Dynamic Pricing in the Presence of Strategic Consumers with "Experience-in-Store-and-Buy-Online"

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Abstract: Experience-in-store-and-buy-online (ESBO) is a popular omni-channel strategy. This paper studies the effects of inspection service provision on the interactions of a dynamicpricing retailer and strategic consumers, i.e., the effects of the ESBO initiative on store operations. Selling a seasonal product over two periods, the cross-channel retailer may allow consumers to inspect this product offline only in the first period (first-period inspection) or in both periods (two-period inspection). Hence, consumers make decisions on both when to purchase and how to purchase. First, we find that allowing first-period inspection makes the retailer better off. The retailer will price the product higher in the first period but probably lower in the second period. Even so, more consumers will purchase in the first period: that is, allowing first-period inspection can somewhat deter strategic deferral. In the presence of first-period inspection, some consumers may engage in intertemporal showrooming behavior (i.e., inspect the product offline in the first period but defer online purchase to the second period). This seemingly negative intertemporal showrooming behavior benefits the retailer. Compared to first-period inspection, allowing two-period inspection increases the retailer's prices in both periods as well as profit, provided that inspection is *definitely available* in the second period. On the other hand, when inspection is possible in the second period, it may be profitable to allow inspection only in the first period.

Keywords: OR in marketing, dynamic pricing, strategic consumers, resolving uncertainty, intertemporal showrooming

1. Introduction

Strategic consumers are the main object of the game we study, whose strategy is usually reflected in the fact that they are rational and forward-looking. Strategic consumers may delay their purchase in anticipation of a future markdown even if the current price of the product is lower than their own reservation price, which means they will make decisions to maximizing their utility including such an intertemporal purchasing decision. The importance of strategic consumer behavior in shaping firms' pricing decisions has been widely recognized by practitioners and academics alike: to defend against its negative effects, firms are investing heavily in price-optimization algorithms (e.g., Schlosser 2004), while the literature has produced a number of managerial insights regarding how firms should adjust their approach to dynamic pricing (e.g., Aviv and Pazgal 2008, Besbes and Lobel 2015, Cachon and Feldman 2015, Mersereau and Zhang 2012, Su 2007). Apart from pricing, the effects of strategic consumer behavior also influence on a range of other operational decisions; examples include decisions pertaining to stocking quantities (Liu and van Ryzin 2008), inventory display formats (Yin et al. 2009), the implementation of quick-response and fast-fashion practices (Cachon and Swinney 2009, 2011), and the timing of new product launches (Lobel et al. 2016), to name but a few. Although existing research examines strategic consumer behavior from different perspectives, it usually does not take into account the product-consumer match, resulting in the creation of physical showrooms as a solution that can well address the matchability issue.

Recent years have witnessed the prevalence of omni-channel retailing, from offline to online or vice versa, improving consumers' shopping experience (Bell et al., 2018b). A traditional brick-and-mortar (B&M) retailer may open an online channel to enrich the fulfilment choices, while an online-first retailer who starting from online and opening stores initially without offline sales stores could build a physical showroom to satisfy consumers' inspection needs. For example, the furniture available at Urban Ladder' stores are only for display and experience the products, but all furniture will be delivered from their warehouse directly. Besides, an existing cross-channel retailer even redesigns its physical stores as showrooms for inspection service provision, like SAMSUNG 837X in NEW YORK. Under each move, showrooming behavior may arise to help consumers to resolve fit-related uncertainty (Swift, 2013). For example, Accenture (2013) found that 71% of shoppers surveyed indicated that they prefer browsing in stores for apparel before purchasing online. The emergence of Bonobos, a men's clothing store, is an example of this new model of retailing-zero inventory experience store with uniform delivery from the warehouse-where they claim that "we will offer a one-on-

¹ https://www.urbanladder.com/furniture-stores?src=g topnav new experience-centres

² https://www.samsung.com/us/explore/metaverse-837x/

one experience with a guide who will go through our full product line with you, and help you place your order and ship it right to you" (Bell et al., 2018b). The same as Under Ladder, and we call this pure showrooming. Although a body of research has shown that such a behavior hurts the B&M retailer (Balakrishnan et al., 2014; Mehra et al., 2018; Jing, 2018), a cross-channel retailer may embrace setting up showrooms to help reduce online product returns (Bell et al., 2018a).

Despite the growing opportunity for physical inspection, little attention has been paid to the length of time for which the inspection service is provided. Whether it is clothing or electronics, or even furniture, there are certain periods of popularity, and once the new products are launched, the layout of the showroom will need to be adjusted in order to free up limited shelf space, and those "out of style" products will no longer be displayed, so that consumers will not be able to do physical inspection, or at least the accessibility to inspection may decrease. But in order to capture more profit, retailers may dynamically change prices-maintaining full prices for their best-selling items while marking down slow sellers over time. Although most retailers still make such pricing decisions manually, many are now deploying sophisticated modeling and optimization software to help support pricing decisions (See Talluri and van Ryzin 2004, Chap. 5.). Such systems have proved quite effective; Ann Taylor, a U.S. women's apparel retailer with over 580 stores nationwide, reported a year-on-year increase in sales by 26% over the Christmas period of 2003 after it implemented markdown optimization software. Success stories like this have led to increased acceptance of model-based approaches to pricing among major retailers (Qian Liu, 2008). In other words, consumers face a trade-off between early purchase, for which product price is relatively high but physical inspection is more available, and delayed purchase, for which product price is relatively low but physical inspection is less available. Thus, the retailer needs to account for such a trade-off when making decisions to optimize its own profit.

Based on such an observation, we considering a retailer who sells products through an online channel and also operates physical showrooms, i.e., physical stores that only display samples, which we call "pure showrooms". In addition, "Experience-in-Store-and-Buy-Online" means customers can only inspect the products in the showrooms but have to purchase them through the online channel. The question of interest is how the retailer should manage the inspection service, as whether to provide inspection and the length of time that would affect consumers' strategic behavior. Our goal is to investigate how the pure showrooms' time influences the strategic interaction between a monopolist firm and a population of consumers. In this paper, we study three cases about the length of time period for which the inspection service is provided, and we will compare the performance of each case by considering the fundamental problem of uncapacitated dynamic pricing. To ease exposition, we assume that a retailer sells a seasonal product over two periods (Aviv and Pazgal, 2008). The product is

assumed to be season in the first period but out of season in the second period. Without access to inspection in two periods, for an expected lower price, a forward-looking consumer may typically delay the purchase until the second period. With inspection opportunities in the first period (i.e., first-period inspection), however, consumers may hesitate over such delay, since showrooming is available in the first period. In this case, a highly strategic consumer may inspect the product offline in the first period but defer the purchase online to the second period, which we refer to as *intertemporal showrooming behavior*. Such behavior appears to exert a negative effect on the retailer. If inspection is available in both periods (i.e., two-period inspection), however, the *intertemporal showrooming behavior* would disappear. In light of these scenarios, we intend to address the following research questions: 1) How would the inspection service affect retailer's pricing? 2) How would the time of providing pure showrooms affect the retailer's profit? 3) What is an optimal length of time from the retailer's perspective?

The model setting we consider is much in the spirit of the seminal paper by Besanko and Winston (1990). There is a monopolist firm selling a new product to a fixed population of strategic consumers, over two periods. Two alternative classes of dynamic-pricing policies may be employed: the firm may either (a) announce the full price path from the beginning of the selling horizon (pre-announced pricing) or (b) announce only the first-period price and delay the second-period price announcement until the beginning of the second period (responsive pricing). Consumers are heterogeneous in their preferences for the product and make adoption decisions to maximize their expected utility. Our addition to this simple model, and the focal point of our analysis, is the product-consumer match, which may be partially resolved by providing pure showrooms.

Because with a physical showroom, information about the product can be understood, this allows for product-consumer matching, encourages consumers to pay a certain access cost to enhance the consumer experience, saves return costs, and increases customer loyalty. With the introduction of physical showrooms, the game between purely online retailers and consumers changes from a simple pricing problem to a complex one driven by the relationship between the cost of visit and the cost of return. For retailers, offering a showroom induces a portion of strategic consumers to purchase products in the first issue, discouraging their strategic behavior, optimizing pricing and increasing profits. For consumers, physical showrooms help them better understand the product.

To explore the impact of showrooms on the game between retailers and consumers, we compare the equilibrium results of the model with those of a benchmark model in which retailers and consumers remain strategic, but we turn off showrooms (i.e., display and sell only online) so that we can compare the first-period inspection with the case in which showrooms are not available, and this comparison yields the following findings.

First, compared to the no-inspection case, allowing first-period inspection benefits the retailer. The retailer prices higher in the first period but probably lower in the second period. Even so, more consumers will purchase in the first period, i.e., allowing first-period inspection can somewhat deter strategic deferral. Second, the seemingly negative intertemporal showrooming behavior benefits the retailer, because consumers with low store-visiting costs are those who are willing to engage in intertemporal showrooming behavior, and who thus can afford higher prices. Third, when inspection service provision is surely available in the second period, the retailer will benefit more from pricing higher in both periods, compared to firstperiod inspection. Given the availability of second-period inspection, fewer consumers purchase in the first period, as some consumers shift to the second period. When inspection service provision is possible in the second period, however, it may be better to offer the inspection service only in the first period. The implication is that when launching new items, the retailer could obviate the inspection uncertainty by announcing clearly on the website that no inspection is available for the future season (e.g., new summer fashions will be online exclusive once autumn arrives, or the current range of furniture will be online exclusive once new series arrive). In general, those main results remain qualitatively unchanged under preannounced pricing: a pricing mode that makes the retailer better off than responsive pricing.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes a parsimonious model, which is analyzed in Section 4. In Section 5, we conduct several extensions, including isolation of intertemporal showrooming behavior and pre-announced pricing. Section 6 concludes. Equilibrium derivation for extensions and all proofs are presented in the Online Appendix.

2. Literature Review

This study is related to three streams of research, namely information service provision, showrooming within omni-channel retailing, and strategic consumer behavior.

First, this work is connected to research on information service provision, as a move to resolve consumers' fit-related uncertainty. Such service could be adopted in both physical stores (e.g., Ofek et al., 2011; Xia et al., 2017; Gao and Su, 2017b) and online channels (e.g., Gao and Su, 2017b; Gallino and Moreno, 2018), helping the retailer to reduce consumer mismatch and product returns. This paper corresponds to the former case in which consumers could go to the physical stores to inspect the product. However, the service under-provision issue may emerge due to the presence of a competing online retailer. Particularly, in the presence of showrooming, the service-providing retailer (i.e., the B&M retailer) may lack the incentive to provide inspection services, as the online retailer free-rides. To increase the sales efforts on the part of the B&M retailer, Basak et al. (2020) study the feasibility of a manufacturer-driven alliance with the B&M retailer, which can combat the adverse impact of

showrooming. Dan et al. (2020) show that product competition may cause increased demand for the online manufacturer (i.e., a manufacturer who establishes online channels to distribute products) and collaboration with offline retailers. Under asymmetric demand information, Li et al. (2019) find that online retailers' competition prompts the offline showroom to increase the level of its inspection service. Kuksov and Liao (2018) and Zhang et al. (2020) investigate the manufacturer's service compensation as an incentive to improve B&M retailers' service level. This paper provides a different angle to this stream of work. Instead of examining the incentive for inspection service provision by a traditional retailer, this work focuses on the timing of inspection service provision by a cross-channel retailer. Our analysis shows that the characteristics of inspection service provision affect the retailer's pricing and profit.

Second, this work belongs to the growing literature on omni-channel retailing. Overall, the extant research focuses on two aspects: order fulfillment methods and information mechanisms (Gao and Su, 2017b). The former refers to the modes by which a consumer places the order and receives the product, such as buy-online-pick-up-in-store (Gao and Su, 2017a; Jin et al., 2018), ship-to-store (Gallino et al., 2017; Boysen et al., 2021), reserve-online-pay-in-store (Jin et al., 2018; Difrancesco et al., 2021; Hauser et al., 2021), and the fulfillment of online orders through stores (Arslan et al., 2021; Bayram and Cesaret, 2021; Janjevic et al., 2021). Specifically, the retailer uses virtual showrooms and/or physical stores to deliver product information. It follows that consumers can make better decisions before purchasing. In particular, the welldocumented showrooming behavior arises when the information mechanism is realized in the physical stores. Considering consumers' valuation uncertainty, Balakrishnan et al. (2014) demonstrate the occurrence of showrooming behavior in equilibrium, which intensifies the retailers' competition and reduces their profits. Jing (2018) shows similar results and further examines the use of price-matching by a B&M retailer to eliminate showrooming behavior. In addition to proving the effectiveness of price-matching when consumers engage in showrooming, Mehra et al. (2018) also analyze product exclusivity as a long-term strategy to counter showrooming behavior. These papers focus on the competition between B&M retailers and online retailers. There are also other scenarios. Considering inventory availability, Gao and Su (2017b) find that physical showrooms may be unprofitable for a cross-channel retailer. In contrast, Bell et al. (2018a) provide empirical evidence that the introduction of showrooms by online-first retailers increases overall demand and operational efficiency. Viejo-Fernández et al. (2020) empirically reveal that showroomers are more likely to purchase products at a high price, explaining retailers' special interest in these customers. Instead of the normal showrooming behavior, Gu and Tayi (2017) explore pseudo-showrooming (i.e., the behavior of inspecting one product at a seller's physical store before buying a related but different product at the same seller's online store) when two horizontally differentiated products are sold respectively online and offline, while Zhang et al. (2020) investigate inter-product showrooming (i.e., the behavior of inspecting one product offline but buying a different or related product online) when a cross-channel retailer sources from two manufacturers. However, all the aforementioned papers fix the setting in a single period. In contrast, in a two-period setting, we introduce *intertemporal showrooming behavior*: consumers do the first part of showrooming (i.e., inspect the product offline) in the first period and then finish the online purchase in the second period. Moreover, we show that this seemingly negative behavior does not weaken the retailer's attempt to provide inspection only in the first period.

Third, this paper is also related to the literature on managing strategic consumer behavior. This stream of research typically employs dynamic pricing policies, including responsive pricing and pre-announced pricing (Su, 2007; Aviv and Pazgal, 2008; Aviv et al., 2019; Yu et al., 2020). Moreover, the general consensus is that the latter policy is more effective than the former, although Cachon and Swinney (2009) and Papanastasiou and Savva (2017) are two notable exceptions. Our results correspond to the conventional wisdom. However, in contrast to the previous literature, consumers are strategic along two dimensions in our model: that is, they strategically decide when to purchase and how to purchase. A more recent work, by Aflaki and Swinney (2020), also investigates a two-dimensional situation, but their focus is on inventory pooling rather than inspection service provision.

3. Model

Consider a cross-channel retailer selling a seasonal product over two periods, with price p_1 and p_2 respectively. The product incurs a constant margin cost $^{\mathcal{C}}$. There is a unit mass of consumers with idiosyncratic valuation $^{\mathcal{V}}$ for the product's observable features (e.g., color, style, brand), where $_{\mathcal{V} \square \ U \ 0,1}$. However, for products like furniture, even if consumers are

well informed about the design through online browsing, they are still uninformed about the unobservable attributes (e.g., texture, material). Simply put, without inspection, consumers are uncertain about whether the product fits their needs. The fit-related uncertainty is independent of the customer's private valuation (e.g., Huang et al., 2018; McWilliams, 2012). Moreover, a consumer who discovers a mismatch does not purchase. Formally, we assume that a consumer with a good fit obtains value v while a consumer finding a bad fit obtains zero value. The product matches its need with probability θ and does not with probability $1-\theta$ (e.g., Jing, 2018; Kuksov and Liao, 2018). This distribution is a common knowledge. Each consumer demands at most one unit product during the whole selling period.

To resolve its match uncertainty, a consumer can inspect the product at the retailer's offline showrooms (if available), which incurs visiting cost h_s . In reality, consumers may have different visiting costs due to varying travel distances. For simplicity, we assume that h_s is

uniformly distributed in [0,1]. Without loss of generality, we normalize the cost of buying online to zero. Under the omni-channel strategy of offline information delivery and online product fulfillment, the retailer may selectively display some products in the showrooms because of limited shelf space and costly sales assistance. To focus on the information delivery role of the physical showrooms, we only consider the case where there are very limited showrooms and the showrooms are intended mainly for product display rather than product fulfillment³ (e.g., the furniture available at Urban Ladder⁴ stores is only for display and inspection). Note that we aim to analyze how the availability of product inspection affects the retailer's dynamic pricing in the presence of strategic consumers, disregarding which channel fulfills purchases. Without ambiguity, we assume that those who visit the physical showrooms uniformly fulfill their orders online (i.e., the typical showrooming behavior). We use *informed* purchase to denote purchasing with inspection and uninformed purchase otherwise. Without inspection, consumers have to learn about possible mismatch after receiving the product. In this case, any consumer discovering a mismatch ex post gains a full refund by returning the product, which meanwhile incurs the additional hassle cost h_r . For the retailer, the cost incurred by a product return is assumed to be zero.

We first consider a case in which the product will be online exclusive in the second period. In other words, consumers are only able to inspect the product in the first period (i.e., firstperiod inspection). This assumption is based on the observation that a physical store usually displays selected products due to its limited space⁵ and updates its display regularly. This operation is commonly conducted for some fashion products that are rapidly updated, such as apparel.⁶ In this case, a strategic consumer considers not only whether to make an informed purchase but also whether to defer the purchase. We denote by $\delta \in (0,1)$ the consumers' strategic degree (i.e., a measure of patience), which is a discount factor applied to second-period purchase (Papanastasiou and Savva, 2017). On the other hand, we assume that the retailer does not discount second-period profit—that is, the retailer is more patient than consumers (Cachon and Swinney, 2011). To reveal the effect of the timing of the inspection service provision, we also consider the case in which inspection is available in both periods (two-period inspection).

Regarding the inspection service provision, we do not include the associated cost in our analysis, while a stream of research on motivating traditional B&M retailers' service provision does (e.g., Kuksov and Liao, 2018; Zhang et al., 2020). Instead, we focus on the timing of the

³ We note that some retailers, such as Zara and Ikea, provide their products offline for both selling and display

⁵ https://venturebeat.com/2017/04/29/heres-what-amazons-new-stores-will-look-like/

⁶ We observed that on online shopping platform like Tmall.com, some apparel brands tag "new arrivals, available offline" on their seasonal products. One can also see discounted apparel tagged with "online exclusive" at Gap online.

inspection service provision, given an already operated physical store. Following the literature on managing consumers' strategic behavior (e.g., Papanastasiou and Savva, 2017), we consider two pricing strategies, with responsive pricing in the main model and pre-announced pricing as an extension.

Table 1. Summary of Notations

Symbol	Description	
<i>i</i> = 1, 2	Product selling periods	
p	Retail price of the product	
С	The retailer's marginal cost	
v	Consumers' value for the product, where $v \square U [0,1]$	
θ	The probability with which the product matches consumers' needs	
$h_{_{r}}$	The hassle cost incurred by returning the product	
$h_{_{s}}$	Visiting cost: i.e., the hassle cost incurred by visiting the physical store	
δ	Consumers' strategic degree, where $\delta \in (0,1)$	
arphi	The probability that the product will be available for inspection in the second period	
EU	Consumers' expected utility	
π	The retailer's profit	
n	The number of consumers who make purchases	
N	The superscript/subscript indicates the case that the retailer does not provide an	
14	inspection service	
S	The superscript/subscript indicates the case that the retailer provides an inspection	
S	service only in the first period	
В	The superscript/subscript indicates the case that the retailer provides inspection	
D	service in both periods (BP means that the inspection is possible in the second period)	
D	The superscript/subscript indicates the case that the consumers are distracted when the	
2	retailer provides an inspection service only in the first period	
*	The superscript/subscript indicates the equilibrium results	

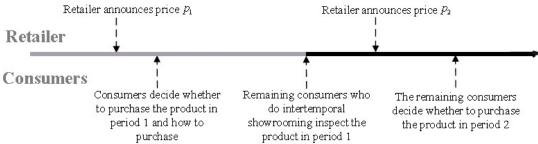


Figure 1. The sequence of actions under responsive pricing (first-period inspection available).

We summarize below the sequence of the game when first-period inspection is available (see Figure 1). At the beginning of the first period, the retailer announces its first-period price

 p_1 under responsive pricing. Observing the price, consumers need to decide whether to purchase. Or not a consumer makes a first-period purchase only if the following two conditions are satisfied simultaneously: (i) its expected utility from first-period purchase is non-negative, and (ii) its expected utility from first-period purchase is not lower than the expected utility from second-period purchase (discounted). Meanwhile, consumers decide how to purchase given the availability of first-period inspection. Those consumers who defer their purchases may inspect the product before entering into the second period (see the analysis in Section 4.2), from which the *intertemporal showrooming behavior* arises. As the remaining consumers reach the second period, the retailer then sets its second-period price p_2 . Any remaining consumers tend to make a second-period purchase as long as the product's expected utility is non-negative. The retailer seeks to maximize its overall expected profit.

4. Analysis

We begin with the benchmark (i.e., the No Inspection case) in which product inspection is not accessible. Then, we move on to the First-period Inspection case and the Two-period Inspection case. Throughout the analysis, we consider the *threshold purchasing policy* (Besanko and Winston, 1990; Papanastasiou and Savva, 2017). Given any first-period price, consumers with valuation over $\overline{v_1}$ will make purchases and a portion of the remaining consumers will make second-period purchases, while others will leave the market.

4.1. No Inspection (N)

We conduct a backward induction analysis to derive the equilibrium results. To ensure that the retailer's prices are higher than the marginal cost, throughout the paper, we assume that $c < 1 - \frac{1 - \theta}{\theta} h_r$ (i.e., $(1 - c)\theta - (1 - \theta)h_r > 0$).

In the second period, we assume that the total mass of the remaining consumers is $\overline{v_1}^N$. Thus, for some $\overline{v_1}^N \in [0,1]$, the second-period consumers are distributed uniformly among $\left[0,\overline{v_1}^N\right]$. Given the second-period price p_2^N , a consumer will purchase only if $EU_2^N = \theta\left(v - p_2^N\right) - \left(1 - \theta\right)h_r \ge 0$, which leads to second-period purchasing threshold $\overline{v_2}^N = p_2^N + \frac{1 - \theta}{\theta}h_r$. Then the retailer maximizes its second-period profit $\pi_2^N = \theta\left(\overline{v_1}^N - p_2^N - \frac{1 - \theta}{\theta}h_r\right)\left(p_2^N - c\right)$ by choosing $p_2^N = \frac{\overline{v_1}^N + c}{2} - \frac{1 - \theta}{2\theta}h_r$. The subgame's equilibrium in the second period is clearly unique for any given $\overline{v_1}^N$.

Now consider the first period. The retailer and consumers will anticipate the equilibrium of the second period subgame. Given the first period price p_1^N , the consumers will form beliefs over $\overline{v_1}^N$ and $p_2^N(\overline{v_1}^N)$, which are correct in equilibrium. Then, a consumer makes a first-period purchase only if the following two conditions are satisfied simultaneously: (i) $EU_1^N = \theta(v - p_1^N) - (1 - \theta)h_r \ge 0$ and (ii) $EU_1^N \ge \delta \left[\theta(v - p_2^N) - (1 - \theta)h_r\right]$. Accordingly, one can readily derive the purchasing threshold:

$$\overline{v}_{1}^{N}(p_{1}^{N}) = \begin{cases} \frac{2\theta p_{1}^{N} + (2-\delta)(1-\theta)h_{r} - \delta\theta c}{(2-\delta)\theta} & \text{if } p_{1}^{N} \leq \frac{2-\delta(1-c)}{2} - \frac{(2-\delta)(1-\theta)h_{r}}{2\theta}, \\ 1 & \text{if } p_{1}^{N} > \frac{2-\delta(1-c)}{2} - \frac{(2-\delta)(1-\theta)h_{r}}{2\theta}. \end{cases}$$

If the first-period price is not too high, consumers with higher expected utility make a purchase in this period while others defer their purchase; otherwise, all consumers will postpone their purchase until the second period. Anticipating the consumers' above response and the second-period outcome, the retailer maximizes its overall profit:

$$\pi_{N}\left(p_{1}^{N}\right) = \theta\left(1 - \overline{v_{1}}^{N}\right)\left(p_{1}^{N} - c\right) + \frac{\theta}{4}\left(\overline{v_{1}}^{N} - \frac{1 - \theta}{\theta}h_{r} - c\right)^{2}$$
with
$$p_{1}^{N} = \frac{\left(2 - \delta\right)^{2}\left(\theta - \left(1 - \theta\right)h_{r}\right) + \left(2 - \delta^{2}\right)\theta c}{2\left(3 - 2\delta\right)\theta}.$$

Lemma 1. In the absence of inspection, the retailer's optimal responsive pricing path is:

$$p_1^{N*} = \frac{(2-\delta)^2 (\theta - (1-\theta)h_r) + (2-\delta^2)\theta c}{2(3-2\delta)\theta}, \text{ and } p_2^{N*} = \frac{(2-\delta + (4-3\delta)c)\theta - (2-\delta)(1-\theta)h_r}{2(3-2\delta)\theta}.$$

The retailer's optimal profit is $\pi_N^* = \frac{\left(2-\delta\right)^2 \left(\theta-\left(1-\theta\right)h_r-\theta c\right)^2}{4\left(3-2\delta\right)\theta}$.

Note that for $p_1^N > \frac{2 - \delta(1 - c)}{2} - \frac{(2 - \delta)(1 - \theta)h_r}{2\theta}$, the optimal profit is $\pi_N = \frac{1}{2} - \frac{1}{2} + \frac{1}{$

$$\frac{\left(\left(1-c\right)\theta-\left(1-\theta\right)h_{r}\right)^{2}}{4\theta} \text{ , which is lower than } \pi_{N}^{*} \text{ for } \delta \in \left(0,1\right) \text{ . Moreover, } p_{1}^{N^{*}} \leq 1$$

$$\frac{2-\delta(1-c)}{2} - \frac{(2-\delta)(1-\theta)h_r}{2\theta}$$
. This means that $p_1^{N^*}$ is the global maximizer of the profit

function, since $\pi_N(p_1^N)$ is concave and continuous. Based on the optimal prices, the first- and second-period purchasing thresholds are respectively

$$\overline{v_1}^N = \frac{\left(2 - \delta + (1 - \delta)c\right)\theta + (1 - \delta)(1 - \theta)h_r}{(3 - 2\delta)\theta} \quad \text{and} \quad$$

$$\overline{v}_{2}^{N} = \frac{\left(2 - \delta + \left(4 - 3\delta\right)c\right)\theta + \left(4 - 3\delta\right)\left(1 - \theta\right)h_{r}}{2\left(3 - 2\delta\right)\theta}.$$

We can readily show that the equilibrium price path is decreasing (i.e., $p_1^N \ge p_2^N$). With consumers becoming more strategic (i.e., as δ increases), the first-period price decreases while the second-period price increases, approaching each other, and the retailer's profit declines.

4.2. First-period Inspection (S)

Given the availability of offline inspection, a consumer at the beginning of the first period will consider not only when to purchase but also whether to inspect the product before purchasing. In other words, the strategic behavior involves two aspects regarding a purchase. A highly strategic consumer, for instance, may attempt to inspect the product in the first period but defer purchase until the second period. We refer to such behavior as *strategic and informed deferring* (i.e., *intertemporal showrooming behavior*) to distinguish it from the traditional strategic deferring. In doing so, the consumer incurs a visiting cost h_s in the first period but avoids the mismatch cost $(1-\theta)h_r$ in the second period. The corresponding expected utility discounted to the first period is $\delta\theta(v-p_2)-h_s$.

To clarify the possible scenarios regarding consumers' purchasing options, we first investigate the situation when a consumer is willing to inspect the product in the first period but defer purchase until the second period (i.e., the consumer expects to make an informed purchase in the second period). Note that the consumer makes such a decision in the first period. It follows that the consumer is willing to engage in intertemporal showrooming if $\delta\theta \left(v-p_2\right)-h_s\geq 0$. That is to say, those with valuation v no less than $p_2+\frac{h_s}{\delta\theta}$ (i.e., $v\geq p_2+\frac{h_s}{\delta\theta}$) are willing to inspect in advance, and to make a second-period purchase (if they do not purchase in the first period).

Apart from the above informed purchase in the second period, a consumer can also purchase without first-period inspection provided that $v \ge p_2 + \frac{1-\theta}{\theta}h_r$. Hence, two options are possible in the second period. First, if $h_s \le \delta (1-\theta)h_r$, consumers prefer to make an informed purchase in the second period. Thus, if not purchasing in the first period, those who are willing to engage in intertemporal showrooming (i.e., $v \ge p_2 + \frac{h_s}{\delta \theta}$) will make purchases in the second period; while those who are unwilling to engage in intertemporal showrooming (i.e.,

 $v < p_2 + \frac{h_s}{\delta\theta}$) will leave the market, since they are not willing to make an uninformed purchase in the second period (see $\delta\theta(v-p_2)-\delta(1-\theta)h_r \leq \delta\theta(v-p_2)-h_s < 0$). Therefore, in this case, a consumer's willingness to engage in intertemporal showrooming determines the purchasing threshold in the second period (i.e., $p_2 + \frac{h_s}{\delta\theta}$). In contrast, if $h_s > \delta(1-\theta)h_r$, consumers expect to make an uninformed purchase in the second period. However, among those unwilling to engage in intertemporal showrooming (i.e., $v < p_2 + \frac{h_s}{\delta\theta}$), a portion (i.e., $p_2 + \frac{(1-\theta)h_r}{\theta} \leq v < p_2 + \frac{h_s}{\delta\theta}$) will still be willing to make second-period purchases. Moreover, those who are willing to engage in intertemporal showrooming ($v \geq p_2 + \frac{h_s}{\delta\theta}$) will prefer to make an uninformed purchase (since $\delta[\theta(v-p_2)-(1-\theta)h_r] > \delta\theta(v-p_2)-h_s$). Thus, in this case, a consumer's willingness to make an uninformed purchase results in the purchasing threshold in the second period (i.e., $p_2 + \frac{(1-\theta)h_r}{\theta}$).

Now we discuss the first-period purchasing options. For any consumer, before considering strategic deferring, they also have two potential choices: informed purchase and uninformed purchase. The corresponding expected utilities are respectively $\theta(v-p_1)-h_s$ and $\theta(v-p_1)-(1-\theta)h_r$. Given the two aforementioned scenarios in the second period, a consumer may fall into one of the following three scenarios (see Table 2). Specifically, if $h_s > (1-\theta)h_r$, to decide on when to purchase, a strategic consumer's tradeoff is between $\theta(v-p_1)-(1-\theta)h_r$ and $\delta[\theta(v-p_2)-(1-\theta)h_r]$. The remaining consumers will make a purchase in the second period provided that $v \ge p_2 + \frac{(1-\theta)h_r}{\theta}$. Second, if $\delta(1-\theta)h_r < h_s \le (1-\theta)h_r$, a consumer compares $\delta(v-p_1)-h_s$ with $\delta[\theta(v-p_2)-(1-\theta)h_r]$. Similarly, among the remaining consumers, those with $v \ge p_2 + \frac{(1-\theta)h_r}{\theta}$ will make an uninformed purchase in the second period. Third, if $h_s \le \delta(1-\theta)h_r$, the trade-off is between $\delta(v-p_1)-h_s$ and $\delta(v-p_2)-h_s$ since a consumer expects to make an informed purchase in the second period. Among the remaining consumers, those with $\delta(1-\theta)h_r$ 0 will first inspect the product and then enter the second period.

Table 2. Scenarios in the Presence of First-Period Inspection

Scenarios and Conditions	Possible Purchase Choices and Expected Utility		
Scenarios and Conditions	First-period Purchase	Second-period Purchase	
$SL: h_{s} \leq \delta (1-\theta) h_{r}$	$\theta(v-p_1)-h_s$	$\delta\theta(v-p_2)-h_s$	
SI: $\delta(1-\theta)h_r < h_s \le (1-\theta)h_r$	$\theta(v-p_{_1})-h_{_s}$	$\delta \left[\theta \left(v-p_{2}\right)-\left(1-\theta\right)h_{r}\right]$	
SH: $h_s > (1-\theta)h_r$	$\theta(v-p_1)-(1-\theta)h_r$	$\delta \left[\theta \left(v-p_{2}\right)-\left(1-\theta\right)h_{r}\right]$	

Overall, the relationship between visiting cost h_s and mismatch cost $(1-\theta)h_r$ drives those shopping options. The detailed equilibrium analysis can be found in Appendix A.⁷ **Lemma 2.** In the presence of first-period inspection, the retailer's optimal responsive pricing path is:

$$p_1^{S*} = \frac{2((2-\delta)^2 + (2-\delta^2)c)\theta + (2-2\delta^2 + \delta^3)(1-\theta)^2 h_r^2 - 2(2-\delta)^2 (1-\theta)h_r}{4(3-2\delta)\theta},$$
and
$$p_2^{S*} = \frac{2((2-\delta) + (4-3\delta)c)\theta - (2-4\delta+\delta^2)(1-\theta)^2 h_r^2 - 2(2-\delta)(1-\theta)h_r}{4(3-2\delta)\theta}.$$

The retailer's optimal profit is:

$$\pi_{S}^{*} = \frac{1}{16(3-2\delta)\theta} \left\{ \left(4 - 4\delta + \delta^{4} \right) \left(1 - \theta \right)^{4} h_{r}^{4} - 4 \left(2 - 2\delta^{2} + \delta^{3} \right) \left(1 - \theta \right)^{3} h_{r}^{3} + 4 \left(4 + 2(1-c)\theta - 4\delta + \left(1 - 2(1-c)\theta \right) \delta^{2} + \left(1 - c \right) \theta \delta^{3} \right) \left(1 - \theta \right)^{2} h_{r}^{2} - 8 \left(2 - \delta \right)^{2} \left(1 - c \right) \theta \left(1 - \theta \right) h_{r} + 4 \left(2 - \delta \right)^{2} \left(1 - c \right)^{2} \theta^{2} \right\}.$$

Proposition 1. In the presence of first-period inspection, (a) the retailer will price higher in the first period and probably lower in the second period, i.e., $p_1^{S^*} > p_1^{N^*}$; $p_2^{S^*} < p_2^{N^*}$ if $\delta \in (0,2-\sqrt{2})$ while $p_2^{S^*} > p_2^{N^*}$ if $\delta \in (2-\sqrt{2},1)$; (b) the retailer will be better off, i.e., $\pi_S^* > \pi_N^*$; (c) the retailer will achieve more sales in the first period but probably fewer sales in the second period, but in total it will capture more purchases, i.e., $n_1^S > n_1^N$; $n_2^S < n_2^N$ if $\delta \in (0,2-\sqrt{2})$ while $n_2^S > n_2^N$ if $\delta \in (2-\sqrt{2},1)$; $n_2^S > n_2^N$.

Proposition 1 summarizes the comparison between the benchmark and the case of firstperiod inspection. As consumers are strategic, they typically tend to defer purchase until the

⁷ Note that we only consider the case where first-period demand is nonnegative for each scenario. In the subsequent analysis, we have incorporated the constraints to ensure that the first-period purchasing thresholds are smaller than 1. Moreover, in Appendix A, we investigate another two cases where the retailer does not target all consumers in the first period and where the retailer targets no consumers in the first period. The former case might make the retailer better off, but it is hardly likely; the latter case makes the retailer worse off.

second period. However, when inspection is available in the first period, more consumers will make first-period purchases. In other words, allowing first-period inspection in part deters consumers' strategic behavior. Moreover, the retailer captures more consumers in total. This indicates that the retailer can deliver more value to society (Shum et al., 2017). Intuitively, to induce more sales in the first period, the retailer should either lower price p_1 or increase price p_2 . But Proposition 1 suggests that the retailer increases its first-period price and may lower its second-period price. This is because product inspection enables the retailer to adjust price more flexibly. Given any price p_1 , a consumer with low visiting cost (relative to mismatch cost) will be more likely to make a first-period purchase when inspection is available than when it is not. Thus, lifting the first-period price may not decrease the retailer's first-period profit. In the second period, the retailer may also benefit more. When consumers are relatively more strategic, the retailer sets a higher second-period price but captures more second-period purchases. Note that while the retailer benefits from providing first-period inspection, our model does not take into account the fixed cost of inspection service. However, our model could have implications for the retailer who has already spent a considerable amount of money (in the form of physical infrastructure).

We denote by n the number of purchases realized. Figure 2 illustrates the variation in purchasing numbers and purchasing behaviors. The left-hand plots show that there exist twoway consumer shifts between the two periods, while the right-hand plots display specifically four types of shifts. First, some consumers switch between "buy" and "not buy" (e.g., "NB-2IB" refers to shifting from "Not Buying" to "Second-period, Informed Buying"). Second, some consumers switch between the two periods without changing information condition (e.g., "1UB-2UB" means shifting from first-period purchase to second-period purchase while remaining uninformed). Third, consumers may shift from uninformed purchase to informed purchase in the same period (e.g., "2UB \rightarrow 2IB"). The last case occurs when some originally making uninformed purchases in the second period now make informed purchases in the first period (i.e., "2UB→1IB"). As some consumers who were originally leaving the market now make second-period purchases, the retailer captures a larger segment of the consumer population. Moreover, those with relatively low visiting costs make informed purchases, enabling the retailer to price higher in the first period. However, some of those with relatively high visiting costs who originally make first-period purchases are forced to enter the second period. Nonetheless, inspection provision could deter strategic delay overall, as inspection is more attractive to those with low visiting costs.

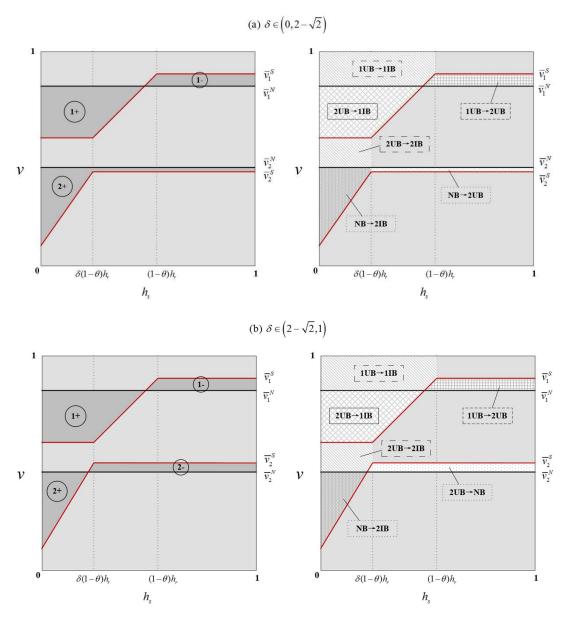


Figure 2. Comparison of equilibrium market segmentation between first-period inspection and no inspection. *Note:* The figure is not numerical but is directly based on the equilibrium results. To show the subtle details, this figure is not drawn exactly to scale.

4.3 Two-Period Inspection (B)

So far, we have focused on the case where inspection is only available in the first period, if at all. In practice, however, some products may be available offline for both periods. The question of interest is whether delivering the uncertainty-resolving service across two periods is better for the retailer. In this case, a consumer will incur the visiting cost in the second period to make an informed purchase in this period. Then, one can readily divide consumers' purchasing options into two scenarios by weighing *visiting cost* against *mismatch cost*. More specifically, in both periods, the expected utility (yet to be discounted) for making a purchase

is either $\theta(v-p_i)-h_s$ or $\theta(v-p_i)-(1-\theta)h_r$. Thus, if $h_s \leq (1-\theta)h_r$, a strategic consumer compares $\theta(v-p_1)-h_s$ with $\delta\left[\theta(v-p_2)-h_s\right]$ to make the purchasing decision. Otherwise, the trade-off is between $\theta(v-p_1)-(1-\theta)h_r$ and $\delta\left[\theta(v-p_2)-(1-\theta)h_r\right]$. The detailed equilibrium results are given in Online Appendix A.

Table 3. Scenarios in the Presence of Two-Period Inspection

Scenarios and Conditions	Possible Purchase Choices and Expected Utility		
Scenarios and Conditions	First-period Purchase	Second-period Purchase	
BL: $h_s \le (1 - \theta)h_r$	$\theta(v-p_1)-h_s$	$\delta[\theta(v-p_2)-h_s]$	
BH: $h_s > (1 - \theta)h_r$	$\theta(v-p_1)-(1-\theta)h_r$	$\delta[\theta(v-p_2)-(1-\theta)h_r]$	

Proposition 2. When inspection is available in both periods, (a) the retailer will price higher in both periods and benefit more than when inspection is available only in the first period: i.e., $p_1^{B^*} > p_1^{S^*}$, $p_2^{B^*} > p_2^{S^*}$ and $\pi_B^* > \pi_S^*$; (b) the retailer will achieve fewer sales in the first period but more sales in the second period, and may achieve more sales in total: i.e., $n_1^B < n_1^S$; $n_2^B > n_2^S$; $n_2^B > n_2^S$ if $\delta \in (0,2/3)$ while $n_2^B < n_2^S$ if $\delta \in (2/3,1)$.

Compared to the case of first-period inspection, the retailer will price higher in the first period to target fewer consumers. Meanwhile, the retailer can also price higher in the second period, because consumers with relatively low visiting costs can now make informed purchases without intertemporal showrooming in the second period. Intuitively, allowing second-period inspection will induce more consumers to purchase in the second period and may make the retailer worse off, since the second-period price is lower than the first-period price. However, this intuition is not valid. This is because those with visiting costs in the middle range (i.e., $\delta(1-\theta)h_r < h_s \le (1-\theta)h_r$) may switch from uninformed purchases to informed purchases, improving their expected net utility. Thus, the retailer can profit more by raising its secondperiod price as well as its first-period price. Similarly, Figure 3 describes the specific variation regarding the purchasing number and purchasing behavior. Also, we identify a two-way switch between the two periods and four types of shifts. Generally, by offering an inspection service in the second period, some consumers with visiting costs in the middle range will join the market, while some with relatively high visiting costs will leave the market. This implies that a two-period inspection service tends to be less beneficial to those located far from the physical stores, since they are less likely to engage in showrooming.

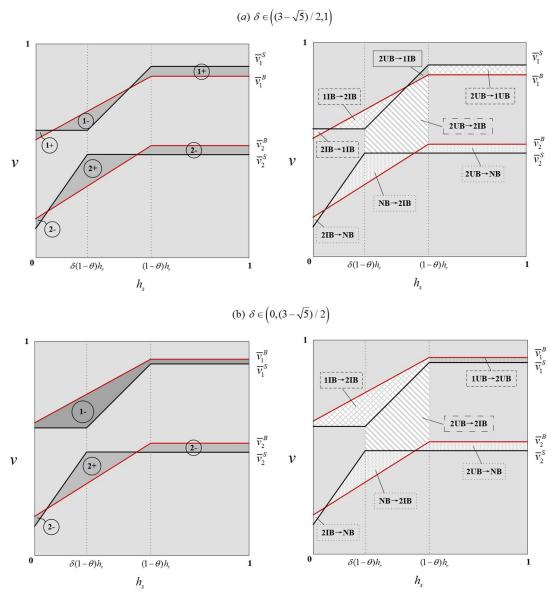


Figure 3. Comparison of equilibrium market segmentation between two-period inspection provision and first-period inspection provision. *Note:* The figure is not numerical but is directly based on the equilibrium results. To show the subtle details, t

his figure is not drawn exactly to scale.

Table 4. Scenarios in the Presence of Possible Two-Period Inspection

Scenarios and Condition	Possible Purchase Choices and Expected Utility		
Scenarios and Condition	First-period Purchase	Second-period Purchase	
BPL: $h_s \le \varphi(1-\theta)h_r$	$\theta(v-p_1)-h_s$	$\delta[\theta(v-p_2)-(1-\varphi)(1-\theta)h_r-h_s]$	
BPI: $\varphi(1-\theta)h_r < h_s \le$	$\theta(v-p_1)-h_s$	$\delta\theta(v-p_1)-(1-\theta)h_r$	
$(1-\theta)h_r$	$\theta(\nu-p_1)-n_s$	$\partial\theta(\nu-\rho_1)-(1-\theta)n_r$	
BPH: $h_s > (1 - \theta)h_r$	$\theta(v-p_1)-(1-\theta)h_r$	$\delta\theta(v-p_1)-(1-\theta)h_r$	

Proposition 3. Suppose that $\varphi \in (0,1)$. There exist φ and δ such that $\pi_{BP}^* < \pi_S^*$. Specifically, if $\varphi \in (\sqrt{\delta},1)$, $\pi_{BP}^* > \pi_S^*$; if $\varphi \in (0,\sqrt{\delta})$, $\pi_{BP}^* < \pi_S^*$.

Proposition 2 justifies the inspection service provision in both periods without accounting for the occupation of limited space in real life. To continue providing inspection service in the second period, some store space needs to be left for out-of-season products. However, the occupied space might otherwise be used for other new arrivals. Such tradeoff affects whether to provide inspection in the second period. In practice, as the season changes, the retailer typically replaces out-of-season items with new arrivals in the store. As a result, consumers may gain no information when attempting to inspect the product in the second period. The likelihood is not minimal if the availability of offline inspection is not announced online. To model this situation, we further assume that inspection in the second period is satisfied with probability $\varphi \in (0,1)$ and unsatisfied with probability $1-\varphi$. We use BP to denote this scenario. Note that $\varphi=1$ indicates that the second-period inspection is ensured, corresponding to the aforementioned analysis.

In this case, when failing to resolve the uncertainty offline, a consumer can still make a purchase online provided that the expected utility is nonnegative. Consequently, if the consumer attempts to visit a physical store to make a second-period purchase, the expected utility will be $\delta \left[\theta \left(v-p_2^{BP}\right)-(1-\varphi)\left(1-\theta\right)h_r-h_s\right]$. If the consumer directly purchases online, the expected utility is $\delta \left[\theta \left(v-p_2^{BP}\right)-(1-\theta)h_r\right]$. As with Gao and Su (2017b), the visiting cost is incurred before consumers inspect the product. The equilibrium results are included in Online Appendix A. Interestingly, Proposition 3 shows that given the uncertainty of inspection availability in the second period, the retailer is likely to prefer first-period inspection.

To summarize, the above results combined may imply a tradeoff for the inspection service: if the retailer can guarantee its uncertainty-resolving service in the second period, it is better to provide two-period inspection; otherwise, first-period inspection may be more secure. Based on their shopping experience, consumers may anticipate the decreasing path of inspection availability. To obviate consumers' anticipation, when launching new items, the retailer could announce clearly on the website or the online platform that no inspection is available for the future season (e.g., new summer fashions will be online exclusive once autumn arrives).

5. Extensions

⁸ We observe that even if a number of retailers adopt omni-channel operation in China, not many of them provide information on whether the product is available offline, not to mention the exact information on which physical store supplies the product.

5.1 No Intertemporal Showrooming

Recall that in the case of first-period inspection, some consumers can make second-period informed purchases, even though inspection is not allowed in the second period. That is, consumers may inspect the product in advance (before entering the second period) if they are willing to do so. In reality, however, consumers may be absent-minded in the sense that they forget the outcome of fit-related information. It follows that consumers may be disinclined to inspect the product in advance. Thus, consumers' possible strategic behavior will be reduced to the normal case—strategic deferring. We use D to denote this case and the equilibrium results are given in Online Appendix A.

Proposition 4. (i) Compared to the case of first-period inspection, if intertemporal showrooming does not exist, the retailer will price lower in both periods and benefit less. Mathematically, (a) $p_1^{S^*} > p_1^{D^*}$ and $p_2^{S^*} > p_2^{D^*}$; (b) $\pi_S^* > \pi_D^*$.

This consideration allows us to isolate how *intertemporal showrooming* affects the retailer's pricing and profit. The concerning case may be that $\pi_D^* > \pi_{BP}^* > \pi_S^*$. In other words, due to the *intertemporal showrooming*, providing a first-period inspection service becomes less profitable than providing a two-period inspection service. Proposition 4 suggests that such case will not occur, because *intertemporal showrooming* benefits the retailer. One may intuit that when consumers engage in intertemporal showrooming, the retailer benefits less, as the second-period price is lower. The presence of intertemporal showrooming, however, induces the retailer to raise its second-period price. This is because consumers who are willing to inspect in advance are those who have low visiting costs, and hence can afford the higher price.

5.2 Pre-announced Pricing

The existing literature on managing strategic consumer behavior typically considers the pre-announced pricing policy as an alternative to responsive pricing. To align with this stream of literature, we also investigate pre-announced pricing. While Papanastasiou and Savva (2017) find the reverse case in the presence of social learning, the conventional wisdom is that pre-announced pricing is more effective than responsive pricing (e.g., Aviv and Pazgal, 2008). In our model, this consensus is also valid, irrespective of the inspection provision strategy. Furthermore, Proposition 5 indicates that the qualitative results under responsive pricing remain unchanged. For the detailed equilibrium results, one can refer to Online Appendix A.

⁹ Extant literature on consumers' strategic behavior, such as the work of Chakraborty and Swinney (2019), assumes consumers to be distracted: i.e., some of them may forget to return to the purchase after intentional delay. To isolate the effect of *strategic and informed deferring*, we similarly consider the case in which consumers waive inspection in advance because of the "distractedness" when returning to purchase.

Proposition 5. Under pre-announced pricing, (a) when inspection is available in the first period, the retailer will price higher in the first period but perhaps lower in the second period, and will benefit more than when inspection is not available; (b) when inspection is available in both periods, the retailer will price higher in both periods, and will benefit more than when inspection is available only in the first period.

6. Managerial Implications

Based on our model, we have used quantitative analysis to gain some qualitative managerial implications. Not only did we get some conventional conclusions, but also some counterintuitive ones, which can have more or less significant effects on the retailer's operations.

First, by comparing the results of the case of N and S, the following managerial implications are obtained. Due to the existence of product mismatch problem, uninformed purchase online may decrease the utility of consumers, so retailers can solve this problem by providing offline showrooms. Under some assumptions, the result, from the model, is that the retailer will be better off when providing first-period inspection. In other words, allowing first-period inspection in part deters consumers' strategic behavior. Moreover, the Proposition 1 in this study suggests that the retailer increases its first-period price and may lower second-period price in the presence of first-period inspection, which is different from intuition.

Second, there are some other managerial implications by adding the case of B and BP. On the one hand, if inspection provision is ensured in the second period, allowing two-period inspection increases the retailer's prices and profit in both periods. On the other hand, when inspection availability in the second period follows a decreasing path (i.e., inspection is possible in the second period), it may be better to make inspection available only in the first period.

To summarize, in the context of the prevalence of omni-channel retailing, retailers may be able to increase their profit by providing pure showrooms through some appropriate strategies. For instance, for clothing and electronic products, which are updated quickly and take up little shelf space, more showrooms can be available to reduce the consumer's visiting cost and attract them to do offline inspection. While, for a product like furniture, it may be better for retailers to expand the size of the showrooms. This is because showrooms do not require space to store inventory, and they could be large enough to provide two-period inspections.

7. Conclusion

In this paper, we utilize a parsimonious model to analyze the effects of inspection provision on the interactions of a dynamic-pricing retailer and strategic consumers. In the presence of inspection provision, consumers' strategic behavior involves two dimensions: when to purchase (i.e., whether to purchase in the first period or the second period) and how to purchase (i.e., whether to engage in showrooming or not). For comparison, we consider four cases: no inspection is available; inspection is only available in the first period; inspection is ensured in both periods; and inspection is possible in the second period. Our results demonstrate that the characteristics of inspection provision affect the retailer's pricing and profit.

First, we find that allowing first-period inspection benefits the retailer. The retailer will price higher in the first period but probably lower in the second period. Even so, more consumers will purchase in the first period: that is, allowing first-period inspection can somewhat deter strategic deferral. This is because inspection provision can help to divide consumers according to their heterogeneous shopping costs. In the presence of first-period inspection, some consumers may engage in intertemporal showrooming. We find that the seemingly negative intertemporal showrooming behavior turns out to benefit the retailer.

To compare with first-period inspection, we also consider two-period inspection. On the one hand, if inspection provision is ensured in the second period, allowing two-period inspection increases the retailer's prices and profit in both periods. On the other hand, when inspection availability in the second period follows a decreasing path (i.e., inspection is possible in the second period), it may be better to make inspection available only in the first period.

A limitation of the study is the missing consideration of partial availability of a seasonal product towards the end of its season. Future research should integrate the stock-out possibility into the model, since it is another important factor that affects a strategic consumer's purchasing decision, particularly intertemporal showrooming. Another limitation is the cost of providing inspection services, which may have significant implications for the retailer. Further studies could consider the fixed cost of building and maintaining a B&M showroom as well as the opportunity cost of using limited shelf space for new arrivals in the second period.

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Dynamic Pricing when Providing Strategic Consumers with Showrooming Opportunities

Online Appendix A: equilibrium results

1 Analysis for Heterogeneous Visiting Cost

1.1 Target all consumers in both periods (S)

Note that when h_s is uniformly distributed in [0,1], the retailer needs to consider one integrated case in which consumers fall into the three scenarios (i.e., SL, SI, SH) in Section 4 according to their heterogeneous visiting costs. Figure A1 illustrates the general segments of the population in the market. Though facing the same price, consumers in each scenario have different purchasing behaviors, leading to different purchasing thresholds. Thus, the expected demand in each period is composed of three parts. Note that we consider the case where first-period demand is nonnegative for each scenario. In the next subsection, we will investigate the cases where those with relatively high visiting costs may purchase only in the second period.

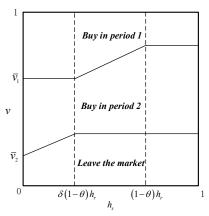


Figure A1. Market segmentation with two-dimensional heterogeneous consumers.

First, consider the second-period subgame. Since h_s is a random variable, the purchasing threshold is not constant; instead, it may be a function of h_s . Following the analysis in the basic model, one can readily show that $\overline{v}_2^L = p_2^S + \frac{h_s}{\theta \delta}$, $\overline{v}_2^I = p_2^S + \frac{1-\theta}{\theta}h_r$ and $\overline{v}_2^H = p_2^S + \frac{1-\theta}{\theta}h_r$. However, the purchasing threshold in the first period is not as clear. We use $\overline{v}_1(h_s)$ to denote it. More specifically, by comparing $EU_1 = \theta(v - p_1) - h_s$ with $EU_2 = \delta\theta(v - p_2) - h_s$, we assume that $\overline{v}_1^L = ah_s + b$ for scenario SL, where a = 0; by comparing $EU_1 = \theta(v - p_1) - h_s$ with $EU_2 = \delta \left[\theta(v - p_2) - (1-\theta)h_r\right]$, we assume that $\overline{v}_1^I = fh_s + g$ for

scenario SI; by comparing $EU_1=\theta\big(v-p_1\big)-\big(1-\theta\big)h_r$ with $EU_2=\delta\Big[\theta\big(v-p_2\big)-\big(1-\theta\big)h_r\Big]$, we assume that $\overline{v}_1^H=jh_s+k$ for scenario SH, where j=0. Then one can obtain the second-period demand:

$$\begin{split} D_2 &= \int_0^{\delta(1-\theta)h_r} \theta \bigg(b - p_2^S - \frac{h_s}{\theta \delta}\bigg) dh_s + \int_{\delta(1-\theta)h_r}^{(1-\theta)h_r} \theta \bigg(fh_s + g - p_2^S - \frac{1-\theta}{\theta}h_r\bigg) dh_s + \int_{(1-\theta)h_r}^1 \theta \bigg(k - p_2^S - \frac{1-\theta}{\theta}h_r\bigg) dh_s \\ &= \theta \delta \bigg(b - p_2^S\bigg) \Big(1-\theta\bigg)h_r - \frac{1}{2\delta} \Big(\delta \Big(1-\theta\Big)h_r\Big)^2 + \frac{\theta f}{2} \Big(1-\delta^2\Big) \Big(1-\theta\Big)^2 h_r^2 \\ &+ \theta \bigg(g - p_2^S - \frac{1-\theta}{\theta}h_r\bigg) \Big(1-\delta\Big) \Big(1-\theta\Big)h_r + \theta \Big(1-\Big(1-\theta\Big)h_r\Big) \bigg(k - p_2^S - \frac{1-\theta}{\theta}h_r\bigg). \end{split}$$

The retailer chooses \hat{p}_2^s to maximize $\pi_2(p_2^s) = (p_2^s - c)D_2$, where:

$$\hat{p}_{2}^{S} = \frac{k+c}{2} - \frac{\left(1+k\theta-g\left(1-\delta\right)\theta-b\delta\theta\right)\left(1-\theta\right)h_{r}}{2\theta} + \frac{\left(\delta+f\theta-f\theta\delta^{2}\right)\left(1-\theta\right)^{2}h_{r}^{2}}{4\theta}.$$

In the first period, the retailer and consumers anticipate the second-period subgame. Given a first-period price p_1^S and consumers' beliefs over \hat{p}_2^S , one can derive the purchasing thresholds in the first period, i.e.:

$$\overline{v_1}^L = \frac{p_1^S - \delta \hat{p}_2^S}{1 - \delta}, \quad \overline{v_1}^I = \frac{\theta p_1^S - \delta \theta \hat{p}_2^S + h_s - \delta (1 - \theta) h_r}{(1 - \delta) \theta}, \text{ and}$$

$$\overline{v_1}^H = \frac{\theta p_1^S - \delta \theta \hat{p}_2^S + (1 - \delta) (1 - \theta) h_r}{(1 - \delta) \theta}.$$

Comparing them with the above assumptions regarding thresholds, one can readily derive:

$$b = \frac{p_1^s - \delta \hat{p}_2^s}{1 - \delta}, \quad f = \frac{1}{(1 - \delta)\theta}, \quad g = \frac{\theta p_1^s - \delta \theta \hat{p}_2^s - \delta (1 - \theta) h_r}{(1 - \delta)\theta},$$
and
$$k = \frac{\theta p_1^s - \delta \theta \hat{p}_2^s + (1 - \delta)(1 - \theta) h_r}{(1 - \delta)\theta}.$$

Solve b, f, g, k, and \hat{p}_2^s simultaneously and we can derive:

$$\hat{p}_{2}^{s} = \frac{2\theta \left(p_{1}^{s} + (1-\delta)c\right) - (1-\delta)(1-\theta)^{2} h_{r}^{2}}{2(2-\delta)\theta}, \quad b = \frac{2\theta \left(2p_{1}^{s} - \delta c\right) + \delta(1-\theta)^{2} h_{r}^{2}}{2(2-\delta)\theta},$$

$$f = \frac{1}{(1-\delta)\theta}, \quad g = \frac{2\theta (1-\delta)(2p_{1}^{s} - \delta c) + \delta(1-\delta)(1-\theta)^{2} h_{r}^{2} - 2\delta(2-\delta)(1-\theta) h_{r}}{2(2-\delta)(1-\delta)\theta},$$

$$k = \frac{2\theta \left(2p_{1}^{s} - \delta c\right) + \delta(1-\theta)^{2} h_{r}^{2} + 2(2-\delta)(1-\theta) h_{r}}{2(2-\delta)\theta}.$$

Then, we can obtain:

$$\begin{split} \hat{\pi}_{2}\left(p_{1}^{s}\right) &= \frac{\left(2\theta\left(p_{1}^{s}-c\right)-\left(1-\delta\right)\left(1-\theta\right)^{2}h_{r}^{2}\right)^{2}}{4\left(2-\delta\right)^{2}\theta}\,, \\ &\overline{v}_{1}^{L}\left(p_{1}^{s}\right) = \frac{2\theta\left(2p_{1}^{s}-\delta c\right)+\delta\left(1-\theta\right)^{2}h_{r}^{2}}{2\left(2-\delta\right)\theta}\,, \\ &\overline{v}_{1}^{I}\left(p_{1}^{s}\right) = \frac{1}{\left(1-\delta\right)\theta}h_{s} + \frac{2\theta\left(1-\delta\right)\left(2p_{1}^{s}-\delta c\right)+\delta\left(1-\delta\right)\left(1-\theta\right)^{2}h_{r}^{2}-2\delta\left(2-\delta\right)\left(1-\theta\right)h_{r}}{2\left(2-\delta\right)\left(1-\delta\right)\theta}\,, \\ &\overline{v}_{1}^{H}\left(p_{1}^{s}\right) = \frac{2\theta\left(2p_{1}^{s}-\delta c\right)+\delta\left(1-\theta\right)^{2}h_{r}^{2}+2\left(2-\delta\right)\left(1-\theta\right)h_{r}}{2\left(2-\delta\right)\theta}\,. \end{split}$$

Similarly, the first-period demand is:

$$D_{1} = \int_{0}^{\delta(1-\theta)h_{r}} \theta\left(1-\overline{v_{1}}^{L}\left(p_{1}^{S}\right)\right) dh_{s} + \int_{\delta(1-\theta)h_{r}}^{(1-\theta)h_{r}} \theta\left(1-\overline{v_{1}}^{I}\left(p_{1}^{S}\right)\right) dh_{s} + \int_{(1-\theta)h_{r}}^{1} \theta\left(1-\overline{v_{1}}^{H}\left(p_{1}^{S}\right)\right) dh_{s}$$

At the beginning of the game, the retailer then chooses p_1^s to maximize its overall profit:

$$\pi_{S} = (p_{1}^{S} - c)D_{1} + \frac{(2\theta(p_{1}^{S} - c) - (1 - \delta)(1 - \theta)^{2}h_{r}^{2})^{2}}{4(2 - \delta)^{2}\theta}.$$

The equilibrium results are:

$$p_{1}^{S*} = \frac{2((2-\delta)^{2} + (2-\delta^{2})c)\theta + (2-2\delta^{2} + \delta^{3})(1-\theta)^{2}h_{r}^{2} - 2(2-\delta)^{2}(1-\theta)h_{r}}{4(3-2\delta)\theta},$$

$$p_{2}^{S*} = \frac{2((2-\delta) + (4-3\delta)c)\theta - (2-4\delta+\delta^{2})(1-\theta)^{2}h_{r}^{2} - 2(2-\delta)(1-\theta)h_{r}}{4(3-2\delta)\theta},$$

$$\pi_{S}^{*} = \frac{1}{16(3-2\delta)\theta} \left\{ (4-4\delta+\delta^{4})(1-\theta)^{4}h_{r}^{4} - 4(2-2\delta^{2} + \delta^{3})(1-\theta)^{3}h_{r}^{3} + 4(4+2(1-c)\theta-4\delta+(1-2(1-c)\theta)\delta^{2} + (1-c)\theta\delta^{3})(1-\theta)^{2}h_{r}^{2} - 8(2-\delta)^{2}(1-c)\theta(1-\theta)h_{r} + 4(2-\delta)^{2}(1-c)^{2}\theta^{2} \right\}.$$

Furthermore, we have:

$$\overline{v_{1}}^{L} = \frac{2\theta(2-\delta+(1-\delta)c)+(1+2\delta-\delta^{2})(1-\theta)^{2}h_{r}^{2}-2(2-\delta)(1-\theta)h_{r}}{2(3-2\delta)\theta},$$

$$\overline{v_{1}}^{I} = \frac{2(3-2\delta)h_{s}+2\theta(1-\delta)(2-\delta+(1-\delta)c)+(1+\delta-3\delta^{2}+\delta^{3})(1-\theta)^{2}h_{r}^{2}-2(2-\delta^{2})(1-\theta)h_{r}}{2(3-2\delta)(1-\delta)\theta},$$

$$\overline{v_{1}}^{H} = \frac{2\theta(2-\delta+(1-\delta)c)+(1+2\delta-\delta^{2})(1-\theta)^{2}h_{r}^{2}+2(1-\delta)(1-\theta)h_{r}}{2(3-2\delta)\theta},$$

$$\overline{v}_{2}^{L} = \frac{h_{s}}{\theta \delta} + \frac{2((2-\delta)+(4-3\delta)c)\theta - (2-4\delta+\delta^{2})(1-\theta)^{2}h_{r}^{2} - 2(2-\delta)(1-\theta)h_{r}}{4(3-2\delta)\theta}$$

and

$$\overline{v}_{2}^{I} = \overline{v}_{2}^{H} = \frac{\left(1-\theta\right)h_{r}}{\theta} + \frac{2\left(\left(2-\delta\right)+\left(4-3\delta\right)c\right)\theta - \left(2-4\delta+\delta^{2}\right)\left(1-\theta\right)^{2}h_{r}^{2} - 2\left(2-\delta\right)\left(1-\theta\right)h_{r}}{4\left(3-2\delta\right)\theta}.$$

1.2 Not target all consumers in both periods (O&T)

As we show in the above subsection, those with relatively high visiting costs share higher first-period purchasing thresholds: that is, they are less likely to endure higher price. Thus, the retailer can enhance the first-period price to not target all consumers in both periods. Specifically, if $\overline{h}_s \leq \delta(1-\theta)h_r$, the retailer sells to no consumers in the first period; if $\delta(1-\theta)h_r < \overline{h}_s \leq (1-\theta)h_r$, the retailer will not target those along $[\overline{h}_s,1]$ in the first period. We use O and T to denote the above two cases, respectively. The following analysis will show that the main case (i.e., targeting all consumers in both periods) is less dominant. Note that other considerations are unchanged.

Target no consumers in the first period

This strategy indicates that $\overline{v}_1^L\left(p_1^s\right) = \overline{v}_1^I\left(p_1^s\right) = \overline{v}_1^H\left(p_1^s\right) = 1$: i.e., the retailer's first-period price is higher than $p_1^S \mid_{\overline{v}_1^L\left(p_1^s\right) = 1} = \frac{2\left(2-\left(1-c\right)\delta\right)\theta - \delta\left(1-\theta\right)^2h_r^2}{4\theta}$. Then, the retailer chooses p_2^O to maximize its profit $\pi_O = \left(p_2^O - c\right)D_O$, where:

$$D_O = \theta \left(\int_0^{\delta(1-\theta)h_r} \left(1 - p_2^O - \frac{h_s}{\theta \delta} \right) dh_s + \int_{\delta(1-\theta)h_r}^1 \left(1 - p_2^O - \frac{(1-\theta)h_r}{\theta} \right) dh_s \right).$$

One can readily derive that $p_2^{O^*} = \frac{2(1+c)\theta - 2(1-\theta)h_r + \delta(1-\theta)^2 h_r^2}{4\theta}$. The optimal profit

is:

$$\pi_{O}^{*} = \frac{\left(2(1-c)\theta - 2(1-\theta)h_{r} + \delta(1-\theta)^{2}h_{r}^{2}\right)^{2}}{16\theta}.$$

Then, we have:

$$\pi_{S}^{*} - \pi_{O}^{*} = \frac{(1-\delta)^{2} (2(1-c)\theta - 2(1-\theta)h_{r} + (2+\delta)(1-\theta)^{2}h_{r}^{2})^{2}}{16(3-2\delta)\theta} > 0.$$

Similar to the basic model, when no consumers participate in the first period, the retailer cannot be better off.

Target partial consumers in the first period

Now we consider the case in which the retailer targets those along $\left[\overline{h}_s,1\right]$ only in the second period, where $\delta(1-\theta)h_r < \overline{h}_s \le (1-\theta)h_r$. Note that $\overline{h}_s = (1-\theta)h_r$ means that those along $\left[0,(1-\theta)h_r\right]$ are sold in both periods, i.e., $\overline{v}_1^I\left(p_1^s\right)\big|_{h_s=(1-\theta)h_r}=\overline{v}_1^H\left(p_1^s\right)=1$; while $\overline{h}_s = \delta(1-\theta)h_r$ implies that no consumers are sold the item in the first period, because $\overline{v}_1^L\left(p_1^s\right) = \overline{v}_1^I\left(p_1^s\right)\big|_{h_s=\delta(1-\theta)h_r}=1$ (see Figure A1). Generally, we define:

$$\overline{v}_{1}^{I}\left(p_{1}^{S},\overline{h}_{s}\right) = \frac{1}{\left(1-\delta\right)\theta}\overline{h}_{s} + \frac{2\theta\left(1-\delta\right)\left(2p_{1}^{S}-\delta c\right) + \delta\left(1-\delta\right)\left(1-\theta\right)^{2}h_{r}^{2} - 2\delta\left(2-\delta\right)\left(1-\theta\right)h_{r}}{2\left(2-\delta\right)\left(1-\delta\right)\theta}.$$

This indicates that the retailer's first period price is higher than:

$$p_1^S \mid_{\overline{v}_1^I\left(p_1^S,\overline{h}_s\right)=1} = \frac{2\left(1-\delta\right)\left(2-\left(1-c\right)\delta\right)\theta - \delta\left(1-\delta\right)\left(1-\theta\right)^2h_r^2 + 2\delta\left(2-\delta\right)\left(1-\theta\right)h_r - 2\left(2-\delta\right)\overline{h}_s}{4\left(1-\delta\right)\theta}.$$

In the second period, the demand is formulated as:

$$D_2 = \int_0^{\delta(1-\theta)h_r} \theta \left(\overline{v_1}^{OL} - p_2^S - \frac{h_s}{\theta \delta} \right) dh_s + \int_{\delta(1-\theta)h_r}^{\overline{h}_s} \theta \left(\overline{v_1}^{OI} - p_2^S - \frac{1-\theta}{\theta} h_r \right) dh_s + \int_{\overline{h}_s}^1 \theta \left(1 - p_2^S - \frac{1-\theta}{\theta} h_r \right) dh_s.$$

Accordingly, the first-period demand is formulated as:

$$D_1 = \int_0^{\delta(1-\theta)h_r} \theta\left(1-\overline{v_1}^{OL}\right) dh_s + \int_{\delta(1-\theta)h}^{\overline{h_s}} \theta\left(1-\overline{v_1}^{OI}\right) dh_s.$$

One can conduct a similar analysis to Subsection 1.1 and derive the equilibrium results as follows:

$$\begin{split} p_1^{T^*} &= \frac{1}{4\theta \overline{h}_s \left(4 - 4\delta + (2\delta - 1)\overline{h}_s\right)} \Big\{ \delta \left(1 - \theta\right)^2 h_r^2 \Big[\left(2 - 4\delta^2 + 2\delta\right) \overline{h}_s + \delta^2 \overline{h}_s^2 - 4 (1 - \delta)\delta \Big] - \\ 2 \Big[2 + \left(2 - \overline{h}_s\right) \delta^2 - 2 \left(2 - \overline{h}_s\right) \delta \Big] (1 - \theta) h_r \overline{h}_s + 2 \Big[2 (1 - \delta) (3 - \delta + \delta c + c)\theta + (1 - \delta) \overline{h}_s^2 + \\ \left((2 + 4\theta) \delta - 2 (1 + \theta) - (1 - c)\theta \delta^2 \right) \overline{h}_s \Big] \overline{h}_s \Big\}, \\ p_2^{T^*} &= \frac{1}{4\theta \left(4 - 4\delta + (2\delta - 1)\overline{h}_s\right)} \Big\{ \delta \left(1 - \theta\right)^2 h_r^2 \left(4 - \delta \left(2 - \overline{h}_s\right)\right) - 2 (1 - \theta) h_r \left(4 - \delta \left(4 - 3\overline{h}_s\right)\right) + \\ 2 \Big[\overline{h}_s^2 + \left((3 + c)\delta - 2\right)\theta \overline{h}_s + 4 (1 - \delta) (1 + c)\theta \Big] \Big\}, \\ \pi_T^* &= \frac{1}{16\theta \overline{h}_s \left(4 - 4\delta + (2\delta - 1)\overline{h}_s\right)} \Big\{ \delta^2 \left(1 - \theta\right)^4 h_r^4 \Big[4\delta^2 + 4 \left(1 - \delta^2\right) \overline{h}_s + \delta^2 \overline{h}_s^2 \Big] - \\ 4\delta \left(1 - \theta\right)^3 h_r^3 \overline{h}_s \Big[4 + \delta^2 \left(2 - \overline{h}_s\right) - 2\delta \left(1 - \overline{h}_s\right) \Big] + 4 (1 - \theta)^2 h_r^2 \overline{h}_s \Big[4 + \delta^3 \theta \left(2 - \overline{h}_s\right) (1 - c) + \\ \delta \left(4\theta + \overline{h}_s^2 + 2 \left(2 - \theta (1 - c) + 2\right) \overline{h}_s - 4\theta c - 4\right) + \delta^2 \left(\left(3 + 4 \left(1 - c\right)\theta\right) \overline{h}_s - \overline{h}_s^2 - 6 \left(1 - c\right)\theta\right) \Big] - \\ \end{split}$$

$$8h(1-\theta)\overline{h}_{s} \left[(1+\delta)\overline{h}_{s}^{2} + (\delta^{2} + 2\delta - 2)(1-c)\theta\overline{h}_{s} + 4(1-\delta)(1-c)\theta \right] + 4\overline{h}_{s} \left[\overline{h}_{s}^{3} - 2(1-\delta)(1-c)\theta\overline{h}_{s}^{2} + (1-c)^{2}\delta^{2}\theta^{2}\overline{h}_{s} + 4(1-\delta)(1-c)^{2}\theta^{2} \right] \right\}.$$

Due to the complexity, we then conduct numerical analysis to compare π_T^* with π_S^* . Figure A2 illustrates the result, from which one can see that π_T^* can indeed be larger than π_S^* . Nonetheless, this is unlikely to happen. The consumers' degree of strategy needs to be high enough, making more consumers belong to $[0, \delta(1-\theta)h_r]$. Thus, for other extensions, we only consider the main case to focus on the effects of provision timing.

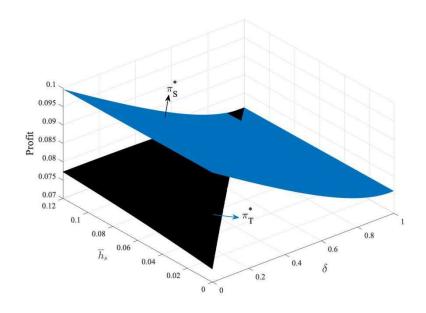


Figure A2. Comparison between π_T^* and π_S^* . Parameter values: $\theta = 0.6$, $h_r = 0.3$, c = 0.1.

2 Equilibrium when inspection is allowed in both periods

2.1 Inspection is definitely available in the second period (B)

When inspection is available in two periods, consumers fall into two scenarios depending on their heterogeneous visiting costs. The analysis is similar to before and we only present the equilibrium results as follows:

$$p_{1}^{B^{*}} = \frac{2((2-\delta)^{2} + (2-\delta^{2})c)\theta + (2-\delta)^{2}(1-\theta)^{2}h_{r}^{2} - 2(2-\delta)^{2}(1-\theta)h_{r}}{4(3-2\delta)\theta},$$

$$p_{2}^{B^{*}} = \frac{2((2-\delta) + (4-3\delta)c)\theta + (2-\delta)(1-\theta)^{2}h_{r}^{2} - 2(2-\delta)(1-\theta)h_{r}}{4(3-2\delta)\theta},$$

$$\pi_{B}^{*} = \frac{(2-\delta)^{2}((1-\theta)^{2}h_{r}^{2} + 2(1-c)\theta - 2(1-\theta)h_{r})^{2}}{16(3-2\delta)\theta},$$

$$\overline{v_1}^{BL} = \frac{h_s}{\theta} + \frac{2((2-\delta)+(1-\delta)c)\theta + (2-\delta)(1-\theta)^2 h_r^2 - 2(2-\delta)(1-\theta)h_r}{2(3-2\delta)\theta},$$

$$\overline{v_1}^{BH} = \frac{2((2-\delta)+(1-\delta)c)\theta + (2-\delta)(1-\theta)^2 h_r^2 + 2(1-\delta)(1-\theta)h_r}{2(3-2\delta)\theta},$$

$$\overline{v_2}^{BL} = \frac{h_s}{\theta} + \frac{2((2-\delta)+(4-3\delta)c)\theta + (2-\delta)(1-\theta)^2 h_r^2 - 2(2-\delta)(1-\theta)h_r}{4(3-2\delta)\theta},$$

$$\overline{v_2}^{BH} = \frac{(1-\theta)h_r}{\theta} + \frac{2((2-\delta)+(4-3\delta)c)\theta + (2-\delta)(1-\theta)^2 h_r^2 - 2(2-\delta)(1-\theta)h_r}{4(3-2\delta)\theta}.$$

2.2 Inspection is probably available in the second period (BP)

In this section, we further assume that inspection in the second period is satisfied with probability $\varphi \in (0,1)$ and unsatisfied with probability $1-\varphi$. Then, consumers fall into three scenarios depending on their heterogeneous visiting costs. With $h_s \leq \varphi(1-\theta)h_r$, consumers expect to make an informed purchase in either period; those with $\varphi(1-\theta)h_r < h_s \leq (1-\theta)h_r$ expect to make an uninformed purchase in the second period; those with $h_s > (1-\theta)h_r$ expect to make an uninformed purchase in either period. By conducting a similar analysis, one can derive the following results:

$$\begin{split} p_{1}^{\textit{BP}^{*}} &= \frac{2 \Big(\big(2 - \delta \big)^{2} + \big(2 - \delta^{2} \big) c \Big) \theta + \Big(\varphi^{2} \delta^{2} + 2 \Big(1 + \varphi^{2} \Big) \big(1 - \delta \big) \Big) \big(1 - \theta \big)^{2} \, h_{r}^{2} - 2 \big(2 - \delta \big)^{2} \, \big(1 - \theta \big) h_{r}}{4 \big(3 - 2 \delta \big) \theta}, \\ p_{2}^{\textit{BP}^{*}} &= \frac{2 \big(\big(2 - \delta \big) + \big(4 - 3 \delta \big) c \big) \theta - \Big(2 - \big(4 - \delta \big) \varphi^{2} \big) \big(1 - \theta \big)^{2} \, h_{r}^{2} - 2 \big(2 - \delta \big) \big(1 - \theta \big) h_{r}}{4 \big(3 - 2 \delta \big) \theta}, \\ \pi_{\textit{BP}}^{*} &= \frac{1}{16 \big(3 - 2 \delta \big) \theta} \Big\{ \Big[4 - 4 \big(1 + \delta \big) \varphi^{2} + \Big(4 + \delta^{2} \Big) \varphi^{4} \Big] \big(1 - \theta \big)^{4} \, h_{r}^{4} - 4 \Big(2 \Big(1 + \varphi^{2} \Big) - 2 \delta \Big(1 + \varphi^{2} \Big) + \delta^{2} \varphi^{2} \Big] \big(1 - \theta \big)^{3} \, h_{r}^{3} + 4 \big(1 - \theta \big)^{2} \, h_{r}^{2} \Big[\Big(1 + \big(1 - c \big) \theta \varphi^{2} \Big) \delta^{2} + 2 \big(1 - \delta \big) \Big(2 + \big(1 - c \big) \theta \Big(1 + \varphi^{2} \Big) \Big) \Big] - 8 \big(2 - \delta \big)^{2} \, \big(1 - c \big) \theta \, \big(1 - \theta \big) h_{r} + 4 \big(2 - \delta \big)^{2} \, \big(1 - c \big)^{2} \, \theta^{2} \Big\}, \\ \overline{V}_{1}^{\textit{BPI}} &= \frac{h_{s}}{\theta} + \frac{2 - \delta + \big(1 - \delta \big) c}{3 - 2 \delta} + \frac{\Big(1 + \big(1 - 3 \delta + \delta^{2} \big) \varphi^{2} \Big) \big(1 - \theta \big)^{2} \, h_{r}^{2} - 2 \big(2 - 3 \delta \varphi - \big(1 - 2 \varphi \big) \delta^{2} \Big) \big(1 - \theta \big) h_{r}}{2 \big(1 - \delta \big) (3 - 2 \delta \big) \theta}, \\ \overline{V}_{1}^{\textit{BPI}} &= \frac{h_{s}}{(1 - \delta) \theta} + \frac{2 - \delta + \big(1 - \delta \big) c}{3 - 2 \delta} + \frac{\Big(1 + \big(1 - 3 \delta + \delta^{2} \big) \varphi^{2} \Big) \big(1 - \theta \big)^{2} \, h_{r}^{2} - 2 \big(2 - \delta^{2} \big) \big(1 - \theta \big) h_{r}}{2 \big(1 - \delta \big) (3 - 2 \delta \big) \theta}, \end{split}$$

$$\begin{split} \overline{v_{1}}^{BPH} &= \frac{2 - \delta + \left(1 - \delta\right)c}{3 - 2\delta} + \frac{\left(1 + \left(1 - 3\delta + \delta^{2}\right)\varphi^{2}\right)\left(1 - \theta\right)^{2}h_{r}^{2} + 2\left(1 - \delta\right)^{2}\left(1 - \theta\right)h_{r}}{2\left(1 - \delta\right)\left(3 - 2\delta\right)\theta}, \\ \overline{v_{2}}^{BPL} &= \frac{h_{s} + \left(1 - \varphi\right)\left(1 - \theta\right)h_{r}}{\theta} + \\ &= \frac{2\left(\left(2 - \delta\right) + \left(4 - 3\delta\right)c\right)\theta - \left(2 - \left(4 - \delta\right)\varphi^{2}\right)\left(1 - \theta\right)^{2}h_{r}^{2} - 2\left(2 - \delta\right)\left(1 - \theta\right)h_{r}}{4\left(3 - 2\delta\right)\theta}, \\ \overline{v_{2}}^{BPI} &= \frac{\left(1 - \theta\right)h_{r}}{\theta} + \frac{2\left(\left(2 - \delta\right) + \left(4 - 3\delta\right)c\right)\theta - \left(2 - \left(4 - \delta\right)\varphi^{2}\right)\left(1 - \theta\right)^{2}h_{r}^{2} - 2\left(2 - \delta\right)\left(1 - \theta\right)h_{r}}{4\left(3 - 2\delta\right)\theta}, \\ \overline{v_{2}}^{BPH} &= \frac{\left(1 - \theta\right)h_{r}}{\theta} + \frac{2\left(\left(2 - \delta\right) + \left(4 - 3\delta\right)c\right)\theta - \left(2 - \left(4 - \delta\right)\varphi^{2}\right)\left(1 - \theta\right)^{2}h_{r}^{2} - 2\left(2 - \delta\right)\left(1 - \theta\right)h_{r}}{4\left(3 - 2\delta\right)\theta}. \end{split}$$

3 Equilibrium for Strategic but Distracted Consumers (D)

Note that if consumers do not inspect the product in advance, they will fall into two scenarios depending on their heterogeneous visiting costs. Specifically, they will expect to either make an informed purchase or an uninformed purchase in the first period, but only an uninformed purchase in the second period.

Table A1. Scenarios in the Presence of Distracted Consumers

Scenarios and Conditions	Possible Purchase Choices and Expected Utility		
Scenarios and Conditions	First-period Purchase	Second-period Purchase	
DL: $h_s \le (1 - \theta)h_r$	$\theta(v-p_1)-h_s$	$\delta[\theta(v-p_2)-(1-\theta)h_r]$	
DH: $h_s > (1 - \theta)h_r$	$\theta(v-p_1)-(1-\theta)h_r$	$\delta[\theta(v-p_2)-(1-\theta)h_r]$	

The equilibrium results are as follows:

$$\begin{split} p_{1}^{D^{*}} &= \frac{\left(\left(2 - \delta \right)^{2} + \left(2 - \delta^{2} \right) c \right) \theta + \left(1 - \delta \right) \left(1 - \theta \right)^{2} h_{r}^{2} - \left(2 - \delta \right)^{2} \left(1 - \theta \right) h_{r}}{2 \left(3 - 2 \delta \right) \theta}, \\ p_{2}^{D^{*}} &= \frac{\left(\left(2 - \delta \right) + \left(4 - 3 \delta \right) c \right) \theta - \left(1 - \theta \right)^{2} h_{r}^{2} - \left(2 - \delta \right) \left(1 - \theta \right) h_{r}}{2 \left(3 - 2 \delta \right) \theta}, \\ \pi_{D}^{*} &= \frac{1}{4 \left(3 - 2 \delta \right) \theta} \left\{ \left(1 - \theta \right)^{4} h_{r}^{4} - 2 \left(1 - \delta \right) \left(1 - \theta \right)^{3} h_{r}^{3} + \left(1 - \theta \right)^{2} h_{r}^{2} \left[4 + \delta^{2} - 4 \delta + 2 \left(1 - c \right) \theta - 2 \left(1 - c \right) \theta \delta \right] - 2 \left(2 - \delta \right)^{2} \left(1 - c \right) \theta \left(1 - \theta \right) h_{r} + \left(2 - \delta \right)^{2} \left(1 - c \right)^{2} \theta^{2} \right\}, \\ \overline{v}_{1}^{DL} &= \frac{h_{s}}{\left(1 - \delta \right) \theta} + \frac{2 \left(1 - \delta \right) \left(\left(2 - \delta \right) + \left(1 - \delta \right) c \right) \theta + \left(1 - \theta \right)^{2} h_{r}^{2} - 2 \left(2 - \delta^{2} \right) \left(1 - \theta \right) h_{r}}{2 \left(1 - \delta \right) \left(3 - 2 \delta \right) \theta}, \end{split}$$

$$\overline{v}_{1}^{DH} = \frac{2(1-\delta)((2-\delta)+(1-\delta)c)\theta+(1-\theta)^{2}h_{r}^{2}+2(1-\delta)^{2}(1-\theta)h_{r}}{2(1-\delta)(3-2\delta)\theta},$$

$$\overline{v}_{2}^{D} = \frac{(1-\theta)h_{r}}{\theta} + \frac{((2-\delta)+(4-3\delta)c)\theta-(1-\theta)^{2}h_{r}^{2}-(2-\delta)(1-\theta)h_{r}}{2(3-2\delta)\theta}.$$

4 Analysis for Pre-announced Pricing (A)

4.1 No Inspection (AN)

Under the pre-announced pricing policy, the derivation of equilibrium results is different. Given any pre-announced price plan $\left\{p_1^{AN},p_2^{AN}\right\}$, a consumer will make a first-period purchase as long as $v \ge \overline{v_1}^{AN}$, where:

$$\overline{v_1}^{AN} = \begin{cases} p_1^{AN} + \frac{\left(1-\theta\right)h_r}{\theta} & \text{if } p_1^{AN} \leq p_2^{AN}, \\ \frac{\theta p_1^{AN} - \delta\theta p_2^{AN} + \left(1-\delta\right)\left(1-\theta\right)h_r}{\left(1-\delta\right)\theta} & \text{if } p_1^{AN} > p_2^{AN} \text{ and } p_1^{AN} - \delta p_2^{AN} \leq \left(1-\delta\right)\left(1-\frac{\left(1-\theta\right)h_r}{\theta}\right), \\ 1 & \text{if } p_1^{AN} > p_2^{AN} \text{ and } p_1^{AN} - \delta p_2^{AN} > \left(1-\delta\right)\left(1-\frac{\left(1-\theta\right)h_r}{\theta}\right). \end{cases}$$

If $p_1^{AN} \le p_2^{AN}$, any consumer with non-negative utility will purchase in the first period; if $p_1^{AN} > p_2^{AN}$ and p_2^{AN} is not much lower, a segment of consumers with high valuation will purchase in the first period while some of the remaining consumers will make second-period purchases; if p_2^{AN} is significantly lower than p_1^{AN} , no consumers will purchase in the first period. Considering consumers' response, the retailer will choose $\{p_1^{AN*}, p_2^{AN*}\}$ to maximize its overall profit:

$$\pi_{AN}\left(p_1^{AN}, p_2^{AN}\right) = \theta\left(1 - \overline{v_1}^{AN}\right)\left(p_1^{AN} - c\right) + \theta\left(\overline{v_1}^{AN} - p_2^{AN} - \frac{1 - \theta}{\theta}h_r\right)\left(p_2^{AN} - c\right).$$

The equilibrium results are as follows:

$$p_1^{AN*} = \frac{\left(2 + \left(1 + \delta\right)c\right)\theta - 2\left(1 - \theta\right)h_r}{\left(3 + \delta\right)\theta}, \quad p_2^{AN*} = \frac{\left(1 + 2c + \delta\right)\theta - \left(1 + \delta\right)\left(1 - \theta\right)h_r}{\left(3 + \delta\right)\theta},$$
and
$$\pi_{AN}^* = \frac{\left(\left(1 - c\right)\theta - \left(1 - \theta\right)h_r\right)^2}{\left(3 + \delta\right)\theta}.$$

One can readily show that the profit is smaller when demand only occurs in one period.

4.2 Inspection Available in the First Period (AS)

When inspection is available in the first period, as with that under responsive pricing, consumers will fall into three scenarios depending on their heterogeneous visiting costs. Note that we only consider the case where first-period demand is nonnegative for each scenario. By similar analysis, one can derive:

$$p_{1}^{AS*} = \frac{2(2 + (1 + \delta)c)\theta + (1 + \delta)(1 - \theta)^{2} h_{r}^{2} - 4(1 - \theta)h_{r}}{2(3 + \delta)\theta},$$

$$p_{2}^{AS*} = \frac{2(1 + 2c + \delta)\theta + (2\delta + \delta^{2} - 1)(1 - \theta)^{2} h_{r}^{2} - 2(1 + \delta)(1 - \theta)h_{r}}{2(3 + \delta)\theta},$$

$$\pi_{AS}^{*} = \frac{1}{4(3 + \delta)\theta} \left\{ (1 - \theta)^{4} h_{r}^{4} - 2(1 + \delta)(1 - \theta)^{3} h_{r}^{3} + 2(2 + (1 - c)(1 + \delta)\theta)(1 - \theta)^{2} h_{r}^{2} - 8(1 - c)\theta(1 - \theta)h_{r} + 4(1 - c)^{2} \theta^{2} \right\}.$$

4.3 Inspection Available in both Periods (AB and ABP)

When inspection is also available in the second period, the equilibrium results are:

$$p_{1}^{AB^{*}} = \frac{\left(2 + (1 + \delta)c\right)\theta + (1 - \theta)^{2} h_{r}^{2} - 2(1 - \theta)h_{r}}{(3 + \delta)\theta},$$

$$p_{2}^{AB^{*}} = \frac{2(1 + 2c + \delta)\theta + (1 + \delta)(1 - \theta)^{2} h_{r}^{2} - 2(1 + \delta)(1 - \theta)h_{r}}{2(3 + \delta)\theta},$$

$$\pi_{AB}^{*} = \frac{\left(2((1 - c)\theta - (1 - \theta)h_{r}) + (1 - \theta)^{2} h_{r}^{2}\right)^{2}}{4(3 + \delta)\theta}.$$

Considering inspection probability $\varphi \in (0,1)$ in the second period, we present the equilibrium results as follows:

$$\begin{split} p_1^{ABP^*} &= \frac{2 \left(2 + (1 + \delta)c\right)\theta + \left(1 + \varphi^2\right) \left(1 - \theta\right)^2 h_r^2 - 4 \left(1 - \theta\right) h_r}{2 \left(3 + \delta\right)\theta}, \\ p_2^{ABP^*} &= \frac{2 \left(1 + 2c + \delta\right)\theta + \left((2 + \delta)\varphi^2 - 1\right) \left(1 - \theta\right)^2 h_r^2 - 2 \left(1 + \delta\right) \left(1 - \theta\right) h_r}{2 \left(3 + \delta\right)\theta}, \\ \pi_{ABP}^* &= \frac{1}{4 \left(1 - \delta\right) \left(3 + \delta\right)\theta} \left\{ \left(1 - \left(1 + \delta\right)\varphi^2 + \varphi^4\right) \left(1 - \theta\right)^4 h_r^4 - 2 \left(1 - \delta\right) \left(1 + \varphi^2\right) \left(1 - \theta\right)^3 h_r^3 + 2 \left(1 - \delta\right) \left(2 + \left(1 + \varphi^2\right) \left(1 - c\right)\theta\right) \left(1 - \theta\right)^2 h_r^2 - 8 \left(1 - \delta\right) \left(1 - c\right)\theta \left(1 - \theta\right) h_r + 4 \left(1 - \delta\right) \left(1 - c\right)^2 \theta^2 \right\}. \end{split}$$

Online Appendix B: proofs

Proof of Proposition 1:

(1) It is clear that
$$p_1^{S^*} - p_1^{N^*} = \frac{\left(2 - 2\delta^2 + \delta^3\right)\left(1 - \theta\right)^2 h_r^2}{4\left(3 - 2\delta\right)\theta} > 0$$
. Given that $p_2^{S^*} - p_2^{N^*} = \frac{\left(2 - 2\delta^2 + \delta^3\right)\left(1 - \theta\right)^2 h_r^2}{4\left(3 - 2\delta\right)\theta} > 0$.

$$\frac{-\left(2-4\delta+\delta^{2}\right)\left(1-\theta\right)^{2}h_{r}^{2}}{4\left(3-2\delta\right)\theta}, \text{ it is clear that } p_{2}^{S^{*}}-p_{2}^{N^{*}}>0 \text{ if } 2-4\delta+\delta^{2}<0, \text{ namely,}$$

$$\delta \in (2 - \sqrt{2}, 1)$$
 and $p_2^{S^*} - p_2^{N^*} < 0$ if $\delta \in (0, 2 - \sqrt{2})$.

Note that we have assumed that $\frac{(1-c)\theta-(1-\theta)h_r}{(1-\theta)^2h_r^2} > 0$ throughout the paper. Here, we

add the additional assumptions for scenario S, which may be used for the subsequent analysis.

Define $H = \frac{(1-c)\theta - (1-\theta)h_r}{(1-\theta)^2 h_r^2}$. First, to ensure that $p_2^{s*} > c$, we need to further assume that

$$H > \left[\frac{2 - 4\delta + \delta^2}{4 - 2\delta} \right]^+, \text{ where } (x)^+ = \max\{x, 0\}. \text{ The rationale is as follows: if } 2 - 4\delta + \delta^2$$

> 0, then we have a more binding constraint to ensure that $p_2^{S^*} > c$; if $2-4\delta+\delta^2 < 0$, H>0 is enough to ensure $p_2^{S^*} > c$.

Furthermore, we need consider the constraint to ensure that the optimal first-period price is able to induce all consumers with $h_s \in [0,1]$ to possibly participate in first-period purchases.

Specifically, we need
$$\overline{v}_1^H(p_1^s) = \frac{2\theta(2p_1^s - \delta c) + \delta(1-\theta)^2 h_r^2 + 2(2-\delta)(1-\theta)h_r}{2(2-\delta)\theta} < 1$$
. That is,

the first-period optimal price should satisfy that $P_1^{s^*} < P_1^{s} \mid_{\overline{v}_1^H(p_1^s)=1}$. Define

$$\overline{p}_{1}^{s} = p_{1}^{s} |_{\overline{v}_{1}^{H}=1} = \frac{2(2-(1-c)\delta)\theta - 2(2-\delta)(1-\theta)h_{r} - \delta(1-\theta)^{2}h_{r}^{2}}{4\theta}.$$

One can show that:

$$p_{1}^{s*} - \overline{p}_{1}^{s} = \frac{(2-\delta)((1+2\delta-\delta^{2})(1-\theta)^{2}h_{r}^{2} - 2(1-\delta)((1-c)\theta - (1-\theta)h_{r}))}{4(3-2\delta)\theta}.$$

To let
$$p_1^{s^*} - \overline{p}_1^s < 0$$
, we need $H > \frac{1 + 2\delta - \delta^2}{2(1 - \delta)}$. Note that $\frac{1 + 2\delta - \delta^2}{2(1 - \delta)} > \left[\frac{2 - 4\delta + \delta^2}{4 - 2\delta}\right]^+$, thus,

we overall assume that $H > \frac{1+2\delta-\delta^2}{2(1-\delta)}$.

(2) It is clear that:

$$\pi_{S}^{*} - \pi_{N}^{*} = \frac{\left(1 - \theta\right)^{2} h_{r}^{2} \left(4\left(2 - 2\delta^{2} + \delta^{3}\right)\left(\left(1 - c\right)\theta - \left(1 - \theta\right)h_{r}\right) + \left(4 - 4\delta + \delta^{4}\right)\left(1 - \theta\right)^{2} h_{r}^{2}\right)}{16\left(3 - 2\delta\right)\theta} > 0.$$

(3) One can show that:

$$n_{2}^{s} = \frac{2(2-\delta)((1-c)\theta - (1-\theta)h_{r}) - (2-4\delta + \delta^{2})(1-\theta)^{2}h_{r}^{2}}{4(3-2\delta)},$$

$$n_{2}^{N} = \frac{(2-\delta)((1-c)\theta - (1-\theta)h_{r})}{2(3-2\delta)},$$

$$n_{3}^{S} = \left(\theta - (1-\theta)h_{r} - \frac{2((2-\delta) + (4-3\delta)c)\theta - (2-4\delta + \delta^{2})(1-\theta)^{2}h_{r}^{2} - 2(2-\delta)(1-\theta)h_{r}}{4(3-2\delta)}\right) + \frac{\delta(1-\theta)^{2}}{2}h_{r}^{2},$$

and $n^{s} = \theta - \frac{\left(2 - \delta + \left(4 - 3\delta\right)c\right)\theta + \left(4 - 3\delta\right)\left(1 - \theta\right)h_{r}}{2\left(3 - 2\delta\right)}$

Then, one can show that:

$$n_2^S - n_2^N = \frac{-\left(2 - 4\delta + \delta^2\right)\left(1 - \theta\right)^2 h_r^2}{4\left(3 - 2\delta\right)},$$

$$n^S - n^N = \frac{\left(2 + 2\delta - 3\delta^2\right)\left(1 - \theta\right)^2 h_r^2}{4\left(3 - 2\delta\right)},$$

and

$$n_1^S - n_1^N = (n^S - n^N) - (n_2^S - n_2^N) = \frac{(4 - 2\delta - 2\delta^2)(1 - \theta)^2 h_r^2}{4(3 - 2\delta)}.$$

It is clear that $n_1^S - n_1^N > 0$; $n_2^S - n_2^N > 0$ if $\delta \in (2 - \sqrt{2}, 1)$ and $n_2^S - n_2^N < 0$ if $\delta \in (0, 2 - \sqrt{2})$; $n^S - n^N > 0$.

Proof of Proposition 2:

(1) It is clear that
$$p_1^{B^*} - p_1^{S^*} = \frac{\left(2\left(1-\delta\right)^2 + \delta^2\left(1-\delta\right)\right)\left(1-\theta\right)^2 h_r^2}{4\left(3-2\delta\right)\theta} > 0$$
 and $p_2^{B^*} - p_2^{S^*} = \frac{\left(2\left(1-\delta\right)^2 + \delta^2\left(1-\delta\right)\right)\left(1-\theta\right)^2 h_r^2}{4\left(3-2\delta\right)\theta} > 0$

$$\frac{\left(4-\delta\right)\left(1-\delta\right)\left(1-\theta\right)^{2}h_{r}^{2}}{4\left(3-2\delta\right)\theta}>0.$$

(2) Given that $(1-c)\theta - (1-\theta)h_r > 0$, one can readily derive that $\pi_B^* - \pi_S^* = \frac{(1-\delta)(1-\theta)^2 h_r^2 \left(4(2-2\delta+\delta^2)((1-c)\theta - (1-\theta)h_r) + (\delta^2+\delta^3)(1-\theta)^2 h_r^2\right)}{16(3-2\delta)\theta} > 0$.

(3) One can show that:

$$n_{2}^{B} = \frac{(2-\delta)(2(1-c)\theta - 2(1-\theta)h_{r} + (1-\theta)^{2}h_{r}^{2})}{4(3-2\delta)},$$

$$n_{2}^{S} = \frac{(2-\delta)(2(1-c)\theta - 2(1-\theta)h_{r}) - (2-4\delta+\delta^{2})(1-\theta)^{2}h_{r}^{2}}{4(3-2\delta)},$$

$$n^{B} = \left(\theta - \left(1 - \theta\right)h_{r} - \frac{2\left(\left(2 - \delta\right) + \left(4 - 3\delta\right)c\right)\theta + \left(2 - \delta\right)\left(1 - \theta\right)^{2}h_{r}^{2} - 2\left(2 - \delta\right)\left(1 - \theta\right)h_{r}}{4\left(3 - 2\delta\right)}\right) + \frac{\left(1 - \theta\right)^{2}h_{r}^{2}}{2},$$

$$n^{S} = \left(\theta - \left(1 - \theta\right)h_{r} - \frac{2\left(\left(2 - \delta\right) + \left(4 - 3\delta\right)c\right)\theta - \left(2 - 4\delta + \delta^{2}\right)\left(1 - \theta\right)^{2}h_{r}^{2} - 2\left(2 - \delta\right)\left(1 - \theta\right)h_{r}}{4\left(3 - 2\delta\right)}\right) + \frac{\delta\left(1 - \theta\right)^{2}h_{r}^{2}}{2},$$

$$n^{B} - n^{S} = \frac{\left(2 - 5\delta + 3\delta^{2}\right)\left(1 - \theta\right)^{2}h_{r}^{2}}{4\left(3 - 2\delta\right)},$$
and
$$n_{2}^{B} - n_{2}^{S} = \frac{\left(4 - 5\delta + \delta^{2}\right)\left(1 - \theta\right)^{2}h_{r}^{2}}{4\left(3 - 2\delta\right)}.$$

Then, we have $n^B - n^S > 0$ when $2 - 5\delta + 3\delta^2 > 0$, i.e., $\delta < \frac{2}{3}$; $n_2^B - n_2^S > 0$; $n_1^B - n_1^S = 0$

$$(n^B - n^S) - (n_2^B - n_2^S) = \frac{-(2 - 2\delta^2)(1 - \theta)^2 h_r^2}{4(3 - 2\delta)} < 0. \quad \Box$$

Proof of Proposition 3:

Compare the optimal profits and we have:

$$\pi_{BP}^* - \pi_S^* = \frac{\left(\varphi^2 - \delta\right)\left(1 - \theta\right)^2 h_r^2}{16\left(3 - 2\delta\right)\theta} \left(4\left(2 - 2\delta + \delta^2\right)\left(\left(1 - c\right)\theta - \left(1 - \theta\right)h_r\right) - \left(4 - \delta^3 - \varphi^2\delta^2 - 4\varphi^2\right)\left(1 - \theta\right)^2 h_r^2\right).$$

Similar to that in proof of Proposition 1, we first add the assumptions for scenario BP. To

ensure that
$$p_2^{BP^*} > c$$
, we need $H > \left[\frac{2 - (4 - \delta) \varphi^2}{4 - 2\delta} \right]^+$. Moreover, $p_1^{BP^*} < p_1^{BP} |_{\nabla_1^{BPH} (p_1^{BP}) = 1}$

should be satisfied as we consider nonnegative first-period demand for $h_s \in [0,1]$. Note that:

$$\overline{p}_{1}^{BP} = p_{1}^{BP} \Big|_{\overline{v}_{1}^{BPH}(p_{1}^{BP}) = 1} = \frac{2(1-\delta)(2-(1-c)\delta)\theta - 2(2-3\delta+\delta^{2})(1-\theta)h_{r} - \delta(1-\varphi^{2})(1-\theta)^{2}h_{r}^{2}}{4(1-\delta)\theta}.$$

Then, one can readily derive that:

$$p_1^{BP^*} - \overline{p}_1^{BP} = \frac{(2-\delta)(2(1-\delta)^2((1-\theta)h_r - (1-c)\theta) + (1+(1-3\delta+\delta^2)\varphi^2)(1-\theta)^2h_r^2)}{4(1-\delta)(3-2\delta)\theta}.$$

Let $p_1^{BP^*} < \overline{p}_1^{BP}$ and we have $H > \frac{1 + (1 - 3\delta + \delta^2)\varphi^2}{2(1 - \delta)^2}$. Overall, we assume that $H > \frac{1 + (1 - 3\delta + \delta^2)\varphi^2}{2(1 - \delta)^2}$

$$\frac{1 + \left(1 - 3\delta + \delta^2\right)\varphi^2}{2\left(1 - \delta\right)^2} \text{ as } \frac{1 + \left(1 - 3\delta + \delta^2\right)\varphi^2}{2\left(1 - \delta\right)^2} > \left[\frac{2 - \left(4 - \delta\right)\varphi^2}{4 - 2\delta}\right]^+. \text{ Scenario } S \text{ and scenario } BP$$

combined lead to the constraint:
$$H > \max \left\{ \frac{1 + \left(1 - 3\delta + \delta^2\right) \varphi^2}{2\left(1 - \delta\right)^2}, \frac{1 + 2\delta - \delta^2}{2\left(1 - \delta\right)} \right\}.$$

Next, we conclude the comparison between the above profits. First, consider $\varphi^2 - \delta > 0$

i.e.,
$$\varphi > \sqrt{\delta}$$
. To let $\pi_{BP}^* - \pi_S^* < 0$, we need $H < \left\lceil \frac{4 - 4\varphi^2 - \delta^2 \varphi^2 - \delta^3}{4(2 - 2\delta + \delta^2)} \right\rceil^+$. Note that:

$$\frac{1+\left(1-3\delta+\delta^2\right)\varphi^2}{2\left(1-\delta\right)^2} > \left\lceil \frac{4-4\varphi^2-\delta^2\varphi^2-\delta^3}{4\left(2-2\delta+\delta^2\right)} \right\rceil^+.$$

Given the above constraint, we clearly have $\pi_{BP}^* > \pi_S^*$ in this case. Second, consider

$$\varphi^2 - \delta < 0$$
, i.e., $\varphi < \sqrt{\delta}$. To let $\pi_{BP}^* - \pi_S^* < 0$, we need $H > \left[\frac{4 - 4\varphi^2 - \delta^2 \varphi^2 - \delta^3}{4(2 - 2\delta + \delta^2)} \right]^+$. In this

case, we clearly have $\pi_{\mathit{BP}}^* < \pi_{\mathit{S}}^*$ given the constraint. This completes the proof. \square

Proof of Proposition 4:

(1) One can readily derive that
$$p_1^{S^*} - p_1^{D^*} = \frac{\left(2\delta - 2\delta^2 + \delta^3\right)\left(1 - \theta\right)^2 h_r^2}{4\left(3 - 2\delta\right)\theta} > 0$$
 and $p_2^{S^*} - p_2^{D^*} = \frac{\left(2\delta - 2\delta^2 + \delta^3\right)\left(1 - \theta\right)^2 h_r^2}{4\left(3 - 2\delta\right)\theta} > 0$

$$\frac{\left(4\delta-\delta^2\right)\left(1-\theta\right)^2h_r^2}{4(3-2\delta)\theta}>0.$$

(2) One can show that:

$$\pi_{D}^{*} - \pi_{S}^{*} = \frac{\delta (1-\theta)^{2} h_{r}^{2} (4(2-2\delta+\delta^{2})((1-\theta)h_{r}-(1-c)\theta)+(4-\delta^{3})(1-\theta)^{2} h_{r}^{2})}{16(3-2\delta)\theta}.$$

Similarly, we first add the assumptions for scenario D. Considering $p_2^{D^*} > c$, we need $H > \frac{1}{2-\delta}$. Moreover, $P_1^{D^*} < P_1^D|_{\overline{V_1^{DH}}(p_1^D)=1}$ should be satisfied. Note that:

$$\overline{p}_{1}^{D} = p_{1}^{D} \mid_{\overline{v}_{1}^{DH}\left(p_{1}^{D}\right) = 1} = \frac{2(1-\delta)(2-(1-c)\delta)\theta - 2(2-3\delta+\delta^{2})(1-\theta)h_{r} - \delta(1-\theta)^{2}h_{r}^{2}}{4(1-\delta)\theta}.$$

Then, one can readily derive that:

$$p_{1}^{D^{*}} - \overline{p}_{1}^{D} = \frac{(2-\delta)(2(1-\delta)^{2}((1-\theta)h_{r} - (1-c)\theta) + (1-\theta)^{2}h_{r}^{2})}{4(1-\delta)(3-2\delta)\theta}.$$

Let $p_1^{D^*} < \overline{p}_1^D$ and we have $H > \frac{1}{2(1-\delta)^2}$. Overall, we assume that $H > \frac{1}{2(1-\delta)^2}$ as

 $\frac{1}{2(1-\delta)^2} > \frac{1}{2-\delta}$. Scenario S and scenario D combined lead to the constraint: H > 1

$$\max \left\{ \frac{1+2\delta-\delta^2}{2(1-\delta)}, \frac{1}{2(1-\delta)^2} \right\}.$$

Next, we conclude the comparison between the above profits. To let $\pi_D^* - \pi_S^* < 0$, we

need
$$H > \frac{4-\delta^3}{4\left(2-2\delta+\delta^2\right)}$$
. Note that $\frac{1}{2\left(1-\delta\right)^2} > \frac{4-\delta^3}{4\left(2-2\delta+\delta^2\right)}$, hence we must have

 $\pi_D^* - \pi_S^* < 0$ given the above constraints.

Proof of Proposition 5:

(1) It is straightforward to show that $p_1^{AS^*} > p_1^{AN^*}$; $p_2^{AS^*} > p_2^{AN^*}$ if $\delta \in (\sqrt{2} - 1, 1)$ and $p_2^{AS^*} < p_2^{AN^*}$ if $\delta \in (0, \sqrt{2} - 1)$.

One can also show that:

$$\pi_{AS}^* - \pi_{AN}^* = \frac{\left(1 - \theta\right)^2 h_r^2}{4(3 + \delta)\theta} \left(\left(1 - \theta\right)^2 h_r^2 + 2\left(1 + \delta\right) \left(\left(1 - c\right)\theta - \left(1 - \theta\right)h_r \right) \right) > 0.$$

(2) When $\varphi = 1$, one can show that $p_1^{AS^*} - p_1^{AB^*} = \frac{(\delta - 1)(1 - \theta)^2 h_r^2}{2(3 + \delta)\theta} < 0$ and $p_2^{AS^*} - p_2^{AB^*} = \frac{(\delta - 1)(1 - \theta)^2 h_r^2}{2(3 + \delta)\theta} < 0$

$$\frac{\left(\delta+\delta^2-2\right)\left(1-\theta\right)^2h_r^2}{2\left(3+\delta\right)\theta}<0.$$

When $\varphi = 1$, we have:

$$\pi_{AS}^* - \pi_{AB}^* = \frac{(1-\delta)(1-\theta)^2 h_r^2 ((1-\theta)h_r - (1-c)\theta)}{2(3+\delta)\theta}.$$

Given that $(1-\theta)h_r - (1-c)\theta < 0$, it is clear that $\pi_{AS}^* - \pi_{AB}^* < 0$.