Endec-Decoder-Based N-Step Model Predictive Control: Detectability, Stability and Optimization ⋆

Yan Song [a], Zidong Wang [b,c,∗], Lei Zou [c], Shuai Liu [d]

[a] *Department of Control Science and Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China*

[b] *College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China*

[c] *Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, UK*

[d] *College of Science, the Shanghai Key Lab of Modern Optical System, University of Shanghai for Science and Technology, Shanghai 200093, China*

Abstract

In this paper, the N-step model predictive control problem is investigated for a class of networked control systems with limited communication capacity. By resorting to the dynamic uniform quantization method, a novel observer-based endec-decoder is put forward in order to accommodate the digital transmission requirement. In this sense, the state reconstructed by the observer is coded into certain codewords and then transmitted to the controller via a bandwidth-limited network. The aim of the problem addressed is to co-design the observer-based endec-decoder and the N-step model predictive controller such that the underlying system is detectable and asymptotically stable. By solving certain offline "min-max" optimization problems with matrix inequality constraints, a series of one step sets and the terminal constraint set are derived as well as the desired offline controller parameters. Then, in order to improve the convergence speed of the closed-loop system, a recursive algorithm is developed to design the online controller based on the results obtained from the offline optimization problems. Finally, a numerical example is given to demonstrate the validity of the proposed control scheme.

Key words: N-step model predictive control; networked control systems; dynamic uniform quantization; observer-based endec-decoder scheme; detectability.

1 Introduction

In the past few decades, model predictive control (MPC) strategy has gained much attention from a variety of engineering fields due to its great potential in handling optimal control problems with hard constraints, see e.g. [10, 22, 39, 48]. In practical applications, the desired control inputs of the MPC strategy are calculated by solving a certain moving-horizon online optimization problem. Accordingly, the computation complexity of such an optimization issue plays a crucial role in evaluating the performance of the MPC scheme. To this end, much research effort has been devoted to the topic of developing an efficient MPC strategy to reduce the online computation burden, thereby leading to the natural emergence of the so-called *N-step MPC* strategy [11, 33, 38]. The purpose of N-step MPC scheme is to design a finite number of control inputs such that the system states in an initial feasible region can be steered into a prescribed terminal constraint set within N time steps. Note that with the increase of the step number N, the initial feasible region becomes large at the cost of the computation complexity accordingly. Therefore, how to develop a simple yet effective N-step MPC strategy to make a right compromise between the initial feasible region and the computation complexity constitutes one of our motivations.

With the rapid development of computer science and communication technology, networked control systems (NCSs) have found successful applications in various fields owing to their low cost, flexibility and portability, see e.g. [3, 8, 12, 21, 27, 41–43, 47]. Nevertheless, the utilization of communication networks would also pose great challenges on the controller design due mainly to the existence of the so-called networked-induced phenomena [4–6, 32, 37]. More specifically, due to the limited bandwidth of the communication channel, simultaneous transmissions of a large amount of data would lead to heavy network burden, thus inevitably causing unexpected network-induced phenomena that could deteriorate the system performance [16, 17]. In order to achieve the desired control performance, the underlying network-induced phenomena should be considered in the design of the controller. In this case, the traditional MPC strategy under the assumption of the perfect communication environment would be no longer valid for the bandwidth-constrained NCSs. Recently, a number of research attempts have been made on MPC for NCSs with communication constraints and some representative achievements have been obtained, see e.g. [28–30, 40].

In NCSs, the inherently limited communication resource, however, brings in certain requirements on the

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∗ Corresponding author.

Email addresses: sonya@usst.edu.cn (Yan Song), Zidong.Wang@brunel.ac.uk (Zidong Wang), Lei.Zou@brunel.ac.uk (Lei Zou), liushuai871030@163.com (Shuai Liu).

data delivery, e.g., the amount (bits) of the data transferred per time unit should not exceed a specified threshold. For this purpose, particular research attention has been paid on the so-called coding-decoding scheme due primarily to its three evident merits of 1) compressing data before being transmitted, 2) reducing the network transmission burden and 3) improving network security. Therefore, it is both theoretically important and practically significant to launch an investigation on the state estimation/control problems subject to the coding-decoding mechanism, and some elegant results have been reported, see [9,13,14,18,24,25,31,34,35,44] and references therein. However, a thorough literature survey has revealed that the coding-decoding-based MPC problem has not been fully investigated yet owing probably to the difficulties in guaranteeing the recursive feasibility of the moving-horizon online optimization and the asymptotical stability of the closed-loop system. Consequently, we are motivated to develop a novel coding-decoding-based MPC technique to ensure the desired control performance in a resource-constrained network environment.

Within the coding-decoding scheme framework, since the coded data usually only carry partial information of the raw data, the information distortion would inevitably occur in the decoded signals. In this case, the resulting error (also called decoding error) is the main source of the degradation of the control performance. From a technical viewpoint, the so-called detectability, as a primary performance index, has been recently proposed to examine the convergence of the decoding error. For this purpose, a dynamic-quantizer-based coding-decoding technique has been developed in [14] by encoding the error of the current state (or estimate state) and the last decoded value. Such a scheme has been extended in [44] to the descriptor systems and in [34] for discrete-time dynamical networks with packet dropouts. It is worth pointing out that, despite the stirred research interests in developing coding-decoding schemes, there is still much room for its extensive applications, for example, making some improvements on the existing coding-decoding algorithms suitable for more complicated control strategies, which constitutes another motivation of our present research.

According to the above analysis, the technical challenges of the underlying N-step MPC problem via a coding-decoding scheme can be identified as the three points: 1) how to design an appropriate coding-decoding technique revealing the impact of the decoding error on the resulted control performance? 2) how to derive the desired control inputs ensuring that the state can be steered into the terminal constraint set within the required N time steps? and 3) how to develop effective methodologies guaranteeing the detectability and asymptotical stability of the addressed closed-loop system?

In response to above identified challenges, we endeavor to co-design the observer-based coding-decoding scheme and the N-step MPC strategy such that the expected control performance is achieved in a limited-bandwidth network. The main contributions can be highlighted as follows: *1) the observer-based coding-decoding technique is, for the first time, proposed for the N-step MPC problem in a resource-constrained network environment; 2) the upper bound of the decoding error resulting from the coding-decoding scheme is thoroughly analyzed, and a dedicatedly constructed objective function is adopted to reflect the impact of this norm-bounded decoding error on the controller design issue; and 3) sufficient conditions are provided for guaranteeing the recursive feasibility of the algorithm and the asymptotical stability of the addressed system.*

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. $\mathbb{Z}_{\geq 0}(\mathbb{Z}_{>0})$ and $\mathbb{R}_{\geq 0}(\mathbb{R}_{>0})$ are used to denote the set of all nonnegative integers (positive integers) and the set of all nonnegative real numbers (positive real numbers), respectively. I and 0 represent the identity and zero matrices of compatible dimensions, respectively. P^T represents the transpose of P. The shorthand diag$\{\cdots\}$ denotes a block diagonal matrix. For $x \in \mathbb{R}^n$, $\|x\| = \sqrt{x^T x}$ and $\|x\|_\infty = \max\{|x_i|, 1 \leq i \leq n\}$. For a vector y, $y > 0$ means that every element of y is greater than 0. In symmetric block matrices, the symbol "$*$" is used as an ellipsis for the terms induced by symmetry. For two square matrices X, Y, $X \geq Y$ (especially, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (especially, positive-definite). For a time-varying variable $x(k) \in \mathbb{R}^n$, $x(k+n|k)$ is the prediction at the future time instant $k+n$ ($n \in \mathbb{Z}_{\geq 0}$) based on its value at the current time instant k, and $x(k|k) \triangleq x(k)$. $\lambda_{\max}\{\cdot\}$ (respectively, $\lambda_{\min}\{\cdot\}$)) means the largest (respectively, smallest) eigenvalue of "$\cdot$". $[\centerdot]_i$ denotes the ith element of a vector or the ith row of a matrix. $[\centerdot]_{ii}$ denotes the ith diagonal element of a matrix "$\centerdot$".

2 Problem Formulation and Preliminaries

2.1 NCSs via an observer-based endec-decoder scheme

Consider the following linear discrete-time system:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) & \text{(1a)} \\ y(k) = Cx(k) & \text{(1b)} \\ x(0) = x_0 \in \mathcal{X}_0 & \text{(1c)} \end{cases}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the system state, $y(k) \in \mathbb{R}^{n_y}$ is the measured output, and $u(k) \in \mathbb{R}^{n_u}$ is the control input, respectively. A, B and C are known matrices with appropriate dimensions. x_0 is an initial state which belongs to a known ellipsoid set denoted by $\mathcal{X}_0 \triangleq \{x \in \mathbb{R}^{n_x} | x^T \Theta^{-1} x \leq 1\}$, where Θ is a given positive-definite matrix.

To better reflect the practice, the following hard constraints on system states and inputs are taken into consideration:

$$\begin{cases} |[u(k)]_s| \leq [\vec{u}]_s & s \in \{1, \cdots, n_u\} & \text{(2a)} \\ |[\Psi]_l x(k)| \leq [\vec{x}]_l & l \in \{1, \cdots, h\} & \text{(2b)} \end{cases}$$

where $\Psi \in \mathbb{R}^{h\times n_x}$ is a known matrix; $\vec{u} > 0$ and $\vec{x} > 0$ are the known vectors.

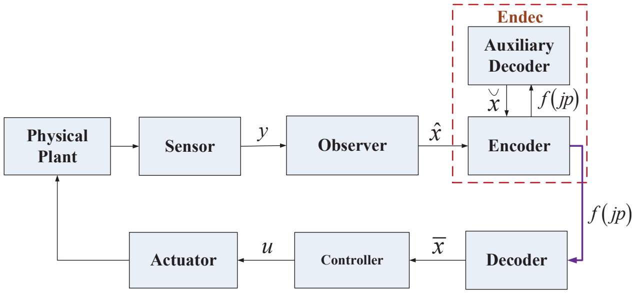

Fig. 1. The structure of the NCS with an observer-based coding-decoding scheme.

For reducing communication burden as well as guaranteeing information security during data transmission, an observer-based coding-decoding scheme is employed from the sensor to the controller in a bandwidth-constrained network circumstance. The state observer of such a scheme is constructed as follows:

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L\left(y(k) - C\hat{x}(k)\right) & \text{(3a)} \\ \hat{x}(0) = \hat{x}_0 \in \mathcal{X}_0 & \text{(3b)} \end{cases}$$

where $\hat{x}(k) \in \mathbb{R}^{n_x}$ is the state estimate with the given initial value $\hat{x}_0$ and L is the observer gain to be designed.

In this paper, the difference coding technique is adopted, which means that the difference between the observer state and the encoder state (which is calculated based on the recently generated *decoding value*). In this sense, to obtain the decoding value at the encoder side and avoid the data postback, the encoder side is equipped with an auxiliary decoder that has the same dynamics as the actual decoder (the word "actual" will be omitted in the sequel when no confusion can arise). In particular, the encoder and the auxiliary decoder, as a whole, are called "endec". The structure of the closed-loop system with the observer-based endec-decoder scheme is depicted in Fig. 1.

Assumption 1 *The coding period is the same as the decoding period (denoted as p), and both coding and decoding are operated simultaneously.*

According to the above assumptions, the coding-decoding mechanism is stated by

$$\textbf{Coding}: \begin{cases} \tilde{x}(jp) = \mathscr{F}(\check{x}(jp)) \\ f(jp) = \mathcal{F}\left(\hat{x}(jp) - \tilde{x}(jp)\right) \end{cases} \quad (4)$$

$$\textbf{Decoding}: \zeta(jp) = \mathcal{G}\left(f(jp)\right) \quad (5)$$

for $j = 1, 2, \cdots$, where jp ($j \in \mathbb{Z}_{>0}$) denotes the coding/decoding time instants, $\check{x}(jp)$ and $\tilde{x}(jp)$ represent the states of the auxiliary decoder and the encoder, respectively. At each time instant jp, the error $\hat{x}(jp) - \tilde{x}(jp)$ is coded. $\mathcal{F}(\cdot)$ and $\mathcal{G}(\cdot)$ are the coding and decoding functions to be designed, respectively. $f(jp)$ is the codeword generated by the encoder at the coding time instant jp, and $\zeta(jp)$ is the corresponding decoding value defined as $\zeta(jp) \triangleq \left[\bar{x}^T(jp)\ \bar{x}^T(jp+1)\ \cdots\ \bar{x}^T((j+1)p-1)\right]^T$ where $\bar{x}(\cdot)$ is the output of the decoder. The mappings $\mathscr{F}(\cdot)$, $\mathcal{F}(\cdot)$ and $\mathcal{G}(\cdot)$ will be designed later.

Remark 1 *The observer-based coding-decoding procedure can be divided into the following two steps. Step 1: at the coding time instant jp ($j \in \mathbb{Z}_{>0}$), $\hat{x}(jp) - \tilde{x}(jp)$ is coded into certain codewords and then are transmitted to the auxiliary decoder and the decoder. Step 2: the decoded value $\bar{x}(k)$ generated by the decoder is transmitted to the controller.*

2.2 *N-step MPC scheme*

Let us first introduce the offline N-step MPC strategy. Based on the decoding value, the corresponding control laws are given by

$$u(k+n|k) = \begin{cases} K_n\bar{x}(k+n|k), & 0 \le n < N \\ K_N\bar{x}(k+n|k), & n \ge N \end{cases} \quad (6)$$

where K_n ($0 \le n \le N$) are the controller gains to be designed.

Denoting $e(k) \triangleq x(k) - \bar{x}(k)$ as the decoding error , the following cost function is constructed:

$$J_N(k) \triangleq \sum_{n=0}^{N-1} l(x(k+n|k), u(k+n|k)) + V_N(x(k+n|k)) \quad (7)$$

where N denotes the prediction horizon, the control horizon and the optimization horizon. $l(x(k+n|k), u(k+n|k))$ and $V_N(x(k+N|k))$ denote, respectively, the stage cost and the terminal cost defined by

$$V_N(x(k+n|k)) \triangleq x^T(k+n|k)P_Nx(k+n|k) \quad (8)$$

$$\begin{aligned} l(x(k+n|k), u(k+n|k)) \triangleq{}& \|x(k+n|k)\|_Q^2 \\ &+ \|u(k+n|k)\|_R^2 - \tau\,\|e(k+n|k)\|^2 \end{aligned} \quad (9)$$

where $\tau > 0$ is a known scalar, Q and R are the known positive-definite weighting matrices, and P_N is a positive-definite matrix to be designed.

According to the approach adopted in [22], the controller parameters can be solved by suppressing the cost function (7) subjected to constraints (2a)-(2b), thus giving rise to the following offline optimization problem (OP).

OP 1a: Calculate the desired controller parameters K_i ($i = 1, 2, \cdots, N$) by suppressing the cost function $J_N(k)$ subject to the following constraints:

$$\begin{cases} x(k+n+1|k) = Ax(k+n|k) \\ \quad + Bu(k+n|k), n = 0, \cdots, N-1 & \text{(10a)} \\ \left|[u(k+n|k)]_s\right| \le [\vec{u}]_s, n = 0, \cdots, N-1 & \text{(10b)} \\ \left|[\Psi]_l x(k+n|k)\right| \le [\vec{x}]_l, n = 1, \cdots, N & \text{(10c)} \\ x(k) \in \Omega_N,\ k \ge N & \text{(10d)} \end{cases}$$

where Ω_N is the so-called terminal constraint set to be designed, which is vitally important to guarantee the stability of the system by means of the MPC strategy.

OP 1b: Calculate the desired control input $u(k)$ at each time instant k by suppressing the cost function $J_N(k)$ subject to the constraints (10a)-(10d).

2.3 Preliminaries

Definition 1 *A set* $\mathbf{X}$ *is said to be a positively invariant (PI) set if, for any* $x(k) \in \mathbf{X}$ *and* $k \in \mathbb{Z}_{\geq 0}$*, there is* $x(k+1) \in \mathbf{X}$.

Definition 2 *[38] The set* $\mathcal{Q}(\Omega)$ *is called a one-step set (OSS) if all the states which belong to it can be steered into* Ω *by an admissible control* u*, i.e.,* $\mathcal{Q}(\Omega) \triangleq \{x(k) \in \mathbb{R}^{n_x} | \exists\, u \in \mathbb{R}^{n_u}, x(k+1) \in \Omega\}$.

Definition 3 *[38] Let a terminal constraint set* Ω *be given. If state predictions* $x(k+n|k)$ $(0 \leq n < N)$*, with a series of constrained controllers determined by the MPC approach, can be steered into* Ω *within* N *steps, i.e.* $x(k+N|k) \in \Omega$*, then such an MPC strategy is named by* N*-step MPC strategy.*

Definition 4 *System (1a)-(1c) is said to be detectable with a communication channel of capacity* $\mathcal{C}$ *if there exists a pair of endec and decoder such that*

$$\lim_{k\to\infty} \|e(k)\| = 0 \tag{11}$$

for any solution of system (1a)-(1c).

Definition 5 *System (1a)-(1c) under the control law (6) is said to be asymptotically stable with a communication channel of capacity* $\mathcal{C}$ *if there exists a pair of endec and decoder such that*

$$\lim_{k\to\infty} \|x(k)\| = 0 \tag{12}$$

for any solution of system (1a)-(1c).

In this paper, the main purpose is to co-design an observer-based endec-decoder scheme and a set of desired controllers in the framework of N-step MPC such that system (1a)-(1c) under the control law (6) with hard constraints (2a)-(2b) is detectable and asymptotically stable. More specifically, the following two requirements are simultaneously satisfied:

- R1) design an effective observer-based endec-decoder scheme such that the system (1a)-(1c) is detectable;
- R2) under the condition R1), establish sufficient conditions to guarantee that the system (1a)-(1c) is asymptotically stable in the framework of offline N-step MPC strategy and the online N-step MPC strategy, and the desired controller inputs can be derived by solving certain optimization problems.

3 Detectability via the observer-based endec-decoder scheme

3.1 Observer design

Lemma 1 *Considering system (1a)-(1c), if there exist a positive-definite matrix* Z*, a matrix* Y *and a positive scalar* $0 < \beta < 1$ *such that*

$$\begin{bmatrix} (1-\beta)Z & * \\ ZA - YC & Z \end{bmatrix} > 0, \tag{13}$$

then there exist $p \in \mathbb{Z}_{>0}$ *and a constant* α $(0 < \alpha < 1)$ *satisfying*

$$\|x(k+p) - \hat{x}(k+p)\|_\infty \leq \alpha \|x(k) - \hat{x}(k)\|_\infty . \tag{14}$$

Furthermore, the desired observer gain is obtained by

$$L = Z^{-1}Y. \tag{15}$$

Proof: Defining $\eta(k) \triangleq x(k) - \hat{x}(k)$, it follows immediately from (1a) and (3a) that

$$\eta(k+1) = (A - LC)\eta(k). \tag{16}$$

Choose the following Lyapunov function:

$$\bar{V}(k) = \eta^T(k) Z \eta(k). \tag{17}$$

Taking the difference along (16) yields

$$\begin{aligned} \Delta\bar{V}(k) &= \eta^T(k+1)Z\eta(k+1) - \eta^T(k)Z\eta(k) \\ &= \eta^T(k)((A-LC)^T Z (A-LC) - Z)\eta(k). \end{aligned} \tag{18}$$

Adding the term $\beta\bar{V}(k)$ $(0 < \beta < 1)$ to both sides of (18), we have

$$\begin{aligned} \Delta\bar{V}(k) + \beta\bar{V}(k) = \eta^T(k)((A-LC)^T Z(A-LC) \\ -(1-\beta)Z)\eta(k). \end{aligned} \tag{19}$$

In terms of (13) and (19), the following inequality is true:

$$\Delta\bar{V}(k) + \beta\bar{V}(k) \leq 0, \tag{20}$$

which implies $\bar{V}(k+1) \leq (1-\beta)\bar{V}(k)$. Obviously, one has $\bar{V}(k+p) \leq (1-\beta)\bar{V}(k+p-1) \leq (1-\beta)^2\bar{V}(k+p-2) \leq \cdots \leq (1-\beta)^p\bar{V}(k)$.

It is easily seen that

$$\begin{aligned} \lambda_{\min}\{Z\} \|\eta(k+p)\|_\infty &\leq \bar{V}(k+p) \\ &\leq n_x(1-\beta)^p \lambda_{\max}\{Z\} \|\eta(k)\|_\infty , \end{aligned} \tag{21}$$

which implies that

$$\|\eta(k+p)\|_\infty \leq \alpha \|\eta(k)\|_\infty , \tag{22}$$

where $\alpha = n_x(1-\beta)^p \frac{\lambda_{\max}\{Z\}}{\lambda_{\min}\{Z\}}$, n_x is the dimension of system state $x(k)$. Thus, there exists a proper p such that $0 < \alpha < 1$. The proof is now complete.

3.2 Endec-decoder scheme design

Lemma 2 *If there exist a positive-definite matrix* W *and a scalar* $\gamma > 0$ *such that*

$$\begin{bmatrix} W - \gamma I & * \\ WA & W \end{bmatrix} > 0, \tag{23}$$

then for any two solutions $x_1(k)$ *and* $x_2(k)$ *of the system (1a)-(1c), we have*

$$\|x_1(k+1) - x_2(k+1)\|_\infty \leq \mu \|x_1(k) - x_2(k)\|_\infty \tag{24}$$

where $\mu = \frac{n_x(1-\gamma)\lambda_{\max}\{W\}}{\lambda_{\min}\{W\}}$.

Proof: The proof procedure is similar to that of Lemma 1 and is thus omitted.

To facilitate the design of the endec-decoder scheme, the uniform quantization method is provided as follows.

For a given scaling parameter $a > 0$ and a given integer $q \in \mathbb{Z}_{>0}$, denote by $\mathcal{B}_a = \{\theta \in \mathbb{R}^{n_x} : \|\theta\|_\infty \leq a\}$ a quantization region, which is uniformly partitioned into q^{n_x} hypercubes. By using the similar partition technique as [25], for each $i \in \{1, 2, \cdots, n_x\}$, the corresponding ith component of the vector θ, i.e. θ_i, can be partitioned into the following q segments:

$$H_1^i(a) \triangleq \left\{\theta_i \middle| \theta_i \in \left[-a, -a+\frac{2a}{q}\right)\right\}$$
$$H_2^i(a) \triangleq \left\{\theta_i \middle| \theta_i \in \left[-a+\frac{2a}{q}, -a+\frac{4a}{q}\right)\right\}$$
$$\vdots$$
$$H_q^i(a) \triangleq \left\{\theta_i \middle| \theta_i \in \left[a-\frac{2a}{q}, a\right]\right\} \quad (25)$$

where each segment $H_j^i(a)$ $(j = 1, 2, \cdots, q)$ corresponds to a certain number j. Then, for any $\theta \in \mathcal{B}_a$, there exists a unique set of integers $\{\nu_1, \nu_2, \cdots, \nu_{n_x}\}$ where $\nu_i \in \{1, 2, \cdots, q\}$, $i = 1, 2, \cdots, n_x$ such that $\theta \in H^1_{\nu_1}(a) \times H^2_{\nu_2}(a) \times \cdots \times H^{n_x}_{\nu_{n_x}}(a)$. With respect to the set of integers $\{\nu_1, \nu_2, \cdots, \nu_{n_x}\}$, the vector defined by

$$\zeta_a(\nu_1, \nu_2, \cdots, \nu_{n_x}) \triangleq \left[-a+\frac{a(2\nu_1-1)}{q} \quad \cdots \quad -a+\frac{a(2\nu_{n_x}-1)}{q}\right]^T \quad (26)$$

is the center of the hypercube $H^1_{\nu_1}(a) \times H^2_{\nu_2}(a) \times \cdots \times H^{n_x}_{\nu_{n_x}}(a)$. According to the above analysis, it is easily seen that

$$\|\theta - \zeta_a(\nu_1, \nu_2, \cdots, \nu_{n_x})\|_\infty \le \frac{a}{q}. \quad (27)$$

Next, we are going to consider the design of the endec-decoder scheme based on the derived scalars α and μ. For the subsequent development, as in [44], some necessary definitions related to the adopted endec-decoder scheme are presented as follows:

$$\begin{cases} r_0 = \sup_{x_0 \in \mathcal{X}_0} \|x_0\|_\infty \\ a(p) = (2\alpha + \mu^p) r_0 \\ a((j+1)p) = \left(2\alpha^{j+1} + \mu^p \alpha^j\right) r_0 + \mu^p \dfrac{a(jp)}{q}, \ j \in \mathbb{Z}_{>0}. \end{cases} \quad (28)$$

Define the error as $\tilde{e}(jp) \triangleq \hat{x}(jp) - \tilde{x}(jp)$. Next, we are ready to propose the uniform-quantization-based endec-decoder scheme.

Endec: For $\tilde{e}(jp) \in H^1_{\nu_1}(a(jp)) \times H^2_{\nu_2}(a(jp)) \times \cdots \times H^{n_x}_{\nu_{n_x}}(a(jp)) \subseteq \mathcal{B}_{a(jp)}$, we have

- Encoder:

$$\begin{cases} \tilde{x}(0) = 0 \\ \tilde{x}(jp) = A\breve{x}(jp-1) + Bu(jp-1) \\ f(jp) = \{\nu_1, \nu_2, \cdots, \nu_{n_x}\}. \end{cases} \quad (29)$$

- Auxiliary decoder:

$$\begin{cases} \breve{x}(0) = 0 \\ \breve{x}(k+1) = A\breve{x}(k) + Bu(k), k \ne jp-1 \\ \breve{x}(jp) = A\breve{x}(jp-1) + Bu(jp-1) \\ \qquad + \zeta_{a(jp)}(\nu_1, \nu_2, \cdots, \nu_{n_x}) \end{cases} \quad (30)$$

Decoder:

$$\begin{cases} \bar{x}(0) = 0 \\ \bar{x}(k+1) = A\bar{x}(k) + Bu(k), \quad k \ne jp-1 \\ \bar{x}(jp) = \tilde{x}(jp) + \zeta_{a(jp)}(\nu_1, \nu_2, \cdots, \nu_{n_x}). \end{cases} \quad (31)$$

Remark 2 *In most of the different coding techniques (see e.g. [25, 36, 44]), it is necessary for the encoders to obtain the states of decoders via the data postbacks. Such a resource-consuming transmission manner is unrealistic in practical engineering. To overcome such an obstacle, an endec consisting of an auxiliary decoder and an encoder is employed to replace the traditional encoder, where the auxiliary decoder is utilized to generate the required decoder's states. In this way, the proposed endec can not only retain the same advantages as the traditional encoders but also save more communication resources. Without loss of generality, the initial states of endec and decoder are assumed to be zero.*

Lemma 3 *For all $j \ge 1$, the endec-decoder scheme described by (29)-(31) satisfies*

$$\|\tilde{e}(jp)\|_\infty \le a(jp). \quad (32)$$

Proof: The *mathematical induction* is utilized to prove this lemma, which is divided into the following two steps.

i) For the case $j = 1$, it follows from Lemmas 1 and 2 that

$$\begin{aligned} \|\tilde{e}(p)\|_\infty &= \|\hat{x}(p) - \tilde{x}(p)\|_\infty \\ &\le \|\hat{x}(p) - x(p)\|_\infty + \|x(p) - \tilde{x}(p)\|_\infty \\ &\le 2\alpha r_0 + \mu^p \|x(0) - \breve{x}(0)\|_\infty \\ &\le (2\alpha + \mu^p) r_0 = a(p). \end{aligned} \quad (33)$$

ii) Assume that (32) holds for j, we need to prove that it also holds for $j+1$. From Lemmas 1 and 2, it is inferred from (27) that:

$$\begin{aligned} &\|\tilde{e}((j+1)p)\|_\infty \\ &= \|\hat{x}((j+1)p) - \tilde{x}((j+1)p)\|_\infty \\ &\le \|\hat{x}((j+1)p) - x((j+1)p)\|_\infty \\ &\quad + \|x((j+1)p) - \tilde{x}((j+1)p)\|_\infty \\ &\le 2\alpha^{j+1} r_0 + \mu^p \alpha^j r_0 + \mu^p \frac{a(jp)}{q} = a((j+1)p). \end{aligned} \quad (34)$$

Thus, (32) holds for all $j \ge 1$, which completes the proof.

Theorem 1 *Suppose that there exist two positive scalars β and γ, two positive-definite matrices Z and W, and a matrix Y such that the matrix inequalities (13) and (23) are solvable. Then, then system (1a)-(1c) is detectable via the proposed endec-decoder scheme (29)-(31) if the following condition holds*

$$q > \mu^p \quad (35)$$

where μ is defined in Lemma 2.

Proof: i) For $k = jp$, $j \ge 1$, one easily obtains from $0 < \alpha < 1$ and condition (35) that $\lim_{j\to\infty} a(jp) = 0$. On the other hand, it follows from (29)-(31), Lemmas 1 and 3 that

$$
\begin{aligned}
&\|e(jp)\|_\infty \\
&= \|(x(jp) - \hat{x}(jp)) + (\hat{x}(jp) - \tilde{x}(jp)) \\
&\quad + (\tilde{x}(jp) - \bar{x}(jp))\|_\infty \\
&\le \|x(jp) - \hat{x}(jp)\|_\infty + \|\tilde{e}(jp)\|_\infty \\
&\quad + \|\tilde{x}(jp) - \check{x}(jp)\|_\infty \\
&\le 2\alpha^j r_0 + \|\tilde{e}(jp)\|_\infty + \frac{a(jp)}{q}.
\end{aligned} \tag{36}
$$

Since $x_0 \in \mathcal{X}_0$ from (1c) and $r_0 = \sup_{x_0 \in \mathcal{X}_0} \|x_0\|_\infty$ from (28), r_0 has an upper bound. Thus, based on $0 < \alpha < 1$ in Lemma 1, we have $\lim_{j\to\infty} a(jp) = 0$. In addition, the coding/decoding period p and the scalar μ are bounded, it can be easily obtained from (28) that $\lim_{j\to\infty} a((j+1)p) = \lim_{j\to\infty} \mu^p \frac{a(jp)}{q}$. Due to $q > \mu^p$ in (35), we have $\lim_{j\to\infty} a(jp) = 0$. According to the above discussions, and with the help of $\|\tilde{e}(jp)\|_\infty \le a(jp)$ in Lemma 3, we can conclude that $\lim_{j\to\infty}(2\alpha^j r_0 + \|\tilde{e}(jp)\|_\infty + \frac{a(jp)}{q}) = 0$, which implies $\lim_{j\to\infty} \|e(jp)\|_\infty = 0$.

ii) For $k \in (jp, (j+1)p)$, it is easily seen that $x(k)$ and $\bar{x}(k)$ are two solutions of system (1a)-(1c). In terms of Lemma 2, one has $\|e(k)\|_\infty \le \mu^{k-jp} \|e(jp)\|_\infty$, which means that the decoding error $\|e(k)\|_\infty$ is also bounded at the non-coding time instant and approaches zero when k goes to infinity, i.e. $\lim_{k\to\infty} \|e(k)\|_\infty = 0$.

According to the above analysis, we conclude that system (1a)-(1c) is detectable via the proposed endec-decoder (29)-(31) if the condition (35) holds. The proof is thus complete.

Let's discuss the upper bound of $\|e(k)\|^2$. For $k = jp$, $j > 1$, based on (28), $0 < \alpha < 1$ and (35), we have

$$
\begin{aligned}
a(jp) &= \left(2\alpha^j + \mu^p \alpha^{j-1}\right) r_0 + \frac{\mu^p}{q} a((j-1)p) \\
&= \alpha^{j-1} a(p) + \frac{\mu^p}{q} a((j-1)p) \le \frac{a(p)}{1-\alpha}.
\end{aligned} \tag{37}
$$

Then, keeping $0 < \alpha < 1$, (32) and (37) in mind, it can be learned from (36) that

$$
\begin{aligned}
\|e(jp)\|_\infty &\le 2\alpha^j r_0 + \|\tilde{e}(jp)\|_\infty + \frac{a(jp)}{q} \\
&\le 2r_0 + \frac{q+1}{q(1-\alpha)}(2\alpha + \mu^p) r_0.
\end{aligned} \tag{38}
$$

For $k \in (jp, (j+1)p)$, with the help of Lemma 2 and (38), we obtain

$$
\begin{aligned}
\|e(k)\|_\infty &\le \mu^{k-jp} \|e(jp)\|_\infty \\
&\le \tilde{\mu} \left(2r_0 + \frac{q+1}{q(1-\alpha)} (2\alpha + \mu^p) r_0\right)
\end{aligned} \tag{39}
$$

where $\tilde{\mu} = \max\{\mu^p, 1\}$. Thus, it can be inferred from (38) and (39) that

$$
\begin{aligned}
\|e(k)\|^2 &\le n_x \left(\|e(k)\|_\infty\right)^2 \\
&\le n_x \tilde{\mu} \left(2r_0 + \frac{q+1}{q(1-\alpha)} (2\alpha + \mu^p) r_0\right) \\
&\le \chi
\end{aligned} \tag{40}
$$

where $\chi \triangleq \left\lceil n_x \tilde{\mu} \left(2r_0 + \frac{q+1}{q(1-\alpha)} (2\alpha + \mu^p) r_0\right)\right\rceil$ with $\lceil \cdot \rceil$ denoting an operation to round up "·" to an integer.

4 N-step MPC strategy

The design of the offline N-step MPC strategy is divided into two steps in this section: *Step 1*: establish sufficient conditions to ensure the existence of the required terminal constraint set; *Step 2*: design the controller parameters to guarantee that the system state can be steered into the proposed terminal constraint set with N steps. Substituting (6) into (1a)-(1c) results in the following closed-loop system:

$$
\begin{aligned}
x(k+n+1|k) = (A + BK_n)x(k+n|k) \\
-BK_n e(k+n|k).
\end{aligned} \tag{41}
$$

Note that the decoding error on prediction horizon satisfies the constraint (40), i.e., $\|e(k+n|k)\|^2 \le \chi$.

4.1 Terminal constraint set

As suggested by [22,38], the set Ω_N is said to be a terminal constraint set for system (1a)-(1c) under the control law (6) if the following two requirements are satisfied:
i) under the constraints (10b)-(10c), Ω_N is a PI set;
ii) for any $x(k+n|k) \in \Omega_N$, the terminal cost function is a local Lyapunov-like function satisfying

$$
\begin{aligned}
&V_N(x(k+n+1|k)) - V_N(x(k+n|k)) \\
&\quad \le -l\left(x(k+n|k), u(k+n|k)\right).
\end{aligned}
$$

Lemma 4 *Define a set $\Omega_N \triangleq \left\{x \in \mathbb{R}^{n_x} \middle| x^T \Phi_N^{-1} x \le 1\right\}$. Ω_N is a PI set if the following condition*

$$
V_N\left(x\left(k+n+1|k\right)\right) - V_N(x(k+n|k) \le 0 \tag{42}
$$

holds under the constraint

$$
\frac{1}{\chi} \|e(k+n|k)\|^2 \le \frac{1}{\rho} \|x(k+n|k)\|_{P_N}^2.
$$

Proof: The above lemma can be easily obtained along the similar line of [1], and thus is omitted for space saving.

Lemma 5 *Let a matrix Ψ be given. Considering system (41) via the endec-decoder scheme (29)-(31), if there exist positive-definite matrices $\mathbb{X}_N$, $\mathbb{U}_N$, a matrix W_N, and two scalars $\rho > 0$, $0 < \xi < 1$ such that*

$$
\begin{bmatrix}
(1-\xi)\Phi_N & * & * \\
0 & 2\Phi_N - \frac{\chi}{\xi} I & * \\
A\Phi_N + BW_N & -BW_N & \Phi_N
\end{bmatrix} \ge 0, \tag{43}
$$

$$
\begin{bmatrix}
\mathbb{U}_N & * & * \\
W_N^T & \Phi_N & * \\
W_N^T & 0 & 2\Phi_N - \chi I
\end{bmatrix} \ge 0, [\mathbb{U}_N]_{ss} \le [\vec{u}]_s^2, \tag{44}
$$

$$
\begin{bmatrix}
\mathbb{X}_N & * \\
(\Psi \Phi_N)^T & \Phi_N
\end{bmatrix} \ge 0, \quad [\mathbb{X}_N]_{ll} \le [\vec{x}]_l^2 \tag{45}
$$

where $\Phi_N = \rho P_N^{-1}$, then the set Ω_N is a PI set. Furthermore, the desired feedback gain is given by

$$
K_N = W_N \Phi_N^{-1}. \tag{46}
$$

Proof: This lemma can be obtained along the similar line of [30], and thus is omitted for space saving.

4.1.1 Terminal cost function

Lemma 6 *Let weighting matrices $Q > 0$, $R > 0$ and a scalar $\tau > 0$ be given. For system (41) via the observer-based endec-decoder scheme (29)-(31), assume that the matrix inequality*

$$\begin{bmatrix} \Phi_N & * & * & * & * \\ 0 & \tau\Upsilon_N & * & * & * \\ \Gamma_N & -BW_N & \Phi_N & * & * \\ \sqrt{R}W_N & -\sqrt{R}W_N & 0 & \rho I & * \\ \sqrt{Q}\Phi_N & 0 & 0 & 0 & \rho I \end{bmatrix} > 0 \quad (47)$$

holds, where W_N, Φ_N and ρ are defined in Lemma 5, $\Gamma_N = A\Phi_N + BW_N$ and $\Upsilon_N = 2\Phi_N - \rho I$, then one has

$$V_N(x(k+n+1|k)) - V_N(x(k+n|k)) + l\left(x(k+n|k), u(k+n|k)\right) < 0. \quad (48)$$

Proof: This lemma can be obtained along the similar line of [30], and thus is omitted for space saving.

Based on the achievements in Lemmas 5 and 6, the desired controller parameter K_N is derived by

$$\textbf{OP2} \quad \min_{\rho>0, W_N} \ \text{trace}(\Phi_N) \quad \text{s.t. } (43) - (45) \text{ and } (47).$$

4.2 Approximating sets of OSSs

Based on the derived terminal constraint set Ω_N, define a sequence of OSSs as follows:

$$\Omega_i \triangleq \mathcal{Q}(\Omega_{i+1}), \quad i = 0, 1, \cdots, N-1$$

where the mapping $\mathcal{Q}(\cdots)$ is defined in Definition 2. Then, as stated in [38], the state constraint $x(k+N|k) \in \Omega_N$ can be ensured by certain admissible control inputs $u(k+i|k)$ $(i = 0, 1, \cdots, N-1)$ if $x_{k|k} \in \Omega_0$. Note that it is always very difficult to achieve these exact OSSs. Accordingly, in this paper, we use a sequence of ellipsoid sets to "approximate" these desired OSSs. These approximating ellipsoid sets are defined as follows:

$$\mathbb{P}_i \triangleq \left\{ x(k+i|k) \middle| x^T(k+i|k)\Phi_i^{-1}x(k+i|k) \le 1 \right\}, \quad i = 0, 1, \cdots, N,$$

where the matrices Φ_i $(i = 0, 1, \cdots, N-1)$ are positive-definite matrices to be determined later.

One objective of this paper is to design the a sequence of controller parameters K_i $(i = 0, 1, \cdots, N-1)$ such that $x_{k+i|k} \in \mathbb{P}_i$ holds for all $i = 1, 2, \cdots, N$ under the condition $x_{k|k} \in \mathbb{P}_0$. Obviously, it is easy to find that $\mathbb{P}_N = \Omega_N$. Furthermore, it is easy to see that $\mathbb{P}_i \subseteq \mathcal{Q}^{u_i}(\mathbb{P}_{i+1})$ $(i = 0, 1, \cdots, N-1)$.

Lemma 7 *Let a matrix Ψ and scalars $\tilde{\tau}_n \le \tau$ $(n = 0, 1, 2, \cdots, N)$ be given. Considering system (41) via the observer-based endec-decoder scheme (29)-(31), if there exist positive-definite matrices P_n, $\tilde{\mathbb{X}}_n$, $\mathbb{U}_n$, a matrix W_n and a scalar $0 < \tilde{\xi} < 1$, for any $0 \le n \le N-1$ such that*

$$\begin{bmatrix} (1-\tilde{\xi})\Phi_n & * & * \\ 0 & 2\Phi_n - \frac{\chi}{\tilde{\xi}}I & * \\ A\Phi_n + BW_n & -BW_n & \Phi_{n+1} \end{bmatrix} \ge 0, \quad (49)$$

$$\begin{bmatrix} \Phi_n & * & * & * & * \\ 0 & \tilde{\tau}_n(2\Phi_n - \rho I) & * & * & * \\ A\Phi_n + BW_n & -BW_n & \Phi_{n+1} & * & * \\ \sqrt{R}W_n & -\sqrt{R}W_n & 0 & \rho I & * \\ \sqrt{Q}\Phi_n & 0 & 0 & 0 & \rho I \end{bmatrix} > 0, \quad (50)$$

$$\begin{bmatrix} \mathbb{U}_n & * & * \\ W_n^T & \Phi_n & * \\ W_n^T & 0 & 2\Phi_n - \chi I \end{bmatrix} \ge 0, \quad [\mathbb{U}_n]_{ss} \le [\vec{u}]_s^2, \quad (51)$$

$$\begin{bmatrix} \tilde{\mathbb{X}}_n & * \\ (\Psi\Phi_n)^T & \Phi_n \end{bmatrix} \ge 0, \quad \left[\tilde{\mathbb{X}}_n\right]_{ll} \le [\vec{x}]_l^2 \quad (52)$$

$$\Phi_0 \ge \Theta \quad (53)$$

where $s = 1, 2, \cdots, n_u$, $l = 1, 2, \cdots, h$ and $\Phi_n = \rho P_n^{-1}$, ρ is defined Lemma 5, then the set $\mathbb{P}_n$ is an ellipsoidal approximating set of the OSS $\mathcal{Q}^{u_n}(\mathbb{P}_{n+1})$. Furthermore, the condition $\sum_{i=0}^{N-1} l(x(k+i|k), u(k+i|k)) < x^T(k|k)P_n x(k|k)$ holds for any $n \in \{0, 1, \cdots, N-1\}$. The corresponding feedback gain is given by

$$K_n = W_n\Phi_n^{-1}. \quad (54)$$

Proof: The similar line with Lemma 5 can be used to conduct the proof. Due to the limitation of the space, the proof is omitted.

Based on Lemma 7, the following offline optimization problem is proposed to calculate the "best" controller parameters K_i $(i = 0, 1, \cdots, N-1)$:

$$\textbf{OP3} \quad \min_{\{P_n, \rho\}>0, W_n} \ \text{trace}(P_0) \quad \text{s.t. } (49) - (53).$$

4.3 Online optimization algorithms

For system (1a), the prediction model is given by

$$x(k+n+1|k) = Ax(k+n|k) + Bu(k+n|k). \quad (55)$$

Let $M \triangleq \arg\min_n\{x(k) \in \mathbb{P}_n\}$ and $\bar{M} \triangleq \min\{M+1, N\}$. Then, it is easy to see from Lemmas 6-7 that there exist a sequence of control inputs $u(k+i|k)$ such that

$$x^T(k+i+1|k)P_{\bar{M}}x(k+i+1|k) - x^T(k+i|k) \times P_M x(k+i|k) \le l(x(k+i|k), u(k+i|k)) \quad (56)$$

holds for any $i = 0, 1, \cdots, N$ under the constraints (2a) and (2b) if the matrix inequalities in Lemma 6 and Lemma 7 are solvable. Accordingly, it is easy to derive from Lemma 7 that $\sum_{n=1}^{N} l(x(k+n|k), u(k+n|k)) \le x^T(k+1|k)P_{\bar{M}}x(k+1|k)$, holds for all $n = 0, 1, \cdots, N-1$, which means that

$$J_N(k) \triangleq \sigma(k) + \sum_{n=1}^{N} l(x(k+n|k), u(k+n|k))$$
$$< \sigma(k) + x^T(k+1|k) P_{\bar{M}} x(k+1|k) \quad (57)$$

where $\sigma(k) \triangleq x^T(k)Qx(k) + u^T(k)Ru(k)$. In this case, at each time instant k, the cost function $J_N(k)$ can be suppressed by minimizing the term $\sigma(k) + x^T(k+1|k)P_{\bar{M}}x(k+1|k)$ with the optimal control input $u(k)$.

Assume that the matrix inequalities in Lemma 6 and Lemma 7 are solvable. Letting φ be a certain upper bound of $J_N(k)$, the following auxiliary optimization problem can be formulated to solve $u(k)$ [38]:

$$\textbf{OP4} \begin{cases} \min\limits_{u(k)} \ \varphi & \text{(58a)} \\ \text{s.t.}\sigma(k) + x^T(k+1|k)P_{\bar{M}}x(k+1|k) \leq \varphi, & \text{(58b)} \\ x(k+1|k) \in \mathbb{P}_{\bar{M}}, & \text{(58c)} \\ \tilde{V}_1(k) - \tilde{V}_0(k) \leq -l(x(k|k), u(k)), & \text{(58d)} \\ |[u(k)]_s| \leq [\vec{u}]_s, & \text{(58e)} \\ |[\Psi]_l x(k+1|k)| \leq [\vec{x}]_l. & \text{(58f)} \end{cases}$$

where $\tilde{V}_1(k) \triangleq x^T(k+1|k)P_{\bar{M}}x(k+1|k)$ and $\tilde{V}_0(k) \triangleq x^T(k|k)P_M x(k|k)$. Noting that some parameters in **OP4** (e.g. $\mathbb{P}_n$, $n = 0, 1, \cdots, N$) are derived based on the results of **OP2** and **OP3**, the feasibility of the online optimization algorithm depends on the conditions given in Lemmas 5-7. Obviously, **OP4** cannot be directly implemented since the corresponding constraints are state-dependent. Next, we transform the conditions of **OP4** into some solvable linear matrix inequalities which can be directly adopted for the controller.

Let us first establish a sufficient condition to guarantee the constraint (58b). By employing $x(k) = e(k) + \bar{x}(k)$ and inequality property, (58b) can be guaranteed by

$$2\bar{x}^T(k)\left(Q + 2A^T P_{\bar{M}} A\right)\bar{x}(k) + 2e^T(k)\left(Q + 2A^T P_{\bar{M}} A\right) \times e(k) + u^T(k)\left(R + 2B^T P_{\bar{M}} B\right)u(k) \leq \varphi. \quad (59)$$

By virtue of (4), (59) holds if

$$2\bar{x}^T(k)\left(Q + 2A^T P_{\bar{M}} A\right)\bar{x}(k) + 2\lambda_{\max}\{\Theta\}\chi + u^T(k)\left(R + 2B^T P_{\bar{M}} B\right)u(k) \leq \varphi \quad (60)$$

where $\Theta = Q + 2A^T P_{\bar{M}} A$. By using the Schur Complement Lemma, (60) holds if and only if

$$\begin{bmatrix} \varphi - 2\lambda_{\max}\{\Theta\}\chi & * & * \\ \left(R + 2B^T P_{\bar{M}} B\right)^{\frac{1}{2}} u(k) & I & * \\ \left(2\left(Q + 2A^T P_{\bar{M}} A\right)\right)^{\frac{1}{2}} \bar{x}(k) & 0 & I \end{bmatrix} \geq 0. \quad (61)$$

Thus, (61) suffices (58b). Furthermore, (58c) implies

$$x^T(k+1|k)\Phi_{\bar{M}}^{-1}x(k+1|k) \leq 1. \quad (62)$$

By means of the S-procedure technique, (62) holds if and only if the following inequality

$$x^T(k+1|k)\Phi_{\bar{M}}^{-1}x(k+1|k) - 1 - \varsigma\left(e^T(k)e(k) - \chi\right) \leq 0 \quad (63)$$

is true with $\varsigma > 0$ under the condition $e^T(k)e(k) \leq \chi$. Substituting (55) and $x(k) = e(k) + \bar{x}(k)$ into (63) yields

$$\left(Ae(k) + A\bar{x}(k) + Bu(k)\right)^T \Phi_{\bar{M}}^{-1}\left(Ae(k) + A\bar{x}(k) + Bu(k)\right) - 1 - \varsigma\left(e^T(k)e(k) - \chi\right) \leq 0. \quad (64)$$

By resorting to the Schur Complement Lemma, (64) is true if and only if

$$\begin{bmatrix} 1 & e^T(k) \end{bmatrix} \begin{bmatrix} 1 - \varsigma\chi & * & * \\ 0 & \varsigma & * \\ A\bar{x}(k) + Bu(k) & A & \Phi_{\bar{M}} \end{bmatrix} \begin{bmatrix} 1 \\ e(k) \end{bmatrix} \geq 0. \quad (65)$$

Obviously, (65) is true if

$$\begin{bmatrix} 1 - \varsigma\chi & * & * \\ 0 & \varsigma & * \\ A\bar{x}(k) + Bu(k) & A & \Phi_{\bar{M}} \end{bmatrix} \geq 0. \quad (66)$$

Thus, (66) guarantees (58c).

On the other hand, along the similar lines, it can be found that the condition (58d) is achieved if the following inequality holds:

$$\begin{bmatrix} 2\lambda_{\max}\{\hat{\Theta}\}\chi & * & * \\ (R + 2B^T P_{\bar{M}} B)^{\frac{1}{2}} u(k) & -I & * \\ (2(Q + 2A^T P_{\bar{M}} A - P_M))^{\frac{1}{2}} \bar{x}(k) & 0 & -I \end{bmatrix} \leq 0 \quad (67)$$

where $\hat{\Theta} = Q + 2A^T P_{\bar{M}} A - P_M - \tau I$.

It can be seen that (58e) and (58f) are satisfied if the following inequalities are true:

$$\begin{bmatrix} \mathbb{F} & * \\ u^T(k) & I \end{bmatrix} \geq 0, \quad [\mathbb{F}]_{ss} \leq [\vec{u}]_s^2, \quad s = 1, 2, \cdots, n_u \quad (68)$$

$$\begin{bmatrix} \mathbb{G} & * \\ (\Psi\Phi_{\bar{M}})^T & \Phi_{\bar{M}} \end{bmatrix} \geq 0, \ [\mathbb{G}]_{ll} \leq [\vec{x}]_l^2, \ l = 1, \cdots, h. \quad (69)$$

According to the above analysis, **OP4** can be transformed into the following optimization problem:

$$\textbf{OP5} \ \min_{u(k)} \ \varphi \quad \text{s.t.} \ (61), (66), (67), (68), (69) \text{ and } (50). \quad (70)$$

4.4 Stability analysis

Theorem 2 *Assume that conditions (13), (23) and (35) are satisfied, and* $(\mathbb{P}_0 \bigcup \mathbb{P}_1 \bigcup \cdots \bigcup \mathbb{P}_{N-1} \bigcup \Omega_N) \bigcap \mathcal{X}_0 \neq \varnothing$*, where* $\varnothing$ *denotes a set containing no element. For system (1a)-(1b), if the offline optimization problems* **OP2** *and* **OP3** *are feasible, then for any initial state* x_0 *satisfying* $x_0 \in (\mathbb{P}_0 \bigcup \mathbb{P}_1 \bigcup \cdots \bigcup \mathbb{P}_{N-1} \bigcup \Omega_N) \bigcap \mathcal{X}_0$*, the online optimization problem* **OP5** *is feasible, and the system state can be steered into the terminal constraint set within N steps. Furthermore, the closed-loop system is asymptotically stable.*

Proof: Let us prove this theorem in two steps.

i) *Feasibility:* Noticing (50) is equivalent to (58b), it can be easily found from (59)-(60) that (61) can be guaranteed by choosing a sufficiently big φ. (66) can be guaranteed by (49) with a proper ς. Constraints (68) and (69) can be guaranteed by hard constraints (61) and (52). Since **OP3** is feasible and $x(k+1|k) = x(k+1)$, for any initial state x_0 satisfying $x_0 \in (\mathbb{P}_0 \bigcup \mathbb{P}_1 \bigcup \cdots \bigcup \mathbb{P}_{N-1} \bigcup \Omega_N) \bigcap \mathcal{X}_0$, the online optimization problem **OP5** is feasible at $k = 1$. Such a procedure can be conducted to all the future instants, then **OP5** is feasible for any $k > 0$. Furthermore, in subsection 4.3, we have shown that a feasible control $u(k)$ can steer the state $x(k) \in \mathbb{P}_M \setminus \mathbb{P}_{M+1}$. Thus, even in the worst case, i.e., $x(k) \in \mathbb{P}_0 \setminus \mathbb{P}_1$, the state can be steered into the terminal constraint set no more than N steps.

ii) *Asymptotical stability:* We only need to prove that the system is asymptotically stable under the feedback gain K_N after it enters the terminal constraint set.

Denote $\tilde{V}_N(x(k)) \triangleq x^T(k)P_N x(k)$. Since **OP2** is feasible and $x(k+1|k) = x(k+1)$, it can be derived from (48) that

$$\begin{aligned}&\tilde{V}_N(x(k+1)) - \tilde{V}_N(x(k))\\ &< -\left(\|x(k)\|_Q^2 + \|u(k)\|_R^2\right) + \tau \|e(k)\|^2, \end{aligned} \tag{71}$$

which implies that $\tilde{V}_N(x(k+1)) - \tilde{V}_N(x(k)) < 0$ holds under the constraint $\|x(k)\|_Q^2 > \tau\|e(k)\|^2$. Accordingly, we have $\tilde{V}_N(x(k+1)) - \tilde{V}_N(x(k)) < 0$ for all $x \notin \tilde{\Omega}_k \triangleq \{x| \|x\|_Q^2 \leq \tau\|e(k)\|^2\}$. All the state outside the set $\tilde{\Omega}_k$ will be steered into the time-varying set $\tilde{\Omega}_k$ asymptotically. Owing to the detectability of system (1a)-(1c), it can be concluded from Definition 4 that $\lim_{k\to\infty} \|e(k)\| = 0$, which implies that $\lim_{k\to\infty} \tilde{\Omega}_k = \{0\}$. Thus, it can be concluded that $\lim_{k\to\infty} x(k) = 0$, which guarantees the asymptotical stability of the closed-loop system. The proof is thus complete.

Next, we are going to find an implementation problem of **OP5** based on the available information (i.e. $\bar{x}(k)$). Considering the definition of $\mathbb{P}_n$, the constraint of $\mathbb{P}_n$ is rewritten as follows:

$$(e(k) + \bar{x}(k))^T \Phi_n^{-1} (e(k) + \bar{x}(k)) \leq 1. \tag{72}$$

Since

$$2\bar{x}^T(k)\Phi_n^{-1}e(k) \leq \bar{x}^T(k)\Phi_n^{-1}\bar{x}(k) + e^T(k)\Phi_n^{-1}e(k) \tag{73}$$

(72) holds if the following equality is true:

$$\bar{x}^T(k)\Phi_n^{-1}\bar{x}(k) + e^T(k)\Phi_n^{-1}e(k) \leq 0.5. \tag{74}$$

By virtue of (4), (74) is ensured by

$$\bar{x}^T(k)\Phi_n^{-1}\bar{x}(k) \leq 0.5 - \lambda_{\max}\{\Phi_n^{-1}\}\chi. \tag{75}$$

Thus, the constraint of $\mathbb{P}_n$ can be guaranteed by (75).

Define the set

$$\mathbb{Q}_n \triangleq \left\{\bar{x}(k) | \bar{x}^T(k)\Phi_n^{-1}\bar{x}(k) \leq 0.5 - \lambda_{\max}\{\Phi_n^{-1}\}\chi\right\}.$$

Obviously, $x(k) \in \mathbb{P}_n$ can be guaranteed if $\bar{x}(k) \in \mathbb{Q}_n$, $n = 0, 1, \cdots, N$.

Algorithm 1

Offline part:

Step 1. Solve the observer gain L and scalars α and β by Lemma 1, and solve the scalar μ by Lemma 2.

Step 2. Compute the terminal constraint set Ω_N and the corresponding feedback gain K_N by **OP2**. Then, calculate $\mathbb{Q}_N$.

Step 3. Obtain the approximating sets of OSSs $\mathbb{P}_i$ and the corresponding feedback gain K_i $(i = 0, \cdots, N-1)$ by **OP3**. Then, derive the set $\mathbb{Q}_i$ according to the proposed OSSs $\mathbb{P}_i$.

Online part:

Step 1. Set the initial instant $k = 0$ and $\bar{x}(0) = 0$.

Step 2. According to $n \triangleq \arg\min_i\{\bar{x}(k) \in \mathbb{Q}_i\}$, compute the value of n.

Step 3. Derive the desired control input $u(k)$ by solving **OP5**. Set $k = k + 1$ and go to *Step 2*.

Remark 3 *So far, we have discussed the N-step MPC problem for a class of linear time-invariant systems via an observer-based coding-decoding scheme. Obviously, the proposed N-step MPC strategy is an* offline-to-online *synthetic algorithm. Compared with the zero-step MPC [28, 29], the N-step MPC can reduce the online computation burden and obtain better practical applicability at a very little sacrifice of the control performance. Noting that the offline part is of great importance for the online implementation of Algorithm 1, since the online part is implemented based on the parameters derived by the offline part (e.g. P_n, $n = 0, 1, \cdots, N$). It can be observed that, in the co-design of the observer-based endec-decoder scheme and the N-step MPC strategy, Algorithm 1 covers all important factors contributing to the complexities: 1) coding/decoding period p, 2) quantization density a, 3) step size N and 4) terminal constraint set Ω_N.*

5 Numerical Examples

Consider the following discrete-time system:

$$\begin{cases} x(k+1) = \begin{bmatrix} 0.94 & -0.72 \\ 0.96 & 0.93 \end{bmatrix} x(k) + \begin{bmatrix} 0.15 & 1.6 \\ 1.6 & -1 \end{bmatrix} u(k) \\ \qquad\qquad \triangleq Ax(k) + Bu(k) \\ y(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(k) \triangleq Cx(k). \end{cases}$$

The upper bounds for hard constraints and the weighting matrices are given by

$$\vec{u} = \begin{bmatrix} 30 \\ 30 \end{bmatrix}, \ \vec{x} = \begin{bmatrix} 50 \\ 50 \end{bmatrix}, \ Q = \begin{bmatrix} 0.005 & 0 \\ 0 & 0.005 \end{bmatrix},$$
$$R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.6 \end{bmatrix}, \quad \Psi = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$

Choose the initially feasible scalars $\xi^* = 0.5$ and $\tilde{\xi}^* = 0.55$, the positive scalars $\tilde{\tau}_n = \tau = 0.8$. From Lemma 1, we can obtain $\beta = 0.5$ and the observer gain is calculated as

$$L = \begin{bmatrix} 0.9334 & -0.0309 \\ -0.0462 & 0.9366 \end{bmatrix}.$$

By setting the decoding period as $p = 2$, one derives from Lemmas 1 and 2 that $\alpha = 0.7471$, $\mu = 2.7290$, $r_0 = 0.086$ and $q = 30$. The norm bound of the decoding error is calculated by $\chi = 50$. By solving **OP2**, the terminal constraint set $\Omega_N = \left\{x \in \mathbb{R}^n | x^T \Phi_N^{-1} x \leq 1\right\}$ and the corresponding feedback gain are obtained as follows:

$$\Phi_N = \begin{bmatrix} 101.8069 & 6.1059 \\ 6.1059 & 138.0573 \end{bmatrix},$$

$$K_N = \begin{bmatrix} -0.5607 & -0.1748 \\ -0.2875 & 0.2811 \end{bmatrix}.$$

By solving the offline optimization **OP3** backwards, it is verified that **OP3** is feasible by letting $N = 17$. Then, we derive a sequence of approximating sets for OSSs. Thus, the number of approximation sets of OSSs is 16. The initial state and its estimate are, respectively, selected as $x(0) = \begin{bmatrix} -20 & 10 \end{bmatrix}^T$ and $\hat{x}(0) = \begin{bmatrix} -10 & 7 \end{bmatrix}^T$.

In the simulation results, Fig. 2 plots the responses of system states and decoder states without control input. Fig. 3 depicts the response of system states with the N-step MPC approach via the observer-based endec-decoder scheme. It is easily seen from Fig. 3 that system states belonging to the initial feasible region are steered into the terminal constraint set quickly and stay inside. Fig. 4 shows the responses of system states, observer states and decoder states, respectively, and Fig. 5 draws the response of decoding errors, which effectively verifies the detectability and the asymptotical stability of the closed-loop system. This example demonstrates the effectiveness of the proposed endec-decoder scheme and the N-step MPC algorithm.

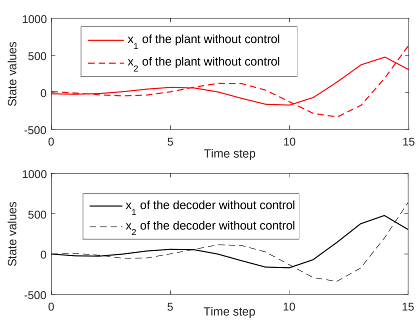

Fig. 2. Responses of plant states and decoder states without control.

6 Conclusion And Future Work

6.1 Conclusion

This paper has addressed the N-step MPC problem for a class of NCSs via a communication channel with limited bandwidths. In order to reduce the communication burden as well as guarantee the security of the data transmission, an endec-decoder scheme has been proposed in

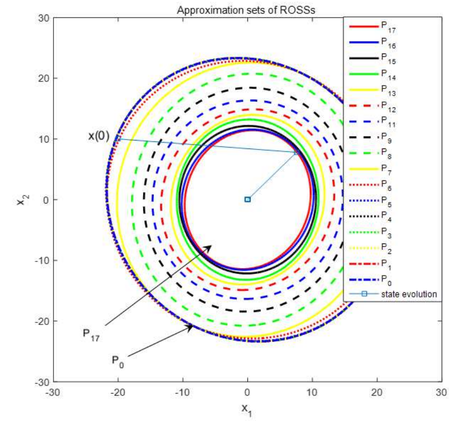

Fig. 3. Response of plant states.

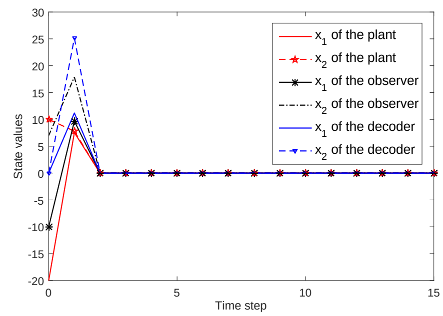

Fig. 4. Responses of plant states, observer states and decoder states, respectively.

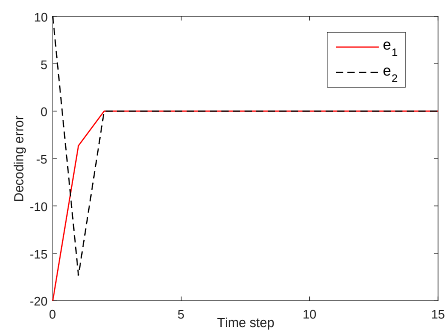

Fig. 5. Response of decoding errors.

the backward channel, where an auxiliary decoder with the same dynamic behavior as the decoder has been utilized in order to obtain the decoding value at the encoder side. By co-designing an observer-based endec-decoder scheme and an N-step MPC strategy, sufficient conditions have been established such that the underlying system is detectable and asymptotically stable. An algorithm including both offline and online parts has been presented to derive a set of desired control laws. In the end, an illustrative example has been utilized to demonstrate the effectiveness of the proposed controller design scheme.

6.2 Future work

In this paper, we have focused on the investigation of the MPC problems for NCSs subject to the limited amount (bits) of data transmission. In our future work, the research topics would include the extension of the main results to 1)the MPC problems of more general systems like uncertain stochastic nonlinear systems [2, 7, 26]; 2) the MPC problems of NCSs with network-induced phenomena like time delay and packet loss [15, 23, 28]; 3) the moving-horizon estimation problem of networked systems [45,46]; and 4) the improvement of the state estimation performance by using some latest optimization algorithms [19, 20].

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