Dynamic Output-Feedback H_∞ Control for Discrete Time-Delayed Systems with Actuator Saturations under Round-Robin Communication Protocol

Yonggang Chen, Zidong Wang*, Fuad E. Alsaadi and Hongjian Liu

Abstract

In this paper, the dynamic output-feedback H_∞ control problem is investigated for discrete-time state-delay systems subject to exogenous disturbances and actuator saturations under the Round-Robin communication protocol. Using the switched system approach, the Lyapunov-Krasovskii functional and the modified sector condition, sufficient conditions are first obtained under which the closed-loop systems can achieve some desirable performance indices such as the boundedness, the H_∞ performance and the stability in the absence of disturbances. Then, the controller is explicitly characterized by means of the solvability of linear matrix inequalities. For the case without time delays, the corresponding conditions are also presented. Subsequently, the optimization problems about the performance indices are discussed. Finally, the benefit and effectiveness of the proposed results are specifically illustrated via two simulation examples.

Index Terms

Dynamic output-feedback control, discrete-time systems, state delays, actuator saturations, Round-Robin protocol.

I. Introduction

Time delays are often encountered in many practical systems such as power systems, chemical process, neural networks and networked control systems (NCSs), which could lead to poor performance and instability of control

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systems. Over the past several decades, an ever-growing research interest has been devoted to the investigation on the time-delay systems, see e.g., [9], [12], [15], [22], [24]–[27], [29], [32], [36], [37]. On the other hand, in most practical feedback control systems, the phenomenon of actuator saturations is unavoidable, which constitutes another source of performance degradation and instability. Some important results about analysis and synthesis of control systems subject to actuator saturations can be found in [31], [42] and the references therein.

In reality, time delays and actuator saturations might coexist in a control system. For a decade, the coupling issue of time delays and actuator saturations has drawn considerable research attention [1], [2], [4], [5], [28], [43]. For example, the semi-global stabilization problem has been addressed in [43] for input-delay systems by using the law gain technique. In [2], [4], the local/regional stabilization problem has been studied by using the Lyapunov-Krasovskii (L-K) functional approach to deal with time delays and the polytopic models to represent saturation nonlinearities. Note that most existing results have been mainly concerned with the design of state feedback controller. In the case that the system state is immeasurable, it becomes necessary to design the output-feedback controller [14], [35]. However, it is worth pointing out that some stringent constrains would have to be imposed on the matrix variables when designing the observer-based controller [35].

Along with the rapid development of network technologies, NCSs have been attracting a recurring research interest in the control community. So far, NCSs have exhibited fascinating advantages (e.g. cost reduction and maintenance convenience) with extensive applications in a variety of practical systems. Nevertheless, the embedding of communication networks in control systems have brought about certain imperfections (e.g., packet dropouts, communication delays, and signal quantization) which result mainly from inherent limited bandwidth [3], [13], [16], [18], [30], [34], [39], [44], [46]. For network with limited bandwidth, a typical way of preventing networked-induced phenomena is to deploy *communication protocols* so as to facilitate multiple (and simultaneous) signal transmissions. According to the popularity, three types of communication protocols have been adopted in industry applications, i.e., Round-Robin (RR) protocol, Try-Once-Discard (TOD) protocol, and Random Access (RA) protocol [48].

In the past few years, the analysis and synthesis problems for NCSs under various communication protocols have gained significant attention and some pioneering results have been reported [8], [10], [11], [19]–[21], [33], [38], [45], [47]. For example, in [20], the stability and L_2-gain analysis problems have been studied for NCSs with communication delays under the RR protocol and, in [47], the moving horizon estimation problem has been addressed for networked systems with state delays under the RR protocol. However, it is worth mentioning that the phenomenon of actuator saturations has been largely overlooked in most existing literature. Note that the results proposed in [21] have dealt with the RR-protocol-based and TOD-protocol-based stability analysis problems for discrete-time systems without state delays and disturbances, where two sensor nodes have been considered.

Based on the above discussions, this paper aims to address the dynamic output-feedback control problem for discrete-time state-delay systems with exogenous disturbances and actuator saturations under the RR communication protocol. Using the switched system approach, the L-K functional as well as the modified sector condition, sufficient conditions are established under which the closed-loop systems can achieve some desirable performance indices including the boundedness, the H_∞ performance and the stability in the absence of disturbances. A simulation example shows that our proposed result can provide a large estimate of admissible initial conditions for the case without transmission delays. In fact, when the time delays, the actuator saturations and the communication protocols are simultaneously involved in a control system, the problem of designing the dynamic output-feedback controller is non-trivial. The main difficulty is how to design and characterize the controller effectively in terms of linear matrix inequalities (LMIs). It should be pointed out that, when the time delay is unknown, the existing dynamic output-feedback design techniques cannot be applicable due to the existence of saturations [14], [15].

The main contributions of the paper are summarized as follows. *1) The dynamic output-feedback H_∞ control problem is studied, for the first time, for discrete-time systems with state delays, exogenous disturbances and actuator saturations under the RR communication protocol, and the corresponding sufficient conditions are established. 2) A switched dynamic output-feedback controller is proposed to reduce the conservatism of the obtained results, and the linearized technique is developed to characterize the controller by means of the solvability of LMIs. 3) The controller is designed directly based on the system and the actual measurement (without resorting to their augmented models [8]), thereby meriting the implementation of the output-feedback controller.*

Notation. The superscript "T" denotes the transpose of a matrix. $P > 0$ (≥ 0) means that P is a real, symmetric and positive definite (positive semi-definite) matrix. $\mathbb{R}^n$ stands for the n-dimensional Euclidean space and $\|\cdot\|$ is the 2-norm of a vector. $G_{(l)}$ is the l-th row of the matrix G and $v_{(l)}$ is the l-th element of the vector v. $\lambda_M(P)$ is the maximum eigenvalue of P. The symmetric terms in a symmetric matrix are denoted by $*$. $\mathbf{I}[a,b] \triangleq \{a, a+1, \ldots, b\}$.

II. Problem formulation

Consider the following linear discrete-time state-delay system subject to actuator saturations:

$$\begin{cases} x(k+1) = Ax(k) + A_d x(k-\tau(k)) + B\mathrm{sat}(u(k)) + D\omega(k), \\ \quad y(k) = Cx(k) + E\omega(k), \\ \quad z(k) = Fx(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, $y(k) \in \mathbb{R}^p$, $z(k) \in \mathbb{R}^q$ and $\omega(k) \in \mathbb{R}^r$ are, respectively, the system state, the control input, the measurement output, the controlled output and the disturbance input. A, A_d, B, C, D, E and F are some real constant matrices. $\tau(k)$ denotes the state delay that is variable and satisfies $0 < \tau_1 \leq \tau(k) \leq \tau_2$. $\mathrm{sat}(u)$ $= [\mathrm{sat}(u_{(1)})\ \ \mathrm{sat}(u_{(2)})\ \ \cdots\ \ \mathrm{sat}(u_{(m)})]^T$ is a vector-valued function representing the actuator saturations, where each component is defined as $\mathrm{sat}(u_{(l)}) = \mathrm{sgn}(u_{(l)})\min\{|u_{(l)}|, \bar{u}_{(l)}\}$ $(l = 1, 2, \ldots, m)$ with the saturation level $\bar{u}_{(l)} > 0$.

The exogenous disturbance $\omega(k)$ of the system (1) is energy-bounded and satisfy the condition

$$\sum_{k=0}^{+\infty} \omega^T(k)\omega(k) \leq \delta\ \ (\delta > 0). \tag{2}$$

In this paper, the measurements are transmitted over a shared communication network. In particular, in order to alleviate the phenomenon of data collision induced by the limited bandwidth of communication network, the RR protocol is adopted here to schedule the network traffic where only one sensor node is allowed to transmit its data at each time instant [20], [47]. As in [47], we rewrite the measurement output $y(k)$ as follows:

$$y(k) = [y_1^T(k),\ y_2^T(k), \ldots,\ \ y_N^T(k)], \tag{3}$$

where N is the number of sensors and $y_i(k) \in \mathbb{R}^{p_i}$ is the measurement corresponding to the i-th sensor ($\Sigma_{i=1}^N p_i = p$).

Let $\sigma(k)$ be the selected node at the instant k. Under the RR protocol, it can be seen that $\sigma(k)$ satisfies the relations $\sigma(k+N) = \sigma(k)$ and $\sigma(k) \in \mathbf{I}[1, N]$ [47]. Here, we assume that $\sigma(k) = k+1$ for $k \in \mathbf{I}[0, N-1]$.

Under the RR communication protocol, the actual measurements $\bar{y}_i(k)$ $(i \in \mathbf{I}[1, N])$ can be represented as

$$\bar{y}_i(k) = \begin{cases} y_i(k),\ \ i = \sigma(k), \\ \bar{y}_i(k-1),\ \ \text{otherwise}. \end{cases} \tag{4}$$

Let $\delta(\cdot)$ be the Kronecker delta function and denote

$$\Phi_{\sigma(k)} = \text{diag}\{\delta(\sigma(k)-1)I, \delta(\sigma(k)-2)I, \ldots, \delta(\sigma(k)-N)I\}. \tag{5}$$

Then, the actual measurement can be rewritten as follows [47]:

$$\bar{y}(k) = \Phi_{\sigma(k)}y(k) + (I - \Phi_{\sigma(k)})\bar{y}(k-1). \tag{6}$$

In this paper, we consider the following dynamic output-feedback controller with an anti-windup loop [14]:

$$\begin{cases} \hat{x}(k+1) = A^c_{\sigma(k)}\hat{x}(k) + B^c_{\sigma(k)}\bar{y}(k) + E^c_{\sigma(k)}(\text{sat}(u(k)) - u(k)), \\ \quad u(k) = C^c_{\sigma(k)}\hat{x}(k) + D^c_{\sigma(k)}\bar{y}(k), \end{cases} \tag{7}$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the controller state, and A^c_i, B^c_i, C^c_i, D^c_i and E^c_i ($i \in \mathbf{I}[1, N]$) are the gain matrices.

Denoting that $\bar{x}(k) \triangleq [x^T(k) \;\; \hat{x}^T(k) \;\; \bar{y}^T(k-1)]^T$, $\psi(u(k)) \triangleq u(k) - \text{sat}(u(k))$, $\tilde{\Phi}_{\sigma(k)} \triangleq I - \Phi_i(k)$, and using (1), (6) and (7), one obtains the following closed-loop system:

$$\begin{cases} \bar{x}(k+1) = \mathcal{A}_{\sigma(k)}\bar{x}(k) + \mathcal{A}_d x(k-\tau(k)) \\ \qquad\qquad - \mathcal{B}_{\sigma(k)}\psi(u(k)) + \mathcal{D}_{\sigma(k)}\omega(k), \\ \quad z(k) = \mathcal{F}\bar{x}(k), \end{cases} \tag{8}$$

where $\mathcal{A}_d \triangleq [A^T_d \;\; 0 \;\; 0]^T$, $\mathcal{F} \triangleq [F \;\; 0 \;\; 0]$, and

$$\mathcal{A}_{\sigma(k)} \triangleq \begin{bmatrix} A + BD^c_{\sigma(k)}\Phi_{\sigma(k)}C & BC^c_{\sigma(k)} & BD^c_{\sigma(k)}\tilde{\Phi}_{\sigma(k)} \\ B^c_{\sigma(k)}\Phi_{\sigma(k)}C & A^c_{\sigma(k)} & B^c_{\sigma(k)}\tilde{\Phi}_{\sigma(k)} \\ \Phi_{\sigma(k)}C & 0 & \tilde{\Phi}_{\sigma(k)} \end{bmatrix},$$

$$\mathcal{B}_{\sigma(k)} \triangleq \begin{bmatrix} B \\ E^c_{\sigma(k)} \\ 0 \end{bmatrix}, \; \mathcal{D}_{\sigma(k)} \triangleq \begin{bmatrix} D + BD^c_{\sigma(k)}\Phi_{\sigma(k)}E \\ B^c_{\sigma(k)}\Phi_{\sigma(k)}E \\ \Phi_{\sigma(k)}E \end{bmatrix}.$$

The initial condition associated with (8) has the form $\phi(k) \triangleq [\phi^T_x(k) \;\; \hat{x}^T(0) \;\; \bar{y}^T(-1)]^T$ ($k \in \mathbf{I}[-\tau_2, 0]$).

In order to handle the nonlinearity $\psi(u)$ appearing in (8), we introduce the modified sector condition.

Lemma 1: [31] Let the vectors $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^m$ be given. If $|v_{(l)}| \le \bar{u}_{(l)}$ ($l \in \mathbf{I}[1, m]$), then for any $m \times m$ positive diagonal matrix H, the following inequality holds:

$$\psi^T(u)H[\psi(u) - u + v] \le 0.$$

Let us define a switched function of the following form:

$$v(k) \triangleq G_{\sigma(k)}\bar{x}(k), \tag{10}$$

where $\sigma(k)$ has the same definition as in (8), and G_i ($i \in \mathbf{I}[1, N]$) are $m \times (2n + p)$ matrices to be determined.

Suppose that the following constraints are satisfied:

$$|v_{(l)}(k)| \le \bar{u}_{(l)}, \; l \in \mathbf{I}[1, m]. \tag{11}$$

Then, for any diagonal matrices $H_i > 0$ ($i \in \mathbf{I}[1, N]$), it follows from Lemma 1 that

$$-2\psi^T(u(k))H_{\sigma(k)}\big[\psi(u(k)) - u(k) + v(k)\big] \ge 0, \tag{12}$$

where $u(k) \triangleq \mathcal{K}_{\sigma(k)}\bar{x}(k) + D^c_{\sigma(k)}\Phi_{\sigma(k)}E\omega(k)$ with $\mathcal{K}_{\sigma(k)} \triangleq [D^c_{\sigma(k)}\Phi_{\sigma(k)}C \quad C^c_{\sigma(k)} \quad D^c_{\sigma(k)}\tilde{\Phi}_{\sigma(k)}]$.

The main objective of this paper is to design the dynamic output-feedback controller (7) such that: 1) all trajectories of the closed-loop system (8) are bounded for admissible initial conditions and non-zero disturbances; 2) the following H_∞ performance constraint is ensured [17], [23]:

$$\sum_{k=0}^{+\infty} z^T(k)z(k) \le \gamma V_{\sigma(0)}(0) + \gamma \sum_{k=0}^{+\infty} \omega^T(k)\omega(k), \tag{13}$$

where $\gamma > 0$ is a scalar and $V_{\sigma(k)}(k)$ is an L-K functional to be selected; and 3) the closed-loop system (8) is locally asymptotically stable in the absence of exogenous disturbances.

Remark 1: Using the time-delay approach, the stability analysis problem has been thoroughly investigated in [21] for discrete-time systems subject to actuator saturations and transmission delays under RR and TOD protocols, where the main results are applicable to the case of two sensor nodes. Different from the approach developed in [21], the switched model (6) is utilized in this paper to represent the measurement output under which the multiple sensor nodes can be conveniently dealt with. In addition, the switched dynamic output-feedback controller (7) is proposed in our paper to reduce unnecessary conservatism of the obtained results.

III. Main Results

For the stability and performance analysis, we select the following switched L-K functional [6], [15]:

$$\begin{aligned} V_{\sigma(k)}(k) =& \bar{x}^T(k)P_{\sigma(k)}\bar{x}(k) + \sum_{i=k-\tau_1}^{k-1} x^T(i)Q_1x(i) + \sum_{i=k-\tau_2}^{k-\tau_1-1} x^T(i)Q_2x(i) \\ &+ \sum_{j=-\tau_2}^{-\tau_1}\sum_{i=k+j}^{k-1} x^T(i)Q_3x(i) + \tau_1 \sum_{j=-\tau_1}^{-1}\sum_{i=k+j}^{k-1} \eta^T(i)\mathcal{I}^T Z_1\mathcal{I}\eta(i) \\ &+ \tilde{\tau}\sum_{j=-\tau_2}^{-\tau_1-1}\sum_{i=k+j}^{k-1} \eta^T(i)\mathcal{I}^T Z_2\mathcal{I}\eta(i) \ (\tilde{\tau} \triangleq \tau_2 - \tau_1), \end{aligned} \tag{14}$$

where $\eta(k) \triangleq \bar{x}(k+1) - \bar{x}(k)$, $\mathcal{I} \triangleq [I \quad 0 \quad 0]$, and $P_i > 0$, $Q_j > 0$, $Z_1 > 0$, $Z_2 > 0$, $i \in \mathbf{I}[1,N]$, $j \in \mathbf{I}[1,3]$.

Theorem 1: Assume that there exist $(2n+p)\times(2n+p)$ matrices $P_i > 0$, T_{1i}, T_{2i}, $n \times n$ matrices $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, L_i, A^c_i, $n \times p$ matrices B^c_i, $m \times n$ matrices C^c_i, $m \times p$ matrices D^c_i, $n \times m$ matrices E^c_i, $m \times (2n+p)$ matrices G_i, $m \times m$ diagonal matrices $H_i > 0$, $i \in \mathbf{I}[1,N]$, and the scalars $\gamma > 0$, $\varpi > 0$, such that $\varpi \le 1/\delta$ and the following matrix inequalities are satisfied:

$$\begin{bmatrix} Z_2 & L_i \\ L_i^T & Z_2 \end{bmatrix} > 0, \ \forall i \in \mathbf{I}[1,N], \tag{15}$$

$$\begin{bmatrix} \Omega(i,i+1) & \tilde{\mathcal{F}}^T \\ \tilde{\mathcal{F}} & -\gamma I \end{bmatrix} < 0, \ \forall i \in \mathbf{I}[1,N-1], \tag{16}$$

$$\begin{bmatrix} \Omega(N,1) & \tilde{\mathcal{F}}^T \\ \tilde{\mathcal{F}} & -\gamma I \end{bmatrix} < 0, \tag{17}$$

$$\begin{bmatrix} \bar{u}^2_{(l)}\varpi & G_{i(l)} \\ G^T_{i(l)} & P_i \end{bmatrix} \ge 0, \ \forall i \in \mathbf{I}[1,N], \forall l \in \mathbf{I}[1,m], \tag{18}$$

where $\tilde{\mathcal{F}} \triangleq [\mathcal{F} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$ and $\Omega(\sigma(k),\sigma(k+1)) \triangleq$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & 0 & \Omega_{15} & \Omega_{16} & \Omega_{17} \\ * & \Omega_{22} & \Omega_{23} & L_{\sigma(k)} & 0 & 0 & 0 \\ * & * & \Omega_{33} & \Omega_{34} & 0 & \Omega_{36} & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & \Omega_{56} & \Omega_{57} \\ * & * & * & * & * & \Omega_{66} & \Omega_{67} \\ * & * & * & * & * & * & -I \end{bmatrix}$$

with

$$\begin{aligned}
\Omega_{11} \triangleq & T_{1\sigma(k)}^T(\mathcal{A}_{\sigma(k)} - I) + (\mathcal{A}_{\sigma(k)} - I)^T T_{1\sigma(k)} - P_{\sigma(k)} \\
& + P_{\sigma(k+1)} + \mathcal{I}^T[Q_1 + (\tilde{\tau}+1)Q_3 - Z_1]\mathcal{I}, \\
\Omega_{12} \triangleq & \mathcal{I}^T Z_1, \ \Omega_{13} \triangleq T_{1\sigma(k)}^T \mathcal{A}_d, \ \Omega_{17} \triangleq T_{1\sigma(k)}^T \mathcal{D}_{\sigma(k)}, \\
\Omega_{15} \triangleq & - T_{1\sigma(k)}^T \mathcal{B}_{\sigma(k)} + \mathcal{K}_{\sigma(k)}^T H_{\sigma(k)}^T - G_{\sigma(k)}^T H_{\sigma(k)}^T, \\
\Omega_{16} \triangleq & - T_{1\sigma(k)}^T + (\mathcal{A}_{\sigma(k)} - I)^T T_{2\sigma(k)} + P_{\sigma(k+1)}, \\
\Omega_{22} \triangleq & - Q_1 + Q_2 - Z_1 - Z_2, \ \Omega_{23} \triangleq Z_2 - L_{\sigma(k)}, \\
\Omega_{33} \triangleq & - Q_3 - 2Z_2 + L_{\sigma(k)} + L_{\sigma(k)}^T, \ \Omega_{34} \triangleq Z_2 - L_{\sigma(k)}, \\
\Omega_{36} \triangleq & \mathcal{A}_d^T T_{2\sigma(k)}, \ \Omega_{44} \triangleq -Q_2 - Z_2, \ \Omega_{55} \triangleq -2H_{\sigma(k)}, \\
\Omega_{56} \triangleq & - \mathcal{B}_{\sigma(k)}^T T_{2\sigma(k)}, \ \Omega_{57} \triangleq H_{\sigma(k)} D_{\sigma(k)}^c \Phi_{\sigma(k)} E, \\
\Omega_{66} \triangleq & - T_{2\sigma(k)} - T_{2\sigma(k)}^T + P_{\sigma(k+1)} \\
& + \mathcal{I}^T(\tau_1^2 Z_1 + \tilde{\tau}^2 Z_2)\mathcal{I}, \ \Omega_{67} \triangleq T_{2\sigma(k)}^T \mathcal{D}_{\sigma(k)}.
\end{aligned}$$

Then, 1) all trajectories of the closed-loop system (8) are bounded for all initial conditions satisfying $V_{\sigma(0)}(0) \leq 1/\varpi - \delta$ and all non-zero disturbances satisfying (2); 2) the H_∞ performance constraint (13) is guaranteed; 3) the system (8) is asymptotically stable for all initial conditions satisfying $V_{\sigma(0)}(0) \leq 1/\varpi$ in the absence of disturbances.

Proof: Denoting $\triangle V_k \triangleq V_{\sigma(k+1)}(k+1) - V_{\sigma(k)}(k)$, by tedious calculations, we have

$$\begin{aligned}
\triangle V_k \leq & [\bar{x}(k) + \eta(k)]^T P_{\sigma(k+1)}[\bar{x}(k) + \eta(k)] - \bar{x}^T(k) P_{\sigma(k)} \bar{x}(k) \\
& + x^T(k)[Q_1 + (\tilde{\tau}+1)Q_3]x(k) + x^T(k-\tau_1) \\
& \times (-Q_1 + Q_2)x(k-\tau_1) - x^T(k-\tau(k))Q_3 x(k-\tau(k)) \\
& - x^T(k-\tau_2)Q_2 x(k-\tau_2) + \eta^T(k)\mathcal{I}^T(\tau_1^2 Z_1 + \tilde{\tau}^2 Z_2)\mathcal{I}\eta(k) \\
& - \tau_1 \sum_{i=k-\tau_1}^{k-1} \eta^T(i)\mathcal{I}^T Z_1 \mathcal{I}\eta(i) - \tilde{\tau} \sum_{i=k-\tau_2}^{k-\tau_1-1} \eta^T(i)\mathcal{I}^T Z_2 \mathcal{I}\eta(i).
\end{aligned} \tag{19}$$

Using the Jensen inequality [12] and noting $\sum_{i=k-\tau_2}^{k-\tau_1-1}(\cdot) = \sum_{i=k-\tau(k)}^{k-\tau_1-1}(\cdot) + \sum_{i=k-\tau_2}^{k-\tau(k)-1}(\cdot)$, it follows that

$$\tau_1 \sum_{i=k-\tau_1}^{k-1} \eta^T(i)\mathcal{I}^T Z_1 \mathcal{I}\eta(i) \geq \zeta_1^T(k) Z_1 \zeta_1(k), \tag{20}$$

$$\tilde{\tau} \sum_{i=k-\tau_2}^{k-\tau_1-1} \eta^T(i)\mathcal{I}^T Z_2 \mathcal{I}\eta(i) \geq [\tilde{\tau}/(\tau(k) - \tau_1)]$$

$$\times\, \zeta_2^T(k) Z_2 \zeta_2(k) + [\tilde{\tau}/(\tau_2 - \tau(k))] \zeta_3^T(k) Z_2 \zeta_3(k), \tag{21}$$

where $\zeta_1(k) \triangleq x(k) - x(k-\tau_1)$, $\zeta_2(k) \triangleq x(k-\tau_1) - x(k-\tau(k))$ and $\zeta_3(k) \triangleq x(k-\tau(k)) - x(k-\tau_2)$.

If there exist matrices L_i ($i \in \mathbf{I}[1,N]$) such that the LMIs (15) hold, the inequality (21) can be modified as [24]

$$\tilde{\tau} \sum_{i=k-\tau_2}^{k-\tau_1-1} \eta^T(i) \mathcal{I}^T Z_2 \mathcal{I} \eta(i) \geq \begin{bmatrix} \zeta_2(k) \\ \zeta_3(k) \end{bmatrix}^T \begin{bmatrix} Z_2 & L_{\sigma(k)} \\ L_{\sigma(k)}^T & Z_2 \end{bmatrix} \begin{bmatrix} \zeta_2(k) \\ \zeta_3(k) \end{bmatrix}. \tag{22}$$

For any matrices T_{1i} and T_{2i} ($i \in \mathbf{I}[1,N]$), from the closed-loop system (8), one obtains the following equality:

$$\begin{aligned} &2[\bar{x}^T(t) T_{1\sigma(k)}^T + \eta^T(k) T_{2\sigma(k)}^T][(\mathcal{A}_{\sigma(k)} - I)\bar{x}(k) - \eta(k) \\ &+ \mathcal{A}_d x(k-\tau(k)) - \mathcal{B}_{\sigma(k)} \psi(u(k)) + \mathcal{D}_{\sigma(k)} \omega(k)] = 0. \end{aligned} \tag{23}$$

Noting the non-negativity of the left-hand sides of (12) and (23), and using (19), (20) and (22), it follows that

$$\begin{aligned} &\triangle V_k + (1/\gamma) z^T(k) z(k) - \omega^T(k)\omega(k) \\ \leq &\triangle V_k + (1/\gamma) z^T(k) z(k) - \omega^T(k)\omega(k) - 2\psi^T(u(k)) H_{\sigma(k)} \\ &\times [\psi(u(k)) - \mathcal{K}_{\sigma(k)}\bar{x}(k) - D_{\sigma(k)}^c \Phi_{\sigma(k)} E\omega(k) + G_{\sigma(k)}\bar{x}(k)] \\ &+ 2[\bar{x}^T(t) T_{1\sigma(k)}^T + \eta^T(k) T_{2\sigma(k)}^T][(\mathcal{A}_{\sigma(k)} - I)\bar{x}(k) - \eta(k) \\ &+ \mathcal{A}_d x(k-\tau(k)) - \mathcal{B}_{\sigma(k)} \psi(u(k)) + \mathcal{D}_{\sigma(k)} \omega(k)] \\ = &\xi^T(k)[\Omega(\sigma(k), \sigma(k+1)) + (1/\gamma)\tilde{\mathcal{F}}^T \tilde{\mathcal{F}}]\xi(k), \end{aligned} \tag{24}$$

where $\xi(k) = [\bar{x}^T(k) \quad x^T(k-\tau_1) \quad x^T(k-\tau(k)) \quad x^T(k-\tau_2) \quad \psi^T(u(k)) \quad \eta^T(k) \quad \omega^T(k)]^T$, and the matrix $\Omega(\sigma(k), \sigma(k+1))$ is denoted in the statement of this theorem.

Applying Schur complement to (16) and (17), we have

$$\Omega(i, i+1) + (1/\gamma)\tilde{\mathcal{F}}^T \tilde{\mathcal{F}} < 0, \ \forall i \in \mathbf{I}[1, N-1], \tag{25}$$

$$\Omega(N, 1) + (1/\gamma)\tilde{\mathcal{F}}^T \tilde{\mathcal{F}} < 0. \tag{26}$$

Then, it is seen from the inequality (24) that

$$\triangle V_k + (1/\gamma) z^T(k) z(k) - \omega^T(k)\omega(k) < 0. \tag{27}$$

Summing up the above inequality from 0 to $k-1$ yields

$$V_{\sigma(k)}(k) - V_{\sigma(0)}(0) + (1/\gamma)\sum_{i=0}^{k-1} z^T(i) z(i) - \sum_{i=0}^{k-1} \omega^T(i)\omega(i) < 0. \tag{28}$$

Applying Schur complement to (18), one obtains the following matrix inequalities:

$$\bar{u}_{(l)}^2 \varpi P_i \geq G_{i(l)}^T G_{i(l)}, \ \forall i \in \mathbf{I}[1, N], \ \forall l \in \mathbf{I}[1, m]. \tag{29}$$

In addition, it is seen from the L-K functional (14) that

$$V_{\sigma(k)}(k) \geq \bar{x}^T(k) P_{\sigma(k)} \bar{x}(k). \tag{30}$$

Using the inequalities (29) and (30), it follows that

$$|v_{(l)}(k)|^2 = |G_{\sigma(k)(l)} \bar{x}(k)|^2 \leq (\bar{u}_{(l)}^2 \varpi) V_{\sigma(k)}(k). \tag{31}$$

For all $\phi(k)$ satisfying $V_{\sigma(0)}(0) \leq 1/\varpi - \delta$ and all non-zero $\omega(k)$ satisfying (2), it is seen from (28) and (31) that the constraints (11) can be guaranteed. Moreover, it follows from (28) and (30) that

$$\bar{x}^T(k)P_{\sigma(k)}\bar{x}(k) \leq V_{\sigma(k)}(k) \leq 1/\varpi. \tag{32}$$

From (32), it is clear that all trajectories of the system (8) belong to the union of the following bounded ellipsoids:

$$\{\bar{x} \in \mathbb{R}^{2n+p} : \bar{x}^T P_i \bar{x} \leq 1/\varpi\}, \ i \in \mathbf{I}[1, N]. \tag{33}$$

In (28), letting $k \to +\infty$ and noting $V_{\sigma(k)}(k) > 0$, it is seen that the H_∞ performance constraint (13) is satisfied.

For the case that $\omega(k) = 0$, from (28) and (31), the constraints (11) can also be guaranteed for all initial conditions $\phi(k)$ satisfying $V_{\sigma(0)}(0) \leq 1/\varpi$. Meanwhile, from (27), we have the following relation:

$$V_{\sigma(k+1)}(k+1) < V_{\sigma(k)}(k) \tag{34}$$

which implies that the closed-loop system (8) is locally asymptotically stable and this completes the proof. ■

If the time delay $\tau(k)$ is not contained in the system (1), the corresponding result can be formulated as follows.

Corollary 1: The conclusions of Theorem 1 are true for the case without time delay if there exist $(2n+p)\times(2n+p)$ matrices $P_i > 0$, T_{1i}, T_{2i}, $n \times n$ matrices A_i^c, $n \times p$ matrices B_i^c, $m \times n$ matrices C_i^c, $m \times p$ matrices D_i^c, $n \times m$ matrices E_i^c, $m \times (2n + p)$ matrices G_i, $m \times m$ diagonal matrices $H_i > 0$, $i \in \mathbf{I}[1, N]$, and the scalars $\gamma > 0$, $\varpi > 0$, such that $\varpi \leq 1/\delta$, (18) and the following matrix inequalities are satisfied:

$$\Gamma(i, i+1) < 0, \ \forall i \in \mathbf{I}[1, N-1]; \ \Gamma(N, 1) < 0, \tag{35}$$

where

$$\Gamma(\sigma(k), \sigma(k+1)) \triangleq \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \mathcal{F}^T \\ * & -2H_{\sigma(k)} & \Gamma_{23} & \Gamma_{24} & 0 \\ * & * & \Gamma_{33} & \Gamma_{34} & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma I \end{bmatrix}$$

with

$$\begin{aligned}
\Gamma_{11} \triangleq & T_{1\sigma(k)}^T(\mathcal{A}_{\sigma(k)} - I) + (\mathcal{A}_{\sigma(k)} - I)^T T_{1\sigma(k)} \\
& + P_{\sigma(k+1)} - P_{\sigma(k)}, \ \Gamma_{14} \triangleq T_{1\sigma(k)}^T \mathcal{D}_{\sigma(k)}, \\
\Gamma_{12} \triangleq & - T_{1\sigma(k)}^T \mathcal{B}_{\sigma(k)} + \mathcal{K}_{\sigma(k)}^T H_{\sigma(k)}^T - G_{\sigma(k)}^T H_{\sigma(k)}^T, \\
\Gamma_{13} \triangleq & - T_{1\sigma(k)}^T + (\mathcal{A}_{\sigma(k)} - I)^T T_{2\sigma(k)} + P_{\sigma(k+1)}, \\
\Gamma_{23} \triangleq & - \mathcal{B}_{\sigma(k)}^T T_{2\sigma(k)}, \ \Gamma_{24} \triangleq H_{\sigma(k)} D_{\sigma(k)}^c \Phi_{\sigma(k)} E, \\
\Gamma_{33} \triangleq & P_{\sigma(k+1)} - T_{2\sigma(k)} - T_{2\sigma(k)}^T, \ \Gamma_{34} \triangleq T_{2\sigma(k)}^T \mathcal{D}_{\sigma(k)}.
\end{aligned}$$

Next, we will address the control design problem. In order to solve the problem in the framework of LMIs, we set $T_{1i} = T$, $T_{2i} = \lambda T_{1i}$ in (16) and (17), and specify the matrices T and T^{-1} as follows [7]:

$$T = \begin{bmatrix} R^T & \mathcal{J}_1 & 0 \\ N^T & \mathcal{J}_2 & 0 \\ 0 & 0 & \mu^{-1}I \end{bmatrix}, \ T^{-1} = \begin{bmatrix} S^T & \mathcal{J}_3 & 0 \\ M^T & \mathcal{J}_4 & 0 \\ 0 & 0 & \mu I \end{bmatrix}, \tag{36}$$

where R, S, M, N, $\mathcal{J}_{\tilde{i}}$ $(\tilde{i}=1,2,3,4)$ are some matrices and μ is a scalar. Then, we define the two matrices

$$X_1 \triangleq \begin{bmatrix} I & S^T & 0 \\ 0 & M^T & 0 \\ 0 & 0 & \mu I \end{bmatrix}, \ X_2 \triangleq \begin{bmatrix} R^T & I & 0 \\ N^T & 0 & 0 \\ 0 & 0 & I \end{bmatrix}. \tag{37}$$

It is easy to verify that $TX_1 = X_2$. Moreover, we denote

$$\begin{aligned} \begin{bmatrix} \bar{A}_i^c & \bar{B}_i^c \\ \bar{C}_i^c & \bar{D}_i^c \end{bmatrix} \triangleq & \begin{bmatrix} N & RB \\ 0 & I \end{bmatrix} \begin{bmatrix} A_i^c - I & B_i^c \\ C_i^c & D_i^c \end{bmatrix} \\ & \times \begin{bmatrix} M^T & 0 \\ \Phi_i C S^T & I \end{bmatrix} + \begin{bmatrix} R(A-I)S^T & 0 \\ 0 & 0 \end{bmatrix}, \end{aligned} \tag{38}$$

$$\bar{E}_i^c \triangleq (RB + NE_i^c)\bar{H}_i, \ Y \triangleq SR^T + MN^T, \tag{39}$$

$$\bar{P}_i \triangleq X_1^T P_i X_1, \ \bar{G}_i \triangleq G_i X_1, \ \bar{H}_i \triangleq H_i^{-1}, \tag{40}$$

$$\Lambda_1 \triangleq \begin{bmatrix} \Lambda_{11} & \bar{A}^c_{\sigma(k)} & \mu \bar{B}^c_{\sigma(k)} \tilde{\Phi}_{\sigma(k)} \\ \Lambda_{12} & \Lambda_{13} & \mu B \bar{D}^c_{\sigma(k)} \tilde{\Phi}_{\sigma(k)} \\ \Lambda_{14} & \Phi_{\sigma(k)} C S^T & -\mu \Phi_{\sigma(k)} \end{bmatrix}, \tag{41}$$

$$\Lambda_2 \triangleq \begin{bmatrix} I & I & 0 \\ S & S & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \Lambda_3 \triangleq \begin{bmatrix} R & Y^T & 0 \\ I & S^T & 0 \\ 0 & 0 & \mu I \end{bmatrix}, \tag{42}$$

$$\Lambda_{11} \triangleq R(A-I) + \bar{B}^c_{\sigma(k)} \Phi_{\sigma(k)} C, \ \Lambda_{14} \triangleq \Phi_{\sigma(k)} C, \tag{43}$$

$$\Lambda_{12} \triangleq A - I + B \bar{D}^c_{\sigma(k)} \Phi_{\sigma(k)} C, \ \mathbb{S} \triangleq [I \quad S^T \quad 0], \tag{44}$$

$$\Lambda_{13} \triangleq (A-I)S^T + B\bar{C}^c_{\sigma(k)}, \ \mathbb{I} \triangleq [I \quad I \quad 0]. \tag{45}$$

Theorem 2: Let the scalars $\lambda > 0$, μ, α, β_1, β_2, β_3 and β_4 be given. Assume that there exist $(2n+p)\times(2n+p)$ matrices $\bar{P}_i > 0$, $n \times n$ matrices $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, L_i, R, S, Y, $\bar{A}_i^c$, $n \times p$ matrices $\bar{B}_i^c$, $m \times n$ matrices $\bar{C}_i^c$, $m \times p$ matrices $\bar{D}_i^c$, $n \times m$ matrices $\bar{E}_i^c$, $m \times (2n+p)$ matrices $\bar{G}_i$, $m \times m$ diagonal matrices $\bar{H}_i > 0$, $i \in \mathbf{I}[1,N]$, and the scalars $\gamma > 0$, $\varpi > 0$, such that $\varpi \le 1/\delta$, and the LMIs (15) and

$$\begin{bmatrix} \bar{\Omega}(i,i+1) & \Psi_1^T \\ \Psi_1 & \Psi_2 \end{bmatrix} < 0, \ \forall i \in \mathbf{I}[1, N-1], \tag{46}$$

$$\begin{bmatrix} \bar{\Omega}(N,1) & \Psi_1^T \\ \Psi_1 & \Psi_2 \end{bmatrix} < 0, \tag{47}$$

$$\begin{bmatrix} \bar{u}^2_{(l)} \varpi & \bar{G}_{i(l)} \\ \bar{G}^T_{i(l)} & \bar{P}_i \end{bmatrix} \ge 0, \ \forall i \in \mathbf{I}[1, N], \forall l \in \mathbf{I}[1, m], \tag{48}$$

are satisfied, where $\bar{\Omega}(\sigma(k),\sigma(k+1)) \triangleq$

$$\begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13} & 0 & \bar{\Omega}_{15} & \bar{\Omega}_{16} & \bar{\Omega}_{17} \\ * & \bar{\Omega}_{22} & \Omega_{23} & L_{\sigma(k)} & 0 & 0 & 0 \\ * & * & \Omega_{33} & \Omega_{34} & 0 & \bar{\Omega}_{36} & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 \\ * & * & * & * & \bar{\Omega}_{55} & \bar{\Omega}_{56} & \bar{\Omega}_{57} \\ * & * & * & * & * & \bar{\Omega}_{66} & \bar{\Omega}_{67} \\ * & * & * & * & * & * & -I \end{bmatrix},$$

$$\Psi_1 \triangleq \begin{bmatrix} F\mathbb{S} & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha\mathbb{I} & -\alpha I & 0 & 0 & 0 & 0 & 0 \\ \mathbb{S} & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\tilde{\tau}+1}\mathbb{S} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tau_1\mathbb{S} & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{\tau}\mathbb{S} & 0 \end{bmatrix},$$

$$\begin{aligned} \Psi_2 \triangleq & \operatorname{diag}\{-\gamma I, -Z_1, \beta_1^2 Q_1 - 2\beta_1 I, \beta_2^2 Q_3 - 2\beta_2 I, \\ & \beta_3^2 Z_1 - 2\beta_3 I, \beta_4^2 Z_2 - 2\beta_4 I\} \end{aligned}$$

with

$$\begin{aligned} \bar{\Omega}_{11} \triangleq & \bar{P}_{\sigma(k+1)} - \bar{P}_{\sigma(k)} + \Lambda_1 + \Lambda_1^T - \alpha(\Lambda_2 + \Lambda_2^T), \\ \bar{\Omega}_{12} \triangleq & \alpha[2I \quad S^T + I \quad 0]^T, \ \bar{\Omega}_{13} \triangleq [A_d^T R^T \quad A_d^T \quad 0]^T, \\ \bar{\Omega}_{15} \triangleq & \begin{bmatrix} (\bar{D}^c_{\sigma(k)}\Phi_{\sigma(k)}C)^T \\ (\bar{C}^c_{\sigma(k)})^T \\ \mu(\bar{D}^c_{\sigma(k)}\tilde{\Phi}_{\sigma(k)})^T \end{bmatrix} - \begin{bmatrix} \bar{E}^c_{\sigma(k)} \\ B\bar{H}_{\sigma(k)} \\ 0 \end{bmatrix} - \bar{G}^T_{\sigma(k)}, \\ \bar{\Omega}_{16} \triangleq & -\Lambda_3 + \lambda\Lambda_1^T + \bar{P}_{\sigma(k+1)}, \ \bar{\Omega}_{36} \triangleq \lambda\bar{\Omega}_{13}^T, \\ \bar{\Omega}_{17} \triangleq & \begin{bmatrix} RD + \bar{B}^c_{\sigma(k)}\Phi_{\sigma(k)}E \\ D + B\bar{D}^c_{\sigma(k)}\Phi_{\sigma(k)}E \\ \Phi_{\sigma(k)}E \end{bmatrix}, \ \bar{\Omega}_{55} \triangleq -2\bar{H}_{\sigma(k)}, \\ \bar{\Omega}_{22} \triangleq & -Q_1 + Q_2 - Z_2 - 2\alpha I, \ \bar{\Omega}_{67} \triangleq \lambda\bar{\Omega}_{17}, \\ \bar{\Omega}_{56} \triangleq & -\lambda[(\bar{E}^c_{\sigma(k)})^T \quad (B\bar{H}_{\sigma(k)})^T \quad 0], \\ \bar{\Omega}_{57} \triangleq & \bar{D}^c_{\sigma(k)}\Phi_{\sigma(k)}E, \ \bar{\Omega}_{66} \triangleq P_{\sigma(k+1)} - \lambda(\Lambda_3 + \Lambda_3^T). \end{aligned}$$

Then, the conclusions of Theorem 1 are true. Moreover, the controller gain is obtained by solving (38) and (39).

Proof: Pre- and post-multiplying (16) and (17) by Π^T and Π, respectively, where $\Pi \triangleq \operatorname{diag}\{X_1, I, I, I, \bar{H}_i, X_1, I, I\}$, and using the notations in (38)-(45), we can obtain the following matrix inequalities:

$$\begin{bmatrix} \check{\Omega}(i,i+1) & \check{\mathcal{F}}^T \\ \check{\mathcal{F}} & -\gamma I \end{bmatrix} < 0, \ \forall i \in \mathbf{I}[1, N-1], \tag{49}$$

$$\begin{bmatrix} \check{\Omega}(N,1) & \check{\mathcal{F}}^T \\ \check{\mathcal{F}} & -\gamma I \end{bmatrix} < 0, \tag{50}$$

where $\check{\mathcal{F}} = [F\mathbb{S} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$ and $\check{\Omega}(\sigma(k), \sigma(k+1)) \triangleq$

$$\begin{bmatrix} \check{\Omega}_{11} & \mathbb{S}^T Z_1 & \bar{\Omega}_{13} & 0 & \bar{\Omega}_{15} & \bar{\Omega}_{16} & \bar{\Omega}_{17} \\ * & \Omega_{22} & \Omega_{23} & L_{\sigma(k)} & 0 & 0 & 0 \\ * & * & \Omega_{33} & \Omega_{34} & 0 & \Omega_{36} & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 \\ * & * & * & * & \bar{\Omega}_{55} & \bar{\Omega}_{56} & \bar{\Omega}_{57} \\ * & * & * & * & * & \check{\Omega}_{66} & \bar{\Omega}_{67} \\ * & * & * & * & * & * & -I \end{bmatrix}$$

with

$$\begin{aligned} \check{\Omega}_{11} &\triangleq \mathbb{S}^T[Q_1 + (\tilde{\tau}+1)Q_3 - Z_1]\mathbb{S} + \bar{P}_{\sigma(k+1)} - \bar{P}_{\sigma(k)} + \Lambda_1 + \Lambda_1^T, \\ \check{\Omega}_{66} &\triangleq P_{\sigma(k+1)} - T_{2\sigma(k)} - T_{2\sigma(k)}^T + \mathbb{S}^T(\tau_1^2 Z_1 + \tilde{\tau}^2 Z_2)\mathbb{S}. \end{aligned}$$

For any matrices Υ_1 and Υ_2, it is seen that

$$(\Upsilon_1 - \alpha Z_1^{-1}\Upsilon_2)^T Z_1 (\Upsilon_1 - \alpha Z_1^{-1}\Upsilon_2) \geq 0. \tag{51}$$

Let us select $\Upsilon_1 = [\mathbb{S} \quad -I]$ and $\Upsilon_2 = [\mathbb{I} \quad -I]$, we have

$$\begin{aligned} \begin{bmatrix} -\mathbb{S}^T Z_1 \mathcal{S} & \mathbb{S}^T Z_1 \\ Z_1\mathbb{S} & -Z_1 \end{bmatrix} &= -\Upsilon_1^T Z_1 \Upsilon_1 \\ &\leq -\alpha(\Upsilon_1^T\Upsilon_2 + \Upsilon_2^T\Upsilon_1) + \alpha^2 \Upsilon_2^T Z_1^{-1} \Upsilon_2. \end{aligned} \tag{52}$$

Similarly, we have the following matrix inequalities:

$$-Q_1^{-1} \leq \beta_1^2 Q_1 - 2\beta_1 I, \quad -Q_3^{-1} \leq \beta_2^2 Q_3 - 2\beta_2 I, \tag{53}$$

$$-Z_1^{-1} \leq \beta_3^2 Z_1 - 2\beta_3 I, \quad -Z_2^{-1} \leq \beta_4^2 Z_2 - 2\beta_4 I. \tag{54}$$

For the LMIs (46) and (47), using the relations (52)-(54) and Schur complement, it can be inferred that the matrix inequalities (49) and (50) are ensured. Pre- and post-multiplying the matrix inequalities (18) by $\mathrm{diag}\{1, X_1^T\}$ and $\mathrm{diag}\{1, X_1\}$, respectively, and using the notations in (40), the LMIs (48) are readily obtained.

Note that, if the LMIs (46) and (47) hold, we have

$$\Lambda_3 + \Lambda_3^T = \begin{bmatrix} R + R^T & I + Y^T & 0 \\ I + Y & S + S^T & 0 \\ 0 & 0 & 2\mu I \end{bmatrix} > 0, \tag{55}$$

which implies that R is invertible. Pre- and post-multiplying (55) by $[R^{-1} \quad -I \quad 0]$ and its transpose yields

$$(Y - SR^T)R^{-T} + R^{-1}(Y - SR^T)^T < 0. \tag{56}$$

From (56), it is seen that the matrix $Y - SR^T$ is non-singular. Applying the technique of singular value decomposition to $Y - SR^T$, the matrices M and N^T can be readily obtained. Moreover, the controller gain can be solved by using the matrix equations (38) and (39). This completes the proof. ■

Remark 2: In order to obtain LMI-based conditions, we have specified that the matrix T in (36) has a special structure. Of course, we can first augment the system (1) and the measurement (6), and then design the controller based on the augmented system [8]. In this case, the standard linearized technique in [7] can be adopted and the special assumption on the matrix T is no longer required. However, it should be pointed out that the technique in [8] will produce a higher-order controller which makes the implementation of controller more difficult. In fact, some simulation examples shows that our proposed technique would not bring much conservatism.

Remark 3: In [14], the non-rational dynamic output controller is designed under which the existing linearized techniques can be directly employed. Nevertheless, when the time delay is unknown, the non-rational dynamic output controller is no longer applicable. In this paper, the matrix inequalities (52)-(54) are utilized to make that the obtained matrix inequalities are linear. However, it is worth mentioning that the introduction of the inequalities (52)-(54) will increase the conservatism. Of course, the modified cone complementary linearization (CCL) algorithm [15] could be used here, but then the optimization of system performance indices will become difficult.

Corresponding to Corollary 1, we can obtain the following result.

Corollary 2: The conclusions of Theorem 1 are true for the case without time delay if there exist $(2n+p)\times(2n+p)$ matrices $\bar{P}_i > 0$, $n \times n$ matrices R, S, Y, $\bar{A}_i^c$, $n \times p$ matrices $\bar{B}_i^c$, $m \times n$ matrices $\bar{C}_i^c$, $m \times p$ matrices $\bar{D}_i^c$, $n \times m$ matrices $\bar{E}_i^c$, $m \times (2n+p)$ matrices $\bar{G}_i$, $m \times m$ diagonal matrices $\bar{H}_i > 0$, $i \in \mathbf{I}[1, N]$, and the scalars $\gamma > 0$, $\varpi > 0$, such that $\varpi \leq 1/\delta$, the LMIs (48) and the following LMIs are satisfied:

$$\bar{\Gamma}(i, i+1) < 0, \;\; \forall i \in \mathbf{I}[1, N-1]; \;\; \bar{\Gamma}(N, 1) < 0, \tag{57}$$

where

$$\bar{\Gamma}(\sigma(k), \sigma(k+1)) \triangleq \begin{bmatrix} \bar{\Gamma}_{11} & \bar{\Gamma}_{12} & \bar{\Gamma}_{13} & \bar{\Gamma}_{14} & \bar{\Gamma}_{15} \\ * & -2\bar{H}_{\sigma(k)} & \bar{\Gamma}_{23} & \bar{\Gamma}_{24} & 0 \\ * & * & \bar{\Gamma}_{33} & \bar{\Gamma}_{34} & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\gamma I \end{bmatrix}$$

with

$$\begin{aligned}
\bar{\Gamma}_{11} \triangleq & \bar{P}_{\sigma(k+1)} - \bar{P}_{\sigma(k)} + \Lambda_1 + \Lambda_1^T, \\
\bar{\Gamma}_{12} \triangleq & \begin{bmatrix} (\bar{D}^c_{\sigma(k)} \Phi_{\sigma(k)} C)^T \\ (\bar{C}^c_{\sigma(k)})^T \\ \mu(\bar{D}^c_{\sigma(k)} \tilde{\Phi}_{\sigma(k)})^T \end{bmatrix} - \begin{bmatrix} \bar{E}^c_{\sigma(k)} \\ B\bar{H}_{\sigma(k)} \\ 0 \end{bmatrix} - \bar{G}^T_{\sigma(k)}, \\
\bar{\Gamma}_{13} \triangleq & -\Lambda_3 + \lambda \Lambda_1^T + \bar{P}_{\sigma(k+1)}, \;\; \bar{\Gamma}_{15} = \mathbb{S}^T F^T, \\
\bar{\Gamma}_{14} \triangleq & \begin{bmatrix} RD + \bar{B}^c_{\sigma(k)} \Phi_{\sigma(k)} E \\ D + B\bar{D}^c_{\sigma(k)} \Phi_{\sigma(k)} E \\ \Phi_{\sigma(k)} E \end{bmatrix}, \;\; \bar{\Gamma}_{24} \triangleq \bar{D}^c_{\sigma(k)} \Phi_{\sigma(k)} E, \\
\bar{\Gamma}_{23} \triangleq & -\lambda [(\bar{E}^c_{\sigma(k)})^T \quad (B\bar{H}_{\sigma(k)})^T \quad 0], \\
\bar{\Gamma}_{33} \triangleq & P_{\sigma(k+1)} - \lambda(\Lambda_3 + \Lambda_3^T), \;\; \bar{\Gamma}_{34} \triangleq \lambda \bar{\Gamma}_{14}.
\end{aligned}$$

Remark 4: In obtaining our main results, the modified sector condition (12) is utilized to handle the nonlinear term $\psi(u)$ induced by saturations. Here, it should be pointed out that the polytopic approach might be more effective in dealing with saturations for the multiple-input systems [31]. Along the similar analysis as this paper, some alternative conditions can be readily obtained by incorporating the polytopic approach handling the saturations. Nevertheless, it is worth mentioning that the polytopic approach could lead to a larger numerical complexity.

Finally, let us deal with the optimization problems involved in our main results. For the case with disturbances, we can first measure the largest disturbance tolerance level δ_M. Then, for a given scalar $\delta \leq \delta_M$, we can obtain the minimum H_∞ performance level γ_m. Note that, under the zero initial condition, the scalar ϖ in (48) should be modified as $1/\delta$. The corresponding optimization problems are easily formulated and thus omitted here.

For the case without disturbances, it is necessary to maximize the set of admissible initial conditions in designing the controller. In this case, the rows and columns involving $\omega(k)$ in LMIs (46), (47) and (57) can be removed. Here, we are interested in estimating the admissible initial conditions $\phi_x(k)$ ($k \in \mathbf{I}[-\tau_2, 0]$) of the system (1) under the assumption that $\hat{x}(0) = \bar{y}(-1) = 0$. As in [4], we assume that the initial conditions $\phi_x(k)$ belong to the set

$$\mathscr{X}_\rho \triangleq \big\{\phi_x(k) : \max_{k\in\mathbf{I}[-\tau_2,0]} \|\phi_x(k)\| \leq \rho_1, \ \max_{k\in\mathbf{I}[-\tau_2,-1]} \|\Delta\phi_x(k)\| \leq \rho_2\big\}, \tag{58}$$

where $\Delta\phi_x(k) \triangleq \phi_x(k+1) - \phi_x(k)$, $\rho_1 > 0$ and $\rho_2 > 0$ are two scalars.

Remark 5: In fact, we can remove the second constraint involved in the set $\mathscr{X}_\rho$. In this case, the initial conditions $\phi_x(k)$ belong to the set $\tilde{\mathscr{X}}_\rho \triangleq \big\{\phi_x(k) : \max_{k\in\mathbf{I}[-\tau_2,0]} \|\phi_x(k)\| \leq \rho\big\}$. Using the fact that $\|\Delta\phi_x(k)\| \leq \|\phi_x(k+1)\| + \|\phi_x(k)\|$, it is seen that the relation $\max_{k\in\mathbf{I}[-\tau_2,-1]} \|\Delta\phi_x(k)\| \leq 2\rho$ is implied in the set $\tilde{\mathscr{X}}_\rho$. Compared with the set $\tilde{\mathscr{X}}_\rho$, it is clear that the set $\mathscr{X}_\rho$ is more flexible in characterizing the admissible initial conditions.

Using the assumption $\sigma(0) = 1$ and noting that $\mathcal{I}P_1\mathcal{I}^T = \mathcal{I}\bar{P}_1\mathcal{I}^T \triangleq P_{111}$, it is seen that the scalars ρ_1 and ρ_2 involved in the initial condition set $\mathscr{X}_\rho$ satisfy the following inequality:

$$V_{\sigma(0)}(0) \leq \psi_1\rho_1^2 + \psi_2\rho_2^2 \leq 1/\varpi, \tag{59}$$

where

$$\begin{aligned}\psi_1 &\triangleq \lambda_M(\mathcal{I}\bar{P}_1\mathcal{I}^T) + \tau_1\lambda_M(Q_1) + \tilde{\tau}\lambda_M(Q_2) + (1/2)(\tilde{\tau}+1)(\tau_1+\tau_2)\lambda_M(Q_3),\\ \psi_2 &\triangleq (1/2)[\tau_1^2(\tau_1+1)\lambda_M(Z_1) + \tilde{\tau}^2(\tau_1+\tau_2+1)\lambda_M(Z_2)].\end{aligned}$$

Let us introduce the following LMIs:

$$\mathcal{I}\bar{P}_1\mathcal{I}^T \triangleq P_{111} \leq \chi_p I, \tag{60}$$

$$Q_1 \leq \chi_{q_1} I, \ Q_2 \leq \chi_{q_2} I, \ Q_3 \leq \chi_{q_3} I, \tag{61}$$

$$Z_1 \leq \chi_{z_1} I, \ Z_2 \leq \chi_{z_2} I. \tag{62}$$

Then, the maximization of the set $\mathscr{X}_\rho$ can be described by the following optimization problem:

Prob. 1. $$\max_{\bar{P}_i,Q_1,Q_2,Q_3,Z_1,Z_2,L_i,R,S,Y,\bar{A}_i^c,\bar{B}_i^c,\bar{C}_i^c,\bar{D}_i^c,\bar{E}_i^c,\bar{G}_i,\bar{H}_i,\chi_p,\chi_{q_1},\chi_{q_2},\chi_{q_3},\chi_{z_1},\chi_{z_2}} \nu,$$
$$s.t., \text{ LMIs (15), (46)} - \text{(48) and (60)} - \text{(62) hold,}$$

where

$$\begin{aligned}\nu \triangleq &\chi_p + \tau_1\chi_{q_1} + \tilde{\tau}\chi_{q_2} + (1/2)(\tilde{\tau}+1)(\tau_1+\tau_2)\chi_{q_3}\\ &+ (1/2)[\tau_1^2(\tau_1+1)\chi_{z_1} + \tilde{\tau}^2(\tau_1+\tau_2+1)\chi_{z_2}].\end{aligned}$$

In case that $\tau(k)$ is not contained in (1), the initial conditions $\phi_x(0) \triangleq x_0$ are assumed to belong to an ellipsoid

$$\tilde{\mathscr{X}}_\varpi \triangleq \big\{x_0 \in \mathbb{R}^n : x_0^T P_{111} x_0 \leq 1/\varpi\big\}. \tag{63}$$

Correspondingly, the maximization of the ellipsoid $\tilde{\mathscr{X}}_{\varpi}$ can be written as follows:

$$\textit{Prob. 2.} \max_{\bar{P}_i, R, S, Y, \bar{A}_i^c, \bar{B}_i^c, \bar{C}_i^c, \bar{D}_i^c, \bar{E}_i^c, \bar{G}_i, \bar{H}_i, \chi_p} \chi_p,$$
$$s.t., \text{ LMIs (48), (57) and (60) hold.}$$

Remark 6: Recently, the dynamic output-feedback control problem has been considered in [40], [41] for discrete-time switched delay systems with actuator saturations. However, the switching $\sigma(k)$ in this paper is different from that in [40], [41]. Moreover, the measurement model (6) makes use of the information of its previous step that is also different from the system outputs in [40], [41]. In addition, the time delays in [40], [41] are assumed to be constant. It is obvious the results in [40], [41] cannot be applied to the system (1) and the model (6).

Remark 7: In the past decade, the stability analysis and control synthesis for NCSs under different communication protocols have received considerable research attention [8], [11], [20], [21], [38], but the actuator saturations have not been taken into account in most existing literature except [21]. Note that the results in [21] are applicable to the case of two sensor nodes for systems without disturbances or state delays. As such, the results obtained in this paper are more general than existing ones in terms of the considered system and the practical applicability.

Remark 8: In order to reduce the potential conservatism, the adjustable parameters λ, μ, α, β_1, β_2, β_3 and β_4 are introduced in Theorem 2. For such parameters, we can first select them through experience to ensure the feasibility of the optimization problems involved in Theorem 2, and then perform the linear search within their neighbours to obtain the optimized performance indices. For the linear search, one can first search the scalar λ by fixing the last 6 parameters, then search the scalar μ by fixing other 6 parameters, and until finally search β_4 by fixing first 6 parameters. To further decrease the conservatism, one can repeat the above search process.

IV. Numerical examples

Example 1: Consider the discrete-time system (1) with the parameters

$$A = \begin{bmatrix} 1 & 0.001 & 0 & 0 \\ 0 & 1 & -0.0005 & 0 \\ 0 & 0 & 1 & 0.001 \\ 0 & 0 & 0.0448 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0.0064 \\ 0 \\ -0.0280 \end{bmatrix},$$
$$A_d = 0, \ C = I, \ D = E = F = 0, \ \bar{u}_{(1)} = 50.$$

The system is a linearized model of the inverted pendulum on a cart [21]. As in [47], we rewrite the measurement output $y(k)$ as (3) with $p_1 = p_2 = N = 2$. By solving Prob. 2 with $\lambda = \varpi = 1$, $\mu = 8 \times 10^8$, and the additional constraints $R \leq 5000I$, $S \leq 5000I$, $\bar{C}_1^c(\bar{C}_1^c)^T \leq 100$, $\bar{D}_1^c(\bar{D}_1^c)^T \leq 100$, we have $\chi_p = 0.0062$ and

$$A_1^c = \begin{bmatrix} 0.0008 & 0.0011 & 0.0000 & 0.0171 \\ -0.0370 & -0.0548 & 0.0027 & -0.8340 \\ -0.0002 & 0.0012 & 0.9921 & 0.1574 \\ 0.0035 & 0.0049 & -0.0070 & 0.9934 \end{bmatrix},$$

$$B_1^c = \begin{bmatrix} -0.9133 & 0.0213 & 0.0000 & 0.0000 \\ -0.0632 & -1.0863 & -0.0000 & -0.0000 \\ -0.0002 & 0.0012 & -0.0000 & -0.0000 \\ 0.0033 & 0.0050 & 0.0000 & 0.0000 \end{bmatrix},$$

$$C_1^c = \begin{bmatrix} 5.6026 & 8.3106 & -0.3886 & 126.4517 \end{bmatrix},$$
$$D_1^c = \begin{bmatrix} 5.3523 & 8.4468 & 0.0001 & 0.0000 \end{bmatrix},$$
$$A_2^c = \begin{bmatrix} 0.9994 & -0.0024 & -0.0001 & -0.0000 \\ 0.0293 & 1.1586 & -0.0002 & -0.0000 \\ 0.0002 & -0.0003 & 0.0000 & 0.0000 \\ -0.0000 & -0.0000 & 0.0000 & -0.0000 \end{bmatrix},$$
$$B_2^c = \begin{bmatrix} 0.0000 & 0.0000 & 0.0086 & 0.0072 \\ -0.0000 & -0.0000 & -0.4217 & -0.3507 \\ -0.0000 & -0.0000 & -0.0032 & -0.0005 \\ 0.0000 & -0.0000 & 0.0004 & -0.0000 \end{bmatrix},$$
$$C_2^c = \begin{bmatrix} -4.4386 & -24.0363 & 0.0051 & 0.0001 \end{bmatrix},$$
$$D_2^c = \begin{bmatrix} 0.0000 & 0.0000 & 64.0069 & 53.1689 \end{bmatrix},$$
$$E_1^c = E_2^c = \begin{bmatrix} 0.0001 & -0.0066 & -0.0000 & 0.0000 \end{bmatrix}^T,$$
$$P_{111} = \begin{bmatrix} 0.0050 & 0.0004 & 0.0000 & -0.0000 \\ 0.0004 & 0.0054 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0062 & 0.0001 \\ -0.0000 & 0.0000 & 0.0001 & 0.0056 \end{bmatrix}.$$

Applying Theorem 1 and Remark 3 in [21] with $\lambda = \beta = 1$, $\mu = 1.02$, $\sigma = 0.01$ and $\eta_m = \eta_M = \tau_M = 0$, it follows that the largest ball of admissible initial conditions is $\|x_0\| < 0.5250$ under the static output-feedback controller with $K = [5.825 \quad 5.883 \quad 24.941 \quad 5.140]$. Noting that our obtained largest ball of the set $\tilde{\mathscr{X}}_\varpi$ is $\|x_0\| \leq 1/\sqrt{\varpi \lambda_M(P_{111})} = 12.6787$, it is obvious that our proposed control strategy can provide a larger estimate of admissible initial conditions. In Fig. 1, the state evolution of the closed-loop system is plotted with $x_0 = [12 \quad -7 \quad -3 \quad -1]^T \in \tilde{\mathscr{X}}_\varpi$. It is seen from Fig. 1 that the stability of the closed-loop system can be ensured.

For this example, applying Theorem 1 with $\bar{A}_i^c \triangleq \bar{A}^c$, $\bar{B}_i^c \triangleq \bar{B}^c$, $\bar{C}_i^c \triangleq \bar{C}^c$, $\bar{D}_i^c \triangleq \bar{D}^c$, $\bar{E}_i^c \triangleq \bar{E}^c$ $(i \in \mathbf{I}[1, N])$, and the same parameter selection and the same additional constraints as above, we cannot find the feasible solution, which means that the switched controller might be more effective than the non-switched controller.

Example 2: [4] Consider the system (1) and (3) with the following parameters:

$$A = \begin{bmatrix} 1.1 & 0.15 \\ 0.03 & 0.8 \end{bmatrix}, \ A_d = \begin{bmatrix} 0 & -0.1 \\ 0 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 0.1 \end{bmatrix},$$
$$C = I, \ D = E = [0.1 \quad 0.1]^T, \ F = [1 \quad 1], \ \bar{u}_{(1)} = 15,$$
$$\tau_1 = 1, \ \tau_2 = 3, \ p_1 = p_2 = 1, \ N = 2.$$

Under the zero initial condition, applying Theorem 2 with $\lambda = 1.7$, $\mu = 10^4$, $\alpha = 0.15$, $\beta_1 = 0.7$, $\beta_2 = 10^6$, $\beta_3 = 0.1$ and $\beta_4 = 0.4$, we obtain the largest disturbance tolerance level $\delta_M = 4.5018 \times 10^4$. Letting $\delta = 2 \times 10^4 \leq \delta_M$, and applying Theorem 2 again with $\lambda = 1$, $\alpha = 0.1$, $\beta_1 = 0.5$, $\beta_2 = \mu = 10^4$, $\beta_3 = 0.2$ and $\beta_4 = 0.6$, we have the minimum H_∞ performance level $\gamma_m = 0.6812$. Correspondingly, the controller gain matrices are given as

$$A_1^c = \begin{bmatrix} 0.8142 & 0.6020 \\ -0.0014 & -0.0209 \end{bmatrix}, \ B_1^c = \begin{bmatrix} -4.9802 & 0.0003 \\ 11.1554 & -0.0002 \end{bmatrix},$$

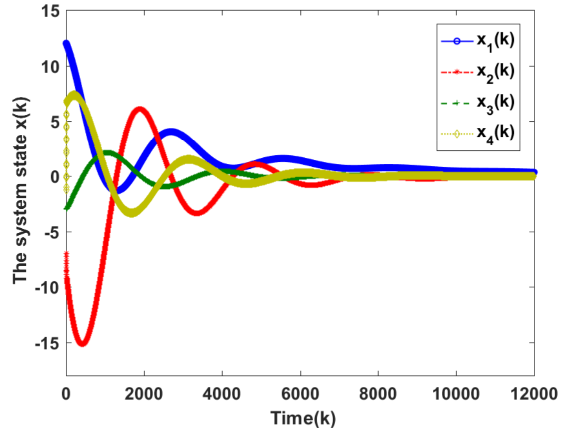

Fig. 1. State evolution under the proposed controller.

$$C_1^c = \begin{bmatrix} -0.0037 & -0.0015 \end{bmatrix}, \ D_1^c = \begin{bmatrix} -0.6304 & -0.0000 \end{bmatrix},$$

$$A_2^c = \begin{bmatrix} 0.1305 & -0.0961 \\ -0.2688 & 0.2599 \end{bmatrix}, \ B_2^c = \begin{bmatrix} -0.0001 & 22.7256 \\ 0.0002 & 9.0595 \end{bmatrix},$$

$$C_2^c = \begin{bmatrix} 0.0126 & -0.0152 \end{bmatrix}, \ D_2^c = \begin{bmatrix} 0.0000 & -0.5115 \end{bmatrix},$$

$$E_1^c = \begin{bmatrix} 4.0386 & 29.3856 \end{bmatrix}^T, \ E_2^c = \begin{bmatrix} -23.8098 & 43.4463 \end{bmatrix}^T.$$

Next, we consider the case that the disturbances $\omega(k)$ are absent. By solving Prob. 1 with $\lambda = 1.2$, $\alpha = 10^{-5}$, $\beta_1 = \beta_3 = \beta_4 = \mu = 10^5$, $\beta_3 = 10^4$, $\varpi = 1$, and the additional constraints $S \leq 300I$, $\bar{C}_1^c(\bar{C}_1^c)^T \leq 10^4$, $\bar{C}_2^c(\bar{C}_2^c)^T \leq 10^4$, we obtain the following controller gain matrices:

$$A_1^c = \begin{bmatrix} 0.5969 & -0.4113 \\ 0.1299 & -0.0323 \end{bmatrix}, \ B_1^c = \begin{bmatrix} -0.0019 & 0.0000 \\ 0.0002 & 0.0000 \end{bmatrix},$$

$$C_1^c = \begin{bmatrix} -122.4041 & 71.0628 \end{bmatrix}, \ D_1^c = \begin{bmatrix} -0.7700 & -0.0019 \end{bmatrix},$$

$$A_2^c = \begin{bmatrix} 0.3552 & -0.7422 \\ 0.0341 & 0.0304 \end{bmatrix}, \ B_2^c = \begin{bmatrix} -0.0000 & 0.0014 \\ -0.0000 & 0.0003 \end{bmatrix},$$

$$C_2^c = \begin{bmatrix} 175.2796 & 64.0781 \end{bmatrix}, \ D_2^c = \begin{bmatrix} -0.0030 & -0.4849 \end{bmatrix},$$

$$E_1^c = \begin{bmatrix} -0.0056 & -0.0017 \end{bmatrix}^T, \ E_2^c = \begin{bmatrix} 0.0030 & 0.0004 \end{bmatrix}^T.$$

Meanwhile, it follows that the scalars ρ_1 and ρ_2 involved in the set $\mathscr{X}_\rho$ satisfy $0.0020\rho_1^2 + 4.8148 \times 10^{-5}\rho_2^2 \leq 1$.

In Figs. 2-3, we plot the state evolutions by using the above obtained controller gains, where the time delay $\tau(k)$ is randomly generated within $[1, 3]$. In Fig. 2, the disturbance is chosen as $\omega(k) = 50$ for $0 \leq k \leq 7$ and $\omega(k) = 0$ for $k \geq 8$, and in Fig. 3, the initial condition is selected as $\phi_x(k) = [21 \quad 7]^T$ $(-3 \leq k \leq 0)$. Noting that the open-loop system is not stable, it is clear from Figs. 2-3 that our proposed control scheme is indeed effective.

Remark 9: In order to avoid excessively high controller gain, the constraints $R \leq 5000I$, $S \leq 5000I$, $\bar{C}_1^c(\bar{C}_1^c)^T \leq 100$, $\bar{D}_1^c(\bar{D}_1^c)^T \leq 100$ are introduced in solving Prob. 2 in Example 1, and the constraints $S \leq 300I$, $\bar{C}_1^c(\bar{C}_1^c)^T \leq 10^4$, $\bar{C}_2^c(\bar{C}_2^c)^T \leq 10^4$ are imposed in solving Prob. 1 in Example 2. Such constraints can be readily written LMIs.

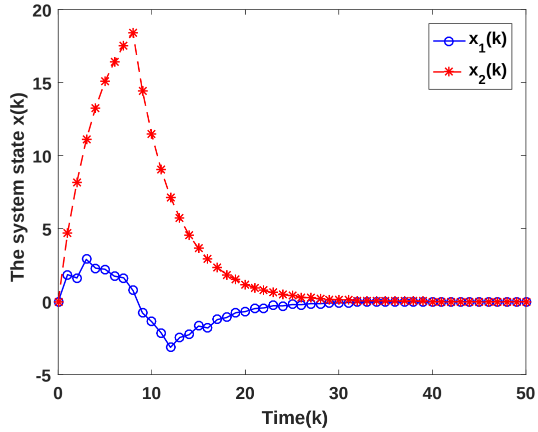

Fig. 2. State evolution without exogenous disturbance.

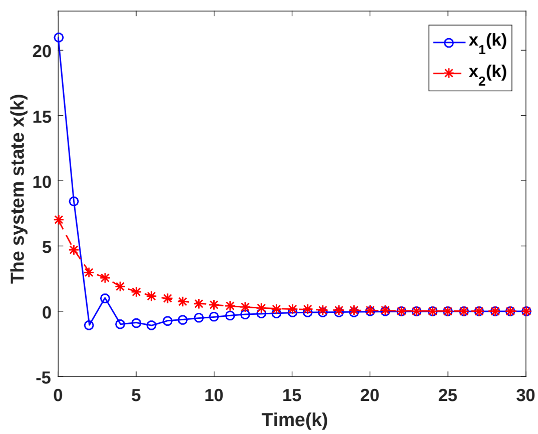

Fig. 3. State evolution with exogenous disturbance.

V. Conclusions

In this paper, the dynamic output-feedback H_∞ control problem has been studied for discrete state-delay systems with exogenous disturbances and actuator saturations under RR protocol. Based on the switched system approach, the L-K functional and the generalized sector condition, a sufficient condition has been proposed under which the closed-loop systems can possess the desirable performance indices. The controller gain has been explicitly characterized in terms of solvability of LMIs. The results concerning the special case without time delay has also been given. Moreover, the optimization problems about the performance indices have been discussed. The less conservatism and feasibility of the obtained results have been shown by simulation examples. Our proposed results can be readily extended to more general systems, such as uncertain systems [4] and T-S fuzzy systems [10], [36].

In this paper, the techniques handling the time delay are conservative to a certain extent. By incorporating the augmented L-K functionals and some advanced inequalities [27], [37], we can obtain less conservative conditions, which is our future work. Moreover, it is worth mentioning that the communication delay is ignored in this paper. If the communication delay is constant, we can modify the model (6) as $\bar{y}(k) = \Phi_{\sigma(k)}y(k-h) + (I - \Phi_{\sigma(k)})\bar{y}(k-1)$, where $h > 0$ denotes the time delay. Then, the corresponding results can be easily obtained. If the communication delay is time-varying, the phenomenon of packet disorders might occur and the further research should be given. In addition, it is seen that the inequalities (52)-(54) are used to obtain LMI-based conditions, which brings the conservatism. A further research direction is to develop some more effective linearized techniques.

VI. Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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