Nonlinear Deformation and Failure Characteristics of Horseshoe-Shaped Tunnel under Varying Principal Stress Direction

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Abstract: To reveal the nonlinear deformation characteristics and failure mechanisms of surrounding rocks of horseshoe-shaped tunnel affected by varying principal stress directions, the physical experiments are carried out based on the similarity theory and control variable method. Simultaneously, on the basis of the statistical strength theory and meso-damage mechanics, a series of 2D numerical models which are able to consider the rock heterogeneity are established to further investigate the mechanical mechanism of damage evolution of surrounding rocks. Seven different kinds of horseshoe-shaped tunnel models are tested and the typical failure modes are analyzed according to the experimental data and numerical simulations. The results show that when the lateral pressure coefficient is small, fractures mainly develop towards the remote maximum principal stress direction; when the pressure difference between the vertical and horizontal directions is small, tunnel surrounding rocks damage seriously; initial damage of surrounding rocks basically occurs at the bottom floor corners and arch shoulders; the process of stress buildup, shadow and transfer is the fundamental mechanical process for the formation of mesoscopic damage and macroscopic failure. Overall, these achievements can provide valuable insights into the nonlinear failure mechanisms of horseshoe-shaped tunnel and will contribute to tunnel support design and stability evaluation in geotechnical engineering.

Keyword: horseshoe-shaped tunnel; principal stress direction; surrounding rock failure; tunnel instability; numerical simulation

1 Introduction

The safety and stability of tunnel surrounding rocks is always regarded as a challenging research topic in the field of geotechnical engineering. To meet the demand of transportation facility construction, resource exploitation, hydroelectric development, energy conservation and so on, a large number of underground projects have been built in the past 50 years. It is worth noting that some current large-scale tunnel projects have to face complicated geological and stress environment, which puts forward high demand and great challenge for the stability analysis theory of surrounding rocks. Generally, ground stress is the fundamental force that causes rock deformation and damage in underground engineering (Hoek 1965). In some early studies (Hoek 1964; Lajtai 1971; Gay 1976; Fakhimi et al. 2002), primary tensile and compression fractures were observed in the experiments conducted on circle tunnel models subjected to uniaxial or multiaxial pressures. Subsequently, many classical theories for the stability analysis of surrounding rocks were proposed, such as the classical pressure theory, granular pressure theory, elastoplastic theory, etc. The development of analysis theories is significantly influenced by the understanding of the interaction between the geostress and surrounding rocks.

The initial stress field of surrounding rocks consists of the gravitational stress field, tectonic stress field, etc. The magnitude and direction of principal stresses vary in different regions because of the superposition of different stress fields. Moreover, because of tectonic factors, the principal stresses are generally neither vertical nor horizontal, but incline with a certain angle. Considering that failure mode is consequentially influenced by the initial stress field (Lajtai and Lajtai 1975), study of the effect of principal stresses on failure patterns of surrounding rocks will provide insights into understanding the failure laws of surrounding rocks under different stress states and help to rationally determine the possibility of instability.

Recently, there has been growing interest in the failure mechanism and stability control method of underground tunnels and many valuable results have been achieved. Charpentier et al. (2003) analyzed the difference between excavation unloading and tectonism by carrying out experiments on the formation of micro near-field cracks of shale in tunnel and proposed that the mechanical mechanism of excavation in underground engineering lies in the stress redistribution. Simona (2014) analyzed the influence factors regarding the opening stability, including the support stiffness, rock damage and tunnel depth, and proposed an analytical approach based on the determination and integration of the rock-lining interface differential equation. Jure and Janko (2014) simulated the effect of overburden and the orientation of anisotropy plane on tunnel deformation and compared their results to the measurements at the

Trojane tunnel (Slovenia). Zhao et al. (2018) discussed the strengthening and isolation strategies to reduce seismic damage to tunnels and thought that the isolation layer is better to collaborate with the strengthening layer and a relatively large modulus and small thickness should be chosen for the isolation layer. Ng et al. (2018) conducted three-dimensional numerical simulations to study the failure rules of circular crossing tunnel affected by the size of existing horseshoe-shaped tunnel and revealed that the mid-plane deformation of the existing tunnel are elongated along the vertical direction and compressed along the horizontal direction.

These research results have promoted the understanding of the failure mechanism of underground tunnels to a certain extent. However, the geological conditions of large-scale underground tunnels are always complicated, and it is generally difficult to detect the whole ground stress field of engineering region accurately. Especially, the influence of principal stress direction on the failure behaviors of tunnels remains unclear. Meanwhile, although circular tunnels are easy for analysis because of the symmetry, horseshoe-shaped tunnels are used more widely in practice due to the convenience for pedestrians and vehicles to pass through.

Additionally, many theoretical models (Tokar 1990; Shao et al. 1994) have been proposed to analyze the influence of mechanical properties on the failure behaviors of rock masses based on the fracture mechanics. However, it is almost incapable to characterize the entire fracture process involving the initiation, propagation and coalescence of micro-cracks through to the formation of a full-scale macrocrack in host rock. Therefore, numerical methods have been used by researches to model the rock progressive failure. Suchowerska et al. (2012) used the finite element method (FEM) to evaluate the roof collapse of underground rectangular cavities for a range of geometries and rock properties. Dhawan et al. (2002) compared FEM results of 2D and 3D modelling of underground openings in heterogeneous rock mass to in-situ measurements. But as a continuum method, FEM has many natural limitations in modelling geological discontinuities such as joints, bedding planes and faults. The distinct element method (DEM), as a useful technique for discontinuous analysis in rock engineering, has been widely used to study the rock failure instability in many cases (Xiang et al. 2018; Shi et al. 2020; Gao et al. 2020). Besides, the discontinuous deformation analysis (DDA) proposed by Shi and Goodman (Shi 1988; Shi and Goodman 1985) is also an appropriate numerical tool for the discontinuous block-system simulation and successfully used in many underground engineering projects (Gong et al. 2018; Xu et al. 2019; Huang et al. 2020). However, almost none of these reported models, including the non-linear rule-based model (Blair and Cook 1998), the lattice model (Chinaia et al. 1997) and the bonded particle model (Potyondy et al. 1996), are able to effectively simulate the progressive fracture process of rocks around underground tunnels characterized by the initiation, propagation and coalescence of cracks, which can be easily modeled by the rock failure process analysis (RFPA) method (Tang 1998; Zhu and Tang 2004; Li and Tang 2021). RFPA has been extensively applied for rock fracture analysis (Gong et al. 2019; Liu et al. 2019; Tang et al. 2020).

In this paper, the experimental tests are conducted to reveal the failure laws of horseshoe-shaped tunnel affected by principal stresses with different directions using seven kinds of physical models. In addition, the stress field variation and the mechanical process of crack initiation and propagation are analyzed numerically based on the RFPA method. The results reveal the nonlinear deformation and failure mechanism of horseshoe-shaped tunnel under varying principal stress directions and will definitely benefit tunnel support design and stability evaluation in geotechnical engineering.

2 Experimental models and loading conditions

Principal stress direction has a great influence on the deformation and failure behaviors of surrounding rocks. By taken the rock properties and geometrical characteristics into account, the physical model tests of horseshoe-shaped tunnels are conducted to explore the related bearing capacity, deformation laws and failure modes under different principal stresses with different directions. Simultaneously, the RWL-3000 servo-controlled testing machine is used, as shown in Fig. 1.

The horseshoe-shaped tunnel is simplified to a physical model with a horseshoe-shaped opening at the center. The model size is 200 mm×200 mm×200 mm. Meanwhile, the width and height of the opening are 30 mm and 45 mm, respectively. The tunnel arch is a semicircle whose radius is 15 mm and the cement mortar is used as the experimental material.

The tunnel models with different inclined angles $\boldsymbol{\theta}$ between the tunnel cross section axis and the vertical direction are made. The inclined angle $\boldsymbol{\theta}$ increases from 0° to 90° gradually by an increment of 15°. After being fixed in the testing machine, the physical model is preloaded. Then, the vertical load $\boldsymbol{p_v}$ is applied on the top. Simultaneously, the horizontal pressure $\boldsymbol{p_h}$ is applied on the right face as well. Namely, the lateral pressure coefficient $\boldsymbol{k}$ is the ratio of horizontal pressure to vertical pressure, as determined by Eq. (1). Note that $\boldsymbol{k}$ can be set to different values. The test model and loading conditions are shown in Fig. 2. The physical models and loading process are shown in Fig. 3.

$$k = \frac{p_h}{p_v} \tag{1}$$

where $\boldsymbol{p_v}$ and $\boldsymbol{p_h}$ are the vertical pressure and the horizontal pressure, respectively. $\boldsymbol{k}$ is termed the lateral pressure coefficient.

3 Numerical analysis method

Based on the continuum mechanics, statistical strength theory and meso-damage mechanics, the RFPA2D method, as a two-dimensional FEM-based method, has been developed to simulate the fracture process of rock-like quasi-brittle materials. The main advantages of the RFPA2D code are that there are no priori assumptions about where and how fracture and failure will appear. Cracks can occur spontaneously, and a variety of mechanisms can be exhibited when certain local stress state satisfies the given strength criteria. During the loading process, the material heterogeneity plays an important role in affecting the nonlinear deformation of rock masses. Actually, in the RFPA calculation, the solid or structure is assumed to be made up of numerous mesoscopic elements to simulate the failure of rock masses. The material properties of these elements conform to the Weibull distribution (Weibull 1951). The probability density function of the Weibull distribution is shown as follows:

$$f(u) = \frac{m}{u_0}(\frac{u}{u_0})^{m-1}exp(-\frac{u}{u_0})^m \tag{2}$$

where $\boldsymbol{u}$ is a specific parameter of mesoscopic elements, such as strength and elastic modulus; $\boldsymbol{u_0}$ is a scale parameter reflecting the average value of the mesoscopic element parameter; $\boldsymbol{m}$ determines the shape of the distribution function curve and represents the degree of material homogeneity. Thus, it is also termed the homogeneity index. It can be summarized from the Weibull distribution that a larger $\boldsymbol{m}$ means a more heterogeneous material and vice versa. Using the probability density function Eq. (2), a heterogeneous medium can be built up numerically with many mesoscopic elements. Meanwhile, the heterogeneous medium produced computationally is analogous to a real specimen tested in the laboratory.

For the RFPA2D code, material medium is analyzed at the mesoscopic scale and the elastic-damage mechanics is used to describe the constitutive law of mesoscopic elements. Initially, the stress-strain relation of an element is considered linear elastic until the given failure criteria are reached, and then, it is modified by softening. The mechanism consisting of tensile-opening and shearing seems to be dominant in comparing with other mechanisms of crack evolution (Meglis et al. 1995). In this code, element damage in tension or shear is initiated when the stress/strain state satisfies the maximum tensile strain criterion or the Mohr-Coulomb criterion, respectively. In the elastic damage mechanics, the elastic modulus of material degrades gradually with the development of damage (Liang et al. 2004). The elastic modulus of damaged element is defined as follows:

$$E = (1 - \omega)E_0 \tag{3}$$

where $\boldsymbol{E}$ and $\boldsymbol{E_0}$ represents the elastic modulus of damaged and undamaged material elements, respectively; $\boldsymbol{\omega}$ represents the damage variable reflecting the damage degree. Note that the element and damage are assumed to be isotropic and elastic in the current code. Hence, $\boldsymbol{E}$, $\boldsymbol{E_0}$ and $\boldsymbol{\omega}$ are all scalars (Tang et al. 2002).

When a mesoscopic element is subjected to a uniaxial stress state (i.e., the maximum tensile or compressive state), the constitutive relation of the element can be illustrated in Fig. 4. In the beginning, the stress-strain curve is linear elastic and no damage exists. Therefore, the damage variable $\boldsymbol{\omega}$ is 0. Once the Mohr-Coulomb criterion with a cutting-off is satisfied, damage occurs in the element.

When the mesoscopic element is under a uniaxial tension stress state, the constitutive relationship, as shown in the third quadrant of Fig. 4, can be expressed as follows:

$$\omega = \begin{cases} 0, & \varepsilon > \varepsilon_{t0} \\ 1 - \frac{\lambda \varepsilon_{t0}}{\varepsilon}, & \varepsilon_{tu} < \varepsilon \le \varepsilon_{t0} \\ 1, & \varepsilon \le \varepsilon_{tu} \end{cases} \tag{4}$$

where $\boldsymbol{\lambda}$ is the residual strength coefficient, defined by $\boldsymbol{f_{tr}} = \boldsymbol{\lambda f_{t0}} = \boldsymbol{\lambda E_0 \varepsilon_{t0}}$; $\boldsymbol{f_{t0}}$ and $\boldsymbol{f_{tr}}$ are the uniaxial tensile strength and residual tensile strength, respectively; $\boldsymbol{\varepsilon_{t0}}$ is the strain at the elastic limit and generally termed the threshold strain; $\boldsymbol{\varepsilon_{tu}}$ is the ultimate tensile strain of the element, describing the state at which the element damages completely. The ultimate tensile strain is defined by $\boldsymbol{\varepsilon_{tu}} = \boldsymbol{\eta \varepsilon_{t0}}$, where $\boldsymbol{\eta}$ is termed the ultimate strain coefficient.

In addition, the damage of mesoscopic element is assumed to be isotropic and elastic under multiaxial stress state. Using the method of extending a one-dimensional constitutive law under uniaxial tension to a complex stress condition (Mazars and Pijaudier 1987), the constitutive described above can be extended to three-dimensional stress states. In a multiaxial stress state, the element still damages in the tensile mode when the equivalent major tensile strain $\bar{\varepsilon}$, which can be determined by Eq. (5), reaches the threshold strain ε_{t0}. This kind of damage is termed the tensile damage.

$$\bar{\varepsilon} = -\sqrt{\langle -\varepsilon_1 \rangle^2 + \langle -\varepsilon_2 \rangle^2 + \langle -\varepsilon_3 \rangle^2} \tag{5}$$

where $\boldsymbol{\varepsilon_1}$, $\boldsymbol{\varepsilon_2}$ and $\boldsymbol{\varepsilon_3}$ are three principal strains, and the term within angular bracket is a function defined by:

$$\langle x \rangle = \begin{cases} x, & x \ge 0 \\ 0, & x < 0 \end{cases} \tag{6}$$

When an element is subjected to a multiaxial stress state, the constitutive relation can be further expressed by substituting the calculated strain $\boldsymbol{\varepsilon}$ into Eq. (4) with the equivalent strain $\bar{\varepsilon}$ defined by Eq. (5) and Eq. (6). Then the damage variable $\boldsymbol{\omega}$ can be described as:

$$\omega = \begin{cases} 0, & \bar{\varepsilon} > \varepsilon_{t0} \\ 1 - \frac{\lambda \varepsilon_{t0}}{\bar{\varepsilon}}, & \varepsilon_{tu} < \bar{\varepsilon} \leq \varepsilon_{t0} \\ 1, & \bar{\varepsilon} \leq \varepsilon_{tu} \end{cases} \tag{7}$$

In order to judge if damage will occur when an element is subjected to a compression-shear stress state, the Mohr-Coulomb criterion is chosen to define the corresponding damage threshold and can be expressed as follows:

$$\sigma_1 - \frac{1+si}{1-sin\varphi}\sigma_3 \geq f_{c0} \tag{8}$$

where $\boldsymbol{\sigma_1}$ and $\boldsymbol{\sigma_3}$ are the maximum and minimum principal stresses, respectively; $\boldsymbol{f_{c0}}$ is the uniaxial compressive strength; $\boldsymbol{\varphi}$ is the internal friction angle. This kind of damage which appears when the stress state of an element satisfies the Mohr-Coulomb criterion, expressed by Eq. (8), is termed the shear damage.

Following the similar way as for the uniaxial tension, if an element is subjected to uniaxial compression but damaged due to the Mohr-Coulomb criterion, the damage variable $\boldsymbol{\omega}$ can be expressed as follows:

$$\omega = \begin{cases} 0, & \varepsilon < \varepsilon_{c0} \\ 1 - \frac{\lambda \varepsilon_{c0}}{\varepsilon}, & \varepsilon \geq \varepsilon_{c0} \end{cases} \tag{9}$$

where $\boldsymbol{\lambda}$ is the residual strength coefficient. It is assumed that $\boldsymbol{f_{cr}/f_{c0}} = \boldsymbol{f_{tr}/f_{t0}} = \boldsymbol{\lambda}$ is true when an element is subjected to uniaxial tension or uniaxial compression.

If an element is subjected to a multiaxial stress state and the Mohr-Coulomb criterion is satisfied, damage will occur. Meanwhile, it is necessary to add the influence of intermediate principal stress on the damage evolution. When this criterion is satisfied, the maximum principal (compressive) strain ε_{c0} can be attained at the peak of the maximum principal (compressive) stress value, as follows:

$$\varepsilon_{c0} = \frac{1}{E_0}\left[f_{c0} + \frac{1+si}{1-sin\varphi}\sigma_3 - \mu(\sigma_1 + \sigma_2)\right] \tag{10}$$

where $\boldsymbol{\sigma_1}$, $\boldsymbol{\sigma_2}$ and $\boldsymbol{\sigma_3}$ represent the maximum, intermediate and minimum principal stresses, respectively; $\boldsymbol{E_0}$ is the elastic modulus of undamaged material element; $\boldsymbol{f_{c0}}$ is the uniaxial compressive strength; $\boldsymbol{\varphi}$ is the internal friction angle of mesoscopic element.

The shear damage evolution is considered only to be related to the maximum compressive principal strain $\boldsymbol{\varepsilon_1}$. Thus, for an element subjected to a multiaxial stress state, the damage variable can be easily obtained by substituting the strain $\boldsymbol{\varepsilon}$ in Eq. (9) with the maximum compressive principal strain $\boldsymbol{\varepsilon_1}$, as follows:

$$\omega = \begin{cases} 0, & \varepsilon_1 < \varepsilon_{c0} \\ 1 - \frac{\lambda \varepsilon_{c0}}{\varepsilon_1}, & \varepsilon_1 \geq \varepsilon_{c0} \end{cases} \tag{11}$$

4 Experimental results and analysis

4.1 Effect of the principal stress direction on tunnel failure modes

4.1.1 The lateral pressure coefficient k = 0.125

When the lateral pressure coefficient $\boldsymbol{k}$ = 0.125, the failure modes are shown in Fig. 5. It can be seen that the tunnel presents a variety of failure laws with the inclined angle changing. When the angle $\boldsymbol{\theta}$ is 0°, damage can be found in the two side walls of the horseshoe-shaped tunnel firstly. After that, the cracks develop towards the inner rock gradually. The two side walls are threatened by the developing cracks and the two main rupture zones form in the walls. In the middle of the vault and floor, two cracks generate and develop upwards and downwards, respectively. When the angle $\boldsymbol{\theta}$ is 15°, damage occurs at the left corner of the bottom floor firstly. Then, two main cracks develop from the corner to the inner along different directions, which results in a large fracture zone. In addition, a crack generates at the right arch foot and threatens the safety of the tunnel. Then, there is a crack growth forming in the middle of the vault, which is similar to the previous case.

When $\boldsymbol{\theta}$ is 30°, the fracture zone at the left corner of the tunnel floor splits into two cracks. But then, they develop along the opposite directions. At the same time, two cracks appear in the right arch foot. One grows towards the right directly and the other develops upwards gradually which is nearly parallel to the crack that generates in the left spandrel. When $\boldsymbol{\theta}$ is 45°, it can be seen that two fracture zones, which threaten the safety of the tunnel, form in the right spandrel and the left bottom corner, respectively. Because the left bottom fracture zone is relatively large and may lead to the tunnel collapse together with the cracks occurring in the left side wall, the corresponding supporting treatments should be considered reasonably in this case.

When $\boldsymbol{\theta}$ is 60°, attention should be paid to the two cracks that may affect the safety of the bottom floor. One crack generates in the middle of the floor and develops to the left horizontally to a certain distance. But then, its growth path is changed to be obliquely upward, which results in the formation of a potential movable block in the left half part of the floor. The other crack whose scale is relatively small occurs at the right bottom floor corner. Meanwhile, a crack appears in the right arch shoulder. Unlike previous failure modes, a small crack occurs in the left spandrel as well. When $\boldsymbol{\theta}$ is 75°, obviously, different failure characteristics are indicated in Fig. 5. Two cracks whose paths are almost perpendicular to the side wall appear at the left bottom corner and the left arch foot, respectively and extend obliquely upwards in parallel. Then, a rupture zone forms near the left side wall. Simultaneously, the vault also damages and splits into a large single fracture developing obliquely downwards.

When $\boldsymbol{\theta}$ is 90°, it should be noted that a crack occurs at the left corner of the tunnel floor, but then it develops downward to the left and promotes the formation of the damage zone near the bottom floor. Moreover, there is another crack generating at the lower part of the left side wall and developing upwards into the inner rock. Also, a crack appears in the vault. However, its scale is relatively small.

4.1.2 The lateral pressure coefficient k = 0.5

When the lateral pressure coefficient $\boldsymbol{k}$ = 0.5, the pressure difference between the vertical direction and the horizontal direction reduces because of the increase of the horizontal stress. The relevant failure patterns can be seen in Fig. 6. When $\boldsymbol{\theta}$ is 0°, the lower parts of the side walls damage firstly and then two cracks occur at the bottom corners of the tunnel floor. At first, they develop into the inner surrounding rock in parallel. After that, the left crack keeps propagating downwards, but the right crack changes its path to the right gradually due to the material heterogeneity. Meanwhile, the fracture zones form along the crack growth paths. When $\boldsymbol{\theta}$ is 15°, the rock at the right arch foot damages seriously. A large fracture occurs and develops towards the right almost straightly. In addition, the left spandrel damages as well, and the rupture zone splits into two fractures developing upwards into the inner rock gradually. Obviously, in this case, much attention should be paid to the tunnel vault roof and it should be supported by related measures rationally. Besides, a crack occurs at the right corner of the tunnel floor, but the scale is relatively small.

When $\boldsymbol{\theta}$ is 30°, several cracks appear at the left corner of the floor. Among them, a main crack develops towards the left, but the growth path changes gradually to upside, i.e., along the direction of the maximum principal stress of the remote field. Also, another crack propagates downwards, nearly along the remote major principal stress direction. Simultaneously, a small crack occurs in the right part of the vault. When $\boldsymbol{\theta}$ is 45°, a rupture zone forms in the roof where three fractures develop upwards nearly in parallel, which poses a significant threat to the tunnel stability. In addition, there is a fracture cutting the rock under the tunnel floor, which leads to the loosening deformation of the floor. Under the loosening rock mass, a rupture zone occurs, which definitely increase the level of the tunnel damage and deformation.

When $\boldsymbol{\theta}$ is 60°, the rock near the left corner of the bottom floor is damaged firstly. It leads to the formation of a damage zone. Then, a clear crack occurs and develops upwards from the damage zone. Besides, the appearance of a crack that is nearly parallel to the bottom floor results in the rock mass loosening and threatens the floor safety. In addition, a small crack generates in the damage zone which forms near the middle of the roof and develops upwards along the remote major principal stress direction. When $\boldsymbol{\theta}$ is 75°, it is clear that the rock mass near the right corner of the floor damages seriously. A crack appears in the damage zone, and after growing obliquely downwards to a certain extent, it splits into two cracks that develop towards different directions. The one whose scale is larger propagates back to cut the rock mass under the tunnel floor and poses a threat to the tunnel safety. Besides, the middle of the vault is dangerous because of the downward propagation of a large fracture.

When $\boldsymbol{\theta}$ is 90°, it can be seen from Fig. 6 that both the vault and the floor damage seriously. Specifically, two cracks occur in the middle of the roof. One crack develops to the right and the related damage zone forms along its growth path. The other crack grows towards the upper left, leads to the loosening deformation of the surrounding rock and threatens the side wall security. At the same time, a large fracture zone forms at the left corner of the tunnel floor and causes serious damage to the deep rock.

4.2 Numerical analysis of the tunnel failure process

In this study, to reveal the mechanical mechanism of the damage and fracture evolution of surrounding rocks of horseshoe-shaped tunnels, a series of numerical simulations are conducted using the RFPA2D code. The rock physical and mechanical properties including the elastic modulus ($\boldsymbol{E}$), the uniaxial compressive strength ($\boldsymbol{f_c}$), the poisson's rate ($\boldsymbol{\mu}$), the internal friction angle ($\boldsymbol{\phi}$), the tension-compression ratio ($\boldsymbol{\alpha}$), the residual strength coefficient ($\boldsymbol{\lambda}$) and the homogeneity index ($\boldsymbol{m}$) are listed in Table 1. The model size and loading conditions are shown in Fig. 2. Note that only the tunnel cross section is used to establish the numerical model. The vertical pressure $\boldsymbol{p}$ increases by an increment of 0.1MPa per step. The lateral pressure $\boldsymbol{kp}$ depends on the vertical pressure $\boldsymbol{p}$ and the lateral pressure coefficient $\boldsymbol{k}$. After calculation, the stress field variation of the host rock and the mechanical process of crack initiation and propagation can be obtained by the RFPA2D code.

4.2.1 The lateral pressure coefficient $\boldsymbol{k}$ = 0.125

When the lateral pressure coefficient $\boldsymbol{k}$ = 0.125, the numerical simulations are carried out to analyze the tunnel failure process and mechanism, and the related shear stress fields are shown in Fig. 7. When $\boldsymbol{\theta}$ is 0°, the compressive stress is mainly concentrated in the surrounding rock near the two side walls. Under the effect of high compressive stress, the shear failure firstly occurs in the two side walls and controls the stability of the surrounding rocks. By contrast, the tensile stress concentration areas appear in the middle of the vault roof and the floor, and lead to the rock damage there. Besides, due to the heterogeneity of the material, the rupture zones at the right and left walls are not symmetric. A fracture generates at the left floor corner. When $\boldsymbol{\theta}$ is 15°, the compressive stress is concentrated in the right spandrel and the left tunnel floor corner. Especially, it induces the large-scale fracture zone near the left part of the surrounding rock. In addition, the tensile stress in the roof and the right bottom corner promotes the development of micro cracks.

When $\boldsymbol{\theta}$ is 30°, it is similar to the case when $\boldsymbol{\theta}$ is 15° that the high compressive stress is in the right spandrel and the left bottom corner, and the high tensile stress is in the vault roof and the right bottom corner. But because of the increase of the inclined angle $\boldsymbol{\theta}$, the stress concentration near the upper part of the right-side wall is more remarkable, which leads to much deeper damage. When $\boldsymbol{\theta}$ is 45°, the reasons for the two main cracks are evidently different. The large compressive stress value occurs at the left floor corner and induces the crack growth. By contrast, the tensile stress concentration near the left spandrel is significant, which results in tensile damage.

When $\boldsymbol{\theta}$ is 60°, the surrounding rock near the left floor corner damages seriously due to the high compressive stress, and the damage zone in the right spandrel is also the compressive stress concentration area. Moreover, the high tensile stress causes the crack initiation and propagation at the left arch foot and the right floor corner. When $\boldsymbol{\theta}$ is 75°, obviously, the two parallel cracks inside the left side wall of the tunnel are caused by different factors, i.e., the compressive stress and tensile stress, respectively. In addition, the location of the tensile stress concentration in the right-side wall is changed to be just above the right floor corner along the wall.

When $\boldsymbol{\theta}$ is 90°, the failure mode of the tunnel is more symmetrical, but not completely symmetrical because of the material heterogeneity. Under the effect of the compressive stress, two fractures generate in the rocks at the floor corners and they develop upwards and downwards, respectively. Also, the roof damages due to the compressive stress. Furthermore, there are the tensile stress concentration zones occurring in the middle of the two side walls.

4.2.2 The lateral pressure coefficient $\boldsymbol{k}$ = 0.5

When the lateral pressure coefficient $\boldsymbol{k}$ = 0.5, the tunnel shows different failure patterns due to the decrease of the pressure difference between the vertical direction and the horizontal direction. The relevant failure patterns are shown in Fig.8. When $\boldsymbol{\theta}$ is 0°, the high compressive stress is concentrated in the two side walls, which makes the surrounding rock damages gradually with the stress accumulating, stress releasing and stress transferring process. The rocks near the floor corners are most dangerous. When $\boldsymbol{\theta}$ is 15°, because of the large value of the compressive stress, the shear damage zones form in the left side wall and the right spandrel. Meanwhile, there is a tensile stress concentration appearing in the middle of the roof vault.

When $\boldsymbol{\theta}$ is 30°, the cracks generate in the surrounding rock mass near the right spandrel and the lower part of the left side wall due to the large compressive stress concentration. The high stress releases after rock mass getting damaged. Then, the released stress is transferred into the inner rock and stress builds up again. When $\boldsymbol{\theta}$ is 45°, compared with the previous case, the compressive stress concentration area in the right arch foot moves to the middle of the roof and the other concentration area near the left bottom corner of the tunnel changes its location under the floor to a certain extent.

When $\boldsymbol{\theta}$ is 60°, the high compressive stress results in the formation of the deep damage in the vault. Unlike the previous cases, the large compressive stress is concentrated under the floor and consequently, the floor rocks are damaged seriously, which threatens the safety of the tunnel. However, the rupture zone under the floor is not uniform because the compressive stress of the left part of the floor is much higher than the right part. When $\boldsymbol{\theta}$ is 75°, the large compressive stress concentration occurring in the vault induces a main crack developing into the surrounding rock gradually. Besides, because of the relatively uniform compressive stress distribution under the floor, the rupture zone full of micro cracks is more average.

When $\boldsymbol{\theta}$ is 90°, in the middle of the two side walls, there are small tensile stress concentrations. But more importantly, the damage degree of the vault roof is more serious than the previous cases under the effect of the compressive stress concentration. In addition, the high compressive stress leads to the formation of a 'V' shaped fracture zone under the floor.

4.3 Effect of the lateral pressure coefficient

In order to reveal the influence of the lateral pressure coefficient $\boldsymbol{k}$ on the failure behaviors of horseshoe-shaped tunnel, the tunnel model with an inclined angle $\boldsymbol{\theta}$ = 30° is chosen as the typical model and the corresponding experiments are conducted when the lateral pressure coefficient $\boldsymbol{k}$ is set to be 0.125, 0.25, 0.5 and 0.75, respectively. Meanwhile, limited by the performance of the pressure testing machine, the test cannot be carried out when the lateral pressure coefficient $\boldsymbol{k}$ = 1. The experimental results are shown in Fig. 9.

From Fig. 9, it can be seen that when the lateral pressure coefficient $\boldsymbol{k}$ = 0.125, the right arch foot is damaged deeply, and the damage zone splits into two main cracks. One develops to the right and the other develops upwards roughly along the remote maximum principal stress direction. Also, the left floor corner damages seriously and two cracks generate and propagate to different directions. It is worth noting that two cracks occur in the left parts of the vault and the bottom floor, and then develop upwards and downwards, respectively. When the lateral pressure coefficient $\boldsymbol{k}$ = 0.25, the vault is dangerous because of the formation of the 'V' shaped loosening rock mass in the left part of the roof. Meanwhile, two cracks propagate from the damage zone in the right arch shoulder, which is similar to the case when $\boldsymbol{k}$ = 0.125. In addition, two main fractures occur at the left and right corners of the floor, respectively and develop almost in parallel in the early stage, which may cause the floor loosening deformation.

When the lateral pressure coefficient $\boldsymbol{k}$ = 0.5, with the pressure difference between the vertical and horizontal directions reducing, the cracks are mainly concentrated in the right arch shoulder and the left floor corner. Clearly, the crack generating in the right spandrel develops slightly obliquely upwards and to the right. However, the scale of this crack is smaller than the two main cracks appearing at the left corner of the bottom floor. The two main cracks contribute to the tunnel collapse to a great extent. When the lateral pressure coefficient $\boldsymbol{k}$ = 0.75, the right part of the roof is damaged seriously. At the same time, attention should be paid to the left floor corner where a large-scale fracture generates. The fracture develops downwards into the surrounding rock and finally promotes the destruction of the tunnel. In addition, there is a small crack occurring in the roof and it develops upwards gradually.

Simultaneously, the simulations are carried out to study the relevant mechanical mechanism. The results are shown in Fig. 10. When the lateral pressure coefficient $\boldsymbol{k}$ = 0.125, the compressive stress is mainly concentrated in the right spandrel and the left bottom floor corner, and it causes serious damage to the surrounding rock. With the process of the stress buildup, stress shadow and stress transfer, the high compressive stress induces the fractures to develop into the inner rock gradually. Moreover, there are two distinct tensile stress concentrations appearing in the left part of the tunnel vault and the right part of the bottom floor. When the lateral pressure coefficient $\boldsymbol{k}$ = 0.25, because the value of the compressive stress in the right spandrel is large, the shear damage happens, and two groups of cracks appear but develop towards two opposite directions. Meanwhile, it is the high compressive stress that causes the fractures containing many micro cracks near the left bottom corner under the floor. Furthermore, the tensile stress concentrations still exist in the left part of the roof and the right part of the floor, but are not significant.

When the lateral pressure coefficient $\boldsymbol{k}$ = 0.5, the compressive stress concentration areas occur in the right arch shoulder and the lower part of the left side wall. When the stress state of local rock mass reaches the failure criterion, the accumulated stress releases and rebuilds up in the inner rock. This process repeats and leads to the formation of fractures. In particular, the supporting measures should be considered to reinforce the lower part of the left side wall. When the lateral pressure coefficient $\boldsymbol{k}$ = 0.75, under the effect of the larger horizontal lateral pressure, the stress field changes. The stress concentration in the arch foot moves to the middle of the roof to a certain degree. Meanwhile, the concentration in the lower part of the left side wall moves to the left floor corner. Due to the high compressive stress, the right arch shoulder damages seriously. In addition, the compressive stress concentration exists at the right corner as well.

4.4 Effect of the principal stress direction on the overall stability

Because the vertical load $\boldsymbol{p_v}$ applied on the top of the model increases step by step, the critical pressure value when the whole tunnel collapses can be recorded to reflect the stability of the tunnel. The numerical tunnel models with different inclined angles $\boldsymbol{\theta}$ are calculated by RFPA under different lateral pressure coefficients $\boldsymbol{k}$ and the line graph of the critical pressure changing with the tunnel inclined angle $\boldsymbol{\theta}$ is shown in Fig. 11. Although the tunnel stability shows very complex rules, there is still certain regularity.

Overall, the variation trend of the critical pressure can be classified into three different types when the angle $\boldsymbol{\theta}$ changes from 0° to 45°. When the lateral pressure coefficients $\boldsymbol{k}$ = 0.25, 0.5 and 0.75, the tunnel critical pressure value drops firstly and then increases; when the lateral pressure coefficient $\boldsymbol{k}$ = 0.125, the maximum pressure value rises firstly, but then decreases dramatically; when the lateral pressure coefficient $\boldsymbol{k}$ = 1, the maximum press value almost remains at the same level only with a slight fluctuation.

In contrast, when the angle $\boldsymbol{\theta}$ changes from 45° to 90°, the critical pressure tendencies are different from one another. When the lateral pressure coefficient $\boldsymbol{k}$ = 0.125, the pressure remains at nearly 29 MPa with the angle $\boldsymbol{\theta}$ changing from 45° to 75°, but then it grows to 31 MPa when $\boldsymbol{\theta}$ = 90°; When $\boldsymbol{k}$ = 0.25, the critical value declines from 35 MPa to 29.5 MPa. After that, it goes up back to 35 MPa gradually; When $\boldsymbol{k}$ = 0.5, the value of the critical pressure $\boldsymbol{p}$ fluctuates around 34 MPa; When $\boldsymbol{k}$ = 0.75, the critical pressure falls down remarkably from 40 MPa to 32.5 MPa with the angle $\boldsymbol{\theta}$ increasing from 45° to 75°. Then, it increases to 38.5 MPa when $\boldsymbol{\theta}$ = 90°; When $\boldsymbol{k}$ = 1.0, the critical pressure value remains at around 36 MPa basically.

5 Conclusion

In this study, the mechanical tests and numerical simulations are conducted to reveal the effect of principal stress directions on the nonlinear failure behaviors of horseshoe-shaped tunnels. Clearly, based on the similarity theory and controlling variable method, seven different kinds of tunnel models are tested and the typical failure modes are analyzed according to the experimental results. Simultaneously, on the basis of the continuum mechanics, statistical strength theory and meso-damage mechanics, the RFPA2D numerical models are established for further understanding the mechanical mechanisms of the damage and fracture evolution of surrounding rocks. The major findings and conclusions can be summarized as follows:

With the load increasing gradually, the initial damage of surrounding rocks mainly occurs at the bottom floor corners or arch shoulders of horseshoe-shaped tunnel. Then, cracks continuously develop into the inner rock no matter what the inclined angle between the remote major principal stress and the tunnel cross section axis is, leading to the macroscopic instability of the tunnel.

When the lateral pressure coefficient is small, the fractures that will threaten the safety of the tunnel mainly develop towards the remote maximum principal stress direction. Meanwhile, when the lateral pressure coefficient is relatively large, i.e., the pressure difference between the vertical direction and the horizontal direction is small, the surrounding rocks around the tunnel damage seriously and the direction of crack propagation is affected by the tunnel inclined angle to a certain degree.

Furthermore, when the tunnel inclined angle is 30°, the high tensile stress concentrations in the left spandrel and the right floor corner of the tunnel decrease gradually with the lateral pressure coefficient increasing. In addition, the initiation location and propagation direction of the shear cracks are increasingly clear, and it is the shear damage occurring at the left floor corner and the right arch shoulder that results in the final tunnel collapse to a great extent.

Besides, the continuous process of stress buildup, stress shadow and stress transfer repeats and leads to the formation of the mesoscopic damage and macroscopic failure. The high stresses release after rock mass getting damaged. Then, the released stresses are transferred into the inner rock and stresses build up again. This process induces the fractures to develop into the inner rock gradually. Hence, it is the fundamental mechanical process underlying the tunnel instability.

Additionally, the tunnel stability shows complex laws under the influence of principal stress directions. When the tunnel inclined angle changes from 0° to 45°, the variation trend of the critical pressure values can be classified into three different types; when the tunnel inclined angle changes from 45° to 90°, the variation range of the critical pressure is relatively large when the lateral pressure coefficients are 0.75 and 0.25, while the critical pressure basically remains at the same level when the lateral pressure coefficients are 0.125, 0.5 and 1.0.

Data Availability Statement

All data, models, and code that support the findings of this study are available from the corresponding author upon reasonable request.

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Declarations

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Table List

Table 1 Rock physical and mechanical parameters employed in simulation

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$\boldsymbol{E}$(GPa)	$\boldsymbol{f_c}$(MPa)	$\boldsymbol{\mu}$	$\boldsymbol{\phi}$(°)	$\boldsymbol{\alpha}$	$\boldsymbol{\lambda}$	$\boldsymbol{m}$
60	200	0.25	30	0.1	0.1	3

Figure List