MODELLING PERSISTENCE AND NON-LINEARITIES IN THE US TREASURY 10-YEAR BOND YIELDS

Guglielmo Maria Caporale
Brunel University London, United Kingdom

Luis Alberiko Gil-Alana
University of Navarra, Pamplona, Spain
and Universidad Francisco de Vitoria, Madrid, Spain

and

OlaOluwa Simon Yaya
University of Ibadan, Ibadan, Nigeria
Centre for Econometrics and Applied Research, Ibadan, Nigeria

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Abstract

This paper analyses persistence and non-linearities in quarterly and monthly US Treasury 10-year bond yields over the period 1962-2021 using two different fractional integration approaches including Chebyshev polynomials and Fourier functions respectively. The results for both quarterly and monthly data provide evidence of non-linear structures and mean reversion (i.e., of transitory effects of shocks) under the assumption of autocorrelated errors.

Keywords: Non-linearities; Chebyshev polynomials; Fourier functions; persistence; US Treasury 10-year bond yields

JEL Classification: C22, E43

Corresponding author: Professor Guglielmo Maria Caporale, Department of Economics and Finance, Brunel University London, Uxbridge, Middlesex, UB8 3PH, UK. Email: Guglielmo-Maria.Caporale@brunel.ac.uk; https://orcid.org/0000-0002-0144-4135

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1. Introduction

A well-known stylised fact about the behaviour of interest rates is their high degree of persistence. This is an important issue that has implications for both the design of monetary policy and the empirical validity of different interest rate theories, such as consumption-based asset pricing models and the Fisher effect, and thus it has been extensively investigated in the literature. In the early stages, the framework most commonly adopted was based on the I(0)/I(1) dichotomy. Examples of such studies include: Cox et al. (1985), who argued that US short-term nominal interest rates can be characterised as a stationary and mean-reverting I(0) process; Campbell and Shiller (1987), who instead reached the conclusion that both short- and long-term US bond yields exhibit unit root behaviour, with shocks having permanent effects; Karanasos et al. (2006), who found stationarity of US monthly interest rates over the period from1876 to 2000.

Subsequently, it became apparent that unit root tests have very low power against fractional alternatives owing to over- or under-differencing (Diebold and Rudebusch, 1991; Hassler and Wolters, 1993; Lee and Schmidt, 1996; etc.)[1], and therefore fractional integration techniques started being used to examine the degree of persistence of interest rates. Studies using this approach include Lai (1997), Phillips (1998) and Tsay (2000), who analysed US real interest rates (see also Barkoulas and Baum, 1997; Meade and Maier, 2003; Gil-Alana, 2004a,b), and Couchman et al. (2006), who examined a broader sample including data from sixteen countries. More recently, Caporale and Gil-Alana (2009) found that the estimated degree of persistence of the US Federal Funds effective rate is affected by the assumptions about the behaviour of the underlying residuals, and

[1] Kramer (1998) showed that ADF test (Dickey and Fuller, 1979) is consistent under fractional alternatives if the order of the autoregression does not tend to infinity too fast; however, his simulation results indicate that this is not the case in finite samples.

Caporale and Gil-Alana (2016, 2017) modelled long memory as well as cyclical behaviour in both the Euribor and the Fed Funds rate.

All the above studies are based on the assumption of linearity. However, it has become increasingly clear that fractional integration is possibly linked to the existence of non-linearities (see Granger and Hyung, 2004; Deo et al., 2006; etc.). The latter have become a particularly important issue following the onset of the global financial crisis (GFC), as a result of which many countries adopted unconventional monetary policy measures such as Quantitative Easing (QE) and interest rates were cut sharply creating the so-called zero lower bound (ZLB) problem, namely a situation characterised by a liquidity trap and limited central bank's ability to boost growth. Given the results in the more recent econometrics literature and the current economic environment, it is therefore essential to use a modelling framework that captures both persistence and non-linearities in interest rates. These issues are addressed in a recent paper by Caporale et al. (2022) who analyse the stochastic behaviour of monthly 10-year US Treasury bond yields using a fractional integration model for persistence that also allows for non-linearities in the form of Chebyshev polynomials; their findings confirm the presence of both features in the data (though there is evidence of mean reversion).

The present study extends the work of Caporale et al. (2022) by (i) including in the sample the latest available observations, (ii) carrying out the analysis at both the monthly and quarterly frequency (as opposed to monthly only), and (iii) also using a second approach for modelling non-linearities which is based on Fourier functions – the latter two as robustness checks. The structure of the paper is the following: Section 2 describes the methodology; Section 3 discusses the data and the empirical results, and Section 4 offers some concluding remarks.

2. Methodology

The general fractional integration approach considers the model

$$y_t = \rho(\theta; t) + x_t; \qquad (1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{1}$$

where y_t is the observed series of size T; $\rho(\theta; t)x_t$ is a non-linear function of time that depends on a vector of unknown parameters θ; L is the lag operator, i.e., ($L^k x_t = x_{t-k}$), and the parameter d stands for the order of integration of the series that can be any real value.

First, we assume that ρ is a function of Chebyshev polynomials in time, which implies that Eq. (1) can be re-written as:

$$y_t = \sum_{i=0}^{m} \theta_i P_{iT}(t) + x_t; \qquad (1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{2}$$

with m indicating the order of the Chebyshev polynomial $P_{i,T}(t)$ defined as:

$$P_{0,T}(t) = 1,$$

$$P_{i,T}(t) = \sqrt{2}\cos\left(i\pi(t-0.5)/T\right), \qquad t = 1, 2, \ldots, T; \quad i = 1, 2, \ldots \tag{3}$$

Bierens (1997) and Tomasevic and Stanivuk (2009) argue that it is possible to approximate highly non-linear trends with rather low degree polynomials. If $m = 0$ the model contains an intercept, if $m = 1$ it also includes a linear trend, and if $m > 1$ it becomes non-linear - the higher m is the less linear the approximated deterministic component becomes.[2] Specifically, we use the method developed in Cuestas and Gil-Alana (2016) which is essentially an extension to the non-linear case of the Robinson's (1994) fractional integration (linear) approach.

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[2] See Hamming (1973) and Smyth (1998) for a detailed description of these polynomials.

Equation (1) can be extended to include additional forms of non-linear deterministic terms, also capturing both non-linear cycles and persistence in the time series, as in the following model proposed in Gil-Alana and Yaya (2021), where $\rho(\theta; t)$ is defined as:

$$\rho(\theta;t) = \alpha + \beta t + \sum_{k=1}^{n} \lambda_k \sin(2\pi kt/T) + \sum_{k=1}^{n} \gamma_k \cos(2\pi kt/T); \qquad n \le T/2; \quad t = 1, 2, ..., T \quad (4)$$

where α and β are the intercept and the coefficient on a linear trend, respectively; λ_k and γ_k are parameters corresponding to the amplitude and displacement of the Fourier form which induces non-linearity, and the Fourier form expansion has frequency n with k being a given frequency set equal to 1, 2, …. T, and T is the number of observations. Thus, the significance of the parameters λ_k and/or γ_k (for all k) implies the presence of non-linearity in the process.

3. Data and Empirical Results

The series used for the analysis are non-seasonally adjusted quarterly and monthly market yields on US Treasury bonds with 10-Year maturity, expressed in percentage. The monthly data span the period from January 1962 to November 2021, while the corresponding period for the quarterly ones goes from 1962 Q1 to 2021 Q3. The source is the Federal Reserve Economic Data (FRED) online database at https://fred.stlouisfed.org. Figure 1 contains plots of the two series, both of which rose sharply in the early part of the sample period and peaked in 1982 before starting to decrease and reaching very low levels in recent years.

Insert Figure 1 about here

As a first step we carry out standard unit root tests, more precisely the ADF (Dickey and Fuller, 1979) and PP (Phillips and Perron, 1988) ones. The results displayed in Table 1 imply non-rejection of the unit root null at both frequencies. Given the well-known low power of such tests (Diebold and Rudebusch, 1991), next we use fractional integration methods to measure the degree of persistence of the series. We follow in turn each of the two non-linear modelling approaches described before (the results are reported in Table 2 and 3 respectively), and also consider two alternative specifications for the errors assuming that they are either white noise or autocorrelated (see panel i) and ii) in the tables); in the latter case a non-parametric approach based on the exponential spectral model of Bloomfield (1973) is adopted. When using the first non-linear approach based on Chebyshev polynomials (see Table 2), we find that, under the assumption of white noise errors, the Chebyshev coefficients are not significantly different from 0, the estimates of d being slightly higher than 1 at both frequencies; also, the unit root null hypothesis cannot be rejected for the quarterly series, but it is rejected in favour of $d > 1$ with monthly data. However, when allowing for autocorrelated disturbances, a different picture emerges: the orders of integration are 0.69 for the quarterly data, and 0.79 for the monthly data, and in both cases the I(1) hypothesis is rejected in favour of mean reversion ($d < 1$), which implies that shocks have transitory effects. Moreover, there is evidence of non-linear structures since both θ_2 and θ_3 are statistically significant for both series.

Insert Tables 2 and 3 about here

The results based on the second non-linear approach based on Fourier transforms (see Table 3) are qualitatively similar. Under the assumption of white noise errors, the estimates of d are slightly above 1 (1.12 and 1.18 respectively for the quarterly and monthly data); the I(1) hypothesis cannot be rejected with quarterly data while d is found to be significantly higher than 1 with monthly data, and no deterministic term is found to be statistically significant. However, when allowing for autocorrelation, d is significantly smaller than 1 (0.90 for the quarterly data, and 0.80 for the monthly case), and the non-linear trends appear to be significant (specifically, the two coefficients for sin(t) and cos(t) with quarterly data, and sin(t) for the monthly one, are significant). Thus, the results for the two non-linear specifications considered are broadly consistent, in both cases mean reversion and non-linearity being found with autocorrelated errors.

4. Conclusions

This paper analyses persistence and non-linearities in quarterly and monthly US Treasury 10-year bond yields over the period 1962-2021 using two different fractional integration approaches including Chebyshev polynomials and Fourier functions respectively. The results provide evidence of non-linear structures and mean reversion (i.e., of transitory effects of shocks) at both frequencies under the assumption of autocorrelated errors. These findings complement and confirm those reported by Caporale et al. (2022) in a previous study using only the first of those two approaches to modelling non-linearities and analysing only monthly data.

Future work could extend the analysis of the present paper in various ways. For instance, stochastic nonlinear structures (such as Markov Switching models) rather than deterministic ones could be considered. Cyclical fractional integration methods could also be used to investigate possible cyclical patterns.

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Figure 1: Quarterly and Monthly 10-year US Treasury bond yields

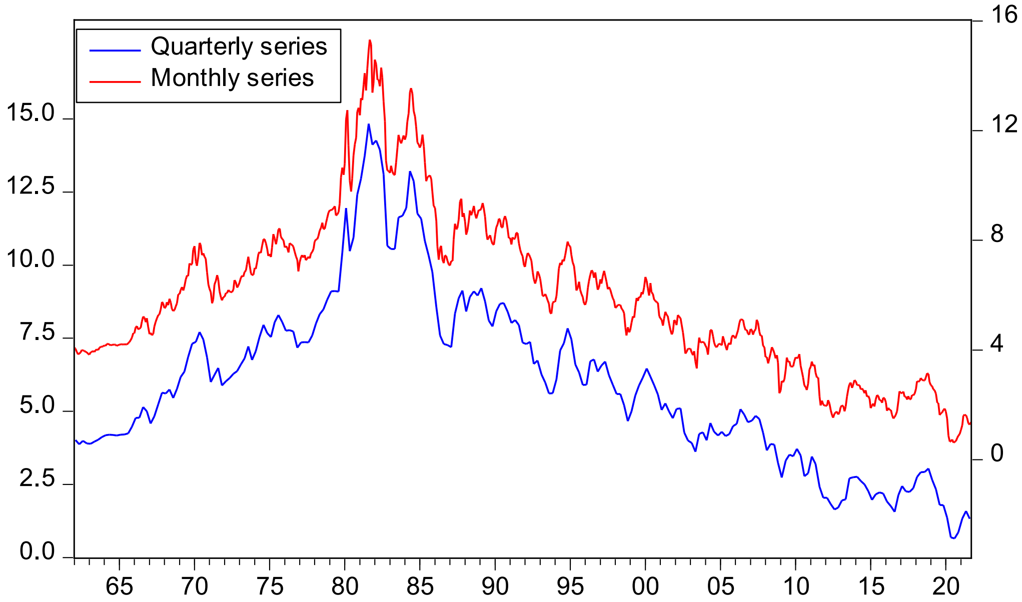

Table 1: Unit root tests

Quarterly series			
	None	Intercept only	Intercept with trend
ADF	-0.7970 [1]	-1.2413 [1]	-2.3144 [1]
PP	-0.7698 [4]	-1.1015 [5]	-2.1405 [4]
Monthly series			
	None	Intercept only	Intercept with trend
ADF	-0.7371 [2]	-1.0734 [2]	-2.1588 [2]
PP	-0.7702 [2]	-1.1433 [2]	-2.1422 [1]

Note: t-statistics for unit root tests are reported with optimal lags and bandwidth parameters for the ADF and PP tests respectively.

Table 2: Non-linear I(d) model with Chebyshev's polynomials in time

i) White noise errors					
Series	d (95% band)	θ_0 (t-value)	θ_1 (t-value)	θ_2 (t-value)	θ_3 (t-value)
QUARTERLY	1.12 (0.99, 1.29)	5.844 (0.80)	1.599 (0.39)	-1.811 (-1.01)	-1.070 (-0.94)
MONTHLY	1.18 (1.10, 1.28)	5.879 (0.53)	1.621 (0.23)	-1.815 (-0.64)	-1.068 (-0.61)
ii) Autocorrelated errors					
Series	d (95% band)	θ_0 (t-value)	θ_1 (t-value)	θ_2 (t-value)	θ_3 (t-value)
QUARTERLY	0.69 (0.48, 0.95)	**5.702 (5.94)**	**1.753 (3.19)**	**-1.910 (-4.99)**	**-1.030 (-3.51)**
MONTHLY	0.79 (0.70, 0.88)	**5.690 (4.64)**	**1.766 (2.46)**	**-1.883 (-4.16)**	**-1.038 (-3.13)**

In bold the parameters whose estimates are significant with t-statistics in parenthesis.

Table 3: Non-linear I(d) model with Fourier functions in time

i) White noise errors					
Series	d (95% band)	c (t-value)	t (t-value)	$\sin_k$ (t-value)	$\cos_k$ (t-value)
QUARTERLY	1.128 (0.997, 1.259)	1.654 (0.380)	-0.007 (0.119)	2.014 (0.778)	-2.644 (-1.010)
MONTHLY	1.183 (1.107, 1.259)	-5.232 (-0.508)	0.014 (0.334)	6.039 (0.840)	4.040 (0.384)
ii) Autocorrelated errors					
Series	d (95% band)	c (t-value)	t (t-value)	$\sin_k$ (t-value)	$\cos_k$ (t-value)
QUARTERLY	0.406 (0.096, 0.715)	0.4994 (0.584)	-0.0057 (-0.798)	**2.247 (2.78)**	**-2.790 (-4.54)**
MONTHLY	0.807 (0.656, 0.958)	**-6.8266 (-1.66)**	0.0128 (1.03)	**5.9632 (2.84)**	4.1365 (1.14)

In bold the parameters whose estimates are significant with t-statistics in parenthesis.