New Journal and we have not received input yet 33 (2022) 101381

Contents lists available at ScienceDirect

Thermal Science and Engineering Progress

journal homepage: www.sciencedirect.com/journal/thermal-science-and-engineering-progress

An efficient optimization of an irreversible Ericsson refrigeration cycle based on thermo-ecological criteria

Parth Prajapati [a], Vivek Patel [a], Hussam Jouhara [b,c,*]

[a] Department of Mechanical Engineering, Pandit Deendayal Energy University, Gandhinagar 382421, Gujarat, India
[b] Heat Pipe and Thermal Management Research Group, College of Engineering, Design and Physical Sciences, Brunel University London, UB8 3PH, UK
[c] Vytautas Magnus University, Studentu Str. 11, LT-53362, Akademija, Kaunas Distr, Lithuania

A R T I C L E I N F O

Keywords:
Ericsson cycle
Coefficient of performance
Cooling load
Ecological function
Optimization

A B S T R A C T

In the present work, a multi-objective optimization study of an irreversible Ericsson cycle is carried out to evaluate a cryogenic refrigerator system based on thermal and ecological performance. The coefficient of performance, cooling load and input power are considered as objective functions along with an ecological objective function and the effect of variables like the effectiveness of the heat sink heat exchanger, the effectiveness of the heat source heat exchanger, the temperature ratio of heat sink and heat source and the capacitance rate of the heat sink and heat source. Objective functions are optimized using a heat transfer search optimization algorithm for two different scenarios, and the Pareto front is obtained. The decision making method namely TOPSIS is adopted to identify the best solution among numerous optimal solutions for each scenario. The effect of design variables on thermo-ecological objective functions is explored and presented. A maximum cooling load of 3.27 kW at heat sink temperature of 172 K is obtained for a cryogenic refrigerator operating on an irreversible Ericsson cycle at an input power of 3.91 kW with a COP of the system as 0.84. Finally, the distribution of the design variables during the optimization is identified and presented.

Introduction

Heat engines work on the principle of utilizing thermal energy from a high-temperature thermal reservoir (heat source) and converting it into useful power while the remaining energy gets rejected to a low-temperature reservoir (heat sink) [1,2]. To reverse the cycle, an external amount of work should be done on the system to transfer energy from a low-temperature reservoir to a high-temperature reservoir [3]. Such a concept is beneficial for refrigeration systems because of their high energy conversion efficiency, and it is easily adaptable in practical engineering problems [4]. One such system is the Ericsson refrigeration system which works on the basis of the Ericsson cycle to produce low temperatures. The efficiency of the Ericsson cycle is in-line with Carnot cycle and Sterling cycle and hence it can be adopted for low temperature production. Several researchers have made an effort to use the Ericsson cycle to design practical systems wherein magnetic material or gas is used as the working fluid.

The Ericsson cycle is similar to the Carnot cycle, except that two isentropic processes are replaced by two isobaric regeneration processes. A heat exchanger regarded as a thermal energy storage device, working as a regenerator, is used to absorb and supply heat energy to the working fluid at constant pressure [5]. The working principle of an Ericsson refrigeration cycle is presented in Fig. 1. Steady flow devices, a compressor and a turbine, are co-axially coupled on a same shaft and used for executing isothermal compression and expansion processes. Heat reservoirs of defined temperatures, namely the heat source and the heat sink, are used to supply energy to the working fluid during expansion and absorb energy from the working fluid during the compression process. Thermal reservoirs are used to ensure isothermal processes are carried out within thermodynamic limits. However ideal cycles, which are reversible in nature, require an infinite time or an infinite area of heat exchanger for the process to occur completely reversibly, which is not feasible practically. Hence, a definite time period is allowed for the heat transfer process, which incurs irreversibility in the system and entropy generation [6].

The concept of an ecological function, which is defined in this paper in Section “Thermal model”, was introduced by Angulo-Brown [7] to study the heat transfer in Carnot engines for a finite duration of time. The Carnot cycle works between the extreme temperatures T_H and T_L and, when the ambient temperature is not the same as the lower

* Corresponding author.
E-mail address: hussam.jouhara@brunel.ac.uk (H. Jouhara).

https://doi.org/10.1016/j.tsep.2022.101381
Received 29 April 2022; Received in revised form 14 June 2022; Accepted 16 June 2022
Available online 20 June 2022

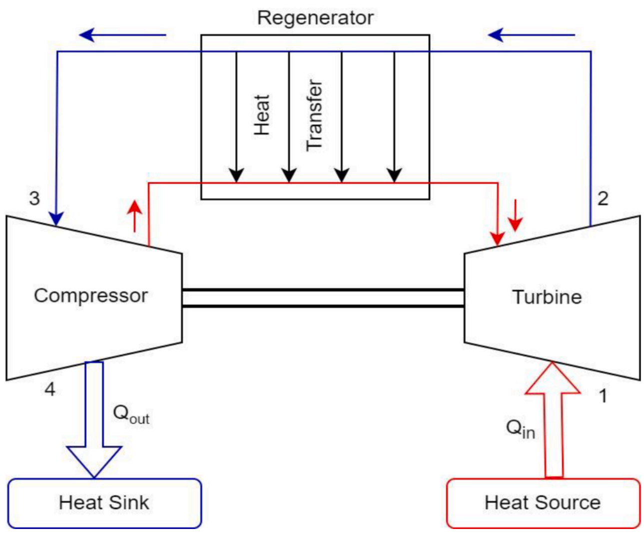

Fig. 1. Working principle of Ericsson cycle.

temperature (heat sink) T_L, Yan [8] suggested using the ambient temperature to calculate the ecological function. Numerous researchers like Heng Wu [9], Ahmadi [10], Tyagi [11,12], have used the concept of ecological functions in different power cycles.

In real world engineering problems, system performance is affected by multiple factors and it is preferred to operate the system at optimum condition. Multi-objective optimization offers an optimum solution for individual design variables, however it is difficult to find when the objective functions are of conflicting nature [13]. The multi-objective optimization technique offers fine tuning of the design parameters, thus attaining the goal of either maximizing or minimizing the objective function [14]. A series of numerous optimal solutions can be plotted on the Pareto front where each of the objective functions is satisfied within acceptable limits or constraints. The Pareto front offers a range of optimum solutions to the user for multiple design variables to be used for the operation of the system. Heat transfer search (HTS) optimization is one such novel optimization technique which mimics the laws of thermodynamics to optimize the objective functions with reference to three modes of heat transfer namely conduction, convection and radiation [15,16].

During the multi-objective optimization, the Pareto front offers multiple optimal solutions satisfying multiple predefined objectives. Selecting an ideal optimum solution refers to multiple attribute decision making (MADM) methodology and it is imperative to define an ideal operating condition for the system. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is an easy and uncomplicated ranking method developed by Hwang and Yoon [17] to select alternative optimal solutions from the most positive ideal solutions. The MADM method is widely adopted because of its elementary underlying concept that the best solution is always nearer to the ideal solution and farthest from the negative solution [18]. During decision making situations TOPSIS holds its importance and applications in various fields like manufacturing decision making [19], technology and strategy selection [20], environmental and sustainable assessment [21,22] and many more.

In the current work, a multi-objective optimization study of an irreversible Ericsson refrigeration cycle is carried out with the objective

Table 1
Bounds of design variable for optimization.

Design Variable	Lower Bound	Upper Bound
Heat source capacitance rate, kW/K	0.8	1.5
Heat sink capacitance rate, kW/K	0.8	1.5
Temperature ratio	1.7	1.9
Effectiveness of hot side heat exchanger	0.8	0.9
Effectiveness of cold side heat exchanger	0.8	0.9
Temperature of heat sink, K	172	177

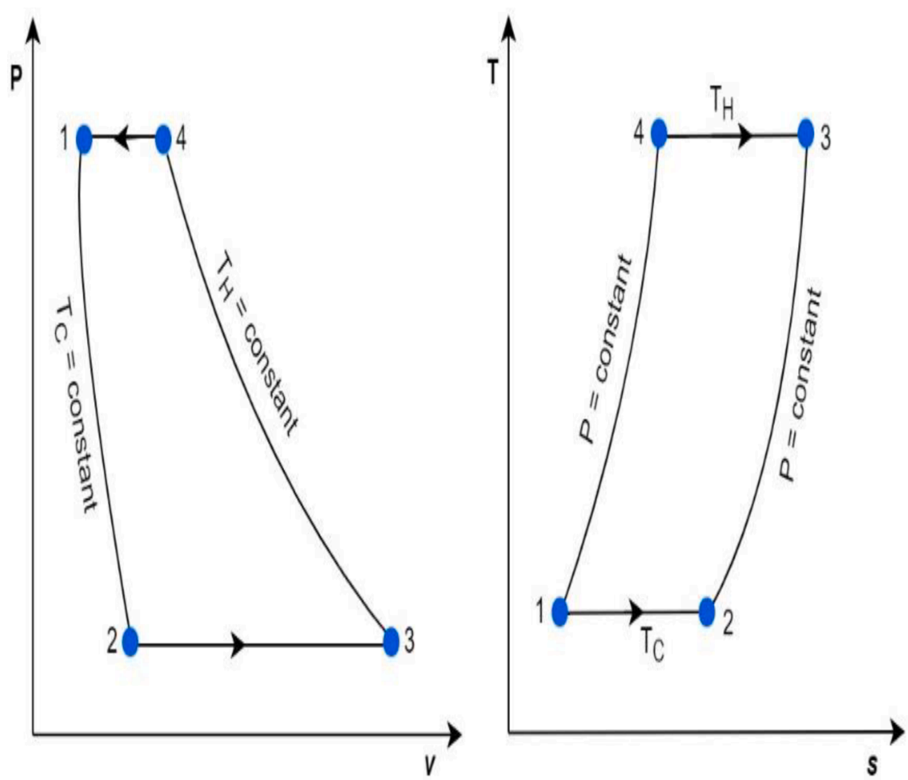

Fig. 2. P-v and T-s diagram of ideal Ericsson cycle.

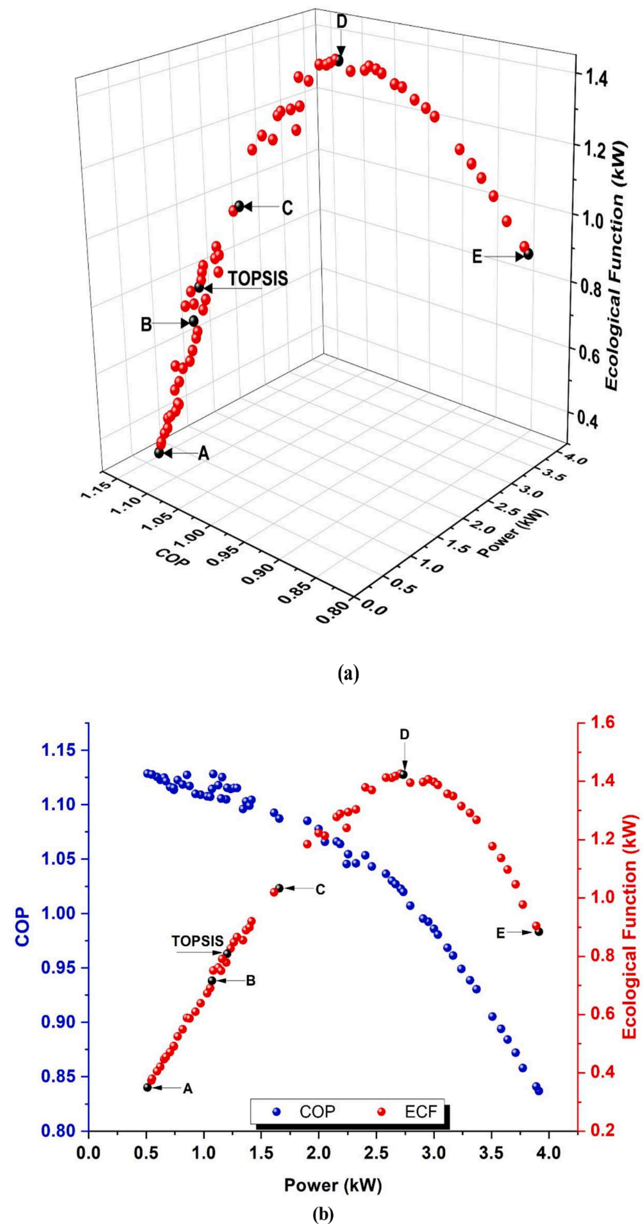

Fig. 3. (a) and (b) Pareto optimal curve for COP, Power and Ecological function – Scenario 1.

of maximizing the COP, cooling load and ecological function with minimum input power. The design variables considered for the study are heat capacitance rate of heat source and heat sink, temperature ratio, effectiveness of the heat exchanger on the heat source side and the heat sink side and the temperature of the cold side. A heat transfer search optimization algorithm is used to obtain multiple optimum solutions in the form of the Pareto front which satisfies the constraints of the design variables and the objective functions within permissible limits. The TOPSIS selection criterion is implemented to select the best solution among numerous optimal solutions and closest to the best positive solution. The population distribution of the design variables is plotted to study the spread within the bounds. A sensitivity analysis of the variables is carried out for the selected Pareto points to study the effect on the objective functions.

The work's novelty lies in considering ecological optimization during the multi-objective study along with the thermodynamic optimization. An ecological study in the form of ecological coefficient of performance and ecological objective function enables researchers to

Table 2
Optimal parameters for design points (A) - (E) during multi-objective optimization for Scenario 1.

Variables/Objective Functions	Optimal design points					
	A	B	C	D	E	TOPSIS
Heat sink temperature, K	172	172.27	172.29	172.6	172	172.21
Temperature ratio *(x)*	1.70	1.71	1.72	1.76	1.88	1.71
Heat source capacitance rate, kW/K	0.80	1.07	1.26	1.49	1.49	1.28
Heat sink capacitance rate, kW/K	0.80	1.19	1.37	1.47	1.49	1.27
Effectiveness of cold side heat exchanger	0.80	0.86	0.86	0.90	0.89	0.87
Effectiveness of hot side heat exchanger	0.80	0.85	0.84	0.89	0.87	0.86
Power, kW	**0.51**	1.07	1.66	2.73	3.91	1.20
Coefficient of Performance	**1.13**	1.11	1.09	1.02	0.84	1.12
Ecological function, kW	0.35	0.72	1.03	**1.42**	0.88	0.81

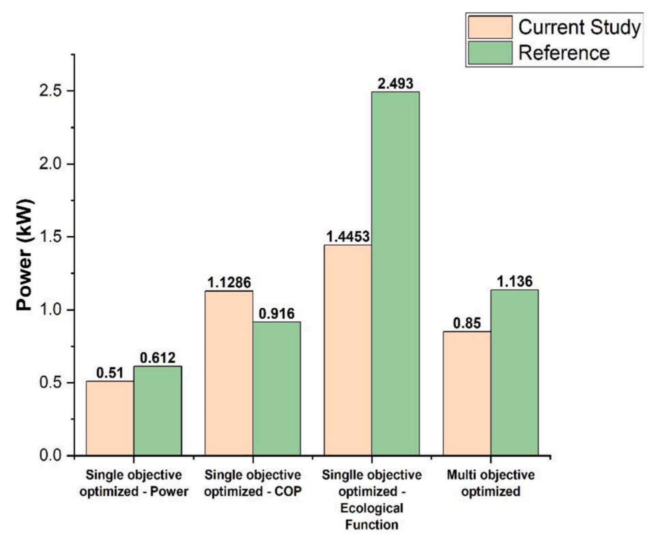

Fig. 4. Comparison of input powers for current study and reference study.

understand the entropy generation rate and energy losses in the irreversible Ericsson refrigeration cycle. Thermodynamic optimization focuses on the system's cooling load, input power, and COP. In contrast, ecological optimization focuses on the available energy. A novel approach is implemented and presented for a thermodynamic system by considering the thermo-ecological criteria.

The current work is presented in the following manner. Section "System description" consists of a system description of the Ericsson refrigeration cycle and explains the working principles of the ideal cycle. Section "Thermal model" presents a detailed thermal model of Ericsson cycle to calculate input power, coefficient of performance (COP), cooling load and ecological function (ECF). Section "Heat Transfer Search (HTS) optimization algorithm" describes the heat transfer search optimization algorithm and Section "Results and Discussion" presents results of multi-objective optimization, population distribution and sensitivity analysis of design variables. The major findings of the work are presented and concluded in Section "Conclusion".

System description

The Ericsson cycle is a closed thermodynamic cycle which uses magnetic material or gas as the working fluid for energy transfer [23].

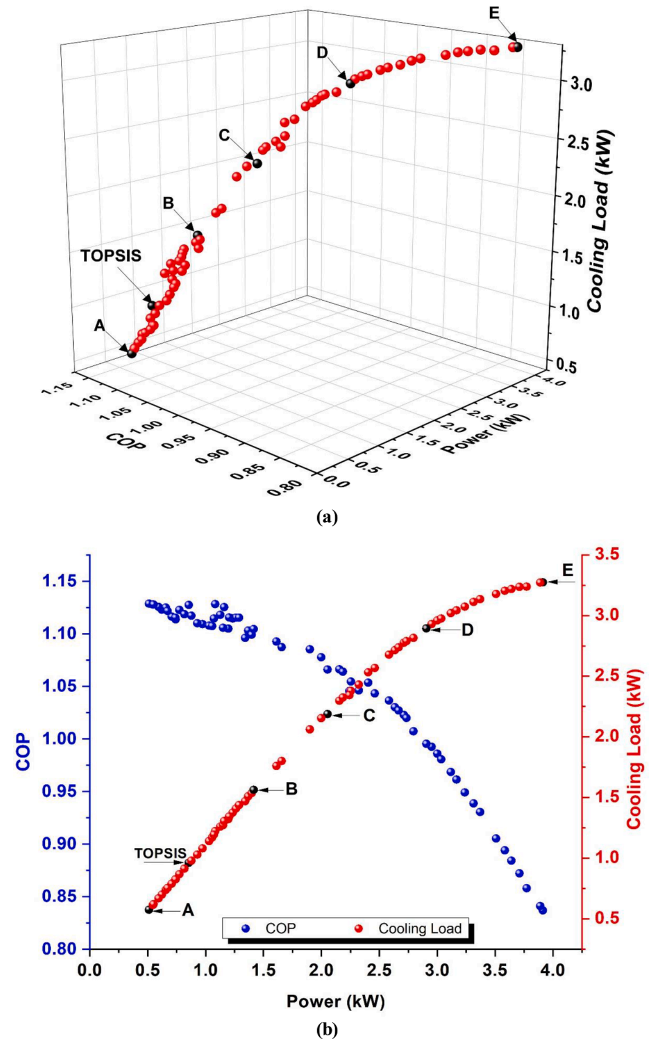

Fig. 5. (a) and (b) Pareto optimal curve for COP, Power and Cooling load – Scenario 2.

Table 3
Optimal parameters for design points *(A)* - *(E)* during multi objective optimization for Scenario 2.

Variables/Objective Functions	Optimal design points					
	A	B	C	D	E	TOPSIS
Heat sink temperature, K	172	172.52	172.8	172.54	172	172.16
Temperature ratio *(x)*	1.70	1.71	1.73	1.77	1.88	1.70
Heat source capacitance rate, kW/K	0.8	1.17	1.32	1.48	1.49	1.16
Heat sink capacitance rate, kW/K	0.8	1.36	1.34	1.46	1.49	1.16
Effectiveness of cold side heat exchanger	0.8	0.81	0.87	0.89	0.89	0.86
Effectiveness of hot side heat exchanger	0.8	0.88	0.86	0.89	0.87	0.85
Power, kW	**0.51**	1.42	2.05	2.91	3.91	0.85
Coefficient of Performance	**1.13**	1.10	1.07	1	0.84	1.13
Cooling Load, kW	0.58	1.56	2.19	2.89	**3.27**	0.96

Fig. 2 shows the P-v and T-s diagrams of an ideal Ericsson cycle using an ideal gas as the working fluid. It comprises two isothermal and two isobaric processes [24]. The turbine and compressor are used as steady flow devices to perform isothermal expansion and compression respectively. The ideal expansion process results in a decrease in temperature of the working fluid but a heat source is employed to provide the additional amount of heat required to maintain the isothermal process. The expansion process in the conventional cycle is modeled by a constant temperature heat addition process 1–2 from a heat source at temperature T_c. The heat source is assumed to be of finite heat capacity and during the heat transfer process, the temperature of the heat source reduces from T_{L1} to T_{L2}. A counter flow heat exchanger serving as a regenerator is used as an energy storage device to exchange energy with the working fluid. In a regenerator, energy is transferred to the working fluid during the isobaric heat transfer process 2–3. Similar to the expansion process, the compression is modeled by a constant temperature heat rejection process 3–4 to a heat sink at temperature T_H. The heat sink holds finite heat capacity and during the heat transfer process the temperature changes from T_{H1} to T_{H2}. During the last process 4–1 of the cycle, heat is transferred from the working fluid to the regenerator which is modeled as an isobaric heat exchange process.

An ideal regeneration process (heat exchange between regenerator and working fluid) requires an infinite area of the heat exchanger or an infinite amount of time and it is not practically possible to construct a device working on such processes. Further, during the regeneration process heat losses are encountered, which should be considered during the study of practical cases. Hence such affecting factors enforce the irreversibility in the system and the processes should be executed in finite duration using the concept for practical application.

Thermal model

Thermodynamic analysis is carried out to study the performance of the Ericsson cycle with reference to the aforementioned thermodynamic processes. Heat transfer from the heat source and heat sink at T_H and T_C is denoted as Q_{out} and Q_{in} and calculated as,

$$Q_{out} = T_H(S_3 - S_4) = nT_H R ln\lambda = C_H(T_{H2} - T_{H1})t_H \quad (1)$$

$$Q_{in} = T_L(S_2 - S_1) = nT_C R ln\lambda = C_L(T_{L1} - T_{L2})t_L \quad (2)$$

In the above equations, λ represents ratio of pressures for the isobaric heat exchange process (regeneration), n represents the number of moles of working fluid (gas) and R represents the universal gas constant. Heat capacitance is defined as the product of the mass of an object and its specific heat. For the current study, the heat capacitances of heat sink and heat source are denoted as C_H and C_L, respectively. The heat exchange process has to be executed during a finite time and hence t_H and t_L represents the times of heat rejection and heat addition processes, respectively. As per the fundamentals of thermodynamics [25], heat transfer can also be calculated as,

$$Q_{out} = U_H A_H (LMTD)_H t_H \quad (3)$$

$$Q_{in} = U_L A_L (LMTD)_L t_L \quad (4)$$

In Equations (3) and (4), U_H and U_L are overall heat transfer coefficients for heat sink and heat source respectively; A_H and A_L are areas of thermal reservoirs; $(LMTD)_H$ and $(LMTD)_L$ are logarithmic mean temperature differences on heat sink and heat source sides, respectively. Considering the effectiveness of heat exchangers on cold and hot sides as ε_H and ε_L;

$$\varepsilon_H = 1 - e^{-N_H} \quad (5)$$

$$\varepsilon_L = 1 - e^{-N_L} \quad (6)$$

where N_H and N_L are the number of transfer units used to calculate the rate of heat transfer in heat exchangers. The numbers of transfer units are calculated on the basis of the overall heat transfer coefficient, cross sectional area and heat capacitance and they are expressed as Equations (7) & (8),

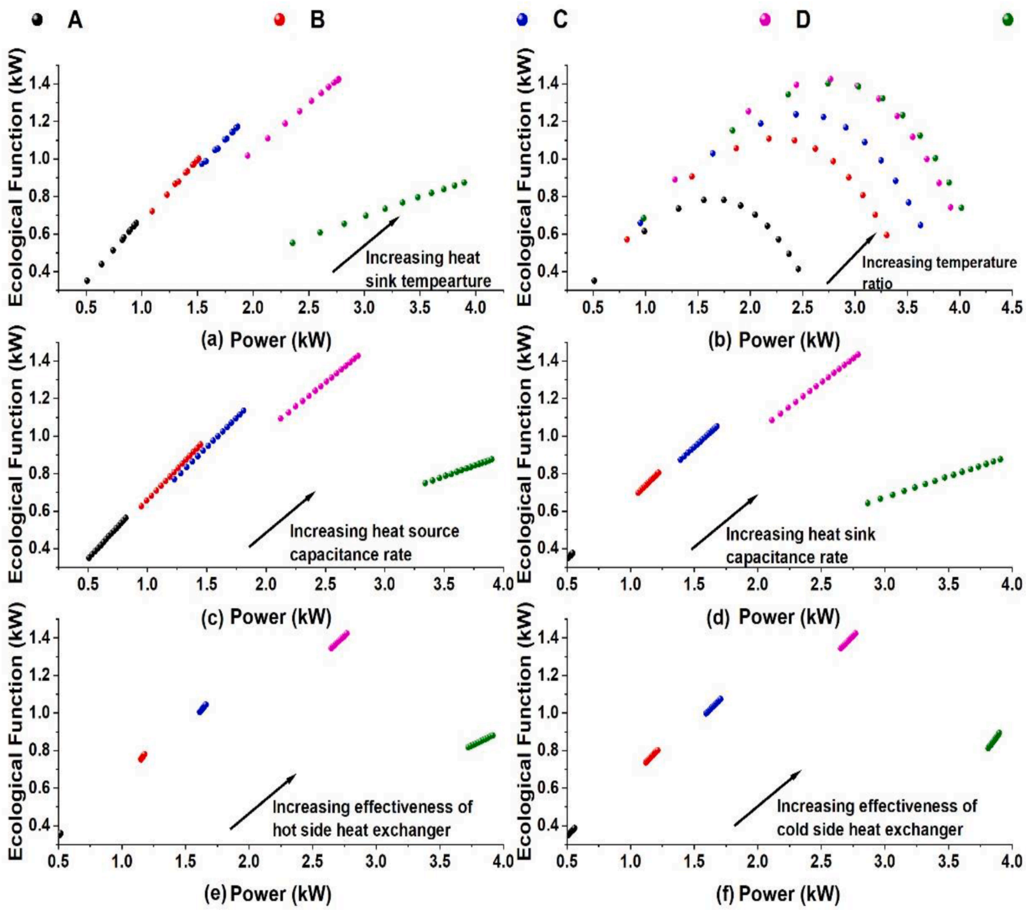

Fig. 6. Sensitivity analysis of design variables for the selected optimal points (A) – (E) of Scenario 1.

$$N_H = \frac{U_H A_H}{C_H} \tag{7}$$

$$N_L = \frac{U_L A_L}{C_L} \tag{8}$$

During the heat transfer process in the regenerator, a finite amount of energy loss is incurred, which is proportional to the temperature difference between heat source and heat sink [26–28].

$$Q_{Reg,loss} = nC_f(1-\varepsilon_R)(T_H - T_L) \tag{9}$$

In Equation (9), C_f is the specific heat of the working fluid, $\mathcal{E}_R$ is the effectiveness of the regenerator. For a completely reversible cycle, heat transfer processes should be executed for an infinite duration of time; however the irreversibility incurred in the system will allow a definite period of time for heat transfer during regeneration. The time for regeneration can be calculated using a proportional constant α, and the temperature difference between heat source and heat sink, as in Equation (9). The total time taken to execute a complete Ericsson cycle is calculated using Equation (10).

$$t_R = t_3 + t_4 = 2\alpha(T_H - T_L) \tag{10}$$

$$t_{cycle} = t_L + t_H + t_R \tag{11}$$

With reference to the irreversibilities incurred in the system, the actual amounts of heat absorbed from the source and of heat transferred to the sink is calculated using Equations (12) and (13).

$$Q_{out,net} = \left(Q_H - Q_{Reg,loss}\right) \tag{12}$$

$$Q_{in,net} = \left(Q_L - Q_{Reg,loss}\right) \tag{13}$$

The power input (P), COP and cooling load (R_L) are important variables to study for a refrigeration system when assessing the thermal performance of the Ericsson cycle. Equations (14) – (17) are used to calculate the aforementioned parameters [29,30].

$$P = \frac{Q_{out,net} - Q_{in,net}}{t_{cycle}} \tag{14}$$

$$R_L = \frac{Q_{in,net}}{t_{cycle}} \tag{15}$$

$$COP = \frac{R_L}{P} \tag{16}$$

For the actual cycle, the entropy generation rate (cooling load loss rate) should be considered and based on the total available energy transfer, the ecological function is defined and can be calculated using Equation (17),

$$E_C = R_L - LT_a\dot{S}_{gen} \tag{17}$$

In the above equation, L represents the dissipation coefficient, T_a represents the ambient temperature and S_{gen} represents the entropy generation rate which is calculated using Equation (18),

$$\dot{S}_{gen} = \frac{1}{t_{cycle}}\left(\frac{Q_{out,net}}{T_{H,mean}} - \frac{Q_{in,net}}{T_{L,mean}}\right) \tag{18}$$

where $T_{H,mean}$ and $T_{L,mean}$ are the mean temperatures of heat source and heat sink during the heat exchange process.

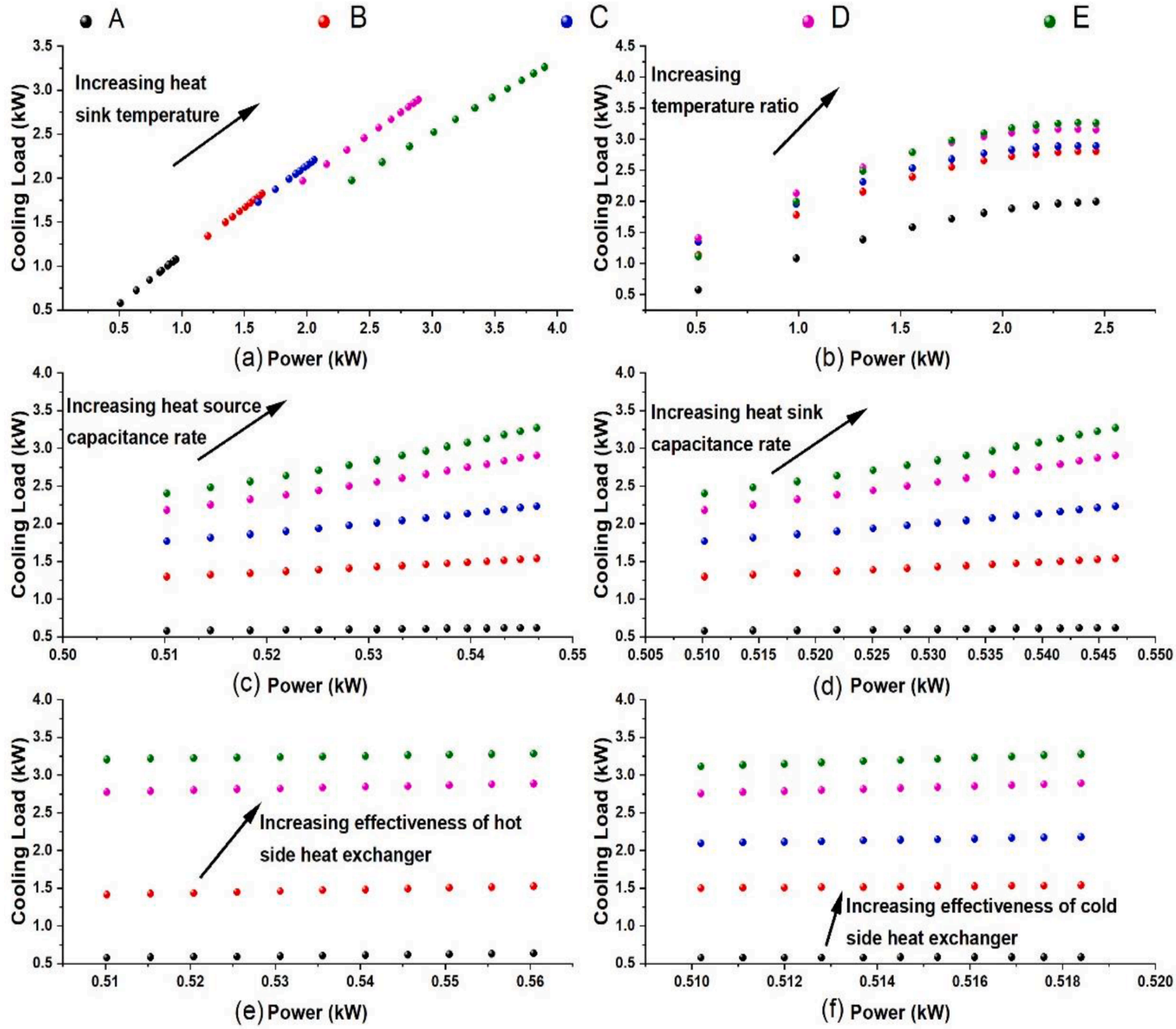

Fig. 7. Sensitivity analysis of design variables for the selected optimal points (A) – (E) of Scenario 2.

Heat transfer search (HTS) optimization algorithm

In the current section, a multi-objective optimization study is carried out using a novel *meta*-heuristic heat transfer search (HTS) optimization algorithm developed by Patel [31]. Based on the modes of heat transfer, viz. conduction, convection and radiation, the optimization is executed for the defined objective functions within the given constraints. In the current study, minimization of input power and maximization of COP and cooling load are considered as objective functions. The HTS algorithm tries to maintain thermal equilibrium between the system and the surroundings and the formulation is executed using 'conduction phase', 'convection phase' and 'radiation phase' [32], which are explained in subsequent sub-sections.

Conduction phase

In the conduction phase, the HTS algorithm imitates conduction heat transfer between higher and lower level energy molecules, which is analogous to high and low objective function values respectively. Solutions are updated using Equation (19) and (20), for '*i*' indicates the population size, '*j*' indicates the design variables, and 'g' indicates the number of generations. CDF represents conduction factor and the probability (R) for executing the conduction phase lies between 0 and 0.3333.

$$A_{j,i}{'} = \begin{cases} A_{k,i} + \left(-R^2 A_{k,i}\right), iff\left(A_j\right) > f\left(A_k\right) \\ A_{j,i} + \left(-R^2 A_{j,i}\right), iff\left(A_j\right) < f\left(A_k\right) \end{cases} ; ifg \leq g_{\max}/CDF \tag{19}$$

$$A_{j,i}{'} = \begin{cases} A_{k,i} + \left(-r_i A_{k,i}\right), iff\left(A_j\right) > f\left(A_k\right) \\ A_{j,i} + \left(-r_i A_{j,i}\right), iff\left(A_j\right) < f\left(A_k\right) \end{cases} ; ifg > g_{\max}/CDF \tag{20}$$

Convection phase

This phase imitates convection heat transfer between the system and its surroundings using Newton's law of cooling. New solutions in the convection phase are updated using Equation (21) and (22) in which COF is the convection factor and the probability (R) of executing this phase lies between 0.6666 and 1. TCF represent the temperature change factor.

$$A_{j,i}{'} = A_{j,i} + R(A_s - TCF\text{x}A_{ms}) \tag{21}$$

$$TCF = \begin{cases} abs(R - r_i), ifg \leq g_{max}/COF \\ round(1 + r_i) + \left(-R^2 A_{j,i}\right), ifg > g_{max}/COF \end{cases} \tag{22}$$

Radiation phase

The radiation phase mimics radiation heat transfer between the system and its surroundings using Stephan Boltzman's law. Similar to

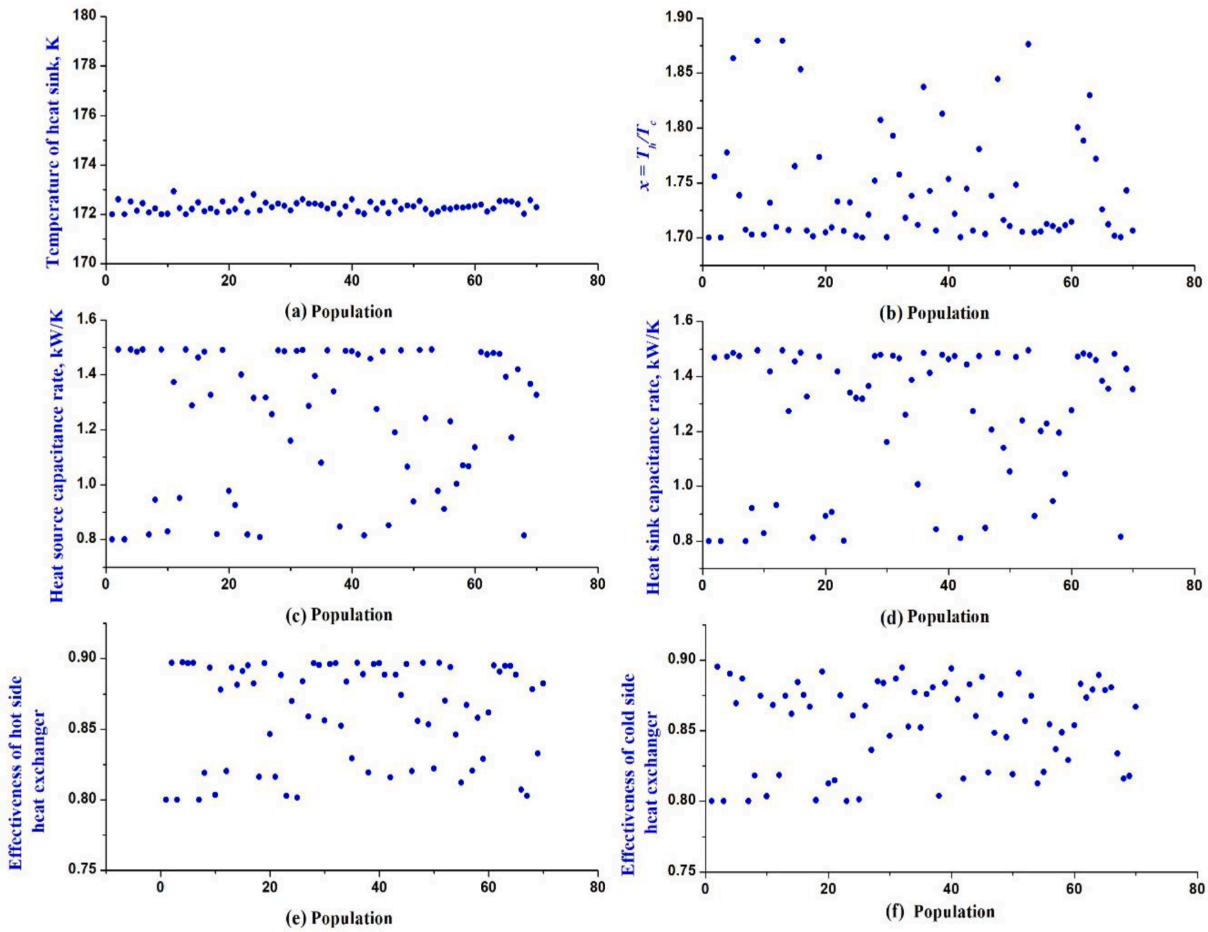

Fig. 8. Distribution of design variables during multi-objective optimization (a) temperature of heat sink, (b) temperature ratio of heat source and heat sink, (c) heat source capacitance rate, (d) heat sink capacitance rate, (e) effectiveness of hot side heat exchanger, (f) effectiveness of cold side heat exchanger.

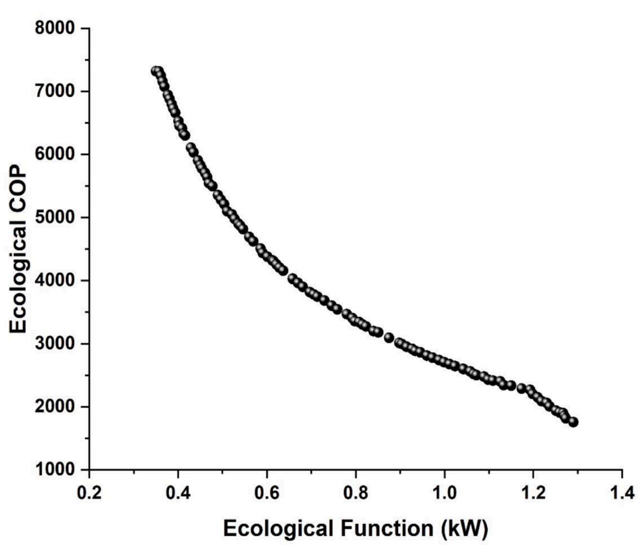

Fig. 9. Pareto optimal curve for ecological function and ecological coefficient of performance.

other phases, the system interacts with the surroundings to establish a state of thermal equilibrium. In this phase, solutions are updated using Equation (23) and (24) wherein RDF is the radiation factor and the probability of executing the radiation phase lies between 0.3333 and 0.6666.

$$A_{j,i}{}' = \begin{cases} A_{j,i} + R\left(A_{k,i} - A_{j,i}\right), iff\left(A_j\right) > f(A_k) \\ A_{j,i} + R\left(A_{j,i} - A_{k,i}\right), iff\left(A_j\right) < f(A_k) \end{cases}; ifg \le g_{\max}/RDF \tag{23}$$

$$A_{j,i}{}' = \begin{cases} A_{j,i} + r_i\left(A_{k,i} - A_{j,i}\right), iff\left(A_j\right) > f(A_k) \\ A_{j,i} + r_i\left(A_{j,i} - A_{k,i}\right), iff\left(A_j\right) < f(A_k) \end{cases}; ifg > g_{\max}/RDF \tag{24}$$

The multi-objective heat transfer search (MOHTS) optimization provides simultaneous solutions for more than one objective function. The MOHTS algorithm incorporates the non-dominated solution and saves the obtained solutions in an external archive [33]. The MOHTS algorithm uses the $\mathcal{E}$-dominance based updating method to check the domination of the solution in the archive [34]. More details about the MOHTS algorithm are available in references [35–37].

Objective functions and decision parameters

Power input, cooling load, COP and ecological function are considered as objective functions for the multi objective optimization study and can be calculated using Equations (14), (15), (16) and (17) respectively. The objective is to maximize the cooling load, COP and ecological function with minimum input power to the system. By considering the constraints as mentioned in Table 1 optimum solutions of objective functions are obtained as represented by the Pareto curve.

Results and discussion

In this section, results obtained from multi-objective optimization for two different scenarios are presented and the findings of the results are discussed. Scenario 1 pertains to the investigation of the effect of design variables on input power, coefficient of performance and ecological function, whereas Scenario 2 explains the effect of design variables on input power, coefficient of performance and cooling load. Objective functions are optimized using the heat transfer search optimization algorithm and the Pareto optimal curve is plotted with multiple optimal solutions. Five optimal points *(A) – (E)* are selected from the optimal curve and their effect on the objective function is presented using sensitivity analysis. Results of the present study is validated by comparing the results discussed by literature already published. Lastly, the population distribution of each design variable is plotted to understand the spread of population during multi-objective optimization to attain the said objectives.

Multi-objective optimization – Scenario 1

The multi-objective optimization using the heat transfer search algorithm is carried out for the Ericsson cycle working between temperatures T_H and T_C with the objective of maximizing the cooling load and COP with minimum input power. The Pareto optimal curve obtained from multiple optimal points is presented in Fig. 3.

Scenario 1 is considered where optimum design variables are selected to maximize the effect of the ecological function and COP and to minimize the input power requirement. Fig. 3 represents the effect of design variables on COP, input power and ecological function. With increase in input power, the ecological function (ECF) tends to increase because of an increased cooling load, however after the maximum point, the ecological function decreases because of high irreversibility at higher input powers. The maximum ECF of 1.42 kW is obtained at 2.73 kW of input power, after which ECF keeps on reducing to 0.885 kW when the input power is 3.91 kW. The effect of design variables on objective functions is also represented using a 3D plot as shown in Fig. 3. With an increase in ECF, the COP of the system reduces because of an increased input power. The maximum COP of 1.13 is obtained when the input power and ECF are 0.58 kW and 0.35 kW respectively.

Five optimal points *(A) – (E)* are selected on the Pareto front and for each optimal point, the effect of design variables on the objective function is studied and presented in a subsequent section. Optimal point (A) refers to the condition of minimum input power whereas optimal point (D) refers to the condition of maximum ecological function.

Design variables pertaining to each of the selected optimal points are tabulated in Table 2. The optimal point is selected using the TOPSIS criterion and the values of objective functions at the selected point are tabulated.

Validation of results

To validate the results of the current study, the work carried out by Ahmadi [26] is considered as a reference and the results obtained are compared with the reference study as presented in Fig. 4.

Different optimization scenarios are considered during the investigation in which input power to the system is minimized for each case like single objective optimization of power, COP, ecological function. Also, a multi objective optimization study is performed and input power for the same is calculated. As demonstrated in the figure, for the scenario of single objective optimization of power, ecological function and multi-obejctive optimization, results show reduced input power by 20%, 42% and 25% respectively. Results prove that the heat transfer search algorithm helps in attaining an improved solution of input power as compared to the NSGA-II algorithm adopted in the reference study. The results obtained hold a motivation to carry out the further investigation of the cooling load of the system, which is presented through Scenario-2 in the subsequent section.

Multi-objective optimization – Scenario 2

The variation of COP and cooling load with respect to the input power is presented as Scenario 2 using a Pareto curve and the conflicting nature of the objectives is observed between input power and cooling load. For an optimum system working on the Ericsson cycle it is always preferred that the system operates at a maximum cooling load with minimum input power. In the present study, for the maximum cooling load of 3.27 kW the input power required by the system is 3.91 kW; whereas for the minimum input power of 0.51 kW, the cooling load obtained is 0.58 kW. The effect of design variables on COP along with cooling load and input power is presented using a 3D plot as shown in Fig. 5. With an increase in cooling load of the system, the input increases and at the same time, the coefficient of performance of the system reduces. Such a type of conflicting nature between the objective functions can be resolved by selecting the optimum operating condition for the system from the Pareto curve.

On the Pareto optimal curve five points are selected *(A) – (E)* and their effect on the objective functions is discussed. It is observed from Fig. 5, optimal point *(A)* refers to the condition of minimum input power required by the system, whereas point *(E)* signifies maximum cooling load condition. The optimal point is selected using the TOPSIS criterion and the values of objective functions at the selected points are tabulated in Table 3.

Sensitivity of design variables

A sensitivity analysis of the selected Pareto optimal points *(A) – (E)* for Scenario 1 from Fig. 3 is carried out and presented in Fig. 6 to understand the influence on the ecological function and input power. Fig. 6 (a) demonstrates the effect of heat sink temperature on the objective function. With an increase in heat sink temperature, the ecological function increases and attains a maximum value of 1.42 kW with an input power of 2.73 kW. With a further increase in heat sink temperature, the ecological function reduces because of an increased input power, which is undesirable.

The temperature ratio of heat source and heat sink has a direct effect on the ecological function up to the maximum value of 1.42 kW and helps in attaining a cooling load of 2.79 kW. It is observed that for each optimal point, the input power required for the system increases and for the system with the minimum requirement of input power, it is necessary to operate the system at point A as shown in Fig. 6 (b). The effect of heat source and heat sink capacitance rate is presented in Fig. 6 (c) and (d). A similar trend is observed for the design variables, which tends to maximize the magnitude of the ecological function. However by doing so, the input power requirement of the system increases substantially and it is necessary to operate the system in accordance with the needs of the system. When the cryogenic refrigerator is operated at optimal point *(A)*, the input power requirement is at a minimum whereas the sysem develops a high cooling load at point *(D)* with a higher input power. The effect of heat exchanger effectiveness is presented in Fig. 6 (e) and (f).

A sensitivity analysis of the selected Pareto optimal points *(A) – (E)* for Scenario 2 from Fig. 5 is carried out and presented in Fig. 7 to understand the influence on cooling load and input power. Fig. 7 (a) represents the effect of increasing heat sink temperature on the input power and cooling load. It is evident that, with an increase in heat sink temperature, the cooling load increases towards the end of the curve at point *(E)*, whereas, with an increase in cooling load, the input power requirement of the system also increases which can be seen from the plot.

A similar trend is observed for an increasing temperature ratio (heat source temperature divided by heat sink temperature) for each optimal point *(A) to (E)* as shown in Fig. 7 (b). For a given input power, the optimized system operates at a higher cooling load at point E as compared to point A. The effects of heat source capacitance rate and heat sink capacitance rate are demonstrated in Fig. 7 (c) & (d). It is evident from the trend, that increasing the capacitance rates, the cooling load increases; however for same input power, optimal point *(E)* provides a higher cooling load as compared to point (*A*). The effectiveness of the heat exchanger helps in efficient heat transfer between the working fluid and heat source and sink. Fig. 7 (e) and (f) shows the effect of effectiveness of hot side and cold sides of the heat exchanger. From the results, it is necessary to operate the system for optimal conditions with respect to point *(E)* for maximum cooling load with minimum input power.

Design variable behaviour

The scattered distribution of design variables is presented in Fig. 8 (a) – (f) for all the population. These distribution plots are obtained for the optimal curve and represent the optimum value of each design variable for different sets of the population. Fig. 8 (a) shows that the optimum value of heat sink temperature remains invariably close to −172℃ or 101 K during the optimization. The effect of this design variable is not so substantial for obtaining the Pareto optimal solutions for input power and cooling load. However Fig. 8 (b) to (f) exhibits dispersed distributions for the design variables for obtaining the Pareto optimal solutions for minimum input power and maximum cooling load. The effect of these variables on the objective functions is noticeable and optimum values are scattered between the given bounds of respective design variables.

Ecological optimization

In the current section, ecological optimization is carried out for the ecological coefficient of performance (ECOP) and ecological objective function (ECF) as presented in Fig. 9. ECOP is calculated based on the net heat output whereas ECF is calculated based on the cooling load of the system. The ecological coeffient of performance is calculated using Equation (25).

$$E_{COP} = \frac{Q_{out,net}}{T_a S_{gen}} \tag{25}$$

Irreversibility generated in the system is considered for calculating both factors and their effect on the system performance is discussed. The objective is to resolve the conflict between two objective functions, for which the system is optimized, for maximum ecological function and ecological coefficient of performance. To obtain the maximum ECF, the system tends to operate at minimum ECOP and similarly when the system tends to work at maximum ECOP, the ECF of the system goes to a minimum. Hence, the Pareto front representing the number of optimal solutions is obtained and any desirable working condition can be selected among the possible solutions. Between the minimum and maximum values of the objective functions it is observed that ECF and ECOP increase by 2.74 and 3.92 times respectively. Further, maximum and minimum values of ECF are obtained as 1.31 and 0.35 respectively; whereas for ECOP they are 7320 and 1485 respectively.

Conclusion

In the present work, a thermo-ecological optimization of an Ericsson cryogenic refrigerator is attempted for multi-objective considerations. Input power, coefficient of performance, cooling load and ECF of the refrigerator are considered as objective functions. The heat sink temperature, temperature ratio, effectiveness of the heat exchanger and heat capacitance rates of sink and source are considered as design variables. Optimization is carried out for two different scenarios with thermo-ecological obectives. The major findings of the investigation are listed below:

- With an increase in input power, cooling load and ecological functions increase, but at the same time, the COP of the refrigerator decreases.
- The maximum cooling load of 3.27 kW for the cryogenic refrigerator is obtained with a input power of 3.91 kW, whereas the maximum COP of 1.13 is obtained for the system when input power and cooling load are 0.51 kW and 0.58 kW respectively.
- A sensitivity analysis is carried out for five selected points *(A)* – *(E)* for both scenarios, and it is observed that the effects of heat capacitance rates of heat sink and source are significant among other variables.
- The maximum and minimum value of ECF are obtained as 1.31 and 0.35 respectively, whereas for ECOP they are 7320 and 1485 respectively during ecological optimization.
- The scattered distribution of all the design variables (except heat sink temperature) represents the conflicting nature of the objective functions.

CRediT authorship contribution statement

Parth Prajapati: Conceptualization, Validation, Investigation, Writing – review & editing, Writing – original draft. **Vivek Patel:** Conceptualization, Writing – review & editing. **Hussam Jouhara:** Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgement

The work is done as part of the collaboration between *Pandit Deendayal Energy University* and the Heat Pipe and Thermal Management Research Group at Brunel University London, UK.

References

[1] H. Jouhara, et al., Thermoelectric generator (TEG) technologies and applications, Int. J. Thermofluids 9 (2021), https://doi.org/10.1016/J.IJFT.2021.100063.
[2] H. Jouhara, A.G. Olabi, Editorial: Industrial waste heat recovery, Energy 160 (2018) 1–2, https://doi.org/10.1016/J.ENERGY.2018.07.013.
[3] T.F. Miller, Parametric modeling of a solid state Ericsson cycle heat engine, Energy 236 (2021), https://doi.org/10.1016/J.ENERGY.2021.121413.
[4] D.A. Blank, C. Wu, Power limit of an endoreversible Ericsson cycle with regeneration, Energy Convers. Manag. 37 (1) (1996) 59–66, https://doi.org/10.1016/0196-8904(95)00020-E.
[5] S.C. Kaushik, S. Kumar, Finite time thermodynamic evaluation of irreversible Ericsson and Stirling heat engines, Energy Convers. Manag. 42 (3) (2001) 295–312, https://doi.org/10.1016/S0196-8904(00)00063-7.
[6] J. Chen, J.A. Schouten, The comprehensive influence of several major irreversibilities on the performance of an Ericsson heat engine, Appl. Therm. Eng. 19 (5) (1999) 555–564, https://doi.org/10.1016/S1359-4311(98)00059-3.
[7] F. Angulo-Brown, An ecological optimization criterion for finite-time heat engines, J. Appl. Phys. 69 (11) (1991) 7465–7469, https://doi.org/10.1063/1.347562.
[8] Z. Yan, 'Comment on "an ecological optimization criterion for finite-time heat engines" [J. Appl. Phys. 69, 7465, (1991)]', *J. Appl. Phys.*, vol. 73, no. 7, p. 3583, 1993, 10.1063/1.354041.
[9] H. Wu, Y. Ge, L. Chen, H. Feng, Power, efficiency, ecological function and ecological coefficient of performance optimizations of irreversible Diesel cycle based on finite piston speed, Energy 216 (2021), https://doi.org/10.1016/J.ENERGY.2020.119235.
[10] M.H. Ahmadi, M.A. Ahmadi, M. Mehrpooya, M. Sameti, Thermo-ecological analysis and optimization performance of an irreversible three-heat-source absorption heat pump, Energy Convers. Manag. 90 (2015) 175–183, https://doi.org/10.1016/J.ENCONMAN.2014.11.021.

[11] [11]S. K. Tyagi, S. C. Kaushik, and V. Tiwari, 'Ecological Optimization and Parametric Study of an Irreversible Regenerative Modified Brayton Cycle with Isothermal Heat Addition', *Entropy 2003, Vol. 5, Pages 377-390*, vol. 5, no. 5, pp. 377–390, Dec. 2003, 10.3390/E5050377.

[12] S.K. Tyagi, S.C. Kaushik, R. Salhotra, Ecological optimization and performance study of irreversible Stirling and Ericsson heat engines, J. Phys. D. Appl. Phys. 35 (20) (2002) 2668–2675, https://doi.org/10.1088/0022-3727/35/20/330.

[13] V. K. Patel and B. D. Raja, 'Comparative Performance of Recent Advanced Optimization Algorithms for Minimum Energy Requirement Solutions in Water Pump Switching Network', *Arch. Comput. Methods Eng. 2020 283*, vol. 28, no. 3, pp. 1545–1559, Apr. 2020, 10.1007/S11831-020-09429-X.

[14] P.P. Prajapati, V.K. Patel, Thermo-economic optimization of a nanofluid based organic Rankine cycle: A multi-objective study and analysis, Therm. Sci. Eng. Prog. 17 (2020), https://doi.org/10.1016/j.tsep.2019.100381.

[15] P.P. Prajapati, V.K. Patel, Comparative analysis of nanofluid-based Organic Rankine Cycle through thermoeconomic optimization, Heat Transfer - Asian Research 48 (7) (2019) 3013–3038, https://doi.org/10.1002/htj.21528.

[16] P. Prajapati and V. Patel, 'Multi-objective optimization of CuO based organic Rankine cycle operated using R245ca', in *E3S Web of Conferences*, 2019, vol. 116, 10.1051/e3sconf/201911600062.

[17] C.-L. Hwang and K. Yoon, 'Multiple Attribute Decision Making', vol. 186, 1981, 10.1007/978-3-642-48318-9.

[18] S. Chakraborty, TOPSIS and Modified TOPSIS: A comparative analysis, Decis. Anal. J. 2 (2022), https://doi.org/10.1016/J.DAJOUR.2021.100021.

[19] V. P. Agrawal, V. Kohli, and S. Gupta, 'Computer aided robot selection: the "multiple attribute decision making" approach', *https://doi.org/10.1080/00207549108948036*, vol. 29, no. 8, pp. 1629–1644, 2007, 10.1080/00207549108948036.

[20] H. Ture, S. Dogan, and D. Kocak, 'Assessing Euro 2020 Strategy Using Multi-criteria Decision Making Methods: VIKOR and TOPSIS', *Soc. Indic. Res. 2018 1422*, vol. 142, no. 2, pp. 645–665, Jun. 2018, 10.1007/S11205-018-1938-8.

[21] P. Zhou, B.W. Ang, K.L. Poh, Comparing aggregating methods for constructing the composite environmental index: An objective measure, Ecol. Econ. 59 (3) (2006) 305–311, https://doi.org/10.1016/J.ECOLECON.2005.10.018.

[22] L. Zhang, Y. Xu, C.H. Yeh, Y. Liu, D. Zhou, City sustainability evaluation using multi-criteria decision making with objective weights of interdependent criteria, J. Clean. Prod. 131 (2016) 491–499, https://doi.org/10.1016/J.JCLEPRO.2016.04.153.

[23] J. Rosettani, W. Ahmed, P. Geddis, L. Wu, B. Clements, Experimental and numerical investigation of gas-liquid metal two-phase flow pumping, Int. J. Thermofluids 10 (2021), https://doi.org/10.1016/J.IJFT.2021.100092.

[24] B. Yunus, Cengel; Michael, Thermodynamics: An engineering approach, 8th ed., McGraw-Hill, 2015.

[25] A. S. Bergman, T.L; F.P, Incropera; D.P, DeWitt; Lavine, Fundamentals of heat and mass transfer, 7th ed. John Wiley & Sons., 2011.

[26] M.H. Ahmadi, M.A. Ahmadi, Thermodynamic analysis and optimization of an irreversible Ericsson cryogenic refrigerator cycle, Energy Convers. Manag. 89 (2015) 147–155, https://doi.org/10.1016/j.enconman.2014.09.064.

[27] S.K. Tyagi, S.C. Kaushik, M.K. Singhal, Parametric study of irreversible Stirling and Ericsson cryogenic refrigeration cycles, Energy Convers. Manag. 43 (17) (2002) 2297–2309, https://doi.org/10.1016/S0196-8904(01)00181-9.

[28] L. Chen, C. Wu, F. Sun, Cooling load versus COP characteristics for an irreversible air refrigeration cycle, Energy Convers. Manag. 39 (1–2) (1998) 117–125, https://doi.org/10.1016/S0196-8904(96)00119-7.

[29] E. Açikkalp and H. Yamik, 'Limits and Optimization of Power Input or Output of Actual Thermal Cycles', *Entropy 2013, Vol. 15, Pages 3219-3248*, vol. 15, no. 8, pp. 3219–3248, Aug. 2013, 10.3390/E15083309.

[30] E. Açikkalp, Entransy analysis of irreversible Carnot-like heat engine and refrigeration cycles and the relationships among various thermodynamic parameters, Energy Convers. Manag. 80 (2014) 535–542, https://doi.org/10.1016/J.ENCONMAN.2014.01.056.

[31] V.K. Patel, V.J. Savsani, Heat transfer search (HTS): a novel optimization algorithm, Inf. Sci. (Ny) 324 (2015) 217–246, https://doi.org/10.1016/J.INS.2015.06.044.

[32] V.K. Patel, An efficient optimization and comparative analysis of ammonia and methanol heat pipe for satellite application, Energy Convers. Manag. 165 (2018) 382–395, https://doi.org/10.1016/J.ENCONMAN.2018.03.076.

[33] V. Patel, V. Savsani, A. Mudgal, Efficiency, thrust, and fuel consumption optimization of a subsonic/sonic turbojet engine, Energy 144 (2018) 992–1002, https://doi.org/10.1016/J.ENERGY.2017.12.080.

[34] K. Deb, M. Mohan, S. Mishra, Evaluating the ε-Domination Based Multi-Objective Evolutionary Algorithm for a Quick Computation of Pareto-Optimal Solutions, Evol. Comput. 13 (4) (2005) 501–525, https://doi.org/10.1162/106365605774666895.

[35] B.D. Raja, R.L. Jhala, V. Patel, Many-objective optimization of cross-flow plate-fin heat exchanger, Int. J. Therm. Sci. 118 (2017) 320–339, https://doi.org/10.1016/J.IJTHERMALSCI.2017.05.005.

[36] B.D. Raja, R.L. Jhala, V. Patel, Thermal-hydraulic optimization of plate heat exchanger: A multi-objective approach, Int. J. Therm. Sci. 124 (2018) 522–535, https://doi.org/10.1016/J.IJTHERMALSCI.2017.10.035.

[37] B.D. Raja, R.L. Jhala, V. Patel, Many-objective optimization of shell and tube heat exchanger, Therm. Sci. Eng. Prog. 2 (2017) 87–101, https://doi.org/10.1016/J.TSEP.2017.05.003.