



Short communication

Distributional effects of endogenous discounting

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ABSTRACT

This paper studies the effect of endogenous discounting on the distribution of wealth in a Bewley–Huggett economy with an exogenous borrowing constraint. We introduce the Koopmans–Uzawa–Epstein time preferences in the benchmark model of Achdou et al. (2022) and investigate the implications on saving behaviour and wealth distribution across different wealth classes. The results highlight a self-reinforcing redistribution mechanism, through which the endogenous discounting can lead to a higher equilibrium interest rate and a more unequal wealth distribution, in comparison to the benchmark model with constant discount rate.

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1. Introduction

The idea that consumers have preference for advancing the time of future satisfaction has been around since at least Fisher (1930), where the concept of impatience was formalised. The standard discounted expected utility model is built on the assumptions about preferences over payoffs being additive over time, with probabilities attached to all possible states of nature and constant exogenous discounting. Some of the alternative theories have been developed to explain anomalous predictions of the expected utility theory with constant time preferences, while other alternatives emerged from the developments in pure theory of intertemporal choice; see discussion in Frederick et al. (2002) and Backus et al. (2005).

Koopmans–Uzawa–Epstein (KUE) preferences were born from such developments. Koopmans (1960) proposed to define preferences for timing advances entirely in terms of the utility function. Uzawa (1968) introduced a recursive representation of intertemporal utility with endogenous time preference, later extended to the environment with uncertainty by Epstein (1983, 1987). The KUE framework became widely spread in the literature since the 1980s, both in microeconomic and macroeconomic applications, due to its attractive feature of intertemporal interdependencies.

Kouri (1980) and Obstfeld (1981) analysed the implications of KUE preferences for trade and exchange rate dynamics; Sala-i-Martin (1996) noted that introducing them in the AK-model of

growth can help explain the so-called β –convergence. Intertemporal interdependencies generated by recursive preferences were explored in a wide range of models, from business cycles (Mendoza, 1991) and asset pricing (Duffie and Epstein, 1992) to drug addiction (Shi and Epstein, 1993). KUE preferences with increasing marginal impatience naturally induce stationarity in small open economy models (Schmitt-Grohe and Uribe, 2003).¹ While there seems to be no direct empirical evidence of KUE preferences in the literature, an indirect evidence, in the form of intertemporal correlation aversion, was reported by Cheung (2015) and Andersen et al. (2018). As shown by Epstein (1983, 1987) in the stochastic version of the general equilibrium model with KUE preferences, an increasing marginal impatience implies an individual's aversion to the correlation between random consumption levels in any two periods.

The assumption of endogenous impatience, combined with heterogeneity of agents and incomplete markets, can create rich dynamics and a non-trivial wealth distribution (Epstein and Hynes, 1983; Lucas and Stokey, 1984; Farmer and Lahiri, 2005). Intuitively, with KUE preferences, the wealthy, less patient than the poor, have incentives to consume more and save less. Therefore, in the long run, the distribution of wealth should be less dispersed than in the standard case of constant discounting.

¹ The opposite assumption of decreasing marginal impatience would result in instability of an infinitely-lived representative-agent dynamic economy, see Obstfeld (1990). Under this assumption, in an economy with permanently patient and impatient consumers the most patient consumer will accumulate all the wealth. This property is often used to model a permanently binding borrowing constraint in the representative-agent literature; see, for example, Iacoviello (2005).

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Our result is at odds with the above intuition. We show that KUE preferences in a Bewley–Huggett economy (Bewley, 1983; Huggett, 1993) with an exogenous borrowing constraint can generate a stationary wealth distribution that is substantially more dispersed than the distribution in an economy with standard preferences. A similar model but with constant discount rate was studied in Achdou et al. (2022) who showed that in such an economy there is a unique stationary distribution of wealth, while individuals are mobile across levels of consumption, income and wealth. With diminishing marginal impatience in the tails, the wealth distribution in the KUE model is more skewed and has higher kurtosis than with standard preferences. In this sense, it is ‘more unequal’ and, therefore, more empirically relevant than the distribution obtained in the standard model.

The underlying mechanism is the saving behaviour of the ‘middle class’: in this economy they save less than in an economy with constant discounting. This drives up the interest rate and, consequently, the debt of the ‘poor’ who borrow to service the debt. Higher wealth and savings of the ‘rich’ are insufficient to offset this effect.

2. The model

Consider a continuous-time economy with incomplete markets and stochastic shocks to exogenous incomes following a two-state Markov process, as in Huggett (1993). There is a continuum of infinitely-lived individuals with different levels of wealth a and income y . The income follows a two-state Markov process, $y \in \{y_1, y_2\}$ with $y_2 > y_1$. An individual is said to be low-income type when his income is y_1 and high-income type when his income is y_2 . The process jumps from state 1 to state 2 with intensity λ_1 and from state 2 to state 1 with intensity λ_2 , where λ_1 and λ_2 are exogenous constants. The income process is uninsurable, and individuals can only lend or borrow in the form of non-contingent private bonds a at interest rate r determined in equilibrium. Individuals face a borrowing constraint,

$$a \geq \underline{a} \tag{1}$$

where the exogenous borrowing limit \underline{a} is tighter than the ‘natural’ borrowing limit: $-y_1/r < \underline{a} < 0$.

The joint probability distribution of income y_j and wealth a is denoted $G_j(a, t)$, and the corresponding density function is $g_j(a, t)$, $j = 1, 2$. We assume that private bonds are in zero net supply:

$$\int_{\underline{a}}^{\infty} a g_1(a) da + \int_{\underline{a}}^{\infty} a g_2(a) da = 0 \tag{2}$$

Individual preferences are described by a discounted life-time utility function

$$\mathbb{E}_0 \int_0^{\infty} \exp\left(-\int_0^s \rho(c(\tau)) d\tau\right) u(c(s)) ds \tag{3}$$

where the discount rate, ρ , is a function of consumption, c . Function $u(c)$ is strictly increasing and strictly concave. The individual budget constraint is

$$\dot{a} = ra + y - c. \tag{4}$$

Individuals choose consumption path $c(\cdot)$ to maximise the expected life-time utility (3) subject to the borrowing constraint (1), the budget constraint (4), and an exogenously specified income process, taking interest rate as given.

We assume that individual time preference $\rho(c)$ satisfies the following properties: (i) $0 < \rho(c) < \bar{R} < \infty$, (ii) $\rho_c(c) \geq 0$. The first property states that individuals discount future utility and that the discount rate is bounded from above. The second property is the assumption of increasing impatience.

As noted by Obstfeld (1990), there is no agreement on whether impatience rises or falls when consumption rises. In an infinite-horizon model increasing impatience is required to produce a non-degenerate distribution of wealth in the long run (Lucas and Stokey, 1984). Otherwise, an increase in current consumption leads to an increase in the marginal utility of future consumption and, as a result, the optimal savings plan fails to converge.

2.1. Stationary equilibrium

Individual optimal consumption and saving decision is described by the Hamilton–Jacobi–Bellman equation

$$0 = \max_c \left(u(c) - \rho(c) V_j(a) + (ra + y_j - c) \frac{\partial V_j(a)}{\partial a} + \lambda_j (V_{-j}(a) - V_j(a)) \right), \tag{5}$$

and the Kolmogorov Forward equation

$$0 = -\frac{d}{da} (s_j(a) g_j(a)) - \lambda_j g_j(a) + \lambda_{-j} g_{-j}(a), \tag{6}$$

where $V_j(a)$ is the value function, $j = 1, 2$ denotes the state, and index $-j$ means ‘other than j ’. The saving policy function is

$$s_j(a) = ra + y_j - c_j(a). \tag{7}$$

Maximisation in (5) yields:

$$\frac{\partial u(c)}{\partial c} = \frac{\partial \rho(c)}{\partial c} V_j(a) + \frac{\partial V_j(a)}{\partial a}. \tag{8}$$

Term $\frac{\partial \rho(c)}{\partial c} V_j(a)$ captures the effect of endogenous discounting on the agent’s trade-off between consumption and saving. Greater past consumption induces the agent to save less and consume more in the present. At the point of optimality the marginal utility of consumption is lower than the marginal value of savings because in this model $V_j(a) < 0$.

Together with market clearing condition (2), the boundary condition

$$\frac{\partial V_j(\underline{a})}{\partial a} + \frac{\partial \rho(y_j + r\underline{a})}{\partial c} V_j(\underline{a}) \geq \frac{\partial u(y_j + r\underline{a})}{\partial c}, \tag{9}$$

and the normalisation of the joint distribution

$$\int_{\underline{a}}^{\infty} g_1(a) da + \int_{\underline{a}}^{\infty} g_2(a) da = 1, \tag{10}$$

the system of Eqs. (5)–(8) describes the stationary equilibrium. The formal proofs of the properties of the stationary equilibrium are in the Online Appendix.

2.2. Parameterisation

For the benchmark parameterisation we follow Achdou et al. (2022). We set $y_1 = 0.1$, $y_2 = 0.2$, $\lambda_1 = \lambda_2 = 1.2$. This distribution of income has the mean of 0.15, the standard deviation of 0.05, zero skewness, and unit kurtosis. We set the borrowing limit at $\underline{a} = -0.15$.

We assume the CRRA form for the flow utility function,

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \quad \gamma > 0, \tag{11}$$

with $\gamma = 2$.

For the KUE endogenous discount rate we assume the following functional form:

$$\rho(c) = \bar{\rho} + \frac{\kappa}{\pi} \arctan\left(\Theta \frac{\pi}{\kappa} [c - c_0]\right), \tag{12}$$

where parameters $\{\bar{\rho}, \Theta, \kappa, c_0\}$ are all positive. Parameter $\bar{\rho}$ is the discount rate at a certain benchmark level of consumption

c_0 . Parameter $\Theta \equiv \rho_c(c_0)$ determines the slope of the discount rate function at the benchmark consumption level; $\Theta = 0$ corresponds to the case of the constant discount rate (CDR) $\rho(c) = \bar{\rho}$. Parameter κ sets upper and lower limits on the discount rate. With higher κ the discount rate tends to a linear function with slope Θ . We set $\Theta = 0.1$, $\kappa = 0.01$ so that the benchmark shape of the discount rate function is not 'too far' from the constant discount rate, see Panel A in Fig. 1. We calibrate c_0 to yield the average discount factor equal to $\bar{\rho} = 0.05$. This implies the value of $c_0 = 0.1508$, which remains stable across different parameterisations of the model. Such specification makes different parameterisations directly comparable.

We investigate robustness of our results with respect to parameters Θ and κ .

3. Wealth inequality

The benchmark CDR case is discussed in Achdou et al. (2022). Panel B in Fig. 1 depicts the shape of the endogenous discount factor as the function of wealth, along with the benchmark CDR. One can see that individuals with higher wealth are more impatient, but the marginal impatience falls with wealth. Other panels illustrate saving and consumption behaviour and stationary distributions of consumption and wealth. The equilibrium properties can be summarised as follows.

(i) The high-income group are period-savers, while the low-income group are period-dissavers (C). Consumption of both groups rises with wealth. The level of consumption and therefore welfare for the high-income types is higher than for the low-income types for all wealth levels (D).

(ii) Both groups have a bounded wealth support: $\underline{a} \leq a \leq a_{\max}$, but only the low-income group has a positive mass on \underline{a} (E). Once an individual hits this borrowing constraint, he remains there as long as he subsequently draws low income.

(iii) The high-income group has almost symmetric distribution of consumption, while the density of the low-income group has a pronounced long left tail where the individuals consume their total wealth net of interest payments (F). This tail in consumption distribution corresponds to the point-mass on the lower bound \underline{a} in wealth distribution.

Introducing KUE discounting has non-trivial quantitative effect. First, the consumption and wealth distributions have fatter tails compared to those in a CDR economy, with notable increase of the mass on the borrowing constraint; see Table 1. Second, the equilibrium interest rate is noticeably higher under KUE than under CDR. Intuitively, strong income effect leads to self-reinforcing linkage between the higher interest rate and fatter tails, as explained below.

Consider the three wealth classes: the poor, who have negative wealth ($a < 0$), the middle class, who have small positive wealth ($a \gtrsim 0$), and the rich, who have large positive wealth ($a \gg 0$). Because the aggregate assets are zero, the poor constitute about a half of the population and so most important quantitative effects realise on the 'boundary' between the poor and the middle class. One can think about the middle class as being relatively vulnerable to being pushed into the poor after an adverse income shock.² Every period, the poor – who are net borrowers – need to roll over the whole stock of loans they have and so they generate the (stock) demand for loans. The other two classes generate the (stock) supply of loans, as they are net lenders.³ Panels A and

² In this framework, there is no natural point in wealth separating the middle class and the rich. In the numerical exercise we use $|\underline{a}|$ as the cut-off between the middle class and the rich.

³ Note that period-saving and period-dissaving in Panel C of Fig. 1 are determined by whether a change in the wealth stock at a given time is positive or negative, respectively. In contrast, whether an individual is a (net) borrower or lender (as we discuss further using Tables 2 and 3) is determined by whether the individual's stock of debt is, respectively, positive or negative at a given time.

B in Fig. 1 show the location of these wealth classes across the relevant ranges of consumption and wealth.

Table 2 documents partial and general equilibrium effects on consumption c , saving s , demand and supply of loans L^D and L^S , the interest rate r , the discount rate ρ and the proportion of population n for each class, as we describe next.

Consider the effect of change in the discount rate recorded in columns (1)–(3) of Table 2. A change in the discount rate has *substitution effect* on consumption and saving decisions. Consumption of the most of the poor ($a < 0$) falls below the benchmark level, so that in the KUE economy their discount rates are lower than the (benchmark) constant discount rate of $\bar{\rho}$ in the CDR economy. Because of these lower discount rates they consume less and borrow less, as recorded in columns (1) and (2). As a result, for a given interest rate the demand for loans among these individuals falls below that in the CDR case, and this exerts a *downward* pressure on the interest rate, recorded in column (3). Meanwhile, consumption of the middle class ($a \gtrsim 0$) and the rich ($a \gg 0$) is, on average, above the benchmark level, and so their discount rates are higher than the benchmark CDR. With higher discount rate they consume more and save less than they would in the CDR economy. The supply of loans goes down, which puts an *upward* pressure on the interest rate compared to the CDR case; this is recorded in column (3).

Next, consider the effect of higher interest rate on the three classes, recorded in columns (5)–(6) of Table 2. Column (5) shows the *income effect* of higher interest rate on consumption and saving decisions. To service their debt, the poor ($a < 0$) increase borrowing and simultaneously reduce their current consumption. Column (6) shows the partial equilibrium effect: lower current consumption further reduces the discount rate of the poor, and higher borrowing increases the demand for loans thus exerting *upward* pressure on the interest rate. At the same time, with higher interest rate the middle class ($a \gtrsim 0$) and the rich ($a \gg 0$) receive higher return on their saving and increase both saving and current consumption, as shown in column (5). Higher current consumption further increases their discount rate. Higher savings mean higher supply of loans, thus creating *downward* pressure on the interest rate, as recorded in column (6). This behaviour of different wealth classes bends, not tilts, consumption and saving profiles as shown in Fig. 1. The stronger income effect of a high interest rate affects both tails of the distribution and leads to greater wealth inequality.

The relative population share of the poor increases, as some middle-class agents drawing low income decumulate wealth and become borrowers. This further increases the demand for loans and reduces the supply of loans. At the same time, among the net lenders, the rich accumulate wealth at a higher rate (because of the higher return on loans) and so their relative share in the population is higher, while the population share of the middle class is lower, compared to the CDR economy. Thus, the income effect of a high interest rate 'fattens' both tails of the distribution and leads to a greater wealth inequality.

Thus, the self-reinforcing mechanism of a higher interest rate and a higher demand for loans to service past loans results in a higher long-run interest rate, compared to the CDR economy, as recorded for a range of parameters in columns (2)–(6) in Panel I in Table 3. Panel II quantifies column (7) of Table 2 and confirms that, as a result of such changes in the discount rate, in the stationary equilibrium the total demand and total supply for loans are higher in the KUE economy than in the CDR economy. It also shows that within each class the equilibrium effect goes in the same direction for both income types. Panel III reports the population shares.

This self-reinforcing mechanism can only be triggered by increasing impatience ($\Theta > 0$). The driving force is the behaviour

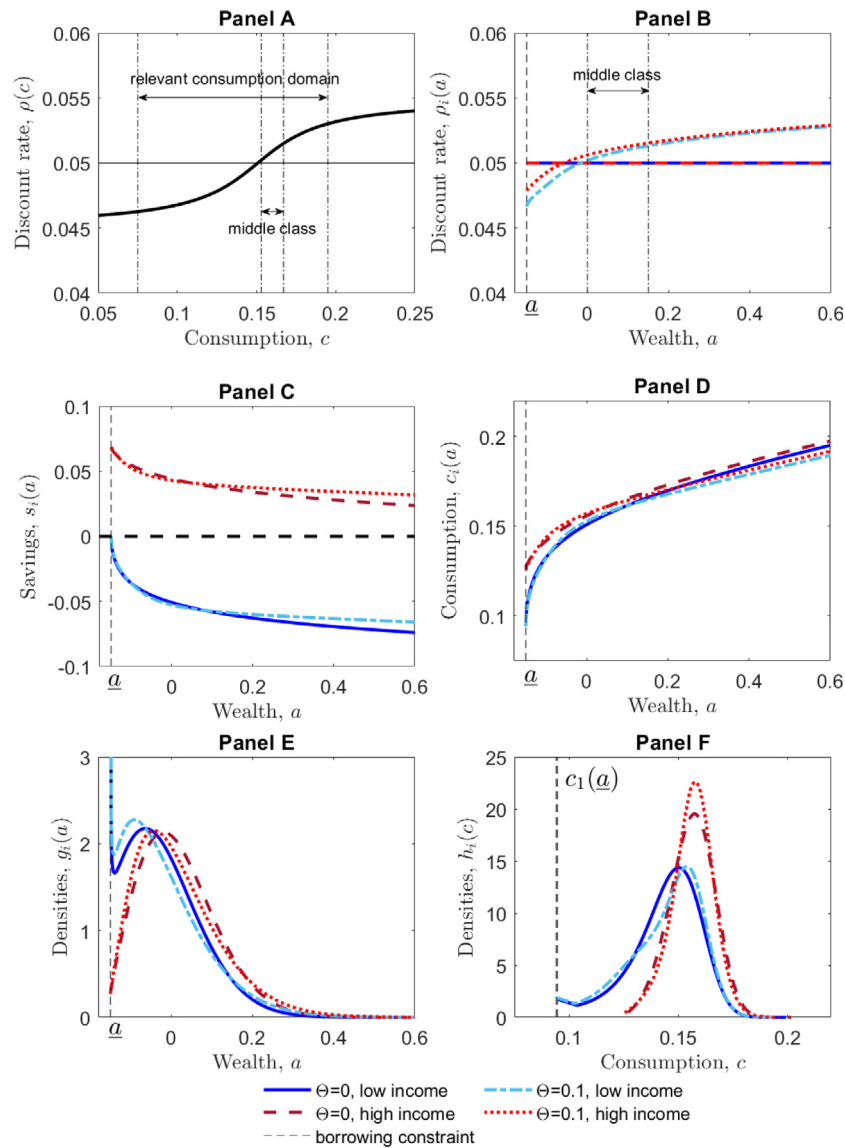


Fig. 1. Stationary Distributions.

Table 1

Numerical characteristics of the stationary distribution. Parameters θ and κ regulate, respectively, the slope and the boundaries of the discount factor function.

	CDR	KUE				
θ	0.0	0.1	0.1	0.1	0.05	0.15
κ	-	0.01	0.005	0.02	0.01	0.01
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Panel I: Sample statistics of the overall wealth distribution</i>						
Mean	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Standard deviation	0.0939	0.1053	0.1136	0.0983	0.0968	0.1194
Skewness	0.6697	1.0366	1.2357	0.8139	0.7657	1.3877
Kurtosis	3.2674	4.3323	4.7919	3.6773	3.5395	5.4235
Mass at the borrowing constraint	0.0151	0.0156	0.0151	0.0155	0.0153	0.0150
Top 10% by wealth, low-income types, $\hat{a}_{L,90}$	0.1074	0.1199	0.1344	0.1095	0.1095	0.1406
Top 10% by wealth, high-income types, $\hat{a}_{H,90}$	0.1448	0.1593	0.1739	0.1489	0.1469	0.1801
<i>Panel II: Standard deviation of consumption distribution</i>						
Low-income group	0.0168	0.0179	0.0187	0.0173	0.0171	0.0194
High-income group	0.0102	0.0096	0.0092	0.0102	0.0102	0.0090
All population	0.0196	0.0204	0.0209	0.0201	0.0199	0.0214

Table 2
General equilibrium effects in KUE economy.

Wealth group	Dis-count rate	Substi-tution effect	Partial eqm. effect	Eqm. Interest rate	Income effect	Partial eqm. effect	General equilibrium effect
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$a < 0$	$\rho \downarrow$	$c \downarrow, s \uparrow$	$L^D \downarrow, r \downarrow$		$s \downarrow, c \downarrow$	$L^D \uparrow, r \uparrow, \rho \downarrow$	$L^D \uparrow, a \downarrow, n \uparrow$
$a \gtrsim 0$	$\rho \uparrow$	$c \uparrow, s \downarrow$	$L^S \downarrow, r \uparrow$	$r \uparrow$	$s \uparrow, c \uparrow$	$L^S \uparrow, r \downarrow, \rho \uparrow$	$L^S \downarrow, a \downarrow, n \downarrow$
$a \gg 0$	$\rho \uparrow$	$c \uparrow, s \downarrow$	$L^S \downarrow, r \uparrow$		$s \uparrow, c \uparrow$	$L^S \uparrow, r \downarrow, \rho \uparrow$	$L^S \uparrow, a \uparrow, n \uparrow$

Table 3
Interest rates and net supply of loans. Parameters Θ and κ regulate, respectively, the slope and the boundaries of the discount factor function.

	CDR	KUE				
Θ	0.0	0.1	0.1	0.1	0.05	0.15
κ	-	0.01	0.005	0.02	0.01	0.01
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Panel I: Interest rate</i>						
Interest rate	0.0352	0.0392	0.0391	0.0389	0.0373	0.0411
<i>Panel II: Net supply of loans, $L^S - L^D$</i>						
The poor, $a < 0$	-0.0382	-0.0414	-0.0443	-0.0392	-0.0387	-0.0460
- low income	-0.0240	-0.0258	-0.0273	-0.0245	-0.0243	-0.0282
- high income	-0.0142	-0.0157	-0.0170	-0.0147	-0.0144	-0.0178
The middle class, $a \gtrsim 0$	0.0227	0.0203	0.0183	0.0220	0.0224	0.0175
- low income	0.0093	0.0085	0.0078	0.0091	0.0092	0.0074
- high income	0.0134	0.0119	0.0106	0.0130	0.0132	0.0101
The rich, $a \gg 0$	0.0155	0.0211	0.0259	0.0172	0.0163	0.0285
- low income	0.0055	0.0081	0.0104	0.0062	0.0059	0.0116
- high income	0.0100	0.0130	0.0156	0.0109	0.0105	0.0169
<i>Panel III: Population shares</i>						
The poor, $a < 0$	0.5509	0.5741	0.5957	0.5575	0.5543	0.6039
The middle class, $a \gtrsim 0$	0.3746	0.3329	0.2965	0.3621	0.3682	0.2829
The rich, $a \gg 0$	0.0746	0.0930	0.1078	0.0803	0.0775	0.1132

of the relatively impatient middle class: they save and lends less, which pushes the interest rate up and, thus, increases both the debt of the poor and the wealth of the rich. In the CDR model this mechanism is absent.⁴ Tables 1 and 3 show that the discussed effects are robust with respect to the shape of discount factor function as described by parameters Θ and κ .

Recall that in our parameterisation, higher Θ makes the discount function steeper in the middle, effectively increasing the impatience of the middle class, which drives up the equilibrium interest rate. This is illustrated in Table 3. One can see that, for a given κ , higher Θ leads to a higher interest rate. Higher interest payments on debt increase the vulnerability of the middle class to the adverse income shock. This leads to a higher share of the poor in the population and a lower share of the middle class. As a result, the wealth distribution becomes more dispersed. The effect is relatively weak: changing Θ by a factor of 1.5–2 around the benchmark value leads to a change in the interest rate by about 0.2 percentage points, while the population shares change by about 2 to 5 percentage points.

Higher κ for a given Θ makes the discount function closer to linear. Simulations reveal that this has a non-monotone but even weaker effect on the equilibrium interest rate. Thus, varying κ by a factor of 2 around the benchmark value leads to a change in the interest rate by about 0.01–0.03 percentage points, whereas the population shares change by about 1 to 3 percentage points. Higher κ leads to a wider gap in impatience between the very poor and the very rich. The former save more, and the latter will consume more. As a result, the wealth distribution with higher κ

becomes relatively less dispersed, but remains substantially more dispersed than in the CDR economy.

To summarise, the distribution of wealth in the KUE economy exhibits a larger inequality in the population and within each income group relative to the CDR economy.

The effect on consumption distribution is less clear-cut. A higher interest rate reduces consumption on the borrowing limit. The KUE reduces consumption inequality for the high-income group and exacerbates it for the low-income group, as shown in Panel III of Table 3.

4. Conclusion

In this paper we examined the distributional consequences of endogenous time preferences in a model with stochastic incomes and uninsurable risk. We have demonstrated that with the KUE time preferences the long-run distribution of wealth is more unequal than in a similar economy with constant discounting. This is driven by a self-reinforcing mechanism of a higher equilibrium interest rate and a higher net demand for loans.

Data availability

No data was used for the research described in the article.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.mathsocsci.2023.01.003>.

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⁴ Additional numerical simulations, available upon request, show that the higher equilibrium interest rate in a CDR model (with higher discount rate) results in virtually the same wealth and consumption distributions as described in column (1) in Table 1. These simulations confirm that the increasing impatience is the key to redistributive effects.

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