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Recursive Minimum-Variance Filter Design for State-Saturated Complex Networks with Uncertain Coupling Strengths Subject to Deception Attacks

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***Abstract*—In this paper, the recursive filtering problem is investigated for state-saturated complex networks (CNs) subject to uncertain coupling strengths (UCSs) and deception attacks. The measurement signals transmitted via the communication network may suffer from deception attacks which are governed by Bernoulli-distributed random variables. The purpose of the problem under consideration is to design a minimum-variance filter for CNs with deception attacks, state saturations and UCSs such that upper bounds on the resulting error covariances are guaranteed. Then, the expected filter gains are acquired via minimizing the traces of such upper bounds and sufficient conditions are established to ensure the exponential mean-square boundedness of the filtering errors. At last, two simulation examples (including a practical application) are exploited to validate the effectiveness of our designed approach.**

***Index Terms*—Complex networks, recursive filtering, state saturations, coupling uncertainties, deception attacks.**

I. Introduction

The past decades have seen a recurring research interest in complex networks (CNs) because of their wide applications in a variety of practical situations such as economics, biology, and social science [29]–[31], [33], [36], [38], [39], [41], [47]. Basically, for many CN applications, it is indispensable for the network states to be utilized to fulfil the requirements for monitoring, approximation and optimization. However, due to the inherent characteristics (e.g. complicated structure and local interaction) of CNs, the network states are usually unavailable to the end user but only the measurement outputs can be accessible. As such, much research enthusiasm has recently been generated towards the state estimation problems for CNs.

Up to now, plenty of filtering/estimation algorithms have been devised for a great variety of CNs (see [5], [18], [27], [45], [48], [50]) with tremendous attention from both academia and industry. In practical engineering, many complex systems have *time-varying* parameters that might be caused by different reasons, e.g. operating point shifting and parameter fluctuation [1], [2]. To address the filtering issues of time-varying CNs, various methods have been devised with examples including the finite-horizon H_∞ filtering and recursive filtering (RF) algorithms, where the latter is most widely studied algorithm that has gained a great deal of research interest [5], [10], [15], [20]–[24].

In CN applications, an underlying assumption is that the coupling strengths among CN nodes can be described as certain *known constants*, see e.g. [30], [35], [40], [45]. This assumption is, unfortunately, quite restrictive in practice. For instance, it has been mentioned in [44] that coupling strengths among CN nodes might fluctuate due to various reasons such as noise disturbance and signal transmission congestion. Thus, the analysis/synthesis problems of CNs with uncertain coupling strengths (UCSs) have stirred certain research attention with preliminary results in [14], [26]. In [14], an adaptive scheme has been put forward to handle synchronization problems for CNs against the network deterioration caused by coupling uncertainties. In [26], some adaptive controllers have been designed for CNs with uncertain coupling matrices.

State saturation has been well recognized as a special kind of nonlinear constraints whose pervasive existence is largely due to the physical limits of the internal states of CNs. In fact, in the CN-related filtering problem, the presence of state saturations has a major impact on the filter performance since the state estimate is saturated. The state saturation phenomenon, if not adequately handled, may lead to performance degradation or even filter instability. In recent years, some initial works have been acquired on filtering problems of state-saturated CNs, see e.g. [8], [19]. For instance, an H_∞ filter has been devised in [19] for state-saturated CNs with distributed delays and quantized measurements and under the event-triggering mechanism. Nevertheless, for time-varying state-saturated CNs with UCSs, relevant results on their RF problems are still scarce.

Along with the prevalence of utilizing industry networks, cyber-security issues have gradually become major concerns as open and unprotected communication networks are vulnerable to cyber attacks launched by adversaries. Recently, the security RF problems under cyber attacks have received much attention with many results available in the literature [4], [7], [17]. For example, the distributed RF method has been proposed in [6] for time-delayed stochastic systems with deception attacks

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and quantization effects. The variance-constrained distributed filtering issue has been solved in [32] for sensor networks under deception attacks. In [43], a filter has been designed for nonlinear systems with time-delays, where both stochastic deception attacks and sensor saturations have been taken into account. However, to date, RF issues for CNs with state saturations and UCSs subject to deception attacks have not been thoroughly discussed yet.

Concluding the discussions made thus far, it is both practically and theoretically significant to cope with the RF issue for state-saturated CNs with UCSs under deception attacks. In order to handle this issue, we are confronted with three main difficulties: 1) how to establish an appropriate state-saturated CN model with coupling uncertainties; 2) how to devise a suitable filter for the concerned CNs; 3) how to guarantee the exponential mean-square boundedness (EMSB) of the filtering errors. Accordingly, our primary contributions are: *1) a novel CN model is proposed to tackle the concurrence of state saturations, UCSs, and deception attacks; 2) upper bounds on error covariances are obtained with filter parameters calculated by utilizing both local and neighboring information; and 3) sufficient conditions are presented to guarantee the EMSB of filtering errors.*

II. Problem Formulation and Preliminaries

Consider a state-saturated CN with UCSs as follows:

$$\begin{cases} x_i^{k+1} = \sigma_i\left(f(x_i^k) + \sum_{j=1}^{N}(\omega_{ij}+\Delta\omega_{ij})\Gamma x_j^k\right) + D_i^k\varpi_i^k \\ y_i^k = G_i^k x_i^k + v_i^k \end{cases} \tag{1}$$

where $x_i^k \in \mathbb{R}^n$ $(i = 1, 2, \ldots, N)$ is the state of the ith node and $y_i^k \in \mathbb{R}^m$ is the associated measurement output. $\Gamma \triangleq \mathrm{diag}\{b_1, b_2, \cdots, b_n\} > 0$ denotes the inner-coupling matrix where $b_s \neq 0$ $(s = 1, 2, \ldots, n)$ is the coupling strength. $f(\cdot)$ denotes a known nonlinear function. $\Omega \triangleq [\omega_{ij}]$ $(j = 1, 2, \ldots, N)$ is the certain coupling strength coefficient where $\omega_{ij} \geq 0$. $\Delta\omega_{ij}$ is the UCS with $|\Delta\omega_{ij}| \leq \varsigma_j$ $(j = 1, \cdots, N)$. The zero-mean white Gaussian noises $\varpi_i^k \in \mathbb{R}^{r_1}$ and $v_i^k \in \mathbb{R}^{r_2}$ have covariances $R_{i\varpi}^k$ and R_{iv}^k, respectively. Assume that ϖ_i^k and v_i^k are mutually uncorrelated for any i and k. D_i^k and G_i^k (G_i^k is invertible) are known time-varying system matrices.

The saturation function $\sigma(\cdot) : \mathbb{R}^n \mapsto \mathbb{R}^n$ is defined as

$$\sigma_i(\pi) \triangleq \begin{bmatrix} \sigma_{i1}(\pi_{i1}) & \sigma_{i2}(\pi_{i2}) & \cdots & \sigma_{in}(\pi_{in}) \end{bmatrix}^T \tag{2}$$

where

$$\pi \triangleq \begin{bmatrix} \pi_{i1} & \pi_{i2} & \cdots & \pi_{in} \end{bmatrix}^T, \tag{3}$$

$$\sigma_{is}(\pi_{is}) \triangleq \mathrm{sgn}(\pi_{is})\mathrm{min}\{\pi_{is}^{\max}, |\pi_{is}|\},\ s = 1, 2, \ldots, n \tag{4}$$

in which $\mathrm{sgn}(\cdot)$ denotes a signum function, and $\pi_{is}^{\max}$ is the s-th element of the vector $\pi_i^{\max}$ representing the saturation level vector.

Let us now introduce the transmission model under deception attacks. In general, the successes of deception attacks launched by the adversaries are dependent on the network conditions and the performance of the protection equipment. Therefore, the deception attacks can be considered as randomly occurring from the defenders' perspective and the transmission signals subject to deception attacks can then be modeled as follows:

$$\begin{cases} \tilde{y}_i^k = y_i^k + \alpha_i^k \zeta_i^k, \\ \zeta_i^k = -y_i^k + \xi_k \end{cases} \tag{5}$$

where $\tilde{y}_i^k$ is the received signal of the i-th node from adjacent node, and $\xi_k \in \mathbb{R}^m$ represents the non-zero signal sent by adversaries satisfying $\|\xi_k\| \leq \theta$ for an arbitrary given positive scalar θ.

The stochastic variable α_i^k is Bernoulli-distributed with the following probabilities:

$$\mathrm{Prob}\{\alpha_i^k = 0\} = 1 - \bar{\alpha}_i, \quad \mathrm{Prob}\{\alpha_i^k = 1\} = \bar{\alpha}_i \tag{6}$$

where $\bar{\alpha}_i \in [0, 1)$ is a known constant.

Letting $\hat{x}_i^{k+1|k}$ and $\hat{x}_i^{k+1|k+1}$ represent, respectively, the predicted and estimated values of x_i^{k+1}, we put forward the following filter:

$$\begin{cases} \hat{x}_i^{k+1|k} = \sigma_i\left(f_i(\hat{x}_i^{k|k}) + \sum_{j=1}^{N}\omega_{ij}\Gamma\hat{x}_j^{k|k}\right) \\ \hat{x}_i^{k+1|k+1} = \hat{x}_i^{k+1|k} + K_i^{k+1}(\tilde{y}_i^{k+1} - G_i^{k+1}\hat{x}_i^{k+1|k}) \end{cases} \tag{7}$$

where K_i^{k+1} is the gain to be determined.

In order to facilitate our analysis, we denote

$$\begin{aligned} e_i^{k+1|k} &\triangleq x_i^{k+1} - \hat{x}_i^{k+1|k}, \\ e_i^{k+1|k+1} &\triangleq x_i^{k+1} - \hat{x}_i^{k+1|k+1}, \\ P_i^{k+1|k} &\triangleq \mathbb{E}\left\{e_i^{k+1|k}\left(e_i^{k+1|k}\right)^T\right\}, \\ P_i^{k+1|k+1} &\triangleq \mathbb{E}\left\{e_i^{k+1|k+1}\left(e_i^{k+1|k+1}\right)^T\right\}. \end{aligned}$$

The main objectives of this paper are to: 1) obtain upper bounds on $P_i^{k+1|k+1}$; 2) calculate the time-varying filter gain through minimizing the trace of the obtained bounds; and 3) discuss the EMSB of the filtering errors.

III. Main Results

In this section, we will design a RF method to estimate states of the state-saturated CNs with UCSs subject to deception attacks. First, we calculate upper bounds on $P_i^{k+1|k+1}$. Then, filter gains K_i^{k+1} are obtained by minimizing the trace of the acquired upper bound. Finally, the EMSB of the filtering errors are discussed.

Before proceeding, we need the following lemmas.

Lemma 1: [24] For vectors $a, b \in \mathbb{R}^n$, the inequality

$$ab^T + ba^T \leq \lambda aa^T + \lambda^{-1}bb^T \tag{8}$$

holds where $\lambda > 0$ is a positive scalar.

Lemma 2: [8] For any $x_1, x_2 \in \mathbb{R}$, there exists $\varepsilon_h \in [0, 1]$ such that

$$\sigma_h(x_1) - \sigma_h(x_2) = \varepsilon_h(x_1 - x_2), \quad h = 1, 2, \ldots, n \tag{9}$$

where $\sigma_h(\cdot)$ is the saturation function defined in (2)-(4).

Lemma 3: [34] For matrices $\mathscr{P}, \mathscr{Q}, \mathscr{R}$ and $\mathscr{S}$, symmetric matrix $Z > 0$ and constant $\epsilon > 0$ satisfying $\mathscr{R}\mathscr{R}^T < I$ and $\epsilon^{-1}I - \mathscr{S}Z\mathscr{S}^T > 0$, the inequality

$$\begin{aligned}&(\mathscr{P} + \mathscr{Q}\mathscr{R}\mathscr{S})Z(\mathscr{P} + \mathscr{Q}\mathscr{R}\mathscr{S})^T \\ \leq&\mathscr{P}(Z^{-1} - \epsilon\mathscr{S}^T\mathscr{S})^{-1}\mathscr{P}^T + \epsilon^{-1}\mathscr{Q}\mathscr{Q}^T \end{aligned} \tag{10}$$

holds.

Lemma 4: [42] For $0 \leq k \leq n$, suppose that $X = X^T > 0$, $\mathcal{M}_k(X) = \mathcal{M}_k^T(X) \in \mathbb{R}^{L\times L}$ and $\mathcal{N}_k(X) = \mathcal{N}_k^T(X) \in \mathbb{R}^{L\times L}$. If there exists $\mathcal{Z} = \mathcal{Z}^T > X$ such that

$$\mathcal{M}_k(X) \geq \mathcal{M}_k(\mathcal{Z}), \quad \mathcal{N}_k(X) \geq \mathcal{M}_k(X), \tag{11}$$

then solutions $\mathcal{R}_k$ and $\mathcal{S}_k$ to

$$\mathcal{R}_k = \mathcal{M}_k(\mathcal{R}_{k-1}),\ \mathcal{S}_k = \mathcal{N}_k(\mathcal{S}_{k-1}),\ \mathcal{R}_0 = \mathcal{S}_0 > 0 \tag{12}$$

satisfy $\mathcal{R}_k \leq \mathcal{S}_k$.

Lemma 5: [28] Given constant matrices $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ where $0 < \mathcal{T}_1 = \mathcal{T}_1^T$ and $0 < \mathcal{T}_2 = \mathcal{T}_2^T$, then $\mathcal{T}_1 - \mathcal{T}_3^T\mathcal{T}_2\mathcal{T}_3 \geq 0$ if and only if

$$\begin{aligned}&\begin{bmatrix} \mathcal{T}_1 & \mathcal{T}_3^T \\ \mathcal{T}_3 & \mathcal{T}_2^{-1}\end{bmatrix} \geq 0, \quad \text{or} \quad \begin{bmatrix} \mathcal{T}_2^{-1} & \mathcal{T}_3 \\ \mathcal{T}_3^T & \mathcal{T}_1\end{bmatrix} \geq 0, \\ &\text{or} \quad \mathcal{T}_2^{-1} - \mathcal{T}_3\mathcal{T}_1^{-1}\mathcal{T}_3^T \geq 0.\end{aligned} \tag{13}$$

Lemma 6: [37] For any stochastic process $V_k(\vartheta_k)$ and real number $\delta_{\min}$, $\delta_{\max}$, $u > 0$ and $0 < \beta \leq 1$, if

$$\delta_{\min}\|\vartheta_k\|^2 \leq V_k(\vartheta_k) \leq \delta_{\max}\|\vartheta_k\|^2 \tag{14}$$

and

$$\mathbb{E}\{V_k(\vartheta_k)|\vartheta_{k-1}\} \leq (1-\beta)V_{k-1}(\vartheta_{k-1}) + u, \tag{15}$$

then the EMSB of ϑ_k is ensured , i.e.,

$$\mathbb{E}\big\{\|\vartheta_k\|^2\big\} \leq \frac{\delta_{\max}}{\delta_{\min}}\mathbb{E}\big\{\|\vartheta_0\|^2\big\}(1-\beta)^k + \frac{u}{\delta_{\min}}\sum_{i=1}^{k}(1-\beta)^i. \tag{16}$$

Theorem 1: The following recursions hold:

$$\begin{aligned}&P_i^{k+1|k} \\ =&D_i^k R_{i\varpi}(D_i^k)^T + \mathbb{E}\big\{\Theta_i^k(F_i^k + M_i^kW_i^k)P_i^{k|k}(F_i^k \\ &+ M_i^kW_i^k)^T\Theta_{i,k}^T\big\} + \sum_{j=1}^N\omega_{ij}\mathbb{E}\big\{\Theta_i^k\big((F_i^k + M_i^kW_i^k)e_i^{k|k} \\ &\times (e_j^{k|k})^T\Gamma^T + \Gamma e_j^{k|k}(e_i^{k|k})^T(F_i^k + M_i^kW_i^k)^T\big)(\Theta_i^k)^T\big\} \\ &+ \sum_{j=1}^N\Delta\omega_{ij}\mathbb{E}\big\{\Theta_i^k\big(\Gamma x_j^k(e_i^{k|k})^T(F_i^k + M_i^kW_i^k)^T \\ &+ (F_i^k + M_i^kW_i^k)e_i^{k|k}(x_j^k)^T\Gamma^T\big)(\Theta_i^k)^T\big\} + \sum_{j=1}^N\sum_{p=1}^N\Delta\omega_{ij} \\ &\times \Delta\omega_{ip}\mathbb{E}\big\{\Theta_i^k\Gamma x_j^k(x_p^k)^T\Gamma^T(\Theta_i^k)^T\big\} + \sum_{j=1}^N\sum_{p=1}^N\omega_{ij}\omega_{ip} \\ &\times \mathbb{E}\big\{\Theta_i^k\Gamma e_p^{k|k}(e_j^{k|k})^T\Gamma^T(\Theta_i^k)^T\big\} + \sum_{j=1}^N\sum_{p=1}^N\Delta\omega_{ij}\omega_{ip} \\ &\times \mathbb{E}\big\{\Theta_i^k\Gamma(x_j^k(e_p^{k|k})^T + e_p^{k|k}x_{j,k}^T)\Gamma^T(\Theta_i^k)^T\big\}\end{aligned} \tag{17}$$

and

$$\begin{aligned}&P_i^{k+1|k+1} \\ =&(I - K_i^{k+1}G_i^{k+1})P_i^{k+1|k}(I - K_i^{k+1}G_i^{k+1})^T \\ &+ (1-\bar{\alpha}_i)K_i^{k+1}R_{iv}^{k+1}(K_i^{k+1})^T + \mathbb{E}\big\{\bar{\alpha}_iK_i^{k+1}\xi_{k+1}\xi_{k+1}^T \\ &\times (K_i^{k+1})^T + \bar{\alpha}_iK_i^{k+1}G_i^{k+1}x_i^{k+1}(x_i^{k+1})^T(G_i^{k+1})^T(K_i^{k+1})^T \\ &+ \bar{\alpha}_i(I - K_i^{k+1}G_i^{k+1})e_i^{k+1|k}(x_i^{k+1})^T(G_i^{k+1})^T(K_i^{k+1})^T \\ &- \bar{\alpha}_iK_i^{k+1}G_i^{k+1}x_i^{k+1}\xi_{k+1}^T(K_i^{k+1})^T + \bar{\alpha}_iK_i^{k+1}G_i^{k+1} \\ &\times x_i^{k+1}(e_i^{k+1|k})^T(I - K_i^{k+1}G_i^{k+1})^T - \bar{\alpha}_iK_i^{k+1}\xi_{k+1} \\ &\times (x_i^{k+1})^T(G_i^{k+1})^T(K_i^{k+1})^T - \bar{\alpha}_i(I - K_i^{k+1}G_i^{k+1}) \\ &\times e_i^{k+1|k}\xi_{k+1}^T(K_i^{k+1})^T - \bar{\alpha}_iK_i^{k+1}\xi_{k+1}(e_i^{k+1|k})^T \\ &\times (I - K_i^{k+1}G_i^{k+1})^T\big\}\end{aligned} \tag{18}$$

where $\Theta_i^k \triangleq \mathrm{diag}\big\{(\varepsilon_i^k)^{(1)}, (\varepsilon_i^k)^{(2)}, \cdots, (\varepsilon_i^k)^{(n)}\big\}$, $(\varepsilon_i^k)^{(\nu)} \in [0,1]$ $(\nu = 1,2,\ldots,n)$ and $F_i^k \triangleq \frac{\partial f(x_i^k)}{\partial x_i^k}|_{x_i^k=\hat{x}_i^{k|k}}$. The scaling matrix M_i^k is problem-dependent and the unknown matrix W_i^k stands for linearization errors and satisfies $W_i^k(W_i^k)^T \leq I$.

Proof: It follows from Lemma 1 and (1) that

$$\begin{aligned}e_i^{k+1|k} =&x_i^{k+1} - \hat{x}_i^{k+1|k} \\ =&\sigma_i\big(f_i(x_i^k) + \sum_{j=1}^N(\omega_{ij} + \Delta\omega_{ij})\Gamma x_j^k\big) \\ &- \sigma_i\big(f_i(\hat{x}_i^{k|k}) + \sum_{j=1}^N\omega_{ij}\Gamma\hat{x}_j^{k|k}\big) + D_i^k\varpi_i^k \\ =&\Theta_i^k\big(f_i(x_i^k) - f_i(\hat{x}_i^{k|k}) + \sum_{j=1}^N\Delta\omega_{ij}\Gamma x_j^k \\ &+ \sum_{j=1}^N\omega_{ij}\Gamma e_j^{k|k}\big) + D_i^k\varpi_i^k.\end{aligned} \tag{19}$$

Expanding $f(x_i^k)$ around $\hat{x}_i^{k|k}$ generates

$$f(x_i^k) = f(\hat{x}_i^{k|k}) + F_i^ke_i^{k|k} + o(|e_i^{k|k}|) \tag{20}$$

where

$$F_i^k \triangleq \frac{\partial f(x_i^k)}{\partial x_i^k}|_{x_i^k=\hat{x}_i^{k|k}}$$

is the Jacobian matrix. $o(|e_i^{k|k}|)$ is the high-order term of Taylor series expression. According to [15], $o(|e_i^{k|k}|)$ is rewritten as

$$o(|e_i^{k|k}|) \triangleq M_i^kW_i^ke_i^{k|k}. \tag{21}$$

Substituting (20)-(21) into (19), we obtain

$$\begin{aligned}e_i^{k+1|k} =&x_i^{k+1} - \hat{x}_i^{k+1|k} \\ =&\Theta_i^k(F_i^k + M_i^kW_i^k)e_i^{k|k} + \Theta_i^k\sum_{j=1}^N\Delta\omega_{ij}\Gamma x_j^k \\ &+ \Theta_i^k\sum_{j=1}^N\omega_{ij}\Gamma e_j^{k|k} + D_i^k\varpi_i^k.\end{aligned} \tag{22}$$

On account of $P_i^{k+1|k} \triangleq \mathbb{E}\big\{e_i^{k+1|k}(e_i^{k+1|k})^T\big\}$, one has

$$P_i^{k+1|k}$$

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$$
\begin{aligned}
=&\mathbb{E}\Big\{\big(\Theta_i^k(F_i^k+M_i^kW_i^k)e_i^{k|k}+\Theta_i^k\sum_{j=1}^{N}\Delta\omega_{ij}\Gamma x_j^k \\
&+\Theta_i^k\sum_{j=1}^{N}\omega_{ij}\Gamma e_j^{k|k}+D_i^k\varpi_i^k\big)\big(\Theta_i^k(F_i^k+M_i^kW_i^k) \\
&\times e_i^{k|k}+\Theta_i^k\sum_{j=1}^{N}\Delta\omega_{ij}\Gamma x_j^k+\Theta_i^k\sum_{j=1}^{N}\omega_{ij}\Gamma e_j^{k|k} \\
&+D_i^k\varpi_i^k\big)^T\Big\}. \qquad (23)
\end{aligned}
$$

Thus, it is easy to obtain (17) from (23).

From (1), (5) and (7), one has

$$
\begin{aligned}
e_i^{k+1|k+1}=&x_i^{k+1}-\hat{x}_i^{k+1|k+1} \\
=&(I-K_i^{k+1}G_i^{k+1})e_i^{k+1|k}+\bar{\alpha}_iK_i^{k+1} \\
&\times(G_i^{k+1}x_i^{k+1}+v_i^{k+1}-\xi_{k+1})+\tilde{\alpha}_i^{k+1} \\
&\times K_i^{k+1}(G_i^{k+1}x_i^{k+1}+v_i^{k+1}-\xi_{k+1}) \\
&-K_i^{k+1}v_i^{k+1} \qquad (24)
\end{aligned}
$$

where $\tilde{\alpha}_i^{k+1}\triangleq\alpha_i^{k+1}-\bar{\alpha}_i$. Therefore, $P_i^{k+1|k+1}$ is derived as

$$
\begin{aligned}
&P_i^{k+1|k+1} \\
=&(I-K_i^{k+1}G_i^{k+1})P_i^{k+1|k}(I-K_i^{k+1}G_i^{k+1})^T \\
&+\mathbb{E}\{\bar{\alpha}_i(I-K_i^{k+1}G_i^{k+1})e_i^{k+1|k}(x_i^{k+1})^T(G_i^{k+1})^T(K_i^{k+1})^T \\
&-\bar{\alpha}_i(I-K_i^{k+1}G_i^{k+1})e_i^{k+1|k}\xi_{k+1}^T(K_i^{k+1})^T \\
&+\bar{\alpha}_i^2K_i^{k+1}G_i^{k+1}x_i^{k+1}(x_i^{k+1})^T(G_i^{k+1})^T(K_i^{k+1})^T \\
&+\bar{\alpha}_iK_i^{k+1}G_i^{k+1}x_i^{k+1}(e_i^{k+1|k})^T(I-K_i^{k+1}G_i^{k+1})^T \\
&-\bar{\alpha}_i^2K_i^{k+1}G_i^{k+1}x_i^{k+1}\xi_{k+1}^T(K_i^{k+1})^T+(\bar{\alpha}_i-1)^2 \\
&\times K_i^{k+1}R_{iv}^{k+1}(K_i^{k+1})^T+\bar{\alpha}_i^2K_i^{k+1}\xi_{k+1}\xi_{k+1}^T(K_i^{k+1})^T \\
&-\bar{\alpha}_iK_i^{k+1}\xi_{k+1}(e_i^{k+1|k})^T(I-K_i^{k+1}G_i^{k+1})^T \\
&-\bar{\alpha}_i^2K_i^{k+1}\xi_{k+1}(x_i^{k+1})^T(G_i^{k+1})^T(K_i^{k+1})^T+\bar{\alpha}_i(1-\bar{\alpha}_i) \\
&\times(K_i^{k+1}G_i^{k+1}x_i^{k+1}(x_i^{k+1})^T(G_i^{k+1})^T(K_i^{k+1})^T \\
&+K_i^{k+1}R_{iv}^{k+1}(K_i^{k+1})^T+K_i^{k+1}\xi_{k+1}\xi_{k+1}^T(K_i^{k+1})^T \\
&-K_i^{k+1}G_i^{k+1}x_i^{k+1}\xi_{k+1}^T(K_i^{k+1})^T \\
&-K_i^{k+1}\xi_{k+1}(x_i^{k+1})^T(G_i^{k+1})^T(K_i^{k+1})^T)\}. \qquad (25)
\end{aligned}
$$

■

Remark 1: It is worth mentioning that some uncertain terms are included in (18) because of the consideration of the deception attacks and the noises, and this makes it impossible to accurately compute $P_i^{k+1|k+1}$ and K_i^{k+1}. To deal with this problem, an alternative method for finding upper bounds on $P_i^{k+1|k+1}$ is proposed through employing mathematical induction, and then the filter gain K_i^{k+1} is obtained via minimizing traces of such upper bounds.

Theorem 2: Let model (1), positive scalars $\lambda_1,\rho_1,\rho_2,\rho_3$ and η and initial conditions $P_i^{0|0}\le\Phi_i^{0|0}$ be given. If

$$
\Phi_i^{k+1|k}=a_i^kI+D_i^kR_{i\varpi}^k(D_i^k)^T \qquad (26)
$$

and

$$
\begin{aligned}
&\Phi_i^{k+1|k+1} \\
=&(1+\bar{\alpha}_i\rho_1+\bar{\alpha}_i\rho_3)(I-K_i^{k+1}G_i^{k+1})\Phi_i^{k+1|k} \\
&\times(I-K_i^{k+1}G_i^{k+1})^T+2\bar{\alpha}_i(1+\rho_1^{-1}+\rho_2)K_i^{k+1} \\
&\times G_i^{k+1}\Phi_i^{k+1|k}(G_i^{k+1})^T(K_i^{k+1})^T+2\bar{\alpha}_i(1+\rho_1^{-1}+\rho_2) \\
&\times K_i^{k+1}G_i^{k+1}\hat{x}_i^{k+1|k}\hat{x}_i^T(k+1|k)(G_i^{k+1})^T(K_i^{k+1})^T \\
&+\bar{\alpha}_i(1+\rho_2^{-1}+\rho_3^{-1})\theta^2K_i^{k+1}(K_i^{k+1})^T \\
&+(1-\bar{\alpha}_i)K_i^{k+1}R_{iv}^{k+1}(K_i^{k+1})^T \qquad (27)
\end{aligned}
$$

admit positive-definite solutions such that, for all $k\ge0$, the constraint $\eta^{-1}I<\Phi_i^{k|k}$ is satisfied, then matrices $\Phi_i^{k+1|k}$ and $\Phi_i^{k+1|k+1}$ are, respectively, the upper bounds on $P_i^{k+1|k}$ and $P_i^{k+1|k+1}$, i.e.,

$$
P_i^{k+1|k}\le\Phi_i^{k+1|k}, \qquad (28)
$$

$$
P_i^{k+1|k+1}\le\Phi_i^{k+1|k+1} \qquad (29)
$$

where

$$
\begin{aligned}
a_i^k\triangleq&\min\{z_i^k,4\bar{\pi}_i\}, \\
z_i^k\triangleq&(1+\lambda_1\bar{\omega}_i+\bar{\varsigma})\mathrm{tr}\big(F_i^k((\Phi_i^{k|k})^{-1}-\eta I)^{-1}(F_i^k)^T+\eta^{-1} \\
&\times M_i^k(M_i^k)^T\big)+(\lambda_1^{-1}+\bar{\varsigma}+\bar{\omega}_i)\sum_{j=1}^{N}\omega_{ij}\mathrm{tr}(\Gamma\Phi_j^{k|k}\Gamma^T) \\
&+2(1+\bar{\omega}_i+\bar{\varsigma})\sum_{j=1}^{N}\varsigma_j\mathrm{tr}\big(\Gamma(\Phi_j^{k|k}+\hat{x}_j^{k|k}(\hat{x}_j^{k|k})^T)\Gamma^T\big), \\
\bar{\pi}_i\triangleq&\sum_{h=1}^{n}(\pi_{ih}^{\max})^2,\quad\bar{\omega}_i\triangleq\sum_{j=1}^{N}\omega_{ij},\quad\bar{\varsigma}\triangleq\sum_{j=1}^{N}\varsigma_j. \qquad (30)
\end{aligned}
$$

The gain matrix K_i^{k+1} is calculated by

$$
K_i^{k+1}=(1+\bar{\alpha}_i\rho_1+\bar{\alpha}_i\rho_3)\Phi_i^{k+1|k}(G_i^{k+1})^T(\Pi_i^{k+1})^{-1} \qquad (31)
$$

where

$$
\begin{aligned}
&\Pi_i^{k+1} \\
\triangleq&G_i^{k+1}\Big((1+\bar{\alpha}_i\rho_1+\bar{\alpha}_i\rho_3)\Phi_i^{k+1|k}+2\bar{\alpha}_i(1+\rho_1^{-1}+\rho_2) \\
&\times(\Phi_i^{k+1|k}+\hat{x}_i^{k+1|k}(\hat{x}_i^{k+1|k})^T)\Big)(G_i^{k+1})^T+\bar{\alpha}_i(1+\rho_2^{-1} \\
&+\rho_3^{-1})\theta^2I+(1-\bar{\alpha}_i)R_{iv}^{k+1}. \qquad (32)
\end{aligned}
$$

Proof: The proof is carried out by resorting to mathematical induction. Assume that $P_i^{k|k}\le\Phi_i^{k|k}$.

Observing the terms in (17), one knows from Lemmas 1-2 that

$$
\begin{aligned}
&\sum_{j=1}^{N}\omega_{ij}\mathbb{E}\Big\{\Theta_i^k\big((F_i^k+M_i^kW_i^k)e_i^{k|k}(e_j^{k|k})^T\Gamma^T \\
&+\Gamma e_j^{k|k}(e_i^{k|k})^T(F_i^k+M_i^kW_i^k)^T\big)(\Theta_i^k)^T\Big\} \\
\le&\sum_{j=1}^{N}\omega_{ij}\Big(\lambda_1(F_i^k+M_i^kW_i^k)P_i^{k|k}(F_i^k+M_i^kW_i^k)^T \\
&+\lambda_1^{-1}\Gamma P_j^{k|k}\Gamma^T\Big), \qquad (33)
\end{aligned}
$$

$$
\sum_{j=1}^{N}\Delta\omega_{ij}\mathbb{E}\Big\{\Theta_i^k\big(\Gamma x_j^k(e_i^{k|k})^T(F_i^k+M_i^kW_i^k)^T
$$

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$$
\begin{aligned}
&\quad + (F_i^k + M_i^k W_i^k) e_i^{k|k} (x_j^k)^T \Gamma^T \big) (\Theta_i^k)^T \Big\} \\
&\leq \sum_{j=1}^{N} \varsigma_j \Big((F_i^k + M_i^k W_i^k) P_i^{k|k} (F_i^k + M_i^k W_i^k)^T \\
&\quad + 2\Gamma (P_j^{k|k} + \hat{x}_j^{k|k} (\hat{x}_j^{k|k})^T) \Gamma^T \Big),
\end{aligned} \tag{34}
$$

and

$$
\begin{aligned}
&\sum_{j=1}^{N} \sum_{p=1}^{N} \Delta\omega_{ij} \Delta\omega_{ip} \mathbb{E}\big\{ \Theta_i^k \Gamma x_j^k (x_p^k)^T \Gamma^T (\Theta_i^k)^T \big\} \\
\leq& \frac{1}{2} \sum_{j=1}^{N} \sum_{p=1}^{N} \Delta\omega_{ij} \Delta\omega_{ip} \mathbb{E}\big\{ \Theta_i^k \Gamma (x_j^k (x_p^k)^T + x_p^k x_{j,k}^T) \Gamma^T (\Theta_i^k)^T \big\} \\
\leq& \frac{1}{2} \sum_{j=1}^{N} \sum_{p=1}^{N} \Delta\omega_{ij} \Delta\omega_{ip} \mathbb{E}\big\{ \Theta_i^k \Gamma (x_j^k x_{j,k}^T + x_p^k (x_p^k)^T) \Gamma^T (\Theta_i^k)^T \big\} \\
\leq& 2\bar{\varsigma} \sum_{j=1}^{N} \varsigma_j \Gamma (P_j^{k|k} + \hat{x}_j^{k|k} (\hat{x}_j^{k|k})^T) \Gamma^T.
\end{aligned} \tag{35}
$$

Similarly, we have

$$
\begin{aligned}
&\sum_{j=1}^{N} \sum_{p=1}^{N} \omega_{ij} \omega_{ip} \mathbb{E}\big\{ \Theta_i^k \Gamma e_p^{k|k} (e_j^{k|k})^T \Gamma^T (\Theta_i^k)^T \big\} \\
\leq& \bar{\omega}_i \sum_{j=1}^{N} \omega_{ij} \Gamma P_j^{k|k} \Gamma^T,
\end{aligned} \tag{36}
$$

and

$$
\begin{aligned}
&\sum_{j=1}^{N} \sum_{p=1}^{N} \Delta\omega_{ij} \omega_{ip} \mathbb{E}\big\{ \Theta_i^k \Gamma (x_j^k (e_p^{k|k})^T + e_p^{k|k} x_{j,k}^T) \Gamma^T (\Theta_i^k)^T \big\} \\
\leq& \sum_{j=1}^{N} \sum_{p=1}^{N} \Delta\omega_{ij} \omega_{ip} \mathbb{E}\big\{ \Theta_i^k \Gamma (2 P_j^{k|k} + 2 \hat{x}_j^{k|k} (\hat{x}_j^{k|k})^T \\
&+ P_p^{k|k}) \Gamma^T (\Theta_i^k)^T \big\} \\
\leq& 2\bar{\omega}_i \sum_{j=1}^{N} \varsigma_j \Gamma (P_j^{k|k} + \hat{x}_j^{k|k} (\hat{x}_j^{k|k})^T) \Gamma^T + \bar{\varsigma} \sum_{j=1}^{N} \omega_{ij} \Gamma P_j^{k|k} \Gamma^T.
\end{aligned} \tag{37}
$$

Substituting (33)-(37) into (17) yields

$$
\begin{aligned}
&P_i^{k+1|k} \\
\leq& D_i^k R_{i\varpi}^k (D_i^k)^T + (1 + \lambda_1 \bar{\omega}_i + \bar{\varsigma}) \mathbb{E}\big\{ (F_i^k + M_i^k W_i^k) \\
&P_i^{k|k} (F_i^k + M_i^k W_i^k)^T \big\} + (\lambda_1^{-1} + \bar{\varsigma} + \bar{\omega}_i) \sum_{j=1}^{N} \omega_{ij} \\
&\times \mathbb{E}\big\{ \Gamma P_j^{k|k} \Gamma^T \big\} + 2(1 + \bar{\omega}_i + \bar{\varsigma}) \sum_{j=1}^{N} \varsigma_j \\
&\times \mathbb{E}\big\{ \Gamma \big(P_j^{k|k} + \hat{x}_j^{k|k} (\hat{x}_j^{k|k})^T \big) \Gamma^T \big\} \\
\leq& D_i^k R_{i\varpi}^k (D_i^k)^T + (1 + \lambda_1 \bar{\omega}_i + \bar{\varsigma}) \mathrm{tr}\big(F_i^k ((P_i^{k|k})^{-1} - \eta I)^{-1} \\
&\times (F_i^k)^T + \eta^{-1} M_i^k (M_i^k)^T \big) I + (\lambda_1^{-1} + \bar{\varsigma} + \bar{\omega}_i) \sum_{j=1}^{N} \omega_{ij} \\
&\times \mathrm{tr}(\Gamma P_j^{k|k} \Gamma^T) I + 2(1 + \bar{\omega}_i + \bar{\varsigma}) \sum_{j=1}^{N} \varsigma_j \mathrm{tr}\big(\Gamma (P_j^{k|k} \\
&+ \hat{x}_j^{k|k} (\hat{x}_j^{k|k})^T) \Gamma^T \big) I \\
=& s_i^k I + D_i^k R_{i\varpi}^k (D_i^k)^T
\end{aligned} \tag{38}
$$

where

$$
\begin{aligned}
s_i^k \triangleq& (1 + \lambda_1 \bar{\omega}_i + \bar{\varsigma}) \mathrm{tr}\big(F_i^k ((P_i^{k|k})^{-1} - \eta I)^{-1} (F_i^k)^T + \eta^{-1} \\
&\times M_i^k (M_i^k)^T \big) + (\lambda_1^{-1} + \bar{\varsigma} + \bar{\omega}_i) \sum_{j=1}^{N} \omega_{ij} \mathrm{tr}(\Gamma P_j^{k|k} \Gamma^T) \\
&+ 2(1 + \bar{\omega}_i + \bar{\varsigma}) \sum_{j=1}^{N} \varsigma_j \mathrm{tr}\big(\Gamma (P_j^{k|k} + \hat{x}_j^{k|k} (\hat{x}_j^{k|k})^T) \Gamma^T \big).
\end{aligned} \tag{39}
$$

Next, using the definition of $\sigma_i(\cdot)$ in (2), we have

$$
\begin{aligned}
&P_i^{k+1|k} \\
=& D_i^k R_{i\varpi}^k (D_i^k)^T + \mathbb{E}\Big\{ \Big(\sigma_i \big(f_i(x_i^k) + \sum_{j=1}^{N} (\omega_{ij} + \Delta\omega_{ij}) \Gamma x_j^k \big) \\
&- \sigma_i \big(f_i(\hat{x}_i^{k|k}) + \sum_{j=1}^{N} \omega_{ij} \Gamma \hat{x}_j^{k|k} \big) \Big) \Big(\sigma_i \big(f_i(x_i^k) + \sum_{j=1}^{N} (\omega_{ij} \\
&+ \Delta\omega_{ij}) \Gamma x_j^k \big) - \sigma_i \big(f_i(\hat{x}_i^{k|k}) + \sum_{j=1}^{N} \omega_{ij} \Gamma \hat{x}_j^{k|k} \big) \Big)^T \Big\} \\
\leq& 4\bar{\pi}_i I + D_i^k R_{i\varpi}^k (D_i^k)^T
\end{aligned} \tag{40}
$$

where $\bar{\pi}_i \triangleq \sum_{h=1}^{n} (\pi_{ih}^{\max})^2$.

From (38)-(40), it is concluded that

$$
P_i^{k+1|k} \leq \min\{ s_i^k, 4\bar{\pi}_i \} I + D_i^k R_{i\varpi}^k (D_i^k)^T. \tag{41}
$$

Together with (26), (30), (41) and the assumption $P_i^{k|k} \leq \Phi_i^{k|k}$, one assures that the inequality (28) holds.

By using Lemma 2 and the fact that $x_i^{k+1} = \hat{x}_i^{k+1|k} + e_i^{k+1|k}$, one has

$$
\begin{aligned}
&P_i^{k+1|k+1} \\
\leq& (I - K_i^{k+1} G_i^{k+1}) P_i^{k+1|k} (I - K_i^{k+1} G_i^{k+1})^T \\
&+ (1 - \bar{\alpha}_i) K_i^{k+1} R_{iv}^{k+1} (K_i^{k+1})^T + \mathbb{E}\big\{ \bar{\alpha}_i K_i^{k+1} \xi_{k+1} \xi_{k+1}^T \\
&\times (K_i^{k+1})^T + \bar{\alpha}_i K_i^{k+1} G_i^{k+1} x_i^{k+1} (K_i^{k+1} G_i^{k+1} x_i^{k+1})^T \\
&+ \bar{\alpha}_i \big(\rho_1 (I - K_i^{k+1} G_i^{k+1}) P_i^{k+1|k} (I - K_i^{k+1} G_i^{k+1})^T \\
&+ \rho_1^{-1} K_i^{k+1} G_i^{k+1} x_i^{k+1} (K_i^{k+1} G_i^{k+1} x_i^{k+1})^T \big) \\
&+ \bar{\alpha}_i \big(\rho_2 K_i^{k+1} G_i^{k+1} x_i^{k+1} (K_i^{k+1} G_i^{k+1} x_i^{k+1})^T \\
&+ \rho_2^{-1} K_i^{k+1} \xi_{k+1} \xi_{k+1}^T (K_i^{k+1})^T \big) + \bar{\alpha}_i \big(\rho_3 (I - K_i^{k+1} \\
&\times G_i^{k+1}) P_i^{k+1|k} (I - K_i^{k+1} G_i^{k+1})^T \\
&+ \rho_3^{-1} K_i^{k+1} \xi_{k+1} \xi_{k+1}^T (K_i^{k+1})^T \big) \big\} \\
\leq& (1 + \bar{\alpha}_i \rho_1 + \bar{\alpha}_i \rho_3) (I - K_i^{k+1} G_i^{k+1}) P_i^{k+1|k} \\
&\times (I - K_i^{k+1} G_i^{k+1})^T + (1 - \bar{\alpha}_i) K_i^{k+1} R_{iv}^{k+1} (K_i^{k+1})^T \\
&+ \bar{\alpha}_i (1 + \rho_2^{-1} + \rho_3^{-1}) \theta^2 K_i^{k+1} (K_i^{k+1})^T \\
&+ 2\bar{\alpha}_i (1 + \rho_1^{-1} + \rho_2) K_i^{k+1} G_i^{k+1} P_i^{k+1|k} (G_i^{k+1})^T (K_i^{k+1})^T \\
&+ 2\bar{\alpha}_i (1 + \rho_1^{-1} + \rho_2) K_i^{k+1} G_i^{k+1} \hat{x}_i^{k+1|k} (\hat{x}_i^{k+1|k})^T \\
&\times (G_i^{k+1})^T (K_i^{k+1})^T.
\end{aligned} \tag{42}
$$

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Applying Lemma 4 to (27) and (42), we draw a conclusion as (29).

From the above discussions, the upper bounds (27) on $P_i^{k+1|k+1}$ are obtained. Moving forward, we arrive at

$$\begin{aligned}&\frac{\partial \text{tr}(\Phi_i^{k+1|k+1})}{\partial K_i^{k+1}}\\ =&2(1+\bar{\alpha}_i\rho_1+\bar{\alpha}_i\rho_3)(-\Phi_i^{k+1|k}(G_i^{k+1})^T+K_i^{k+1}G_i^{k+1}\\ &\times\Phi_i^{k+1|k}(G_i^{k+1})^T)+4\bar{\alpha}_i(1+\rho_1^{-1}+\rho_2)K_i^{k+1}G_i^{k+1}\\ &\times\Phi_i^{k+1|k}(G_i^{k+1})^T+2\bar{\alpha}_i(1+\rho_2^{-1}+\rho_3^{-1})\theta^2K_i^{k+1}\\ &+4\bar{\alpha}_i(1+\rho_1^{-1}+\rho_2)K_i^{k+1}G_i^{k+1}\hat{x}_i^{k+1|k}(\hat{x}_i^{k+1|k})^T(G_i^{k+1})^T\\ &+2(1-\bar{\alpha}_i)K_i^{k+1}R_{iv}^{k+1}\\ =&-2(1+\bar{\alpha}_i\rho_1+\bar{\alpha}_i\rho_3)\Phi_i^{k+1|k}(G_i^{k+1})^T+2K_i^{k+1}\Pi_i^{k+1}\\ =&0. \end{aligned}\tag{43}$$

It is obtained from (43) and the invertibility of Π_i^{k+1} that K_i^{k+1} is computed by (31). ■

Remark 2: In this paper, a novel approach has been presented to calculate the time-varying filter gain for CNs. The main idea of this method is to calculate the filter gain of node i by only using its *local* and neighboring information, thereby reducing the computational complexity. Note that, by using the traditional state-augmentation method, the dimension of the corresponding upper-bound matrix amounts to $nN\times nN$, which would give rise to a heavy computational burden in case of large network size. As such, our recursive scheme is more suitable for online application in practical engineering as compared with the traditional state augmentation approach.

Now, we are ready to discuss the EMSB of $e_i^{k|k}$ under Assumption 1 and Lemmas 7-8.

Assumption 1: The positive scalars $\underline{r}_{i\varpi}$, $\bar{r}_{i\varpi}$, $\underline{\gamma}$, $\bar{\gamma}$, $\underline{d}_i$, $\bar{d}_i$, $\underline{f}_i$, $\bar{f}_i$, $\underline{m}_i$ and $\bar{m}_i$ satisfy

$$\begin{gathered}\underline{r}_{i\varpi}\le\|R_{i\varpi,k+1}\|\le\bar{r}_{i\varpi},\ \underline{\gamma}\le\|\Gamma\|\le\bar{\gamma},\\ \underline{d}_i\le\|D_i^k\|\le\bar{d}_i,\ \underline{f}_i\le\|F_i^k\|\le\bar{f}_i,\ \underline{m}_i\le\|M_i^k\|\le\bar{m}_i\end{gathered}$$

for any i and k.

Lemma 7: Consider the state-saturated CN described by (1)-(6) with filter (7). Then, we have

$$\begin{aligned}\mathcal{H}_i^{k+1}&\le(\Phi_i^{k+1|k})^{-1},\\ (G_i^{k+1})^T\mathcal{K}_i^{k+1}G_i^{k+1}&\le\frac{1}{2\bar{\alpha}_i(1+\rho_1^{-1}+\rho_2)}(\Phi_i^{k+1|k})^{-1},\\ (\Phi_i^{k+1|k})^{-1}&\le\frac{1}{\underline{d}_i^2\underline{r}_{i\varpi}}I\end{aligned}\tag{44}$$

if positive scalars ρ_1 and ρ_2 exist such that

$$\begin{aligned}3+2\rho_1^{-1}+2\rho_2\ge\rho_1,\\ \rho_1\ge2\end{aligned}\tag{45}$$

where

$$\begin{aligned}\mathcal{H}_i^{k+1}&\triangleq(\mathcal{L}_i^{k+1})^T(\Phi_i^{k+1|k+1})^{-1}\mathcal{L}_i^{k+1},\\ \mathcal{K}_i^{k+1}&\triangleq(K_i^{k+1})^T\times(\Phi_i^{k+1|k+1})^{-1}K_i^{k+1},\\ \mathcal{L}_i^{k+1}&\triangleq I-(1-\bar{\alpha}_i)K_i^{k+1}G_i^{k+1}.\end{aligned}$$

Proof: It follows from (26) that

$$\Phi_i^{k+1|k}\ge D_i^kR_{i\varpi}^k(D_i^k)^T,\tag{46}$$

or

$$(\Phi_i^{k+1|k})^{-1}\le\frac{1}{\underline{d}_i^2\underline{r}_{i\varpi}}I.$$

From (27), one knows that

$$\begin{aligned}\Phi_i^{k+1|k+1}\ge&(1+\bar{\alpha}_i\rho_1+\bar{\alpha}_i\rho_3)(I-K_i^{k+1}G_i^{k+1})\Phi_i^{k+1|k}\\ &\times(I-K_i^{k+1}G_i^{k+1})^T+2\bar{\alpha}_i(1+\rho_1^{-1}+\rho_2)\\ &\times K_i^{k+1}G_i^{k+1}\Phi_i^{k+1|k}(G_i^{k+1})^T(K_i^{k+1})^T.\end{aligned}\tag{47}$$

Thus, if the positive scalars ρ_1 and ρ_2 satisfy the condition (45), the inequality

$$\Phi_i^{k+1|k+1}\ge\mathcal{L}_i^{k+1}\Phi_i^{k+1|k}(\mathcal{L}_i^{k+1})^T,\tag{48}$$

holds. Then, by applying Lemma 5, the above inequality (48) holds if and only if

$$(\mathcal{L}_i^{k+1})^T(\Phi_i^{k+1|k+1})^{-1}\mathcal{L}_i^{k+1}\le(\Phi_i^{k+1|k})^{-1},\tag{49}$$

i.e., $\mathcal{H}_i^{k+1}\le(\Phi_i^{k+1|k})^{-1}$.

In addition, (27) tells that

$$\begin{aligned}\Phi_i^{k+1|k+1}\ge&2\bar{\alpha}_i(1+\rho_1^{-1}+\rho_2)K_i^{k+1}G_i^{k+1}\Phi_i^{k+1|k}\\ &\times(G_i^{k+1})^T(K_i^{k+1})^T.\end{aligned}\tag{50}$$

By means of Lemma 5, it is derived from (50) that

$$\begin{aligned}&(G_i^{k+1})^T(K_i^{k+1})^T(\Phi_i^{k+1|k+1})^{-1}K_i^{k+1}G_i^{k+1}\\ \le&\frac{1}{2\bar{\alpha}_i(1+\rho_1^{-1}+\rho_2)}(\Phi_i^{k+1|k})^{-1},\end{aligned}\tag{51}$$

or

$$(G_i^{k+1})^T\mathcal{K}_i^{k+1}G_i^{k+1}\le\frac{1}{2\bar{\alpha}_i(1+\rho_1^{-1}+\rho_2)}(\Phi_i^{k+1|k})^{-1}.$$

Based on Assumption 1 and summarizing the above analysis, Lemma 7 is proven. ■

Lemma 8: Consider the state-saturated CN described by (1)-(6). We have

$$\begin{aligned}&\mathcal{F}_{i,k}^T(\Phi_i^{k+1|k})^{-1}\mathcal{F}_{i,k}\le\frac{1}{1+\lambda_1\bar{\omega}_i+\bar{\varsigma}}(\Phi_i^{k|k})^{-1},\\ &(\Phi_i^{k+1|k})^{-1}\le\frac{1}{1+\lambda_1\bar{\omega}_i+\bar{\varsigma}}\big((F_i^k)^T\big)^{-1}(\Phi_i^{k|k})^{-1}(F_i^k)^{-1}\end{aligned}\tag{52}$$

when $\Phi_i^{k+1|k}=z_i^kI+D_i^kR_{i\varpi}^k(D_i^k)^T$, where $\mathcal{F}_{i,k}\triangleq F_i^k+M_i^kW_i^k$.

Proof: From (26), (30) and Lemma 3, we can easily obtain the following inequalities:

$$\begin{aligned}\Phi_i^{k+1|k}&\ge(1+\lambda_1\bar{\omega}_i+\bar{\varsigma})\mathcal{F}_{i,k}\Phi_i^{k|k}\mathcal{F}_{i,k}^T,\\ \Phi_i^{k+1|k}&\ge(1+\lambda_1\bar{\omega}_i+\bar{\varsigma})F_i^k((\Phi_i^{k|k})^{-1}-\eta I)^{-1}(F_i^k)^T.\end{aligned}\tag{53}$$

According to the definition of F_i^k in (20), it is easy to know that F_i^k is invertible. Based on Lemma 5, we see from (53) that Lemma 8 holds. ■

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Theorem 3: Consider the state-saturated CN depicted by (1). Let λ_1, ρ_1 and ρ_2 be positive scalars satisfying

$$\begin{aligned} 2\rho_2 \geq& \rho_1 - 2\rho_1^{-1} - 3, \\ \rho_1 \geq& 2, \\ \lambda_1 \bar{\varpi}_i \underline{f}_i^2 >& \underline{f}_i^2(5 + 3\bar{\varsigma} + 2\bar{\omega}_i) + N\bar{f}_i^2\bar{\gamma}^2(\bar{\varsigma} + \bar{\omega}_i) \\ &\times (5 + 3N\bar{\varsigma} + 2N\bar{\omega}_i). \end{aligned} \tag{54}$$

Then, the EMSB of $e_i^{k|k}$ is ensured under Assumption 1.

Proof: First, we define the following notations:

$$\begin{aligned} e^{k+1|k} \triangleq& [(e_1^{k+1|k})^T, (e_2^{k+1|k})^T, \cdots, (e_N^{k+1|k})^T]^T, \\ e^{k+1|k+1} \triangleq& [(e_1^{k+1|k+1})^T, (e_2^{k+1|k+1})^T, \cdots, (e_N^{k+1|k+1})^T]^T. \end{aligned}$$

Next, we select a quadratic function as follows:

$$V_k(e^{k|k}) \triangleq \sum_{i=1}^{N} (e_i^{k|k})^T (\Phi_i^{k|k})^{-1} e_i^{k|k}. \tag{55}$$

Substituting (22) and (24) into (55), we obtain

$$\begin{aligned} &\mathbb{E}\{V_{k+1}(e^{k+1|k+1})|e^{k|k}\} \\ =& \sum_{i=1}^{N} (e_i^{k+1|k+1})^T (\Phi_i^{k+1|k+1})^{-1} e_i^{k+1|k+1}. \end{aligned} \tag{56}$$

According to (22), (24) and recalling $x_i^{k+1} = e_i^{k+1|k} + \hat{x}_i^{k+1|k}$, one has

$$\begin{aligned} e_i^{k+1|k+1} =& (\mathcal{L}_i^{k+1} + \tilde{\alpha}_i^{k+1} K_i^{k+1} G_i^{k+1}) \Theta_i^k \big(\mathcal{F}_{i,k} e_i^{k|k} \\ &+ \sum_{j=1}^{N} \Delta\omega_{ij} \Gamma \hat{x}_j^{k|k} + \sum_{j=1}^{N} (\Delta\omega_{ij} + \omega_{ij}) \Gamma e_j^{k|k}\big) \\ &+ (\mathcal{L}_i^{k+1} + \tilde{\alpha}_i^{k+1} K_i^{k+1} G_i^{k+1}) D_i^k \varpi_i^k \\ &+ \bar{\alpha}_i K_i^{k+1} G_i^{k+1} \hat{x}_i^{k+1|k} - (1 - \bar{\alpha}_i) K_i^{k+1} v_i^{k+1} \\ &- \bar{\alpha}_i K_i^{k+1} \xi_{k+1} + \tilde{\alpha}_i^{k+1} K_i^{k+1} (G_i^{k+1} \hat{x}_i^{k+1|k} \\ &+ v_i^{k+1} - \xi_{k+1}). \end{aligned} \tag{57}$$

Calculate $\mathbb{E}\{V_{k+1}(e^{k+1|k+1})|e^{k|k}\}$ along (57) as follows:

$$\begin{aligned} &\mathbb{E}\{V_{k+1}(e^{k+1|k+1})|e^{k|k}\} \\ =& \sum_{i=1}^{N} (e_i^{k|k})^T \mathcal{F}_{i,k}^T (\Theta_i^k)^T \mathcal{G}_i^{k+1} \Theta_i^k \mathcal{F}_{i,k} e_i^{k|k} + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{h=1}^{n} (\Delta\omega_{ij} \\ &+ \omega_{ij})(\Delta\omega_{ih} + \omega_{ih})(e_j^{k|k})^T \Gamma^T (\Theta_i^k)^T \mathcal{G}_i^{k+1} \Theta_i^k \Gamma e_h^{k|k} \\ &+ \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{h=1}^{n} \Delta\omega_{ij}\Delta\omega_{ih} (\hat{x}_j^{k|k})^T \Gamma^T (\Theta_i^k)^T \mathcal{G}_i^{k+1} \Theta_i^k \Gamma \hat{x}_h^{k|k} \\ &+ \sum_{i=1}^{N} \bar{\alpha}_i \xi_{k+1}^T \mathcal{K}_i^{k+1} \xi_{k+1} + \sum_{i=1}^{N} (\varpi_i^k)^T (D_i^k)^T \mathcal{G}_i^{k+1} D_i^k \varpi_i^k \\ &+ \sum_{i=1}^{N} \bar{\alpha}_i (\hat{x}_i^{k+1|k})^T (G_i^{k+1})^T \mathcal{K}_i^{k+1} G_i^{k+1} \hat{x}_i^{k+1|k} \\ &+ \sum_{i=1}^{N} (1 - \bar{\alpha}_i)(v_i^{k+1})^T \mathcal{K}_i^{k+1} v_i^{k+1} \\ &+ 2\sum_{i=1}^{N}\sum_{j=1}^{N} (\Delta\omega_{ij} + \omega_{ij})(e_i^{k|k})^T \mathcal{F}_{i,k}^T (\Theta_i^k)^T \mathcal{G}_i^{k+1} \Theta_i^k \Gamma e_j^{k|k} \\ &+ 2\sum_{i=1}^{N}\sum_{j=1}^{N} \Delta\omega_{ij} (e_i^{k|k})^T \mathcal{F}_{i,k}^T (\Theta_i^k)^T \mathcal{G}_i^{k+1} \Theta_i^k \Gamma \hat{x}_j^{k|k} \\ &+ 2\sum_{i=1}^{N} \bar{\alpha}_i (e_i^{k|k})^T \mathcal{F}_{i,k}^T (\Theta_i^k)^T \mathcal{Q}_i^{k+1} (G_i^{k+1} \hat{x}_i^{k+1|k} - \xi_{k+1}) \\ &- 2\sum_{i=1}^{N} \bar{\alpha}_i (\hat{x}_i^{k+1|k})^T (G_i^{k+1})^T \mathcal{K}_i^{k+1} \xi_{k+1} \\ &+ 2\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{h=1}^{N} \Delta\omega_{ih}(\Delta\omega_{ij} + \omega_{ij})(e_j^{k|k})^T \Gamma^T (\Theta_i^k)^T \mathcal{G}_i^{k+1} \\ &\times \Theta_i^k \Gamma \hat{x}_h^{k|k} \Gamma \hat{x}_h^{k|k} + 2\sum_{i=1}^{N}\sum_{j=1}^{N} \bar{\alpha}_i (\Delta\omega_{ij} + \omega_{ij})(e_j^{k|k})^T \Gamma^T \\ &\times (\Theta_i^k)^T \mathcal{Q}_i^{k+1} (G_i^{k+1} \hat{x}_i^{k+1|k} - \xi_{k+1}) + 2\sum_{i=1}^{N}\sum_{j=1}^{N} \bar{\alpha}_i \Delta\omega_{ij} \\ &\times (\hat{x}_j^{k|k})^T \Gamma^T (\Theta_i^k)^T \mathcal{Q}_i^{k+1} (G_i^{k+1} \hat{x}_i^{k+1|k} - \xi_{k+1}) + 2\sum_{i=1}^{N} \bar{\alpha}_i \\ &\times (1 - \bar{\alpha}_i)(e_i^{k|k})^T \mathcal{F}_{i,k}^T (\Theta_i^k)^T (G_i^{k+1})^T \mathcal{K}_i^{k+1} (G_i^{k+1} \hat{x}_i^{k+1|k} \\ &- \xi_{k+1}) + 2\sum_{i=1}^{N}\sum_{j=1}^{N} \bar{\alpha}_i (1 - \bar{\alpha}_i) \Delta\omega_{ij} (\hat{x}_j^{k|k})^T \Gamma^T (\Theta_i^k)^T \\ &\times (G_i^{k+1})^T \mathcal{K}_i^{k+1} (G_i^{k+1} \hat{x}_i^{k+1|k} - \xi_{k+1}) \\ &+ 2\sum_{i=1}^{N}\sum_{j=1}^{N} \bar{\alpha}_i (1 - \bar{\alpha}_i)(\Delta\omega_{ij} + \omega_{ij})(e_j^{k|k})^T \Gamma^T (\Theta_i^k)^T \\ &\times (G_i^{k+1})^T \mathcal{K}_i^{k+1} (G_i^{k+1} \hat{x}_i^{k+1|k} - \xi_{k+1}) \end{aligned} \tag{58}$$

where $\mathcal{Q}_i^{k+1} \triangleq (\mathcal{L}_i^{k+1})^T (\Phi_i^{k+1|k+1})^{-1} K_i^{k+1}$ and $\mathcal{G}_i^{k+1} \triangleq \mathcal{H}_i^{k+1} + \bar{\alpha}_i(1 - \bar{\alpha}_i)(G_i^{k+1})^T \mathcal{K}_i^{k+1} G_i^{k+1}$.

Observing the terms in (58), one has

$$\begin{aligned} &\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{h=1}^{n} (\Delta\omega_{ij} + \omega_{ij})(\Delta\omega_{ih} + \omega_{ih})(e_j^{k|k})^T \Gamma^T \\ &\times (\Theta_i^k)^T \mathcal{G}_i^{k+1} \Theta_i^k \Gamma e_h^{k|k} \\ \leq& \sum_{i=1}^{N}\sum_{j=1}^{N} N(\Delta\omega_{ij} + \omega_{ij})^2 (e_j^{k|k})^T \Gamma^T (\Theta_i^k)^T \mathcal{G}_i^{k+1} \Theta_i^k \Gamma e_j^{k|k} \\ \leq& \sum_{i=1}^{N}\sum_{j=1}^{N} N(\varsigma_j + \omega_{ij})^2 (e_j^{k|k})^T \Gamma^T \mathcal{G}_i^{k+1} \Gamma e_j^{k|k}, \end{aligned} \tag{59}$$

and

$$\begin{aligned} &\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{h=1}^{n} \Delta\omega_{ij}\Delta\omega_{ih} (\hat{x}_j^{k|k})^T \Gamma^T (\Theta_i^k)^T \mathcal{G}_i^{k+1} \Theta_i^k \Gamma \hat{x}_h^{k|k} \\ \leq& \sum_{i=1}^{N}\sum_{j=1}^{N} N\varsigma_j^2 (\hat{x}_j^{k|k})^T \Gamma^T \mathcal{G}_i^{k+1} \Gamma \hat{x}_j^{k|k}. \end{aligned} \tag{60}$$

For the cross terms in (58), one has

$$2\sum_{i=1}^{N}\sum_{j=1}^{N} (\Delta\omega_{ij} + \omega_{ij})(e_i^{k|k})^T \mathcal{F}_{i,k}^T (\Theta_i^k)^T (\mathcal{G}_i^{k+1})^T \Theta_i^k \Gamma e_j^{k|k}$$

$$
\begin{aligned}
&\leq \sum_{i=1}^{N}\sum_{j=1}^{N}(\varsigma_j+\omega_{ij})(e_i^{k|k})^T\mathcal{F}_{i,k}^T\mathcal{G}_i^{k+1}\mathcal{F}_{i,k}e_i^{k|k}\\
&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}(\varsigma_j+\omega_{ij})(e_j^{k|k})^T\Gamma^T\mathcal{G}_i^{k+1}\Gamma e_j^{k|k}.
\end{aligned}\tag{61}
$$

The remaining cross terms in (58) can be handled similarly. Moreover, substituting (59)-(61) and all the other cross terms into (58), we have

$$
\begin{aligned}
&\mathbb{E}\{V_{k+1}(e^{k+1|k+1})|e^{k|k}\}\\
&\leq \sum_{i=1}^{N}(e_i^{k|k})^T\mathcal{F}_{i,k}^T\big((1+2\bar{\varsigma}+\bar{\omega}_i+2\bar{\alpha}_i)\mathcal{H}_i^{k+1}+\bar{\alpha}_i(1-\bar{\alpha}_i)\\
&\quad\times(3+2\bar{\varsigma}+\bar{\omega}_i)(G_i^{k+1})^T\mathcal{K}_i^{k+1}G_i^{k+1}\big)\mathcal{F}_{i,k}e_i^{k|k}\\
&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}(\varsigma_j+\omega_{ij})(e_j^{k|k})^T\Gamma^T\big((1+2N\varsigma_j+N\omega_{ij}+2\bar{\alpha}_i)\\
&\quad\times\mathcal{H}_i^{k+1}+\bar{\alpha}_i(1-\bar{\alpha}_i)(3+2N\varsigma_j+N\omega_{ij})(G_i^{k+1})^T\mathcal{K}_i^{k+1}\\
&\quad\times G_i^{k+1}\big)\Gamma e_j^{k|k}+\sum_{i=1}^{N}\bar{\alpha}_i\big(2+(1+2\bar{\varsigma}+\bar{\omega}_i)(2-\bar{\alpha}_i)\big)\\
&\quad\times(\xi_{k+1}^T\mathcal{K}_i^{k+1}\xi_{k+1}+(\hat{x}_i^{k+1|k})^T(G_i^{k+1})^T\mathcal{K}_i^{k+1}G_i^{k+1}\hat{x}_i^{k+1|k})\\
&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}\varsigma_j(\hat{x}_j^{k|k})^T\Gamma^T\big((1+2N\varsigma_j+N\omega_{ij}+2\bar{\alpha}_i)\mathcal{H}_i^{k+1}\\
&\quad+\bar{\alpha}_i(1-\bar{\alpha}_i)(3+2N\varsigma_j+N\omega_{ij})(G_i^{k+1})^T\mathcal{K}_i^{k+1}G_i^{k+1}\big)\\
&\quad\times\Gamma\hat{x}_j^{k|k}+\sum_{i=1}^{N}(\varpi_i^k)^T(D_i^k)^T\mathcal{G}_i^{k+1}D_i^k\varpi_i^k\\
&\quad+\sum_{i=1}^{N}(1-\bar{\alpha}_i)(v_i^{k+1})^T\mathcal{K}_i^{k+1}v_i^{k+1}.
\end{aligned}\tag{62}
$$

Now, let us focus on the terms of $\mathcal{H}_i^{k+1}$ and $\mathcal{K}_i^{k+1}$ in (62) by recalling the inequalities (44) in Lemma 7, in which the matrix $\Phi_i^{k+1|k}$ has two possible solutions according to Theorem 2. As such, we shall discuss the EMSB of $e_i^{k|k}$ for both cases.

Case 1: Consider the case that $\Phi_i^{k+1|k} = 4\bar{\pi}_i I + D_i^k R_{i\varpi}^k (D_i^k)^T$. It is obvious from Lemma 7 that $(\Phi_i^{k+1|k})^{-1}$, $\mathcal{H}_i^{k+1}$ and $\mathcal{K}_i^{k+1}$ are all bounded. According to (1) and (7), we know that $\hat{x}_i^{k+1|k}$ and $\hat{x}_i^{k|k}$ are also both bounded. To sum up, the right-hand side of inequality (62) is bounded based on Assumption 1, and thus $e_i^{k|k}$ has the EMSB from Lemma 6.

Case 2: Consider the case that $\Phi_i^{k+1|k} = z_i^k I + D_i^k R_{i\varpi}^k (D_i^k)^T$. Let us recall Lemmas 7-8 and Assumption 1, and then substitute (44) into (62). The inequality (62) is arranged as

$$
\begin{aligned}
&\mathbb{E}\{V_{k+1}(e^{k+1|k+1})|e^{k|k}\}\\
&\leq \sum_{i=1}^{N}(e_i^{k|k})^T\Big(\frac{1+2\bar{\varsigma}+\bar{\omega}_i+2\bar{\alpha}_i}{1+\lambda_1\bar{\omega}_i+\bar{\varsigma}}(\Phi_i^{k|k})^{-1}\\
&\quad+\frac{(1-\bar{\alpha}_i)(3+2\bar{\varsigma}+\bar{\omega}_i)}{2(1+\rho_1^{-1}+\rho_2)(1+\lambda_1\bar{\omega}_i+\bar{\varsigma})}(\Phi_i^{k|k})^{-1}\Big)e_i^{k|k}\\
&\quad+N\sum_{i=1}^{N}(\bar{\varsigma}+\bar{\omega}_i)(e_i^{k|k})^T\Gamma^T\Big(\frac{1+2N\bar{\varsigma}+N\bar{\omega}_i+2\bar{\alpha}_i}{1+\lambda_1\bar{\omega}_i+\bar{\varsigma}}\\
&\quad\times\big((F_i^k)^T\big)^{-1}(\Phi_i^{k|k})^{-1}(F_i^k)^{-1}\\
&\quad+\frac{(1-\bar{\alpha}_i)(3+2N\bar{\varsigma}+N\bar{\omega}_i)}{2(1+\rho_1^{-1}+\rho_2)(1+\lambda_1\bar{\omega}_i+\bar{\varsigma})}\\
&\quad\times\big((F_i^k)^T\big)^{-1}(\Phi_i^{k|k})^{-1}(F_i^k)^{-1}\Big)\Gamma e_i^{k|k}\\
&\quad+\sum_{i=1}^{N}\frac{2+(1+2\bar{\varsigma}+\bar{\omega}_i)(2-\bar{\alpha}_i)}{2(1+\rho_1^{-1}+\rho_2)}(\hat{x}_i^{k+1|k})^T(\Phi_i^{k+1|k})^{-1}\\
&\quad\times\hat{x}_i^{k+1|k}+\sum_{i=1}^{N}\sum_{j=1}^{N}\varsigma_j\big((1+2N\varsigma_j+N\omega_{ij}+2\bar{\alpha}_i)\\
&\quad+\frac{(1-\bar{\alpha}_i)(3+2N\varsigma_j+N\omega_{ij})}{2(1+\rho_1^{-1}+\rho_2)}\big)(\hat{x}_j^{k|k})^T\Gamma^T(\Phi_i^{k+1|k})^{-1}\Gamma\\
&\quad\times\hat{x}_j^{k|k}+\sum_{i=1}^{N}\bar{\alpha}_i\big(2+(1+2\bar{\varsigma}+\bar{\omega}_i)(2-\bar{\alpha}_i)\big)\xi_{k+1}^T\mathcal{K}_i^{k+1}\\
&\quad\times\xi_{k+1}+\sum_{i=1}^{N}(\varpi_i^k)^T(D_i^k)^T\mathcal{G}_i^{k+1}D_i^k\varpi_i^k\\
&\quad+\sum_{i=1}^{N}(1-\bar{\alpha}_i)(v_i^{k+1})^T\mathcal{K}_i^{k+1}v_i^{k+1}\\
&\leq\sum_{i=1}^{N}p_i(e_i^{k|k})^T(\Phi_i^{k|k})^{-1}e_i^{k|k}+\sum_{i=1}^{N}\frac{2+2\bar{\varsigma}+\bar{\omega}_i}{1+\rho_1^{-1}+\rho_2}(\hat{x}_i^{k+1|k})^T\\
&\quad\times(\Phi_i^{k+1|k})^{-1}\hat{x}_i^{k+1|k}\\
&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}2\varsigma_j(2+2N\varsigma_j+N\omega_{ij}+\bar{\alpha}_i)\\
&\quad\times(\hat{x}_j^{k|k})^T\Gamma^T(\Phi_i^{k+1|k})^{-1}\Gamma\hat{x}_j^{k|k}+\sum_{i=1}^{N}2\bar{\alpha}_i(2+2\bar{\varsigma}+\bar{\omega}_i)\\
&\quad\times\xi_{k+1}^T\mathcal{K}_i^{k+1}\xi_{k+1}+\sum_{i=1}^{N}(\varpi_i^k)^T(D_i^k)^T\big((\Phi_i^{k+1|k})^{-1}\\
&\quad+\bar{\alpha}_i(1-\bar{\alpha}_i)(G_i^{k+1})^T\mathcal{K}_i^{k+1}G_i^{k+1}\big)D_i^k\varpi_i^k\\
&\quad+\sum_{i=1}^{N}(1-\bar{\alpha}_i)(v_i^{k+1})^T\mathcal{K}_i^{k+1}v_i^{k+1}
\end{aligned}\tag{63}
$$

where

$$
\begin{aligned}
p_i\triangleq&\frac{1}{\lambda_1\bar{\omega}_i\underline{f}_i^2}\big(\underline{f}_i^2(5+3\bar{\varsigma}+2\bar{\omega}_i)+N\bar{f}_i^2\bar{\gamma}^2(\bar{\varsigma}+\bar{\omega}_i)\\
&\times(5+3N\bar{\varsigma}+2N\bar{\omega}_i)\big).
\end{aligned}\tag{64}
$$

Moreover, according to (44) and Assumption 1, we know that $(\Phi_i^{k+1|k})^{-1}$ and $\mathcal{K}_i^{k+1}$ are both bounded. In consideration of the boundedness of $\hat{x}_i^{k+1|k}$ and $\hat{x}_i^{k|k}$, we conclude that the sum of the last five terms on the right-hand side of (63) is bounded, i.e., there exists a positive scalar t_0 such that

$$
\mathbb{E}\{V_{k+1}(e^{k+1|k+1})|e^{k|k}\}\leq\sum_{i=1}^{N}p_i(e_i^{k|k})^T\ (\Phi_i^{k|k})^{-1}e_i^{k|k}+t_0.\tag{65}
$$

To this end, it follows from (45), (64), (65) and Lemma 6 that $e_i^{k|k}$ has the EMSB under conditions (54). ■

Remark 3: In this paper, the RF problem has been addressed for state-saturated CNs with UCSs and deception

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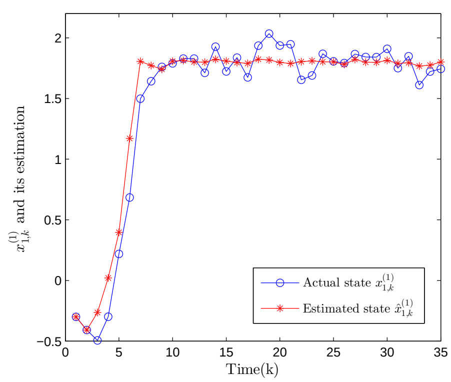
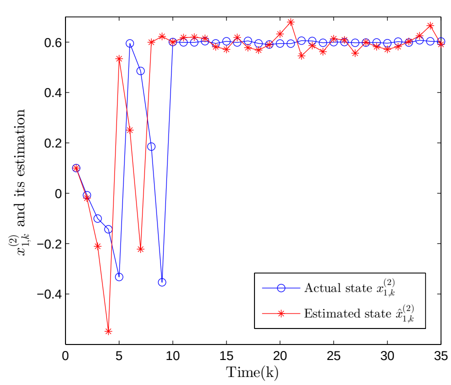
attacks, and the RF algorithm has been designed by the aid of the mathematical induction method. Note that our approach only uses the local and neighboring information to compute the gain for each node. In Theorem 2, a novel RF algorithm has been presented by finding upper bounds on error covariances. In Theorem 3, sufficient conditions have been acquired to guarantee the exponential boundedness of our RF scheme. At last, sufficient conditions have been established to ensure the EMSB of filtering errors $e_i^{k|k}$.

Remark 4: The paper solves the RF issue for CNs with state saturations and UCSs suffering from deception attacks. The primary features of our approach are highlighted as follows: 1) the state saturations, the UCSs, and the deception attacks are simultaneously considered in a unified framework; and 2) the EMSB of filtering errors is analyzed. Thus, the RF scheme developed would have not only theoretical importance but also practical significance.



(a) $x_{1,k}^{(1)}$ and its estimate $\hat{x}_{1,k}^{(1)}$.



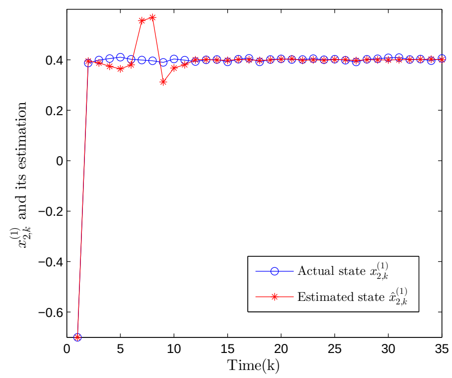
(b) $x_{1,k}^{(2)}$ and its estimate.

Fig. 1: $x_{1,k}$ and its estimates.

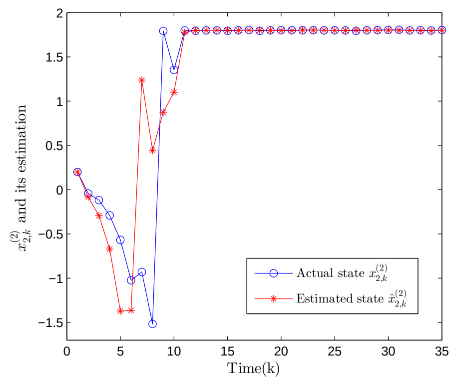
IV. Numerical Example

Example 1: Consider model (1) with parameters given as follows:

$$f(x_i^k) = \begin{bmatrix} (x_i^k)^{(1)} + \sin((x_i^k)^{(1)}(x_i^k)^{(2)}) \\ 0.5(x_i^k)^{(2)} + \sin((x_i^k)^{(1)}(x_i^k)^{(2)}) \end{bmatrix},$$



(a) $x_{2,k}^{(1)}$ and its estimate.



(b) $x_{2,k}^{(2)}$ and its estimate.

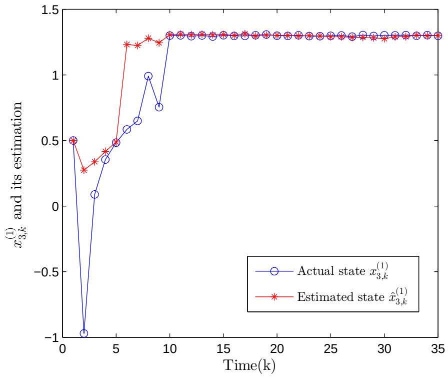
Fig. 2: $x_{2,k}$ and its estimate.

$$\Omega = \begin{bmatrix} 0.2 & 0.7 & 0.5 & 0.5 \\ 0.8 & 0.7 & 0.4 & 0.6 \\ 0.6 & 0.5 & 0.7 & 0.3 \\ 0.4 & 0.8 & 0.6 & 0.7 \end{bmatrix}.$$

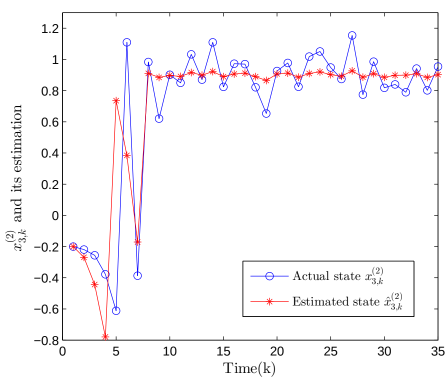
The upper bound of the uncertain term $\Delta\omega_{ij}$ is $\varsigma_j = [0.1 \quad 0.2 \quad 0.2 \quad 0.1]^T$ $(j = 1, 2, 3, 4)$.

$$R_{1v}^k = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, \; R_{2v}^k = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$R_{3v}^k = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.1 \end{bmatrix}, \; R_{4v}^k = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.2 \end{bmatrix}.$$
$$\Gamma = \mathrm{diag}\{0.5, 0.5\}, \; R_{i\varpi}^k = \mathrm{diag}\{0.5, 0.3, 0.4, 0.2\},$$
$$G_1^k = \begin{bmatrix} 0.6 & 0.25 \\ 0.3 & 0.2 \end{bmatrix}, \; G_2^k = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.2 \end{bmatrix},$$
$$G_3^k = \begin{bmatrix} 0.8 & -1.2 \\ 0.6 & 0.8 \end{bmatrix}, \; G_4^k = \begin{bmatrix} 0.85 & 0.95 \\ 0.5 & 0.3 \end{bmatrix},$$
$$D_1^k = \begin{bmatrix} -0.5 \\ 0.02 \end{bmatrix}, \; D_2^k = \begin{bmatrix} -0.03 \\ -0.02 \end{bmatrix},$$
$$D_3^k = \begin{bmatrix} 0.02 \\ -0.6 \end{bmatrix}, \; D_4^k = \begin{bmatrix} -0.4 \\ 0.01 \end{bmatrix}.$$

The initial state values are $x_1^0 = [-0.3 \quad 0.1]^T$, $x_2^0 = [-0.7 \quad 0.2]^T$, $x_3^0 = [0.5 \quad -0.2]^T$ and $x_4^0 = [0.3 \quad -0.25]^T$. Suppose that the saturation levels are $\pi_1^{\max} = [1.8 \quad 0.6]^T$, $\pi_2^{\max} = [0.4 \quad 1.8]^T$, $\pi_3^{\max} = [1.3 \quad 0.9]^T$ and $\pi_4^{\max} =$

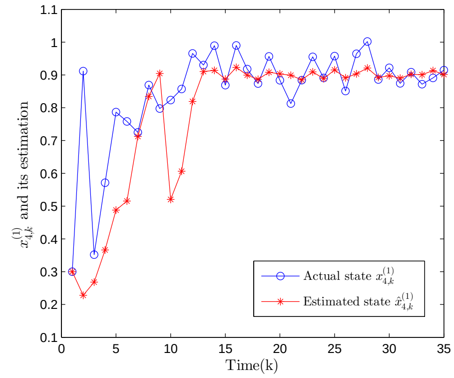


(a) $x^{(1)}_{3,k}$ and its estimate.

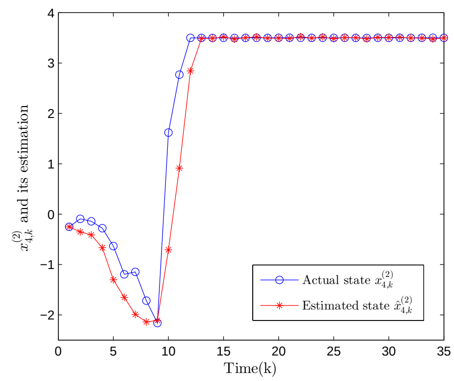


(b) $x^{(2)}_{3,k}$ and its estimate.

Fig. 3: $x_{3,k}$ and its estimate.



(a) $x^{(1)}_{4,k}$ and its estimate.



(b) $x^{(2)}_{4,k}$ and its estimate.

Fig. 4: $x_{4,k}$ and its estimate.

$[0.9 \quad 3.5]^T$. Choose the positive scalars $\rho_3 = 0.8$, $\eta = 0.9$, $\theta = 0.2$, $\bar{\alpha}_i = 0.6$ $(i = 1, 2, 3, 4)$. Other parameters are selected as $\rho_1 = 10$, $\rho_2 = 30$ and $\lambda_1 = 280$ according to Theorem 3.

Figs. 1-4 illustrate the state and their estimation curves for four nodes, which show that the developed filters have achieved the desired performance for the state-saturated CNs subject to deception attacks.

Example 2: In practical application, the states of the robot are constrained from the position and orientation [11]. To explain the practicability of our scheme, consider the indoor localization problem for mobile robots networks [3], which are described by

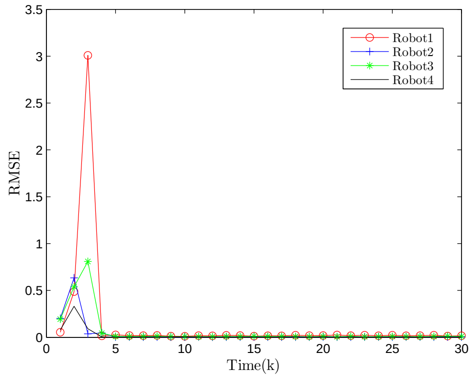
$$\begin{cases} \begin{bmatrix} x_i^{k+1} \\ \kappa_i^{k+1} \\ \psi_i^{k+1} \end{bmatrix} = \sigma_i\Bigg(\begin{bmatrix} x_i^k + W_i^k \cos\psi_i^k \\ \kappa_i^k + W_i^k \sin\psi_i^k \\ \psi_i^k + \tau_i^k \end{bmatrix} + \sum_{j=1}^{4}(\omega_{ij} \\ \qquad + \Delta\omega_{ij}) \begin{bmatrix} x_j^k \\ \kappa_j^k \\ \psi_j^k \end{bmatrix} \Bigg) + \begin{bmatrix} (\varpi_i^k)^x \\ (\varpi_i^k)^\kappa \\ (\varpi_i^k)^\psi \end{bmatrix} \\ y_i^k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_i^k \\ \kappa_i^k \\ \psi_i^k \end{bmatrix} + v_i^k \end{cases}$$

where (x_i^k, κ_i^k) and ψ_i^k are the position and the orientation of the i-th robot, respectively. (W_i^k, τ_i^k) is the velocity vector. The saturation levels are $\pi_1^{\max} = [13.8 \ 4.6 \ 11.1]^T$, $\pi_2^{\max} = [9.4 \ 15.8 \ 7.4]^T$, $\pi_3^{\max} = [5.3 \ 6.9 \ 12.8]^T$ and $\pi_4^{\max} = [7.9 \ 5.5 \ 15]^T$. $\varpi_i^k = [(\varpi_i^k)^x \ (\varpi_i^k)^\kappa \ (\varpi_i^k)^\psi]^T$, $R_{i\varpi}^k = \mathrm{diag}\{0.3, 0.3, 0.3, 0.3\}$ and $R_{iv}^k = \mathrm{diag}\{0.2, 0.2\}$. The initial state values are $x_1^0 = [-0.3 \quad 0.1 \quad 0.2]^T$, $x_2^0 = [-0.7 \quad 0.2 \quad -0.1]^T$, $x_3^0 = [0.5 \quad -0.2 \quad 0.3]^T$ and $x_4^0 = [0.3 \quad -0.25 \quad 0.1]^T$. Other parameters are selected as the same as those in Example 1.

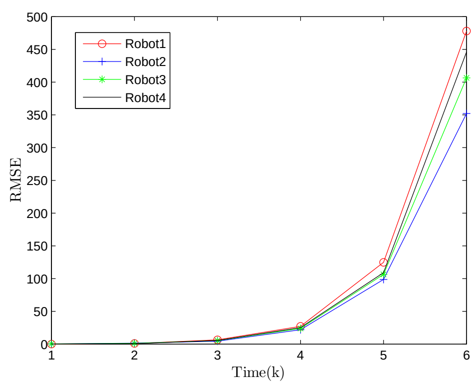
Fig. 5 shows the root-mean-square error (RMSE) curves in position of four robots, where Fig. 5(a) and Fig. 5(b) are the simulation results with and without taking into account the state saturation, respectively. From Fig. 5(b), one observes that the RMSE is unstable when the state saturation is not considered, which illustrates the negative impact of the state saturation on the filtering performance. Considering Fig. 5(a) with Fig. 5(b), it is clearly observed that, even under deception attacks, the performance has been greatly improved when state saturations are considered during design process. This verifies the practicability of our filter.

V. Conclusions

This paper has coped with the state-saturated RF issue for CNs with UCSs under deception attacks. Upper bounds on filtering errors have been acquired and filter gains have been determined through minimizing traces of these bounds. Subsequently, the EMSB of filtering errors has been analyzed. At



(a) RMSE in position for filter when considering state saturations.



(b) RMSE in position for filter without considering state saturations.

Fig. 5: RMSE in position.

last, simulations have been provided to illustrate the usefulness of the developed RF algorithm. Future research directions will involve the RF issue for more complex systems under various network scheduling protocols, the consensus problem for discrete-time CNs, the fault estimation problem based on various communication mechanisms and the control issue for time-varying CNs [9], [12], [13], [16], [25], [46], [49], [51].

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