

On State Estimation for Discrete Time-Delayed Memristive Neural Networks under the WTOD Protocol: A Resilient Set-Membership Approach

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Abstract—In this paper, a resilient set-membership approach is put forward to deal with the state estimation problem for a sort of discrete-time memristive neural networks (DMNNs) with hybrid time-delays under the Weighted Try-Once-Discard protocol (WTODP). The WTODP is utilized to mitigate unnecessary network congestion occurring in the channel between the DMNNs and the state estimator. In order to ensure resilience against possible realization errors, the estimator gain is permitted to undergo some norm-bounded parameter drifts. Our objective is to design a resilient set-membership estimator (RSME) that is capable of resisting gain variations and unknown-but-bounded noises by confining the estimation error to certain ellipsoidal regions. By resorting to the recursive matrix inequality technique, sufficient conditions are acquired for the existence of the expected RSME and, subsequently, an optimization problem is formalized by minimizing the constraint ellipsoid (with respect to the estimation error) under the WTODP. At last, numerical simulation is carried out to validate the usefulness of the RSME.

Index Terms—Discrete-time memristive neural networks, set-membership state estimation, hybrid time-delays, resilient state estimation, weighted try-once-discard protocol.

I. INTRODUCTION

The past decade has seen a recurring research interest in memristive neural networks (MNNs) because of their extensive applications in various subjects, e.g. recognition of patterns [33], processing of digital images [11], and brain

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emulation [26]. The key feature of MNNs is their strong dependence of the connection weights (CWs) on the neuron states, which leads to complicated dynamic behaviors of MNNs as compared to the conventional recurrent neural networks (RNNs). Accordingly, the analysis problems (e.g. stability, convergence and synchronization) of many kinds of dynamics of MNNs have aroused much interest, and some pioneering work has been available in [37], [45]. Almost all existing results, however, have been concerned with the continuous-time MNNs despite the nowadays popularity of the digitization of information sequences in engineering practice, and this gives rise to the necessity of utilizing discrete-time RNNs when handling sequence-based missions. Up to now, little attention has been drawn onto the DMNNs simply because of essential difficulties in dealing with switching behaviors that are inherently state-dependent in the discrete-time setting.

State estimation is a fundamental yet crucial research issue in engineering applications on neural networks (NNs) [16], [27], [29], [40], [43], [44]. In fact, for many NN applications, it is indispensable for the network states to be utilized to fulfil the requirements for monitoring, approximation and optimization [4], [8], [32]. Unfortunately, due mainly to resource constraints and technical limits, the network states are usually unavailable to the end user but only the measurement outputs can be accessible. In addition, when analyzing DMNN-related dynamics, the frequently adopted analysis techniques (e.g. the nonsmooth analysis method and differential inclusions theory) for continuous-time MNNs become futile, and this leads to an urgent demand for developing new techniques appropriate for DMNNs. As such, much research enthusiasm has recently been attracted towards the state estimation problems for DMNNs suffering from hybrid (i.e. both discrete and distributed) time-delays with some inspiring results published in the recent literature [18].

In practice, MNNs can be realized via very large scale integration circuits (VLSICs) where the CWs are executed via memristors [37]. Basically speaking, VLSICs are easily susceptible to both device noises (e.g. the flicker, shot and thermal noises) and ambient noises (e.g. the crosstalk, power and/or ground bounce, and substrate noises). Particularly, the ambient noises, if not adequately addressed, could significantly degrade the VLSIC performance and reliability [31]. From a practical point of view, we assume the ambient noises to be deterministic, unknown-but-bounded within certain ellipsoidal regions (CERs). In this case, the set-membership (ellipsoidal) filtering method becomes especially suitable to constrain sys-

tem states in CERs so as to assure satisfactory state estimation performance [17]. Although a vast body of set-membership state estimation work has been presented (see e.g. [20], [49]), relevant results on MNNs have been scattered, let alone the discrete-time setting is also involved.

It should be pointed out that, even though some attempts have been initiated on the state estimation issues for MNNs (see e.g. [18], [19]), little attention has been paid to the issue of limited communication under resource constraints between the estimator and the network output [3]. Because of the large size of MNNs and the high-degree complexity of the to-do tasks, the volume of the network output data could become considerably high, which has posed great challenges (e.g. data collisions and communication congestion) onto the transmission networks of limited capacity. To handle the network-induced challenges, an effective measure that has been favorably taken by industry is to leverage the communication scheduling protocols in order to regulate the network traffic, see [5], [28], [34], [50], [51]. Among various protocols that have been in use so far, the Weighted Try-Once-Discard protocol (WTODP) has proven to be a particularly attractive scheduling strategy in allocating network resources because of its dynamic scheduling behavior based on the significance of different missions [30]. As such, it is of practical significance to explore how the WTODP can be leveraged to coordinate the considerable data transmission between the MNN and the estimator, and this constitutes one of our motivations in the present investigation.

The biological or circuit implementation of NNs is often confronted by various types of time-delays for the reasons of finite signal transmission and switching speeds in biological systems [1], [12], [15], [39], [47]. It has been acknowledged that time-delays can cause undesirable system performance including instability and oscillation, and thus plenty of research effort has been spent on analyzing the dynamical behaviors of NNs with various time-delays (constant, time-varying, discrete, distributed, mixed delays) where the majority build themselves on existing techniques such as the descriptor model transformation [6], integral inequality [14], etc. For the stability issue of delayed *continuous-time* MNNs, fruitful results have been published by combining the proceeding techniques and the set-valued mapping theory, see [37]. However, when it comes to the delayed DMNNs, corresponding results have been rarely reported due typically to the mathematical challenges embedded in disposing of the coupling among the state-dependent parameters, the time-delays, the scheduling protocol, and the estimator. Accordingly, one of the primary motivations here is to examine the set-membership state estimation problem (SEP) for delayed DMNNs under the WTODP.

When realizing filter/controller algorithms in traditional system design, an implicit assumption is that filter/controller gains are *exactly* implemented. This assumption, unfortunately, goes against the engineering practice as the actual execution of the filter/controller gains might undergo parameter variations induced by many factors including the analogue-digital conversion, round-off errors, and finite resolution of instruments [21]. Such kind of gain variations/drifts, though

possibly small, would undoubtedly impair the corresponding controller/filter performance [13], [38]. As such, a growing body of work has been reported on the design of the *resilient* state estimation algorithms that are of insensitivity to the estimator gain variations [35], [41], [42], [46]. Nevertheless, to our best knowledge, very few results have been acquired so far on the resilient SEPs for DMNNs, not to mention the case where the hybrid time-delays (HTDs) (consisting both discrete and distributed time-delays), unknown-but-bounded noises (UBBNs) and WTODP are all involved.

In this paper, we aim at developing a resilient set-membership estimator (RSME) for DMNNs with HTDs under the WTODP. *The primary contributions we make in this paper are outlined as threefold.* 1) A RSME is developed to mirror the alleviation of the adverse impact induced by the random gain variation in the actual estimator implementation. 2) A unified estimation scheme is built to cope with the mathematical complexities resulting from the state-dependent switching behaviors, HTDs, WTODP and gain variations. 3) In the designed filtering framework, the filter gain is obtained with the help of a bank of recursive matrix inequalities (RMIs) whose solutions are provided by standard software packages.

The rest of this article is highlighted as follows. Section II formulates the concerned filtering problem. Section III presents the design procedure of the filter whose existence is guaranteed by the derivation of some sufficient conditions. An example is given in Section IV to show the feasibility of our RSME, and some remarks are concluded in Section V.

Notation \mathbb{R}^n and $\mathbb{R}^{n \times m}$ define, respectively, the n -dimensional Euclidean space and the set of all $n \times m$ real matrices. \mathbb{N}^+ denotes the set of all nonnegative integers. I denotes identity matrices with compatible dimensions. $A > B$ (or $A \geq B$) implies that $A - B$ is positive-definite (or semi-positive-definite). A^T represents the transpose of A . $\text{diag}\{\cdots\}$ is a block-diagonal matrix. $\delta(\cdot) \in \{0, 1\}$ is the Dirac delta function.

II. PROBLEM FORMULATION

A. Memristive Neural Network Model

Consider a typical DMNN with HTDs:

$$\begin{cases} z(t+1) = D(z(t))z(t) + A(z(t))f(z(t)) \\ \quad + B(z(t))g(z(t-\tau_1)) \\ \quad + C(z(t)) \sum_{\iota=1}^{\tau_2} \mu_\iota h(z(t-\iota)) + L_1 v(t), \\ y(t) = Mz(t) + L_2 v(t), \\ z(\varsigma) = \phi_0(\varsigma), \quad \varsigma = -\tau, -\tau+1, \dots, -1, 0 \end{cases} \quad (1)$$

where

$$\begin{aligned} z(t) &= [z_1(t) \quad z_2(t) \quad \cdots \quad z_n(t)]^T, \\ y(t) &= [y_1(t) \quad y_2(t) \quad \cdots \quad y_m(t)]^T, \\ v(t) &= [v_1(t) \quad v_2(t) \quad \cdots \quad v_q(t)]^T \\ D(z(t)) &= \text{diag}\{d_1(z_1(t)), d_2(z_2(t)), \dots, d_n(z_n(t))\} \end{aligned}$$

are the neuron state vector, the measurement output vector, the exogenous disturbance, and the self-feedback matrix, respectively; $A(z(t)) = (a_{ij}(z_i(t)))_{n \times n}$, $B(z(t)) = (b_{ij}(z_i(t)))_{n \times n}$ and $C(z(t)) = (c_{ij}(z_i(t)))_{n \times n}$ are CWs; and

$$\begin{aligned} f(z(t)) &\triangleq [f_1(z_1(t)) \quad f_2(z_2(t)) \quad \cdots \quad f_n(z_n(t))]^T, \\ g(z(t)) &\triangleq [g_1(z_1(t)) \quad g_2(z_2(t)) \quad \cdots \quad g_n(z_n(t))]^T, \\ h(z(t)) &\triangleq [h_1(z_1(t)) \quad h_2(z_2(t)) \quad \cdots \quad h_n(z_n(t))]^T \end{aligned}$$

are the nonlinear neuron activation functions (AFs); τ_1 and τ_2 are the constant discrete and distributed time-delays, respectively; $\phi_0(\varsigma)$ ($\varsigma = -\tau, -\tau+1, \dots, -1, 0$) are initial conditions with $\tau \triangleq \max\{\tau_1, \tau_2\}$; L_1, L_2 and M are known matrices of compatible dimensions.

Similar to [18], the state-dependent functions $d_i(z_i(t))$, $a_{ij}(z_i(t))$, $b_{ij}(z_i(t))$ and $c_{ij}(z_i(t))$ are

$$\begin{aligned} d_i(z_i(\cdot)) &= \begin{cases} \hat{d}_i, & |z_i(\cdot)| > \ell_i, \\ \check{d}_i, & |z_i(\cdot)| \leq \ell_i, \end{cases} \\ \kappa_{ij}(z_i(\cdot)) &= \begin{cases} \hat{\kappa}_{ij}, & |z_i(\cdot)| > \ell_i, \\ \check{\kappa}_{ij}, & |z_i(\cdot)| \leq \ell_i, \end{cases} \end{aligned} \quad (2)$$

where $\kappa \in \{a, b, c\}$, the switching jumps satisfy $\ell_i > 0$, $|\hat{d}_i| < 1$, $|\check{d}_i| < 1$, and $\hat{\kappa}_{ij}$ and $\check{\kappa}_{ij}$ are known constants.

Based on (2), we first define following switching functions (SFs):

$$\begin{aligned} \vartheta_{ii}^d(z_i(\cdot)) &\triangleq \begin{cases} 1, & |z_i(\cdot)| > \ell_i, \\ -1, & |z_i(\cdot)| \leq \ell_i, \end{cases} \\ \vartheta_{ij}^\kappa(z_i(\cdot)) &\triangleq \begin{cases} 1, & |z_i(\cdot)| > \ell_i, \\ -1, & |z_i(\cdot)| \leq \ell_i, \end{cases} \end{aligned} \quad (3)$$

and $\vartheta_{ij}^d(z_i(t)) \triangleq 0$ for $i \neq j$.

Denote

$$\begin{aligned} \underline{d}_i &\triangleq \min\{\hat{d}_i, \check{d}_i\}, \\ \bar{d}_i &\triangleq \max\{\hat{d}_i, \check{d}_i\}, \\ \underline{\kappa}_{ij} &\triangleq \min\{\hat{\kappa}_{ij}, \check{\kappa}_{ij}\}, \\ \bar{\kappa}_{ij} &\triangleq \max\{\hat{\kappa}_{ij}, \check{\kappa}_{ij}\}. \end{aligned}$$

In addition, we introduce the following matrices:

$$\begin{aligned} \underline{D} &\triangleq \text{diag}\{\underline{d}_1, \underline{d}_2, \dots, \underline{d}_n\}, \\ \bar{D} &\triangleq \text{diag}\{\bar{d}_1, \bar{d}_2, \dots, \bar{d}_n\}, \\ D_0 &\triangleq (\underline{D} + \bar{D})/2, \\ \underline{S} &\triangleq (\underline{\kappa}_{ij})_{n \times n}, \\ \bar{S} &\triangleq (\bar{\kappa}_{ij})_{n \times n}, \\ S_0 &\triangleq (\underline{S} + \bar{S})/2 \end{aligned}$$

where $S \in \{A, B, C\}$. Thus, $D(z(t))$, $A(z(t))$, $B(z(t))$ and $C(z(t))$ are rewritten as

$$\begin{aligned} D(z(t)) &= D_0 + \Delta D(t), & A(z(t)) &= A_0 + \Delta A(t), \\ B(z(t)) &= B_0 + \Delta B(t), & C(z(t)) &= C_0 + \Delta C(t) \end{aligned} \quad (4)$$

with

$$\Delta D(z(t)) \triangleq \mathcal{H}_d \vartheta^d(z(t)) E_d,$$

$$\Delta A(z(t)) \triangleq \mathcal{H}_a \vartheta^a(z(t)) E_a,$$

$$\Delta B(z(t)) \triangleq \mathcal{H}_b \vartheta^b(z(t)) E_b,$$

$$\Delta C(z(t)) \triangleq \mathcal{H}_c \vartheta^c(z(t)) E_c$$

where

$$\begin{aligned} \mathcal{H}_r &\triangleq [t_{11}^r e_1 \quad \cdots \quad t_{1n}^r e_1 \quad \cdots \quad t_{n1}^r e_n \quad \cdots \quad t_{nn}^r e_n] \\ E_r^T &\triangleq [t_{11}^r e_1 \quad \cdots \quad t_{1n}^r e_n \quad \cdots \quad t_{n1}^r e_1 \quad \cdots \quad t_{nn}^r e_n] \\ \vartheta^r(z(t)) &\triangleq \text{diag}\{\vartheta_{11}^r(z_1(t)), \dots, \vartheta_{1n}^r(z_1(t)), \dots, \\ &\quad \vartheta_{n1}^r(z_n(t)), \dots, \vartheta_{nn}^r(z_n(t))\}, \\ t_{ij}^r &\triangleq (|\bar{r}_{ij} - \underline{r}_{ij}|/2)^{\frac{1}{2}}, \end{aligned}$$

$r \in \{d, a, b, c\}$, and $e_i \in \mathbb{R}^n$ is a column vector with 1 being its the i th entry and 0 being others. Apparently, $\vartheta^r(z(t))$ satisfies $\vartheta^{rT}(z(t))\vartheta^r(z(t)) \leq I$.

Remark 1: In case of computer-based calculations, experiments and simulations, continuous-time networks are often required to be discretized in the implementation process, and this merits the necessity of studying the DMNNs. Note that available results on dynamics analysis problems of DMNNs have been really scattered. Additionally, owing to the state-dependent feature of the memristive CWs, it is mathematically difficult to analyze the dynamics of MNNs, and conventional methods cannot be directly employed to investigate the stability and estimation issues for DMNNs. Thanks to the novel SFs (3), we are able to convert the MNN (1) into an equivalent one with parametric uncertainties, on which some traditional robust analysis approaches could be ideally applied.

B. The WTOP Protocol

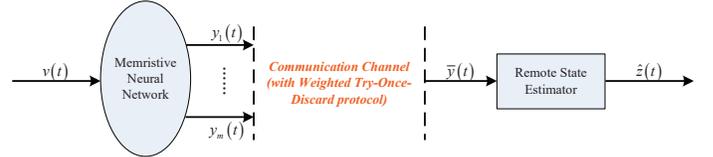


Fig. 1. SEP for an MNN under the WTOP protocol.

As shown in Fig. 1, the measurements $y(t)$ (as the output of the MNN) are sent to the remote estimator via constrained transmission channels. To prevent the data from collisions and maximize the efficiency of data utilization, the WTOPD is applied to schedule the data transmission between the MNN and estimator. Under the WTOPD, each entry of $y(t)$ has the priority that is directly proportional to the norm of the error between the current measurement and the last storing measurement. At each time instant, only the entry with the highest priority is allowed to be updated. If there are more than one entry being assigned with the highest priority, we can choose an arbitrary one for updating.

Define

$$\bar{y}(t) \triangleq [\bar{y}_1(t) \quad \bar{y}_2(t) \quad \cdots \quad \bar{y}_m(t)]^T$$

as the received measurement output by the estimator, where $\bar{y}_s(t)$ ($s = 1, 2, \dots, m$) is the s th entry of $\bar{y}(t)$. Let σ_s be the known weight of the s th component, and $\bar{h}(t) \in \{1, 2, \dots, m\}$

be the entry activated at time t . Then, $\tilde{h}(t)$ can be determined by

$$\tilde{h}(t) = \arg \max_{1 \leq s \leq m} (y(t) - \bar{y}(t-1))^T \bar{Q}_s (y(t) - \bar{y}(t-1)) \quad (5)$$

where $\bar{Q}_s \triangleq Q \Psi_s$, $Q \triangleq \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_m\}$ and $\Psi_s \triangleq \text{diag}\{\delta(s-1), \delta(s-2), \dots, \delta(s-m)\}$.

Denote

$$\begin{aligned} \tilde{z}(t) &\triangleq [z^T(t) \quad \bar{y}^T(t-1)]^T, \\ \tilde{f}(\tilde{z}(t)) &\triangleq [f^T(z(t)) \quad 0]^T, \\ \tilde{g}(\tilde{z}(t-\tau_1)) &\triangleq [g^T(z(t-\tau_1)) \quad 0]^T, \\ \tilde{h}(\tilde{z}(t-\iota)) &\triangleq [h^T(z(t-\iota)) \quad 0]^T, \\ \phi_2(\varsigma) &\triangleq [\phi_0^T(\varsigma) \quad \phi_1^T]^T, \\ \tilde{v}(t) &\triangleq [v^T(t) \quad v^T(t)]^T. \end{aligned}$$

According to the WTODP and the zero order-holders strategy, the actually received measurement can be expressed by

$$\bar{y}(t) = \Psi_{\tilde{h}(t)} y(t) + (I_m - \Psi_{\tilde{h}(t)}) \bar{y}(t-1), \quad (6)$$

and $\bar{y}(t) = \phi_1$ for $t < 0$ where ϕ_1 is known. On the basis of (5), the DMNN in (1) is rewritten as:

$$\begin{cases} \tilde{z}(t+1) = \mathcal{D}(t) \tilde{z}(t) + \mathcal{A}(t) \tilde{f}(\tilde{z}(t)) + \mathcal{B}(t) \tilde{g}(\tilde{z}(t-\tau_1)) \\ \quad + \mathcal{C}(t) \sum_{\iota=1}^{\tau_2} \mu_\iota \tilde{h}(\tilde{z}(t-\iota)) + \mathcal{L}_1 \tilde{v}(t), \\ \bar{y}(t) = \mathcal{M} \tilde{z}(t) + \mathcal{L}_2 v(t), \\ \tilde{z}(\varsigma) = \phi_2(\varsigma), \quad \varsigma = -\tau, -\tau+1, \dots, -1, 0 \end{cases} \quad (7)$$

where

$$\begin{aligned} \mathcal{D}(t) &\triangleq \begin{bmatrix} D(z(t)) & 0 \\ \Psi_{\tilde{h}(t)} M & I_m - \Psi_{\tilde{h}(t)} \end{bmatrix}, \\ \mathcal{A}(t) &\triangleq \text{diag}\{A(z(t)), 0\}, \\ \mathcal{B}(t) &\triangleq \text{diag}\{B(z(t)), 0\}, \\ \mathcal{C}(t) &\triangleq \text{diag}\{C(z(t)), 0\}, \\ \mathcal{L}_1 &\triangleq \text{diag}\{L_1, L_2\}, \\ \mathcal{L}_2 &\triangleq \Psi_{\tilde{h}(t)} L_2, \\ \mathcal{M} &\triangleq [\Psi_{\tilde{h}(t)} M \quad I_m - \Psi_{\tilde{h}(t)}]. \end{aligned}$$

Based on (7), the following RSME is constructed:

$$\begin{cases} \hat{\tilde{z}}(t+1) = \bar{\mathcal{D}} \hat{\tilde{z}}(t) + \bar{\mathcal{A}} \tilde{f}(\hat{\tilde{z}}(t)) \\ \quad + \bar{\mathcal{B}} \tilde{g}(\hat{\tilde{z}}(t-\tau_1)) + \bar{\mathcal{C}} \sum_{\iota=1}^{\tau_2} \mu_\iota \tilde{h}(\hat{\tilde{z}}(t-\iota)) \\ \quad + (\mathcal{K}(t) + \Delta \mathcal{K}(t)) (\bar{y}(t) - \mathcal{M} \hat{\tilde{z}}(t)), \\ \hat{\tilde{z}}(\varsigma) = 0, \quad \varsigma = -\tau, -\tau+1, \dots, -1, 0 \end{cases} \quad (8)$$

where

$$\begin{aligned} \hat{\tilde{z}}(t) &\triangleq \begin{bmatrix} \hat{\tilde{z}}(t) \\ \hat{\bar{y}}(t-1) \end{bmatrix}, \\ \bar{\mathcal{D}} &\triangleq \begin{bmatrix} D_0 & 0 \\ \Psi_{\tilde{h}(t)} M & I_m - \Psi_{\tilde{h}(t)} \end{bmatrix}, \\ \bar{\mathcal{A}} &\triangleq \text{diag}\{A_0, 0\}, \end{aligned}$$

$$\begin{aligned} \bar{\mathcal{B}} &\triangleq \text{diag}\{B_0, 0\}, \\ \bar{\mathcal{C}} &\triangleq \text{diag}\{C_0, 0\}, \\ \mathcal{K}(t) &\triangleq [K_z^T(t) \quad K_y^T(t)]^T, \\ \Delta \mathcal{K}(t) &\triangleq [\Delta K_z^T(t) \quad \Delta K_y^T(t)]^T. \end{aligned}$$

Here, $\hat{\tilde{z}}(t) \in \mathbb{R}^{n+m}$ is the estimate of $\tilde{z}(t)$, $K_z(t) \in \mathbb{R}^{n \times m}$ and $K_y(t) \in \mathbb{R}^{m \times m}$ are parameters to be determined. $\Delta K_z(t)$ and $\Delta K_y(t)$ are parameter variations satisfying

$$\Delta K_z(t) = H_z F_z(t) T_z, \quad \Delta K_y(t) = H_y F_y(t) T_y$$

where H_z , H_y , T_z and T_y are known matrices, and the unknown matrices F_z and F_y satisfy $F_z^T(t) F_z(t) \leq I$ and $F_y^T(t) F_y(t) \leq I$.

Let

$$\begin{aligned} e(t) &\triangleq \tilde{z}(t) - \hat{\tilde{z}}(t), \\ \xi(t) &\triangleq [\tilde{v}^T(t) \quad v^T(t)]^T, \\ F(e(t)) &\triangleq \tilde{f}(\tilde{z}(t)) - \tilde{f}(\hat{\tilde{z}}(t)), \\ G(e(t-\tau_1)) &\triangleq \tilde{g}(\tilde{z}(t-\tau_1)) - \tilde{g}(\hat{\tilde{z}}(t-\tau_1)) \\ H(e(t-\iota)) &\triangleq \tilde{h}(\tilde{z}(t-\iota)) - \tilde{h}(\hat{\tilde{z}}(t-\iota)). \end{aligned}$$

Then, the estimation error system with respect to (7) and (8) is

$$\begin{aligned} e(t+1) &= (\bar{\mathcal{D}} - \tilde{\mathcal{K}}(t) \mathcal{M}) e(t) + \Delta \mathcal{D}(t) \tilde{z}(t) + \bar{\mathcal{A}} F(e(t)) \\ &\quad + \Delta \mathcal{A}(t) \tilde{f}(\tilde{z}(t)) + \bar{\mathcal{B}} G(e(t-\tau_1)) \\ &\quad + \Delta \mathcal{B}(t) \tilde{g}(\tilde{z}(t-\tau_1)) + \sum_{\iota=1}^{\tau_2} \mu_\iota \bar{\mathcal{C}} H(e(t-\iota)) \\ &\quad + \sum_{\iota=1}^{\tau_2} \mu_\iota \Delta \mathcal{C}(t) \tilde{h}(\tilde{z}(t-\iota)) + \mathcal{L} \xi(t) \end{aligned} \quad (9)$$

where

$$\begin{aligned} \Delta \mathcal{D}(t) &\triangleq \begin{bmatrix} \Delta D(z(t)) & 0 \\ 0 & 0 \end{bmatrix}, \\ \Delta \mathcal{A}(t) &\triangleq \begin{bmatrix} \Delta A(z(t)) & 0 \\ 0 & 0 \end{bmatrix}, \\ \Delta \mathcal{B}(t) &\triangleq \begin{bmatrix} \Delta B(z(t)) & 0 \\ 0 & 0 \end{bmatrix}, \\ \Delta \mathcal{C}(t) &\triangleq \begin{bmatrix} \Delta C(z(t)) & 0 \\ 0 & 0 \end{bmatrix}, \\ \mathcal{L} &\triangleq [\mathcal{L}_1 \quad -\tilde{\mathcal{K}}(t) \mathcal{L}_2], \\ \tilde{\mathcal{K}}(t) &\triangleq \mathcal{K}(t) + \Delta \mathcal{K}(t). \end{aligned}$$

In what follows, we define vectors

$$\begin{aligned} \gamma(t) &\triangleq [\tilde{z}^T(t) \quad e^T(t)]^T, \\ \tilde{\xi}(t) &\triangleq [\tilde{v}^T(t) \quad \xi^T(t)]^T, \\ \tilde{F}(\gamma(t)) &\triangleq [\tilde{f}^T(\tilde{z}(t)) \quad F^T(e(t))]^T, \\ \tilde{G}(\gamma(t-\tau_1)) &\triangleq [\tilde{g}^T(\tilde{z}(t-\tau_1)) \quad G^T(e(t-\tau_1))]^T, \\ \tilde{H}(\gamma(t-\iota)) &\triangleq [\tilde{h}^T(\tilde{z}(t-\iota)) \quad H^T(e(t-\iota))]^T, \end{aligned}$$

and then have the augmented system

$$\gamma(t+1) = \tilde{\mathcal{D}}(t) \gamma(t) + \tilde{\mathcal{A}}(t) \tilde{F}(\gamma(t)) + \tilde{\mathcal{B}}(t) \tilde{G}(\gamma(t-\tau_1))$$

$$+ \tilde{\mathcal{C}}(t) \sum_{\iota=1}^{\tau_2} \mu_{\iota} \tilde{H}(\gamma(t-\iota)) + \tilde{\mathcal{L}}\tilde{\xi}(t) \quad (10)$$

where

$$\begin{aligned} \tilde{\mathcal{D}}(t) &\triangleq \bar{\mathcal{D}}_{\gamma} + \Delta\mathcal{D}_{\gamma}(t), \\ \tilde{\mathcal{A}}(t) &\triangleq \bar{\mathcal{A}}_{\gamma} + \Delta\mathcal{A}_{\gamma}(t), \\ \tilde{\mathcal{B}}(t) &\triangleq \bar{\mathcal{B}}_{\gamma} + \Delta\mathcal{B}_{\gamma}(t), \\ \tilde{\mathcal{C}}(t) &\triangleq \bar{\mathcal{C}}_{\gamma} + \Delta\mathcal{C}_{\gamma}(t), \\ \tilde{\mathcal{L}} &\triangleq \bar{\mathcal{L}}_{\gamma} + \Delta\mathcal{L}_{\gamma}, \\ \bar{\mathcal{D}}_{\gamma} &\triangleq \text{diag}\{\bar{\mathcal{D}}, \bar{\mathcal{D}} - \mathcal{K}(t)\mathcal{M}\}, \\ \bar{\mathcal{A}}_{\gamma} &\triangleq \text{diag}\{\bar{\mathcal{A}}, \bar{\mathcal{A}}\}, \\ \bar{\mathcal{B}}_{\gamma} &\triangleq \text{diag}\{\bar{\mathcal{B}}, \bar{\mathcal{B}}\}, \\ \bar{\mathcal{C}}_{\gamma} &\triangleq \text{diag}\{\bar{\mathcal{C}}, \bar{\mathcal{C}}\}, \\ \bar{\mathcal{L}} &\triangleq [\mathcal{L}_1 \quad -\mathcal{K}(t)\mathcal{L}_2], \\ \bar{\mathcal{L}}_{\gamma} &\triangleq \text{diag}\{\mathcal{L}_1, \bar{\mathcal{L}}\}, \\ \Delta\mathcal{L}_{\gamma} &\triangleq \text{diag}\{0, \Delta\bar{\mathcal{L}}\}, \\ \Delta\bar{\mathcal{L}} &\triangleq [0 \quad -\Delta\mathcal{K}(t)\mathcal{L}_2], \\ \Delta\mathcal{D}_{\gamma}(t) &\triangleq \begin{bmatrix} \Delta\mathcal{D}(t) & 0 \\ \Delta\mathcal{D}(t) & -\Delta\mathcal{K}(t)\mathcal{M} \end{bmatrix}, \\ \Delta\mathcal{C}_{\gamma}(t) &\triangleq \begin{bmatrix} \Delta\mathcal{C}(t) & 0 \\ \Delta\mathcal{C}(t) & 0 \end{bmatrix}, \\ \Delta\mathcal{A}_{\gamma}(t) &\triangleq \begin{bmatrix} \Delta\mathcal{A}(t) & 0 \\ \Delta\mathcal{A}(t) & 0 \end{bmatrix}, \\ \Delta\mathcal{B}_{\gamma}(t) &\triangleq \begin{bmatrix} \Delta\mathcal{B}(t) & 0 \\ \Delta\mathcal{B}(t) & 0 \end{bmatrix}. \end{aligned}$$

For presentation convenience, we now introduce the following assumptions and definitions.

Assumption 1: $\phi_2(\varsigma)$ ($\varsigma = -\tau, -\tau+1, \dots, -1, 0$) satisfy

$$\phi_2^T(\varsigma)P^{-1}(\varsigma)\phi_2(\varsigma) \leq 1 \quad (11)$$

where matrices $P(\varsigma)$ ($\varsigma = -\tau, -\tau+1, \dots, -1, 0$) are known and positive-definite.

Assumption 2: The external stochastic disturbance $v(t)$ satisfies the following condition

$$v^T(t)\Gamma_v^{-1}(t)v(t) \leq 1 \quad (12)$$

where matrix $\Gamma_v(t)$ is known and positive-definite.

Assumption 3: The neuron AFs $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ satisfy $f(0) = g(0) = h(0) = 0$ and following Lipschitz conditions:

$$\begin{aligned} \|f(s) - f(t)\| &\leq \|\Gamma_1(s-t)\|, \\ \|g(s) - g(t)\| &\leq \|\Gamma_2(s-t)\|, \\ \|h(s) - h(t)\| &\leq \|\Gamma_3(s-t)\| \end{aligned} \quad (13)$$

for all $s, t \in \mathbb{R}^n$, where Γ_1 , Γ_2 and Γ_3 are known constant matrices.

Definition 1: Let the matrix (ellipsoid matrices) sequence $P(t) \in \mathbb{R}^{(n+m) \times (n+m)}$ ($t \in \mathbb{N}^+$) be given. System (10) is said to meet the $P(t)$ -dependent constraint if

$$\mathcal{R}(t) \triangleq \gamma^T(t)P^{-1}(t)\gamma(t) \leq 1 \quad (14)$$

holds for $t \in \mathbb{N}^+$.

In this paper, we are set to design a RSME that is capable of confining the estimates of the DMNN (1) to a CER under the UNBBNs. Such an aim is accomplished in two steps. First, for the given matrix sequence $\{P(t)\}_{t \in \mathbb{N}^+}$, we like to find the sufficient condition that ensures that the RSME exists and, subsequently, (10) meets the $P(t)$ -dependent constraint (14). Second, we like to minimize the trace of $P(t)$ through appropriately selecting $\mathcal{K}(t)$ that satisfies the aforementioned sufficient condition.

III. MAIN RESULTS

This section aims at establishing sufficient conditions that guarantee that the $P(t)$ -dependent constraint (14) is satisfied by system (10). Then, a recursive algorithm is proposed to determine $\mathcal{K}(t)$ under the WTOPD. To start with, we present the following useful lemmas to benefit the subsequent derivation.

Lemma 1: [2] Let $\varphi_0(\cdot), \varphi_1(\cdot), \dots, \varphi_p(\cdot)$ be quadratic functions of $\mathbf{s} \in \mathbb{R}^n$: $\varphi_j(\mathbf{s}) \triangleq \mathbf{s}^T Z_j \mathbf{s}$ ($j = 0, 1, \dots, p$) and $Z_j = Z_j^T$. If there exist $\varrho_1 \geq 0, \varrho_2 \geq 0, \dots, \varrho_p \geq 0$ such that $Z_0 - \sum_{i=1}^p \varrho_i Z_i \leq 0$, then

$$\varphi_1(\mathbf{s}) \leq 0, \varphi_2(\mathbf{s}) \leq 0, \dots, \varphi_p(\mathbf{s}) \leq 0 \rightarrow \varphi_0(\mathbf{s}) \leq 0.$$

Lemma 2: [2] Given matrices $\Omega_1, \Omega_2, \Omega_3$, $\Omega_1 = \Omega_1^T$ and $\Omega_2 > 0$, then

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$$

iff

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0.$$

Lemma 3: [36] Given matrices $\mathcal{N} = \mathcal{N}^T, \mathcal{H}, \mathcal{E}$ and $\mathcal{F}^T \mathcal{F} \leq I$, then

$$\mathcal{N} + \mathcal{H}\mathcal{F}\mathcal{E} + (\mathcal{H}\mathcal{F}\mathcal{E})^T < 0$$

holds iff there exists scalar $\mu > 0$ such that

$$\mathcal{N} + \mu\mathcal{H}\mathcal{H}^T + \mu^{-1}\mathcal{E}^T\mathcal{E} < 0$$

or

$$\begin{bmatrix} \mathcal{N} & \mu\mathcal{H} & \mathcal{E}^T \\ * & -\mu I & 0 \\ * & * & -\mu I \end{bmatrix} < 0.$$

For the convenience of presentation, we denote

$$\tilde{\mathcal{H}}_r \triangleq [\mathcal{H}_r^T \quad 0]^T, \quad \tilde{E}_r \triangleq [E_r \quad 0], \quad P(t) \triangleq L(t)L^T(t).$$

The following theorem gives a sufficient condition for the solvability of the concerned SEP.

Theorem 1: Consider the DMNN (1) with given estimator (8) and matrices sequences $\{P(t) > 0\}_{t \in \mathbb{N}^+}$. If there exist $\mathcal{K}(t)$, $\varepsilon(t)$, $\lambda_i(t)$ ($i \in \{1, 2, 3, 4\}$) and $\epsilon_s(t)$ ($s \in \{1, 2, \dots, m\}, t \in \mathbb{N}^+$) satisfying

$$\begin{bmatrix} \tilde{\Omega}(t) & * & * \\ \varepsilon(t)\tilde{\mathcal{H}}^T & -\varepsilon(t)I & * \\ \tilde{\xi} & 0 & -\varepsilon(t)I \end{bmatrix} < 0 \quad (15)$$

where

$$\tilde{\Omega}(t) \triangleq \begin{bmatrix} -\Lambda(t) & * \\ \tilde{\Phi}(t) & -P(t+1) \end{bmatrix},$$

$$\begin{aligned}\bar{\Phi}(t) &\triangleq [0 \quad \bar{D}_\gamma L(t) \quad 0 \quad 0 \quad \bar{L}_\gamma \quad \bar{A}_\gamma \quad \bar{B}_\gamma \quad \bar{C}_\gamma], \\ \Lambda(t) &\triangleq \text{diag}\{1 - \lambda_1(t) - \lambda_2(t) - 5\lambda_3(t), \\ &\quad \lambda_1(t)I - \lambda_4(t)L^T(t)\tilde{\Gamma}_1^T\tilde{\Gamma}_1L(t), \\ &\quad \lambda_2(t)I - \lambda_4(t)L^T(t - \tau_1)\tilde{\Gamma}_2^T\tilde{\Gamma}_2L(t - \tau_1), \\ &\quad \lambda_4(t)\Omega_0(t), \lambda_3(t)\tilde{\Gamma}_v^{-1}(t), \lambda_4(t)I, \lambda_4(t)I, \lambda_4(t)I\} \\ &\quad + \mathcal{N}^T(t) \sum_{s=1}^m \epsilon_s(t) (Q(\Psi_s - \Psi_{h(t)})) \mathcal{N}(t),\end{aligned}$$

$$\tilde{\mathcal{H}} \triangleq \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\mathcal{H}}_D^T P^T(t) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\mathcal{H}}_A^T P^T(t) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\mathcal{H}}_B^T P^T(t) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\mathcal{H}}_C^T P^T(t) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\mathcal{H}}_L^T P^T(t) \end{bmatrix}^T,$$

$$\tilde{\mathcal{E}} \triangleq \begin{bmatrix} 0 & \tilde{E}_D L(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{E}_A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tilde{E}_B & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{E}_C & 0 \\ 0 & 0 & 0 & 0 & T_C & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\mathcal{H}}_D \triangleq \begin{bmatrix} \tilde{\mathcal{H}}_d & 0 \\ \tilde{\mathcal{H}}_d & -\mathcal{H}_K \end{bmatrix},$$

$$\tilde{\mathcal{H}}_A \triangleq \begin{bmatrix} \tilde{\mathcal{H}}_a & 0 \\ \tilde{\mathcal{H}}_a & 0 \end{bmatrix},$$

$$\tilde{\mathcal{H}}_B \triangleq \begin{bmatrix} \tilde{\mathcal{H}}_b & 0 \\ \tilde{\mathcal{H}}_b & 0 \end{bmatrix},$$

$$\tilde{\mathcal{H}}_C \triangleq \begin{bmatrix} \tilde{\mathcal{H}}_c & 0 \\ \tilde{\mathcal{H}}_c & 0 \end{bmatrix},$$

$$\tilde{\mathcal{H}}_L \triangleq \begin{bmatrix} 0 \\ -\mathcal{H}_K \end{bmatrix},$$

$$\tilde{E}_D \triangleq \text{diag}\{\tilde{E}_d, T_K \mathcal{M}\}, \quad \tilde{E}_A \triangleq \text{diag}\{\tilde{E}_a, 0\},$$

$$\tilde{E}_B \triangleq \text{diag}\{\tilde{E}_b, 0\},$$

$$\tilde{E}_C \triangleq \text{diag}\{\tilde{E}_c, 0\},$$

$$T_K \triangleq [T_x^T \quad T_y^T]^T,$$

$$T_C \triangleq [0 \quad 0 \quad T_K \mathcal{L}_2],$$

$$\mathcal{H}_K \triangleq \text{diag}\{H_x, H_y\},$$

$$\bar{\mathcal{N}} \triangleq [M \quad -I \quad 0],$$

$$\tilde{\mathcal{L}} \triangleq [L_2 \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\bar{\mu} \triangleq \sum_{\iota=1}^{\tau_2} \mu_\iota,$$

$$\mathcal{N}(t) \triangleq [0 \quad \bar{\mathcal{N}}L(t) \quad 0 \quad 0 \quad \tilde{\mathcal{L}} \quad 0 \quad 0 \quad 0],$$

$$\Omega_0(t) \triangleq \bar{\mu} \text{diag}\{\mu_1 \tilde{\Gamma}_3^T \tilde{\Gamma}_3, \mu_2 \tilde{\Gamma}_3^T \tilde{\Gamma}_3, \dots, \mu_{\tau_2} \tilde{\Gamma}_3^T \tilde{\Gamma}_3\},$$

$$\tilde{\Gamma}_1 \triangleq \text{diag}\{\Gamma_1, \Gamma_1, \Gamma_1, \Gamma_1\},$$

$$\tilde{\Gamma}_2 \triangleq \text{diag}\{\Gamma_2, \Gamma_2, \Gamma_2, \Gamma_2\},$$

$$\tilde{\Gamma}_3 \triangleq \text{diag}\{\Gamma_3, \Gamma_3, \Gamma_3, \Gamma_3\},$$

$$\tilde{\Gamma}_v(t) \triangleq I_5 \otimes \Gamma_v(t),$$

then system (10) satisfies $P(t)$ -dependent constraints with $\mathcal{K}(t)$ ($t \in \mathbb{N}^+$).

Proof: We prove this Theorem by mathematical induction. First, for $t = 0$, based at Assumption 1, we have

$$\mathcal{R}(0) = \gamma^T(0)P^{-1}(0)\gamma(0) = \phi_2^T(0)P^{-1}(0)\phi_2(0) \leq 1. \quad (16)$$

Supposing that $\mathcal{R}(t) \leq 1$ holds at time t , let us prove that $\mathcal{R}(t+1) \leq 1$ holds at time $t+1$.

As $\mathcal{R}(t) \leq 1$ and $P(t) = L(t)L^T(t)$, we know that there exists a vector $\tilde{\gamma}(i)$ with $\|\tilde{\gamma}(i)\| \leq 1$ such that $\gamma(i) = L(i)\tilde{\gamma}(i)$ for $i \in \{0, 1, \dots, t\}$ by referring to [7]. Next, we show that the solution $P(t+1)$ of inequality(15) guarantees $\mathcal{R}(t+1) \leq 1$.

Denoting

$$\tilde{\gamma}_{\tau_2} \triangleq [\gamma^T(t-1) \quad \gamma^T(t-2) \quad \dots \quad \gamma^T(t-\tau_2)]^T \quad (17)$$

$$\begin{aligned}\tilde{\gamma}(t) &\triangleq [1 \quad \tilde{\gamma}^T(t) \quad \tilde{\gamma}^T(t-\tau_1) \quad \tilde{\gamma}_{\tau_2}^T(t) \quad \tilde{\xi}^T(t) \\ &\quad \tilde{F}^T(\gamma(t)) \quad \tilde{G}^T(\gamma(t-\tau_1)) \\ &\quad \sum_{\iota=1}^{\tau_2} \mu_\iota \tilde{H}^T(\gamma(t-\iota))]^T, \quad (18)\end{aligned}$$

we have

$$\gamma(t+1) = \Phi(t)\tilde{\gamma}(t) \quad (19)$$

and thus

$$\begin{aligned}&\gamma^T(t+1)P^{-1}(t+1)\gamma(t+1) - 1 \\ &= \tilde{\gamma}^T(t) (\Phi^T(t)P^{-1}(t+1)\Phi(t) \\ &\quad - \text{diag}\{1, 0, 0, 0, 0, 0, 0, 0\}) \tilde{\gamma}(t)\end{aligned} \quad (20)$$

where

$$\Phi(t) \triangleq [0 \quad \tilde{D}(t)L(t) \quad 0 \quad 0 \quad \tilde{L} \quad \tilde{A}(t) \quad \tilde{B}(t) \quad \tilde{C}(t)].$$

It follows from Assumption 2 and the WTODP (5) that

$$\begin{cases} \|\tilde{\gamma}(t)\|^2 \leq 1, \\ \|\tilde{\gamma}(t-\tau_1)\|^2 \leq 1, \\ \tilde{\xi}^T(t)\tilde{\Gamma}_v^{-1}(t)\tilde{\xi}(t) \leq 5, \\ (y(t) - \bar{y}(t-1))^T Q(\Psi_s - \Psi_{h(t)}) \\ \quad \times (y(t) - \bar{y}(t-1)) \leq 0, \quad s = 1, 2, \dots, m \end{cases} \quad (21)$$

which, by (17)-(21), is rearranged as follows (in terms of $\tilde{\gamma}(t)$):

$$\begin{cases} \tilde{\gamma}^T(t) \text{diag}\{-1, I, 0, 0, 0, 0, 0, 0\} \tilde{\gamma}(t) \leq 0, \\ \tilde{\gamma}^T(t) \text{diag}\{-1, 0, I, 0, 0, 0, 0, 0\} \tilde{\gamma}(t) \leq 0, \\ \tilde{\gamma}^T(t) \text{diag}\{-5, 0, 0, 0, \tilde{\Gamma}_v^{-1}(t), 0, 0, 0\} \tilde{\gamma}(t) \leq 0, \\ \tilde{\gamma}^T(t) \mathcal{N}^T(t) Q(\Psi_s - \Psi_{h(t)}) \\ \quad \times \mathcal{N}(t) \tilde{\gamma}(t) \leq 0, \quad s = 1, 2, \dots, m. \end{cases} \quad (22)$$

Noticing Assumption 3, we have

$$\tilde{F}^T(\gamma(t))\tilde{F}(\gamma(t)) \leq \gamma^T(t)\Gamma_1^T\Gamma_1\gamma(t), \quad (23)$$

$$\begin{aligned}\tilde{G}^T(\gamma(t-\tau_1))\tilde{G}(\gamma(t-\tau_1)) \\ \leq \gamma^T(t-\tau_1)\Gamma_2^T\Gamma_2\gamma(t-\tau_1), \quad (24)\end{aligned}$$

and

$$\begin{aligned}
& \left(\sum_{\iota=1}^{\tau_2} \mu_\iota \tilde{H}(\gamma(t-\iota)) \right)^T \left(\sum_{\iota=1}^{\tau_2} \mu_\iota \tilde{H}(\gamma(t-\iota)) \right) \\
& \leq \left(\sum_{\iota=1}^{\tau_2} \mu_\iota \right) \sum_{\iota=1}^{\tau_2} \mu_\iota \tilde{H}^T(\gamma(t-\iota)) \tilde{H}(\gamma(t-\iota)) \\
& \leq \bar{\mu} \sum_{\iota=1}^{\tau_2} \mu_\iota \gamma^T(t-\iota) \Gamma_3^T \Gamma_3 \gamma(t-\iota) \\
& = \bar{\mu} \tilde{\gamma}_{\tau_2}^T(t) \Omega_0(t) \tilde{\gamma}_{\tau_2}(t).
\end{aligned} \tag{25}$$

Furthermore, (23)–(25) are expressed as

$$\tilde{F}^T(\gamma(t)) \tilde{F}(\gamma(t)) \leq \tilde{\gamma}^T(t) \Omega_1(t) \tilde{\gamma}(t) \leq 0, \tag{26}$$

$$\tilde{G}^T(\gamma(t-\tau_1)) \tilde{G}(\gamma(t-\tau_1)) \leq \tilde{\gamma}^T(t) \Omega_2(t) \tilde{\gamma}(t) \leq 0, \tag{27}$$

$$\begin{aligned}
& \left(\sum_{\iota=1}^{\tau_2} \mu_\iota \tilde{H}(\gamma(t-\iota)) \right)^T \left(\sum_{\iota=1}^{\tau_2} \mu_\iota \tilde{H}(\gamma(t-\iota)) \right) \\
& \leq \tilde{\gamma}^T(t) \Omega_3(t) \tilde{\gamma}(t) \leq 0,
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
\Omega_1(t) & \triangleq \text{diag}\{0, -L^T(t) \Gamma_1^T \Gamma_1 L(t), 0, 0, 0, I, 0, 0\}, \\
\Omega_2(t) & \triangleq \text{diag}\{0, 0, -L^T(t-\tau_1) \Gamma_2^T \Gamma_2 L(t-\tau_1), 0, 0, 0, I, 0\}, \\
\Omega_3(t) & \triangleq \text{diag}\{0, 0, 0, \Omega_0(t), 0, 0, 0, I\}.
\end{aligned}$$

with $\Omega_0(t)$ being defined in (15). Then,

$$\tilde{\gamma}^T(t) \Omega(t) \tilde{\gamma}(t) \leq 0 \tag{29}$$

where

$$\begin{aligned}
\Omega(t) & \triangleq \text{diag}\{0, -L^T(t) \Gamma_1^T \Gamma_1 L(t), \\
& \quad -L^T(t-\tau_1) \Gamma_2^T \Gamma_2 L(t-\tau_1), \Omega_0(t), 0, I, I, I\}.
\end{aligned}$$

Using Lemma 1, if there exist positive scalars $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$ and $\epsilon_s(t)$ ($s = 1, 2, \dots, m$) such that

$$\begin{aligned}
& \Phi^T(t) P^{-1}(t+1) \Phi(t) \\
& - \lambda_1(t) \text{diag}\{-1, I, 0, 0, 0, 0, 0, 0\} \\
& - \lambda_2(t) \text{diag}\{-1, 0, I, 0, 0, 0, 0, 0\} \\
& - \lambda_3(t) \text{diag}\{-5, 0, 0, 0, \tilde{\Gamma}_v^{-1}(t), 0, 0, 0\} \\
& - \lambda_4(t) \Omega(t) - \text{diag}\{1, 0, 0, 0, 0, 0, 0, 0\} \\
& - \mathcal{N}^T(t) \sum_{s=1}^m \epsilon_s(t) (Q(\Psi_s - \Psi_{h(t)})) \mathcal{N}(t) \leq 0,
\end{aligned} \tag{30}$$

then inequality $\gamma^T(t+1) P^{-1}(t+1) \gamma(t+1) - 1 \leq 0$ holds by referring to inequalities (22) and (29).

Let us convert (30) into the compact form below:

$$\Phi^T(t) P^{-1}(t+1) \Phi(t) - \Lambda(t) \leq 0. \tag{31}$$

Then, in view of (31) and Lemma 2, we obtain

$$\begin{bmatrix} -\Lambda(t) & * \\ \Phi(t) & -P(t+1) \end{bmatrix} \leq 0. \tag{32}$$

Furthermore, to eliminate uncertainties embedded in (32), we rewrite (32) as

$$\begin{bmatrix} -\Lambda(t) & * \\ \Phi(t) & -P(t+1) \end{bmatrix} + \tilde{\mathcal{H}} \tilde{\mathcal{F}} \tilde{\mathcal{E}} + \left(\tilde{\mathcal{H}} \tilde{\mathcal{F}} \tilde{\mathcal{E}} \right)^T \leq 0 \tag{33}$$

where

$$\tilde{\mathcal{F}} \triangleq \begin{bmatrix} \mathcal{F}_{\mathcal{D}} & 0 & 0 & 0 & 0 \\ 0 & \vartheta^a(z(t)) & 0 & 0 & 0 \\ 0 & 0 & \vartheta^b(z(t)) & 0 & 0 \\ 0 & 0 & 0 & \vartheta^c(z(t)) & 0 \\ 0 & 0 & 0 & 0 & \mathcal{F}_{\mathcal{K}} \end{bmatrix},$$

$$\mathcal{F}_{\mathcal{D}} \triangleq \text{diag}\{\vartheta^d(z(t)), \mathcal{F}_{\mathcal{K}}\}$$

$$\mathcal{F}_{\mathcal{K}} \triangleq \text{diag}\{F_z, F_y\}.$$

By resorting to Lemma 3, we observe that (15) is ensured by (33). Thus, it can be obtained from (15) that $\mathcal{R}(t+1) \leq 1$. This completes the induction. Hence, (14) is guaranteed with parameter $\mathcal{K}(t)$. ■

Theorem 1 guarantees the existence of a CER that embraces all possible $z(t+1)$, where the state estimate obtained from (8) serves as the center of ellipsoid.

In the following corollary, by exerting the convex optimization approach, we will present an iterative algorithm to optimize this ellipsoid and compute the optimal state estimator gain.

Corollary 1: The constraint matrix $P(t)$ on system (10) is minimized (in the sense of the matrix trace) if there exist $\mathcal{K}(t)$, $\varepsilon(t)$, $\lambda_i(t)$ ($i \in \{1, 2, 3, 4\}$) and $\epsilon_s(t)$ ($s \in \{1, 2, \dots, m\}$, $t \in \mathbb{N}^+$) such that the following optimization problem

$$\min_{(P(t+1), \mathcal{K}(t), \ell(t))} \text{tr}\{P(t+1)\} \tag{34}$$

is solvable s.t. (15), where

$$\begin{aligned}
\ell(t) & \triangleq \{\varepsilon(t)\} \cup \{\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t)\} \\
& \quad \cup \{\epsilon_1(t), \epsilon_2(t), \dots, \epsilon_m(t)\}.
\end{aligned}$$

Proof: The proof is easily accessible from Theorem 1 and is therefore omitted for brevity. ■

It should be mentioned that inequalities (15), are linear to variables $P(t)$, $\mathcal{K}(t)$, $\varepsilon(t)$, $\lambda_i(t)$ ($i \in \{1, 2, 3, 4\}$) and $\epsilon_s(t)$ ($s \in \{1, 2, \dots, m\}$). Actually, it is seen from Corollary 1 that the aforementioned optimization problem can be well addressed via standard semi-definite programming techniques. Therefore, by means of Corollary 1, we can summarize the computational algorithm for the state estimation gains $\{\mathcal{K}(t) > 0\}_{t \in \mathbb{N}^+}$ in Algorithm 1 as follows.

Algorithm 1 Computational Algorithm for $\{\mathcal{K}(t) > 0\}_{t \in \mathbb{N}^+}$

Step 1. Initialization: Set $t = 0$ and given N and $P(\varsigma)$.

Step 2. Calculate $L(\varsigma)$ based on $P(\varsigma) = L(\varsigma) L^T(\varsigma)$ for $\varsigma = t - \tau_1$ and $t - \tau_2 \leq \varsigma \leq t$.

Step 3. Solve (34) s. t. (15). Then, $\mathcal{K}(t)$ and $P(t+1)$ are obtained based on the solution of (34).

Step 4. Set $t = t + 1$. If $t > N$, exit. Otherwise, go to Step 2.

Remark 2: In the paper, the set-membership SEP for DMNNs has been effectively coped with under HTDs and the WTODP. One can observe from Theorem 1 and Algorithm 1 that, in the pursuit of the RSME, all significant factors (including the state-dependent parameters, HTDs, noise information, estimation accuracy, WTODP) are fully reflected in the above analysis, and a comprehensive framework is formulated, under

which the desired estimator gains are derived by taking into account factors that sophisticate the concerned model.

Remark 3: The state estimation problem for artificial neural networks has received a large amount of research attention and a large body of results has been available in the literature. In comparison with the existing literature, the main results of this paper exhibits the following distinctive features: 1) the set-membership state estimation problem discussed in this paper is new in the sense that the random gain variation and the WTODP are taken into careful consideration; 2) a novel yet unified estimation scheme is developed to tackle the mathematical complexities stemming from the state-dependent switching behaviors and hybrid time-delays; and 3) the design algorithm of the desired filters is recursive and can be realized online.

IV. AN ILLUSTRATIVE EXAMPLE

This section provides a demonstrative example to validate the proposed estimation approach.

Consider (1) with parameters:

$$\begin{aligned} d_1(z_1(\cdot)) &= \begin{cases} 0.990, & |z_1(\cdot)| > 0.3, \\ 0.930, & |z_1(\cdot)| \leq 0.3, \end{cases} \\ d_2(z_2(\cdot)) &= \begin{cases} 0.420, & |z_2(\cdot)| > 0.3, \\ 0.860, & |z_2(\cdot)| \leq 0.3, \end{cases} \\ a_{11}(z_1(\cdot)) &= \begin{cases} 0.060, & |z_1(\cdot)| > 0.3, \\ 0.050, & |z_1(\cdot)| \leq 0.3, \end{cases} \\ a_{12}(z_1(\cdot)) &= \begin{cases} -0.330, & |z_1(\cdot)| > 0.3, \\ -0.350, & |z_1(\cdot)| \leq 0.3, \end{cases} \\ a_{21}(z_2(\cdot)) &= \begin{cases} -0.230, & |z_2(\cdot)| > 0.3, \\ -0.200, & |z_2(\cdot)| \leq 0.3, \end{cases} \\ a_{22}(z_2(\cdot)) &= \begin{cases} 0.060, & |z_2(\cdot)| > 0.3, \\ 0.040, & |z_2(\cdot)| \leq 0.3, \end{cases} \\ b_{11}(z_1(\cdot)) &= \begin{cases} -0.300, & |z_1(\cdot)| > 0.3, \\ 0.240, & |z_1(\cdot)| \leq 0.3, \end{cases} \\ b_{12}(z_1(\cdot)) &= \begin{cases} 0.005, & |z_1(\cdot)| > 0.3, \\ 0.015, & |z_1(\cdot)| \leq 0.3, \end{cases} \\ b_{21}(z_2(\cdot)) &= \begin{cases} -0.310, & |z_2(\cdot)| > 0.3, \\ 0.270, & |z_2(\cdot)| \leq 0.3, \end{cases} \\ b_{22}(z_2(\cdot)) &= \begin{cases} 0.020, & |z_2(\cdot)| > 0.3, \\ 0.060, & |z_2(\cdot)| \leq 0.3, \end{cases} \\ c_{11}(z_1(\cdot)) &= \begin{cases} 0.015, & |z_1(\cdot)| > 0.3, \\ 0.045, & |z_1(\cdot)| \leq 0.3, \end{cases} \\ c_{12}(z_1(\cdot)) &= \begin{cases} 0.050, & |z_1(\cdot)| > 0.3, \\ 0.010, & |z_1(\cdot)| \leq 0.3, \end{cases} \\ c_{21}(z_2(\cdot)) &= \begin{cases} 0.260, & |z_2(\cdot)| > 0.3, \\ -0.300, & |z_2(\cdot)| \leq 0.3, \end{cases} \end{aligned}$$

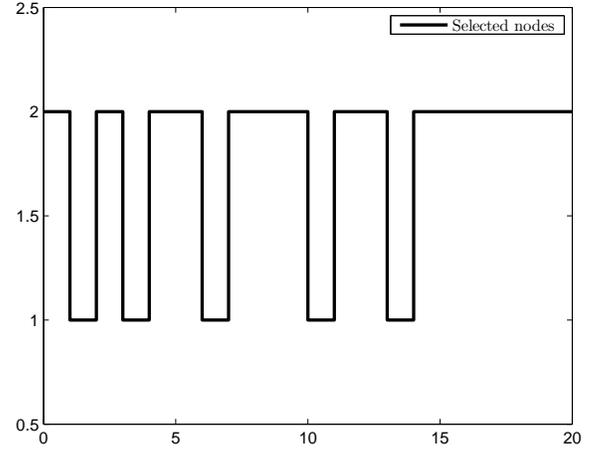


Fig. 2. The selected nodes of MNNs

$$\begin{aligned} c_{22}(z_2(\cdot)) &= \begin{cases} 0.015, & |z_2(\cdot)| > 0.3, \\ 0.035, & |z_2(\cdot)| \leq 0.3, \end{cases} \\ \Gamma_1 = \Gamma_2 = \Gamma_3 &= \begin{bmatrix} 0.25 & 0 \\ 0 & 0.36 \end{bmatrix}, \\ \tau_1 = 1, \tau_2 &= 3. \end{aligned}$$

The weight values in the WTODP are $\sigma_1 = 1.0$ and $\sigma_2 = 0.8$. The bounded noise is set as $v(t) = \sqrt{0.02} \cos(t)$, and thus we have $\Gamma_v(t) = 0.02I$ for $t \in \mathbb{N}^+$. Furthermore, $\phi_2(\varsigma) = [3 \ 3 \ 1 \ 1]^T$, $P(\varsigma) = \text{diag}\{9, 9, 1, 1, 9, 9, 1, 1\}$ ($\varsigma = -\tau, -\tau + 1, \dots, 0$).

Using Matlab YALMIP 3.0 Toolbox, the optimization problem (34) is solved subject to (15), and the corresponding demonstration results are given in Figs. 2-6. Fig. 2 reveals the impact from the neural states on the selected neural nodes, while Fig. 3 and Fig. 4 plot the actual and estimated trajectories of $z_1(t), z_2(t)$ and their estimation errors, respectively. The actual and estimated values of $\bar{y}_1(t), \bar{y}_2(t)$ and their estimation errors are respectively displayed in Fig. 5 and Fig. 6. The obtained results demonstrates that our proposed estimation scheme is indeed effective.

V. CONCLUSIONS

The set-membership SEP has been solved in this paper for DMNNs with HTDS under WTODP. The WTODP has been utilized to mitigate the unnecessary network congestion occurred in the channel between the DMNNs and estimator. To cater for possible execution errors, estimator gains are contaminated by bounded parameter fluctuations. A RSME has then been devised to achieve the estimate of the DMNN susceptible to the UBBNs and WTOD. By means of the RMIs, sufficient conditions have been pinned down to ensure the existence of the desired RSME. Then, an optimization problem has been formulated by minimizing the CER (with respect to estimation errors) under the WTODP. Finally, simulation results illustrate the feasibility of our RSME. Further research topics would

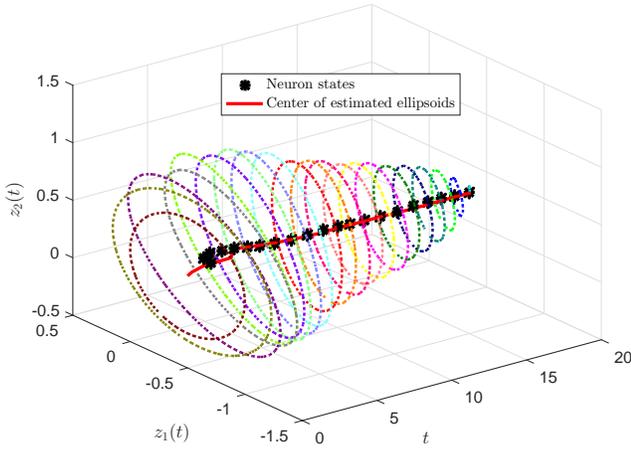


Fig. 3. The actual and estimated trajectories of $z_1(t)$ and $z_2(t)$

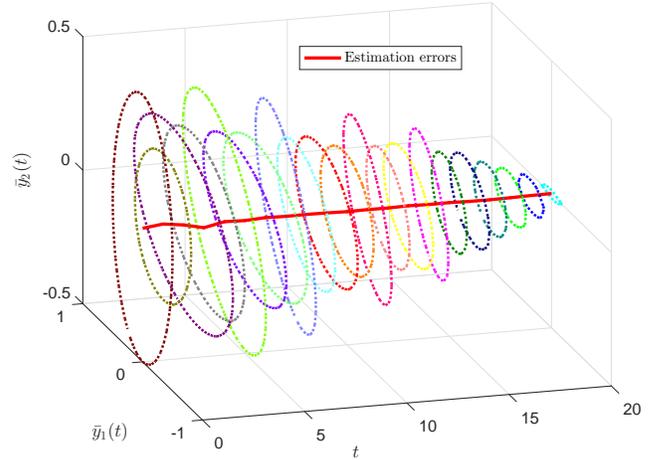


Fig. 6. The estimation errors of $\bar{y}_1(t)$ and $\bar{y}_2(t)$

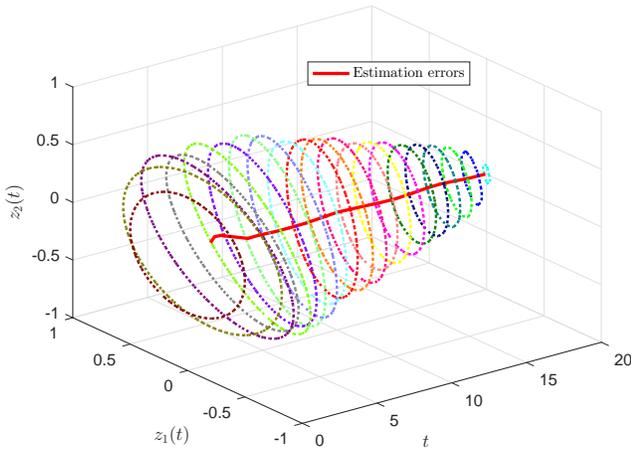


Fig. 4. The estimation errors of $z_1(t)$ and $z_2(t)$

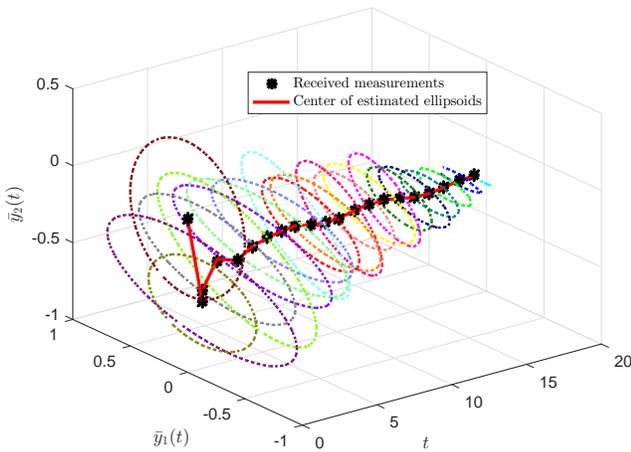


Fig. 5. The actual and estimated values of $\bar{y}_1(t)$ and $\bar{y}_2(t)$

include the extension of the main results to 1) the partial-nodes-based set-membership SEP of complex networks under communication protocols [22], [23], [48]; 2) the recursive SEP of DMNN with network-enhanced complexities [9], [10]; and 3) the improvement of the state estimation performance by using some latest optimization algorithms [24], [25].

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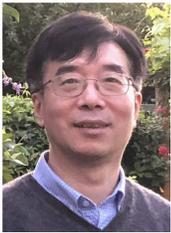
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