Investigation into the Multi-Stage Mechanical Damage Behavior of Columnar Jointed Basalts with Different Meso-Constitutive Relations and Model Sizes

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Abstract: The mechanical characteristics of columnar jointed basalts (CJBs) are generally considered to be subject to size effects. They display complex mechanical behaviors under lateral pressure, which may be related to the variation in the rock meso properties. In this study, many nonuniform CJB models with various rock meso-constitutive relations and model sizes were established, and their influences on the strength and deformation properties, multi-stage mechanical damage behavior, and acoustic emission characteristics of CJBs were investigated. The results show that, as the residual strength coefficient increases, the compressive strength of CJBs rises and the equivalent deformation modulus of CJBs decreases or increases slightly; with an increase in the of model size, the compressive strength slightly decreases at first and then varies moderately, and the equivalent deformation modulus shows a fluctuating trend. Furthermore, the macro stress–strain relationships can be divided into different stages according to the corresponding damage behaviors. The strains and energy accumulations before peak stress was reached were further analyzed. The results greatly improve our understanding of the collapse process of CJBs and the instability precursors of related structures.

Keywords: columnar jointed basalts; damage behavior; acoustic emission; rock meso-constitutive relation; size effect; numerical simulation

1. Introduction

Columnar jointed basalts (CJBs) are rock masses with regular fractures produced through magma cooling. They are distributed widely around the world and can be found in Northern Ireland, Scotland, Brazil, China, Siberia, Mexico, India, Australia, the United States, etc. [1–5]. They are characterized by regular hexagonal prisms or other irregular prisms, and the mechanical properties are generally considered to be affected by size effects [6]. Additionally, they display various strength or deformation characteristics with or without lateral pressure, which may be related to the variation in the rock meso properties. In recent decades, CJBs have been encountered in numerous engineering projects in China, including the Hunan Town, Guandi, Xiluodu, Tongjiezi, Ertan, Baihetan, Wudongde, Jinanqiao hydropower stations and so on [7-10]. Some field photographs [11-12] of CJBs are shown in Figure 1.
Many scholars [8,13-15] have investigated the anisotropy, size effect, and lateral pressure influence of CJBs. Although valuable results have been obtained in field tests involving CJBs, the environments of engineered rock structures in field tests are often affected by many factors, and rock specimens are affected by disturbances. The creep characteristics of columnar jointed rock masses (CJRMs) in the dam zone were studied by Wei et al. [13], but the effects of rock meso property variation were difficult to identify when using their research method. Furthermore, due to limitations in terms of experimental equipment and techniques, it is very difficult to carry out large sample tests. For CJBs at the Baihetan Hydropower Station, a drilling experiment was conducted in the excavation damage zone (EDZ) as well as in undisturbed rocks by Sun et al. [14]. Nevertheless, the samples were susceptible to borehole disturbance. Based on an in situ, large-scale blasting test, the release laws of the acoustic emissions (AEs) were analyzed by Xiang et al. [8], and the unloading processes and relaxation mechanisms of CJBs were revealed. Zhang et al. [4] performed a series of field tests to investigate the EDZ of CJRMs that involved monitoring ultrasonic P waves. Jiang et al. [15] conducted a crack observation and time-dependent failure experiment on CJRMs, but in their research the damage evolution at different stages was difficult to comprehensively display. Dou et al. [2] carried out a field test to investigate consolidation grouting in the CJB foundation of dams. Nevertheless, in the above studies the rock mass occurrence environment was too complex for in situ tests, and it is difficult to take the influence of rock meso properties into account in systematic research.

![Figure 1. The observed CJBs in the field: (a) Baihetan hydroelectric station, China [11]; (b) Giant’s Causeway, Northern Ireland [12]](image)

In physical laboratory tests on CJBs (or CJRMs), some useful results reflecting their mechanical properties have been obtained [1,11-12]. However, there are still difficulties when considering the mechanical parameters of actual joint surfaces in physical laboratory tests. Additionally, when there are many experimental scheme configurations and specimens, these tests can become time-consuming and uneconomical. Ji et al. [11] performed conventional triaxial compression tests to investigate the anisotropic strength and deformation behaviors of CJRMs in a confined state, but the effects of variations in rock meso properties under confining pressure remain unclear. Zhang et al. [16]
conducted a cyclic triaxial loading–unloading experiment to reveal the anisotropic behavior of CJRMs. However, due to the limitations of the laboratory equipment, the size effect of specimens was not studied fully. Uniaxial and triaxial compression experiments were carried out by Lu et al. [12] to determine the fracture mechanisms of CJRMs. Nevertheless, the changes in the rock meso properties during the experiment were generally difficult to observe in their research. The failure mechanism, stress–strain curve, and peak strengths of CJRMs were investigated by Zhu et al. [17]. However, the progressive damage behaviors of specimens were not analyzed in their research. Laboratory triaxial seepage tests were conducted by He et al. [7,18] to clarify the seepage features of CJRMs under different stress states. Xu et al. [1] carried out unloading experiments on similar material specimens, and acoustic wave data were monitored. In the above studies, the influence of size effect was not considered properly, and the damage evolution of CJRMs with different rock meso properties under lateral pressure is not yet fully understood.

In the numerical simulation of CJRMs (or CJRMs), insightful findings have been obtained [4,19-20]. However, there are few systematic studies on the effects of model size, rock meso-constitutive relations, and model boundaries on the damage behavior of CJRMs. Meng et al. [6] conducted a uniform simulation to reveal the influence of columnar joint set on the mechanical parameters of rocks, but the damage process of CJRMs was not considered. On the basis of the three-dimensional (3D) finite difference method, an elastoplastic numerical model of CJRMs was proposed by Yu et al. [19], and the excavation-induced relaxation zones of the diversion tunnel were reproduced. However, the size effect of CJRMs was not taken into account in their research. Wang et al. [21] performed a series of simulations involving the cracking mechanisms and energy release laws of CJRMs using digital image correlation (DIC) and the rock failure process analysis method (RFPA). Nevertheless, the influence of rock meso-constitutive relations on the damage characteristics of CJRMs under lateral pressure remains unclear. A 3D numerical analysis was conducted by Zhang et al. [4] to analyze the unloading-induced damage mechanisms of CJRMs. Additionally, a numerical simulation was performed by Zhang et al. [22] to investigate the mechanism of stress–structural collapse in CJRMs under high stress environments. Li et al. [23] investigated the transient thermoelastic fracture problems of rocks by a high-accuracy numerical manifold method. In the above studies, the damage evolution of CJRMs affected by rock meso properties and model sizes were not systematically studied. Cheng et al. [3] investigated the flow and permeability properties of irregular CJRMs numerically. However, the above studies did not comprehensively consider the influence of the sizes of established numerical models, rock meso-constitutive relations, and model boundaries on the damage processes and AEs energy evolutions of CJRs.

In this paper, to analyze the multi-stage mechanical damage behavior of CJBs with different rock meso-constitutive relations and model sizes under lateral pressure, continuum mechanics and damage mechanics were applied, digital images of CJB specimens were transformed into corresponding heterogeneous models by DIC, and a series of simulations were conducted. Then, the numerical tests were compared to the physical tests to verify the rationality and reliability of our results. Additionally, the mechanical damage evolutions of CJBs under lateral pressure were reproduced. The influence of rock meso-constitutive relations, model sizes, and model boundaries on the multi-stage mechanical damage behavior of CJBs was also investigated.

2. Method

2.1. Basic principles
The main advantages of the RFPA approach are that there are no assumptions about the location where new fractures will initiate and the way in which the newly formed or existing fractures will develop [24-26]. The effectiveness of this method has been verified by many benchmark studies [27-29]. This approach has been used to assess slope safety, tunnel stability, and the mechanical properties of rocks in many studies [30-33]. Thus, it is an effective method for modelling rock mechanics. In order to build up the computable numerical model, a digital image consisting of square pixels is transformed into the necessary node coordinates and element materials by DIC. Based on gray threshold segmentation, each element corresponding to a pixel is assigned joint or rock material properties. According to these principles, the schematic diagram for transforming a digital image into a heterogeneous numerical model is presented in Figure 2a. At the same time, the stress–strain relationship of elements subject to a uniaxial stress state is depicted in Figure 2b. According to Mazars and Pijaudier-Cabot [34], the constitutive relation under uniaxial stress can be extended to a 3D stress state.

![Figure 2](image)

**Figure 2.** (a) The process of transforming a digital image into a computable heterogeneous model; (b) the stress/strain relation of an element subject to uniaxial stress (Note: $m$ is the heterogeneity coefficient [24])

### 2.2. Benchmark

The experimental test by Ke *et al.* [35] was adopted to examine the established computational method. They made columns by mixing cement, fine sand, water, and water reducer at a mass ratio of 1.000:0.500:0.350:0.002.
hexagonal prism with a section diameter and length of 10 mm and 50 mm, respectively, was selected to simulate the actual column. White cement slurry with a water–cement ratio of 0.400:1.000 was used to bond columns, which simulated the joint surface. The rock mass specimens were regular 50 mm×50 mm quadrangular prism samples with a height of 100 mm. The experiment was carried out using a CSS-3940YJ rock mechanics servo testing machine. A loading method with a constant displacement rate of 0.05 mm/min was adopted until the failure of the specimens.

The plane strain assumption was adopted. The size of the rectangular specimens was 50 mm×50 mm×100 mm, and the diameter of the hexagonal prisms within the samples was 10 mm. The direction I orthogonal to the column axis was considered. The finite element model was established by processing the digital image by DIC. Namely, the identification of joints and matrices was conducted by DIC. The joints were simulated by the elements with relatively low strength and elastic modulus. After recognizing joint elements and matrix elements using the threshold segmentation, the corresponding material parameters were assigned, respectively. The physico-mechanical parameter values from the literature [1,6,13,15-16] are presented in Table 1.

<table>
<thead>
<tr>
<th>Material type</th>
<th>Heterogeneity index</th>
<th>Elastic modulus (GPa)</th>
<th>Uniaxial compressive strength (MPa)</th>
<th>Poisson ratio</th>
<th>Friction angle (°)</th>
<th>Ratio of compression to tension</th>
<th>Coefficient of residual strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>5</td>
<td>60</td>
<td>120</td>
<td>0.2</td>
<td>56.15</td>
<td>10</td>
<td>0.1</td>
</tr>
<tr>
<td>Joint</td>
<td>5</td>
<td>15</td>
<td>30</td>
<td>0.25</td>
<td>36</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 3. Comparison between numerical and experimental results: (a) the numerical x-direction displacement contour; (b) the numerical AEs diagram; (c) the experimental failure mode [35]

Because of the material heterogeneity, the elastic moduli or strengths of adjacent elements are not equal, and their distribution was assumed to obey the Weibull distribution [36] in this study. Through this way, the heterogeneity of rock mass was considered. The shape parameter of the Weibull distribution is termed the heterogeneity index. The larger the homogeneity index is, the more homogeneous the rock material is, and vice versa. A displacement load ratio of 0.005 mm/step was applied to the top surface of the samples, and the bottom was fixed along the normal direction. The numerical results were compared to the results of the physical experiments, as displayed in Figure 3. Note that the blue dots in the AEs diagrams of the numerical tests represent the tensile failures, and the red dots...
represent the compressive failures. It can be seen from Figure 3 that the numerical results agree with the experimental results satisfactorily.

2.3. Model Configuration

In terms of model size, seven cases were considered, i.e., 0.5 m×0.5 m, 1.0 m×1.0 m, 2.0 m×2.0 m, 3.0 m×3.0 m, 4.0 m×4.0 m, 6.0 m×6.0 m, and 8.0 m×8.0 m. The column diameter inside the specimen was 20 cm. The size of the inner finite elements kept the same. For instance, the element number of the 8 m×8 m sample was 4,326,400. The residual strength coefficient (RSC) of the CJBs was set to 0.1, 0.5, and 1 sequentially, as can be seen in Figure 4.

Figure 4. Rock meso-constitutive relations with different residual strength coefficients

Figure 5. Model setting under lateral pressure: (a) direction I of the digital image; (b) the finite element model along direction II; (c) the plane strain case; (d) the case between plane stress and plane strain

The other physical and mechanical parameters of the rock and joint are the same as those listed in Table 1. In terms of the non-uniformity of the mechanical properties of the rock and joint, it is assumed that they obey the given Weibull distribution, and the heterogeneity index is taken to be 5. Figure 2a shows a locally enlarged diagram of a
specimen. Figure 5 presents the typical setting and boundary conditions of a CJB specimen under lateral pressure. Note that directions I and II are considered. For the plane strain case, the displacement of the model plane is constrained to 0 m along the positive and negative normal directions, in the same manner as the internal surrounding rocks along a tunnel axis, as can be seen in Figure 5d. For the case between plane stress and plane strain, the displacement of the model plane is constrained to 0 m along the positive normal direction, in the same manner as rocks near a tunnel wall, as can be seen in Figure 5e. The principle of transforming a digital image of CJSs into a finite element mesh can be seen in Section 2.1. The lateral pressures of 4 MPa and 8 MPa are considered. Displacement-controlled loading is applied to the upper surface of the model vertically until the macro failure of the sample.

3. Result and Analysis

3.1. The effects of rock meso-constitutive relations and model size under lateral pressure

3.1.1. The plane strain case

(1) Compressive strength (CS)

Figure 6. Compressive strengths of the CJBs with different rock meso-constitutive relations and model sizes under the plane strain case: (a)–(c) direction I when the lateral pressures = 4 MPa and 8 MPa; (d)–(f) directions I and II when the lateral pressure = 8 MPa
From Figure 6, it can be seen that when the RSC = 1 and the lateral pressure = 4 MPa, the critical value of size effect for the CS of CJBs is 4 m, while for the other cases the critical value of size effect for the CS of CJBs is around 6 m. Meanwhile, the CS of CJBs grows with an increase in the rock RSC, and the CS of CJBs increases significantly when the lateral pressure rises from 4 MPa to 8 MPa. In particular, for CJBs with model sizes of 0.5–4 m, the growth ratio of the CS of CJBs is in line with the growth in lateral pressure.

Figure 6d–f shows the CSs of the CJBs with different rock RSCs and model sizes, from which it can be seen that with a growth in model size, the CS slightly decreases firstly and then changes moderately. At the same time, the critical value of size effect for the CS of CJBs is about 6 m, and the CS of CJBs rises with a growth in the rock RSC. Additionally, except for individual cases, the CS of CJBs along direction II is higher than that along direction I.

(2) Equivalent deformation modulus (EDM)

**Figure 7.** Equivalent deformation moduli of the CJBs with different rock meso-constitutive relations and model sizes under the plane strain case: (a)–(c) direction I when the lateral pressures = 4 MPa and 8 MPa; (d)–(f) directions I and II when the lateral pressure = 8 MPa.
Figure 7a–c shows the EDMs of the CJBs with different RSCs (i.e., 0.1, 0.5, and 1.0) and model sizes. From the perspective of model size, the EDM fluctuates slightly with a growth in the model size. At the same time, with an increase in the RSC, the EDM of CJBs decreases slightly. In terms of lateral pressure, the EDM of CJBs grows slightly with an increase in lateral pressure.

From Figures 7d–f, we can see that with an increase in model size the EDM fluctuates, but the variation range is relatively small. For instance, for the CJBs with a rock RSC of 0.1 along direction I perpendicular to the column axis, the ratio of the maximum value to the minimum value of the EDMs is 1.068; for the CJBs with a rock RSC of 1.0 along direction II perpendicular to the column axis, the ratio of the maximum value to the minimum value of the EDMs is 1.161. Furthermore, with a growth in the rock RSC, the EDMs of CJBs change slightly, and except for a few special cases the EDM along direction II perpendicular to the column axis is slightly larger than that along direction I.

3.1.2. Model boundaries

Figure 8. For the CJBs with different boundary conditions and model sizes: (a), (b) the CSs of the CJBs along directions I and II; (c), (d) the EDMs of the CJBs along directions I and II.
In Figure 8, case I represents the case between plane stress and plane strain and case II represents the plane strain case. As can be seen in Figure 8a, along direction I perpendicular to the column axis, with an increase in the model size, the CS of CJBs drops at first and then changes moderately, and the whole variation range is relatively small for the case between plane stress and plane strain. Furthermore, the ratio of the maximum CS to the minimum CS is 1.076. By contrast, the CS of CJBs decreases at first and then varies moderately, and the whole variation range is relatively large for the plane strain case. At the same time, the ratio of the maximum CS to the minimum CS is 1.729. From Figure 8b, it can be seen that the variation trend of CS with the growth of the model size is similar along direction I and direction II perpendicular to the column axis. At the same time, for the plane strain case, the ratio of the maximum value to the minimum value of CSs is 1.407. Additionally, as can be seen in Figure 8a,b, the CSs of CJBs for the plane strain case are relatively high in comparison with the case between plane stress and plane strain. When the model size is 0.5–4 m, the CS difference between the two kinds of model boundaries is especially obvious.

As can be seen in Figure 8c, for direction I, the EDM of CJBs shows the trend of first growth, slow change, second growth, and then gentle variation for the case between plane stress and plane strain. Furthermore, the ratio of the maximum EDM to the minimum EDM is 1.141. For the plane strain case, the EDM of CJBs displays a fluctuating trend, and the ratio of the maximum EDM to the minimum EDM is 1.107. As is displayed in Figure 8d, the EDM of CJBs fluctuates as the model size rises up for direction II perpendicular to the column axis and the two boundary conditions. The ratio of the maximum value to the minimum value of EDMs is 1.035 for the case between plane stress and plane strain. Furthermore, the ratio of the maximum value to the minimum value of EDMs is 1.056 for the plane strain case. Moreover, as is depicted in Figure 8c–d, the EDMs of CJBs for the plane strain case are larger than in the case between plane stress and plane strain.

### 3.2. The multi-stage mechanical and damage behavior and energy evolution of the CJBs

#### 3.2.1. The plane strain case

1. Meso-constitutive relation

![Figure 9. Different stages of the stress–strain curve of CJBs.](image)

In Figure 9, stage I, II, and III are the stress growth stage, the stress drop stage, and the residual strength stage, respectively. Figure 10 presents the various mechanical stages of the CJBs with different rock meso-constitutive
relations (i.e., the RSCs = 0.1, 0.5, and 1.0) and model sizes of 4 m and 8 m. As can be seen in Figure 10a,b, for the CJBs with a model size of 4 m and RSC of 0.1 along direction I when the lateral pressure = 4 MPa and 8 MPa, stages I and II are obvious; when the RSC is 0.5 or 1.0, stages I, II, and III are all obvious on the stress–strain curves. As can be seen in Figure 10c,d, for the CJBs with a model size of 4 m and RSCs of 0.1, 0.5, and 1.0 along direction II when the lateral pressure = 4 MPa, stages I, II, and III are reproduced on the stress–strain curves. When the lateral pressure = 8 MPa and the RSC of rock is 0.1 or 1.0, stages I and II are obvious on the stress–strain curves; when the RSC of rock is 0.5, stage I, stage II, and a short stage III can be observed before the final failure.

![Figure 10](image)

**Figure 10.** The stress–strain relationships of the CJBs under plane strain: (a), (b) for the CJBs with a model size of 4 m along direction I under the lateral pressures of 4 MPa and 8 MPa; (c), (d) for the CJBs with a model size of 4 m along direction II; (e), (f) for the CJBs with a model size of 8 m along direction II

As can be seen in Figure 10e,f, for the CJBs with a model size of 8 m and RSC of 0.1 along direction II when the lateral pressure = 4 MPa, there are only stages I and II on the stress–strain curve; when the RSC is 0.5 or 1.0, there are stage I, stage II, and a short stage III on the stress–strain curves. When the lateral pressure is set to 8 MPa and the RSC is 0.1 or 0.5, there are only stages I and II on the stress–strain curves; when the RSC is 1.0, there are stages I, II, and III on the stress–strain curve.

Table 2 displays the multi-stage damage behaviors of the CJBs with a model size of 4 m and RSCs of 0.1, 0.5, and 1.0 along directions I and II perpendicular to the column axis when the lateral pressure = 8 MPa under plane strain. When the rock RSC = 0.1 along direction I, during stage I, the vertical joints inside the specimen are damaged; during stage II, the columns at the upper part of the sample are damaged and broken. When the rock RSC = 0.5,
during stages II and III, an M-type damaged area gradually occurs at the upper part of the specimen. When the rock RSC = 1.0, during stages II and III, the damaged areas at the upper part of the sample show strip characteristics.

Table 2. The multi-stage damage behaviors of the CJBs with a model size of 4 m and different rock meso-constitutive relations along directions I and II when the lateral pressure = 8 MPa under plane strain

<table>
<thead>
<tr>
<th>Stage</th>
<th>Stage I</th>
<th>Stage II</th>
<th>Stage III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direction I, RSC = 0.1</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
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<tr>
<td>Direction I, RSC = 0.5</td>
<td><img src="image4" alt="Image" /></td>
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</tr>
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<td><img src="image8" alt="Image" /></td>
<td><img src="image9" alt="Image" /></td>
</tr>
<tr>
<td>Direction II, RSC = 0.1</td>
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<td><img src="image11" alt="Image" /></td>
<td><img src="image12" alt="Image" /></td>
</tr>
<tr>
<td>Direction II, RSC = 0.5</td>
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<td><img src="image14" alt="Image" /></td>
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<tr>
<td>Direction II, RSC = 1.0</td>
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<td><img src="image17" alt="Image" /></td>
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</tbody>
</table>

As can be seen in Table 2, for direction II when the RSC is 0.1, during stage I, the oblique joints inside the specimen are damaged, and some vertical joints near the top of the specimen are damaged and broken to a certain extent; during stage II, a V-shaped damaged zone forms at the upper middle part of the sample. Furthermore, damaged areas also appear at both the upper left and upper right parts of the sample. When the rock RSC = 0.5, the damage evolution of the specimen is similar to the specimen with a rock RSC of 0.1. However, the damage evolution of the former continues to stage III, while the damage evolution of the latter only develops to stage II. When the rock RSC = 1.0, during stage I, damage develops at the oblique joints, and the vertical joints within the upper part of the sample are damaged and broken; during stage II, there are several strip zones damaged and fractured at the upper part of the sample, one of which propagates to the middle left part of the specimen.

From Table 3, it can be seen that along direction I when the RSC is 0.1, during stage I, the vertical joints at the upper part of the sample are damaged, and the columns at the upper middle part of the sample begin to show damage; during stage II, the damaged and crushed range at the upper middle part of the sample further develops, and a damaged
zone also appears at the upper left and right parts of the sample. When the rock RSC = 0.5, the damage evolution of
the specimen is similar, but the damage range of the former is smaller than the latter. When the rock RSC = 1.0,
during stage I, the columns are damaged at the upper middle part of the sample, and strip-shaped damage areas are
developed; during stage II, the damage zone at the upper part of the specimen grows, reaching the upper right part of
the sample; during stage III, the damaged areas appear at the upper left part of the sample, and the breakage of the
damaged zone intensifies.

Table 3. The multi-stage damage behaviors of the CJBs with a model size of 8 m and different rock meso-
constitutive relations along directions I and II when the lateral pressure = 8 MPa under plane strain

<table>
<thead>
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<th>Stage</th>
<th>Stage I</th>
<th>Stage II</th>
<th>Stage III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direction I, RSC = 0.1</td>
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<td><img src="image2.png" alt="Image" /></td>
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<tr>
<td>Direction I, RSC = 0.5</td>
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<tr>
<td>Direction I, RSC = 1.0</td>
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<td><img src="image9.png" alt="Image" /></td>
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<tr>
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</table>

As is depicted in Table 3, when the rock RSC = 0.1 along direction II, the damage evolution of the specimen is
similar to that of the specimen with a rock RSC of 0.1 along direction I. When the rock RSC = 0.5, during stage II,
several strip-shaped damaged zones gradually form inside the specimen. Additionally, obvious damaged areas appear
at the middle right part of the specimen. When the rock RSC = 1.0, during stage I, damaged areas gradually develop
at the upper middle part of the specimen; during stage II, an inverted V-shaped damage zone gradually forms inside
the specimen; during stage III, the damaged zones occur at the upper left and middle right parts of the specimen.
Figure 11. Energy evolutions of the CJBs with different rock meso-constitutive relations under plane strain: (a), (b) CJBs with a model size of 4 m along direction I under the lateral pressures of 4 MPa and 8 MPa; (c), (d) CJBs with a model size of 4 m along direction II; (e), (f) CJBs with a model size of 8 m along direction II.

Figure 11 shows the energy evolutions of the CJBs with different RSCs of 0.1, 0.5, and 1.0 and model sizes of 4 m and 8 m. As can be seen in Figure 11a, for a model size of 4 m and a lateral pressure of 4 MPa along direction I, with an increase in the rock RSC, the peak value of AEs energy drops gradually. Furthermore, when the rock RSC is 0.1, the AEs energy basically presents a single-peak shape distribution; when the rock RSC is 0.5, the AEs energy shows a trend of inapparent double-peak shape distribution; when the rock RSC is 1.0, the AEs energy shows a double-peak shape distribution to a certain extent. Furthermore, when the rock RSC is 0.1 or 0.5, the peak AEs energy value lags behind the former on the strain axis. As can be seen in Figure 11b, for a model size of 4 m and a lateral pressure of 8 MPa along direction I, when the rock RSC is 0.1 or 1.0, the peak values of AEs energy are almost equal; when the rock RSC is 0.5, the peak AEs energy value is relatively small. At the same time, when the rock RSC is 0.1, 0.5, or 1.0, the AEs energy presents a single-peak shape distribution. When the rock RSC is 0.1 or 0.5, the peak values of AEs energy appear roughly in the same order on the strain axis; when the rock RSC is 1, the peak AEs energy value lags slightly on the strain axis.

As can be seen in Figure 11c, with a model size of 4 m and a lateral pressure of 4 MPa along direction II, as the rock RSC increases, the peak value of AEs energy decreases firstly and then increases. When the rock RSC is 0.1, 0.5, or 1.0, the AEs energy shows a double-peak shape distribution, and with an increase in rock RSC, the spacing between the two peaks broadens. When the rock RSC is 0.1 or 0.5, the order difference between AEs energy peaks on the strain axis is relatively small; when the rock RSC is 1.0, the peak AEs energy value lags obviously on the
strain axis. From Figure 11d, it can be seen that with a model size of 4 m and a lateral pressure of 8 MPa along direction II, when the rock RSC is 0.1 or 0.5, the difference of the peak values of AEs energy is very small; when the rock RSC is 1.0, the peak AEs energy value is low. Moreover, when the rock RSC is 0.1 or 0.5, the AEs energy has a single-peak shape distribution; when the rock RSC is 1.0, the AEs energy displays a double-peak shape distribution. Moreover, the orders between AEs energy peaks on the strain axis are relatively close; when the rock RSC is 1.0, the peak AEs energy value is behind the former on the strain axis.

From Figure 11e, we can see that for a model size of 8 m and a lateral pressure of 4 MPa along direction II, with a growth in rock RSC the peak value of AEs energy decreases. Furthermore, when the rock RSC is 0.1 or 0.5, the AEs energy has a single-peak shape distribution; when the rock RSC is 1.0, the AEs energy shows a multi-peak shape distribution. Furthermore, with a rise in rock RSC, the peak AEs energy value gradually lags on the strain axis. As can be seen in Figure 11f, for a model size of 8 m and a lateral pressure of 8 MPa along direction II, as the rock RSC increases, the peak value of AEs energy decreases first and then grows. When the rock RSC is 0.1, the AEs energy displays a single-peak shape distribution; when the rock RSC is 0.5 or 1, the AEs energy presents a rough triple-peak shape distribution with one small peak and two large peaks. This is mainly because the stress concentrations inside the specimens tend to be the banded distribution with the load increasing, resulting in the gradual development of the banded damage-fracture zones. At the same time, with a growth in rock RSC, the peak AEs energy value also progressively lags on the strain axis.

(2) Model size

Figure 12. The multi-stage mechanical behaviors of the stress–strain curves of the CJBs with different model sizes under plane strain.
As can be seen in Figure 12a, for a model size of 0.5–8 m and a lateral pressure of 4 MPa along direction I, stages I, II, and III can be observed on the stress–strain curves. As can be seen in Figure 12b, for a model size of 0.5–6 m and at a lateral pressure of 8 MPa, there are stages I, II, and III on the stress–strain curves; if the model size is 8 m, there are only stages I and II on the stress–strain curve. As is shown in Figure 12c, for a model size of 0.5–4 m and a lateral pressure of 4 MPa along direction II, there are stages I, II, and III on the stress–strain curves; if the model size increases to 6 m, there are only stages I and II on the stress–strain curve; when the model size is 8 m, there are stage I, II, and a short stage III on the stress–strain curve. As is shown in Figure 12d, for a model size of 0.5 m and 4 m and a lateral pressure of 8 MPa, there are stage I, stage II, and a short stage III on the stress–strain curves; if the model size increases to 1–3 m, there are stages I, II, and III on the stress–strain curves; if the model size reaches 6 m and 8 m, there are only stages I and II on the stress–strain curves.

Table 4. The multi-stage damage behaviors of the CJBs with different model sizes when the RSC = 0.5 and the lateral pressure = 8 MPa along the direction I under plane strain

<table>
<thead>
<tr>
<th>Stage</th>
<th>Model size (m)</th>
<th>Stage I</th>
<th>Stage II</th>
<th>Stage III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td><img src="image4" alt="Image" /></td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td><img src="image7" alt="Image" /></td>
<td><img src="image8" alt="Image" /></td>
<td><img src="image9" alt="Image" /></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td><img src="image10" alt="Image" /></td>
<td><img src="image11" alt="Image" /></td>
<td><img src="image12" alt="Image" /></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td><img src="image13" alt="Image" /></td>
<td><img src="image14" alt="Image" /></td>
<td><img src="image15" alt="Image" /></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td><img src="image16" alt="Image" /></td>
<td><img src="image17" alt="Image" /></td>
<td><img src="image18" alt="Image" /></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td><img src="image19" alt="Image" /></td>
<td><img src="image20" alt="Image" /></td>
<td><img src="image21" alt="Image" /></td>
</tr>
</tbody>
</table>
From Table 4, it can be seen that for a model size of 0.5 m during stage I, the damage appears at vertical joints; during stage II, the cracks develop at the columns and fractures are initiated; during stage III, the breakage of the columns inside the specimen intensifies. For a model size of 1 m, during stage I, the damage mainly occurs at vertical joints; during stage II, the columns within the upper area of the sample are damaged, and fracture initiations appear. At the same time, damage to the columns occurs at the lower part of the specimen; during stage III, the breakage of the columns within the upper area of the sample is aggravated. For a model size of 2 m, during stage II, several columns within the upper and lower areas of the sample are damaged, and then fractures initiate, connect, and form strip damaged zones; during stage III, the breakage of the damaged zones is intensified. For the model sizes of 3 m and 4 m, during stage II, an M-shaped damaged area gradually forms at the upper part of the specimen; during stage III, the crushing of the M-shaped damage zone is aggravated. For model sizes of 6 m and 8 m, during stage I, the damage of the columns at the upper middle part of the sample develop and cracks initiate; during stage II (and stage III), the range of damage and number of fractures increase. Clearly, they also occur at the upper left and right parts of the sample.

Figure 13. The energy evolutions of the CJBs with different model sizes under plane strain.
Figure 13 shows the energy evolutions of the CJBs with different model sizes when the rock RSC = 0.5 and the lateral pressure is 4 MPa and 8 MPa along directions I and II perpendicular to the column axis under plane strain. As can be seen in Figure 13a, for CJBs with a lateral pressure of 4 MPa along direction I, the peak AEs energy value increases with a growth in model size. When the model size is 6 m, the AEs energy presents a double-peak shape distribution; when the model is of other sizes, the AEs energy has a single-peak shape distribution. In terms of the order in which the peak AEs energy values appear on the strain axis, the larger the model size, the farther forward the peak value of AEs energy occurs on the strain axis. As can be seen in Figure 13b, for the CJBs with a lateral pressure of 8 MPa along direction I, the peak value of AEs energy first grows, then decreases, and then increases with an increase in the model size. Furthermore, if the model size equals 6 m, the acoustic emission energy roughly displays a triple-peak distribution; if the model size increases to 8 m, the acoustic emission energy presents a double-peak distribution; when the model is of the other sizes, the AEs energy shows a single-peak distribution.

As can be seen in Figure 13c, for the CJBs with a lateral pressure of 4 MPa along direction II, the peak value of AEs energy grows as the model size increases. When the model size is 4 m, the AEs energy shows a double-peak distribution; when the model is of other sizes, the AEs energy displays a single-peak distribution. In terms of the order in which the peak AEs energy values appear on the strain axis, with an increase in the model size, the peak AEs energy value first moves forward on the strain axis, then lags, and then moves forward. As can be seen in Figure 13d, for the CJBs with a lateral pressure of 8 MPa along direction II, the peak value of AEs energy rises as the model size increases. When the model size is 8 m, the AEs energy presents a multi-peak distribution; when the model is of the other sizes, the AEs energy shows a single-peak distribution.

3.2.2. The case between plane stress and plane strain

Figure 14. Different stages of stress–strain curves of the CJBs with different model sizes under the case between plane stress and plane strain
Figure 14 shows the multi-stage mechanical behaviors of the CJBs when the rock RSC = 0.5 and the lateral pressure = 8 MPa along directions I and II perpendicular to the column axis. From Figure 14a, we can see that when the model size is 0.5–2 m along direction I, there are only stages I and II on the stress–strain curves; if the model size is 3–8 m, there are stage I, stage II, and stage III on the stress–strain curves. Figure 14b indicates that for the model sizes 0.5–4 m along direction II, there are only stages I and II on the stress–strain curves; if the model size increases to 6–8 m, there are stage I, stage II, and stage III on the stress–strain curves.

Table 5. The multi-stage damage of the CJBs with a rock RSC of 0.5 and lateral pressure of 8 MPa along the direction II under the case between plane stress and plane strain

<table>
<thead>
<tr>
<th>Stage</th>
<th>Stage I</th>
<th>Stage II</th>
<th>Stage III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model size = 0.5 m</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Model size = 1 m</td>
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<td></td>
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<tr>
<td>Model size = 2 m</td>
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<tr>
<td>Model size = 3 m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model size = 4 m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model size = 6 m</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model size = 8 m</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Table 5, it can be seen that for the model sizes of 0.5–2 m, during stage I, the horizontal joints inside the specimen are damaged; during stage II, several vertical joints inside the sample become broken. If the model size = 3 m, during stages II and III, the columns at the upper middle part of the sample fail. For a model size of 4 m, during
stage II, the strip damaged zones form inside the specimen. For the model sizes of 6 m and 8 m, during stages II and III, the columns at the upper part of the sample become damaged and broken.

Figure 15. The energy evolution of CJBs with different model sizes under the case between plane stress and plane strain

As can be seen in Figure 15a,b, at a lateral pressure of 8 MPa along directions I and II, the peak value of AEs energy grows with an increase in the model size. Furthermore, the AEs energy displays a single-peak shape distribution, and the peak AEs energy value moves forward on the strain axis slowly, although it is not obvious.

3.3. The AEs energy characteristics of the CJBs with different rock meso-constitutive relations and model sizes under lateral pressure

3.3.1. The plane strain case

(1) Meso-constitutive relation

Figure 16 presents the AEs energy accumulations of the CJBs with different rock meso-constitutive relations (i.e., the RSC equals 0.1, 0.5 and 1, respectively) and model sizes of 4 m and 8 m along directions I and II perpendicular to the column axis when the lateral pressure equals 4 MPa or 8 MPa. In Figure 16a,b, when the model size is 4 m, the rock RSC is 0.1, and the pressure is 4 MPa or 8 MPa along direction I, the AEs energy accumulates slowly in the beginning, and very quickly after that; when the rock RSC is 0.5 or 1, the AEs energy accumulates slowly in the beginning. Later, this process becomes quick, but then slows down again.

As can be seen in Figure 16c,d, when the model size is 4 m, the rock RSC is 0.1–1, and the lateral pressure is 4 MPa along direction II, the AEs energy accumulates slowly in the beginning. Later, it rises more quickly, but then varies slightly. When the rock RSC is 0.1 or 1 and the lateral pressure is 8 MPa, the accumulation of AEs energy changes slightly in the beginning and then grows; when the rock RSC is 0.5, the accumulation of AEs energy varies slightly in the beginning, then increases quickly, and after that varies slightly.
As can be seen in Figure 16e,f, when the model size is 8 m, the rock RSC is 0.1 or 1, and the lateral pressure is 4 MPa along direction II, the AEs energy accumulates slowly in the beginning but speeds up; when the rock RSC is 0.5, the AEs energy accumulates slowly at first, then quickly, and then slowly again. When the lateral pressure is 8 MPa and the rock RSC is 0.1 or 0.5, the AEs energy accumulates slowly in the beginning and then quickly; when the rock RSC is 1, the AEs energy accumulates slowly in the beginning, then quickly, and after that slowly again.

Figure 16. AEs energy accumulations of the CJBs with different rock meso-constitutive relations under plane strain: (a), (b) for a model size of 4 m with lateral pressures of 4 MPa and 8 MPa along direction I; (c), (d) for a model size of 4 m along direction II; (e), (f) for a model size of 8 m along direction II

Figure 17 shows the strains and AEs energy accumulations corresponding to the peak stresses of the CJBs with various rock meso-constitutive relations (i.e., the RSC is 0.1, 0.5, or 1) and model sizes of 4 m and 8 m along directions I and II perpendicular to the column axis. As can be seen in Figure 17a, when the model size is 4 m and the lateral pressure is 4 MPa or 8 MPa along direction I, with the growth in the rock RSC the strain and AEs energy accumulation corresponding to the peak stress is slow at first and then increases. Additionally, with a rise in the lateral pressure, the strain and AEs energy accumulation corresponding to the peak stress grows gradually. As can be seen in Figure 17b, when the model size is 4 m and the lateral pressure is 4 MPa or 8 MPa along direction II, the strain and AEs energy accumulation corresponding to the peak stress grows slowly and then quickly as the rock RSC increases. Moreover, as the lateral pressure increases, the strain and AEs energy accumulation corresponding to the peak stress rise.
As is displayed in Figure 17c, when the model size is 8 m and the lateral pressure is 4 MPa or 8 MPa along direction I, the strain corresponding to the peak stress increases with a growth in rock RSC. Furthermore, when the lateral pressure is 4 MPa, with a rise in rock RSC, the AEs energy accumulation corresponding to peak stress varies slightly at first and then grows; if the lateral pressure increases to 8 MPa, the related AEs energy accumulation corresponding to peak stress increases. As is shown in Figure 17d, with a model size of 8 m and a lateral pressure of 4 MPa or 8 MPa along direction II, the strain corresponding to the peak stress rises gradually as the rock RSC increases. Furthermore, if the lateral pressure equals 4 MPa, the AEs energy accumulation corresponding to peak stress increases with the growth in rock RSC; if the lateral pressure rises to 8 MPa, the related AEs energy accumulation, which corresponds to peak stress, shows an increasing trend.

Figure 17. The strains and AEs energy accumulations corresponding to the peak stresses of the CJBs with different rock meso-constitutive relations under plane strain

(2) Model size

From Figure 18a, we can see that when the model size is 0.5–8 m and the lateral pressure is 4 MPa along direction I, the AEs energy accumulates slowly in the beginning, then quickly, and after that slowly as the load
continues to increase. As is shown in Figure 18b, when the model size is 0.5–6 m and the lateral pressure in 8 MPa, the AEs energy accumulates with a low ratio initially. Later, it rises quickly and then varies slightly. If the model size equals 8 m, the AEs energy accumulation shows a trend of varying slowly and then rising quickly. In addition, with an increase in model size, the growth stage of AEs energy accumulation moves forward on the strain axis. As can be seen in Figure 18c,d, when the model size is 0.5–4 m and the lateral pressure is 4 MPa or 8 MPa along direction II, the AEs energy accumulation changes slowly initially, then grows quickly, and after that varies slowly as the load continues; when the model size is 6 m or 8 m, the accumulated AEs energy displays a trend of changing slowly and then increasing quickly. Moreover, with an increase in the model size, the growth stage of AEs energy accumulation also moves forward on the strain axis.

Figure 18. The AEs energy accumulations of the CJBs with different model sizes under plane strain

According to Figure 19a, when the lateral pressure is 4 MPa or 8 MPa along direction I, with a growth of model size, the strain corresponding to the peak stress decreases firstly and then varies slightly (or increases slightly), while the AEs energy accumulation corresponding to the peak stress rises gradually. Additionally, as the lateral pressure
grows, the energy accumulation of the AEs, which corresponds to peak stress, increases, and the extent to which it increases rises as the model size increases. As is shown in Figure 19b, when the lateral pressure is 4 MPa or 8 MPa along direction II, with an increase in the model size, the strain corresponding to the peak stress changes slightly (or increases slightly), then decreases and then varies slightly, while the AEs energy accumulation corresponding to peak stress grows gradually. Moreover, as the lateral pressure rises, the strain corresponding to peak stress increases, but the extent to which it increases decreases with the growth of the model size. As the lateral pressure increases, the energy accumulation of the AEs, which corresponds to the peak stress, rises, but the extent to which it rises grows firstly and then changes slowly with a growth in the model size.

Figure 19. The strains and AEs energy accumulations corresponding to peak stresses of the CJBs with different model sizes under plane strain

3.3.2. The case between plane stress and plane strain

Figure 20. The AEs energy accumulations of the CJBs with different model sizes under the case between plane stress and plane strain
It can be seen from Figure 20a that for the CJBs with the model size 0.5–8 m along direction I, the energy accumulation of AEs changes slightly at first, then grows quickly, and then varies slowly as the load continues. According to Figure 20b, for the CJBs with a model size of 0.5–6 m along direction II, the AEs energy accumulation also displays a trend of varying slowly then rising quickly and after that changing slightly; when the model size is 8 m, the AEs energy accumulation changes slowly and then grows quickly.

![Image](image.png)

**Figure 21. The strains and AEs energy accumulations corresponding to the peak stresses of the CJBs with different model sizes under the case between plane stress and plane strain**

As can be seen in Figure 21, when direction I is considered, the strain corresponding to the peak stress drops firstly, then grows slightly, and then decreases gradually with a growth in the model size. If direction II is considered, the strain that corresponds to the peak stress fluctuates. Additionally, the specific strain corresponding to the peak stress of the CJBs along direction II is higher than that of the CJBs along direction I. At the same time, the gap grows as the model size increases. Furthermore, according to Figure 21, for the CJBs along directions I and II, the AEs energy accumulation corresponding to the peak stress rises gradually with a growth in the model size. Furthermore, the energy accumulation of AEs corresponding to the peak stress of the CJBs along direction II is larger than that of the CJBs along direction I. At the same time, the gap increases gradually with a growth in the model size.

4. Discussion

4.1. Influence of rock meso-constitutive relation and model size on rock mass strength and deformation under lateral pressure

The influence of rock meso-constitutive relation and model size on rock mass strength and deformation under lateral pressure can be summarized as follows: when the lateral pressure is 8 MPa, the CS of CJBs rises and the EDM of CJBs decreases or increases slightly as the rock RSC increases. Furthermore, the CS of CJBs roughly decreases at first and then varies slightly as the model size increases, while the EDM of CJBs fluctuates, but the variation is relatively small.

On the basis of the discrete fractured network (DFN)/discrete element method (DEM), Niazmandi et al. [37] investigated the strength variation of fractured rock masses with different load conditions. Their research showed that
the strength of the specimen drops at first and then changes slightly as the sample size increases, which agrees with
the results obtained in our study. However, they did not analyze the deformation parameters with different model
sizes, and the influence of the meso-constitutive relations on the deformation parameters was not discussed. Fan et
al. [38] used a 3D particle flow code (PFC) to investigate the strength and deformability of rocks cut by many non-
persistent joint sets, but the effects of lateral pressure and model size were not considered in their research. Liu et al.
[39] adopted the PFC to reveal the anisotropy and scale of the strengths of rocks. Nevertheless, the meso-constitutive
relation could vary under lateral pressure in rock engineering, which was not taken into account in their research.

4.2. Influence of rock meso-constitutive relation and model size on mechanical damage of rock mass under lateral
pressure

In terms of the influence of the rock meso-constitutive relation and model size on the mechanical damage of
rock masses under lateral pressure, we came to the following conclusions: when the model size is 4 m, the lateral
pressure is 8 MPa, and the rock RSC is 0.1 along direction I orthogonal to the column axis, there are obvious stages
I and II on the stress–strain curve; when the rock RSC is 0.5 or 1, there are stage I, stage II, and stage III on the stress–
strain curves. When the model size is 8 m and the rock RSC is 0.1 or 0.5, there are only stages I and II on the stress–
strain curves; when the rock RSC is 1, there are stages I, II, and III on the stress–strain relation. Additionally, the
damage behaviors of the CJBs in different stages are investigated.

Using a DFN/DEM approach, Niazmandi et al. [37] obtained stress–strain curves of fractured rock masses under
various numerical situations, but the damage evolutions of specimens in different stages of stress–strain curves were
not displayed and analyzed, and the influence of rock meso-constitutive relations on them was not investigated. Based
on the numerical method of PFC3D, Fan et al. [38] captured the failure processes of jointed rock specimens at
different axial strain levels. However, the influence of model size and lateral pressure on the failure process of
specimens was not considered in their research. The stress–strain curves and failure modes of defected rock masses
were presented by Liu et al. [39] using PFC, but the failure processes of specimens corresponding to different stages
of stress–strain curves were not further analyzed. Gao et al. [44] applied a new method to obtain displacement vectors
and crack distributions of specimens with bedding plane orientations. Nevertheless, the mechanical behaviors of
specimens in various stages were not proposed or investigated, and the influence of the rock meso-constitutive model
and lateral pressure was not further considered. Stavrou et al. [40] presented the failure patterns of micro-defected
rocks for different lateral pressures and model sizes using the DEM, but the failure characteristics in several stages
were not displayed or described in detail. Yang et al. [42] captured the fracture mode, seepage pathway, and stress field of jointed rocks with different sizes. However, the variation in rock meso-constitutive relation was not taken into account in their research. Yu et al. [45] conducted a numerical study and obtained the fracturing process of inclusions embedded in rock matrix, but the failure evolutions corresponding to different stages of stress-strain relations were not displayed systematically.

### 4.3. Influence of rock meso-constitutive relation and model size on AEs of rock mass under lateral pressure

Based on the results of this paper, the influence of rock meso-constitutive relation and model size on the AEs of rock mass under lateral pressure can be summarized as follows: when the model size is 4 m and the lateral pressure is 4 MPa or 8 MPa along direction II orthogonal to the column axis, the strain and AEs energy accumulation corresponding to the peak stress first increases slightly and then rises quickly. As the model size increases, the strain that corresponds to the peak stress changes slowly at first, then decreases, and after that varies slightly, but the AEs energy accumulation corresponding to peak stress displays a trend of rising gradually.

Fan et al. [38] adopted the PFC3D method to conduct numerical simulations on jointed rock masses. However, the AEs energy and the influence of the rock meso-constitutive relation on it were not further investigated. Based on a synthetic rock mass approach, Gao et al. [44] captured the variations in micro-crack number and axial stress as the axial strain increases, but further data analyses (e.g., finding the AEs energy accumulation that corresponds to the peak stress or an effect analysis of model size) were not performed. Using the RFPA method, Liang et al. [46] conducted numerical simulations on jointed rock masses and presented the AEs figures for their specimens, but the AEs energy accumulation underlying the failure process of the specimen and the influence of the rock meso-constitutive model on it were not displayed or analyzed. Stavrou et al. [40] applied DEM to capture the fracture patterns of micro-defected rocks. However, the crack-induced AEs energy evolution and the influence of model size on it were not investigated deeply in their research. Based on the 3D DEM method, Ma et al. [47] obtained the typical fracture pattern of stochastic rock masses with various sizes, but the influence of the rock meso-constitutive relation on the AEs energy evolution was not studied. Using the DIC technique and an AEs device, Dong et al. [48] presented the features of the strains, AEs, and damage of rock media containing joints. Nevertheless, the AEs energy accumulation that corresponds to the peak stress and the effect of model size were not considered. Cui et al. [49] adopted the jointed finite element technique to study the structural effect of the equivalent elastic modulus of CJRM. However, the influence of the rock meso-constitutive relation and model size on the AEs energy accumulation were not investigated in detail in their research.

### 5. Conclusions

Based on the continuous media mechanics and the statistical damage strength theory, a group of nonuniform CJB models with different rock meso-constitutive relations and model sizes were established. The strength and deformation properties of CJBs subject to various boundary conditions under lateral pressure, along directions I and II orthogonal to the column axis, were studied. Furthermore, the multi-stage mechanical and damage behavior and the evolution of the AEs energy of CJBs with different rock meso-constitutive relations and model sizes under lateral pressure were investigated.
pressure were investigated. Then, the strains and AEs energy accumulations corresponding to peak stresses were analyzed. The main conclusions can be summarized as follows:

1. In terms of rock RSC, the CS of CJBs increased and the EDM of CJBs decreased or increased slightly with an increase in the rock RSC. Furthermore, the CS of CJBs slightly decreased at first and then varied slightly with an increase in the model size, while the EDM of CJBs fluctuated, but the variation was relatively small. The CS and EDM of CJBs under plane strain were relatively high in comparison with those of CJBs under the case between plane stress and plane strain, which was mainly due to the strong constraints of plane strain onto CJBs.

2. When the model size was 4 m, the rock RSC was 0.1, and the lateral pressure was 8 MPa along direction I orthogonal to the column axis, there were two obvious stages (stages I and II) to the stress–strain curve; when the rock RSC was 0.5 or 1, there were three stages (stages I, II, and III) to the stress–strain curves. When the model size was 8 m and the rock RSC was 0.1 or 0.5, there were only two obvious stages (stages I and II) to the stress–strain curves; when the rock RSC was 1, there were three different stages (stages I, II, and III) to the stress–strain curve. This phenomenon was because that the plastic deformation capacity of the rock meso-elements was enhanced with the growth of the RSC, and the overall bearing capacity of CJBs was therefore improved.

3. When the model size was 4 m and the lateral pressure was 4 MPa or 8 MPa along direction II orthogonal to the column axis under plane strain, the strain and AEs energy accumulation corresponding to the peak stress first increased slightly and then rose quickly with an increase in the rock RSC. The reason was that with the growth of rock RSC, the energy stored in and released by CJBs become greater. With a growth in the model size, the strain that corresponded to the peak stress changed slowly at first; then decreased quickly; and after that, varied slightly, but the energy accumulation of the AEs corresponding to the peak stress rose gradually. These results demonstrated that with the growth of the model size, the bearing capacity of CJBs decreased, and the instability precursor occurred easier.

4. For the CJBs along direction I under the case between plane stress and plane strain, the strain corresponding to the peak stress dropped at first, then grew slightly, and then decreased with an increase in the model size. Regarding the CJBs along direction II, the strain corresponding to the peak stress fluctuated. The different changing trends were caused by the different damage development processes affected by the joint directions. Additionally, the AEs energy accumulation corresponding to the peak stress rose gradually as the model size increased.

**Author Contributions**

Conceptualization, Yongyi Wang and Bin Gong; Data curation, Xiaoyu Yang; Formal analysis, Yongyi Wang and Xiaoyu Yang; Funding acquisition, Bin Gong; Software, Chun’an Tang; Supervision, Bin Gong and Chun’an Tang; Visualization, Yongyi Wang; Writing – original draft, Yongyi Wang; Writing – review & editing, Bin Gong. All authors have read and agreed to the published version of the manuscript.

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**Data Availability Statement**
The datasets generated and/or analyzed during the current study are available from the corresponding author upon reasonable request.

**Conflicts of Interest**

The authors declare no conflict of interest.

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