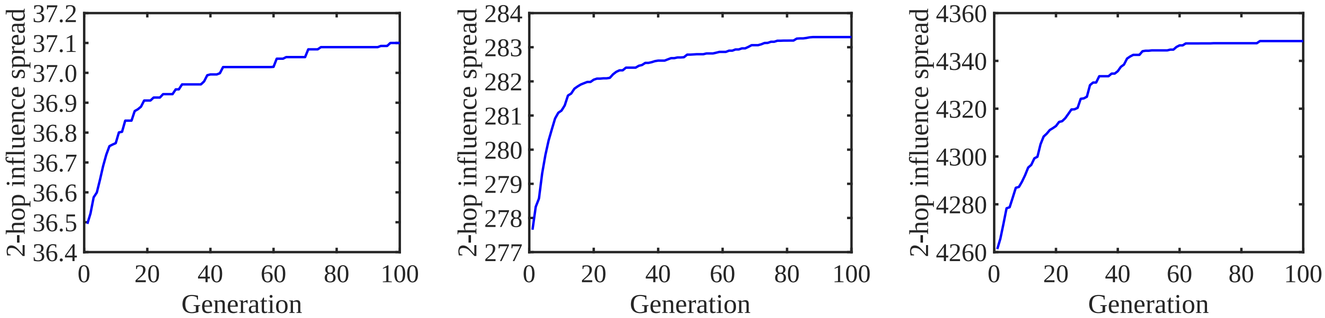

(a) Polbooks-10 (b) Email-30 (c) Wiki-20

Fig. 8. 2-hop influence spread range of EDRL-IM on the Polbook ($k = 10$), Email ($k = 30$) and Wiki ($k = 20$) networks varies with the number of iterations.

nodes selected by the CMA-IM and EDRL-IM algorithms are scattered, and contain some low-degree nodes that connect different connected components. This enables CMA-IM and EDRL-IM to find more influential seed nodes. The main difference of the seed nodes found by the CMA-IM and EDRL-IM is that the seed nodes found by CMA-IM are evenly distributed over all communities. In this case, the influence of nodes within the same community could be overlapped. Therefore, EDRL-IM has a better performance than CMA-IM.

To investigate the convergence of EDRL-IM, Fig. 8 shows the variation of $\mathbf{F}$ values with the number of iterations on the Poolbooks, Email and Wiki networks. The results show that the $\mathbf{F}$ values are increased with the increasing number of iterations when $n_g \leq 60$, which validates the exploration ability of EDRL-IM over the evolution. They also present that EDRL-IM can converge to good solutions within 100 iterations, which demonstrates the convergence and exploitation ability of EDRL-IM in solving the IM problem.

VI. Conclusion

In this article, we have modeled the influence maximization of complex networks as the continuous weight parameter optimization of a deep Q network, and have proposed an evolutionary deep reinforcement learning algorithm (called EDRL-IM) for solving the modeled optimization problem. In EDRL-IM, an evolutionary algorithm has been designed to evolve simultaneously a population of DQNs, each of which can make decisions for the selection of seed nodes through a dynamic markov node selection strategy, while a deep reinforcement learning algorithm has been proposed to accelerate the evolution by using an SGD optimization strategy and integrating all DQNs' training information to optimize the DQN. Experiments on both the GN and LFR benchmark networks and ten real-world networks have shown the advantages of EDRL-IM over the state-of-the-art IM algorithms for finding the influential seed node set, and have validated the generalization of EDRL-IM for tackling different types of networks.

This work was done under the IC influence propagation model, which may be unsuitable certain real scenery such as the influence spreading of social systems under the novel coronavirus (COVID-19), Zika virus and other infectious diseases. In this case, as part of our future work, we will study the IM problem in complex networks under various influence propagation models such as LT, TD, SIR, SEIR, SIQR and SEIQR models, and will use our EDRL-IM solution for tasks of infectious diseases such as disease prediction, target immunity, and analysis of epidemic prevention measures. Moreover, we will study the parallelization of the proposed EDRL-IM, aiming to solve the IM problem in the super-large scale networks which have millions of nodes and billions of edges. In addition, we will extend the proposed EDRL-IM for solving the IM problem in multilayer, temporal, and dynamic networks under with heterogeneous and dynamic influence propagation probability. Finally, inspired by the works [52], we will propose a multitasking evolutionary DRL for simultaneously solving multiple similar optimization problems in complex networks.

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