Recursive Locally Minimum-Variance Filtering for Two-dimensional Systems: When Dynamic Quantization Effect Meets Random Sensor Failure

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***Abstract*—This article deals with the recursive filtering issue for an array of two-dimensional systems with random sensor failures and dynamic quantizations. The phenomenon of sensor failure is introduced whose occurrence is governed by a random variable with known statistical properties. In view of the data transmission over networks of constrained bandwidths, a dynamic quantizer is adopted to compress the raw measurements into the quantized ones. The main objective of this article is to design a recursive filter so that a locally minimal upper bound is ensured on the filtering error variance. To facilitate the filter design, states of the dynamic quantizer and the target plant are integrated into an augmented system, based on which an upper bound is first derived on the filtering error variance and subsequently minimized at each step. The expected filter gain is parameterized by solving some coupled difference equations. Moreover, the monotonicity of the resulting minimum upper bound with regard to the quantization level is discussed and the boundedness analysis is further investigated. Finally, effectiveness of the developed filtering strategy is verified via a simulation example.**

***Index Terms*—Recursive filter, two-dimensional systems, dynamic quantization, sensor failure, monotonicity, boundedness.**

I. Introduction

Over the past few decades, there has been a tremendous amount of research looking at the filtering or state estimation problems in both signal processing and control communities. According to the types of the system noises and the performance specifications, a number of filtering schemes have been thoroughly investigated in the literature and applied in engineering practice, among which the popular ones are H_∞ filtering, set-membership filtering, and minimum-variance filtering algorithms [6], [7], [23], [28], [32], [37], [41], [44]. In particular, the renowned Kalman filter, which aims to characterize the estimation performance in the sense of minimum error variance for exactly known linear systems, has been widely utilized in various scenarios ranging from engineering practice to machine learning. Furthermore, the so-called robust Kalman filter has been developed to enhance the robustness against modeling errors and/or parameter uncertainties, where the main idea is to provide desirable state estimates with guaranteed error variances at each iteration [10], [40]. For some recent results on recursive variance-guaranteed filtering problems, we refer the readers to [4], [19], [25], [26], [42].

In recent years, two-dimensional (2-D) systems have gained an ever-increasing research interest owing to their distinctive feature that the states evolve along two independent directions. Benefiting from the bi-directional signal propagation, 2-D systems are competent in modeling many multi-variable systems with promising applications in water heating, metal rolling, sheet forming, and medical imaging [9], [31]. So far, an abundance of literature has focused on the research topics of stabilization, control and estimation for 2-D systems [1], [30], [33], [38], [39], [43], [47]. In particular, much effort has been made towards the recursive filtering problems for 2-D systems under robust performance requirements [20], [34], [46]. For instance, the robust Kalman filter design issue has been explored in [20] for an array of 2-D stochastic systems with parameter uncertainties and measurement degradations, and the recursive distributed filtering problem has been investigated in [34] for 2-D shift-varying systems over sensor networks with stochastic communication protocol.

The quick revolution of communication technologies has facilitated the implementation of remote filtering algorithms over communication channels. It is noteworthy that the proliferation of communication networks has resulted in various advantages including low cost, simple installation, easy operation, and flexible architecture of networked systems. On the other hand, the inherent bandwidth and unpredictable network load have led to undesired network-induced phenomena (NIP) deserving particular research attention [8], [11]–[14], [45]. One of such NIP is the random sensor failures (RSFs) due to the susceptibility of sensors to *probabilistic* equipment aging/outages and link failures [3], [24]. The occurrence of the RSFs gives rise to imperfect measurements which, in turn, would dramatically deteriorate the filtering performance. Accordingly, the analysis/synthesis issues of RSFs have attracted masses of research attention with many results available in the literature (e.g. [18], [24], [29], [36]). However, the filtering problem for 2-D systems with RSFs has not been adequately addressed yet, which gives rise to one motivation of the present research.

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One way to cope with the communication constraints is to adopt signal quantization for streamlining the data transmission over digital channels. To date, both static and dynamic quantization schemes have been widely employed in practice [16], [21], [22], [27]. More specifically, the quantized signals obtained from static quantizers (e.g. uniform and logarithmic quantizers) are determined by certain persistently fixed thresholds, whilst the states of dynamic quantizers are dynamically adjusted based on a prescribed law [27]. By making use of some adjustable parameters and available measurements, the dynamic quantizers would produce a more intricate yet flexible structure than the static quantizers [2]. On account of the quantization effects, the quantized signals deviate from the original ones, and this gives rise to the data distortion and further affects the dynamical behaviors as well as the filtering performance.

Recently, the filtering problems with dynamic quantization have stirred some initial research interest [5], [15], [35], [48]. For example, the dynamic quantization and rate allocation strategies have been proposed in [15] to estimate the states of certain hidden Markov systems. In [48], the moving horizon estimation scheme has been developed for networked linear systems subject to unknown inputs and dynamic quantization effects. Furthermore, the distributed state estimation problem has been investigated in [35] for discrete time-varying systems with dynamic quantization and event-based mechanism. Nonetheless, the relevant filtering results for 2-D systems with dynamic quantization have been very scattered, not to mention the case where RSFs also become a concern. As such, it is of practical significance to establish a 2-D recursive filter to extract the true states from quantized and degraded measurements, and this comprises the primary motivation of the current study.

Following the above discussions, our attention of this article is on the recursive filtering problem for 2-D systems in the simultaneous presence of sensor failure and dynamic quantization. To address such a problem, three emerging issues that we are facing are summarized as follows: 1) how to develop a proper filtering algorithm for 2-D systems taking into account both the sensor failure and the dynamic quantization? 2) how to design a recursive filter to attain a satisfactory filtering performance regardless of the imprecise measurements? and 3) how to conduct the performance analysis of the devised filter and quantify the impacts from the sensor failure and the dynamic quantization?

To tackle the aforementioned emerging issues, in this article, we make dedicated efforts to contrive recursive filterinng algorithm for the underlying 2-D system. To be more specific, an augmented system is first developed to enable the coexistence of the original system state and the dynamic quantizer state. Then, a sufficient criterion is presented to pledge the existence of an upper bound on the filtering error variance, and the filter gain is then parameterized to accomplish the locally tightest upper bound on the error variance. *The main contributions of this article can be summarized from the following three aspects. 1) The 2-D system under consideration encompasses the dynamic quantization and RSF to reflect the engineering practice suffering from limited channel bandwidth and unreliable data transmission. 2) A novel recursive filtering strategy is established with guaranteed estimation accuracy, where the desired filter and the minimum upper bound are recursively calculated by solving certain difference equations. 3) The monotonicity of the filtering performance is evaluated regarding the dynamic quantization and sensor failure, and the boundedness analysis of the proposed filtering algorithm is further carried out by resorting to some rigorous derivations.*

The remainder of this work is arranged as follows. In Section II, the filtering problem to be addressed is formulated for the considered 2-D system. In Section III, the upper bounds on the error variances are derived and minimized under the designed filter gain, and then the filtering performance is further examined. An illustrative example is shown in Section IV to confirm the validity of the established filtering strategy. Conclusions are lastly drawn in Section V.

Notations: The notation used here is normative. $\mathbb{R}^n$ is the n-dimensional Euclidean space and $\mathbb{Z}$ is the integer set. I indicates the identity matrix and 0 denotes the zero one with appropriate dimensions. Superscripts '-1' and 'T' stand, respectively, for the inverse and transpose operations. For real and symmetric matrices X and Y, $X > Y$ ($X \geq Y$) infers that $X - Y$ is positive definite (nonnegative definite). The notation $[\kappa\ \iota]$ with two integers κ and ι satisfying $\kappa \leq \iota$ represents a finite set $\{\kappa, \kappa+1, \kappa+2, \ldots, \iota\}$. The symbol $\lambda_{\max}\{\cdot\}$ ($\lambda_{\min}\{\cdot\}$) denotes the maximum (minimum) eigenvalue of certain matrix. $\mathbb{E}\{\cdot\}$ and $\mathbb{V}ar\{\cdot\}$ indicate the mathematical expectation and variance operations, respectively. $\mathbb{P}\{\cdot\}$ is the probability of certain event.

II. Problem Formulation

Consider the following 2-D shift-varying system defined on a finite horizon $t, s \in [0\ N]$:

$$\begin{aligned} x(t,s) =& A_1(t,s-1)x(t,s-1) + A_2(t-1,s)x(t-1,s) \\ &+ B_1(t,s-1)w(t,s-1) + B_2(t-1,s)w(t-1,s) \end{aligned} \tag{1}$$

where $x(t,s) \in \mathbb{R}^{n_x}$ is the system state and $w(t,s) \in \mathbb{R}^{n_w}$ is the process noise described by a zero-mean Gaussian white sequence with variance $Q(t,s)$. $A_\ell(t,s)$ and $B_\ell(t,s)$ ($\ell = 1,2$) are known shift-varying matrices with appropriate dimensions. The initial states of system (1) are random variables satisfying $\mathbb{E}\{x(t,0)\} = \mathbf{x}_1(t)$ and $\mathbb{E}\{x(0,s)\} = \mathbf{x}_2(s)$, where $\mathbf{x}_1(t)$ and $\mathbf{x}_2(s)$ are known vectors with $\mathbf{x}_1(0) = \mathbf{x}_2(0)$.

In a networked system, the measurement outputs are sometimes inaccurate due primarily to some unexpected scenarios such as sensor failures and limited channel capacity. Bearing this fact in mind, the degradation and quantization effects on the measurements are considered in this work. To begin with, the measurement model undergoing RSFs is presented as

$$y(t,s) = \gamma(t,s)C(t,s)x(t,s) + v(t,s) \tag{2}$$

where $y(t,s) \in \mathbb{R}^{n_y}$ is the measured signal, $v(t,s) \in \mathbb{R}^{n_y}$ is the measurement white noise with zero mean and variance $R(t,s) > 0$, and $C(t,s)$ is a known matrix. The random variable $\gamma(t,s) \in \mathbb{R}$ characterizing sensor failures takes values on the interval $[0,1]$, and is independently and identically distributed with respect to the indexes t and s, which is governed by a certain probabilistic distribution with $\mathbb{E}\{\gamma(t,s)\}=\bar{\gamma}(t,s)$ and $\mathbb{V}ar\{\gamma(t,s)\} = \hat{\gamma}(t,s)$, where $\bar{\gamma}(t,s)$ and $\hat{\gamma}(t,s)$ are known scalars.

Remark 1: Sensor failures, which might be induced by sensor aging, intermittent sensing, and unreliable communication channels, are one of the frequently encountered phenomena in reality that often take place in a probabilistic manner because of the random/abrupt environmental changes. Such a phenomenon ineluctably leads to undesirable observations subject to stochastic degradations. As such, the random variable $\gamma(t,s)$ (with known statistical information) is introduced into model (2) which, by means of taking different values, is able to describe different levels of deteriorations of the sensor performances.

Next, the measurement subject to dynamic quantization is discussed. The following dynamic quantizer is adopted:

$$\begin{aligned}\psi(t,s)=&D_1(t,s-1)\psi(t,s-1)+D_2(t-1,s)\psi(t-1,s)\\&+E_1(t,s-1)y(t,s-1)+E_2(t-1,s)y(t-1,s)\\&+F_1(t,s-1)\bar{y}(t,s-1)+F_2(t-1,s)\bar{y}(t-1,s)\end{aligned} \tag{3a}$$

$$\bar{y}(t,s)=\mathcal{Q}(D(t,s)\psi(t,s)+E(t,s)y(t,s)) \tag{3b}$$

where $\psi(t,s)\in\mathbb{R}^{n_\psi}$ is the state of the quantizer, $\bar{y}(t,s)\in\mathbb{R}^{n_y}$ is the corresponding output based on the function $\mathcal{Q}(\cdot)$ to be defined later, $D_\ell(t,s)$, $E_\ell(t,s)$, $F_\ell(t,s)$ $(\ell=1,2)$, $D(t,s)$ and $E(t,s)$ are shift-varying matrices of compatible dimensions. The initial states of (3) are given as $\psi(t,0)\equiv\psi(0,s)\equiv 0$.

For a given vector $z\in\mathbb{R}^{n_y}$, the function $\mathcal{Q}(z):\mathbb{R}^{n_y}\to\aleph^{n_y}$ is viewed as a uniform quantizer/mapping expressed as

$$\mathcal{Q}(z)\triangleq\left[\begin{array}{cccc}\eta\mathcal{Q}_1(z_1/\eta) & \eta\mathcal{Q}_2(z_2/\eta) & \ldots & \eta\mathcal{Q}_{n_y}(z_{n_y}/\eta)\end{array}\right]$$

where $\aleph=\{\tau_i|\tau_i=i\eta, i\in\mathbb{Z}\}$ is a discrete subset of $\mathbb{R}$, $\aleph^{n_y}$ is the direct product of the n_y subsets, $\eta>0$ denotes the quantization level of the mapping $\mathcal{Q}(z)$, z_ι ($\iota\in[1\ n_y]$) is the ι-th element of z, and $\mathcal{Q}_\iota(\cdot)$ is a stochastic function that maps certain scalar to its nearest integer according to a uniformly probabilistic distribution. To be specific, when $\tau_i\le z_\iota\le\tau_{i+1}$, the function $\mathcal{Q}_\iota(\cdot)$ obeys the following probability distribution:

$$\begin{aligned}\mathbb{P}\{\mathcal{Q}_\iota(z_\iota/\eta)=i\}=1-\pi_\iota\\ \mathbb{P}\{\mathcal{Q}_\iota(z_\iota/\eta)=i+1\}=\pi_\iota\end{aligned}$$

with $\pi_\iota=(z_\iota-\tau_i)/\eta$ belonging to $[0,1]$. Then, the deviation between the quantized and the raw signals with respect to the ι-th component is denoted as $\Delta_\iota\triangleq\eta\mathcal{Q}_\iota(z_\iota/\eta)-z_\iota$ satisfying

$$\begin{aligned}\mathbb{P}\{\Delta_\iota=-\pi_\iota\eta\}=1-\pi_\iota\\ \mathbb{P}\{\Delta_\iota=(1-\pi_\iota)\eta\}=\pi_\iota.\end{aligned}$$

Hence, based on the property of the uniform quantization [17], the quantized output in (3b) can be further formulated by

$$\bar{y}(t,s)=D(t,s)\psi(t,s)+E(t,s)y(t,s)+\Delta(t,s) \tag{4}$$

where $\Delta(t,s)$ refers to the quantization-induced error having the following statistics:

$$\mathbb{E}\{\Delta(t,s)\}=0,\quad \mathbb{E}\{\Delta(t,s)\Delta^T(t,s)\}\le(\eta^2/4)I.$$

Remark 2: Dynamic quantization effects have been investigated for one-dimensional (1-D) systems in the representative works [2] and [48]. Unfortunately, the dynamic quantizer introduced in the literature cannot be directly used in the 2-D case where the signal evolution is inherently bidirectional, and this demands the development of a new 2-D version of the dynamic quantizer in the present work. As shown in (3), the dynamic quantizer (with a feedback structure) is referred to as a generalized 2-D quantizer which is, for the first time, proposed for 2-D systems. Different from the conventional dynamic quantizer for 1-D systems, the 2-D dynamic quantizer exploits the measurements from both the current instant and the immediate preceding instants along two directions.

Remark 3: In comparison with its static counterpart, the dynamic quantization generates quantized outputs that are dependent on both current and historical measurements. Furthermore, the dynamic quantization in (3) consists of the quantizer state (whose evolution is determined by the historical measurements) and a uniform quantization function (which involves the current measurement information). Obviously, the proposed quantizer can dynamically adjust itself based on the input sequences $\psi(t,s)$, $y(t,s)$, and some adjustable parameters, thereby exhibiting richer dynamics (than the static one) which would then help reduce the conservatism in the system analysis. In addition, the dynamic quantizer (3) will degenerate into the conventional uniform one by just choosing $\psi(t,s)\equiv 0$ and $E(t,s)\equiv I$. To sum up, the proposed dynamic quantizer in a feedback form is a fairly general one with remarkable flexibility, which covers the static uniform quantizer as a special case.

For notational simplicity, let us denote

$$\begin{aligned}\bar{E}_\ell(t,s)&\triangleq E_\ell(t,s)+F_\ell(t,s)E(t,s),\quad \ell=1,2\\ \bar{D}_\ell(t,s)&\triangleq D_\ell(t,s)+F_\ell(t,s)D(t,s)\\ \mathcal{A}_\ell(t,s)&\triangleq\left[\begin{array}{cc}A_\ell(t,s) & 0\\ \gamma(t,s)\bar{E}_\ell(t,s)C(t,s) & \bar{D}_\ell(t,s)\end{array}\right]\\ \bar{\mathcal{A}}_\ell(t,s)&\triangleq\left[\begin{array}{cc}A_\ell(t,s) & 0\\ \bar{\gamma}(t,s)\bar{E}_\ell(t,s)C(t,s) & \bar{D}_\ell(t,s)\end{array}\right]\\ \mathcal{E}_1(t,s)&\triangleq\left[\begin{array}{cc}\gamma(t,s)E(t,s)C(t,s) & D(t,s)\end{array}\right]\\ \bar{\mathcal{E}}_1(t,s)&\triangleq\left[\begin{array}{cc}\bar{\gamma}(t,s)E(t,s)C(t,s) & D(t,s)\end{array}\right]\\ \mathcal{E}_2(t,s)&\triangleq\left[\begin{array}{cc}E(t,s) & I\end{array}\right],\quad \tilde{\gamma}(t,s)\triangleq\gamma(t,s)-\bar{\gamma}(t,s)\\ \mathcal{B}_\ell(t,s)&\triangleq\left[\begin{array}{c}B_\ell(t,s)\\ 0\end{array}\right],\quad \mathcal{F}_\ell(t,s)\triangleq\left[\begin{array}{c}0\\ \hat{F}_\ell(t,s)\end{array}\right],\\ \hat{F}_\ell(t,s)&\triangleq\left[\begin{array}{cc}\bar{E}_\ell(t,s) & F_\ell(t,s)\end{array}\right]\\ \hat{E}_\ell(t,s)&\triangleq\left[\begin{array}{cc}0 & 0\\ \bar{E}_\ell(t,s)C(t,s) & 0\end{array}\right]\\ \hat{E}(t,s)&\triangleq\left[\begin{array}{cc}E(t,s)C(t,s) & 0\end{array}\right].\end{aligned}$$

It is easy to see that $\bar{\mathcal{A}}_\ell(t,s)=\mathbb{E}\{\mathcal{A}_\ell(t,s)\}$ and $\bar{\mathcal{E}}_1(t,s)=\mathbb{E}\{\mathcal{E}_1(t,s)\}$. By further defining

$$\begin{aligned}\bar{x}(t,s)&\triangleq\left[\begin{array}{cc}x^T(t,s) & \psi^T(t,s)\end{array}\right]^T\\ \bar{v}(t,s)&\triangleq\left[\begin{array}{cc}v^T(t,s) & \Delta^T(t,s)\end{array}\right]^T\end{aligned}$$

we have the following augmented system from (1)-(4):

$$\bar{x}(t,s)=\mathcal{A}_1(t,s-1)\bar{x}(t,s-1)+\mathcal{A}_2(t-1,s)$$

$$
\begin{aligned}
&\times \bar{x}(t-1,s)+\mathcal{B}_1(t,s-1)w(t,s-1)\\
&+\mathcal{B}_2(t-1,s)w(t-1,s)+\mathcal{F}_1(t,s-1)\\
&\times \bar{v}(t,s-1)+\mathcal{F}_2(t-1,s)\bar{v}(t-1,s) \quad (5a)\\
\bar{y}(t,s) =&\mathcal{E}_1(t,s)\bar{x}(t,s)+\mathcal{E}_2(t,s)\bar{v}(t,s). \quad (5b)
\end{aligned}
$$

For the augmented system (5), a recursive filter is proposed as follows:

$$
\begin{aligned}
\hat{x}_p(t,s) =&\bar{\mathcal{A}}_1(t,s-1)\hat{x}_u(t,s-1)\\
&+\bar{\mathcal{A}}_2(t-1,s)\hat{x}_u(t-1,s) \quad (6a)\\
\hat{x}_u(t,s) =&\hat{x}_p(t,s)+\mathcal{K}(t,s)\left(\bar{y}(t,s)-\bar{\mathcal{E}}_1(t,s)\hat{x}_p(t,s)\right) \quad (6b)
\end{aligned}
$$

where $\hat{x}_p(t,s)$ and $\hat{x}_u(t,s)$ are, respectively, the one-step prediction and the state estimate of $\bar{x}(t,s)$, and $\mathcal{K}(t,s)$ is the filter gain to be devised. The initial states of (6) are set as

$$
\begin{aligned}
\hat{x}_u(t,0) &= \begin{bmatrix} \mathbf{x}_1^T(t) & \psi^T(t,0) \end{bmatrix}^T\\
\hat{x}_u(0,s) &= \begin{bmatrix} \mathbf{x}_2^T(s) & \psi^T(0,s) \end{bmatrix}^T.
\end{aligned}
$$

Denote $e_p(t,s) \triangleq \bar{x}(t,s)-\hat{x}_p(t,s)$ and $e_u(t,s) \triangleq \bar{x}(t,s)-\hat{x}_u(t,s)$. Subtracting (6) from (5) yields the following error dynamics:

$$
\begin{aligned}
e_p(t,s) =&\bar{\mathcal{A}}_1(t,s-1)e_u(t,s-1)+\bar{\mathcal{A}}_2(t-1,s)\\
&\times e_u(t-1,s)+\tilde{\gamma}(t,s-1)\hat{E}_1(t,s-1)\\
&\times \bar{x}(t,s-1)+\tilde{\gamma}(t-1,s)\hat{E}_2(t-1,s)\\
&\times \bar{x}(t-1,s)+\mathcal{B}_1(t,s-1)w(t,s-1)\\
&+\mathcal{B}_2(t-1,s)w(t-1,s)+\mathcal{F}_1(t,s-1)\\
&\times \bar{v}(t,s-1)+\mathcal{F}_2(t-1,s)\bar{v}(t-1,s) \quad (7a)\\
e_u(t,s) =& \left(I-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)\right)e_p(t,s)-\mathcal{K}(t,s)\\
&\times \left[\tilde{\gamma}(t,s)\hat{E}(t,s)\bar{x}(t,s)+\mathcal{E}_2(t,s)\bar{v}(t,s)\right]. \quad (7b)
\end{aligned}
$$

The following assumption is made throughout this article.

Assumption 1: For $t_j, s_j \in [0\ N]$ with $j \in [1\ 5]$, the process noise $w(t_1,s_1)$, the measurement noise $v(t_2,s_2)$, the sensor failure coefficient $\gamma(t_3,s_3)$, the quantization error $\Delta(t_4,s_4)$, as well as the initial states $x(t_5,0)$ and $x(0,s_5)$ are mutually uncorrelated with each other.

Now, we are ready to introduce the problem to be addressed in this article. For the considered 2-D shift-varying system (1) with dynamic quantization effect (3) and sensor failure (2), the objective of the current work is to devise the recursive filter (6) such that an upper bound of the filtering error variance $\mathbb{E}\left\{e_u(t,s)e_u^T(t,s)\right\}$ is guaranteed and subsequently minimized by properly selecting the filter parameter $\mathcal{K}(t,s)$ at each iteration.

Remark 4: Notice that the quantization error involved in the error dynamics (7) would ineluctably lead to the calculation of the exact filtering error variance inaccessible, not to speak of designing the recursive filter. To this end, an alternative strategy is to derive a locally minimal upper bound on the filtering error variance, and the desired filter gain is to be parameterized for ensuring such a locally minimal upper bound.

III. Main results

In this section, we plan to explore a recursive filter design approach for accomplishing the desired filtering performance for the 2-D system (1). First, some preliminaries are given to formulate the evolutions of the system state's second-order moment and the filtering error variance. Afterwards, an upper bound of the error variance is constructed and the explicit gain parameter is derived to optimize this bound at each step. Finally, the filtering accuracy in relation to the quantization level and the boundedness of the minimum bound are elaborately discussed.

A. Preliminaries

The following lemmas are presented for subsequent developments.

Lemma 1: For any positive scalar $\alpha > 0$ and matrices M, $\bar{M}$ with appropriate dimensions, one has

$$(\alpha^{1/2}M-\alpha^{-1/2}\bar{M})(\alpha^{1/2}M-\alpha^{-1/2}\bar{M})^T \geq 0.$$

Lemma 2: Let $M(t,s)$ and $\bar{M}(t,s)$ be matrices with appropriate dimensions satisfying $M(t,s) \leq \bar{M}(t,s)$ for all $t,s \in [0\ N]$. Assume that $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$ are two symmetric matrix-form functions satisfying the following relationships:

$$
\begin{aligned}
&f(M(t,s-1),M(t-1,s)) \leq f(\bar{M}(t,s-1),\bar{M}(t-1,s))\\
&f(\bar{M}(t,s-1),\bar{M}(t-1,s)) \leq g(\bar{M}(t,s-1),\bar{M}(t-1,s)).
\end{aligned}
$$

Then, the solutions to the following difference equations

$$
\begin{aligned}
Y(t,s) &= f(Y(t,s-1),Y(t-1,s))\\
Z(t,s) &= g(Z(t,s-1),Z(t-1,s))
\end{aligned}
$$

with initial conditions $Y(t,0) \leq Z(t,0)$ and $Y(0,s) \leq Z(0,s)$ satisfy

$$Y(t,s) \leq Z(t,s), \quad t,s \in [0\ N]. \quad (8)$$

Proof: This lemma can be proven based on the mathematical induction method. According to the properties of functions $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$, one obtains from the initial conditions that

$$
\begin{aligned}
Y(1,1) &= f(Y(1,0),Y(0,1))\\
&\leq f(Z(1,0),Z(0,1))\\
&\leq g(Z(1,0),Z(0,1))\\
&= Z(1,1)
\end{aligned}
$$

which means that (8) holds for $(t,s)=(1,1)$. Next, suppose that (8) is valid for $(t,s) \in \{(k,l)|k,l \in [1\ N];\ k+l=\theta\}$ with $\theta \in [2\quad 2N-1]$. To confirm the statement of this lemma, the validity of (8) shall be verified for $(t,s) \in \{(k,l)|k,l \in [1\ N];\ k+l=\theta+1\}$. On account of the properties of $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$ again, together with the above-mentioned hypothesis, one has

$$
\begin{aligned}
Y(t,s) &= f(Y(t,s-1),Y(t-1,s))\\
&\leq f(Z(t,s-1),Z(t-1,s))\\
&\leq g(Z(t,s-1),Z(t-1,s))\\
&= Z(t,s)
\end{aligned}
$$

which ends the proof. ■

For convenience, we set

$$X(t,s) \triangleq \mathbb{E}\left\{\bar{x}(t,s)\bar{x}^T(t,s)\right\}$$
$$P_p(t,s) \triangleq \mathbb{E}\left\{e_p(t,s)e_p^T(t,s)\right\}$$
$$P_u(t,s) \triangleq \mathbb{E}\left\{e_u(t,s)e_u^T(t,s)\right\}.$$

The following lemma provides a bound constraint for the second-order moment of the state $\bar{x}(t,s)$.

Lemma 3: Let $\varsigma > 0$ be a given positive scalar. Assume that there is a set of matrices $\bar{X}(t,s)$ satisfying the following recursion

$$\begin{aligned}\bar{X}(t,s) =&(1+\varsigma)\bar{\mathcal{A}}_1(t,s-1)\bar{X}(t,s-1)\bar{\mathcal{A}}_1^T(t,s-1)\\&+(1+\varsigma^{-1})\bar{\mathcal{A}}_2(t-1,s)\bar{X}(t-1,s)\bar{\mathcal{A}}_2^T(t-1,s)\\&+\hat{\gamma}(t,s-1)\hat{E}_1(t,s-1)\bar{X}(t,s-1)\hat{E}_1^T(t,s-1)\\&+\hat{\gamma}(t-1,s)\hat{E}_2(t-1,s)\bar{X}(t-1,s)\hat{E}_2^T(t-1,s)\\&+Q_1(t,s-1)+Q_2(t-1,s)\end{aligned} \tag{9}$$

with initial constraints

$$\bar{X}(t,0)=X(t,0),\quad \bar{X}(0,s)=X(0,s) \tag{10}$$

where

$$Q_\ell(t,s) \triangleq \mathcal{B}_\ell(t,s)Q(t,s)\mathcal{B}_\ell^T(t,s)+\mathcal{F}_\ell(t,s)R_{\bar{v}}(t,s)\mathcal{F}_\ell^T(t,s)$$
$$R_{\bar{v}}(t,s) \triangleq \mathrm{diag}\left\{R(t,s),(\eta^2/4)I\right\}.$$

Then, the second-order moment of the state $\bar{x}(t,s)$ is bounded by $\bar{X}(t,s)$, that is, $X(t,s)\le\bar{X}(t,s)$.

Proof: Based on the definition of $\bar{v}(t,s)$, we obtain

$$\begin{aligned}\mathbb{E}\left\{\bar{v}(t,s)\bar{v}^T(t,s)\right\} &= \begin{bmatrix} R(t,s) & 0\\ 0 & \mathbb{E}\left\{\Delta(t,s)\Delta^T(t,s)\right\}\end{bmatrix}\\ &\le \begin{bmatrix} R(t,s) & 0\\ 0 & (\eta^2/4)I\end{bmatrix}\\ &= R_{\bar{v}}(t,s).\end{aligned} \tag{11}$$

Recalling the statistical properties of random variables $\gamma(t,s)$, $w(t,s)$, and $\bar{v}(t,s)$, we have

$$\mathbb{E}\left\{\gamma(t,s)w^T(t_0,s_0)\right\}=0,\quad \mathbb{E}\left\{\gamma(t,s)\bar{v}^T(t_0,s_0)\right\}=0$$
$$\mathbb{E}\left\{w(t,s)\bar{v}^T(t_0,s_0)\right\}=0$$

for all $t,s,t_0,s_0\in[0\ N]$ and

$$\mathbb{E}\left\{\bar{x}(t,s)w^T(t_1,s_1)\right\}=0,\quad \mathbb{E}\left\{\bar{x}(t,s)\bar{v}^T(t_1,s_1)\right\}=0$$

for $(t_1,s_1)\in\{(k,l)|k>t \text{ or } l>s\}\cup(t,s)$. Combining the above facts with (5a), we can derive that

$$\begin{aligned}X(t,s) =&\bar{\mathcal{A}}_1(t,s-1)X(t,s-1)\bar{\mathcal{A}}_1^T(t,s-1)\\&+\bar{\mathcal{A}}_2(t-1,s)X(t-1,s)\bar{\mathcal{A}}_2^T(t-1,s)\\&+\bar{\mathcal{A}}_1(t,s-1)\mathbb{E}\left\{\bar{x}(t,s-1)\bar{x}^T(t-1,s)\right\}\\&\times\bar{\mathcal{A}}_2^T(t-1,s)+\bar{\mathcal{A}}_2(t-1,s)\\&\times\mathbb{E}\left\{\bar{x}(t-1,s)\bar{x}^T(t,s-1)\right\}\bar{\mathcal{A}}_1^T(t,s-1)\\&+\hat{\gamma}(t,s-1)\hat{E}_1(t,s-1)X(t,s-1)\hat{E}_1^T(t,s-1)\\&+\hat{\gamma}(t-1,s)\hat{E}_2(t-1,s)X(t-1,s)\hat{E}_2^T(t-1,s)\\&+\mathcal{B}_1(t,s-1)Q(t,s-1)\mathcal{B}_1^T(t,s-1)\\&+\mathcal{B}_2(t-1,s)Q(t-1,s)\mathcal{B}_2^T(t-1,s)\\&+\mathcal{F}_1(t,s-1)\mathbb{E}\left\{\bar{v}(t,s-1)\bar{v}^T(t,s-1)\right\}\\&\times\mathcal{F}_1^T(t,s-1)+\mathcal{F}_2(t-1,s)\\&\times\mathbb{E}\left\{\bar{v}(t-1,s)\bar{v}^T(t-1,s)\right\}\mathcal{F}_2^T(t-1,s)\\ \le&(1+\varsigma)\bar{\mathcal{A}}_1(t,s-1)X(t,s-1)\bar{\mathcal{A}}_1^T(t,s-1)\\&+(1+\varsigma^{-1})\bar{\mathcal{A}}_2(t-1,s)X(t-1,s)\bar{\mathcal{A}}_2^T(t-1,s)\\&+\hat{\gamma}(t,s-1)\hat{E}_1(t,s-1)X(t,s-1)\hat{E}_1^T(t,s-1)\\&+\hat{\gamma}(t-1,s)\hat{E}_2(t-1,s)X(t-1,s)\hat{E}_2^T(t-1,s)\\&+Q_1(t,s-1)+Q_2(t-1,s).\end{aligned}$$

On grounds of the given initial conditions of $\bar{X}(t,s)$ and Lemma 2, it is not difficult to confirm from Lemma 1 that $X(t,s)\le\bar{X}(t,s)$, which completes the proof. ■

The recursions of the error variances are given as follows.

Lemma 4: The one-step prediction error variance $P_p(t,s)$ and the filtering error variance $P_u(t,s)$ are computed as:

$$\begin{aligned}P_p(t,s) =&\bar{\mathcal{A}}_1(t,s-1)P_u(t,s-1)\bar{\mathcal{A}}_1^T(t,s-1)\\&+\bar{\mathcal{A}}_2(t-1,s)P_u(t-1,s)\bar{\mathcal{A}}_2^T(t-1,s)\\&+\bar{\mathcal{A}}_1(t,s-1)\mathbb{E}\left\{e_u(t,s-1)e_u^T(t-1,s)\right\}\\&\times\bar{\mathcal{A}}_2^T(t-1,s)+\bar{\mathcal{A}}_2(t-1,s)\\&\times\mathbb{E}\left\{e_u(t-1,s)e_u^T(t,s-1)\right\}\bar{\mathcal{A}}_1^T(t,s-1)\\&+\Omega_1(t,s-1)+\Omega_1^T(t,s-1)+\Omega_2(t-1,s)\\&+\Omega_2^T(t-1,s)+\hat{Q}_1(t,s-1)+\hat{Q}_2(t-1,s)\end{aligned} \tag{12}$$

$$\begin{aligned}P_u(t,s) =& \big(I-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)\big)P_p(t,s)\big(I-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)\big)^T\\&+\mathcal{K}(t,s)\big[\hat{\gamma}(t,s)\hat{E}(t,s)X(t,s)\hat{E}^T(t,s)\\&+\mathcal{E}_2(t,s)\mathbb{E}\left\{\bar{v}(t,s)\bar{v}^T(t,s)\right\}\mathcal{E}_2^T(t,s)\big]\mathcal{K}^T(t,s)\end{aligned} \tag{13}$$

where, for $\ell=1,2$,

$$\begin{aligned}\Omega_\ell(t,s) \triangleq&\mathbb{E}\Big\{\bar{\mathcal{A}}_\ell(t,s)e_u(t,s)\big(\tilde{\gamma}(t,s)\hat{E}_\ell(t,s)\bar{x}(t,s)\\&+\mathcal{F}_\ell(t,s)\bar{v}(t,s)\big)^T\Big\}\end{aligned} \tag{14}$$

$$\begin{aligned}\hat{Q}_\ell(t,s) \triangleq&\hat{\gamma}(t,s)\hat{E}_\ell(t,s)X(t,s)\hat{E}_\ell^T(t,s)+\mathcal{B}_\ell(t,s)Q(t,s)\\&\times\mathcal{B}_\ell^T(t,s)+\mathcal{F}_\ell(t,s)\mathbb{E}\left\{\bar{v}(t,s)\bar{v}^T(t,s)\right\}\mathcal{F}_\ell^T(t,s).\end{aligned} \tag{15}$$

Proof: It follows from the statistical properties of $\bar{v}(t,s)$ and $e_p(t,s)$ that

$$\mathbb{E}\left\{e_p(t,s)\bar{v}^T(t_1,s_1)\right\}=0$$

for $(t_1,s_1)\in\{(k,l)|k>t \text{ or } l>s\}\cup(t,s)$. The proof is readily concluded from (7) and the detailed derivation is omitted here for space saving purposes. ■

In this subsection, some preliminary results have been given to be used in the subsequent developments. In particular, Lemma 1 shows a fundamental inequality to cope with some cross-terms, Lemma 2 presents a general comparison principle in the 2-D setting to obtain certain upper bounds, Lemma 3 derives a matrix sequence $\bar{X}(t,s)$ as an upper bound on the second-order moment of the state at each iteration, and Lemma 4 provides the recursions of the prediction and the filtering error variances. It is worth noting that, in consequence of the existence of the quantization-induced errors, the analytical solution of the error variance $P_u(t,s)$ in (13) cannot be obtained. As such, we resort to find an upper bound of $P_u(t,s)$ and then optimize it by properly designing the filter gain.

B. The filter design

On the basis of the previous discussions, an upper bound is first calculated on the filtering error variance and, subsequently, the filter is devised to minimize such a bound in this subsection.

Theorem 1: Let ς, μ, α, and β be given positive scalars. Assume that there is a set of nonnegative definite matrices $\bar{X}(t,s)$ and two sets of positive definite matrices $S(t,s)$ and $\Xi(t,s)$ satisfying (9) and the following two recursions

$$\begin{aligned} S(t,s) =&(1+\mu)\bar{\mathcal{A}}_1(t,s-1)\Xi(t,s-1)\bar{\mathcal{A}}_1^T(t,s-1) \\ &+(1+\mu^{-1})\bar{\mathcal{A}}_2(t-1,s)\Xi(t-1,s)\bar{\mathcal{A}}_2^T(t-1,s) \\ &+\bar{Q}_1(t,s-1)+\bar{Q}_2(t-1,s) \\ &+\bar{R}_1(t,s-1)+\bar{R}_2(t-1,s) \end{aligned} \quad (16)$$

$$\begin{aligned} \Xi(t,s) =& \left(I-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)\right)S(t,s)\left(I-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)\right)^T \\ &+\mathcal{K}(t,s)\left[\hat{\gamma}(t,s)\hat{E}(t,s)\bar{X}(t,s)\hat{E}^T(t,s)\right. \\ &\left.+\mathcal{E}_2(t,s)R_{\bar{v}}(t,s)\mathcal{E}_2^T(t,s)\right]\mathcal{K}^T(t,s) \end{aligned} \quad (17)$$

with initial constraints (10) and

$$\Xi(t,0)=P_u(t,0), \quad \Xi(0,s)=P_u(0,s) \quad (18)$$

where

$$\begin{aligned} \bar{Q}_\ell(t,s) \triangleq&(1+\alpha^{-1})\hat{\gamma}(t,s)\hat{E}_\ell(t,s)\bar{X}(t,s)\hat{E}_\ell^T(t,s) \\ &+\mathcal{B}_\ell(t,s)Q(t,s)\mathcal{B}_\ell^T(t,s) \\ \bar{R}_\ell(t,s) \triangleq&\bar{\mathcal{A}}_\ell(t,s)\mathcal{K}(t,s)\left[\alpha\hat{\gamma}(t,s)\hat{E}(t,s)\bar{X}(t,s)\hat{E}^T(t,s)\right. \\ &\left.+\beta\mathcal{E}_2(t,s)R_{\bar{v}}(t,s)\mathcal{E}_2^T(t,s)\right]\mathcal{K}^T(t,s)\bar{\mathcal{A}}_\ell^T(t,s) \\ &+(1+\beta^{-1})\mathcal{F}_\ell(t,s)R_{\bar{v}}(t,s)\mathcal{F}_\ell^T(t,s). \end{aligned}$$

Then, matrices $S(t,s)$ and $\Xi(t,s)$ are always upper bounds for $P_p(t,s)$ and $P_u(t,s)$, respectively, that is,

$$P_p(t,s)\leq S(t,s), \quad P_u(t,s)\leq \Xi(t,s). \quad (19)$$

Moreover, the upper bound $\Xi(t,s)$ is minimized as

$$\Xi(t,s)=S(t,s)-S(t,s)\bar{\mathcal{E}}_1^T(t,s)\hat{R}^{-1}(t,s)\bar{\mathcal{E}}_1(t,s)S(t,s) \quad (20)$$

with the gain parameter

$$\mathcal{K}(t,s)=S(t,s)\bar{\mathcal{E}}_1^T(t,s)\hat{R}^{-1}(t,s) \quad (21)$$

where

$$\begin{aligned} \hat{R}(t,s) \triangleq&\bar{\mathcal{E}}_1(t,s)S(t,s)\bar{\mathcal{E}}_1^T(t,s)+\hat{\gamma}(t,s)\hat{E}(t,s)\bar{X}(t,s) \\ &\times\hat{E}^T(t,s)+\mathcal{E}_2(t,s)R_{\bar{v}}(t,s)\mathcal{E}_2^T(t,s). \end{aligned} \quad (22)$$

Proof: It follows from (7b) and the statistical properties of $\tilde{\gamma}(t,s)$, $\bar{v}(t,s)$, $\bar{x}(t,s)$, and $e_u(t,s)$ that

$$\begin{aligned} &\mathbb{E}\left\{\tilde{\gamma}(t,s)e_u(t,s)\bar{x}^T(t,s)\right\}=-\hat{\gamma}(t,s)\mathcal{K}(t,s)\hat{E}(t,s)X(t,s) \\ &\mathbb{E}\left\{e_u(t,s)\bar{v}^T(t,s)\right\}=-\mathcal{K}(t,s)\mathcal{E}_2(t,s)\mathbb{E}\left\{\bar{v}(t,s)\bar{v}^T(t,s)\right\}. \end{aligned}$$

According to the formulation of $\Omega_\ell(t,s)$ in (14), we obtain easily from Lemmas 1 and 3 as well as (11) that

$$\begin{aligned} &\Omega_\ell(t,s)+\Omega_\ell^T(t,s) \\ &=-\hat{\gamma}(t,s)\bar{\mathcal{A}}_\ell(t,s)\mathcal{K}(t,s)\hat{E}(t,s)X(t,s)\hat{E}_\ell^T(t,s) \\ &\quad-\hat{\gamma}(t,s)\hat{E}_\ell(t,s)X(t,s)\hat{E}^T(t,s)\mathcal{K}^T(t,s)\bar{\mathcal{A}}_\ell^T(t,s) \\ &\quad-\bar{\mathcal{A}}_\ell(t,s)\mathcal{K}(t,s)\mathcal{E}_2(t,s)\mathbb{E}\left\{\bar{v}(t,s)\bar{v}^T(t,s)\right\}\mathcal{F}_\ell^T(t,s) \\ &\quad-\mathcal{F}_\ell(t,s)\mathbb{E}\left\{\bar{v}(t,s)\bar{v}^T(t,s)\right\}\mathcal{E}_2^T(t,s)\mathcal{K}^T(t,s)\bar{\mathcal{A}}_\ell^T(t,s) \\ &\leq\alpha\hat{\gamma}(t,s)\bar{\mathcal{A}}_\ell(t,s)\mathcal{K}(t,s)\hat{E}(t,s)\bar{X}(t,s)\hat{E}^T(t,s)\mathcal{K}^T(t,s) \\ &\quad\times\bar{\mathcal{A}}_\ell^T(t,s)+\alpha^{-1}\hat{\gamma}(t,s)\hat{E}_\ell(t,s)\bar{X}(t,s)\hat{E}_\ell^T(t,s) \\ &\quad+\beta\bar{\mathcal{A}}_\ell(t,s)\mathcal{K}(t,s)\mathcal{E}_2(t,s)R_{\bar{v}}(t,s)\mathcal{E}_2^T(t,s)\mathcal{K}^T(t,s) \\ &\quad\times\bar{\mathcal{A}}_\ell^T(t,s)+\beta^{-1}\mathcal{F}_\ell(t,s)R_{\bar{v}}(t,s)\mathcal{F}_\ell^T(t,s). \end{aligned} \quad (23)$$

In the light of Lemma 3, inequality (11), and expression (15), we also have

$$\begin{aligned} \hat{Q}_\ell(t,s) \leq&\hat{\gamma}(t,s)\hat{E}_\ell(t,s)\bar{X}(t,s)\hat{E}_\ell^T(t,s)+\mathcal{B}_\ell(t,s)Q(t,s) \\ &\times\mathcal{B}_\ell^T(t,s)+\mathcal{F}_\ell(t,s)R_{\bar{v}}(t,s)\mathcal{F}_\ell^T(t,s). \end{aligned} \quad (24)$$

Substituting (23) and (24) into (12), one obtains

$$\begin{aligned} &P_p(t,s) \\ &\leq(1+\mu)\bar{\mathcal{A}}_1(t,s-1)P_u(t,s-1)\bar{\mathcal{A}}_1^T(t,s-1) \\ &\quad+(1+\mu^{-1})\bar{\mathcal{A}}_2(t-1,s)P_u(t-1,s)\bar{\mathcal{A}}_2^T(t-1,s) \\ &\quad+(1+\alpha^{-1})\hat{\gamma}(t,s-1)\hat{E}_1(t,s-1)\bar{X}(t,s-1) \\ &\quad\times\hat{E}_1^T(t,s-1)+(1+\alpha^{-1})\hat{\gamma}(t-1,s)\hat{E}_2(t-1,s) \\ &\quad\times\bar{X}(t-1,s)\hat{E}_2^T(t-1,s)+\bar{\mathcal{A}}_1(t,s-1)\mathcal{K}(t,s-1) \\ &\quad\times\Big[\alpha\hat{\gamma}(t,s-1)\hat{E}(t,s-1)\bar{X}(t,s-1)\hat{E}^T(t,s-1) \\ &\quad+\beta\mathcal{E}_2(t,s-1)R_{\bar{v}}(t,s-1)\mathcal{E}_2^T(t,s-1)\Big]\mathcal{K}^T(t,s-1) \\ &\quad\times\bar{\mathcal{A}}_1^T(t,s-1)+\bar{\mathcal{A}}_2(t-1,s)\mathcal{K}(t-1,s)\Big[\alpha\hat{\gamma}(t-1,s) \\ &\quad\times\hat{E}(t-1,s)\bar{X}(t-1,s)\hat{E}^T(t-1,s)+\beta\mathcal{E}_2(t-1,s) \\ &\quad\times R_{\bar{v}}(t-1,s)\mathcal{E}_2^T(t-1,s)\Big]\mathcal{K}^T(t-1,s)\bar{\mathcal{A}}_2^T(t-1,s) \\ &\quad+(1+\beta^{-1})\mathcal{F}_1(t,s-1)R_{\bar{v}}(t,s-1)\mathcal{F}_1^T(t,s-1) \\ &\quad+(1+\beta^{-1})\mathcal{F}_2(t-1,s)R_{\bar{v}}(t-1,s)\mathcal{F}_2^T(t-1,s) \\ &\quad+\mathcal{B}_1(t,s-1)Q(t,s-1)\mathcal{B}_1^T(t,s-1) \\ &\quad+\mathcal{B}_2(t-1,s)Q(t-1,s)\mathcal{B}_2^T(t-1,s) \\ &\leq(1+\mu)\bar{\mathcal{A}}_1(t,s-1)P_u(t,s-1)\bar{\mathcal{A}}_1^T(t,s-1) \\ &\quad+(1+\mu^{-1})\bar{\mathcal{A}}_2(t-1,s)P_u(t-1,s)\bar{\mathcal{A}}_2^T(t-1,s) \\ &\quad+\bar{Q}_1(t,s-1)+\bar{Q}_2(t-1,s) \\ &\quad+\bar{R}_1(t,s-1)+\bar{R}_2(t-1,s). \end{aligned} \quad (25)$$

Moreover, it follows from equality (13) and Lemma 3 that

$$\begin{aligned} P_u(t,s)\leq&\left(I-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)\right)P_p(t,s)\left(I-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)\right)^T \\ &+\mathcal{K}(t,s)\left[\hat{\gamma}(t,s)\hat{E}(t,s)\bar{X}(t,s)\hat{E}^T(t,s)\right. \\ &\left.+\mathcal{E}_2(t,s)R_{\bar{v}}(t,s)\mathcal{E}_2^T(t,s)\right]\mathcal{K}^T(t,s). \end{aligned} \quad (26)$$

Now, let us prove (19) by using the inductive method. It follows from (16), (25), and the initial conditions that

$$P_p(1,1)-S(1,1)$$

$$\leq (1+\mu)\bar{\mathcal{A}}_1(1,0)(P_u(1,0)-\Xi(1,0))\bar{\mathcal{A}}_1^T(1,0) + (1+\mu^{-1})\bar{\mathcal{A}}_2(0,1)(P_u(0,1)-\Xi(0,1))\bar{\mathcal{A}}_2^T(0,1) \leq 0$$

which, together with (17) and (26), infers

$$\begin{aligned} &P_u(1,1)-\Xi(1,1)\\ &\leq \left(I-\mathcal{K}(1,1)\bar{\mathcal{E}}_1(1,1)\right)(P_p(1,1)-S(1,1))\\ &\quad\times\left(I-\mathcal{K}(1,1)\bar{\mathcal{E}}_1(1,1)\right)^T \leq 0, \end{aligned}$$

namely, (19) holds for $(t,s)\in\{(k,l)|k,l\in[1\ N];\ k+l=2\}$.

Next, assume that (19) is true for $(t,s)\in\{(k,l)|k,l\in[1\ N];\ k+l=\theta\}$ with a certain integer θ. Then, for $(t,s)\in\{(k,l)|k,l\in[1\ N];\ k+l=\theta+1\}$, one obtains

$$\begin{aligned} &P_p(t,s)-S(t,s)\\ &\leq (1+\mu)\bar{\mathcal{A}}_1(t,s-1)(P_u(t,s-1)-\Xi(t,s-1))\\ &\quad\times\bar{\mathcal{A}}_1^T(t,s-1)+(1+\mu^{-1})\bar{\mathcal{A}}_2(t-1,s)\\ &\quad\times(P_u(t-1,s)-\Xi(t-1,s))\bar{\mathcal{A}}_2^T(t-1,s)\leq 0 \end{aligned}$$

which further indicates

$$\begin{aligned} &P_u(t,s)-\Xi(t,s)\\ &\leq\left(I-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)\right)(P_p(t,s)-S(t,s))\\ &\quad\times\left(I-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)\right)^T\leq 0. \end{aligned}$$

Therefore, it is confirmed that (19) holds for all $t,s\in[1\ N]$.

It remains to determine the filter gain that minimizes the upper bound $\Xi(t,s)$. By means of the completing-the-square method, the recursive equation (17) is calculated as

$$\begin{aligned} \Xi(t,s)=&S(t,s)-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)S(t,s)-S(t,s)\bar{\mathcal{E}}_1^T(t,s)\\ &\times\mathcal{K}^T(t,s)+\mathcal{K}(t,s)\big[\bar{\mathcal{E}}_1(t,s)S(t,s)\bar{\mathcal{E}}_1^T(t,s)\\ &+\hat{\gamma}(t,s)\hat{E}(t,s)\bar{X}(t,s)\hat{E}^T(t,s)\\ &+\mathcal{E}_2(t,s)R_{\bar{v}}(t,s)\mathcal{E}_2^T(t,s)\big]\mathcal{K}^T(t,s)\\ =&S(t,s)+\left(\mathcal{K}(t,s)-S(t,s)\bar{\mathcal{E}}_1^T(t,s)\hat{R}^{-1}(t,s)\right)\\ &\times\hat{R}(t,s)\left(\mathcal{K}(t,s)-S(t,s)\bar{\mathcal{E}}_1^T(t,s)\hat{R}^{-1}(t,s)\right)^T\\ &-S(t,s)\bar{\mathcal{E}}_1^T(t,s)\hat{R}^{-1}(t,s)\bar{\mathcal{E}}_1(t,s)S(t,s). \end{aligned}$$

It is obvious that the upper bound $\Xi(t,s)$ attains its minimization

$$\Xi(t,s)=S(t,s)-S(t,s)\bar{\mathcal{E}}_1^T(t,s)\hat{R}^{-1}(t,s)\bar{\mathcal{E}}_1(t,s)S(t,s)$$

by setting the gain as $\mathcal{K}(t,s)=S(t,s)\bar{\mathcal{E}}_1^T(t,s)\hat{R}^{-1}(t,s)$. The proof is now complete. ∎

Remark 5: For the augmented system (5), with the aid of the intensive stochastic analysis and mathematical induction, a sufficient criterion has been given in Theorem 1 for developing an upper bound $\Xi(t,s)$ on the matrix $P_u(t,s)$ in terms of two coupled difference equations. The gain matrix has also been designed to fulfill the minimization of the upper bound at each iteration. It follows from Theorem 1 that the deterministic upper bounds $S(t,s)$ and $\Xi(t,s)$ as well as the filter gain $\mathcal{K}(t,s)$ can be recursively computed by solving the recursions (9), (16), and (17).

Remark 6: Note that $[I_{n_x}\ 0_{n_\psi}]P_u(t,s)[I_{n_x}\ 0_{n_\psi}]^T\leq[I_{n_x}\ 0_{n_\psi}]\Xi(t,s)[I_{n_x}\ 0_{n_\psi}]^T$. For system (1), the estimate of the system state $x(t,s)$ can be computed as $[I_{n_x}\ 0_{n_\psi}]\hat{x}_u(t,s)$. Accordingly, the estimation error is $[I_{n_x}\ 0_{n_\psi}]e_u(t,s)$, and the upper bound on the error variance $[I_{n_x}\ 0_{n_\psi}]P_u(t,s)[I_{n_x}\ 0_{n_\psi}]^T$ is minimized under the determined filter gain.

C. Performance analysis

In this subsection, the influences from the quantization accuracy and the sensor failure on the filtering performance are to be discussed. Furthermore, boundedness of the minimal upper bound will be analyzed with rigorous derivation.

First, we will establish the relationship between the quantization level η and the filtering performance. It is noted from (11) that the parameter η is involved in matrix $R_{\bar{v}}(t,s)$ and is thus contained in $\bar{R}_\ell(t,s)$ ($\ell=1,2$). Accordingly, the following operators are introduced:

$$\begin{aligned} &\mathcal{G}(\eta,Y(t,s-1),Y(t-1,s))\\ &\triangleq(1+\mu)\bar{\mathcal{A}}_1(t,s-1)Y(t,s-1)\bar{\mathcal{A}}_1^T(t,s-1)\\ &\quad+(1+\mu^{-1})\bar{\mathcal{A}}_2(t-1,s)Y(t-1,s)\bar{\mathcal{A}}_2^T(t-1,s)\\ &\quad+\bar{Q}_1(t,s-1)+\bar{Q}_2(t-1,s)\\ &\quad+\bar{R}_1(t,s-1)+\bar{R}_2(t-1,s)\\ &\mathcal{H}(\eta,Z,\mathcal{K}(t,s))\\ &\triangleq\left(I-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)\right)Z\left(I-\mathcal{K}(t,s)\bar{\mathcal{E}}_1(t,s)\right)^T\\ &\quad+\mathcal{K}(t,s)\big[\hat{\gamma}(t,s)\hat{E}(t,s)\bar{X}(t,s)\hat{E}^T(t,s)\\ &\quad+\mathcal{E}_2(t,s)R_{\bar{v}}(t,s)\mathcal{E}_2^T(t,s)\big]\mathcal{K}^T(t,s). \end{aligned}$$

It is not difficult to see that $\mathcal{G}(\eta,Y(t,s-1),Y(t-1,s))$ is nondecreasing with the increase of the parameter η or matrices $Y(t,s-1)$ and $Y(t-1,s)$. Moreover, either a larger matrix Z or a greater value of η (generating a lager $R_{\bar{v}}(t,s)$) will be liable for the growth of $\mathcal{H}(\eta,Z,\mathcal{K}(t,s))$.

For convenience, let us define the desirable filter gain with respect to the quantization level η as $\mathcal{K}_\eta^*(t,s)$, under which the minimum bounds on the error variances are set as $S_\eta^*(t,s)$ and $\Xi_\eta^*(t,s)$. According to (16), (17), and (21), the minimum bounds and the designed filter gain under the prescribed quantization level η can be calculated as:

$$\begin{aligned} \mathcal{K}_\eta^*(t,s)&=S_\eta^*(t,s)\bar{\mathcal{E}}_1^T(t,s)\hat{R}^{-1}(t,s)\\ S_\eta^*(t,s)&=\mathcal{G}(\eta,\Xi_\eta^*(t,s-1),\Xi_\eta^*(t-1,s))\\ \Xi_\eta^*(t,s)&=\mathcal{H}(\eta,S_\eta^*(t,s),\mathcal{K}_\eta^*(t,s)). \end{aligned}$$

Theorem 2: For given positive scalars η_1 and η_2, if $\eta_1\leq\eta_2$, then the following relationship holds

$$\Xi_{\eta_1}^*(t,s)\leq\Xi_{\eta_2}^*(t,s) \tag{27}$$

for all $t,s\in[0\ N]$ with initial conditions $\Xi_{\eta_1}^*(t,0)=\Xi_{\eta_2}^*(t,0)$ and $\Xi_{\eta_1}^*(0,s)=\Xi_{\eta_2}^*(0,s)$.

Proof: The proof of this theorem is conducted by the inductive approach. It follows directly from the initial conditions that (27) is true for $(t,s)\in\{(k,l)|k,l\in[0\ N];\ k+l=1\}$.

Suppose that (27) holds for $(t,s)\in\{(k,l)|k,l\in[1\ N];\ k+l=\theta\}$. Then, for $(t,s)\in\{(k,l)|k,l\in[1\ N];\ k+l=\theta+1\}$, it is easy to check that

$$S_{\eta_1}^*(t,s)=\mathcal{G}(\eta_1,\Xi_{\eta_1}^*(t,s-1),\Xi_{\eta_1}^*(t-1,s))$$

$$\leq \mathcal{G}(\eta_2, \Xi^*_{\eta_2}(t,s-1), \Xi^*_{\eta_2}(t-1,s)) \\ = S^*_{\eta_2}(t,s) \quad (28)$$

and this further indicates

$$\mathcal{H}(\eta_1, S^*_{\eta_1}(t,s), \mathcal{K}^*_{\eta_2}(t,s)) \leq \mathcal{H}(\eta_1, S^*_{\eta_2}(t,s), \mathcal{K}^*_{\eta_2}(t,s)) \\ \leq \mathcal{H}(\eta_2, S^*_{\eta_2}(t,s), \mathcal{K}^*_{\eta_2}(t,s)). \quad (29)$$

In addition, noting that $\mathcal{K}^*_{\eta_1}(t,s)$ ensures the minimal bound $\Xi^*_{\eta_1}(t,s)$, one has

$$\Xi^*_{\eta_1}(t,s) = \mathcal{H}(\eta_1, S^*_{\eta_1}(t,s), \mathcal{K}^*_{\eta_1}(t,s)) \\ \leq \mathcal{H}(\eta_1, S^*_{\eta_1}(t,s), \mathcal{K}^*_{\eta_2}(t,s)). \quad (30)$$

It follows from (28)-(30) that

$$\Xi^*_{\eta_1}(t,s) = \mathcal{H}(\eta_1, S^*_{\eta_1}(t,s), \mathcal{K}^*_{\eta_1}(t,s)) \\ \leq \mathcal{H}(\eta_2, S^*_{\eta_2}(t,s), \mathcal{K}^*_{\eta_2}(t,s)) \\ = \Xi^*_{\eta_2}(t,s)$$

for $(t,s) \in \{(k,l) | k,l \in [1 \ N]; \ k+l = \theta+1\}$. The proof of this theorem is now complete. ■

Remark 7: The monotonicity of the minimal bound regarding the quantization accuracy has been investigated in Theorem 2, where the decrease of η enables the tightening of the bound. In addition to the signal quantization, the sensor failure also plays an important role in guaranteeing a satisfactory filtering performance. Intuitively, the filter performs better with the increase of the coefficient $\bar{\gamma}(t,s)$ accounting for more available information. Owing to the complexity of the analytic expression of the minimal bound caused by the dynamical quantization, the monotonicity issue cannot be directly established for the sensor failure. To better characterize the influence of the sensor failure, a simplified case is considered here. Setting $D(t,s) = 0$ yields $\bar{y}(t,s) = E(t,s)y(t,s) + \Delta(t,s)$ from (4), namely, the quantizer becomes static rather than dynamical. In this case, taking the first variation to $\Xi(t,s)$ with regard to $\bar{\gamma}(t,s)$, the monotonicity of $\Xi(t,s)$ regarding $\bar{\gamma}(t,s)$ (specifically, $\Xi(t,s)$ is nondecreasing with the decline of $\bar{\gamma}(t,s)$) can be proved by some routine computations. The detailed proof is skipped here for the sake of brevity.

Next, the uniform boundedness of the minimum upper bound on the error variance will be analyzed. For this purpose, the following assumption is given.

Assumption 2: For $t,s \in [0 \ N]$ and $\ell = 1,2$, there are positive scalars $\bar{a}_\ell$, $\bar{\lambda}$, $\underline{b}_\ell$, $\bar{b}_\ell$, $\underline{f}_\ell$, $\bar{f}_\ell$, $\underline{e}_\ell$, $\bar{e}_\ell$, $\underline{q}$, $\bar{q}$, $\underline{r}$, $\bar{r}$, $\bar{c}_\ell$, and c that satisfy the following inequalities:

$$\bar{A}_\ell(t,s)\bar{A}^T_\ell(t,s) \leq \bar{a}_\ell I, \quad \bar{X}(t,s) \leq \bar{\lambda} I \\ \underline{b}_\ell I \leq B_\ell(t,s)B^T_\ell(t,s) \leq \bar{b}_\ell I, \quad \underline{q} I \leq Q(t,s) \leq \bar{q} I \\ \underline{f}_\ell I \leq \hat{F}_\ell(t,s)\hat{F}^T_\ell(t,s) \leq \bar{f}_\ell I, \quad \underline{r} I \leq R(t,s) \leq \bar{r} I \\ \underline{e}_1 I \leq \bar{\mathcal{E}}_1(t,s)\bar{\mathcal{E}}^T_1(t,s) \leq \bar{e}_1 I, \\ \underline{e}_2 I \leq \mathcal{E}_2(t,s)\mathcal{E}^T_2(t,s) \leq \bar{e}_2 I \\ \bar{E}_\ell(t,s)C(t,s)C^T(t,s)\bar{E}^T_\ell(t,s) \leq \bar{c}_\ell I \\ E(t,s)C(t,s)C^T(t,s)E^T(t,s) \leq cI.$$

Remark 8: Notice that Assumption 2 implies certain constraints for suppressing the amplitudes of system parameters and noise variances. This assumption is justified from the usual energy-bounded constraints in literally all practical applications. On the other hand, recalling that $\mathcal{E}_2(t,s)\mathcal{E}^T_2(t,s) = E(t,s)E^T(t,s) + I$, there must be a positive scalar $\underline{e}_2$ such that $\underline{e}_2 I \leq \mathcal{E}_2(t,s)\mathcal{E}^T_2(t,s)$. Moreover, the conditions $\underline{b}_\ell I \leq B_\ell(t,s)B^T_\ell(t,s)$ and $\underline{f}_\ell I \leq \hat{F}_\ell(t,s)\hat{F}^T_\ell(t,s)$ indicate that matrices $B_\ell(t,s)$ and $\hat{F}_\ell(t,s)$ are of full row rank, respectively. These conditions are fairly flexible yet general. Roughly speaking, if $\underline{b}_\ell I \leq B_\ell(t,s)B^T_\ell(t,s)$ is not satisfied, a full row rank matrix can be reset. Specifically, it follows from the full rank decomposition that $B_\ell(t,s)w(t,s) = B_{1\ell}(t,s)B_{2\ell}(t,s)w(t,s)$, where $B_{1\ell}(t,s)$ is a redefined matrix of full row rank and $B_{2\ell}(t,s)w(t,s)$ could be regarded as a new process noise.

Denote

$$\underline{r}_{\bar{v}} \triangleq \min\{\underline{r}, \eta^2/4\}, \quad \bar{r}_{\bar{v}} \triangleq \max\{\bar{r}, \eta^2/4\} \\ \breve{Q}_\ell(t,s) \triangleq \mathcal{B}_\ell(t,s)Q(t,s)\mathcal{B}^T_\ell(t,s) \\ + (1+\beta^{-1})\mathcal{F}_\ell(t,s)R_{\bar{v}}(t,s)\mathcal{F}^T_\ell(t,s).$$

Based on Assumption 2 and $R_{\bar{v}}(t,s)$ defined in Lemma 3, one has

$$\underline{r}_{\bar{v}} I \leq R_{\bar{v}}(t,s) \leq \bar{r}_{\bar{v}} I. \quad (31)$$

In addition, the expression of $\breve{Q}_\ell(t,s)$ infers

$$\breve{Q}_\ell(t,s) = \operatorname{diag}\{B_\ell(t,s)Q(t,s)B^T_\ell(t,s), \\ (1+\beta^{-1})\hat{F}_\ell(t,s)R_{\bar{v}}(t,s)\hat{F}^T_\ell(t,s)\} \\ \geq \begin{bmatrix} \underline{q}\underline{b}_\ell I & 0 \\ 0 & (1+\beta^{-1})\underline{r}_{\bar{v}}\underline{f}_\ell I \end{bmatrix} \\ \geq \min\left\{\underline{q}\underline{b}_\ell, (1+\beta^{-1})\underline{r}_{\bar{v}}\underline{f}_\ell\right\} I \triangleq \underline{\phi}_\ell I. \quad (32)$$

Similarly, it is straightforward to see that

$$\breve{Q}_\ell(t,s) \leq \begin{bmatrix} \bar{q}\bar{b}_\ell I & 0 \\ 0 & (1+\beta^{-1})\bar{r}_{\bar{v}}\bar{f}_\ell I \end{bmatrix} \\ \leq \max\{\bar{q}\bar{b}_\ell, (1+\beta^{-1})\bar{r}_{\bar{v}}\bar{f}_\ell\} I \triangleq \bar{\phi}_\ell I. \quad (33)$$

The following theorems demonstrate the boundedness property of the minimal bound $\Xi(t,s)$.

Theorem 3: Under Assumption 2, for all $t,s \in [0 \ N]$, the minimum matrix $\Xi(t,s)$ obeys the following inequality

$$\Xi(t,s) \geq \left[\left(\underline{\phi}_1 + \underline{\phi}_2\right)^{-1} + \underline{e}_2^{-1}\underline{r}_{\bar{v}}^{-1}\bar{e}_1\right]^{-1} I. \quad (34)$$

Proof: It is calculated from (16) and (32) that

$$S(t,s) \geq \breve{Q}_1(t,s-1) + \breve{Q}_2(t-1,s) \geq \underline{\phi}_1 I + \underline{\phi}_2 I$$

which infers

$$S^{-1}(t,s) \leq \left(\underline{\phi}_1 + \underline{\phi}_2\right)^{-1} I. \quad (35)$$

By resorting to Assumption 2 and (31), one has

$$\hat{\gamma}(t,s)\hat{E}(t,s)\bar{X}(t,s)\hat{E}^T(t,s) + \mathcal{E}_2(t,s)R_{\bar{v}}(t,s)\mathcal{E}^T_2(t,s) \\ \geq \mathcal{E}_2(t,s)R_{\bar{v}}(t,s)\mathcal{E}^T_2(t,s) \geq \underline{e}_2\underline{r}_{\bar{v}} I.$$

In the light of the matrix inversion lemma, expressions (20) and (22), and inequality (35), the following relationship is obtained:

$$\begin{aligned}\Xi^{-1}(t,s) =& S^{-1}(t,s) + \bar{\mathcal{E}}_1^T(t,s)\Big[\hat{\gamma}(t,s)\hat{E}(t,s)\bar{X}(t,s)\\ &\times \hat{E}^T(t,s) + \mathcal{E}_2(t,s)R_{\bar{v}}(t,s)\mathcal{E}_2^T(t,s)\Big]^{-1}\bar{\mathcal{E}}_1(t,s)\\ \le& \left(\underline{\phi}_1 + \underline{\phi}_2\right)^{-1} I + \underline{e}_2^{-1}\underline{r}_{\bar{v}}^{-1}\bar{\mathcal{E}}_1^T(t,s)\bar{\mathcal{E}}_1(t,s)\\ \le& \left[\left(\underline{\phi}_1 + \underline{\phi}_2\right)^{-1} + \underline{e}_2^{-1}\underline{r}_{\bar{v}}^{-1}\bar{e}_1\right] I\end{aligned}$$

which completes the proof. ■

Theorem 4: Under Assumption 2, for all $t,s \in [0\ N]$, the minimum matrix $\Xi(t,s)$ obeys the following inequality

$$\Xi(t,s) \le \xi(t,s)I \tag{36}$$

with initial constraints $\Xi(t,0) \le \xi(t,0)I$ and $\Xi(0,s) \le \xi(0,s)I$, where

$$\begin{aligned}\xi(t,s) =& \sum\nolimits_{k=1}^{t} \mu_1\rho(t-k,s-1)\xi(k,0)\\ &+ \sum\nolimits_{l=1}^{s} \mu_2\rho(t-1,s-l)\xi(0,l)\\ &+ \sum\nolimits_{k=0}^{t-1}\sum\nolimits_{l=0}^{s-1} \rho(t-k-1,s-l-1)\\ &\times\left(\beta_1(k+1,l) + \beta_2(k,l+1)\right)\end{aligned} \tag{37}$$

with

$$\mu_1 \triangleq (1+\mu)\bar{a}_1, \quad \mu_2 \triangleq \left(1+\mu^{-1}\right)\bar{a}_2, \quad \rho(0,0) \triangleq 1$$
$$\rho(0,s) \triangleq \mu_1\rho(0,s-1), \quad \rho(t,0) \triangleq \mu_2\rho(t-1,0)$$
$$\rho(t,s) \triangleq \mu_1\rho(t,s-1) + \mu_2\rho(t-1,s)$$
$$\beta_\ell(t,s) \triangleq \bar{\phi}_\ell + (1+\alpha^{-1})\bar{\lambda}\bar{c}_\ell\hat{\gamma}(t,s) + \bar{\xi}_\ell(t,s), \quad \ell = 1,2$$
$$\bar{\xi}_\ell(t,s) \triangleq \left(\alpha\bar{\lambda}c\hat{\gamma}(t,s) + \beta\bar{r}_{\bar{v}}\bar{e}_2\right)\bar{a}_\ell\bar{e}_1\underline{e}_1^{-2}\xi^2(t,s)\left(\underline{\phi}_1 + \underline{\phi}_2\right)^{-2}.$$

Proof: Based on the expressions of $\hat{E}_\ell(t,s)$ and $\hat{E}(t,s)$, it is obtained from Assumption 2 that

$$\begin{aligned}\hat{E}_\ell(t,s)\hat{E}_\ell^T(t,s) &= \mathrm{diag}\left\{0, \bar{E}_\ell(t,s)C(t,s)C^T(t,s)\bar{E}_\ell^T(t,s)\right\}\\ &\le \mathrm{diag}\left\{0, \bar{c}_\ell I\right\} \le \bar{c}_\ell I\end{aligned} \tag{38}$$

$$\hat{E}(t,s)\hat{E}^T(t,s) = E(t,s)C(t,s)C^T(t,s)E^T(t,s) \le cI. \tag{39}$$

Furthermore, we know from (22) and (35) that

$$\hat{R}(t,s) \ge \bar{\mathcal{E}}_1(t,s)S(t,s)\bar{\mathcal{E}}_1^T(t,s) \ge \underline{e}_1\left(\underline{\phi}_1 + \underline{\phi}_2\right)I.$$

Then, recalling the explicit formulation of the gain matrix $\mathcal{K}(t,s)$, we have following inequality

$$\begin{aligned}&\mathcal{K}^T(t,s)\mathcal{K}(t,s)\\ &= \hat{R}^{-1}(t,s)\bar{\mathcal{E}}_1(t,s)S^T(t,s)S(t,s)\bar{\mathcal{E}}_1^T(t,s)\hat{R}^{-1}(t,s)\\ &\le \bar{e}_1\lambda_{\max}^2\{S(t,s)\}\hat{R}^{-1}(t,s)\hat{R}^{-1}(t,s)\\ &\le \bar{e}_1\underline{e}_1^{-2}\left(\underline{\phi}_1 + \underline{\phi}_2\right)^{-2}\lambda_{\max}^2\{S(t,s)\}I.\end{aligned} \tag{40}$$

Moreover, considering the definitions of $\bar{Q}_\ell(t,s)$ and $\bar{R}_\ell(t,s)$ in Theorem 1, we have

$$\begin{aligned}&\bar{Q}_\ell(t,s) + \bar{R}_\ell(t,s)\\ &= \breve{Q}_\ell(t,s) + (1+\alpha^{-1})\hat{\gamma}(t,s)\hat{E}_\ell(t,s)\bar{X}(t,s)\hat{E}_\ell^T(t,s)\\ &\quad + \bar{\mathcal{A}}_\ell(t,s)\mathcal{K}(t,s)\Big[\alpha\hat{\gamma}(t,s)\hat{E}(t,s)\bar{X}(t,s)\hat{E}^T(t,s)\\ &\quad + \beta\mathcal{E}_2(t,s)R_{\bar{v}}(t,s)\mathcal{E}_2^T(t,s)\Big]\mathcal{K}^T(t,s)\bar{\mathcal{A}}_\ell^T(t,s)\\ &\le \bar{\phi}_\ell I + (1+\alpha^{-1})\bar{\lambda}\bar{c}_\ell\hat{\gamma}(t,s)I + \left(\alpha\bar{\lambda}c\hat{\gamma}(t,s) + \beta\bar{r}_{\bar{v}}\bar{e}_2\right)\\ &\quad \times \bar{a}_\ell\bar{e}_1\underline{e}_1^{-2}\left(\underline{\phi}_1 + \underline{\phi}_2\right)^{-2}\lambda_{\max}^2\{S(t,s)\}I\end{aligned} \tag{41}$$

for $t,s \ge 1$, where Assumption 2 and inequalities (31), (33), and (38)-(40) have been utilized in the above derivation. In addition, in the case of $t=0$ or $s=0$, the gain matrix can be set as $\mathcal{K}(t,s) = 0$, which results in

$$\begin{aligned}&\bar{Q}_\ell(t,s) + \bar{R}_\ell(t,s)\\ &= \breve{Q}_\ell(t,s) + (1+\alpha^{-1})\hat{\gamma}(t,s)\hat{E}_\ell(t,s)\bar{X}(t,s)\hat{E}_\ell^T(t,s)\\ &\le \bar{\phi}_\ell I + (1+\alpha^{-1})\bar{\lambda}\bar{c}_\ell\hat{\gamma}(t,s)I\end{aligned} \tag{42}$$

which means the validity of $\bar{Q}_\ell(t,s) + \bar{R}_\ell(t,s) \le \beta_\ell(t,s)I$ for $t=0$ or $s=0$. Thus, it is concluded from (41) and (42) that, if $S(t,s) \le \xi(t,s)I$, then

$$\bar{Q}_\ell(t,s) + \bar{R}_\ell(t,s) \le \beta_\ell(t,s)I \tag{43}$$

holds for all $t,s \in [0\ N]$.

Since $\Xi(t,s) \le S(t,s)$ is valid from (20), the inequality (36) can be confirmed if the assertion $S(t,s) \le \xi(t,s)I$ holds. In the following, this assertion is to be proven by the inductive method. For the initial step, setting $(t,s) = (1,1)$, we have from (16), (42), Assumption 2, and the initial constraints on $\Xi(t,s)$ that

$$\begin{aligned}S(1,1) \le& (1+\mu)\bar{\mathcal{A}}_1(1,0)\Xi(1,0)\bar{\mathcal{A}}_1^T(1,0) + \beta_1(1,0)I\\ &+ (1+\mu^{-1})\bar{\mathcal{A}}_2(0,1)\Xi(0,1)\bar{\mathcal{A}}_2^T(0,1) + \beta_2(0,1)I\\ \le& (1+\mu)\bar{a}_1\xi(1,0)I + (1+\mu^{-1})\bar{a}_2\xi(0,1)I\\ &+ \left(\beta_1(1,0) + \beta_2(0,1)\right)I\\ =& \xi(1,1)I.\end{aligned}$$

Assume that inequality $S(t,s) \le \xi(t,s)I$ is true when $(t,s) \in \{(k,l)|k,l \in [1\ N];\ k+l = \theta\}$ for a given integer θ. This assumption immediately guarantees the correctness of (36) and (43) for $(t,s) \in \{(k,l)|k,l \in [1\ N];\ k+l = \theta\}$. Then, when $(t,s) \in \{(k,l)|k,l \in [1\ N];\ k+l = \theta+1\}$, the following result is obtained from the following algebraic manipulations:

$$\begin{aligned}&S(t,s)\\ &\le (1+\mu)\bar{a}_1\xi(t,s-1)I + (1+\mu^{-1})\bar{a}_2\xi(t-1,s)I\\ &\quad + \left(\beta_1(t,s-1) + \beta_2(t-1,s)\right)I\\ &= \mu_1\Big[\sum\nolimits_{k=1}^{t} \mu_1\rho(t-k,s-2)\xi(k,0)\\ &\quad + \sum\nolimits_{l=1}^{s-1} \mu_2\rho(t-1,s-l-1)\xi(0,l)\\ &\quad + \sum\nolimits_{k=0}^{t-1}\sum\nolimits_{l=0}^{s-2} \rho(t-k-1,s-l-2)\\ &\quad \times\left(\beta_1(k+1,l) + \beta_2(k,l+1)\right)\Big]I\\ &\quad + \mu_2\Big[\sum\nolimits_{k=1}^{t-1} \mu_1\rho(t-k-1,s-1)\xi(k,0)\\ &\quad + \sum\nolimits_{l=1}^{s} \mu_2\rho(t-2,s-l)\xi(0,l)\end{aligned}$$

$$
\begin{aligned}
&+\sum_{k=0}^{t-2}\sum_{l=0}^{s-1}\rho(t-k-2,s-l-1)\\
&\times(\beta_1(k+1,l)+\beta_2(k,l+1))\Big]I\\
&+(\beta_1(t,s-1)+\beta_2(t-1,s))I\\
=\,&\mu_1\Big[\sum_{k=1}^{t-1}(\mu_1\rho(t-k,s-2)\\
&+\mu_2\rho(t-k-1,s-1))\xi(k,0)+\mu_1\rho(0,s-2)\xi(t,0)\Big]I\\
&+\mu_2\Big[\sum_{l=1}^{s-1}(\mu_2\rho(t-2,s-l)\\
&+\mu_1\rho(t-1,s-l-1))\xi(0,l)+\mu_2\rho(t-2,0)\xi(0,s)\Big]I\\
&+\Big[\mu_1\sum_{k=0}^{t-1}\sum_{l=0}^{s-2}\rho(t-k-1,s-l-2)\\
&+\mu_2\sum_{k=0}^{t-2}\sum_{l=0}^{s-1}\rho(t-k-2,s-l-1)\Big]\\
&\times(\beta_1(k+1,l)+\beta_2(k,l+1))\,I\\
&+(\beta_1(t,s-1)+\beta_2(t-1,s))I\\
=\,&\mu_1\Big[\sum_{k=1}^{t-1}\rho(t-k,s-1)\xi(k,0)+\rho(0,s-1)\xi(t,0)\Big]I\\
&+\mu_2\Big[\sum_{l=1}^{s-1}\rho(t-1,s-l)\xi(0,l)+\rho(t-1,0)\xi(0,s)\Big]I\\
&+\sum_{k=0}^{t-2}\sum_{l=0}^{s-2}\rho(t-k-1,s-l-1)\\
&\times(\beta_1(k+1,l)+\beta_2(k,l+1))\,I\\
&+\sum_{k=0}^{t-2}\rho(t-k-1,0)\,(\beta_1(k+1,s-1)+\beta_2(k,s))\,I\\
&+\sum_{l=0}^{s-2}\rho(0,s-l-1)\,(\beta_1(t,l)+\beta_2(t-1,l+1))\,I\\
&+(\beta_1(t,s-1)+\beta_2(t-1,s))I\\
=\,&\sum_{k=1}^{t}\mu_1\rho(t-k,s-1)\xi(k,0)I\\
&+\sum_{l=1}^{s}\mu_2\rho(t-1,s-l)\xi(0,l)I\\
&+\sum_{k=0}^{t-1}\sum_{l=0}^{s-1}\rho(t-k-1,s-l-1)\\
&\times(\beta_1(k+1,l)+\beta_2(k,l+1))\,I=\xi(t,s)I.
\end{aligned}
$$

Therefore, one has $S(t,s)\le\xi(t,s)I$ for all $t,s\in[1\ N]$ and thus the validity of (36) is ensured, and this ends the proof. ■

Remark 9: The evaluation of the filtering performance has been presented in this subsection. To be exact, Theorem 2 looks into the monotonicity of the optimal bound concerning the quantization accuracy, which is in conformity with the engineering practice. Theorems 3-4 show the boundedness of the matrix $\Xi(t,s)$ at each iteration under Assumption 2. The obtained bounds rely on all the factors including the initial constraints, the noise information, the amplitudes of the system parameters, the statistics of the sensor failure, and the quantization level.

Remark 10: Till now, the recursive filtering problem has been systematically studied for 2-D systems with dynamic quantization and sensor failure. An augmented state has been constructed, which embraces and jointly estimates the states of the quantizer and the original system. In contrast to the existing literature, this article exhibits the following distinctive features: 1) a novel dynamic quantization with impressive flexibility is developed for 2-D systems; 2) a new filtering scheme is proposed to withstand the quantization error and the sensor failure with guaranteed filtering performance; and 3) the filtering performance is evaluated with respect to the boundedness and monotonicity issues of the minimum bound.

IV. Numerical Example

In this section, validity of the proposed filtering scheme is examined via a simulation example.

Consider the 2-D system (1) defined over a finite horizon $t,s\in[0\ 60]$ with the following parameters:

$$
\begin{aligned}
&A_1(t,s)=\begin{bmatrix}0.75 & 0.1\cos(t)\\ 0.1 & 0.3+0.1\sin(s)\end{bmatrix}\\
&A_2(t,s)=\begin{bmatrix}0.3 & 0\\ 0 & 0.4-0.1\sin(s)\end{bmatrix}\\
&B_1(t,s)=\begin{bmatrix}0.2-0.1\cos(2t)\\ 0.15e^{-s}\end{bmatrix},\quad B_2(t,s)=\begin{bmatrix}0.1\\ 0.1e^{-2t}\end{bmatrix}\\
&C(t,s)=\begin{bmatrix}-1 & 1+0.3\sin(5(t+s))\end{bmatrix}\\
&D_1(t,s)=0.4-0.15\sin(t),\quad D_2(t,s)=0.25+0.1\cos(2s)\\
&E_1(t,s)=0.5,\quad E_2(t,s)=-0.5,\quad F_2(t,s)=0.1\\
&F_1(t,s)=0.2\sin(t)\cos(s),\quad D(t,s)=E(t,s)=1.
\end{aligned}
$$

The noises $w(t,s)$ and $v(t,s)$ are Gaussian white sequences with respective variances $Q(t,s)=0.16$ and $R(t,s)=0.25$. The random variable $\gamma(t,s)$ is chosen to satisfy the Bernoulli distribution with $\bar{\gamma}(t,s)=0.9$ and $\hat{\gamma}(t,s)=0.09$. The quantization level is given as $\eta=0.1$, and the scaling parameters are taken as $\varsigma=\mu=0.5$ and $\alpha=\beta=1$. In this simulation, we assume that the initial states of system (1) are random vectors whose components obey the uniform distribution over the interval $[-0.3,0.3]$, and thus the expectations and variances of the initial states are calculated as $\mathbf{x}_1(t)=\mathbf{x}_2(s)=[0\ \ 0]^T$ and $P_u(t,0)=P_u(0,s)=0.03I$.

The estimate error is of interest. For $\ell=1,2$, the ℓ-th component of $e_u(t,s)$ is denoted as $e_\ell(t,s)$. Further define

$$\bar{\Xi}_\eta(t,s)\triangleq\operatorname{tr}\left\{[I_{n_x}\ 0_{n_\psi}]\Xi(t,s)[I_{n_x}\ 0_{n_\psi}]^T\right\}$$

under a given quantization level η. As such, $\bar{\Xi}_\eta(t,s)$ indicates the trace of the minimum upper bound on the error variance $[I_{n_x}\ 0_{n_\psi}]P_u(t,s)[I_{n_x}\ 0_{n_\psi}]^T$, which is regarded as an index of the filtering performance.

According to the theoretical results, the filtering algorithm can be recursively carried out to solve the addressed problem, and the corresponding simulation results are presented in Figs. 1–3. Specifically, Figs. 1–2 plot the filtering error trajectories of $e_1(t,s)$ and $e_2(t,s)$, and Fig. 3 depicts the evolution of $\bar{\Xi}_\eta(t,s)$. It is concluded from Figs. 1–3 that the estimation states draw close to their real ones, namely, the proposed filter performs quite well.

In the sequel, we consider different cases to quantitatively demonstrate effects of the quantization accuracy, the RSF, and the process noise.

Case 1: In this case, the influence of the quantization accuracy is presented. Let us reset the quantization level as $\eta_1=0.5$ and $\eta_2=1$, while remaining all the other parameters. The corresponding performance indexes are denoted by $\bar{\Xi}_{\eta_1}(t,s)$ and $\bar{\Xi}_{\eta_2}(t,s)$, respectively. By applying Theorem 1, the relating local minimum upper bounds and the filter gains

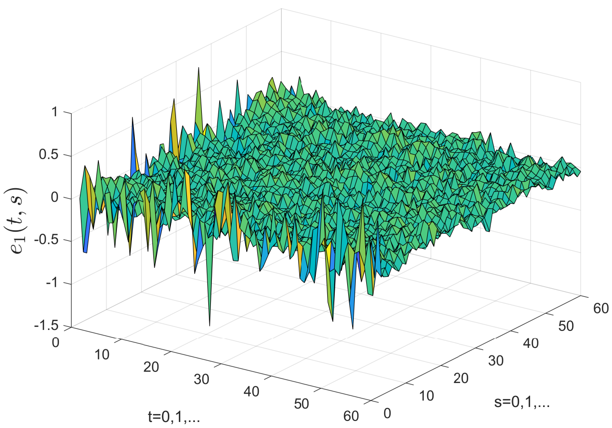

Fig. 1. Trajectory of the filtering error $e_1(t, s)$.

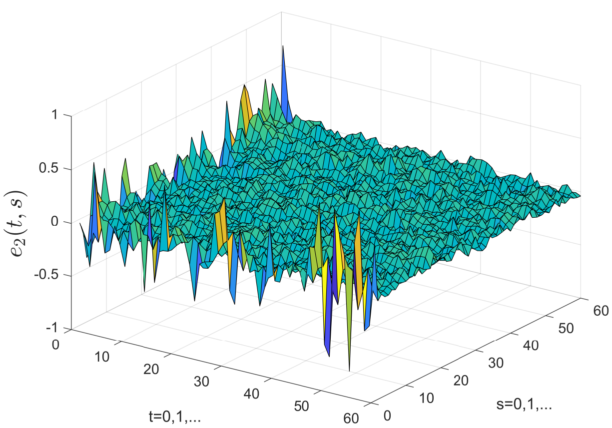

Fig. 2. Trajectory of the filtering error $e_2(t, s)$.

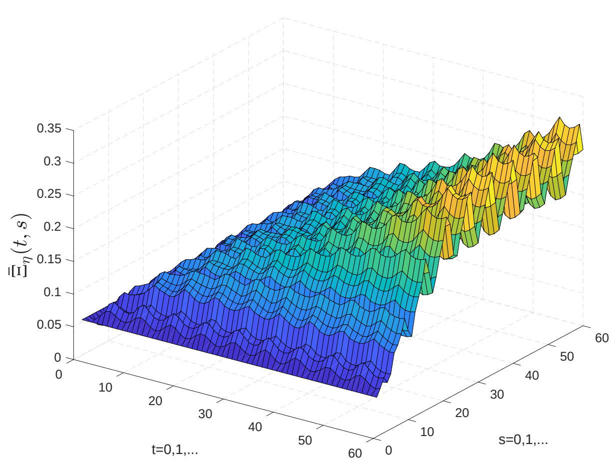

Fig. 3. Trajectory of $\bar{\Xi}_\eta(t, s)$ with $\eta = 0.1$ and $\bar{\gamma}(t, s) = 0.9$.

can also be obtained. The simulation results are shown in Figs. 4–5, where differences of the performance indexes are displayed between different quantization levels. It is easy to see that the increase of the quantization level amplifies the minimum upper bound, which is in accord with the theoretical result.

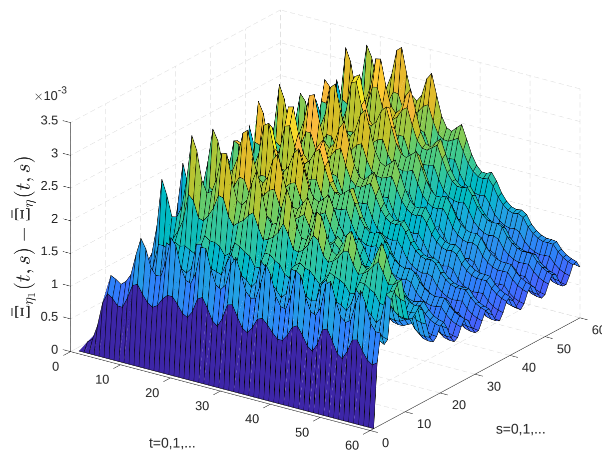

Fig. 4. Trajectory of $\bar{\Xi}_{\eta_1}(t, s) - \bar{\Xi}_\eta(t, s)$.

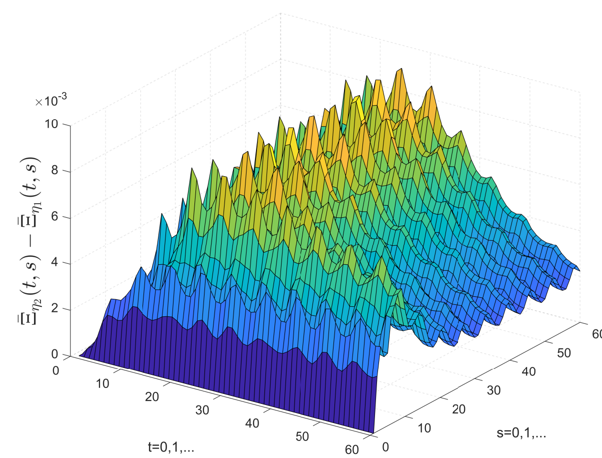

Fig. 5. Trajectory of $\bar{\Xi}_{\eta_2}(t, s) - \bar{\Xi}_{\eta_1}(t, s)$.

Case 2: This case discusses the effect of the RSF on the filtering performance. The value of the RSF coefficient $\bar{\gamma}(t, s)$ is selected as 0.9 and 0.5 in two different scenarios. The simulation results are given in Figs. 3 and 6, which plot the trajectories of the performance index $\bar{\Xi}_\eta(t, s)$ under different $\bar{\gamma}(t, s)$. Obviously, it is witnessed from Figs. 3 and 6 that the increase in the occurrence probability of the sensor failure degrades the filtering performance.

Case 3: The impact of the noise intensity is examined in this case. The noise variance is reset as $Q(t, s) = 0.36$, and the corresponding simulation result is depicted in Fig. 7. A comparison of Figs. 3 and 7 implies that a larger noise variance leads to a worse filtering performance.

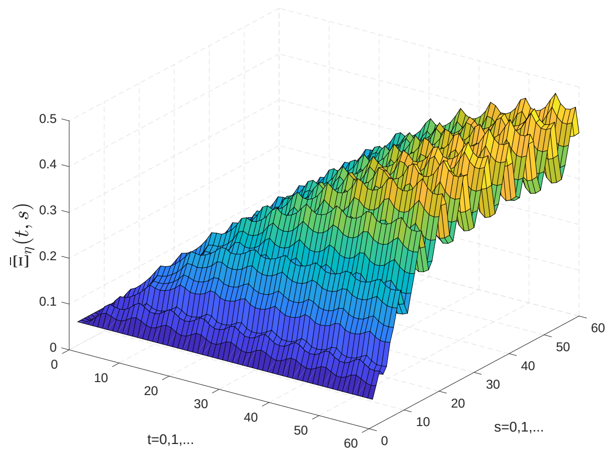

Fig. 6. Trajectory of $\bar{\Xi}_\eta(t,s)$ with $\bar{\gamma}(t,s) = 0.5$.

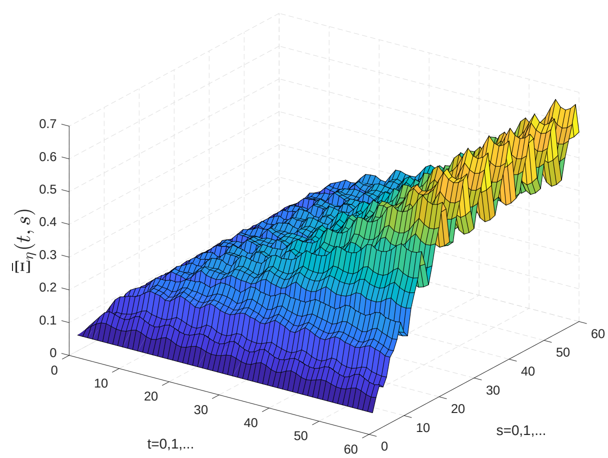

Fig. 7. Trajectory of $\bar{\Xi}_\eta(t,s)$ with $Q(t,s) = 0.36$.

V. Conclusions

In this article, a recursive filtering strategy has been developed for the 2-D system with inaccurate measurements. The measured outputs are subject to the sensor failure modeled by independent and identically distributed random variables with known statistics. The received measurements also undergo quantization effects by exploiting a novel dynamic quantizer. In virtue of the induction and matrix analysis techniques, sufficient criteria have been established to acquire the feasible upper bounds on the error variances and determine the desired recursive filter that ensures the minimal upper bounds. Afterwards, impacts of the quantization accuracy and the sensor failure coefficient on the filtering performance have been expounded. The boundedness of the minimal upper bound has been further illustrated under some mild conditions. Finally, numerical results have been presented to exemplify the validity of the proposed filtering scheme. Our future research topics would include some potential extensions of the proposed methods to cope with 1) the filtering issue for systems with other communication constraints such as coded measurements or cyber-attacks, and 2) the state estimation for more complicated systems including 2-D systems over sensor networks.

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