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Finite-time State Estimation for Delayed Neural Networks with Redundant Delayed Channels

Zhongyi Zhao, Zidong Wang, Lei Zou and Ge Guo

***Abstract*—The finite-time state estimation issue is addressed in this work for discrete time-delayed neural networks. More than one communication channel is utilized to improve the communication performance. The transmission delays of each channel are modeled by a family of stochastic variables which are independent and identically distributed. The main purpose of the current work is to construct an appropriate state estimation scheme under which the corresponding state estimation error dynamics is finite-time bounded in the mean square. By employing the stochastic analysis approach and introducing a special Lyapunov-like functional, we have developed certain sufficient conditions to achieve the prescribed estimation performance. Furthermore, the exact expressions of the achieved estimator parameters are given by solving a special minimization problem subject to certain inequality constraints. Finally, we propose an illustrative simulation to examine the correctness as well as the effectiveness of our proposed state estimation method.**

***Index Terms*—Delayed neural networks, redundant delayed channels, state estimation, finite-time boundedness**

I. Introduction

A tremendous amount of research interest have been witnessed in past several decades in neural networks (NNs) due primarily to their strong self-learning ability adapting to complex environment as well as their application potentials in multi-objective optimization and control issues. So far, plenty of successful applications of NNs have been found in various of practical areas including the automation control, pattern recognition, signal processing and optimization calculation [5], [11], [15], [27], [28], [35]. In recent several years, plenty of important research results have been derived on various analysis issues of the dynamic behaviors for NNs (such as stability, passivity analysis and synchronization), see e.g. [20]–[22], [30], [33]. Furthermore, It is worth pointing out that time delays would always exist in signal transmission for many artificial NNs and the effects of time delays might give rise to undesired oscillation and even the instability of the NNs. Accordingly, the dynamic behaviors of delayed NNs have been extensively considered and a significant number of valuable research results have appeared in the literature, see [7], [16], [23], [24], [31], [44]. Among others, a research topic that has been drawing particular research attention is the state estimation/filtering issue for NNs with various time delays (e.g. distributed, discrete and mixed delays).

As a hot research topic in signal processing and control areas, state estimation/filtering issue has gained significant research interest due primarily to their wide application in industry [2], [29]. For NNs, the state information is always necessary for dealing with certain tasks including optimization and control. Unfortunately, the state information of NNs might not be always fully available (or accessible) due to a number of reasons (e.g. the large network size and the resource constraints). As such, the state estimation problem from available network measurements becomes critically important for successful utilization of NNs in engineering practice. By now, different state estimation schemes have been developed which include, but are not limited to, the well-known Kalman filtering [13], [18], the $\mathcal{H}_\infty$ state estimation [9], [41] and the set-membership state estimation [10], [42] approaches. To mention just a few, the Kalman filtering technique has been recognized as a credible estimation method to deal with linear systems with Gaussian noises, but it might lead to unsatisfactory performance if the external noises are not strictly Gaussian. For the system with energy-bounded noises, the $\mathcal{H}_\infty$ state estimation is an ideal estimation scheme which aims to provide a fixed disturbance attenuation level on the state estimation error. Furthermore, the set-membership state estimation method is able to handle the estimation task for systems with unknown-but-bounded noises, which guarantees that the SEE is confined to certain ellipsoid at each time step.

In most existing results concerning the state estimation issues, the asymptotic (or exponential) stability is the main concern that represents a type of *steady-state* behavior defined in the infinite-time horizon [3], [8]–[10], [12], [37], [41], [42]. Transient properties, on the other hand, are vitally important as well for some engineering applications. In many practical systems, it is always required that the system could achieve certain transient properties (e.g. finite-time convergence) over a finite horizon with guaranteed steady-state properties, and therefore it makes practical sense to study the transient behaviors over the finite time interval. Consequently, the so-called finite-time stability (FTS), finite-time boundedness (FTB) and finite-time tracking (FTT) have recently attracted quite a lot

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of research attention [13], [34], [38]–[40]. Compared with the well-investigated FTS, the aim of the FTB is to ensure that the state trajectory could reach a bounded set within certain given finite time. It is worth mentioning that, in practical engineering, FTB is sometimes more reasonable since the stability might be difficult to achieve due to various reasons such as persistently bounded disturbances. Up to now, various FTB control/estimation problems have started to attract some initial research interest, see e.g. [34], [36], [39]. For instance, in [39], the finite-time state estimation (FTSE) problem for recurrent delayed NNs subject to component-based event-triggered communication has been examined. In order to quantify the estimation performance, a special performance named finite-time bounded in the mean square (FTBMS) has been proposed in [39] where the desired estimator parameter has been obtained through solving a constrained optimization problem. In [36], the $\mathcal{H}_\infty$ control issue has been addressed for a certain type of Markovian jump systems subject to the average dwell time switching, in which the time-varying transition probability is partly unknown. Sufficient conditions for the FTB of the concerned Markovian jump system have been derived under which the system trajectory is enforced to stay within a prescribed bound.

In response to the rapid development of networked communication, more and more signal transmissions are implemented via the communication network. As such, increasing research efforts have been devoted to the analysis and synthesis problems with different network-induced effects including network-induced delays, signal quantization, channel fading, packet dropouts [1], [6], [14], [17], [43], [45]. For example, in [6], partly known distribution transmission delays have been considered and the corresponding $\mathcal{H}_\infty$ filtering issue of networked systems has been studied. For artificial NNs, it is often the case that we are only able to acquire the observations (e.g. the measurement data) transmitted via network channels (e.g. communication networks with limited bandwidth) with certain communication constraints. As such, the state estimation problems for NNs with network-based communication have recently gained particular research attention. In order to enhance the reliability of transmitted information, a novel network-based communication scheme called redundant channel communication has been employed in [14] to cope with the $\mathcal{H}_\infty$ state estimation issue. In such a communication scheme, one more channel is adopted as a redundant one aiming to reduce the possibility of packet losses in the single-channel case. In [43], the distributed $\mathcal{H}_\infty$ filtering problem with redundant channels has been addressed for a type of Markov jump Lur'e systems subject to stochastic switching topologies over sensor networks. Generally speaking, communication over redundant channels is a good scheme to improve the communication performance since more information could be employed for estimation tasks as compared with the signal channel. It is worth mentioning that, current research works about the redundant channels have only considered redundant channels with packet dropouts. As far as the authors' knowledge goes, the research on the redundant channels with transmission delays has not yet been fully studied despite its explicit practical insight in communication and control areas. This leads to the primary motivation of our study.

According to the above discussions, in this work, we shall consider the FTSE problem for a type of delayed NNs with redundant delayed channels. This is a non-trivial task because of the following two identified difficulties: 1) how to design the state estimator for the considered neural networks under the effects of redundant delayed channels? and 2) how to achieve the desired estimator parameters to guarantee that the desired finite-time performance requirement is satisfied? The main purpose of this paper to provide satisfactory answers to these two questions. *Following are the primary contributions of the current work. (1) The state estimation issue is, for the first time, investigated for NNs subject to delayed redundant channels. (2) A novel SE is developed for dealing with the finite-time state estimation (FTSE) issue. (3) An optimization problem is addressed to achieve the desired estimator parameters by minimizing the settling-like time (SLT).*

The remaining parts of this work are summarized as follows. The mathematical model of our considered problem is presented in Section II including the mechanism of redundant delayed channels. Then, in Section III, we achieve sufficient conditions to deal with the FTB problem for the state estimation error (SEE) in the mean square by solving a special constrained optimization issue. Section IV provide an illustrative simulation example to confirm the correctness as well as the effectiveness of the developed estimation method. Section V is a summary of this paper.

Notation: The notation utilized in this work is quite standard. In this work, $\mathbb{Z}^+$ stands for the set of all nonnegative integers. $\mathbb{R}^{n\times m}$ denotes the set of all $n\times m$ real-valued matrices. $\mathbb{R}^n$ represents the n-dimensional Euclidean space. If the dimension of a matrix is not specified, it means that the matrix has compatible dimension. The superscript "T" denotes matrix transposition. 0 and I denotes, respectively, the zero matrix and identity matrix with appropriate dimensions. The notation $S\leq 0$ (respectively, $S<0$) denotes that S is a real symmetric and negative semi-definite (respectively, negative definite) matrix. The notation $\|F\|$ denotes the usual Euclidean norm of vector F. The asterisk "$*$" is utilized to denote a term which is induced by symmetry in symmetric block matrices. $\mathbb{E}\{x\}$ denotes expectation of x. The block-diagonal matrix is denoted by $\mathrm{diag}\{...\}$. $\mathrm{diag}_n\{X\}$ represents the special block-diagonal matrix with the same block X (i.e. $\mathrm{diag}\{\underbrace{X,X,\cdots,X}_{n}\}$). $\mathrm{vec}_n\{X_i\}$ denotes $\begin{bmatrix} X_1^T & X_2^T & \cdots & X_n^T\end{bmatrix}^T$. $\lceil\cdot\rceil$ denotes top integral function. δ denotes Kronecker delta function of which definition is $\delta(i)=\begin{cases}1, & i=0\\ 0, & i\neq 0\end{cases}$.

II. Preliminaries and Problem Statement

In this work, we consider a special type of delayed discrete-time NNs with noise disturbance of the following form:

$$\begin{cases} x(k+1)=Ax(k)+Ff(x(k))+B\omega(k)\\ \qquad\qquad +Gg(x(k-d(k)))+J(k)\\ y(k)=Cx(k)\\ z(k)=Lx(k)\\ x(k)=\phi(k), \quad 0\geq k\geq -\max\{d(k)\}\end{cases} \tag{1}$$

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where $x(k) \in \mathbb{R}^{n_x}$ is the neural state vector; $f(x(k)) = \text{vec}_{n_x}\{f_i(x_i(k))\}$ and $g(x(k)) = \text{vec}_{n_x}\{g_i(x_i(k))\}$ denote the nonlinear activation function; $\omega(k)$ stands for a Gaussian noise satisfying the conditions $\mathbb{E}\{\omega^2(k)\} = 1$ and $\mathbb{E}\{\omega(k)\} = 0$; $J(k) \in \mathbb{R}^{n_x}$ is the external bias; $y(k) \in \mathbb{R}^{n_y}$ represents the output of the neurons; $z(k) \in \mathbb{R}^{n_z}$ means the signal to be estimated; $\phi(k)$ represents the initial condition. $A = \text{diag}\{a_1, a_2, \cdots, a_{n_x}\}$ denotes the state feedback coefficient matrix; G and F denote, respectively, the delayed connection weight matrix and the connection weight matrix; B, C and L are the known real-valued matrices; $d(k)$ characterizes the time-varying discrete time delay.

Assumption 1: For any given positive constant k, the time-delay $d(k)$ satisfies

$$d_m \le d(k) \le d_M \tag{2}$$

in which d_m and d_M are the known nonnegative integers.

Assumption 2: [25] For any $t, s \in \mathbb{R}, s \ne t$, the nonlinear functions g and f in (1) satisfy $g(0) = 0$, $f(0) = 0$ and

$$\begin{aligned} l_i^- &\le \frac{f_i(s) - f_i(t)}{s - t} \le l_i^+; i = 1, 2, \cdots, n_x \\ m_i^- &\le \frac{g_i(s) - g_i(t)}{s - t} \le m_i^+; i = 1, 2, \cdots, n_x \end{aligned} \tag{3}$$

where l_i^- ,l_i^+ ,m_i^-, m_i^+ denote some known scalars.

Remark 1: As is shown in [25], the scalars l_i^+,l_i^1, m_i^+, m_i^1 in Assumption 2 could be zero, negative or positive. As such, it is clear that the nonlinear activation functions are allowed to be non-monotonic, and these functions are more general compared with the usual sigmoid functions.

In order to enhance the communication reliability, in this work, redundant communication channels are utilized to deal with the data transmission between the NN and the remote SE. Without loss of generality, it is supposed that there are l communication channels adopted between the NN and the estimator, which are specifically shown in the Fig.1. We consider the case that the transmission delay would occur in each communication channel. Let the output signal transmitted via the s-th channel be denoted by $\bar{y}_s(k)$ ($s \in \{1, 2, \cdots, l\}$). Obviously, $\bar{y}_s(k)$ could be written as follows:

$$\bar{y}_s(k) = y\big(k - \tau_s(k)\big) + D_s v_s(k) \tag{4}$$

in which $v_s(k)$ represents a Gaussian white noise of the s-th channel satisfying

$$\mathbb{E}\{v_i\} = 0, i = 1, 2, \cdots, l \tag{5}$$

$$\mathbb{E}\{v_i v_j\} = \begin{cases} 1, i = j \\ 0, i \ne j \end{cases}. \tag{6}$$

$\tau_s(k)$ ($k \in \mathbb{Z}^+, s = 1, 2, ..., l$) denotes the transmission delay of the s-th channel which is assumed to be a sequence of independent identically distributed random variables. Let $\tau_s(k) \in S \triangleq \{0, 1, ..., \bar{\tau}\}$ for all the k and s. Furthermore, the occurrence probability of $\tau_s(k) = t$ ($t \in S$) is given by

$$\Pr\{\tau_s(k) = t\} = p_{st}, s = 1, 2, \cdots, l; t = 0, 1, \cdots, \bar{\tau} \tag{7}$$

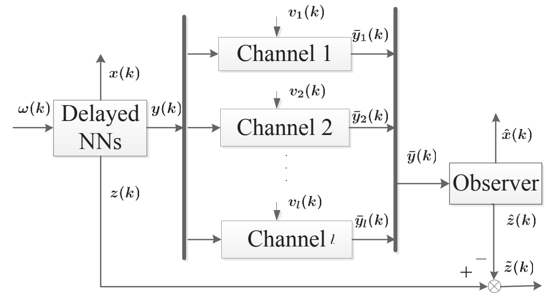

Fig. 1: The estimation scheme with redundant delayed channels.

Then, denoting $\bar{k}_t = k - t$ and using the Kronecker delta function, $\bar{y}_s(k)$ can be reformulated as follows

$$\bar{y}_s(k) = \sum_{t=0}^{\bar{\tau}} \delta\big(t - \tau_s(k)\big) C x(\bar{k}_t) + D_s v_s(k) \tag{8}$$

Obviously, the output signal $\bar{y}_s(k)$ contains both the "distributed delays" and "random variables", which gives rise to the main difficulty in designing the estimator based on the received signal $\bar{y}(k) \triangleq \begin{bmatrix} \bar{y}_1^T(k) & \bar{y}_2^T(k) & \cdots & \bar{y}_l^T(k) \end{bmatrix}^T$.

Remark 2: It is easy to see from the developed measurement model (4) and the probability distribution (7) that the redundant channels could transmit more information than single channel. More specifically, it could be found that the probability of $\bar{y}_s(k) = y(\bar{k}_t) + D_s v_s(k)$ is p_{st}. If only one communication channel is utilized (e.g. only channel 1 is employed), the probability that $\bar{y}(k)$ contains the information about $y(\bar{k}_t)$ is p_{1t}. However, if l communication channels are adopted to transmit data, the probability that $\bar{y}(k)$ contains the information about $y(\bar{k}_t)$ is $\sum_{i=1}^{l} p_{it}$, which is larger than the single-channel case. In other words, as the number of redundant delayed channels increases, the probability of $\bar{y}(k)$ including the information of delayed signal $y_i(\bar{k}_0), y_i(\bar{k}_1), \cdots, y_i(\bar{k}_{\bar{\tau}}), i = 1, 2, ..., l$ would increase, and this enables us to retrieve more useful information. In other words, redundant channels could largely enhance the communication performance between the NN and the remote SE.

To achieve the estimates of the states for the NNs (1), we develop a SE with the following form:

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + Ff\big(\hat{x}(k)\big) + Gg\Big(\hat{x}\big(k - d(k)\big)\Big) \\ \qquad + J(k) + \sum_{s=1}^{l} \Big(K_s\Big(\bar{y}_s(k) - \sum_{t=0}^{\bar{\tau}} \delta\big(t - \tau_s(k)\big) \\ \qquad \times C\hat{x}(\bar{k}_t)\Big)\Big) \\ \hat{z}(k) = L\hat{x}(k) \\ \hat{x}(k) = \hat{\phi}(k), \quad 0 \ge k \ge -d_M \end{cases} \tag{9}$$

where $\hat{x}(k) \in \mathbb{R}^{n_x}$ stands for the estimate of the state $x(k)$ and $\hat{z}(k)$ means the corresponding estimate of the signal $z(k)$; $\hat{\phi}(k)$ is the initial condition; and $K_1, K_2, ..., K_l$ stand for the estimator gain matrices that need to be determined. Then, letting $\tilde{z}(k) = z(k) - \hat{z}(k)$ and $e(k) = x(k) - \hat{x}(k)$, we could

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obtain the estimation dynamics according to (1) and (9) as follows:

$$\begin{cases} e(k+1) = Ae(k) + F\tilde{f}(e(k)) + G\tilde{g}(e(k-d(k))) \\ \qquad - \sum_{s=1}^{l}\Big(K_s\sum_{t=0}^{\bar{\tau}}\delta\big(t-\tau_s(k)\big)Ce(\bar{k}_t)\Big) \\ \qquad + B\omega(k) - \sum_{s=1}^{l}K_sD_sv_s(k) \\ \tilde{z}(k) = Le(k) \end{cases} \tag{10}$$

in which

$$\tilde{g}(e(k)) \triangleq g(x(k)) - g(\hat{x}(k)),$$
$$\tilde{f}(e(k)) \triangleq f(x(k)) - f(\hat{x}(k)).$$

Definition 1: [34] Assume that there exist a time-based function k_* satisfying the following condition:

$$\mathbb{E}\{\|\tilde{z}(k)\|^2\} \leq \varepsilon_*, \forall k \geq k_* \tag{11}$$

where $k_* = k_*(e(0), \varepsilon_*)$ is the SLT function and $\varepsilon_* > 0$ is a given upper bound. Then, the dynamical system (10) is FTBMS.

The purpose of our research is to handle the FTSE issue for a type of discrete time-delayed NNs (1). Specifically, our main attention would be focused on the design issue of the estimator parameter gains K_1, K_2, $\cdots$, K_l such that the SEE dynamics (10) is FTBMS with minimized SLT k_*.

III. Main Results

We firstly achieve the sufficient condition in this Section to ensure that the SEE dynamics (10) is FTBMS. Before giving the main results of our work, let us introduce some necessary lemmas.

Lemma 1: [4] Letting the matrices Y_1, Y_2, Y_3 be given in which $Y_1 = Y_1^T$ and $Y_2 = Y_2^T > 0$, then $Y_1 + Y_3^TY_2^{-1}Y_3 < 0$ if and only if

$$\begin{bmatrix} Y_1 & Y_3^T \\ Y_3 & -Y_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -Y_2 & Y_3 \\ Y_3^T & Y_1 \end{bmatrix} < 0.$$

Lemma 2: [25] Let $\eta = [\eta_1, \eta_2, \cdots, \eta_n]^T \in \mathbb{R}^n$ and $f(\eta) = [f_1(\eta_1), f_2(\eta_2), \cdots, f_n(\eta_n)]^T \in \mathbb{R}^n$ be a continuous nonlinear function satisfying $\iota_p^- \leq \frac{f_i(\epsilon)}{\epsilon} \leq \iota_p^+, \epsilon \neq 0, \epsilon \in \mathbb{R}, 1 \leq p \leq n$ with ι_p^- and ι_p^+ being known scalars. Suppose that $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$ is positive semi-definite. Then

$$\begin{bmatrix} \eta \\ f(\eta) \end{bmatrix}^T \begin{bmatrix} \Lambda M_1 & -\Lambda M_2 \\ -\Lambda M_2 & \Lambda \end{bmatrix} \begin{bmatrix} \eta \\ f(\eta) \end{bmatrix} \leq 0$$

where $M_1 = \mathrm{diag}(\vec{\iota}_1, \vec{\iota}_2, \cdots, \vec{\iota}_n)$, $M_2 = \mathrm{diag}(\bar{\iota}_1, \bar{\iota}_2, \cdots, \bar{\iota}_n)$, $\vec{\iota}_i = \iota_i^+\iota_i^-$ and $\bar{\iota}_i = \iota_i^- + \iota_i^+$.

Theorem 1: Consider the estimator error dynamics (10) and let the positive scalar $0 < \gamma < 1$, estimator gain matrices $K_1, K_2, ..., K_l$ and the desired upper bound of SEE ε_* be given. Then, the dynamical system (10) is FTBMS if there exist $l+4$ positive definite matrices (PDMs) $Q > 0$, $P > 0$, $R_i > 0$ $(i = 1, 2, \cdots, l)$, $\Lambda \triangleq \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_{n_x}) > 0$, $\Gamma \triangleq \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_{n_x}) > 0$ satisfying

$$\Phi < 0 \tag{12}$$

$$L^TL \leq P \tag{13}$$

$$\theta < (1-\gamma)\varepsilon_* \tag{14}$$

where $\bar{\tau}_i = \bar{\tau} + i$ and

$$\Omega_{11} = \begin{bmatrix} \Phi_{00} & \Phi_{01} & \cdots & \Phi_{0\bar{\tau}} \\ * & \Phi_{11} & \cdots & \Phi_{1\bar{\tau}} \\ * & * & \ddots & \vdots \\ * & * & * & \Phi_{\bar{\tau}\bar{\tau}} \end{bmatrix}, \ \Phi = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix},$$

$$\Omega_{12} = \begin{bmatrix} \Phi_{0,\bar{\tau}_1} & \Phi_{0,\bar{\tau}_2} & \Phi_{0,\bar{\tau}_3} \\ \neg_{i,\bar{\tau}_1} & \neg_{i,\bar{\tau}_2} & \neg_{i,\bar{\tau}_3} \end{bmatrix},$$

$$\neg_{i,\bar{\tau}+j} = \mathrm{vec}_{\bar{\tau}}\{\Phi_{i,\bar{\tau}_j}\}, \quad j = 1, 2, 3,$$

$$\Omega_{22} = \begin{bmatrix} \Phi_{\bar{\tau}_1,\bar{\tau}_1} & \Phi_{\bar{\tau}_1,\bar{\tau}_2} & \Phi_{\bar{\tau}_1,\bar{\tau}_3} \\ * & \Phi_{\bar{\tau}_2,\bar{\tau}_2} & \Phi_{\bar{\tau}_2,\bar{\tau}_3} \\ * & * & \Phi_{\bar{\tau}_3,\bar{\tau}_3} \end{bmatrix},$$

$$\begin{aligned} \Phi_{00} = {} & A^TPA + \sum_{i=1}^{l}\bar{\tau}R_i - \gamma P - \Lambda L_1 - \Gamma M_1 \\ & - \sum_{i=1}^{l}p_{i0}A^TPK_iC - \sum_{i=1}^{l}p_{i0}C^TK_i^TPA \\ & + \sum_{i=1}^{l}p_{i0}C^TK_i^TPK_iC \\ & + 2\sum_{1\leq i<j\leq l}p_{i0}p_{j0}C^TK_i^TPK_jC \end{aligned}$$

$$\begin{aligned} \Phi_{st} = {} & -\sum_{i=1}^{l}\gamma^sR_i + \sum_{i=1}^{l}p_{is}C^TK_i^TPK_iC \\ & + 2\sum_{1\leq i<j\leq l}p_{is}p_{jt}C^TK_i^TPK_jC, \\ & s = t \in \{1, 2, \cdots, \bar{\tau}\} \end{aligned}$$

$$\begin{aligned} \Phi_{st} = {} & \sum_{1\leq i<j\leq l}p_{is}p_{jt}C^TK_i^TPK_jC - \sum_{i=1}^{l}p_{it}A^TPK_iC \\ & + \sum_{1\leq i<j\leq l}p_{it}p_{js}C^TK_j^TPK_iC, \\ & s = 0, t = 1, 2, \cdots, \bar{\tau} \end{aligned}$$

$$\begin{aligned} \Phi_{st} = {} & \sum_{1\leq i<j\leq l}p_{is}p_{jt}C^TK_i^TPK_jC \\ & + \sum_{1\leq i<j\leq l}p_{it}p_{js}C^TK_j^TPK_iC, \\ & 1 \leq s < t \leq \bar{\tau} \end{aligned}$$

$$\Phi_{i,\bar{\tau}_1} = \begin{cases} A^TPF - \sum_{s=1}^{l}p_{si}C^TK_s^TPF + \Lambda L_2, & i = 0 \\ -\sum_{s=1}^{l}p_{si}C^TK_s^TPF, & i = 1, 2, \cdots, \bar{\tau} \end{cases}$$

$$\Phi_{i,\bar{\tau}_2} = \begin{cases} \Gamma M_2, & i = 0 \\ 0, & i = 1, 2, \cdots, \bar{\tau} \end{cases}$$

$$\Phi_{i,\bar{\tau}_3} = \begin{cases} A^T PG - \sum_{s=1}^{l} p_{si} C^T K_s^T PG, i = 0 \\ -\sum_{s=1}^{l} p_{si} C^T K_s^T PG, \quad 1 \le i \le \bar{\tau} \end{cases}$$

$\Phi_{\bar{\tau}_1,\bar{\tau}_1} = -\Lambda + F^T PF,\ \Phi_{\bar{\tau}_1,\bar{\tau}_2} = 0,$
$\Phi_{\bar{\tau}_1,\bar{\tau}_3} = F^T PG,\ \Phi_{\bar{\tau}_2,\bar{\tau}_2} = (d_M - d_m + 1)Q - \Gamma,$
$\Phi_{\bar{\tau}_2,\bar{\tau}_3} = 0,\ \Phi_{\bar{\tau}_3,\bar{\tau}_3} = G^T PG - Q,$
$\hat{l}_p = l_p^+ l_p^-, \widehat{l}_p = \frac{l_p^+ + l_p^-}{2}, \hat{m}_p = m_p^+ m_p^-,$
$\widehat{m}_p = \frac{m_p^+ + m_p^-}{2}, p = 1, 2, \cdots, n_x,$
$L_1 = \mathrm{diag}(\hat{l}_1, \hat{l}_2, \cdots, \hat{l}_{n_x}), M_1 = \mathrm{diag}(\hat{m}_1, \hat{m}_2, \cdots, \hat{m}_{n_x}),$
$L_2 = \mathrm{diag}(\widehat{l}_1, \widehat{l}_2, \cdots, \widehat{l}_{n_x}), M_2 = \mathrm{diag}(\widehat{m}_1, \widehat{m}_2, \cdots, \widehat{m}_{n_x}),$
$\theta = B^T PB + \sum_{s=1}^{l} D_s^T K_s^T PK_s D_s.$

In addition, if the condition mentioned above is satisfied, the SLT k_* can be calculated by

$$k_* = \begin{cases} 0, \varepsilon_* \ge \frac{\mathbb{E}\{V(0)\}}{1-\sigma} \\ \lceil \log_\gamma \frac{(1-\sigma)\varepsilon_*}{\mathbb{E}\{V(0)\}} \rceil, \varepsilon_* < \frac{\mathbb{E}\{V(0)\}}{1-\sigma} \end{cases} \quad (15)$$

where $\sigma \in (0, 1)$ is a scalar satisfying

$$\frac{1}{1-\gamma}\Big(\sum_{i=1}^{l} D_i^T K_i^T PK_i D_i + B^T PB\Big) = \sigma\varepsilon_* \quad (16)$$

Proof: Choose a Lyapunov-like functional candidate of the following form:

$$\mathcal{V}(k) = \sum_{j=1}^{4} \mathcal{V}_j(k)$$

where

$$\mathcal{V}_1(k) = e^T(k)Pe(k)$$
$$\mathcal{V}_2(k) = \sum_{j=-d(k)+k}^{-1+k} \gamma^{k-1-j} \tilde{g}^T(e(j))Q\tilde{g}(e(j))$$
$$\mathcal{V}_3(k) = \sum_{i=d_m}^{d_M-1} \sum_{j=-i+k}^{-1+k} \gamma^{k-1-j} \tilde{g}^T(e(j))Q\tilde{g}(e(j))$$
$$\mathcal{V}_4(k) = \sum_{s=1}^{l}\sum_{t=1}^{\bar{\tau}}\sum_{j=-t+k}^{-1+k} \gamma^{k-1-j} e^T(j)R_s e(j)$$

From (10), one obtains that

$$\begin{aligned} &\mathbb{E}\{(1-\gamma)\mathcal{V}_1(k) + \Delta\mathcal{V}_1(k)\} \\ =&\mathbb{E}\{\mathcal{V}_1(k+1) - \gamma\mathcal{V}_1(k)\} \\ =&\mathbb{E}\{e^T(k+1)Pe(k+1) - \gamma e^T(k)Pe(k)\} \\ =&\mathbb{E}\Big\{ e^T(k)(A^T PA - \gamma P)e(k) \\ &+ 2e^T(k)A^T PF\tilde{f}(e(k)) \\ &+ 2e^T(k)A^T PG\tilde{g}(e(k - d(k))) \\ &- 2\sum_{s=1}^{l}\sum_{t=0}^{\bar{\tau}} p_{st} e^T(k)A^T PK_s Ce(\bar{k}_t) \\ &+ \tilde{f}^T(e(k))F^T PF\tilde{f}(e(k)) \\ &+ 2\tilde{f}^T(e(k))F^T PG\tilde{g}(e(k - d(k))) \\ &- 2\sum_{s=1}^{l}\sum_{t=0}^{\bar{\tau}} p_{st}\tilde{f}^T(e(k))F^T PK_s Ce(\bar{k}_t) \\ &+ \tilde{g}^T(e(k - d(k)))G^T PG\tilde{g}(e(k - d(k))) \\ &- 2\sum_{s=1}^{l}\sum_{t=0}^{\bar{\tau}} p_{st}\tilde{g}^T(e(k - d(k)))G^T PK_s Ce(\bar{k}_t) \\ &+ \sum_{s=1}^{l}\sum_{t=0}^{\bar{\tau}} p_{st} e^T(\bar{k}_t)C^T K_s^T PK_s Ce(\bar{k}_t) \\ &+ 2\sum_{1\le i<j\le l}\sum_{s=0}^{\bar{\tau}}\sum_{t=0}^{\bar{\tau}} p_{is}p_{jt} \\ &\times e^T(\bar{k}_s)C^T K_i^T PK_j Ce(\bar{k}_t) \\ &+ B^T PB + \sum_{s=1}^{l} D_s^T K_s^T PK_s D_s \Big\} \end{aligned}$$

$$\begin{aligned} &\mathbb{E}\{(1-\gamma)\mathcal{V}_2(k) + \Delta\mathcal{V}_2(k)\} \\ =&\mathbb{E}\Big\{ \sum_{j=--d(k+1)+k}^{k} \gamma^{k-j}\tilde{g}^T(e(j))Q\tilde{g}(e(j)) \\ &- \gamma\sum_{j=k-d(k)}^{-1+k} \gamma^{k-1-j}\tilde{g}^T(e(j))Q\tilde{g}(e(j)) \Big\} \\ \le&\mathbb{E}\Big\{ \sum_{j=k-d_M+1}^{k-d_m} \gamma^{k-j}\tilde{g}^T(e(j))Q\tilde{g}(e(j)) - \tilde{g}^T(e(k-d(k)))Q \\ &\times \tilde{g}(e(k-d(k))) + \tilde{g}^T(e(k))Q\tilde{g}(e(k)) \Big\} \end{aligned}$$

$$\begin{aligned} &\mathbb{E}\{(1-\gamma)\mathcal{V}_3(k) + \Delta\mathcal{V}_3(k)\} \\ =&\mathbb{E}\Big\{ \sum_{i=d_m}^{-1+d_M}\sum_{j=-i+k+1}^{k} \gamma^{k-j}\tilde{g}^T(e(j))Q\tilde{g}(e(j)) \\ &- \gamma\sum_{i=d_m}^{-1+d_M}\sum_{j=-i+k}^{-1+k} \gamma^{-j+k-1}\tilde{g}^T(e(j))Q\tilde{g}(e(j)) \Big\} \\ =&\mathbb{E}\sum_{i=d_m}^{d_M-1} \Big\{ \tilde{g}^T(e(k))Q\tilde{g}(e(k)) \\ &- \gamma^i\tilde{g}^T(e(\bar{k}_i))Q\tilde{g}(e(\bar{k}_i)) \Big\} \\ =&\mathbb{E}\Big\{ (d_M - d_m)\tilde{g}^T(e(k))Q\tilde{g}(e(k)) \\ &- \sum_{i=d_m}^{d_M-1} \gamma^i\tilde{g}^T(e(\bar{k}_i))Q\tilde{g}(e(\bar{k}_i)) \Big\} \\ =&\mathbb{E}\Big\{ (d_M - d_m)\tilde{g}^T(e(k))Q\tilde{g}(e(k)) \end{aligned}$$

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$$
\left. - \sum_{j=k-d_M+1}^{k-d_m} \gamma^{k-j} \tilde{g}^T(e(j)) Q \tilde{g}(e(j)) \right\}
$$

$$
\begin{aligned}
&\mathbb{E}\{(1-\gamma)\mathcal{V}_4(k) + \Delta\mathcal{V}_4(k)\} \\
=&\mathbb{E}\Big\{ \sum_{s=1}^{l}\sum_{t=1}^{\bar\tau}\sum_{j=-t+k+1}^{k} \gamma^{\bar{k}_j} e^T(j) R_s e(j) \\
&- \gamma \sum_{s=1}^{l}\sum_{t=1}^{\bar\tau}\sum_{j=-t+k}^{-1+k} \gamma^{-j+k-1} e^T(j) R_s e(j) \Big\} \\
=&\mathbb{E}\Big\{ \sum_{s=1}^{l}\sum_{t=1}^{\bar\tau} e^T(k) R_s e(k) \\
&- \sum_{s=1}^{l}\sum_{t=1}^{\bar\tau} \gamma^t e^T(\bar{k}_t) R_s e(\bar{k}_t) \Big\} \\
=&\mathbb{E}\Big\{ \sum_{s=1}^{l} \bar\tau e^T(k) R_s e(k) \\
&- \sum_{s=1}^{l}\sum_{t=1}^{\bar\tau} \gamma^t e^T(\bar{k}_t) R_s e(\bar{k}_t) \Big\}
\end{aligned}
$$

Moreover, it could be obtained from Lemma 2 and Assumption 2 that

$$
\varpi(k)^T \begin{bmatrix} -\Lambda L_1 & \Lambda L_2 \\ * & -\Lambda \end{bmatrix} \varpi(k) \geq 0 \tag{17}
$$

$$
\varpi(k)^T \begin{bmatrix} -\Gamma M_1 & \Gamma M_2 \\ * & -\Gamma \end{bmatrix} \varpi(k) \geq 0 \tag{18}
$$

where $\varpi(k) = \begin{bmatrix} e^T(k) & \tilde{f}^T(e(k)) \end{bmatrix}^T$. As such, it can be obtained according to inequalities (17)-(18) that

$$
\begin{aligned}
&\mathbb{E}\{(1-\gamma)\mathcal{V}(k) + \Delta\mathcal{V}(k)\} \\
=&\sum_{i=1}^{4} \mathbb{E}\{(1-\gamma)\mathcal{V}_i(k) + \Delta\mathcal{V}_i(k)\} \\
\leq&\sum_{i=1}^{4} \mathbb{E}\{(1-\gamma)\mathcal{V}_i(k) + \Delta\mathcal{V}_i(k)\} \\
&+ \mathbb{E}\Big\{ \varpi(k)^T \begin{bmatrix} -\Lambda L_1 & \Lambda L_2 \\ * & -\Lambda \end{bmatrix} \varpi(k) \Big\} \\
&+ \mathbb{E}\Big\{ \varpi(k)^T \begin{bmatrix} -\Gamma M_1 & \Gamma M_2 \\ * & -\Gamma \end{bmatrix} \varpi(k) \Big\} \\
\leq&\mathbb{E}\{\eta^T(k)\Phi\eta(k)\} + B^T P B + \sum_{s=1}^{l} D_s^T K_s^T P K_s D_s
\end{aligned}
$$

where

$$
\eta(k) = \begin{bmatrix} \tilde{e}(k) \\ \tilde{f}(e(k)) \\ \tilde{g}(e(k)) \\ \tilde{g}(e(k-d(k))) \end{bmatrix}, \quad \tilde{e}(k) = \begin{bmatrix} e(\bar{k}_0) \\ e(\bar{k}_1) \\ \vdots \\ e(\bar{k}_{\bar\tau}) \end{bmatrix}.
$$

Based on the condition (12), we have

$$
\mathbb{E}\{(1-\gamma)\mathcal{V}(k) + \Delta\mathcal{V}(k)\} \leq B^T P B + \sum_{s=1}^{l} D_s^T K_s^T P K_s D_s
$$

which implies that

$$
\begin{aligned}
\mathbb{E}\{\mathcal{V}(k)\} \leq&\mathbb{E}\{\gamma\mathcal{V}(\bar{k}_1) + \theta\} \\
\leq&\mathbb{E}\{\gamma^2\mathcal{V}(\bar{k}_2) + \gamma\theta + \theta\} \\
\leq&\mathbb{E}\{\gamma^3\mathcal{V}(\bar{k}_3) + \gamma^2\theta + \gamma\theta + \theta\} \\
\leq&\cdots \leq \mathbb{E}\{\gamma^k\mathcal{V}(0) + \frac{1-\gamma^k}{1-\gamma}\theta\} \\
\leq&\mathbb{E}\{\gamma^k\mathcal{V}(0) + \frac{1}{1-\gamma}\theta\}
\end{aligned}
$$

Let ε_* be a prescribed upper bound of $\mathbb{E}\{\|\tilde{z}(k)\|^2\}$. Then, according to (14), there must exist a $0 < \sigma < 1$ satisfying

$$
\theta = \sigma\varepsilon_*(1-\gamma) \tag{19}
$$

Noticing Definition 1 and (13), we have

$$
\begin{aligned}
\mathbb{E}\{\|\tilde{z}(k)\|^2\} =&\mathbb{E}\{e^T(k) L^T L e(k)\} \\
\leq&\mathbb{E}\{\mathcal{V}(k)\} \\
\leq&\mathbb{E}\{\gamma^k\mathcal{V}(0) + \sigma\varepsilon_*\}
\end{aligned}
$$

which means that the SLT function k_* can be derived as follows:

$$
k_* = \begin{cases} 0, \varepsilon_* \geq \frac{\mathbb{E}\{V(0)\}}{1-\sigma} \\ \lceil \log_\gamma \frac{(1-\sigma)\varepsilon_*}{\mathbb{E}\{V(0)\}} \rceil, \varepsilon_* < \frac{\mathbb{E}\{V(0)\}}{1-\sigma} \end{cases}
$$

The proof is complete now. ■

By now, we have accomplished the analysis task in Theorem 1. Next, we are going to move onto the design issue of the estimator parameters K_i $(i = 1, 2, \cdots, l)$.

Theorem 2: Consider the estimator error dynamics (10). Let the positive scalar $0 < \gamma < 1$ and the desired upper bound ε_* of SEE be given. Then, the dynamical system (10) is FTBMS if there exist $l+4$ PDMs $Q > 0$, $P > 0$, $R_i > 0$ $(1 \leq i \leq l)$, $\Gamma \triangleq \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_{n_x}) > 0$, $\Lambda \triangleq \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_{n_x}) > 0$, and l matrices $Z_1, Z_2, \cdots, Z_l$ satisfying

$$
\Xi < 0 \tag{20}
$$

$$
L^T L \leq P \tag{21}
$$

$$
\Omega < 0 \tag{22}
$$

where

$$
\bar\Xi = -P, \quad \Xi = \begin{bmatrix} \Xi_{11} & * \\ \Xi_{21} & \Xi_{22} \end{bmatrix}, \quad \Xi_{11} = \begin{bmatrix} \tilde\Sigma_{11} & * \\ \tilde\Sigma_{21} & \tilde\Sigma_{22} \end{bmatrix},
$$

$$
\Xi_{21} = \begin{bmatrix} \tilde\Xi^T & \tilde\Xi_0^T & \cdots & \tilde\Xi_{\bar\tau}^T & \tilde\Xi_{1l}^T & \cdots & \tilde\Xi_{l-1,l}^T \end{bmatrix}^T,
$$

$$
\Xi_{22} = \mathrm{diag}\left\{ \bar\Xi, \bar\Xi_0, \cdots, \bar\Xi_{\bar\tau}, \bar\Xi_{1l}, \cdots, \bar\Xi_{l-1,l} \right\},
$$

$$
\tilde\Sigma_{11} = \begin{bmatrix}
\gimel_{00} & * & * & * & * & * \\
\gimel_{10} & \gimel_{11} & * & * & * & * \\
\gimel_{20} & 0 & \gimel_{22} & * & * & * \\
\gimel_{30} & 0 & 0 & \gimel_{33} & * & * \\
\vdots & \vdots & \vdots & \vdots & \ddots & * \\
\Pi_{\bar\tau 0} & 0 & 0 & 0 & \cdots & \gimel_{\bar\tau\bar\tau}
\end{bmatrix},
$$

$$
\tilde\Sigma_{21} = \begin{bmatrix}
\gimel_{\bar\tau_1,0} & \gimel_{\bar\tau_1,1} & \cdots & \gimel_{\bar\tau_1,\bar\tau} \\
\gimel_{\bar\tau_2,0} & \gimel_{\bar\tau_2,1} & \cdots & \gimel_{\bar\tau_2,\bar\tau} \\
\gimel_{\bar\tau_3,0} & \gimel_{\bar\tau_3,1} & \cdots & \gimel_{\bar\tau_3,\bar\tau}
\end{bmatrix},
$$

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$$\tilde{\Sigma}_{22} = \begin{bmatrix} \daleth_{\bar{\tau}_1,\bar{\tau}_1} & * & * \\ \daleth_{\bar{\tau}_2,\bar{\tau}_1} & \daleth_{\bar{\tau}_2,\bar{\tau}_2} & * \\ \daleth_{\bar{\tau}_3,\bar{\tau}_1} & \daleth_{\bar{\tau}_3,\bar{\tau}_2} & \daleth_{\bar{\tau}_3,\bar{\tau}_3} \end{bmatrix},$$

$$\daleth_{ij} = \begin{cases} -\gamma P + \sum_{s=1}^{l} \bar{\tau} R_s - \Lambda L_1 - \Gamma M_1 \\ -\sum_{s=1}^{l} p_{si} A^T Z_s C - \sum_{s=1}^{l} p_{si} C^T Z_s^T A, i, j = 0 \\ -\sum_{s=1}^{l} \gamma^i R_s, \quad i = j, i, j = 1, 2, \ldots, \bar{\tau} \\ -\sum_{s=1}^{l} p_{si} C^T Z_s^T A, \quad j = 0, i = 1, 2, \ldots, \bar{\tau} \end{cases}$$

$$\daleth_{\bar{\tau}_1,i} = \begin{cases} -\sum_{s=1}^{l} p_{si} F^T Z_s C + \Lambda L_2, \quad i = 0 \\ -\sum_{s=1}^{l} p_{si} F^T Z_s C, i = 1, 2, \ldots, \bar{\tau} \\ -\Lambda, \quad i = \bar{\tau} + 1 \end{cases}$$

$$\daleth_{\bar{\tau}_2,i} = \begin{cases} \Gamma M_2, \quad i = 0 \\ 0, \quad 1 \le i \le \bar{\tau}_1 \\ (d_M - d_m + 1) Q - \Gamma, \quad i = \bar{\tau}_2 \end{cases}$$

$$\daleth_{\bar{\tau}_3,i} = \begin{cases} -\sum_{s=1}^{l} p_{si} G^T Z_s C, \quad 1 \le i \le \bar{\tau} \\ 0, \quad i = \bar{\tau}_1, \bar{\tau}_2 \\ -Q, \quad i = \bar{\tau}_3 \end{cases}$$

$$\tilde{\Xi} = \begin{bmatrix} PA & 0 & \cdots & 0 & PF & 0 & PG \end{bmatrix}$$

$$\tilde{\Xi}_0 = \begin{bmatrix} \sqrt{p_{10}} Z_1 C & 0 & \cdots & 0 & 0 & 0 & 0 \\ \sqrt{p_{20}} Z_2 C & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sqrt{p_{l0}} Z_l C & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Xi}_0 = \mathrm{diag}_l\{-P\},$$

$$\tilde{\Xi}_1 = \begin{bmatrix} 0 & \sqrt{p_{11}} Z_1 C & \cdots & 0 & 0 & 0 & 0 \\ 0 & \sqrt{p_{21}} Z_2 C & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \sqrt{p_{l1}} Z_l C & \cdots & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Xi}_1 = \mathrm{diag}_l\{-P\},$$

$$\vdots$$

$$\tilde{\Xi}_{\bar{\tau}} = \begin{bmatrix} 0 & 0 & \cdots & \sqrt{p_{1\bar{\tau}}} Z_1 C & 0 & 0 & 0 \\ 0 & 0 & \cdots & \sqrt{p_{2\bar{\tau}}} Z_2 C & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \sqrt{p_{l\bar{\tau}}} Z_l C & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Xi}_{\bar{\tau}} = \mathrm{diag}_l\{-P\},$$

$$\tilde{\Xi}_{il} = \begin{bmatrix} p_{(i+1)0} Z_{i+1} C & p_{(i+1)1} Z_{i+1} C & \cdots & p_{(i+1)\bar{\tau}} Z_{i+1} C & 0 & 0 & 0 \\ p_{i0} Z_i C & p_{i1} Z_i C & \cdots & p_{i\bar{\tau}} Z_i C & 0 & 0 & 0 \\ p_{(i+2)0} Z_{i+2} C & p_{(i+2)1} Z_{i+2} C & \cdots & p_{(i+2)\bar{\tau}} Z_{i+2} C & 0 & 0 & 0 \\ p_{i0} Z_i C & p_{i1} Z_i C & \cdots & p_{i\bar{\tau}} Z_i C & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ p_{l0} Z_l C & p_{l1} Z_l C & \cdots & p_{l\bar{\tau}} Z_l C & 0 & 0 & 0 \\ p_{i0} Z_i C & p_{i1} Z_i C & \cdots & p_{i\bar{\tau}} Z_i C & 0 & 0 & 0 \end{bmatrix}, \quad i = 1, 2, \cdots, l-1,$$

$$\bar{\Xi}_{il} = \mathrm{diag}_{2(l-i)}\{-P\}, \quad i = 1, 2, \cdots, l-1,$$

$$\Omega = \begin{bmatrix} B^T P B - (1-\gamma)\varepsilon_* & * & * & * & * \\ Z_1 D_1 & -P & * & * & * \\ Z_2 D_2 & 0 & -P & * & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Z_l D_l & 0 & 0 & 0 & -P \end{bmatrix}$$

Furthermore, if $(P, Q, R_1, R_2, \cdots, R_l, \Lambda, \Gamma, Z_1, Z_2, \cdots, Z_l)$ is a feasible solution of (20)-(22), then the parameters of the admissible finite-time SE can be acquired through matrices $Z_i (i = 1, 2, \cdots, l)$ as follows:

$$K_i = P^{-1} Z_i, i = 1, 2, \cdots, l \tag{23}$$

Proof: First, we denote

$$\begin{aligned} \alpha_{i0} &= \begin{bmatrix} \sqrt{p_{i0}} K_i C & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix} \\ \alpha_{i1} &= \begin{bmatrix} 0 & \sqrt{p_{i1}} K_i C & \cdots & 0 & 0 & 0 & 0 \end{bmatrix} \\ &\vdots \\ \alpha_{i\bar{\tau}} &= \begin{bmatrix} 0 & 0 & \cdots & \sqrt{p_{i\bar{\tau}}} K_i C & 0 & 0 & 0 \end{bmatrix} \\ & i = 1, 2, ..., l \\ \alpha_j &= \begin{bmatrix} \alpha_{1j}^T & \alpha_{2j}^T & \cdots & \alpha_{lj}^T \end{bmatrix}^T \\ & j = 0, 1, ..., \bar{\tau} \\ \beta_{ij} &= \begin{bmatrix} p_{j0} K_j C & p_{j1} K_j C & \cdots & p_{j\bar{\tau}} K_j C & 0 & 0 & 0 \\ p_{i0} K_i C & p_{i1} K_i C & \cdots & p_{i\bar{\tau}} K_i C & 0 & 0 & 0 \end{bmatrix} \\ & 1 \le i < j \le l \\ \beta_i &= \begin{bmatrix} \beta_{i,i+1}^T & \beta_{i,i+2}^T & \cdots & \beta_{i,l}^T \end{bmatrix}^T, \quad 1 \le i \le l-1 \\ \Theta &= \begin{bmatrix} \tilde{\Xi}^T & \alpha_0^T & \cdots & \alpha_{\bar{\tau}}^T & \beta_1^T & \cdots & \beta_{l-1}^T \end{bmatrix} \\ \varrho &= l(l-1) + (\bar{\tau}+1) l + 1 \\ \Upsilon &= \mathrm{diag}_{\varrho}\{P, P, \cdots, P\} \end{aligned}$$

Using Lemma 1 and applying the change of variables through $Z_j = P K_j \ (j = 1, 2, \cdots, l)$, it can be seen that $\Phi = \Xi_{11} + \Theta^T \Upsilon \Theta = \Xi_{11} - (\Upsilon\Theta)^T (-\Upsilon)^{-1} (\Upsilon\Theta) < 0$ is guaranteed by the LMI (20). Note also that (14) can be rewritten as $\sum_{i=1}^{l} D_i^T K_i^T P K_i D_i + B^T P B - (1-\gamma)\varepsilon_* < 0$, which is guaranteed by the LMI (22) from Lemma 1. According to Theorem 1, the SEE dynamics is FTBMS. The proof of this theorem is now complete. ■

Having designed the finite-time estimator, we are now going to aim at solving an optimization problem for the SEE dynamics (10), that is, we would like to minimize the SLT.

Theorem 3: Let the prescribed positive scalar $0 < \gamma < 1$, the desired upper bound ε_* of SEE and the upper bound of $\|e(i)\|^2$ and $\|\tilde{g}(e(i))\|^2$ $(i = 0, -1, \cdots, -d_M)$, $\vec{e}_i$ and $\tilde{g}(\vec{e}_i)$, be given. The SEE dynamics (10) is FTBMS if there exist $2l + 6$ PDMs $Q > 0$, $P > 0$, $R_i > 0$ $(1 \le i \le l)$, $\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_{n_x}) > 0$, $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_{n_x}) > 0$, $S_P > 0$, $S_Q > 0$, $S_{R_i} > 0$ $(1 \le i \le l)$ and l matrices Z_1, Z_2, $\cdots$, Z_l such that the optimization problem

$$\min_{\bar{X}, S_{\bar{X}}, \Lambda, \Gamma, Z_1, Z_2, \cdots, Z_l} \mathrm{trace}\{S_P + S_Q + \sum_{i=1}^{l} S_{R_i}\} \tag{24}$$

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with constraints (20), (21), (22) and constraints

$$\begin{bmatrix}-S_P & P^T\\ P & -I\end{bmatrix}<0, \begin{bmatrix}-S_Q & Q^T\\ Q & -I\end{bmatrix}<0, \\ \begin{bmatrix}-S_{R_i} & {R_i}^T\\ R_i & -I\end{bmatrix}<0, i=1,2,\cdots,l \tag{25}$$

has feasible solution, where $\bar{X}$ and $S_{\bar{X}}$ denote, respectively, set $\{P,Q,R_1,R_2,\cdots,R_l\}$ and set $\{S_P,S_Q,S_{R_1},S_{R_2},\cdots,S_{R_l}\}$. Furthermore, when (24) is feasible, the estimator parameters can be given by (23) and the upper bound of minimum SLT k_*, $\vec{k}_*$, can be calculated by

$$\vec{k}_*=\log_\gamma\frac{(1-\sigma)\varepsilon_*}{\mathbb{E}\{\nu\}} \tag{26}$$

where

$$\begin{aligned}\nu=&\mathbb{E}\{\|P\|_F\cdot\vec{e}_0\\ &+\sum_{j=-d_M}^{-1}\gamma^{-1-j}(d_M-d_m+1)\|Q\|_F\cdot\tilde{g}(\vec{e}_j)\\ &+\sum_{s=1}^{l}\sum_{t=1}^{\bar{\tau}}\sum_{j=-t}^{-1}\gamma^{-1-j}\|R_s\|_F\cdot\vec{e}_j\}\end{aligned} \tag{27}$$

Proof: According to Theorem 2, it can be proved that the dynamical system (10) is FTBMS and (23) is the explicit expression of the desired estimator parameters. Moreover, by using Lemma 1, constraint (25) is equivalent to $P^TP\le S_P$, $Q^TQ\le S_Q$, $R_i^TR_i\le S_{R_i}$ $(1\le i\le l)$. Noticing the fact that, for a positive-definite matrix P and a vector x, $x^TPx\le\|P\|_F\cdot\|x\|^2=\sqrt{\text{trace}(P^TP)}\cdot\|x\|^2$ and the form of $\mathbb{E}\{\mathcal{V}(0)\}$, we can gain the conclusion that the optimization problem (24) could achieve the optimization of $\mathbb{E}\{\mathcal{V}(0)\}$. According to Theorem 1, the SLT function k_* can be calculated by (15) and one can find that the smaller the $\mathbb{E}\{\mathcal{V}(0)\}$, the minimum the SLT k_*. This completes the proof of this theorem. ■

Remark 3: In the above theorem, we have proposed sufficient conditions to achieve the FTB of the SEE dynamics in the mean square with the optimized SLT function through an optimization problem with particular solution matrices. Moreover, the explicit expressions of estimator parameters for (9) and the optimized SLT have been given in the meanwhile.

Remark 4: In the past decades, fault detection and fault-tolerant control have gained more and more research interest due to their obvious significance [19].In this paper, we have investigated the case that the probability distribution of transmission delay existing in each channel is known. Note that, the delay step considered is bounded, which means that the channel failure and sensor faults have not been considered here. One of our future research topics is to extended our main results to the finite-time state estimation problem subject to sensor faults or channel failure by adopting some adequate fault detection methods.

IV. Numerical Example

We shall give an illustrative numerical simulation in this section to confirm the correctness and effectiveness of the proposed theorem. Consider the NNs (1) and the output model (4) with the following parameters:

$$A=\begin{bmatrix}1&0&0\\0&0.28&0\\0&0&0.1\end{bmatrix}, F=\begin{bmatrix}0.1&0.05&0.03\\0.06&0.1&0.05\\0.05&0.1&0.08\end{bmatrix},$$
$$G=\begin{bmatrix}0.03&0.07&0.1\\0.02&0.03&0.04\\0.06&0.02&0.04\end{bmatrix}, B=\begin{bmatrix}0.05\\0.02\\0.01\end{bmatrix},$$
$$C=\begin{bmatrix}0.2&0.4&0.3\\0.25&0.3&0.2\end{bmatrix}, L=\begin{bmatrix}1.5&1&0.5\\0.9&1&2\end{bmatrix},$$
$$D_1=\begin{bmatrix}0.01\\0.02\end{bmatrix}, D_2=\begin{bmatrix}0.01\\0.02\end{bmatrix}, D_3=\begin{bmatrix}0.01\\0.02\end{bmatrix},$$
$$J(k)=\begin{bmatrix}0.08\cos(k)\\0.05\sin(k)\\0.06\cos(k)\end{bmatrix}, d(k)=\text{mod}(\frac{k}{2}).$$

The activation function is chosen as

$$\begin{aligned}f(x(k))=&\big[\tanh\big(0.3x_1(k)\big)\quad\tanh\big(0.1x_2(k)\big)\\ &\tanh\big(-0.2x_3(k)\big)\big]^T\\ g(x(k))=&\big[\tanh\big(0.2x_1(k)\big)\quad\tanh\big(-0.7x_2(k)\big)\\ &\tanh\big(0.5x_3(k)\big)\big]^T\end{aligned}$$

and it can then be calculated that

$$L_1=\text{diag}\{-1,-1,-1\}, M_1=\text{diag}\{-1,-1,-1\},$$
$$L_2=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}, M_2=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}.$$

Assume that there are three communication channels between the NN and estimator. The corresponding stochastic transmission delay $\tau_i(k)$ $(i=1,2,3)$ satisfy the following probability distribution:

$$\begin{aligned}&\Pr\{\tau_i(k)=0\}=0.85,\quad\Pr\{\tau_i(k)=1\}=0.1,\\ &\Pr\{\tau_i(k)=2\}=0.05\end{aligned}$$

Let $\gamma=0.6$ and $\varepsilon_*=0.5$. According to the results in Theorem 3, we adopt the MATLAB LMI toolbox to cope with the optimization problem (24) whose solution matrices have prescribed particular structure with the LMI constraints (20), (21) and (22). Then, the estimator gain matrices K_1,K_2 and K_3 can be achieved as follows:

$$K_1=\begin{bmatrix}-1.8618&2.7192\\0.0326&0.0298\\-0.0471&0.0789\end{bmatrix}$$
$$K_2=\begin{bmatrix}-1.8618&2.7192\\0.0326&0.0298\\-0.0471&0.0789\end{bmatrix}$$
$$K_3=\begin{bmatrix}-1.8618&2.7192\\0.0326&0.0298\\-0.0471&0.0789\end{bmatrix}$$

The corresponding simulation results are given in Figs. 2-5, in which Figs. 2-3 plot the state trajectories of $z_i(k)$

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$(i=1,2)$ and their estimations $\hat{z}_i(k)$ $(i=1,2)$ with the initial condition $x(0) = \begin{bmatrix} 3 & -5 & 1.6 \end{bmatrix}^T$, $\hat{x}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, $\phi(i) = \hat{\phi}(i) = 0$ $(i < 0)$. Fig. 4 shows the dynamical evolution of the SEE $\tilde{z}(k)$. Fig. 5 shows the Euclidean norm of the SEE $\tilde{z}(k)$ and the prescribed upper bound ε_*. The SLT function k_* can be computed as $k_* = 16$, and it is easy to see that the square of Euclidean norm of the SEE $\tilde{z}(k)$ stays below its upper bound ε_* in Fig. 5. For comparison purposes, the influence of redundant delayed channels on estimation performance have been observed in TABLE I, where N stands for the number of delayed channels and k_* represents the SLT function. It could be found from this simulation that increasing the number of redundant delayed channels would help lead to the reduction of the SLT k_*, which proves the effectiveness of our proposed estimator design algorithm. The impact of the communication channel parameters on estimation performance is given in TABLE II, where

$$ave \triangleq \frac{1}{\vec{N}\cdot\vec{M}}\sum_{i=1}^{\vec{N}}\sum_{j=1}^{\vec{M}}\|e(j)\|^2$$

$$\bar{t} \triangleq \frac{1}{\vec{N}}\sum_{i=1}^{\vec{N}} t_i$$

with $\vec{N}$, $\vec{M}$ and t_i being respectively the number of the simulation trials, the step size and the running time of the i-th simulation trial. The transmission delay $\tau_i(k)$ $(i=1,2,3)$ of three cases are shown as follows:

- case 1: $\Pr\{\tau_i(k)=0\}=0.85$, $\Pr\{\tau_i(k)=1\}=0.1$, $\Pr\{\tau_i(k)=2\}=0.05$
- case 2: $\Pr\{\tau_i(k)=0\}=0.9$, $\Pr\{\tau_i(k)=1\}=0.1$, $\Pr\{\tau_i(k)=2\}=0$
- case 3: $\Pr\{\tau_i(k)=0\}=0.95$, $\Pr\{\tau_i(k)=1\}=0.025$, $\Pr\{\tau_i(k)=2\}=0.025$

Through TABLE II, the effectiveness of our proposed design method has been further proved.

TABLE I: The relationship between the number N of redundant delayed channels and the upper bound $\vec{k}_*$ of SLT function k_*

N	$\vec{k}_*$
3	16.4847
4	13.8552
5	13.6816

TABLE II: The influence of the communication channels' parameters on estimation performance

performance index	$\vec{k}_*$	ave	$\bar{t}$
case 1	16.4847	0.0642	8.1180
case 2	13.7452	0.0543	1.7856
case 3	13.2471	0.0465	3.1821

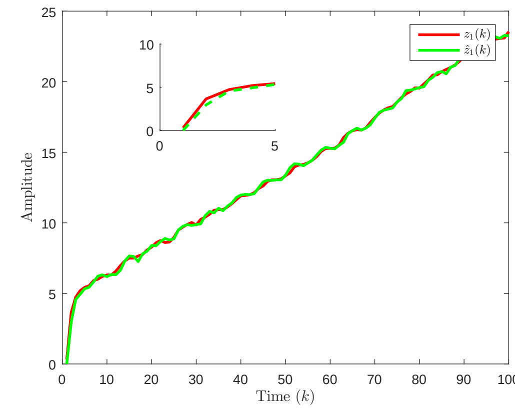

Fig. 2: The state evolutions of $z_1(k)$ and $\hat{z}_1(k)$.

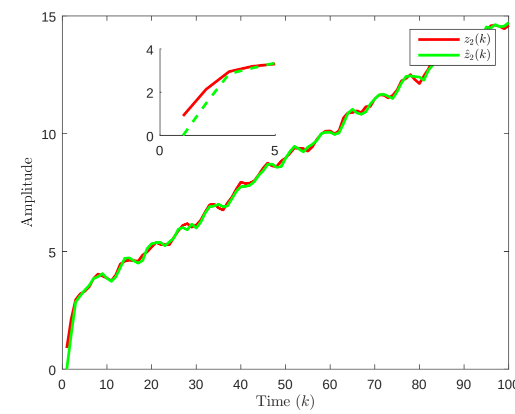

Fig. 3: The state evolutions of $z_2(k)$ and $\hat{z}_2(k)$.

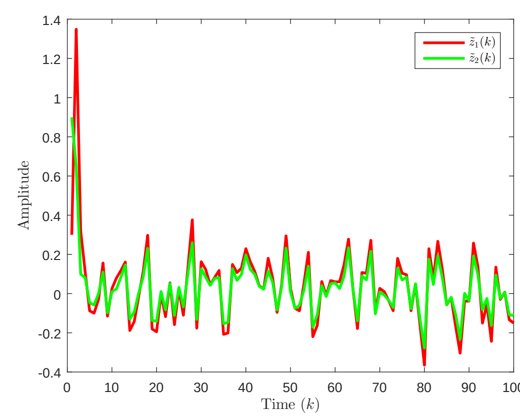

Fig. 4: The components $\tilde{z}_i(k)$ $(i=1,2)$.

V. Conclusion

The FTSE problem has been addressed in this work for a type of delayed NNs subject to redundant delayed channels. To improve communication performance, redundant channels have been used to design the finite-time SE with novel structure and the time delay phenomena existing in redundant channels have been concerned. By introducing a special Lyapunov-like functional corresponding to SEE dynamics and using the stochastic analysis technology, we have achieved sufficient conditions ensuring that the SEE dynamics actualizes FTBMS. Then, the desired estimator gains have been given by solving a special constrained optimization problem. Finally, a

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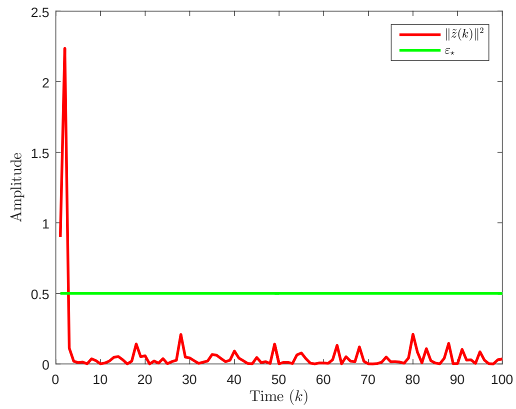

Fig. 5: The size of SEE $\tilde{z}(k)$ and its upper bound $\varepsilon_\star$.

numerical simulation has been adopted to show the correctness and the effectiveness of the developed design method of finite-time estimator. Further research topics include the $\mathcal{H}_\infty$ control problem and $\mathcal{H}_\infty$ filtering problem with redundant delayed channels/protocols [9], [10], [26], [32], [46]–[48]

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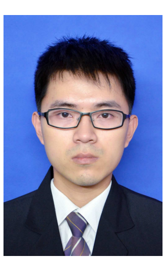

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