{i,k}^{44} =& -\mathcal{P}_{i,k+1}, \quad \Omega_{i,k}^{55} = -\gamma^2 \mathcal{T}_{i,k}, \quad \Omega_{i,k}^{45} = \bar{\mathcal{B}}_k + \bar{\mathcal{D}}_{i,k},\\
\Omega_{i,k}^{16} =& [\bar{\mathcal{A}}_{1,k}^T \ \dots \ \bar{\mathcal{A}}_{r,k}^T], \quad \Omega_{i,k}^{66} = -\mathrm{diag}_r\{\mathcal{P}_{i,k+1}\},\\
\bar{\mathcal{A}}_{0,k} =& \mathrm{diag}\{P_{i,k+1}^1 A_{0,k}, P_{i,k+1}^2 A_{0,k}\},\\
\bar{\mathcal{D}}_{i,k} =& \mathrm{diag}\{0, -E_{i,k} p_{i,k} D_{i,k}\},\\
\bar{\mathcal{A}}_{t,k} =& \begin{bmatrix} P_{i,k+1}^1 A_{t,k} & 0\\ P_{i,k+1}^2 A_{t,k} & 0\end{bmatrix}, \quad \bar{\mathcal{B}}_k = \begin{bmatrix} P_{i,k+1}^1 B_k & 0\\ P_{i,k+1}^2 B_k & 0\end{bmatrix},\\
\bar{\mathcal{C}}_{i,k} =& \begin{bmatrix} 0 & 0\\ E_{i,k}(1 - p_{i,k}) C_{i,k} & -E_{i,k} C_{i,k}\end{bmatrix},\\
\bar{\mathcal{E}}_{i,k} =& \begin{bmatrix} 0 & 0\\ 0 & E_{i,k}(1 - p_{i,k}) \varpi_i\end{bmatrix},\\
\Delta_{i,k}^{11} =& \mathcal{F}^T Q_{i,k} \mathcal{F} + \mathcal{F}^T R_{i,k} \mathcal{F} - \rho_{i,k} \mathcal{P}_{i,k},\\
\Delta_{i,k}^{12} =& -\big[\underbrace{\mathcal{F}^T R_{i,k} \mathcal{F} \ \dots \ \mathcal{F}^T R_{i,k} \mathcal{F}}_{p_i}\big], \quad \Delta_{i,k}^{34} = \bar{\mathcal{B}}_k,\\
\Delta_{i,k}^{22} =& -\mathrm{diag}\Big\{\frac{\alpha_{j_{i1},k}}{1 + q_{j_{i1}}} \mathcal{P}_{j_{i1},k} - \mathcal{F}^T R_{i,k} \mathcal{F}, \dots,\\
&\frac{\alpha_{j_{ip_i},k}}{1 + q_{j_{ip_i}}} \mathcal{P}_{j_{ip_i},k} - \mathcal{F}^T R_{i,k} \mathcal{F}\Big\},\\
\Delta_{i,k}^{13} =& \Big(\bar{\mathcal{A}}_{0,k} - \sum_{j\in\mathcal{N}_i} \bar{\mathcal{K}}_{ij,k}\Big)^T, \quad \Delta_{i,k}^{33} = -\mathcal{P}_{i,k+1},\\
\Delta_{i,k}^{23} =& [\bar{\mathcal{K}}_{ij_1,k} \ \dots \ \bar{\mathcal{K}}_{ij_{p_i},k}]^T, \quad \Delta_{i,k}^{44} = -\gamma^2 \mathcal{T}_{i,k},\\
\Delta_{i,k}^{55} =& -\mathrm{diag}_r\{\mathcal{P}_{i,k+1}\}, \quad \Delta_{i,k}^{15} = [\bar{\mathcal{A}}_{1,k}^T \ \dots \ \bar{\mathcal{A}}_{r,k}^T].
\end{aligned}$$

Moreover, the desired filter gains are given by

$$L_{i,k} = (P_{i,k+1}^2)^{-1} E_{i,k} \quad \text{and} \quad K_{ij,k} = (P_{i,k+1}^2)^{-1} F_{ij,k}. \tag{38}$$

*Proof:* Performing congruence transformation $\mathrm{diag}\Big\{I, I, I, \mathcal{P}_{i,k+1}^{-1}, I, \underbrace{\mathcal{P}_{i,k+1}^{-1}, \dots, \mathcal{P}_{i,k+1}^{-1}}_{r}\Big\}$ to (36a) leads to

$$\begin{bmatrix} \Omega_{i,k}^{11} & \Omega_{i,k}^{12} & 0 & \bar{\Omega}_{i,k}^{14} & 0 & \bar{\Omega}_{i,k}^{16}\\ * & \Omega_{i,k}^{22} & 0 & \bar{\Omega}_{i,k}^{24} & 0 & 0\\ * & * & \Omega_{i,k}^{33} & \bar{\Omega}_{i,k}^{34} & 0 & 0\\ * & * & * & \bar{\Omega}_{i,k}^{44} & \bar{\Omega}_{i,k}^{45} & 0\\ * & * & * & * & \bar{\Omega}_{i,k}^{55} & 0\\ * & * & * & * & * & \bar{\Omega}_{i,k}^{66}\end{bmatrix} < 0, \tag{39}$$

and, similarly, using the congruence transformation $\mathrm{diag}\Big\{I, I, \mathcal{P}_{i,k+1}^{-1}, I, \underbrace{\mathcal{P}_{i,k+1}^{-1}, \dots, \mathcal{P}_{i,k+1}^{-1}}_{r}\Big\}$ to (37a), we have

$$\begin{bmatrix} \Delta_{i,k}^{11} & \Delta_{i,k}^{12} & \bar{\Delta}_{i,k}^{13} & 0 & \bar{\Delta}_{i,k}^{15}\\ * & \Delta_{i,k}^{22} & \bar{\Delta}_{i,k}^{23} & 0 & 0\\ * & * & \bar{\Delta}_{i,k}^{33} & \bar{\Delta}_{i,k}^{34} & 0\\ * & * & * & \Delta_{i,k}^{44} & 0\\ * & * & * & * & \bar{\Delta}_{i,k}^{55}\end{bmatrix} < 0, \tag{40}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.
Citation information: DOI10.1109/TSMC.2019.2907649, IEEE Transactions on Systems, Man, and Cybernetics: Systems


where

$$\begin{aligned}
\bar{\Omega}_{i,k}^{14} =& \Big(\mathcal{A}_{0,k} + \mathcal{C}_{i,k} - \sum_{j\in\mathcal{N}_i} \mathcal{K}_{ij,k}\Big)^T, \quad \bar{\Omega}_{i,k}^{34} = \mathcal{E}_{i,k}^T,\\
\bar{\Omega}_{i,k}^{24} =& [\mathcal{K}_{ij_1,k} \ \ldots \ \mathcal{K}_{ij_{p_i},k}]^T, \quad \bar{\Omega}_{i,k}^{44} = -\mathcal{P}_{i,k+1}^{-1},\\
\bar{\Omega}_{i,k}^{45} =& \mathcal{B}_k + \mathcal{D}_{i,k}, \quad \bar{\Omega}_{i,k}^{16} = [\mathcal{A}_{1,k} \ \mathcal{A}_{2,k} \ \ldots \ \mathcal{A}_{r,k}],\\
\bar{\Omega}_{i,k}^{55} =& -\gamma^2 \mathcal{T}_{i,k}, \quad \bar{\Omega}_{i,k}^{66} = -\mathrm{diag}_r\{\mathcal{P}_{i,k+1}^{-1}\},\\
\bar{\Delta}_{i,k}^{13} =& \Big(\mathcal{A}_{0,k} - \sum_{j\in\mathcal{N}_i} \mathcal{K}_{ij,k}\Big)^T, \quad \bar{\Delta}_{i,k}^{33} = -\mathcal{P}_{i,k+1}^{-1},\\
\bar{\Delta}_{i,k}^{23} =& [\mathcal{K}_{ij_1,k} \ \ldots \ \mathcal{K}_{ij_{p_i},k}]^T, \quad \bar{\Delta}_{i,k}^{34} = \mathcal{B}_k,\\
\bar{\Delta}_{i,k}^{15} =& [\mathcal{A}_{1,k} \ \mathcal{A}_{2,k} \ \ldots \ \mathcal{A}_{r,k}], \quad \bar{\Delta}_{i,k}^{55} = -\mathrm{diag}_r\{\mathcal{P}_{i,k+1}^{-1}\}.
\end{aligned}$$

Then, it is easy to verify the following relationships

$$\begin{aligned}
\Omega_{i,k}^{14} &= \bar{\Omega}_{i,k}^{14}\mathcal{P}_{i,k+1}, \quad \Omega_{i,k}^{24} = \bar{\Omega}_{i,k}^{24}\mathcal{P}_{i,k+1},\\
\Omega_{i,k}^{34} &= \bar{\Omega}_{i,k}^{34}\mathcal{P}_{i,k+1}, \quad \Omega_{i,k}^{44} = \mathcal{P}_{i,k+1}\bar{\Omega}_{i,k}^{44}\mathcal{P}_{i,k+1},\\
\Omega_{i,k}^{45} &= \mathcal{P}_{i,k+1}\bar{\Omega}_{i,k}^{45}, \quad \Omega_{i,k}^{16} = \mathcal{P}_{i,k+1}\bar{\Omega}_{i,k}^{16},\\
\Omega_{i,k}^{66} &= \mathrm{diag}_r\{\mathcal{P}_{i,k+1}\}\bar{\Omega}_{i,k}^{66}\mathrm{diag}_r\{\mathcal{P}_{i,k+1}\},\\
\Delta_{i,k}^{13} &= \bar{\Delta}_{i,k}^{13}\mathcal{P}_{i,k+1}, \quad \Delta_{i,k}^{23} = \bar{\Delta}_{i,k}^{23}\mathcal{P}_{i,k+1},\\
\Delta_{i,k}^{33} &= \mathcal{P}_{i,k+1}\bar{\Delta}_{i,k}^{33}\mathcal{P}_{i,k+1}, \quad \Delta_{i,k}^{34} = \mathcal{P}_{i,k+1}\bar{\Delta}_{i,k}^{34},\\
\Delta_{i,k}^{55} &= \mathrm{diag}_r\{\mathcal{P}_{i,k+1}\}\bar{\Delta}_{i,k}^{55}\mathrm{diag}_r\{\mathcal{P}_{i,k+1}\},\\
\Delta_{i,k}^{15} &= \bar{\Delta}_{i,k}^{15}\mathcal{P}_{i,k+1}, \quad E_{i,k} = P_{i,k+1}^2 L_{i,k},\\
F_{ij,k} &= P_{i,k+1}^2 K_{ij,k}.
\end{aligned}$$

In terms of the Schur Complement Lemma, we immediately draw the conclusion from (39) and (40) that the conditions (14a) and (15a) are satisfied. In addition, (36b) and (37b) hold if and only if (14c) and (15b) are true, and the rest of the proof follows directly from Theorem 1. The proof is thus complete. ■

*Remark 2:* This paper investigates the partial-nodes-based distributed filtering problem for a class of discrete-time systems with censored measurements. A set of deterministic indicative variables is introduced to describe the censored measurements and a new $H_\infty$ performance index is proposed to better reflect the impact of censored measurements on the filtering performance. With the aid of the local performance analysis method, sufficient conditions are established for two kinds of nodes (i.e. having or not having available measurement outputs) such that the prescribed $H_\infty$ performance index is achieved. Compared with the existing results, the developed filtering scheme has the following distinguished features: 1) a censored-related term is involved in the $H_\infty$ performance index in an exponential function form; 2) the designed algorithm is shown to be valid, respectively, for nodes with or without the sensing capabilities, which implies that our algorithm has the desired flexibility; and 3) a local design method is applied to deal with the filtering issue in the distributed sense, which confirms that our algorithm achieves the scalability.

## IV. A Numerical Example

In this section, a numerical example is carried out to illustrate the validity of the proposed distributed filter design scheme.

For given an SN with 5 nodes, its topology is represented by a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathscr{A})$, where the set of nodes is $\mathcal{V} = \{1,2,3,4,5\}$ and the set of edges is $\mathcal{E} = \{(1,4),(2,1),(3,2),(4,3),(5,4),(5,1),(5,3)\}$, as shown in Fig.1. Obviously, it can be inferred that $p_1 = p_2 = p_3 = p_4 = 1$, $p_5 = 3$, and $q_1 = q_3 = q_4 = 2$, $q_2 = 1$, $q_5 = 0$. In this example, assume that the measurement output of node 5 is not available.

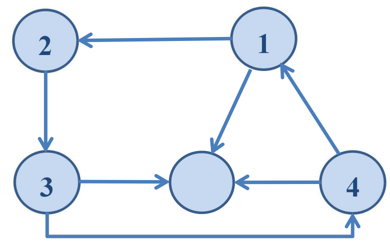


Fig. 1: Topology of a Sensor Network.

The parameters of the target plant is set as:

$$\begin{aligned}
A_k &= \begin{bmatrix} -0.80 & 0.15 \\ 0.15 & 0.21\sin(6k) \end{bmatrix}, \quad B_k = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\\
A_{1,k} &= \begin{bmatrix} -0.80 & 0.15 \\ 0.15 & 0.21\sin(6k) \end{bmatrix},\\
C_{i,k} &= \begin{bmatrix} 0.8 \\ 1.2\sin((i+1)k) \end{bmatrix}^T, \quad D_{i,k} = 1.
\end{aligned}$$

In addition, the other parameters are, respectively, set as $n = 21$, $\gamma^2 = 0.16$, $\alpha_{i,k} \equiv 0.5$, $\mathcal{P}_{i,0} = \mathrm{diag}\{I, 5I\}$, $U_i = 100\mathcal{P}_{i,0}$, $\mathcal{T}_{i,k} \equiv \mathrm{diag}\{1,1\}$, $\Gamma_i = -0.1$ and $\bar{\mathbf{W}}_{i,k} = 1$.

By using the YALMIP toolbox in MATLAB software, all the desired filter gains are recursively calculated according to Theorem 2. The initial values are set as follows: $x_0 = [0.3 \ \ -0.5]^T$, $\hat{x}_{1,0} = [0.6 \ \ -0.7]^T$, $\hat{x}_{2,0} = [0.1 \ \ -0.2]^T$, $\hat{x}_{3,0} = [0.8 \ \ -0.5]^T$, $\hat{x}_{4,0} = [-0.3 \ \ -0.5]^T$ and $\hat{x}_{5,0} = [0.3 \ 0.5]^T$. The disturbances are chosen as $w_k = 0.1\cos(5k)$ and $\xi_{i,k} = 0.1\cos(5k)$, respectively.

The simulation results are shown in Figs. 2-6, where the state trajectories of the plant and their estimates are displayed in Figs. 2-3, the censored measurements $y_{i,k}$ $(i = 1,\ldots,4)$ are depicted in Fig. 4, the norms of the consensus errors $\sum_{j\in\mathcal{N}_i} \|e_{j,k} - e_{i,k}\|^2$ are given in Fig. 5 and the norms of the filtering errors $\|e_{i,k}\|^2$ are plotted in Fig. 6. It is observed from all the simulation results that the proposed distributed filtering scheme performs very well.

## V. Conclusion

In this paper, we have investigated the scalable distributed $H_\infty$-consensus filtering problem for a class of discrete time-varying systems with multiplicative noises and censored measurements over sensor networks where only partial nodes have the ability to conduct the measurement task. A set of deterministic indicative variables has been employed to characterize the censored measurements. A novel $H_\infty$-consensus performance index has been established to reflect the impacts of the censored measurements on the filtering

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI10.1109/TSMC.2019.2907649, IEEE Transactions on Systems, Man, and Cybernetics: Systems


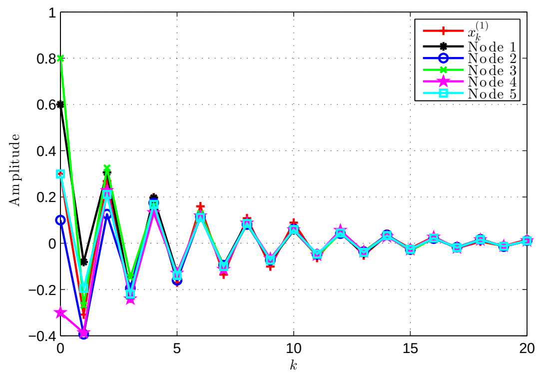


Fig. 2: State $x_k^{(1)}$ and its estimates $\hat{x}_{i,k}^{(1)}$.

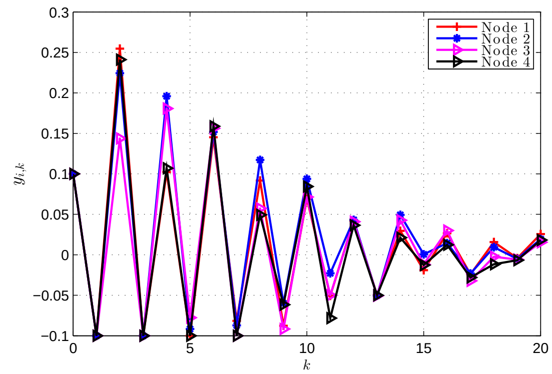


Fig. 4: Censored measurements $y_{i,k}$.

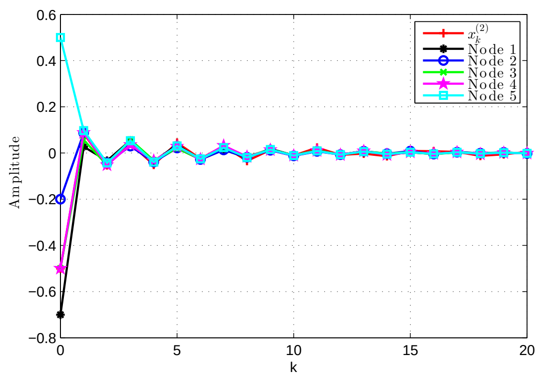


Fig. 3: State $x_k^{(2)}$ and its estimates $\hat{x}_{i,k}^{(2)}$.

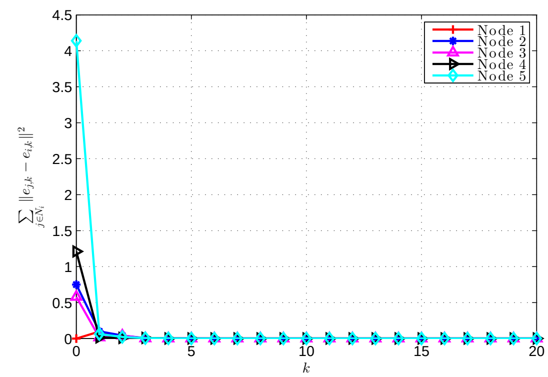


Fig. 5: Sum of the norms of consensus errors $\sum_{j\in\mathcal{N}_i}\|e_{j,k}-e_{i,k}\|^2$.

error dynamics. By using the local performance analysis method, a sufficient condition has been established so as to meet the prescribed $H_\infty$-consensus performance requirement. Furthermore, the desired filter gains have been recursively calculated in a distributed manner. Finally, the effectiveness of the main results has been illustrated by a simulation example. The future research topics would include the extension of the main results of this paper to more complex target plants with engineering-oriented performance specifications (see e.g. [4], [14], [32]–[34], [36], [40]) and to the dual distributed control problems (see [23], [24], [35], [44]).

## References


[1] B. Allik, C. Miller, M. J. Piovoso, and R. Zurakowski, "The Tobit Kalman filter: An estimator for censored measurements", *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 1, pp. 365–371, 2016.

[2] B. Allik, C. Miller, M. J. Piovoso, and R. Zurakowski, "Estimation of saturated data using the Tobit Kalman filter", in *Proc. Amer. Control Conf.*, Portland, Oregon, USA, 2014, pp. 4151–4156.

[3] Y. Chen, Z. Wang, Y. Yuan and P. Date, Distributed $H_\infty$ filtering for switched stochastic delayed systems over sensor networks with fading measurements, *IEEE Transactions on Cybernetics*, in press, DOI:10.1109/TCYB.2018.2852290.

[4] D. Ding, Z. Wang, Q.-L. Han, and G. Wei, "Security control for discrete-time stochastic nonlinear systems subject to deception attacks", *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 5, pp. 779-789, May 2018.

[5] D. Ding, Z. Wang, D. W. C. Ho, and G. Wei, "Distributed recursive filtering for stochastic systems under uniform quantizations and deception attacks through sensor networks", *Automatica*, vol. 78, pp. 231–240, 2017.

[6] H. Dong, X. Bu, N. Hou, Y. Liu, Fuad E. Alsaadi, and T. Hayat, "Event-triggered distributed state estimation for a class of time-varying systems over sensor networks with redundant channels", *Inform. Fusion*, vol. 36, pp. 243–250, 2017.

[7] H. Dong, Z. Wang, and H. Gao, "Distributed $H_\infty$ filtering for a class of Markovian jump nonlinear time-delay systems over lossy sensor networks", *IEEE Trans. Ind. Electron.*, vol. 60, no. 10, pp. 4665–4672, 2013.

[8] X. Ge and Q.-L. Han, "Distributed event-triggered $H_\infty$ filtering over sensor networks with communication delays", *Inf. Sci.*, vol. 291, pp. 128–142, 2015.

[9] X. Ge and Q.-L. Han, "Distributed sampled-data asynchronous $H_\infty$ filtering of Markovian jump linear systems over sensor networks", *Signal Process.*, vol. 127, pp. 86–99, Oct. 2016.

[10] H. Geng, Z. Wang, Y. Liang, Y. Cheng, and F. E. Alsaadi, "Tobit Kalman filter with fading measurements", *Signal Process.*, vol. 140, pp. 60–68, 2017.

[11] H. Geng, Z. Wang, Y. Liang, Y. Cheng, and F. E. Alsaadi, "Tobit Kalman filter with time-correlated multiplicative sensor noises under redundant channel transmission", *IEEE Sens. J.*, vol. 17, no. 24, pp. 8367–8377, 2017.

[12] F. Han, G. Wei, D. Ding, and Y. Song, "Local condition-based consensus filtering with stochastic nonlinearities and multiple missing measurements", *IEEE Trans. Auto. Control*, vol. 62, no. 9, pp. 4784–4790, 2017.

[13] F. Han, Y. Song, S. Zhang, and W. Li, "Local condition-based finite-horizon distributed $H_\infty$-consensus filtering for random parameter system with event-triggering protocols", *Neurocomputing*, vol. 219, pp. 221–231, 2017.

[14] X. He, Z. Wang, Y. Liu, L. Qin, and D.H. Zhou, Fault tolerant control for an Internet-based three-tank system: accommodation to sensor bias

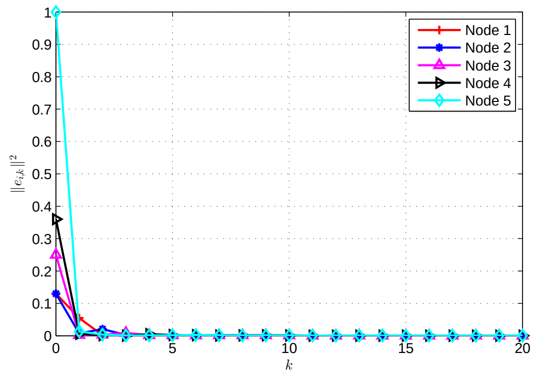


Fig. 6: Norms of the filtering errors $\|e_{i,k}\|^2$.

faults, *IEEE Trans. Ind. Electron.*, vol. 64, no. 3, pp. 2266–2275, Mar. 2017.

[15] C. Huang, D. W. C. Ho, and J. Lu, "Partial-information-based distributed filtering in two-targets tracking sensor networks", *IEEE Trans. Circuits Syst. I*, vol. 59, no. 4, pp. 820–832, Apr. 2012.

[16] W. Li, Y. Jia, and J. Du, "Tobit Kalman filter with time-correlated multiplicative measurement noise", *IET Control Theory Appl.*, vol. 11, no. 1, pp. 122–128, 2017.

[17] J. Li, G. Wei, D. Ding, and Y. Li, "Set-membership filtering for discrete time-varying nonlinear systems with censored measurements under Round-Robin protocol", *Neurocomputing*, vol. 281, pp. 20–26, 2018.

[18] S. Liu, Z. Wang, G. Wei and M. Li, Distributed set-membership filtering for multi-rate systems under the Round-Robin scheduling over sensor networks, *IEEE Transactions on Cybernetics*, (ISSN 2168-2267) accepted for publication.

[19] Y. Liu, Z. Wang, Y. Yuan, and W. Liu, "Event-triggered partial-nodes-based state estimation for delayed complex networks with bounded distributed delays", *IEEE Trans. Syst., Man, Cybern., Syst.*, DOI: 10.1109/TSMC.2017.2720121.

[20] Y. Liu, Z. Wang, Y. Yuan, and F. E. Alsaadi, "Partial-nodes-based state estimation for complex networks with unbounded distributed delays", *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 8, pp. 3906-3912, Aug. 2018.

[21] S. Liu, Z. Wang, G. Wei, and M. Li, "Distributed set-membership filtering for multirate systems under the Round-Robin scheduling over sensor networks", *IEEE Trans. Cybern.*, DOI: 10.1109/TCYB.2018.2885653.

[22] Q. Lu, Q. L. Han, B. Zhang, D. Liu, and S. Liu, "Cooperative control of mobile sensor networks for environmental monitoring: An event-triggered finite-time control scheme", *IEEE Trans. Cybern.*, vol. 47, no. 12, pp. 4134–4147, Dec. 2017.

[23] L. Ma, Z. Wang, and H-K. Lam, "Mean-Square $H_\infty$ consensus control for a class of nonlinear time-varying stochastic multiagent systems: the finite-horizon case", *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no.7, pp. 1050–1060, 2017.

[24] L. Ma, Z. Wang, Q.-L. Han, and Y. Liu, "Consensus control of stochastic multi-agent systems: a survey", *Sci. China, Inform. Sci.*, vol. 60, no. 12, No. 120201, 2017.

[25] J. M. Pak, C. K. Ahn, P. Shi, Y. S. Shmaliy, and M. T. Lim, "Distributed hybrid particle/FIR filtering for mitigating NLOS effects in TOA-based localization using wireless sensor networks", *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 5182–5191, Jun. 2017.

[26] B. Shen, Z. Wang,and X. Liu, "A stochastic sampled-data approach to distributed $H_\infty$ filtering in sensor networks", *IEEE Trans. Circuits Syst. I*, vol. 58, no. 9, pp. 2237–2246, 2011.

[27] B. Shen, Z. Wang, and Y. Hung, "Distributed $H_\infty$-consensus filtering in sensor networks with multiple missing measurements: The finite-horizon case", *Automatica*, vol. 46, no. 10, pp. 1682–1688, 2010.

[28] Y. Song, G. Wei, and G. Yang, "Distributed $H_\infty$ filtering for a class of sensor networks with uncertain rates of packet losses", *Signal Process.*, vol. 104, pp. 143–151, 2014.

[29] X. Su, L. Wu, and P. Shi, "Sensor networks with random link failures: distributed filtering for T-S fuzzy systems", *IEEE Trans. Ind. Inform.*, vol. 9, no. 3, pp. 1739–1750, 2013.

[30] S. Sun, F. Peng, and H. Lin, "Distributed asynchronous fusion estimator for stochastic uncertain systems with multiple sensors of different fading measurement rates", *IEEE Trans. Signal Process.*, vol. 66, no. 3, pp. 641–653, Feb. 2018.

[31] S. Sun, H. Lin, J. Ma, and X. Li, "Multi-sensor distributed fusion estimation with applications in networked systems: A review paper", *Inform. Fusion*, vol. 38, pp. 122–134, Nov. 2017.

[32] X. Wan, Z. Wang, Q.-L. Han and M. Wu, Finite-time $H_\infty$ state estimation for discrete time-delayed genetic regulatory networks under stochastic communication protocols, *IEEE Transactions on Circuits and Systems-Part I*, vol. 65, no. 10, pp. 3481–3491, 2018.

[33] X. Wan, Z. Wang, M. Wu and X. Liu, State estimation for discrete time-delayed genetic regulatory networks with stochastic noises under the Round-Robin protocols, *IEEE Transactions on NanoBioscience*, vol. 17, no. 2, pp. 145–154, 2018.

[34] F. Wang, Z. Wang, J. Liang, and X. Liu, "Resilient filtering for linear time-varying repetitive processes under uniform quantizations and Round-Robin protocols", *IEEE Trans. Circuits Syst. I*, vol. 65, no. 9, pp. 2992-3004, Sept. 2018.

[35] L. Wang, Z. Wang, G. Wei, and F. E. Alsaadi, "Observer-based consensus control for discrete-time multi-agent systems with coding-decoding communication protocol", *IEEE Trans. Cybern.*, DOI: 10.1109/TCYB.2018.2863664.

[36] L. Wang, Z. Wang, G. Wei and F. E. Alsaadi, Finite-time state estimation for recurrent delayed neural networks with component-based event-triggering protocol, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 4, pp. 1046–1057, 2018.

[37] C. Wen, Z. Wang, Q. Liu, and F. E. Alsaadi, Recursive distributed filtering for a class of state-saturated systems with fading measurements and quantization effects, *IEEE Trans. Syst., Man, Cybern, Syst.*, vol. 48, no. 6, pp. 930–941, Jun. 2018.

[38] H. Yan, Q. Yang, H. Zhang, F. Yang, and X. Zhan, "Distributed $H_\infty$ state estimation for a class of filtering networks with time-varying switching topologies and packet losses", *IEEE Trans. Syst., Man, Cybern, Syst.*, vol. 48, no. 12, pp. 2047–2057, 2018.

[39] W. Yu, G. Chen, Z. Wang, and W. Yang, "Distributed consensus filtering in sensor networks", *IEEE Trans. Syst., Man, Cybern. B. Cybern.*, vol. 39, no. 6, pp. 1568–1577, Dec. 2009.

[40] Y. Yuan, Z. Wang, and L. Guo, Event-triggered strategy design for discrete-time nonlinear quadratic games with disturbance compensations: the noncooperative case, *IEEE Trans. Syst., Man, and Cybern, Syst.*, vol. 48, no. 11, pp. 1885–1896, Nov. 2018.

[41] D. Zhang, P. Shi, Q. Wang, and L. Yu, "Distributed non-fragile filtering for T-S fuzzy systems with event-based communications", *Fuzzy Sets and Syst.*, vol. 306, pp. 137–152, 2017.

[42] D. Zhang, Z. Xu, H. R. Karimi, and Q. Wang, "Distributed filtering for switched linear systems with sensor networks in presence of packet dropouts and quantization", *IEEE Trans. Circuits and Syst., I*, vol. 64, no. 10, pp. 2783–2796, Oct. 2017.

[43] H. Zhang, Q. Hong, H. Yan, F. Yang, and G. Guo, "Event-based distributed $H_\infty$ filtering networks of 2-DOF quarter-car suspension systems", *IEEE Trans. Ind. Informat.*, vol. 13, no. 1, pp. 312–321, 2017.

[44] X. M. Zhang, Q.-L. Han, and X. Ge, "A novel finite-sum inequality-based method for robust $H_\infty$ control of uncertain discrete-time Takagi-Sugeno fuzzy systems with interval-like time-varying delays", *IEEE Trans. Cybern.*, vol. 48, no. 9, pp. 2569–2582, 2018.

[45] Y. Zhu, L. Zhang, and W. Zheng, "Distributed $H_\infty$ filtering for a class of discrete-time Markov jump Lur'e systems with redundant channels", *IEEE Trans. Ind. Electron.*, vol. 63, no. 3, pp. 1876–1885, 2016.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI10.1109/TSMC.2019.2907649, IEEE Transactions on Systems, Man, and Cybernetics: Systems


**Fei Han** (M'17) received the B.Sc. degree in mathematics and applied mathematics from China University of Ming and Technology, Xuzhou, China, in 2003 and the M.Sc. degree in applied mathematics from Henan Normal University, Xinxiang, China, in 2009, and the Ph.D. degree in system analysis and integration from University of Shanghai for Science and Technology, Shanghai, China, in 2017. From March 2018 to June 2018, he was a Senior Research Assistant in the Department of Electronic Engineering, the City University of Hong Kong.

He is currently an Associate Professor with the Institute of Complex Systems and Advanced Control, Northeast Petroleum University, Daqing, China. His current research interests include distributed control and filtering.

**Zidong Wang** (SM'03-F'14) was born in Jiangsu, China, in 1966. He received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in electrical engineering in 1994, both from Nanjing University of Science and Technology, Nanjing, China.

He is currently a Professor of Dynamical Systems and Computing in the Department of Computer Science, Brunel University London, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany and the UK. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published around 200 papers in IEEE Transactions and around 60 papers in Automatica. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, William Mong Visiting Research Fellowship of Hong Kong.

Prof. Wang serves (or has served) as the Editor-in-Chief for Neurocomputing, the Deputy Editor-in-Chief for International Journal of Systems Science, and an Associate Editor for 12 international journals, including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics-Part C. He is a Fellow of the IEEE, a Fellow of the Royal Statistical Society and a member of program committee for many international conferences.

**Hongli Dong** (SM'16) received the Ph.D. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2012. From July 2009 to January 2010, she was a Research Assistant in the Department of Applied Mathematics, City University of Hong Kong, Hong Kong. From October 2010 to January 2011, she was a Research Assistant in the Department of Mechanical Engineering, University of Hong Kong. From January 2011 to January 2012, she was a Visiting Scholar in the Department of Information Systems and Computing, Brunel University, London, U.K. From November 2012 to October 2014, she was an Alexander von Humboldt Research Fellow at the University of Duisburg-Essen, Duisburg, Germany. She is currently a Professor with the Institute of Complex Systems and Advanced Control, Northeast Petroleum University, Daqing, China. She is also the Director of the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Daqing, China.

Dr. Dong's current research interests include robust control and networked control systems. She is a very active reviewer for many international journals.

**Hongjian Liu** received the B.Sc. degree in applied mathematics from Anhui University, Hefei, China, in 2003 and the M.Sc. degree in detection technology and automation equipments from Anhui Polytechnic University, Wuhu, China, in 2009, and the Ph.D. degree in control theory and control engineering from Donghua University, Shanghai, China, in 2018.

He is currently an Associate Professor in the School of Mathematics and Physics, Anhui Polytechnic University, Wuhu, China. His current research interests include filtering theory, memristive neural networks and network communication systems. He is a very active reviewer for many international journals.