SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

Unknown-input-observer-based approach to dynamic event-triggered fault estimation for Markovian jump systems with time-varying delay

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Abstract This paper investigates the unknown-input-observer-based fault estimation problem for a class of discrete-time-delay Markovian jump systems under the dynamic event-triggered transmission scheme. The dynamic event-triggered mechanism is used to decide whether the current information should be transmitted to the estimator or not to save the limited communication resources. This study aims to design an event-based fault estimator such that the estimation error is exponentially ultimately bounded in the mean square. By adopting the Lyapunov-Krasovskii functional approach, sufficient conditions are obtained to guarantee the existence of the desired estimator to achieve the prescribed performance requirement. The estimator gains are derived based on the convex optimization technique. A numerical example is provided to illustrate the effectiveness of the developed estimator design scheme.

Keywords fault estimation, unknown input observer, Markovian jump systems, dynamic event-triggered mechanism, time-varying delays

Citation Du X T, Zou L, Zhao Z Y, et al. Unknown-input-observer-based approach to dynamic event-triggered fault estimation for Markovian jump systems with time-varying delay. Sci China Inf Sci, for review

1 Introduction

Recent industrial systems are becoming more expensive and complex, leading to increasing demand for reliability and safety. The so-called faults, generated when the characteristic attribute or parameter of the system deviates from the standard condition, would potentially lead to a severe threat to the industrial systems reliability and safety. Thus, giving rise to fault diagnosis requirements issues (e.g., fault detection, fault estimation, and fault isolation). As an advanced fault diagnosis technique, the main idea of fault estimation is to estimate the size and type of the encountered fault signal. Such a technique is of great importance for online fault-tolerant control and real-time decision-making. In recent decades, the problem of fault estimation has attracted significant attention, and many studies have been reported in the literature, see e.g., [1–8].

Model-based fault estimation is one of the most investigated fault estimation techniques developed based on various observer techniques, such as the adaptive observer technique [9], sliding mode observer technique [10–12], and other observer-based techniques [13, 14]. Among these techniques, the unknown-input-observer-based (UIO-based) fault estimation is an effective scheme aimed to generate the desired estimates by decoupling the undesired disturbances/uncertainties from the estimation process, thereby, reducing/eliminating the effects of such disturbances/uncertainties on the estimation performance. So far, UIO-based fault estimation method has received research on systems subject to an unknown input (e.g., disturbances or uncertainties) [15, 16].

In practical applications, the dynamical behaviors of certain systems may suffer from switching changes, whose properties are often modeled by Markov processes. Markovian jump systems (MJSs) are usually used to model these systems. The corresponding research results have been widely applied in different

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engineering fields, such as power systems, communication systems, and aircraft control, see e.g., [17–22]. Compared with other systems without switching features, the fault estimation for MJSs is more challenging since the estimation performance depends on the switching behavior. It should be mentioned that Most of the existing results about MJSs consider the control and filtering problems, while the fault estimation issues of MJSs have gained little attention, that is due mainly to the difficulties in performance analysis on the fault estimation. Thus, the motivation of this study is to shorten such a gap.

Most existing fault estimation schemes are typically designed as the time-triggered ones, where the signal transmissions between the system and estimator are triggered once the measurements are generated by sensors, thereby, leading to a heavy communication load. An attractive alternative way is to design the fault estimation scheme based on an event-triggered nature, where the signals are only transmitted when necessary to achieve better resource utilization efficiency. Such kind of signal transmission scheme is called event-triggered transmission. It is an on-demand non-periodic signal transmission method to reduce the transmissions over the network while ensuring satisfactory system performance. The core idea of the event-triggered strategy is to design a reasonable event generator to determine whether the current signal sampled by sensors should be transmitted or not. Thus, the event-triggered transmission scheme is undoubtedly an effective solution to reduce energy consumption and has been widely used in the analysis/design of various systems. Recently, event-based fault diagnosis issues have attracted significant attention [23–30]. Among them, the event-triggered fault detection issues for nonlinear network systems with time-delay has been studied [23]. In [24], an event-triggered mechanism has been introduced to save resources in dealing with the problems of fault detection and fault isolation for discrete-time linear systems. In [25], an event-triggered mechanism has been comprehensively investigated on multi-target fault detection, isolation, and control.

It is worth noting that the static event-triggered mechanisms are widely used, where the triggering thresholds (or threshold parameters) are constant values. The static event-triggered mechanism could lead to unnecessary data transmissions. Recently, the so-called dynamic event-triggered mechanism has attracted significant attention [31–36]. Among them, the dynamic event-triggered mechanism has been introduced and studied for continuous-time nonlinear system [33]. It is shown in [34, 35] that the fault detection problems for network systems have been investigated. In [37], the triggering frequency under dynamic event-triggered mechanisms is significantly reduced compared with the static event-triggered case. Thus, energy utilization is efficient. The dynamic event-triggered mechanisms further reduce the system's triggering times and improve the energy utilization efficiency compared with the static event-triggered cases. Therefore, it is a natural idea to introduce the dynamic event-triggered mechanism into a broader application prospect. However, as far as we know, the dynamic event-triggered mechanism has not received enough attention to fault estimation issues.

Motivated by the above studies, we aim to address the UIO-based fault estimation problem for discrete-time MJSs with time-delay under dynamic event-triggered schemes. The main contributions of this paper are highlighted as follows:

(1) The fault estimation problem is, for the first time, investigated for discrete-time MJSs with time-varying delay under dynamic event-triggered schemes.

(2) A novel UIO-based fault estimator is designed to ensure satisfactory fault estimation performance subject to the unknown fault signal.

(3) Sufficient conditions are obtained to guarantee the existence of the desired estimator.

(4) The required estimator parameters are derived by solving a convex optimization problem.

2 Problem statement

Consider the following class of discrete-time-delay Markovian jump systems

$$\begin{cases} x_{k+1} = A_{\theta_k} x_k + A_{\tau\theta_k} x_{k-\tau_k} + B_{\theta_k}\omega_k + E_{\theta_k} f_k \\ y_k = C_{\theta_k} x_k + D_{\theta_k}\nu_k \\ x_k = \varphi_k, \ k \in [-\bar{\tau}, 0] \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the system state; $y_k \in \mathbb{R}^{n_y}$ denotes the measurement output; $\omega_k \in \mathbb{R}^{n_\omega}$ and $\nu_k \in \mathbb{R}^{n_\nu}$ represent the process and measurement noises, respectively; $f_k \in \mathbb{R}^{n_f}$ stands for the fault to be estimated; τ_k denotes the time-varying delay; φ_k is the initial condition in $[-\bar{\tau}, 0]$, where $\bar{\tau}$ is positive integer. It is assumed that the process noise ω_k and the measurement noise ν_k belong to $l_\infty[0,+\infty)$. $\{\theta_k\}$ is a discrete-time homogeneous Markov chain taking values in a finite state space $S=\{1,2,\cdots,N\}$ with a transition probability matrix $\Pi=[\pi_{ij}]$, for $\theta_k=i$, $\theta_{k+1}=j$, and one has

$$\pi_{ij}=\Pr\{\theta_{k+1}=j|\theta_k=i\}$$

where $\pi_{ij}\geqslant 0$, $\forall\, i,j\in S$ and $\sum_{j=1}^{N}\pi_{ij}=1$. We denote the matrices associated with $\theta_k=i\in S$ by

$$A_{\theta_k}\triangleq A_i, A_{\tau\theta_k}\triangleq A_{\tau i}, B_{\theta_k}\triangleq B_i,$$
$$E_{\theta_k}\triangleq E_i, C_{\theta_k}\triangleq C_i, D_{\theta_k}\triangleq D_i$$

where $A_i, A_{\tau i}, B_i, E_i, C_i, D_i$ are all known real constant matrices with appropriate dimensions.

Assumption 1. $\|\omega_k\|_\infty<d_1$, $\|\nu_k\|_\infty<d_2$, where d_1 and d_2 are known positive constants.

Assumption 2. $\mathrm{rank}(E_i)=\mathrm{rank}(C_iE_i)$, $\mathrm{rank}(E_i^TC_i^T,I)=\mathrm{rank}(E_i^TC_i^T)$.

Assumption 3. τ_k is time varying and satisfies $0<\underline{\tau}\leqslant\tau_k\leqslant\bar{\tau}$, where $\bar{\tau}$ and $\underline{\tau}$ are constant positive scalars representing the upper and lower bounds, respectively.

Remark 1. As is widely known, the phenomenon of time-delays happens in many real systems, and in the past few decades, a great number of results have been reported in the literature concerning the research on time-delay systems, see e.g. [38–46]. In this work, in order to make our research more general, we consider the fault estimation problem for time-delay systems. Note that the results of our work can be easily extended to the delay-free systems by setting $\bar{\tau}=\underline{\tau}=0$.

In this paper, the signal transmissions between the fault estimator and the plant are implemented via a communication network with limited network bandwidth. In order to avoid unnecessary signal transmissions and reduce the communication burden, in this work, the signal transmissions are implemented according to an event-triggered manner. A so-called dynamic event-triggered mechanism is put forward to cope with the problem, which would determine whether the current measurement signal should be transmitted or not. The triggering instants are determined the following triggering condition

$$t_{s+1}=\min\{k|k>t_s,\frac{1}{\theta}\eta_k+\sigma-\varepsilon_k^T\varepsilon_k\leqslant 0\} \tag{2}$$

where k is the sampling instant, $k=0,1,\ldots,\infty$; t_s and t_{s+1} are the triggering instants with $s=\{0,1,2,\ldots\}$ denoting the triggering period, and the initial triggering instant $t_0=0$; σ and θ are given event parameters, which are positive scalars; ε_k is defined by $\varepsilon_k\triangleq y_k-y_{t_s}$, which represents the error between the current sampled-data and the latest triggered sensor data; y_k and y_{t_s} represent the current sampled-data and the latest triggered one, respectively. Once the event-triggering condition (2) is satisfied, the event generator releases y_k to the communication channel and store y_k as $y_{t_{s+1}}$; otherwise, the corresponding sampled data packet is discarded purposely; η_k is an internal dynamical variable satisfying

$$\eta_{k+1}=\lambda\eta_k+\sigma-\varepsilon_k^T\varepsilon_k \tag{3}$$

where $\lambda\in(0,1)$ is a given constant and $\eta_0\geqslant 0$ stands for the initial condition.

Remark 2. We are aware that the triggering thresholds of the static event-triggered mechanisms are preset constant values, which probably cause unnecessary data processing. However, the thresholds of the dynamic event-triggered mechanisms can be dynamically adjusted according to the system. In this case, the triggering times can be further decreased on the basis of the static type so as to save more system resources. From (2), it is obvious that when the parameter θ tends to infinity, the triggering condition (3) becomes the static one.

Remark 3. According to the triggering schemes (2)-(3), we have $\frac{1}{\theta}\eta_k+\sigma-\varepsilon_k^T\varepsilon_k\geqslant 0$, $\eta_{k+1}\geqslant(\lambda-\frac{1}{\theta})\eta_k\geqslant\ldots\geqslant(\lambda-\frac{1}{\theta})^{k+1}\eta_0$. Obviously, the internal dynamic variable $\eta_k\geqslant 0$ for all $k\geqslant 0$ if $\eta_0>0$, $\lambda\theta\geqslant 1$, which guarantees $\sigma-\varepsilon_k^T\varepsilon_k\geqslant 0$. Therefore, the triggering times are reduced and the energy can be saved potentially.

Based on the transmission of the event-triggered mechanisms, the signal receiver can be modelled as follows

$$\bar{y}_k\triangleq y_{t_s}, k\in[t_s,t_{s+1}) \tag{4}$$

where $\bar{y}_k$ is latest available observation of fault estimator at time step k; t_s denotes the event-triggered transmission instant and $[t_s, t_{s+1})$ is the holding interval.

Based on this, an UIO-based fault estimator is constructed as follows

$$\begin{cases} z_{k+1} = F_i z_k + F_{\tau i} z_{k-\tau_k} + K_i \bar{y}_k + K_{\tau i} \bar{y}_{k-\tau_k} \\ \hat{x}_{k+1} = z_{k+1} + H_i \bar{y}_{k+1} \\ \hat{f}_k = M_i(\bar{y}_{k+1} - C_i A_i \hat{x}_k - C_i A_{\tau i} \hat{x}_{k-\tau_k}) \end{cases} \tag{5}$$

where $z_k \in \mathbb{R}^{n_z}$ is the filter state; $\bar{y}_k \in \mathbb{R}^{n_{\bar{y}}}$ denotes the received signal; $\hat{x}_k \in \mathbb{R}^{n_x}$ is the estimation of x_k; $\hat{f}_k$ stands for the fault estimation. The matrices F_i, $F_{\tau i}$, K_i, $K_{\tau i}$, H_i, M_i are the filter parameters with appropriate dimensions to be designed later.

Let $e_k = x_k - \hat{x}_k$. It follows from (1) and (5) that

$$\begin{aligned} e_{k+1} =& x_{k+1} - \hat{x}_{k+1} \\ =& x_{k+1} - (F_i z_k + F_{\tau i} z_{k-\tau_k} + K_i \bar{y}_k + K_{\tau i} \bar{y}_{k-\tau_k}) - H_i \bar{y}_{k+1} \\ =& x_{k+1} - F_i(\hat{x}_k - H_i \bar{y}_k) - F_{\tau i}(\hat{x}_{k-\tau_k} - H_i \bar{y}_{k-\tau_k}) - K_i \bar{y}_k - K_{\tau i} \bar{y}_{k-\tau_k} - H_i \bar{y}_{k+1} \\ =& x_{k+1} - F_i(x_k - e_k) + F_i H_i \bar{y}_k + F_{\tau i} H_i \bar{y}_{k-\tau_k} - F_{\tau i}(x_{k-\tau_k} - e_{k-\tau_k}) - K_i \bar{y}_k \\ & - K_{\tau i} \bar{y}_{k-\tau_k} - H_i \bar{y}_{k+1} \\ =& x_{k+1} - F_i x_k + F_i e_k + F_i H_i \bar{y}_k - F_{\tau i} x_{k-\tau_k} + F_{\tau i} e_{k-\tau_k} + F_{\tau i} H_i \bar{y}_{k-\tau_k} - K_i \bar{y}_k \\ & - K_{\tau i} \bar{y}_{k-\tau_k} - H_i \bar{y}_{k+1}. \end{aligned} \tag{6}$$

where $\bar{y}_k$ can be rewritten as

$$\bar{y}_k = y_k - \varepsilon_k. \tag{7}$$

Let

$$K_i = K_{i1} + K_{i2}, \quad K_{\tau i} = K_{\tau i1} + K_{\tau i2}. \tag{8}$$

Substituting (7)-(8) into (6) yields

$$\begin{aligned} e_{k+1} =& x_{k+1} - F_i x_k + F_i e_k + F_i H_i \bar{y}_k - F_{\tau i} x_{k-\tau_k} + F_{\tau i} e_{k-\tau_k} + F_{\tau i} H_i \bar{y}_{k-\tau_k} - (K_{i1} + K_{i2}) \\ & \times \bar{y}_k - (K_{\tau i1} + K_{\tau i2}) K_{\tau i} \bar{y}_{k-\tau_k} - H_i \bar{y}_{k+1} \\ =& F_i e_k + F_{\tau i} e_{k-\tau_k} + x_{k+1} - F_i x_k - F_{\tau i} x_{k-\tau_k} - K_{i1} \bar{y}_k - K_{\tau i1} \bar{y}_{k-\tau_k} + (F_i H_i - K_{i2}) \\ & \times \bar{y}_k + (F_{\tau i} H_i - K_{\tau i2}) \bar{y}_{k-\tau_k} - H_i \bar{y}_{k+1} \\ =& F_i e_k + F_{\tau i} e_{k-\tau_k} + x_{k+1} - F_i x_k - F_{\tau i} x_{k-\tau_k} - K_{i1}(y_k - \varepsilon_k) - K_{\tau i1}(y_{k-\tau_k} - \varepsilon_{k-\tau_k}) \\ & + (F_i H_i - K_{i2}) \bar{y}_k + (F_{\tau i} H_i - K_{\tau i2}) \bar{y}_{k-\tau_k} - H_i(y_{k+1} - \varepsilon_{k+1}). \end{aligned} \tag{9}$$

Observing (1), we can rewritten (9) as

$$\begin{aligned} e_{k+1} =& F_i e_k + F_{\tau i} e_{k-\tau_k} + x_{k+1} - F_i x_k - F_{\tau i} x_{k-\tau_k} - K_{i1}(C_i x_k + D_i \nu_k - \varepsilon_k) - K_{\tau i1} \\ & \times (C_i x_{k-\tau_k} + D_i \nu_{k-\tau_k} - \varepsilon_{k-\tau_k}) + (F_i H_i - K_{i2}) \bar{y}_k + (F_{\tau i} H_i - K_{\tau i2}) \bar{y}_{k-\tau_k} \\ & - H_i(C_i x_{k+1} + D_i \nu_{k+1} - \varepsilon_{k+1}) \\ =& F_i e_k + F_{\tau i} e_{k-\tau_k} + (I - H_i C_i)(A_i x_k + A_{\tau i} x_{k-\tau_k} + B_i \omega_k + E_i f_k) - F_i x_k \\ & - F_{\tau i} x_{k-\tau_k} - K_{i1} C_i x_k - K_{i1} D_i \nu_k + K_{i1} \varepsilon_k - K_{\tau i1} C_i x_{k-\tau_k} - K_{\tau i1} D_i \nu_{k-\tau_k} \\ & + K_{\tau i1} \varepsilon_{k-\tau_k} + (F_i H_i - K_{i2}) \bar{y}_k + (F_{\tau i} H_i - K_{\tau i2}) \bar{y}_{k-\tau_k} - H_i D_i \nu_{k+1} + H_i \varepsilon_{k+1} \\ =& F_i e_k + F_{\tau i} e_{k-\tau_k} + \big((I - H_i C_i) A_i - K_{i1} C_i - F_i\big) x_k + \big((I - H_i C_i) A_{\tau i} - K_{\tau i1} C_i \\ & - F_{\tau i}\big) x_{k-\tau_k} + (I - H_i C_i) B_i \omega_k + (I - H_i C_i) E_i f_k + (F_i H_i - K_{i2}) \bar{y}_k + (F_{\tau i} H_i \\ & - K_{\tau i2}) \bar{y}_{k-\tau_k} - K_{i1} D_i \nu_k - K_{\tau i1} D_i \nu_{k-\tau_k} - H_i D_i \nu_{k+1} + K_{i1} \varepsilon_k + K_{\tau i1} \varepsilon_{k-\tau_k} \\ & + H_i \varepsilon_{k+1}. \end{aligned} \tag{10}$$

Noting $H_i C_i E_i = E_i$, we further have

$$e_{k+1} = F_i e_k + F_{\tau i} e_{k-\tau_k} + \big((I - H_i C_i) A_i - K_{i1} C_i - F_i\big) x_k + \big((I - H_i C_i) A_{\tau i} - K_{\tau i1} C_i$$

$$- F_{\tau i})x_{k-\tau_k} + (I - H_iC_i)B_i\omega_k + (F_iH_i - K_{i2})\bar{y}_k + (F_{\tau i}H_i - K_{\tau i2})\bar{y}_{k-\tau_k} - K_{i1} \times D_i\nu_k - K_{\tau i1}D_i\nu_{k-\tau_k} - H_iD_i\nu_{k+1} + K_{i1}\varepsilon_k + K_{\tau i1}\varepsilon_{k-\tau_k} + H_i\varepsilon_{k+1}. \tag{11}$$

Thus, the state estimation error e_{k+1} is fully decoupled from fault f_k, which implies that the attenuation of disturbance ν_k and ω_k plays an important role in guaranteeing the estimation performance.

It is noted that, since $H_iC_iE_i = E_i$, the equation (10) can be solved when $\mathrm{rank}(C_iE_i) = \mathrm{rank}(E_i)$, the matrix H_i can be calculated by

$$H_i = E_i[(C_iE_i)^T(C_iE_i)]^{-1}(C_iE_i)^T. \tag{12}$$

In view of (1), (5) and (11), we have

$$\begin{aligned}\hat{f}_k &= M_i(y_{k+1} - \varepsilon_{k+1} - C_iA_i\hat{x}_k - C_iA_{\tau i}\hat{x}_{k-\tau_k})\\ &= M_i(C_ix_{k+1} + D_i\nu_{k+1} - \varepsilon_{k+1} - C_iA_i\hat{x}_k - C_iA_{\tau i}\hat{x}_{k-\tau_k})\\ &= M_i\big(C_i(A_ix_k + A_{\tau i}x_{k-\tau_k} + B_i\omega_k + E_if_k) + D_i\nu_{k+1} - \varepsilon_{k+1} - C_iA_i\hat{x}_k - C_iA_{\tau i}\hat{x}_{k-\tau_k}\big)\\ &= M_i(C_iA_ie_k + C_iA_{\tau i}e_{k-\tau_k} + C_iB_i\omega_k + C_iE_if_k + D_i\nu_{k+1} - \varepsilon_{k+1}).\end{aligned} \tag{13}$$

In the case of $M_iC_iE_i = I$, we rewrite $\hat{f}_k$ as

$$\hat{f}_k = M_iC_iA_ie_k + M_iC_iA_{\tau i}e_{k-\tau_k} + M_iC_iB_i\omega_k + f_k + M_iD_i\nu_{k+1} - M_i\varepsilon_{k+1}. \tag{14}$$

Let $\widetilde{f}_k = \hat{f}_k - f_k$. We have

$$\widetilde{f}_k = M_iC_iA_ie_k + M_iC_iA_{\tau i}e_{k-\tau_k} + M_iC_iB_i\omega_k + M_iD_i\nu_{k+1} - M_i\varepsilon_{k+1}. \tag{15}$$

It is easy to find from our developed filter (5) that the estimation error on the fault signal is independent on f_k by setting $M_iC_iE_i = I$. The rationality of such a setting is to guarantee that the fault estimation performance is completely unimpeded by the fault signal (which might be unbounded). Note that the condition $M_iC_iE_i = I$ holds if and only if $\mathrm{rank}(E_i^TC_i^T, I) = \mathrm{rank}(E_i^TC_i^T)$. As such, according to Assumption 1, the value of the matrix M_i can be calculated by

$$M_i = [(C_iE_i)^T(C_iE_i)]^{-1}(C_iE_i)^T. \tag{16}$$

Moreover, the interpretation of the exponentially ultimate bound is introduced in the following definition.

Definition 1. The solution of the system (10) is said to be exponentially ultimately bounded in mean square if there exist constants $\alpha > 0$, $0 < \beta < 1$ and $\bar{l} > 0$ such that

$$\mathbb{E}\{\|e_k\|^2\} \leqslant \alpha\beta^k + l_k, \text{ and } \lim_{k\to+\infty} l_k = \bar{l}. \tag{17}$$

It is obvious that, with Assumption 1, the fault estimation error $\tilde{f}_k$ is bounded if and only if e_k is bounded. At this regard, the main objective of this paper is to design a fault estimator in the form of (5) such that the estimation error is exponentially ultimately bounded and the design of the estimator gains satisfy (17).

3 Main results

In this section, sufficient conditions are established to guarantee the stability and exponentially ultimately bounded constraints on the estimation error of state and fault. Before proceeding further, the following parameters are introduced firstly.

Consider the estimator (5) for the system (1) with fault and event-triggered measurements. If (8) and the following relationships hold

$$\begin{cases} E_i = H_iC_iE_i\\ T_i = I - H_iC_i\\ F_i = A_i - H_iC_iA_i - K_{i1}C_i\\ F_{\tau i1} = A_{\tau i} - H_iC_iA_{\tau i} - K_{\tau i1}C_i\\ K_{i2} = F_iH_i\\ K_{\tau i2} = F_{\tau i}H_i \end{cases} \tag{18}$$

then the state error estimation (11) reduces to

$$\begin{aligned} e_{k+1} =& F_i e_k + F_{\tau i} e_{k-\tau_k} + T_i B_i \omega_k - K_{i1} D \nu_k - K_{\tau i1} D_i \nu_{k-\tau_k} - H_i D_i \nu_{k+1} + K_{i1}\varepsilon_k \\ &+ K_{\tau i1}\varepsilon_{k-\tau_k} + H_i \varepsilon_{k+1} \end{aligned} \tag{19}$$

where F_i, $F_{\tau i}$, T_i, K_{i1}, $K_{\tau i1}$, H_i are parameters to be designed.

Theorem 1. Consider the system (1) with the given parameters λ, θ of the dynamical event-triggered mechanism (2)-(3) and the gains K_i $(i = 1, 2, \cdots, N)$. The estimation error system (15) is exponentially ultimately bounded if there exist positive scalars p, q, positive definite matrices $P_i \in \mathbb{R}^{n_p \times n_p}$ $(i = 1, 2, \cdots, N)$, $Q \in \mathbb{R}^{n_q \times n_q}$, and positive scalars κ_j $(j = 1, 2, \cdots, 5)$ satisfying

$$\Gamma_i \triangleq \begin{bmatrix} \Sigma_i & \Phi_i \\ * & -\bar{P}_i \end{bmatrix} < 0, \quad i = 1, 2, \cdots, N \tag{20}$$

where

$$\Sigma_i \triangleq \begin{bmatrix} -P_i + (1+\bar{\tau}+\underline{\tau})Q & 0 & 0 & 0 & 0 \\ * & -Q & 0 & 0 & 0 \\ * & * & \Lambda_{11} & 0 & 0 \\ * & * & * & \Lambda_{22} & 0 \\ * & * & * & * & \frac{\lambda-q}{\theta} I \end{bmatrix}$$

$$\Lambda_{11} \triangleq \mathrm{diag}\{-\kappa_1 I, -\kappa_2 I, -\kappa_3 I, -\kappa_4 I, -\frac{p+\kappa_5}{\theta} I, -\frac{1}{\theta} I, -\kappa_5 I\},\ \Lambda_{22} \triangleq \frac{\kappa_5\lambda + p(\lambda-1) + (1+\bar{\tau}-\underline{\tau})q}{\theta} I$$

$$\Phi_i \triangleq \begin{bmatrix} \Phi_{i1} & \Phi_{i2} \end{bmatrix}^T,\ \Phi_{i1} \triangleq \begin{bmatrix} \bar{P}_i F_i & \bar{P}_i F_{\tau i} & \bar{P}_i T_i B_i & -\bar{P}_i K_{i1} D_i & -\bar{P}_i K_{\tau i1} D_i & -\bar{P}_i H_i D_i \end{bmatrix}$$

$$\Phi_{i2} \triangleq \begin{bmatrix} -\bar{P}_i K_{i1} & -\bar{P}_i K_{\tau i1} & \bar{P}_i H_i & 0 & 0 \end{bmatrix},\ \bar{P}_i \triangleq \sum_{j=1}^{N} \pi_{ij} P_j,$$

$$\rho \triangleq \frac{\sigma}{\theta}\Big(p + 1 + \kappa_5(1+\theta)\Big) + \kappa_1 d_1^2 n_\omega + (\kappa_2+\kappa_3+\kappa_4) d_2^2 n_\nu.$$

Besides, if the inequality (20) holds, then, the ultimate bound of the error (19) can be given as

$$\bar{l} \triangleq \frac{u_0}{b_5(u_0-1)}\rho \tag{21}$$

with

$$\begin{aligned} & a \triangleq \lambda_{\max}(\Sigma_i + \Phi_i \bar{P}_i^{-1} \Phi_i^T),\ b_1 \triangleq \max\{(1+\bar{\tau}-\underline{\tau})\lambda_{\max}(Q), (1+\bar{\tau}-\underline{\tau})q\} \\ & b_2 \triangleq \max\{\lambda_{\max}(P_i), \frac{p}{\theta}\},\ b_3 \triangleq ua + (u-1)b_2 + b_1\bar{\tau}u^{\bar{\tau}} \\ & b_4 \triangleq \max\{b_1(1+\bar{\tau}), b_2(1+\bar{\tau})\},\ b_5 \triangleq \min\{\lambda_{\min}(P_i), \frac{p}{\theta}\} \\ & u_0 a + (u_0-1)b_2 + b_1\bar{\tau}u^{\bar{\tau}} = 0. \end{aligned}$$

Proof. Choose the following candidate functional

$$\begin{aligned} V_k &\triangleq V_{1,k} + V_{2,k} \\ V_{1,k} &\triangleq e_k^T P_{\theta_k} e_k + \sum_{l=k-\tau_k}^{k-1} e_l^T Q e_l + \sum_{m=-\bar{\tau}+1}^{-\underline{\tau}} \sum_{l=k+m}^{k-1} e_l^T Q e_l \\ V_{2,k} &\triangleq \frac{1}{\theta}(p\eta_k + \sum_{l=k-\tau_k}^{k-1} q\eta_l + \sum_{m=-\bar{\tau}+1}^{-\underline{\tau}} \sum_{l=k+m}^{k-1} q\eta_l) \end{aligned} \tag{22}$$

The difference ΔV_k of the system (1) is calculated as follows

$$
\begin{aligned}
E\{\Delta V_{1,k}|\theta_k=i\} &= E\{V_{1,k+1}-V_{1,k}|\theta_k=i\} \\
&= E\{e_{k+1}^T P_{\theta_{k+1}} e_{k+1} - e_k^T P_{\theta_k} e_k + \sum_{l=k+1-\tau_{k+1}}^{k} e_l^T Q e_l - \sum_{l=k-\tau_k}^{k-1} e_l^T Q e_l \\
&\quad + \sum_{m=-\bar{\tau}+1}^{-\underline{\tau}} \sum_{l=k+1+m}^{k} e_l^T Q e_l - \sum_{m=-\bar{\tau}+1}^{-\underline{\tau}} \sum_{l=k+m}^{k-1} e_l^T Q e_l |\theta_k=i\} \\
&= e_{k+1}^T (\sum_{j=1}^{N} \pi_{ij} P_j) e_{k+1} - e_k^T P_i e_k + e_k^T Q e_k + \sum_{l=k+1-\tau_{k+1}}^{k-1} e_l^T Q e_l \\
&\quad - \sum_{l=k+1-\tau_k}^{k-1} e_l^T Q e_l - e_{k-\tau_k}^T Q e_{k-\tau_k} + \sum_{m=-\bar{\tau}+1}^{-\underline{\tau}} (e_k^T Q e_k - e_{k+m}^T Q e_{k+m}) \\
&\leqslant e_{k+1}^T (\sum_{j=1}^{N} \pi_{ij} P_j) e_{k+1} - e_k^T P_i e_k + e_k^T Q e_k + \sum_{l=k+1-\bar{\tau}}^{k-1} e_l^T Q e_l \\
&\quad - \sum_{l=k+1-\underline{\tau}}^{k-1} e_l^T Q e_l - e_{k-\tau_k}^T Q e_{k-\tau_k} + \sum_{l=k-\bar{\tau}+1}^{k-\underline{\tau}} (e_k^T Q e_k - e_l^T Q e_l) \\
&= e_{k+1}^T \bar{P}_i e_{k+1} - e_k^T P_i e_k + e_k^T Q e_k - e_{k-\tau_k}^T Q e_{k-\tau_k} + \sum_{l=k+1-\bar{\tau}}^{k-\underline{\tau}} e_l^T Q e_l \\
&\quad + (\bar{\tau}-\underline{\tau}) e_k^T Q e_k - \sum_{l=k-\bar{\tau}+1}^{k-\underline{\tau}} e_l^T Q e_l \\
&= e_{k+1}^T \bar{P}_i e_{k+1} + e_k^T \big(-P_i + (1+\bar{\tau}-\underline{\tau})Q\big) e_k - e_{k-\tau_k}^T Q e_{k-\tau_k}.
\end{aligned} \tag{23}
$$

$$
\begin{aligned}
E\{\Delta V_{2,k}|\theta_k=i\} &= E\{V_{2,k+1}-V_{2,k}|\theta_k=i\} \\
&= \frac{1}{\theta}(p\eta_{k+1} - p\eta_k + \sum_{l=k+1-\tau_{k+1}}^{k} q\eta_l - \sum_{l=k-\tau_k}^{k-1} q\eta_l + \sum_{m=-\bar{\tau}+1}^{-\underline{\tau}} \sum_{l=k+1+m}^{k} q\eta_l \\
&\quad - \sum_{m=-\bar{\tau}+1}^{-\underline{\tau}} \sum_{l=k+m}^{k-1} q\eta_l) \\
&= \frac{1}{\theta}\big(p\eta_{k+1} - p\eta_k + q\eta_k + \sum_{l=k+1-\tau_{k+1}}^{k-1} q\eta_l - q\eta_{k-\tau_k} - \sum_{l=k+1-\tau_k}^{k-1} q\eta_l \\
&\quad + \sum_{m=-\bar{\tau}+1}^{-\underline{\tau}} (q\eta_k - q\eta_{k+m})\big) \\
&\leqslant \frac{1}{\theta}\big(p\eta_{k+1} - p\eta_k + q\eta_k - q\eta_{k-\tau_k} + \sum_{l=k+1-\bar{\tau}}^{k-1} q\eta_l - \sum_{l=k+1-\underline{\tau}}^{k-1} q\eta_l \\
&\quad + (\bar{\tau}-\underline{\tau}) q\eta_k - \sum_{l=k+1-\bar{\tau}}^{k-\underline{\tau}} q\eta_l\big) \\
&= \frac{1}{\theta}\big(p\eta_{k+1} - p\eta_k + q\eta_k - q\eta_{k-\tau_k} + (\bar{\tau}-\underline{\tau}) q\eta_k\big) \\
&\leqslant \frac{1}{\theta}\big(p(\lambda\eta_k + \sigma - \varepsilon_k^T \varepsilon_k) - p\eta_k + (1+\bar{\tau}-\underline{\tau}) q\eta_k - q\eta_{k-\tau_k} \\
&\quad + (\lambda\eta_{k-\tau_k} + \sigma - \varepsilon_{k-\tau_k}^T \varepsilon_{k-\tau_k})\big)
\end{aligned}
$$

$$= q_1\eta_k + q_2\eta_{k-\tau_k} - \frac{p}{\theta}\varepsilon_k^T\varepsilon_k - \frac{1}{\theta}\varepsilon_{k-\tau_k}^T\varepsilon_{k-\tau_k} + \frac{p+1}{\theta}\sigma \tag{24}$$

where

$$q_1 = \frac{p(\lambda-1)+(1+\bar{\tau}-\underline{\tau})q}{\theta},\ q_2 = \frac{\lambda-q}{\theta}.$$

Furthermore, by Assumption 1, the inequalities $\|\omega_k\|_\infty < d_1$, $\|\nu_k\|_\infty < d_2$ can be rewritten as

$$\begin{cases} \kappa_1(d_1^2 n_\omega - \omega_k^T\omega_k) \geqslant 0 \\ \kappa_2(d_2^2 n_\nu - \nu_k^T\nu_k) \geqslant 0 \\ \kappa_3(d_2^2 n_\nu - \nu_{k-\tau_k}^T\nu_{k-\tau_k}) \geqslant 0 \\ \kappa_4(d_2^2 n_\nu - \nu_{k+1}^T\nu_{k+1}) \geqslant 0, \end{cases} \tag{25}$$

and according the triggering conditions (2) and (3), one has

$$\begin{aligned} \kappa_5(\frac{1}{\theta}\eta_{k+1} + \sigma - \varepsilon_{k+1}^T\varepsilon_{k+1}) &= \kappa_5(\frac{1}{\theta}(\lambda\eta_k + \sigma - \varepsilon_k^T\varepsilon_k) + \sigma - \varepsilon_{k+1}^T\varepsilon_{k+1}) \\ &= \kappa_5(\frac{\lambda}{\theta}\eta_k - \frac{1}{\theta}\varepsilon_k^T\varepsilon_k - \varepsilon_{k+1}^T\varepsilon_{k+1} + (1+\frac{1}{\theta})\sigma) > 0. \end{aligned} \tag{26}$$

Considering (23)-(26) and noticing (19), it can be known that

$$\begin{aligned} E\{\Delta V_k|\theta_k = i\} \leqslant\ & (F_i e_k + F_{\tau i} e_{k-\tau_k} + T_i B_i \omega_k - K_{i1} D_i \nu_k - K_{\tau i1} D_i \nu_{k-\tau_k} - H_i D_i \nu_{k+1} \\ & + K_{i1}\varepsilon_k + K_{\tau i1}\varepsilon_{k-\tau_k} + H_i\varepsilon_{k+1})^T \bar{P}_i (F_i e_k + F_{\tau i} e_{k-\tau_k} + T_i B_i \omega_k \\ & - K_{i1} D_i \nu_k - K_{\tau i1} D_i \nu_{k-\tau_k} - H_i D_i \nu_{k+1} + K_{i1}\varepsilon_k + K_{\tau i1}\varepsilon_{k-\tau_k} + H_i\varepsilon_{k+1}) \\ & + e_k^T\big(-P_i + (1+\bar{\tau}-\underline{\tau})Q\big)e_k - e_{k-\tau_k}^T Q e_{k-\tau_k} \\ & + q_1\eta_k + q_2\eta_{k-\tau_k} - \frac{p}{\theta}\varepsilon_k^T\varepsilon_k - \frac{1}{\theta}\varepsilon_{k-\tau_k}^T\varepsilon_{k-\tau_k} + \frac{p+1}{\theta}\sigma \\ & + \kappa_1(d_1^2 n_\omega - \omega_k^T\omega_k) + \kappa_2(d_2^2 n_\nu - \nu_k^T\nu_k) + \kappa_3(d_2^2 n_\nu - \nu_{k-\tau_k}^T\nu_{k-\tau_k}) \\ & + \kappa_4(d_2^2 n_\nu - \nu_{k+1}^T\nu_{k+1}) + \kappa_5(\frac{\lambda}{\theta}\eta_k - \frac{1}{\theta}\varepsilon_k^T\varepsilon_k - \varepsilon_{k+1}^T\varepsilon_{k+1} + (1+\frac{1}{\theta})\sigma) \\ =\ & {\xi_k}^T(\Sigma_i + \Phi_i\bar{P}_i^{-1}\Phi_i^T)\xi_k + \frac{\sigma}{\theta}\Big(p+1+\kappa_5(1+\theta)\Big) \\ & + \kappa_1 d_1^2 n_\omega + (\kappa_2+\kappa_3+\kappa_4)d_2^2 n_\nu \\ =\ & {\xi_k}^T(\Sigma_i + \Phi_i\bar{P}_i^{-1}\Phi_i^T)\xi_k + \rho \end{aligned} \tag{27}$$

where

$$\begin{aligned} &\xi_k \triangleq \begin{bmatrix}\xi_{1,k} & \xi_{2,k}\end{bmatrix}^T \\ &\xi_{1,k} \triangleq \begin{bmatrix} e_k^T & e_{k-\tau_k}^T & \omega_k^T & \nu_k^T & \nu_{k-\tau_k}^T & \nu_{k+1}^T & \varepsilon_k^T & \varepsilon_{k-\tau_k}^T & \varepsilon_{k+1}^T \end{bmatrix},\ \xi_{2,k} \triangleq \begin{bmatrix} \eta_k^{\frac{1}{2}} & \eta_{k-\tau_k}^{\frac{1}{2}} \end{bmatrix}. \end{aligned}$$

By the Schur complement lemma, the inequality (20) is true if and only if the following condition holds

$$\Sigma_i + \Phi_i{\bar{P}_i}^{-1}{\Phi_i}^T < 0. \tag{28}$$

so we further have

$$E\{\Delta V_k|\theta_k = i\} \leqslant a\|\phi_k\|^2 + \rho \tag{29}$$

where

$$\phi_k \triangleq \begin{bmatrix} e_k^T & \eta_k^{\frac{1}{2}} \end{bmatrix}^T. \tag{30}$$

From (22), it can be obtained that

$$V_k \leqslant (1+\bar{\tau}+\underline{\tau})\lambda_{\max}(Q)\sum_{l=k-\bar{\tau}}^{k-1}\|e_l\|^2+\lambda_{\max}(P_i)\|e_k\|^2+(1+\bar{\tau}+\underline{\tau})q\sum_{l=k-\bar{\tau}}^{k-1}\eta_l+\frac{p}{\theta}\eta_k$$
$$= b_1\sum_{l=k-\bar{\tau}}^{k-1}\|\phi_l\|^2+b_2\|\phi_k\|^2. \tag{31}$$

For a given scalar $u>1$, we have

$$\begin{aligned} u^{k+1}V_{k+1}-u^kV_k &= u^{k+1}(V_{k+1}-V_k)+(u-1)u^kV_k \\ &\leqslant u^{k+1}(a\|\phi_k\|^2+\rho)+(u-1)u^k(b_1\sum_{l=k-\bar{\tau}}^{k-1}\|\phi_l\|^2+b_2\|\phi_k\|^2) \\ &= u^k\big(ua+(u-1)b_2\big)\|\phi_k\|^2+(u-1)u^kb_1\sum_{l=k-\bar{\tau}}^{k-1}\|\phi_l\|^2+u^{k+1}\rho. \end{aligned} \tag{32}$$

Let $s\geqslant\bar{\tau}+1$ be a positive integer. Summing up k from 0 to $s-1$ on both sides of the (32), we have

$$\begin{aligned} \sum_{k=1}^{s-1}E\{u^{k+1}V_{k+1}-u^kV_k|\theta_k=i\} = \big(ua+(u-1)b_2\big)\sum_{k=1}^{s-1}u^k\|\phi_k\|^2+(u-1)b_1 \\ \times\sum_{k=1}^{s-1}\sum_{l=k-\bar{\tau}}^{k-1}u^k\|\phi_l\|^2+\sum_{k=1}^{s-1}u^{k+1}\rho. \end{aligned} \tag{33}$$

By simplifing (33), one has

$$\begin{aligned} u^sV_s-V_0 &\leqslant \big(ua+(u-1)b_2\big)\sum_{k=1}^{s-1}u^k\|\phi_k\|^2+b_1\frac{\bar{\tau}u^{\bar{\tau}}}{u-1}\max_{-\bar{\tau}\leqslant l\leqslant 0}\|\phi_l\|^2(u-1) \\ &\quad +(u-1)b_1\bar{\tau}u^{\bar{\tau}}\sum_{l=0}^{s-1}u^l\|\phi_l\|^2+\frac{u(1-u^s)}{1-u}\rho \\ &= b_1\bar{\tau}u^{\bar{\tau}}\max_{-\bar{\tau}\leqslant l\leqslant 0}\|\phi_l\|^2+b_3\sum_{k=0}^{s-1}u^k\|\phi_k\|^2+\frac{u(1-u^s)}{1-u}\rho \end{aligned} \tag{34}$$

where $b_3\triangleq ua+(u-1)(b_2+b_1\bar{\tau}u^{\bar{\tau}})$.

When $u=1$, $b_3=a<0$, and $u=+\infty$, $b_3=+\infty$, we can acquire a scalar $u_0>1$ such that $b_3=0$. Thus, we eventually obtain

$$u_0{}^sV_s-V_0\leqslant b_1\bar{\tau}u_0^{\bar{\tau}}\max_{-\bar{\tau}\leqslant l\leqslant 0}\|\phi_l\|^2+\frac{u_0(1-u_0^s)}{1-u_0}\rho. \tag{35}$$

From (23), it is obvious that

$$V_0 \leqslant b_1\sum_{l=-\bar{\tau}}^{-1}\|\phi_l\|^2+b_2\|\phi_0\|^2\leqslant b_4\max_{-\bar{\tau}\leqslant l\leqslant 0}\|\phi_l\|^2 \tag{36}$$

where $b_4\triangleq\max\{(\bar{\tau}+1)b_1,(\bar{\tau}+1)b_2\}$. Moreover, according to the definition of the V_k one has

$$V_s \geqslant b_5\|\phi_s\|^2 \tag{37}$$

where $b_5 \triangleq \min\{\lambda_{\min}(P_i), \frac{p}{\theta}\}$. Substituting (36) and (37) into (35), we have

$$\|\phi_s\|^2 \leqslant \frac{b_1\bar{\tau}u_0^{\bar{\tau}} + b_4}{b_5 u_0^s} \max_{-\bar{\tau}\leqslant l\leqslant 0} \|\phi_l\|^2 + \frac{u_0(1-u_0^s)}{b_5 u_0^s(1-u_0)}\rho. \tag{38}$$

Finally, we have

$$\mathbb{E}\{\|e(k)\|^2\} \leqslant \|\phi_k\|^2 \leqslant \alpha\beta^k + l_k, \text{ and } \lim_{k\to+\infty} l_k = \bar{l}. \tag{39}$$

By Definition 1, it is obvious that the error (19) is exponentially ultimately bounded when letting $\alpha = \frac{b_1\bar{\tau}u_0^{\bar{\tau}}+b_4}{b_5}$, $\beta = \frac{1}{u_0}$, $l_k = \frac{u_0(1-u_0^s)}{b_5u_0^s(1-u_0)}\rho$. Besides, the ultimate bound is

$$\bar{l} = \lim_{k\to+\infty} l_k = \frac{u_0}{b_5(u_0-1)}\rho. \tag{40}$$

So far, e_k has been proved exponentially ultimately bounded already.

For (15), it is easy to see that $\tilde{f}_k$ is relevant with e_k, $e_{k-\tau_k}$, ω_k, ν_{k+1} and ε_{k+1}. Furthermore, note that ω_k and v_{k+1} are bounded, which implies that if the dynamics of e_k, $e_{k-\tau_k}$ and ε_{k+1} are exponentially ultimately bounded, then the dynamics of $\tilde{f}_k$ will be also exponentially ultimately bounded.

Furthermore, from (31) and (39) it is easy to see that V_k and e_k are exponentially ultimately bounded. Considering the dynamical function (2) we have introduced yet, one has

$$\frac{1}{\theta}\eta_k + \sigma - \varepsilon_k^T\varepsilon_k \geqslant 0 \tag{41}$$

i.e.,

$$\varepsilon_k^T\varepsilon_k \leqslant \frac{1}{\theta}\eta_k + \sigma. \tag{42}$$

According to (22), it is obvious that $\frac{1}{\theta}\eta_k$ is exponentially ultimately bounded, for (42) we know that ε_k is exponentially ultimately bounded. Thus, we have proved that the estimation error of fault $\widetilde{f}_k$ is exponentially ultimately bounded. The proof is now complete.

Based on the performance analysis in Theorem 1, the design method of the observer will be given in the next theorem.

Theorem 2. Consider the system (1) with the given parameters λ, θ of the dynamical event-triggered mechanism (2)-(3). The error dynamics (15) is exponentially ultimately bounded if there exist positive scalars p, q, positive definite matrices P_i $(i = 1, 2, \cdots, N)$, Q, matrices $X_i, X_{\tau i}$ $(i = 1, 2, \cdots, N)$ and positive scalars κ_j $(j = 1, 2, \cdots, 5)$ satisfying

$$\Xi_i \triangleq \begin{bmatrix} \Sigma_i & \Omega_i \\ * & -\bar{P}_i \end{bmatrix} < 0, \quad i = 1, 2, \cdots, N \tag{43}$$

where

$$\Omega_i \triangleq \begin{bmatrix} \Omega_{i1} & \Omega_{i2} \end{bmatrix}^T, \quad \bar{P}_i \triangleq \sum_{j=1}^{N} \pi_{ij}P_j,$$
$$\Omega_{i1} \triangleq \begin{bmatrix} (\bar{P}_iF_i)^T & (\bar{P}_iF_{\tau i})^T & (\bar{P}_iT_iB_i)^T & -(X_iD_i)^T & -(X_{\tau i}D_i)^T & -(\bar{P}_iH_iD_i)^T \end{bmatrix},$$
$$\Omega_{i2} \triangleq \begin{bmatrix} X_i^T & X_{\tau i}^T & (\bar{P}_iH_i)^T & 0 & 0 \end{bmatrix},$$

and Σ_i is given in Theorem 1. In this case, the required observer parameters are given by

$$K_{i1} = P_i^{-1}X_i, \quad K_{\tau i1} = P_i^{-1}X_{\tau i}, \ i = 1, 2, \cdots, N. \tag{44}$$

Proof. By adopting the Schur Complement Lemma and letting $P_iK_{i1} \triangleq X_i$, $P_iK_{\tau i1} \triangleq X_{\tau i}$, we can see that (20) holds if and only if (43) is fulfilled. The proof is now completed.

Corollary 1. For given parameters λ, θ of the dynamical event-triggered mechanism (2)-(3), the error dynamics (15) is exponentially ultimately bounded if there exist positive scalars p, q, positive definite matrices P_i $(i = 1, 2, \cdots, N)$, Q, matrices $X_i, X_{\tau i}$ $(i = 1, 2, \cdots, N)$ and positive scalars κ_j $(j = 1, 2, \cdots, 5)$ such that (43) holds. Moreover, the minimum of the asymptotic upper bound of $\mathbb{E}\{\|e_k\|^2\}$ can be obtained by solving the following optimization problem

$$\min\left\{\frac{\sigma}{\theta}\Big(p + 1 + \kappa_5(1+\theta)\Big) + \kappa_1 d_1^2 n_\omega + (\kappa_2 + \kappa_3 + \kappa_4) d_2^2 n_\nu\right\} \tag{45}$$

subject to (43).

Proof. According to Theorem 2, the results of this corollary are obvious, and thus the corresponding proof is omitted.

Until now, a desired observer is designed in Theorem 2 such that the system (5) achieves exponentially ultimately bound under the dynamic event-triggered mechanism (2)-(3).

4 Simulation results

In this section, a numerical example is given to demonstrate the correctness and effectiveness of our developed estimator approach.

Example: Consider a class of discrete-time-delay Markovian jump system described by (1) with the following parameters:

$$\begin{aligned}
A_1 &= \begin{bmatrix} 0.1 & 0.2 \\ 0.5 & -0.1 \end{bmatrix}, A_{\tau 1} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 & -0.3 \end{bmatrix}, C_1 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 0.73 & 0 \\ 0 & -0.2 \end{bmatrix}, A_{\tau 2} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2 & -0.4 \end{bmatrix}, C_2 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \\
E_1 &= \begin{bmatrix} 2 & 2 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 1.5 \end{bmatrix}, D_1 = 0.1, D_2 = 0.2.
\end{aligned}$$

Suppose that the discrete time homogeneous Markov chain takes values in a finite state space $S = \{1, 2\}$ with a transition probability matrix

$$\Pi = \begin{bmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{bmatrix}.$$

The time-varying delay is taken as $\tau_k = 2 + \sin(k\pi)$, from which we have $\bar{\tau} = 3$ and $\underline{\tau} = 1$. The noises are given as $\omega_k = 0.1\sin(0.5k)$ and $\nu_k = 0.1\sin(0.5k)$, respectively. For the dynamic triggering conditions (2) and (3), the initial value of the internal dynamic variable is set as $\eta_0 = 0$, and the threshold is chosen as $\sigma = 0.1$ and other parameters are taken as $\lambda = 0.1$, $\theta = 20$.

In the simulation, we consider the following piecewise fault:

$$f_k = \begin{cases} 2.5\sin(k), & 10 \leqslant k < 20 \\ f_{k-1} + 0.8\sin(1.5k), & 20 \leqslant k < 30 \\ f_{k-1} - 1, & 30 \leqslant k < 40 \\ 0, & \text{otherwise.} \end{cases}$$

By utilizing the MATLAB LMI Toolbox, a set of feasible solutions of the inequalities (20), (43) are acquired as follows

$$\begin{aligned}
K_1 &= \begin{bmatrix} -0.1875 & 0.0625 \end{bmatrix}, K_2 = \begin{bmatrix} 1.7438 & -0.8719 \end{bmatrix}, \\
K_{\tau 1} &= \begin{bmatrix} -0.0188 & 0.0063 \end{bmatrix}, K_{\tau 2} = \begin{bmatrix} -0.0187 & 0.0062 \end{bmatrix}.
\end{aligned}$$

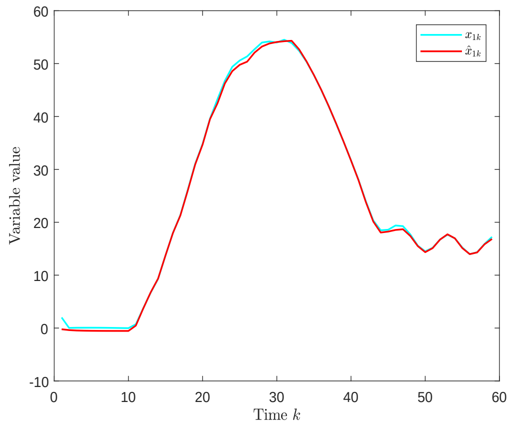

Figure 1 The trajectories of x_{1k} and $\hat{x}_{1k}$

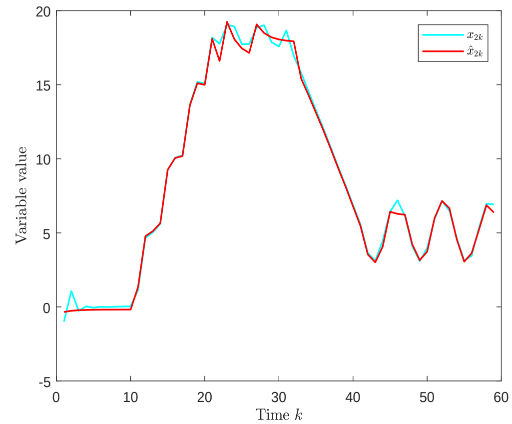

Figure 2 The trajectories of x_{2k} and $\hat{x}_{2k}$

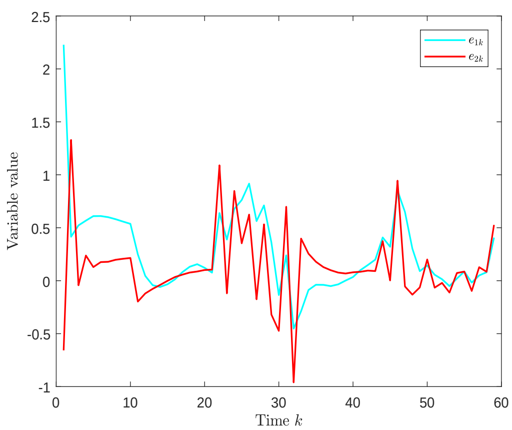

Figure 3 The trajectories of the state estimation error e_{1k} and e_{2k}

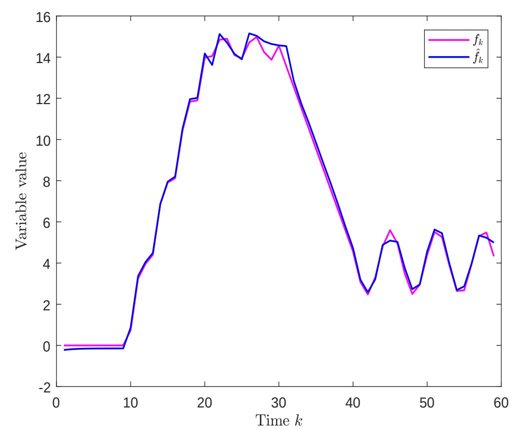

Figure 4 The trajectories of f_k and $\hat{f}_k$

Set the simulation runs length to be 60. Based on the derived observer gain matrices, simulation results are shown in Figures 1- 6. Figures 1- 2 concern the state trajectories and their estimates, respectively. Figure 3 plots the estimation error trajectory of the state. Figure 4 shows the fault trajectory and its estimate. Figure 5 displays the estimation error trajectory of the fault. All simulation results confirm that the estimation performance is well achieved.

In order to show the superiority of the dynamic event-triggered mechanisms, the dynamic triggering results are shown in Figure 6. From Figure 6, it can be seen that, by introducing the dynamic event-triggered mechanism, the information transmission is reduced and network resources are saved effectively.

5 Conclusion

In this paper, the UIO-based fault estimation problem has been addressed for a class of discrete MJSs with time-varying delay subject to the dynamic event-triggered schemes. A discrete-time version dynamic event-triggering mechanism has been proposed to save energy. The fault estimator has been constructed based on the UIO method. By adopting an appropriate Lyapunov-Krasovskii functional, sufficient conditions have been established for the desired estimators to guarantee exponentially ultimate bound on the estimation error. The estimator gains have been calculated by solving a set of matrix inequalities. Finally, numerical examples have been provided to illustrate the correctness and effectiveness of the proposed

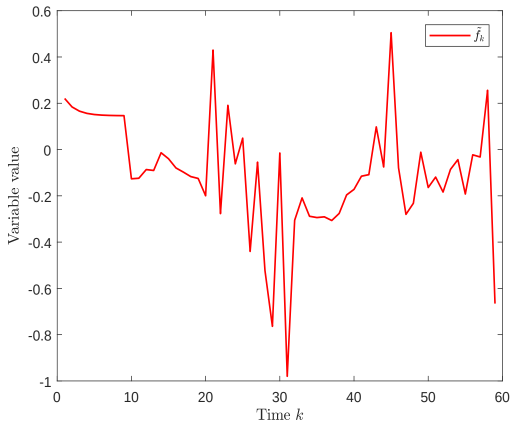

Figure 5 The trajectory of the fault estimation error $\tilde{f}_k$

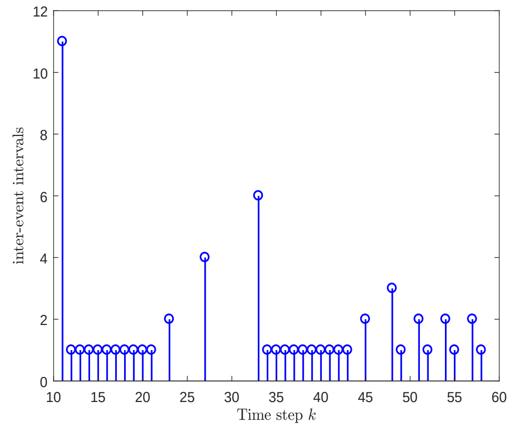

Figure 6 The dynamic triggering instants for estimator

estimation method.

The event-based fault diagnosis problem is still a hot yet challenging research area. Further research topics include the extension of the main results to the fault diagnosis problem for time-varying systems subject to the dynamic event-triggered mechanism, which has important theoretical significance and broad application prospects.

Acknowledgements This work was supported in part by the National Natural Science Foundation of China (Grants Nos. 61873149, 61703245 and 61733009).

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